Simplified Alonso-Mills-Muth Model with a Monopoly Vendor

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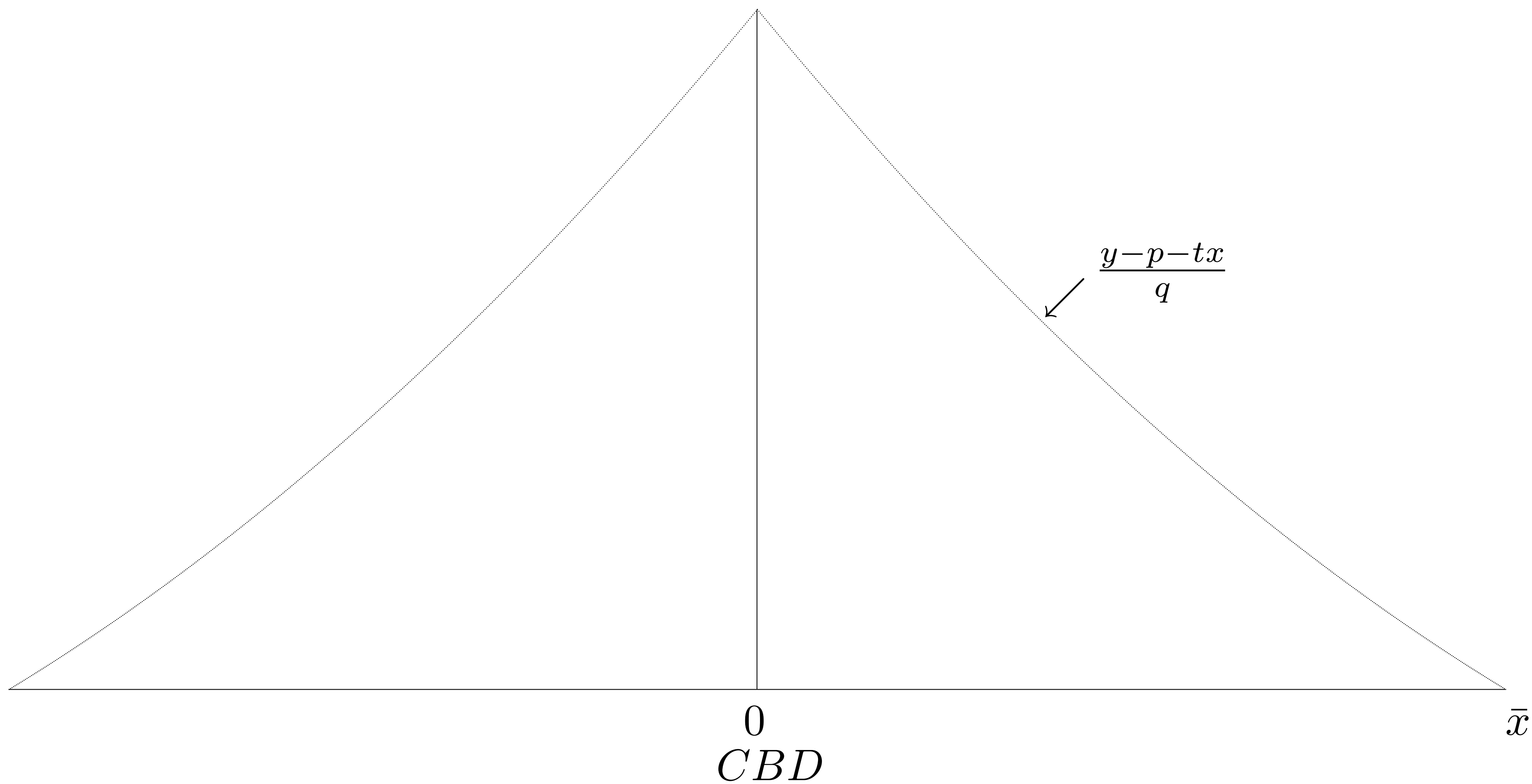
abstract

