R&D Competition between Public and Private Sectors

Ikuo Ishibashi  
Department of Economics, Aoyama Gakuin University  
and  
Toshihiro Matsumura  
Institute of Social Science, University of Tokyo  

April 5, 2004

Abstract

We investigate a mixed market where a welfare-maximizing public research institute competes against profit-maximizing private firms. We investigate R&D competition by using a standard model of patent race where each firm chooses both its innovation size and expenditure. We find that the innovation size (expenditure of investment) chosen by the welfare-maximizing public institute is too small (too large) from the viewpoint of social welfare, so the government should control public institute appropriately.

JEL classification numbers: D43, H42, L13

Key words: public research institution, mixed market, patent race, innovation size, R&D expenditure
1 Introduction

Studies of mixed markets, where state-owned welfare-maximizing public firms compete against profit-maximizing private firms, have become increasingly popular in recent years.\(^1\) Mixed oligopolies are common in developed, developing, and former communist transitional economies. Typical examples are the package and overnight-delivery industries or services provided formerly by the public sectors in the US; airline, automobile, steal, banking, and electric powers in the EU (especially in France and formerly UK); banking, house loan, life insurance, broadcasting, education, medical cares, and overnight delivery in Japan. Similar examples are widely observed in many countries.

In this paper we take a close look at R&D competition between national institutes and private firms. Although cooperation between public and private sectors in R&D is often observed, competition between them also widely exists. A typical example is competition around ‘genome project’ between Celera Genomics and public institutions such as the National Institutes of Health (NIH). The US government started Human Genome Project in 1986, and international cooperation followed in 1990, where public institutes of many countries played important roles. In 1998 Celera Genomics started a similar project and aimed to protect its fruits by patents. In earlier stages, public sectors supporting international human genome project were very afraid of the behavior of Celera Genomics. If a private firm monopolizes such very important knowledge, it must become a serious obstacle for the future progress of human science and bio-technology and for the development of important industries such as medical and agricultural industries; resulting in the huge loss of world-wide welfare in future. The international project accelerated its research in order to prevent the monopoly use of genome information by Celera Genomics.

The purpose of international genome project was not to make current profits, but to make the information public and to improve world-wide welfare in future by the open use of this information. On the other hand, the purpose of Celera Genomics was to make profits. We can regard competition

\(^1\) For pioneering works in mixed oligopolies, see Merrill and Schneider (1966) and Harris and Wiens (1980). See also Börs (1986, 1991), Vickers and Yarrow (1988) and De Fraja and Delbono (1990) for excellent surveys. For a recent work of dynamic competition in mixed markets, see Wen and Sasaki (2001).
between Celera Genomics and public institutes as a typical example of mixed markets.\(^2\)

There are many other examples of R&D competition between public sectors and private firms. The joint project by several Japanese national universities translated a technology of high quality LED to Nichia Corporation, but it did not allow Nichia to use this technology exclusively. If Nichia Corporation had developed this technology itself, it would have been able to obtain the patent and use it exclusively. In this sense, the research by national universities prevented the exclusive use of this important technology.\(^3\) Another example of national university in Japan is on embryonic stem cells. Institute for Frontier Medical Sciences of Kyoto University develops the technology of producing human embryonic stem cells and now it plan to provide them without charges, while other institutes now provide them with charges. In many other fields such as space, agriculture, and bio-technologies, competitions between public research institutes and private firms are widely observed. Delbono and Denicolò (1993), Pal and White (1998), Poyango-Theotoky (1998), Nishimori and Ogawa (2002), and Matsumura and Matsushima (2003) investigated R&D competition between public and private sectors. As well as other traditional researches on R&D competition in pure markets, they discussed the efficient amount of R&D investment and neglected other important aspects of R&D competition such as innovation size or technological choice, which recently have been investigated intensively. In the examples we present above, these aspects also play important roles. In order to incorporate the problem of innovation size as well as investment level, we investigate a patent race model in the mixed market. We use the model of O’Donoghue (1998) discussing patent races in a pure market. In the model firms choose both the innovation size and R&D spending. In our model the public firm chooses them so as to maximize welfare, while the private firm chooses them so as to maximize its own profits. We show that the innovation size chosen by the welfare-maximizing public firm is too small and R&D spending is too large from the viewpoint of social welfare. This result implies that the government, not the public research institute, should choose spending of R&D even if the

\(^2\) Ultimately, NIH and Celera Genomics cooperated and now NIH is a customer of Genomics. However, without any doubt, in earlier stages, they competed with each other.

\(^3\) In fact, Nichia Corporation holds many patents on blue LED, and it chooses not to license them to its rivals and uses them exclusively.
public institute is the welfare maximizer and never requires the wasteful research funds. Another implication is the public institute should choose very innovative research and the government should give the research institute strong incentives for choosing such a very innovative project beyond the welfare-maximizing innovation size.