One important but unrealistic assumption in Alonso-Mills-Muth (AMM) model is that the composite good is ubiquitous and thus zero shopping cost for residents in an open city. In a simplified AMM model, assume that the composite good is only sold by a monopoly vendor inside the city and thus a positive shopping cost is inevitable for residents. This paper shows that the vendor will locate at the city boundary in both "one-group" and "two-group" models. In contrast to the symmetric land rent pattern in the AMM model, the current model shows an asymmetric land rent pattern in equilibrium. Moreover, it is shown that the central business district (CBD) is the social optimal location for the vendor from the standpoint of minimizing the average transportation cost.

Alonso (1964), Mills (1967), and Muth (1969) (AMM model):

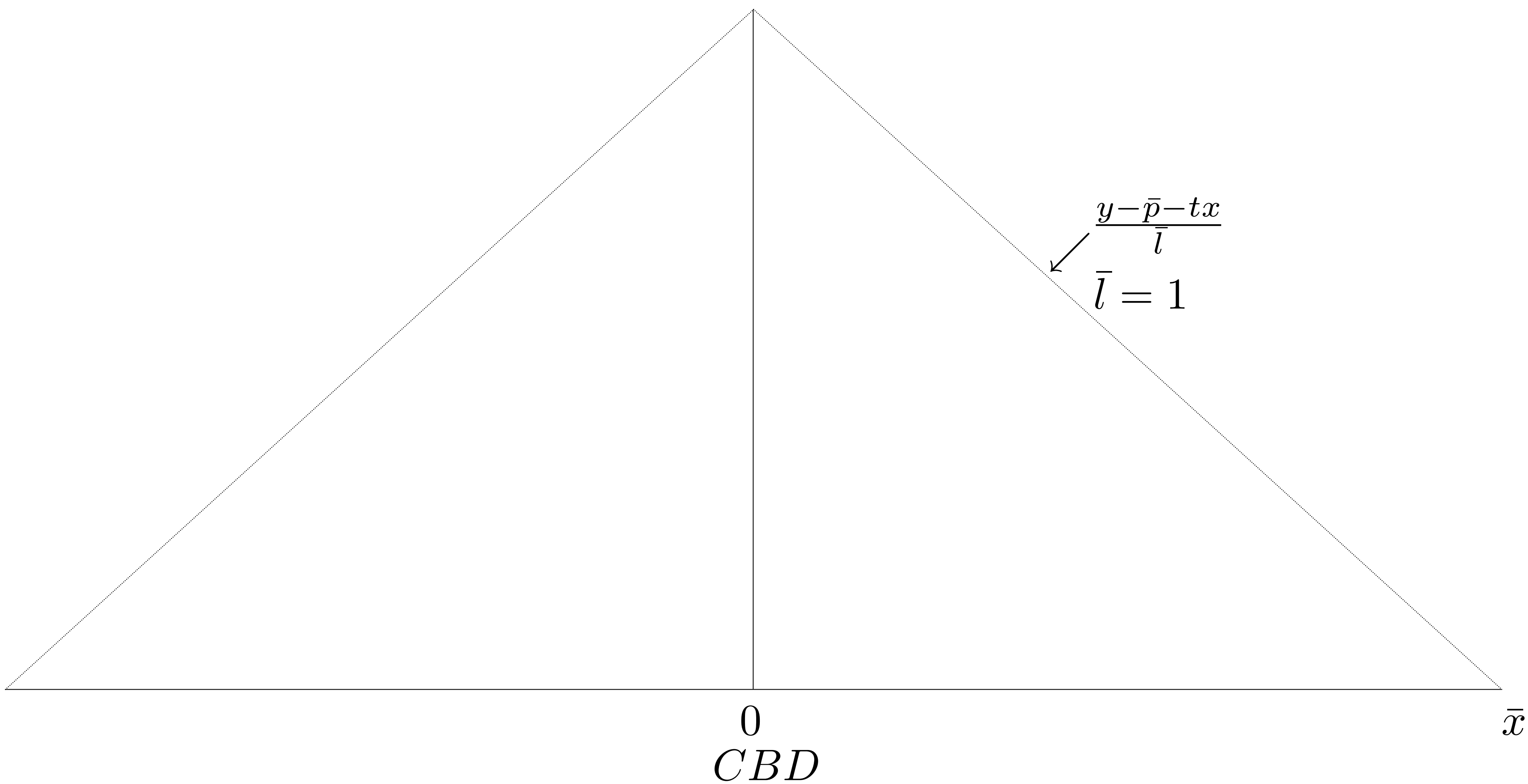
1. Each resident earns identical income (y).
2. Each resident commute to CBD for job with (tx).
3. Each resident consume land and composite good.
4. The composite good is ubiquitous.



AMM model

Original AMM model:
land and composite good are substitutable.

Simplified AMM model:
fixed consumption of land and composite good.



Simplified AMM

Motivation:

The composite good may not ubiquitous, and thus people must do shopping with some costs as well as commuting costs.

The Model

1. A small open linear city.

2. All land is owned by an absentee land-owner.

3. Each resident lives inside the city with one unit of land.

4. Each resident must commute to CBD for his job everyday, with unit transport cost t.

5. Each resident consumes one unit of a composite good produced by a monopoly vendor locate at a point z.

6. Each resident goes shopping to z with a unit shopping cost k.

7. When $k = 0$, the current model reduced to original AMM model.

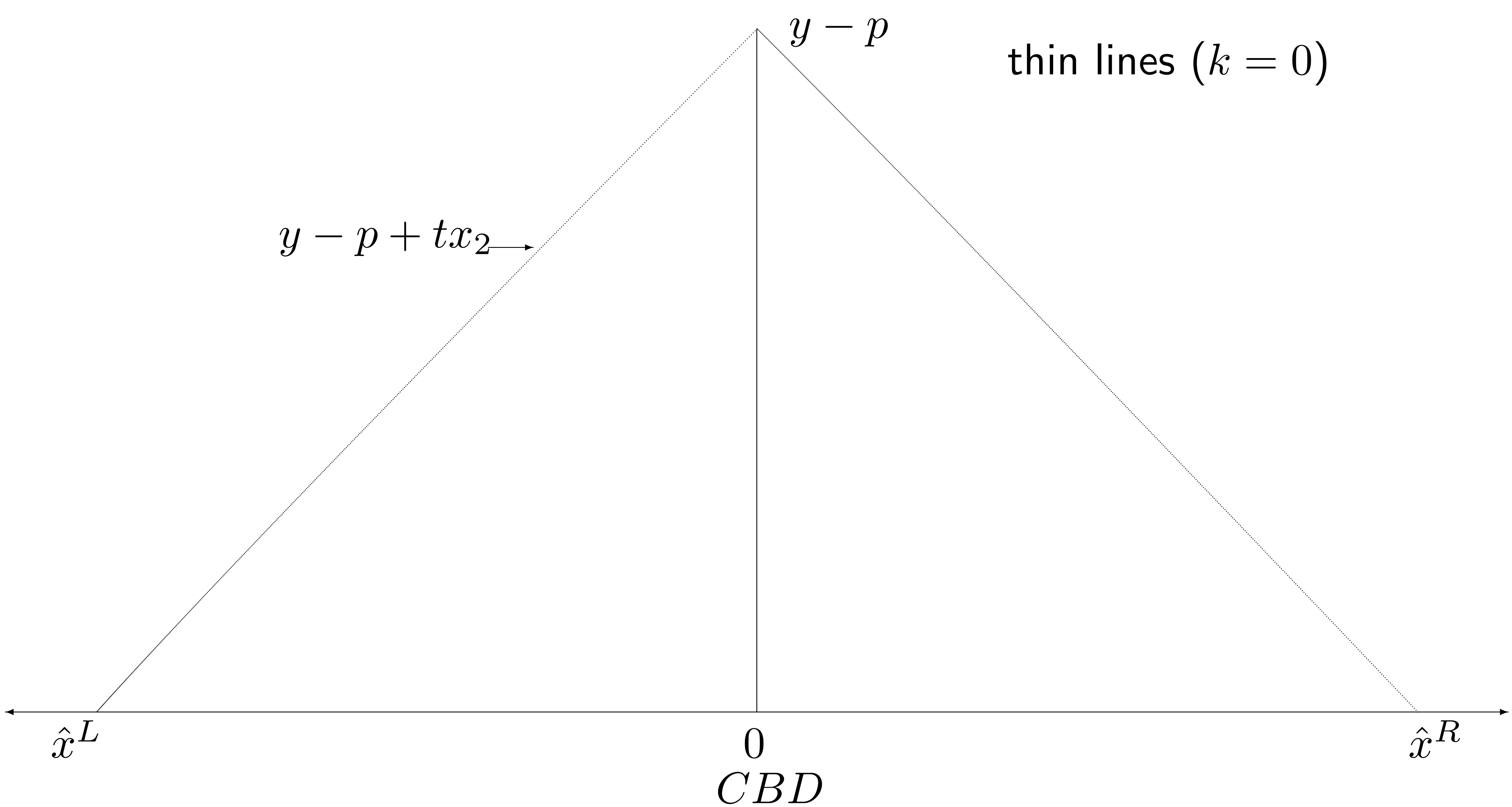
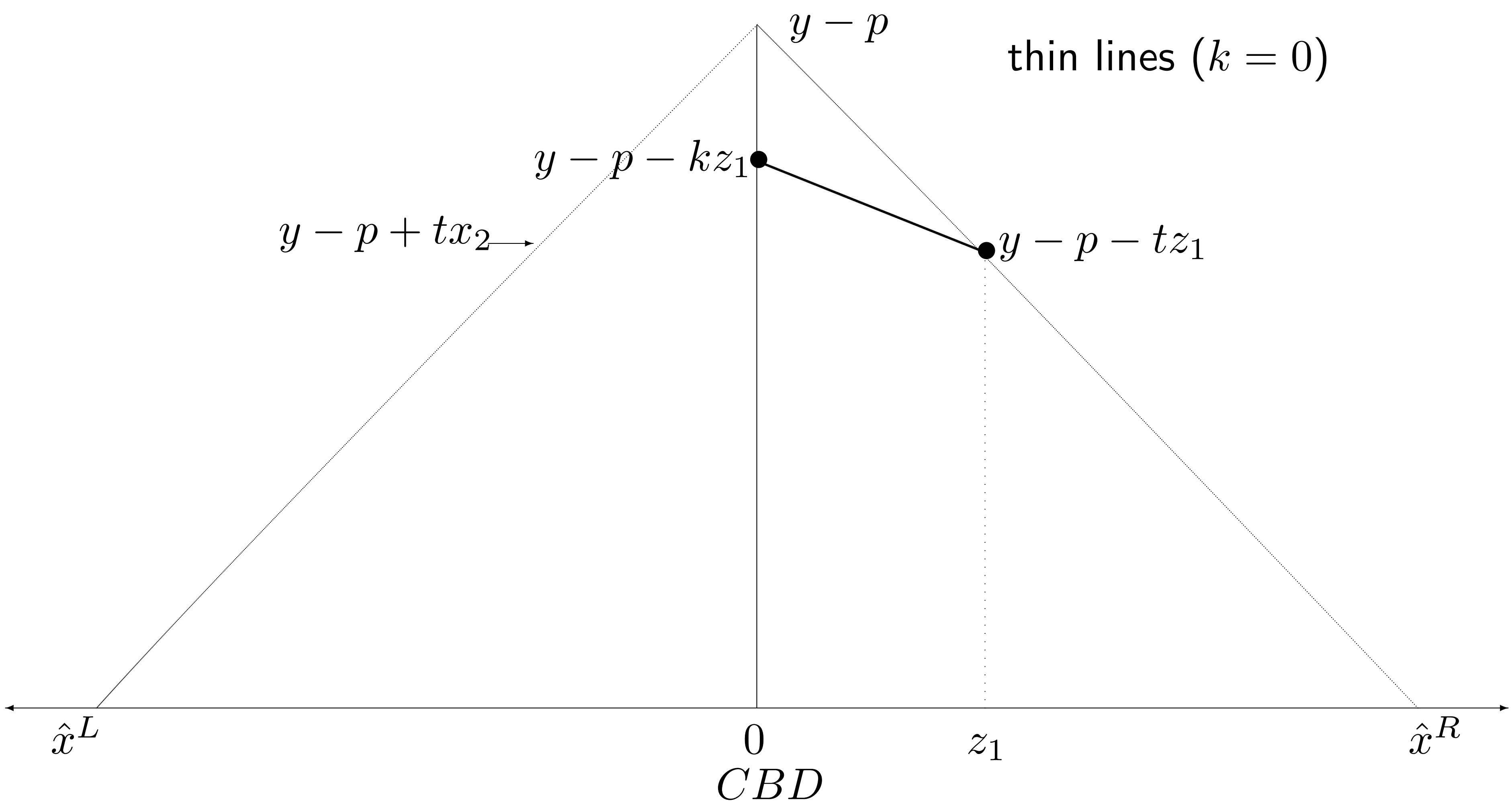