Our work is also related to the recent public finance literature on government versus private provision. Corneo and Rob (2003) and Francois (2000) explicitly consider contract between employer and employees and show that a private firms provide a larger incentive intensity. At first glance, a high-powered incentive contract improves productivity of the firm, so private provision is more efficient. They show, however, that it is possible that public provision is more efficient than private provision. Our model is quite different from theirs. They explicitly consider how to elicit worker’s effort, while we ignore this aspect. On the other hand, we consider competition between public and private firms and consider strategic interaction of two firms. Although the model formulation is quite different, one similar result is obtained: pure private provision is not always efficient than public provision.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 considers the social optimum as a benchmark. Section 4 investigates the equilibrium outcomes in the model. Section 5 discusses the role of the government. Section 6 concludes the paper.

2 Model

The basic structure is from O’Donoghue (1998). We describe an infinite time-horizon game with an infinite sequence of innovations. Time is continuous and the common interest rate is $r$. There are two firms, firm 1 and firm 2, competing in R&D. The two firms are identical except for their objectives. Firm 1 is the state-owned public research institute maximizing social welfare, while firm 2 is the private firm maximizing its own profits.\footnote{See also Atkinson and Halvorsen (1986) for the empirical study on electrical producers in the US.}

\footnote{In this paper we do not allow the government to nationalize all firms. As pointed out by Merrill and Schneider (1966) and De Fraja and Delbono (1990), the most efficient outcome is achieved by the nationalization of all firms in the case where nationalization does not change the costs of firms (i.e., no X-inefficiency in the public firm exists). The need}
At any moment in time, the firms sell their products in an output market. Marginal cost of production is normalized to zero. Their products are different only in qualities. The space of quality for each firm is $[0, \infty)$ and the initial qualities for both firms are 0. We name a firm with a higher quality as the leader and that with a lower quality as the follower. Let $\Gamma(\geq 0)$ denote the quality gap between the leader and the follower. The quality of the product of each firm is determined according to the following stationary R&D process. At any moment in time, each firm determines two variables; a level of R&D spending and the innovation size it targets. Innovation occurs according to a Poisson process. If a firm targets $\Delta \in [0, \infty)$ and spends $x \in [0, \infty)$ for the innovation, the probability that the firm has the innovation before the elapsed time $\tau$ is $F(\tau) = 1 - e^{-g(x)h(\Delta)\tau}$, where $g : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ and $h : \mathbb{R}_+ \rightarrow \mathbb{R}_+$. We assume that $g' > 0$, $g'' < 0$, $h' < 0$, and $h'' < 0$. The probability density of $F(\tau)$ is given by $f(\tau) = g(x)h(\Delta)e^{-g(x)h(\Delta)\tau}$. The assumptions on $g(\cdot)$ and $h(\cdot)$ imply two things; (a) the positive relationship between R&D spending and the probability of success given the target size and (b) the negative relationship between the target size and the probability of success given the R&D spending.

If a firm succeeds in an innovation $\Delta$, it acquires the patent and discloses all information of the innovation. Both firms obtain the same knowledge and skill to produce, but only the firm succeeding in the latest innovation produces the highest quality product since the innovation is protected by the patent until the rival develops more advanced technology. The firm succeeding in the latest innovation becomes the leader with the quality gap $\Delta$ if it was the follower and the leader with the quality gap $\Gamma + \Delta$ if it was the leader with the quality gap $\Gamma$. In short, once either firm succeeds in innovation, the firm becomes the leader regardless of the past role of the firm. The only difference is that if the leader succeeds in innovation again, the leader’s quality gap is the past gap $\Gamma$ plus the

---

for an analysis of mixed oligopoly lies in the fact that it is impossible or undesirable, for political or economic reasons, to nationalize an entire sector. For example, without a competitor public firms may lose the incentive to improve their costs; resulting in a loss of social welfare. Thus we neglect the possibility of nationalizing all firms.

According to the terminologies in O’Donoghue (1998), our assumptions are interpreted as the strongest possible protection from imitation (infinite patent life and complete lagging breadth) and no protection from future innovations (no patentability requirement and no leading breadth). See O’Donoghue (1998) for a detailed discussion on the characteristics of patent.
current innovation size \( \Delta \).

We now describe the static competition in the output market. At all times each consumer buys at most one unit and receives utility \( q - p \) from consuming a product with quality \( q \) at price \( p \). Given that firms compete in price, the leader with quality gap \( \Gamma \) can charge price \( p = \Gamma \). Hence the profit function is \( \Pi(\Gamma) = \Gamma \).\(^7\) Firm 1 always follows marginal cost pricing, while firm 2 chooses its price so as to maximize its own profits.\(^8\)