Figure 1: Bid rents with one vendor



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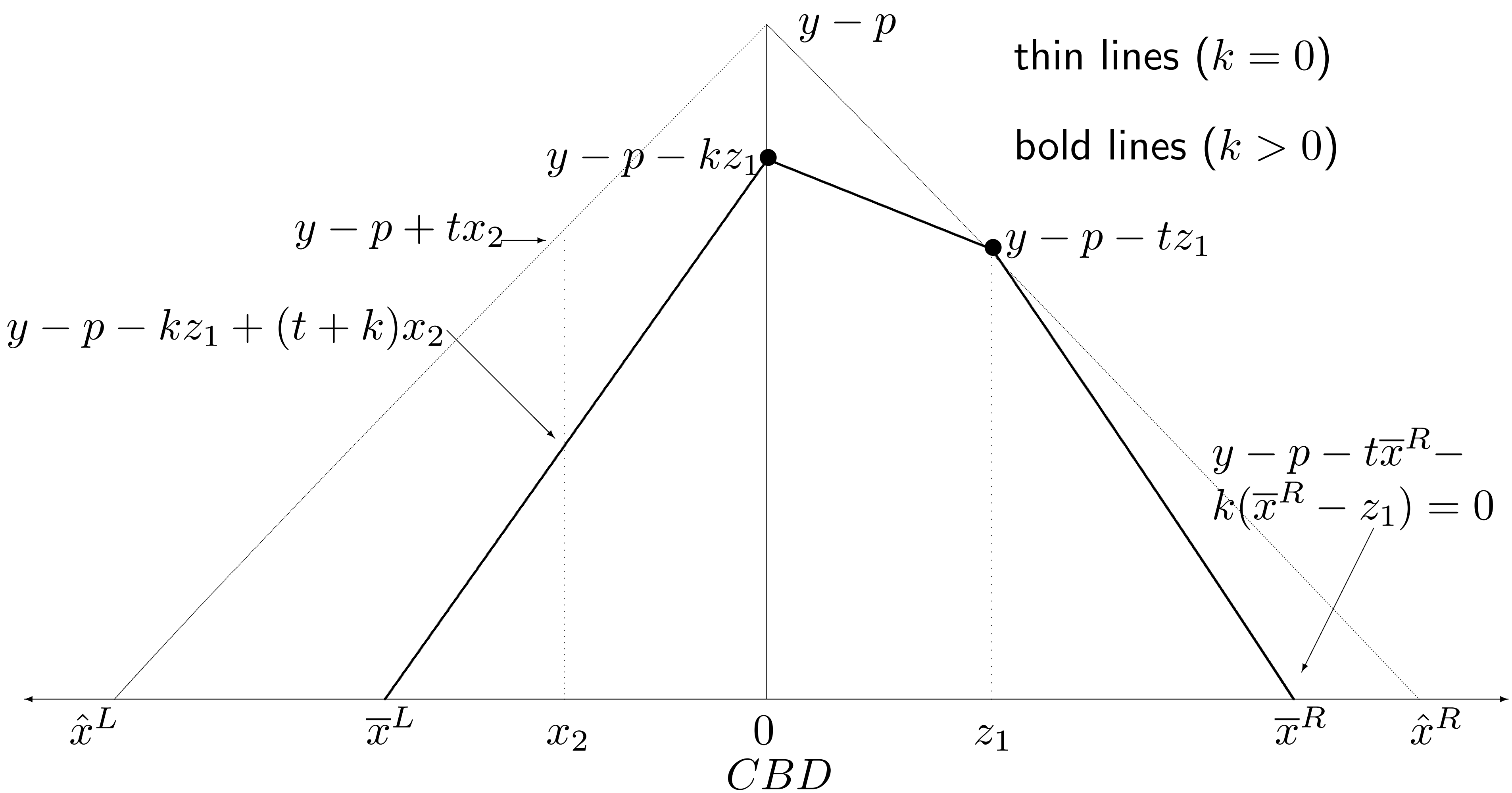