3 Social optimum

In this section we discuss the social optimum outcome. Let \( q(t) \) be the quality of the leader at time \( t \). Also let \( x_i(t) \) and \( \Delta_i(t) \) be firm \( i \)'s R&D spending and the target innovation size at time \( t \). Let \( W \) and \( EW \) denote total social welfare and the expected value of \( W \), respectively. They are given by

\[
W = \int_0^\infty q(\tau)e^{-r\tau}d\tau - \int_0^\infty (x_1(\tau) + x_2(\tau))e^{-r\tau}d\tau, \tag{1}
\]

\[
EW = \int_0^\infty \{ \sum_{i=1}^{2} (-x_i(\tau) + g(x_i(\tau))h(\Delta_i(\tau))\frac{\Delta_i(\tau)}{r}) \} e^{-r\tau}d\tau, \tag{2}
\]

where we derive (2) from the assumption of the stationary Poisson process. Since the situations are stationary, the optimal behaviors for social planner are time independent. The social planner maximizes \( EW \) with respect to \( \Delta_1, \Delta_2, x_1 \) and \( x_2 \). Let superscript * denote the social optimum. Since the situation is symmetric between two firms \( \Delta_1^* = \Delta_2^* \) and \( x_1^* = x_2^* \). So we simply denote \( \Delta^* \equiv \Delta_1^* = \Delta_2^* \) and \( x^* \equiv x_1^* = x_2^* \). The first-best actions \((x^*, \Delta^*)\) for the social planner are derived

\(^7\) We assume that there is no deadweight loss in the output market. That is, the leader’s monopoly power on the quality gap \( \Gamma \) does not cause the static inefficiency. We make this assumption so as to focus on possible inefficiencies of dynamic context, which have been intensively discussed in the literature of patent protection. This assumption makes the following analysis drastically simple. We believe that the essence of our results remains true without this assumption. We will present more detailed discussions of this problem in Section 6. Note that O’Donoghue (1998) considered static deadweight loss, but he also ignored it when he defined social optimum and discussed welfare implications.

\(^8\) In this paper we assume that the public firm directly produces its products. Some readers may suspect that it is unrealistic. If we formulate the following settings, we can obtain exactly the same results without considering the direct production by the public firm: There is at least one other firm which can produce products and does not engage in R&D. Firm 1 does not produce and allow all firms to use its patent without any patent fees, and firms are faced with Bertrand-type competition.
from the following equations:

\[ h(\Delta) + h'(\Delta)\Delta = 0, \]  
\[ -1 + g'(x)h(\Delta)(\frac{\Delta}{r}) = 0. \]  

4 Mixed Market

In this section, we derive Markov perfect equilibria where each firm \( i \) chooses its own \( \Delta_i \) and \( x_i \). Let superscript \( M \) denote the equilibrium outcome in the mixed market. By the definition of Markov perfect equilibrium, \( \Delta_i \) and \( x_i \) are time independent. Let \( x_{M_i}^L(\Gamma) \) denote the equilibrium R&D spending of firm \( i \) when it is the leader with the quality gap \( \Gamma \) and \( x_{M_i}^R(\Gamma) \) denote that of the follower. Let \( \Delta_{M_i}^L(\Gamma) \) denote the equilibrium innovation size of firm \( i \) when it is the leader with the quality gap \( \Gamma \) and \( \Delta_{M_i}^R(\Gamma) \) denote that of the follower. Potentially, these could depend on \( \Gamma \) and whether or not firm \( i \) is the leader. As we show below, however, there exists an equilibrium in which these depend on neither \( \Gamma \) nor the role of firm \( i \) (whether or not it is the leader). From here on, we restrict our attention to such an equilibrium. We now show the existence of the equilibrium.

First, we discuss the behavior of the public firm (firm 1). Suppose that \( x_{M_1}^L = x_{M_2}^L = x_{M_2}^R \), \( \Delta_{M_1}^L = \Delta_{M_2}^L = \Delta_{M_2}^R \) and these are independent of \( \Gamma \). Then we show that \( x_{M_1}^L = x_{M_1}^R, \Delta_{M_1}^L = \Delta_{M_2}^L \) and these are independent of \( \Gamma \). Firm 1 maximizes (2). The first order conditions for the optimality is exactly the same as those for the social planner regardless of \( x_{M_2}^L \) and \( \Delta_{M_2}^L \), and they are given by (3) and (4). Obviously, the equilibrium behaviors of firm 1 depend on neither its role (either the leader or the follower) nor \( \Gamma \). Firm 1 chooses \( x_1 = x^* \) and \( \Delta_1 = \Delta^* \).

Next, we consider the behavior of the private firm (firm 2). Suppose that \( x_{M_1}^L = x_{M_1}^R \) and \( \Delta_{M_1}^L = \Delta_{M_1}^R \) and these are independent of \( \Gamma \). Then we show that \( x_{M_2}^L = x_{M_2}^R \) and \( \Delta_{M_2}^L = \Delta_{M_2}^R \) and these are independent of \( \Gamma \).

Suppose that firm 2 is the follower with the quality gap \( \Gamma \). Firm 2’s instantaneous profit is 0 until it succeeds in innovation. If it succeeds in the innovation \( \Delta_2 \), the instantaneous profit becomes \( \Delta_2 \). If it succeeds again before the success of firm 1, its instantaneous profit becomes 2\( \Delta_2 \). In other words, a success by firm 2 increases its instantaneous profit by \( \Delta_2 \) until firm 1 succeeds in
innovation. A success by firm 1 makes the instantaneous profit of firm 2 zero until firm 2 succeeds in another innovation. Thus, the cost of innovation at any moment is $g(x_2)h(\Delta_2)\Delta_2/(r + g(x_1)h(\Delta_1))$, where $g(x_2)h(\Delta_2)$ is the instantaneous probability of the success, $\Delta_2$ is the instantaneous gain of the success, and $r + g(x_1)h(\Delta_1)$ is the discount rate. Under these conditions firm 2 maximizes (5) with respect to $\Delta_2$ and $x_2$:

$$g(x_2)h(\Delta_2)\Delta_2/(r + g(x_1)h(\Delta_1)) - x_2.$$  

(5)

Since (5) does not depend on $\Gamma$, neither $x^M_{F2}$ nor $\Delta^M_{F1}$ is dependent on $\Gamma$. The first order conditions are given by

$$g'(x_2)h(\Delta_2)\Delta_2/(r + g(x_1)h(\Delta_1)) = 1,$$

(6)

$$h'(\Delta_2)\Delta_2 + h(\Delta_2) = 0.$$  

(7)

From (3) and (7) we have that $\Delta^M_{F2} = \Delta^*$. Since $g'' < 0$, comparing (4) to (6) yields $x^M_{F2} < x^*$.  

We now suppose that firm 2 is the leader with the quality gap $\Gamma$. Firm 2’s instantaneous profit is $\Gamma$ until it succeeds in innovation. If it succeeds in the innovation $\Delta_2$, the instantaneous profit becomes $\Gamma + \Delta_2$. If it succeeds again before the success of firm 1, its instantaneous profit becomes $\Gamma + 2\Delta_2$. In other words, a success by firm 2 increases its instantaneous profit by $\Delta_2$ until firm 1 succeeds in innovation. A success by firm 1 makes the instantaneous profit of firm 2 zero until firm 2 succeeds in another innovation. Thus, the cost of innovation is at any moment $x_2$ and its expected gain of the investment is $g(x_2)h(\Delta_2)\Delta_2/(r + g(x_1)h(\Delta_1))$. These are exactly the same as when firm 2 is the follower. Thus, $x^M_{L2} = x^M_{F2}$ and $\Delta^M_{L2} = \Delta^M_{F2}$.  

Proposition 1 summarizes the above discussions.

**Proposition 1:** (i) There exists an equilibrium in which $x^M_{Li}, x^M_{Fi}, \Delta^M_{Li}$, and $\Delta^M_{Fi}$ do not depend on $\Gamma$, and (ii) in the equilibrium $x^M_{L1} = x^M_{F1} = x^*$, $x^M_{L2} = x^M_{F2} < x^*$, and $\Delta^M_{L1} = \Delta^M_{F1} = \Delta^*$ ($i = 1, 2$).

The most important point of Proposition 1 is that the public firm chooses exactly the same behavior as the social planner, while the private firm chooses the R&D spending too small from the viewpoint of social welfare. An innovation improves welfare forever, and the private firm obtains the benefit
only until its rival succeeds in another innovation. Thus private benefit of the innovation is smaller than the social benefit of it, so the R&D spending of the private firm becomes too small.

Some readers may suspect that the above discussions are too verbal and lack the required accuracy. In Appendix A we prove Proposition 1 rigorously by using the techniques of dynamic programming.

5 Public firm under the control of the government

In this section we present our main results. We consider the situations where the government can control the behavior of public research institute directly. In the previous section we assume that firm 1 (public research institute) chooses both R&D spending $x_1$ and the innovation size $\Delta_1$. In reality, the budget constraint is imposed by the government, so firm 1 cannot freely choose $x_1$. We now suppose that the government chooses $x_1 = b$ before the game. After observing $b$ firm 1 chooses $\Delta_1$ and firm 2 chooses $\Delta_2$ and $x_2$.

**Proposition 2:** (i) There exists $b < x^*$ improving the social welfare, while (ii) $b > x^*$ never improves welfare.

**Proof:** See Appendix B.

Proposition 2 states that the public firm should chooses a smaller R&D spending than the welfare-maximizing level. The intuition behind Proposition 2 is as follows. Suppose that $x_1 = x^*$. Suppose that the government forces firm 1 to decrease $x_1$ slightly. By the envelope theorem, we obtain that a slight decrease in $x_1$ does not directly reduce \( EW \) (i.e., $\partial EW / \partial x_1 = 0$). A decrease in $x_1$ reduces the probability of success of the innovation by firm 1. It increases the average duration periods for which firm 2 obtains profits through one innovation, so it induces firm 2 to spend more for innovations. In short, a decrease in $x_1$ indirectly increases $x_2$. Since $x_2 < x_2^*$, an increase in $x_2$ improves $EW$. Therefore, choosing a smaller $x_1$ improves welfare.

This result indicates that there may be a serious welfare loss in sectors where the public firm is strong and where the competitive pressure from the public firm forces the private firms to operate.

---

\(^9\) Some readers might think that $x_1 \leq b$ rather than $x_1 = b$ is natural. We can show that the constraint $x_1 \leq b$ becomes binding (res. non-binding) if $b < x^*$ ($b > x^*$). As Proposition 2 (ii) states, the government never sets $b > x^*$, so assuming $x_1 = b$ is innocuous.
at inefficiently small levels of R&D. We think that Japanese telecommunication and broadcasting industries are typical examples. In Japan, NTT (Nippon Telegraph and Telephone) and NHK (Nippon Hoso Kyokai) are leading firms in telecommunication and broadcasting industries, respectively. NHK develops many of technological progress of broadcasting such as Hi-vision and hold many patents, while R&D by private companies are quite poor. We do not think that it is because Japanese private firms are incompetent. For example, many Japanese private firms play leading role on R&D for broadcasting cameras, which is not main target of NHK’s investment. In Japanese telephone and telecommunication industry, NTT has the strongest research institute and private firms’ R&D level was quite low. Private firms such as Fujitsu, NEC, and Oki, which were main players in semiconductor markets, play quite limited roles in Japanese telephone and telecommunication industry before NTT was privatized. Now NTT becomes a private firm, and several Japanese private firms make intensive R&D investments in this field. These are just anecdotal stories and are not rigorous empirical results. We believe, however, that these well indicates potential importance of our results.

Regardless of b the welfare maximizing public firm chooses $\Delta_1 = \Delta^*$. We now discuss the situation where the government can control $\Delta_1$ as well as $x_1$. Suppose that the government makes the public firm choose $x_1 = b$ and $\Delta_1 = S$.

**Proposition 3** (i) There exists $S > \Delta^*$ which improves welfare, and (ii) if $S < \Delta^*$, it does not improve welfare.

**Proof:** See Appendix C.

Proposition 3 states that the government should give the public institute additional incentives to make more innovative activities even if the public institute is a welfare-maximizer. This result implies that the public firm should choose more innovative research than the welfare-maximizing level. The intuition behind Proposition 3 is as follows. Suppose that $\Delta_1 = \Delta^*$. Suppose that the government forces firm 1 to increase $\Delta_1$ slightly. By the envelope theorem, we obtain that a slight increase in $\Delta_1$ does not reduce $EW$ (i.e., $\partial EW/\partial \Delta_1 = 0$). Since an increase in $\Delta_1$ decreases the probability of success by firm 1, it increases $x_2$. Since $x_2 < x_2^*$ an increase in $x_2$ improves $EW$. Therefore, a slight increase in $\Delta_1$ from $\Delta^*$ always improves welfare.
6 Concluding remarks

In this paper we investigate a mixed market where a welfare-maximizing public research institute competes against a profit maximizing private firm. We find that the public firms should choose a larger innovation size and to a smaller R&D spending than the welfare-maximizing level (which is derived given R&D levels of the private firm) so as to induce a larger R&D spending by the private firm.

We now briefly compare welfare of the mixed market discussed in Section 4 with that of the pure market where both firms are private as discussed by O’Donoghue (1998). Privatization of the public firm has two countervailing effects. On the one hand, privatization of the public firm significantly reduces the R&D spending of the firm and the welfare loss might appear. On the other hand, the reduction of $x_1$ increases $x_2$, so it might improve welfare. Whether or not the former effect dominates the latter effect is ambiguous, and we cannot present a clear cut result on the desirability of privatization. However, if we consider the situation discussed in Section 5 (i.e., the government directly control the behaviors of the public firm before the private firm chooses its behaviors), privatization never improves welfare.

In this paper we neglect static deadweight loss. If we introduce static deadweight loss, the equilibrium behaviors might depend on $\Gamma$ and firm’s role (the leader or the follower). Thus our Proposition 1(i) does not hold true, and the analysis become much more complicated. However, we believe that our main results Propositions 2 and 3 still hold true because our intuition behind these Propositions seems valid even when the static deadweight loss is introduced. In this paper we restrict our attention to progressive innovation. In the context of patent race, non-progressive R&D investments so as to avoid the infringement of others’ patents, which is discussed by Cardon and Sasaki (1998), are also important. Introducing this discussion remains for future research.

\footnote{The effect of privatization is intensively discussed in the literature of mixed markets. See Anderson et al. (1997), De Fraja and Delbono (1989), Fjell and Pal (1996) and Matsumura (1998). If we allow partial privatization discussed by Matsumura (1998) we can show that it improves welfare. For the discussion of partial privatization see also Bös (1991) and Fershtman (1990).}
Appendix A

Proof of Proposition 1: In our model, the state variables are the quality gap $\Gamma$ and the role of each firm (leader or the follower). Let $V_{Li}(\Gamma)$ be the continuation payoff function (the value function in the standard terminology of dynamic programming) of firm $i$ when it is the leader and the current quality gap is $\Gamma$. Similarly, we define $V_{Fi}(\Gamma)$ as a value function of firm $i$ when it is the follower. Strategies are functions of the state variables. Let $x_{Li}(\Gamma)$ and $\Delta_{Li}(\Gamma)$ be the R&D spending and the innovation size chosen by firm $i$ when it is the leader and the current quality gap is $\Gamma$, and let $x_{Fi}(\Gamma)$ and $\Delta_{Fi}(\Gamma)$ be those when it is the follower. Continuation payoffs in Markov perfect equilibria must satisfy the following functional equations:

$$rV_{L1}(\Gamma) = \max_{x_{L1}(\Gamma), \Delta_{L1}(\Gamma)} \left\{ g(x_{L1}(\Gamma)) h(\Delta_{L1}(\Gamma)) \left( \frac{\Delta_{L1}(\Gamma)}{r} + V_{L1}(\Gamma + \Delta_{L1}(\Gamma)) - V_{L1}(\Gamma) \right) + g(x_{F2}(\Gamma)) h(\Delta_{F2}(\Gamma)) \left( \frac{\Delta_{F2}(\Gamma)}{r} + V_{F1}(\Gamma + \Delta_{F2}(\Gamma)) - V_{F1}(\Gamma) \right) - x_{L1}(\Gamma) - x_{F2}(\Gamma) \right\}, \quad (8)$$

$$rV_{F1}(\Gamma) = \max_{x_{F1}(\Gamma), \Delta_{F1}(\Gamma)} \left\{ g(x_{F1}(\Gamma)) h(\Delta_{F1}(\Gamma)) \left( \frac{\Delta_{F1}(\Gamma)}{r} + V_{L1}(\Delta_{F1}(\Gamma)) - V_{F1}(\Gamma) \right) + g(x_{L2}(\Gamma)) h(\Delta_{L2}(\Gamma)) \left( \frac{\Delta_{L2}(\Gamma)}{r} + V_{F1}(\Gamma + \Delta_{L2}(\Gamma)) - V_{F1}(\Gamma) \right) - x_{F1}(\Gamma) - x_{L2}(\Gamma) \right\}, \quad (9)$$

$$rV_{L2}(\Gamma) = \max_{x_{L2}(\Gamma), \Delta_{L2}(\Gamma)} \left\{ g(x_{L2}(\Gamma)) h(\Delta_{L2}(\Gamma))(V_{L2}(\Gamma + \Delta_{L2}(\Gamma)) - V_{L2}(\Gamma)) + g(x_{F1}(\Gamma)) h(\Delta_{F1}(\Gamma))(V_{F2}(\Delta_{F1}(\Gamma)) - V_{F2}(\Gamma)) + \Gamma - x_{L2}(\Gamma) \right\}, \quad (10)$$

$$rV_{F2}(\Gamma) = \max_{x_{F2}(\Gamma), \Delta_{F2}(\Gamma)} \left\{ g(x_{F2}(\Gamma)) h(\Delta_{F2}(\Gamma))(V_{L2}(\Delta_{F2}(\Gamma)) - V_{F2}(\Gamma)) + g(x_{L1}(\Gamma)) h(\Delta_{L1}(\Gamma))(V_{F2}(\Gamma + \Delta_{L1}(\Gamma)) - V_{F2}(\Gamma)) - x_{F2}(\Gamma) \right\}. \quad (11)$$

We define $x_2^M$ as the solution of the following equation:

$$g'(x) = \frac{r + g(x^*) h(\Delta^*)}{h(\Delta^*) \Delta^*}. \quad (12)$$

From assumptions on $g(\cdot)$ ($g' > 0$ and $g'' < 0$), we have that $x_2^M$ is uniquely determined. We should note that $x_2^M$ is smaller than $x^*$ because $x^*$ is characterized as a solution to

$$g'(x) = \frac{r}{h(\Delta^*) \Delta^*}. \quad (13)$$

We prove the existence of a Markov perfect equilibrium such that
(a) $V_{L1}(\Gamma) = V_{F1}(\Gamma) = \frac{1}{r}(-x^* - x^M_2 + g(x^*)h(\Delta^*) \frac{\Delta}{r} + g(x^M_2)h(\Delta^*) \frac{\Delta}{r})$

(b) $V_{L2}(\Gamma) = A\Gamma + B, \quad V_{F2}(\Gamma) = B$

(c) $x^M_{F1}(\Gamma) = x^M_{L1}(\Gamma) = x^*$

(d) $x^M_{F2}(\Gamma) = x^M_{L2}(\Gamma) = x^M_2$, and

(e) $\Delta^M_{F_i}(\Gamma) = \Delta^M_{L_i}(\Gamma) = \Delta^* (i = 1, 2)$,

where $A \equiv 1/(r + g(x^*)h(\Delta^*))$ and $B \equiv \Delta^*/(r + g(x^*)h(\Delta^*))$. We prove that the above is a Markov perfect equilibrium by the following standard recursive method of solving dynamic programming: First, we suppose that the continuation payoff (value function) is given by (a) and (b), and then we show that (c), (d), and (e) become the optimal behaviors of each firm (step 1). Next, we suppose that each firm follows (c), (d), and (e), and then we show that in fact the continuation payoffs are given by (a) and (b) (step 2).

First we take step 1. Substituting (a) and (b) into the right-hand sides of (8)–(11) yields

$$\max_{x_{L1}(\Gamma), \Delta_{L1}(\Gamma)} \{g(x_{L1}(\Gamma))h(\Delta_{L1}(\Gamma)) \frac{\Delta_{L1}(\Gamma)}{r} + g(x_{F2}(\Gamma))h(\Delta_{F2}(\Gamma)) \frac{\Delta_{F2}(\Gamma)}{r} - x_{L1}(\Gamma) - x_{F2}(\Gamma)\},$$  

(13)

$$\max_{x_{F1}(\Gamma), \Delta_{F1}(\Gamma)} \{g(x_{F1}(\Gamma))h(\Delta_{F1}(\Gamma)) \frac{\Delta_{F1}(\Gamma)}{r} + g(x_{L2}(\Gamma))h(\Delta_{L2}(\Gamma)) \frac{\Delta_{L2}(\Gamma)}{r} - x_{F1}(\Gamma) - x_{L2}(\Gamma)\},$$  

(14)

$$\max_{x_{L2}(\Gamma), \Delta_{L2}(\Gamma)} \{g(x_{L2}(\Gamma))h(\Delta_{L2}(\Gamma))A\Delta_{L2}(\Gamma) - g(x_{F1}(\Gamma))h(\Delta_{F1}(\Gamma))A\Gamma - x_{L2}(\Gamma)\},$$  

(15)

$$\max_{x_{F2}(\Gamma), \Delta_{F2}(\Gamma)} \{g(x_{F2}(\Gamma))h(\Delta_{F2}(\Gamma))A\Delta_{F2}(\Gamma) - x_{F2}(\Gamma)\}.$$  

(16)

From (13) and (14), it is clear that $(x_{L1}(\Gamma), \Delta_{L1}(\Gamma))$ and $(x_{F1}(\Gamma), \Delta_{F1}(\Gamma))$ maximize the same function

$$-x + g(x)h(\Delta) \frac{\Delta}{r}.$$  

(17)

Since this function does not depend on $\Gamma$, $x_{L1}(\Gamma) = x_{F1}(\Gamma) = C$ and $\Delta_{L1}(\Gamma) = \Delta_{F1}(\Gamma) = D$ must hold for all $\Gamma$ ($C$ and $D$ are constants). Then firm 1 maximizes (17) with respect to $x$ and $\Delta$. The first order conditions for the optimality are:

$$-1 + g'(x)h(\Delta) \frac{\Delta}{r} = 0,$$

$$\frac{g(x)}{r}(h'(\Delta) \Delta + h(\Delta)) = 0.$$
The second order conditions are satisfied under the assumptions made in the paper. Since \( g(x)/r > 0 \), these conditions are the same as those of social optimum (see (3) and (4)). Therefore we get

\[ x_M^1(\Gamma) = x_L^1(\Gamma) = x^* \text{ and } \Delta_M^1(\Gamma) = \Delta_L^1(\Gamma) = \Delta^*. \]

Similarly, (15) and (16) imply that both \((x_L^2(\Gamma), \Delta_L^2(\Gamma))\) and \((x_F^2(\Gamma), \Delta_F^2(\Gamma))\) maximize the same function

\[ -x + g(x)h(\Delta)A\Delta. \]  

Then firm 2 maximizes (18) with respect to \( x \) and \( \Delta \). The first order conditions for the optimality are:

\[
-1 + g'(x)h(\Delta)A\Delta = 0 \quad (19)
\]

\[
g(x)A(h'(\Delta)\Delta + h(\Delta)) = 0. \quad (20)
\]

The second order conditions are also satisfied. Since \( g(x)A > 0 \), (20) is the same as the condition for \( \Delta^* \). Therefore we get \( \Delta_M^2(\Gamma) = \Delta_L^2(\Gamma) = \Delta^* \).

Given this result, (19) can be rewritten as

\[
g'(x) \frac{h(\Delta^*)\Delta^*}{r + g(x^*)h(\Delta^*)} = 1.
\]

This is the equation specified in the definition of \( x_M^2 \) (see (12)). Therefore we get \( x_M^2(\Gamma) = x_L^2(\Gamma) = x_M^2 \).

Finally, we take step 2. Substituting the above results into the right-hand sides of (8)–(11), it is easily checked that each equality holds. Thus, (a) and (b) are true. Q.E.D.

Appendix B

**Proof of Proposition 2:** Even when the public firm is force to choose \( x_1 = b \), the basic dynamic structure does not change except for fixing the R&D spending of the public firm to \( b \). Therefore, we omit the proof of the existence of a Markov perfect equilibrium and simply specify the continuation payoffs and equilibrium strategies.

There exists a Markov perfect equilibrium such that
Given this Markov perfect equilibrium, we prove Proposition 2 as follows:

Expected social welfare is exactly the same as $V_{L1} = V_{F1}$, so $EW$ is

$$\frac{1}{r}(-b - x_C^2 + g(b)h(\Delta^*)\Delta^* + g(x_C^2)h(\Delta^*)\Delta^*).$$

Differentiating it by $b$ yields

$$\frac{\partial EW}{\partial x_1} + \frac{\partial EW}{\partial x_2} \frac{dx_C^2}{db} + \frac{\partial EW}{\partial \Delta^*} \frac{d\Delta^*}{db}.$$

From the first order condition of the social optimum, we obtain $\partial EW/\partial x_1 = 0$ evaluated at $b = x^*$. Moreover, we get $\partial EW/\partial x_2 > 0$ since the investment of the private firm (firm 2) must be smaller than the social optimal level. (Note that $\partial EW/\partial x_2$ becomes exactly the same as the left-hand side of (4) evaluated at $\Delta = \Delta^*$.) From (21) we have

$$\frac{dx_C^2}{db} = \frac{g'(x_C^2)(\Delta^*)\Delta^*}{r},$$

where the inequality is derived from $g' > 0$ and $g'' < 0$. From the argument in the proof of Proposition 1, the public firm chooses $\Delta^*$ regardless of $b$. So $(\partial EW/\partial \Delta^*)(d\Delta^*/db) = 0$.

These imply that a slight reduction of $b$ from $x^*$ always improves social welfare. Thus (i) is proved.
Whenever $b > x^*$, reducing $b$ improves social welfare since both $\partial EW / \partial x_1$ and $(\partial EW / \partial x_2)(dx_2^C/db)$ becomes negative. Thus (ii) is proved. Q.E.D.

Appendix C

Proof of Proposition 3: When the government can control $\Delta_1$ as well as $x_1$, a public firm’s maximization problem disappears. However, we still have the same dynamic structure for a private firm as in the proof of Proposition 1. So, we omit the proof again and say that there exists a Markov perfect equilibrium such that

(a) $V_{L2}(\Gamma) = A''\Gamma + B''$, $V_{F2}(\Gamma) = B''$,

(b) $x^D_{F2}(\Gamma) = x^D_{L2}(\Gamma) = x^D_2$, and

(c) $\Delta^D_{F2}(\Gamma) = \Delta^D_{L2}(\Gamma) = \Delta^*$,

where superscript $D$ means equilibrium strategies and $x^D_2$, $A''$, and $B''$ are characterized by

\[ g'(x^D_2) = \frac{r + g(b)h(S)}{h(\Delta^*)\Delta^*} \]
\[ A'' = \frac{1}{r + g(b)h(S)} \]
\[ B'' = \frac{1}{r}(-x^D_2 + g(x^D_2)h(\Delta^*) \Delta^* \frac{1}{r + g(b)h(S)}) \]

Given this Markov perfect equilibrium, we prove Proposition 3 as follows. Substituting $\Delta_1 = S, x_1 = b, \Delta_2 = \Delta^*$, and $x_2 = x^D_2$ into (2) yields

\[ EW = \frac{1}{r}\left(-b - x^D_2 + \frac{g(b)h(S)}{r} + \frac{g(x^D_2)h(\Delta^*)\Delta^*}{r}\right) \]  \hspace{1cm} (25)

Differentiating (25) by $S$ yields

\[ \frac{dEW}{dS} = \frac{\partial EW}{\partial x_2} \frac{dx_2^D}{dS} + \frac{\partial EW}{\partial S} = \frac{\partial EW}{\partial x_2} \frac{dx_2^D}{dS} + \frac{g(b)}{r^2}(h'(S)S + h(S)). \]

By the same reason as in the proof of Proposition 2, $\partial EW / \partial x_2$ is positive. From the definition of $x^D_2$ above, $x^D_2$ is an increasing function of $S$, i.e., $dx^D_2/dS > 0$. Evaluated at $S = \Delta^*$, from (3) we obtain $\partial EW / \partial S = 0$. These imply that slightly increasing $S$ from $\Delta^*$ improves social welfare. Thus (i) is proved. Moreover, at any value $S < \Delta^*$, we get $\partial EW / \partial S > 0$. This implies that increasing $S$ always improves the social welfare. Thus (ii) is proved. Q.E.D.
References