Figure 1: Bid rents with one vendor

The zero shopping cost in AMM model can be interpreted by two viewpoints: First, there are ubiquitous competitive vendors (price takers), and thus shopping cost is zero. Vendors play no roles in the model. Second, there is only one monopoly vendor somewhere in the city and the composite good price is determined by a simple cost-mark-up pricing (thus the vendor play a passive role in th model and every resident faces the same price, say 1 dollar per unit of the composite good) with zero unit shopping cost ($k = 0$). The current model adopts the second viewpoint and allows the monopoly choosing a mill pricing based on profit maximization. Thus, more realistically, the monopoly vendor plays an active role in the current model. Precisely speaking, the AMM model is based on the first viewpoints. For the sake of comparison and without loss of generality, this paper assume $k = 0$ representing the AMM model.

One-group

Equilibrium analysis:

$$\max \ \pi = p(\overline{x}^R - \overline{x}^L) - (y - p - tz) \tag{4}$$
$$s.t. \quad 0 \leq z \leq \overline{x}^R.$$

$$\overline{x}^{R^*} = \frac{y}{2t} = z, \quad \text{and} \quad \overline{x}^{L^*} = \frac{-y(t-k)}{2t(t+k)}.$$

$$N^* = \overline{x}^{R^*} - \overline{x}^{L^*} = \frac{y}{t+k}. \tag{13}$$

$$\pi^* = \frac{y^2}{2(t+k)}. \tag{14}$$

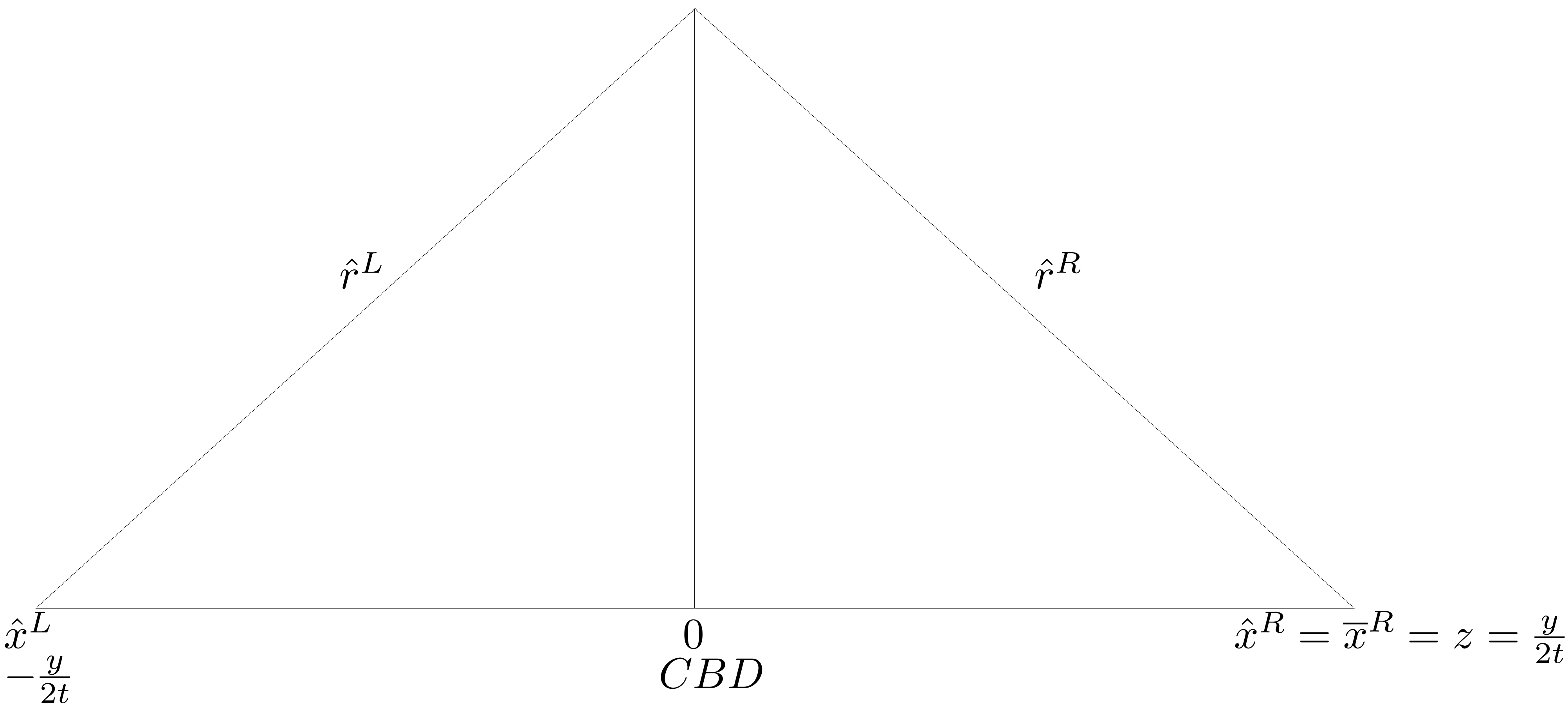


Figure 2: The equilibrium land rent pattern in the one-group model

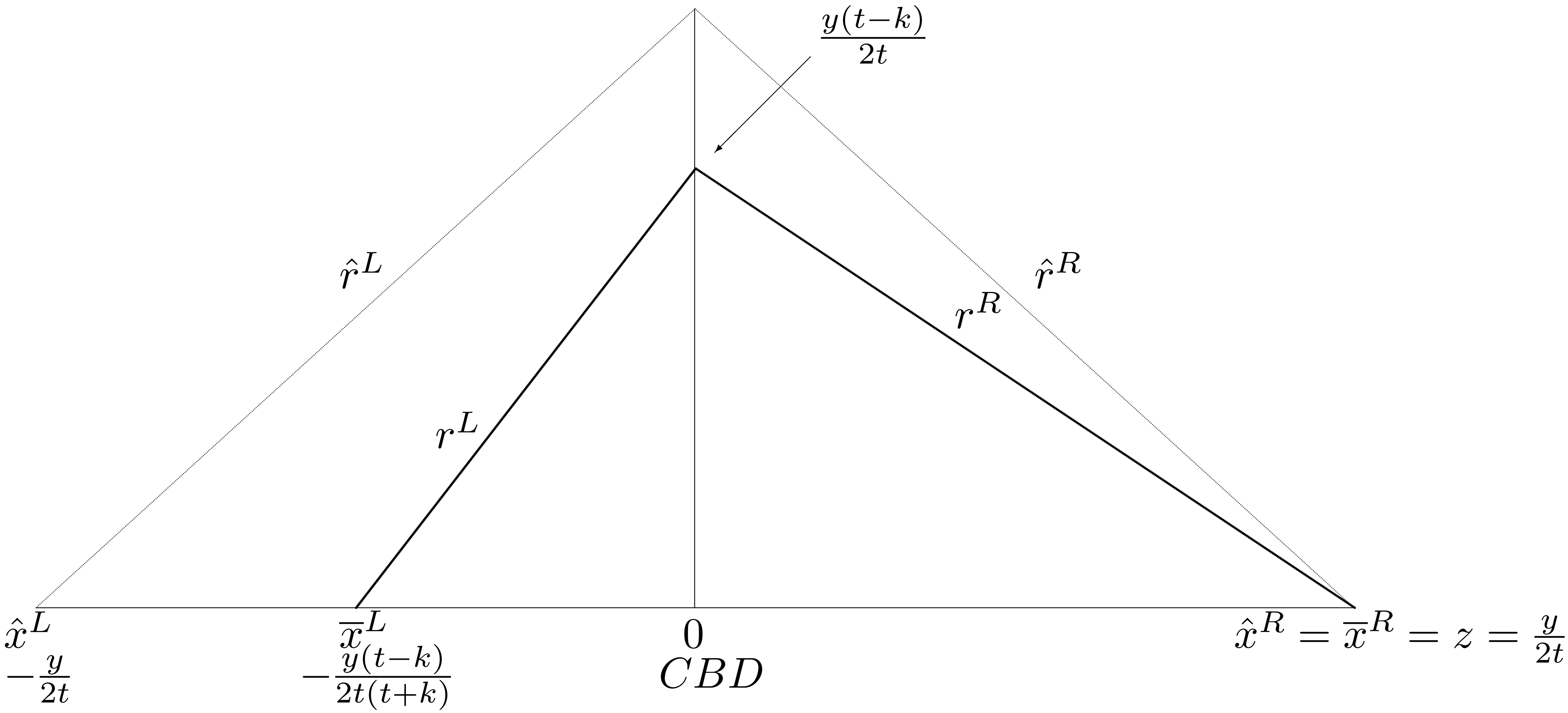


Figure 2: The equilibrium land rent pattern in the one-group model

$$ATC = \Bigg[t \left(\int_0^{\overline{x}^R} m dm + \int_0^{-\overline{x}^L} m dm \right) \tag{16}$$

$$+ k \left(\int_0^z (z - m) dm + \int_z^{\overline{x}^R} (m - z) dm + \int_0^{-\overline{x}^L} (z + m) dm \right) \Bigg] / (\overline{x}^R - \overline{x}^L),$$

$$ATC^* = \frac{y\,(t + k)}{4t}. \tag{17}$$

One-group

Location regulation:

Stage 1: Z_g

Stage 2: P_g

Solving $\partial\pi_g/\partial p_g = 0$, yields

$$p_g^* = \frac{1}{4}(2y + t + k) \tag{18}$$

Minimize ATC

$$L = ATC + \lambda(z_g - \overline{x}^R), \qquad \lambda \geq 0.$$

$$ATC_g^* = \frac{1}{8}(2y - t - k).$$

Proposition 1. *The monopoly vendor in the simplified AMM model will locate at one of the city boundary and pay zero land rent. However, the social optimal location which minimizes the average transportation cost is at* CBD.

Proposition 2. *The size of total population under optimal location regulation ($z_g^* = 0$) is smaller than the population size under market equilibrium. The monopoly vendor earns less profit under optimal location regulation than without location regulation.*

Two-group model

1. Each resident of the poor (group 1) consume $\alpha < 1$ unit of land, while each one of the rich (group 2) consume one unit of land.

2. Each one of the poor earns y_1 income, each one of the rich earns y_2 income.

$$y_1 < y_2$$

3. The size of the vendor always bids one unit of land for its store.

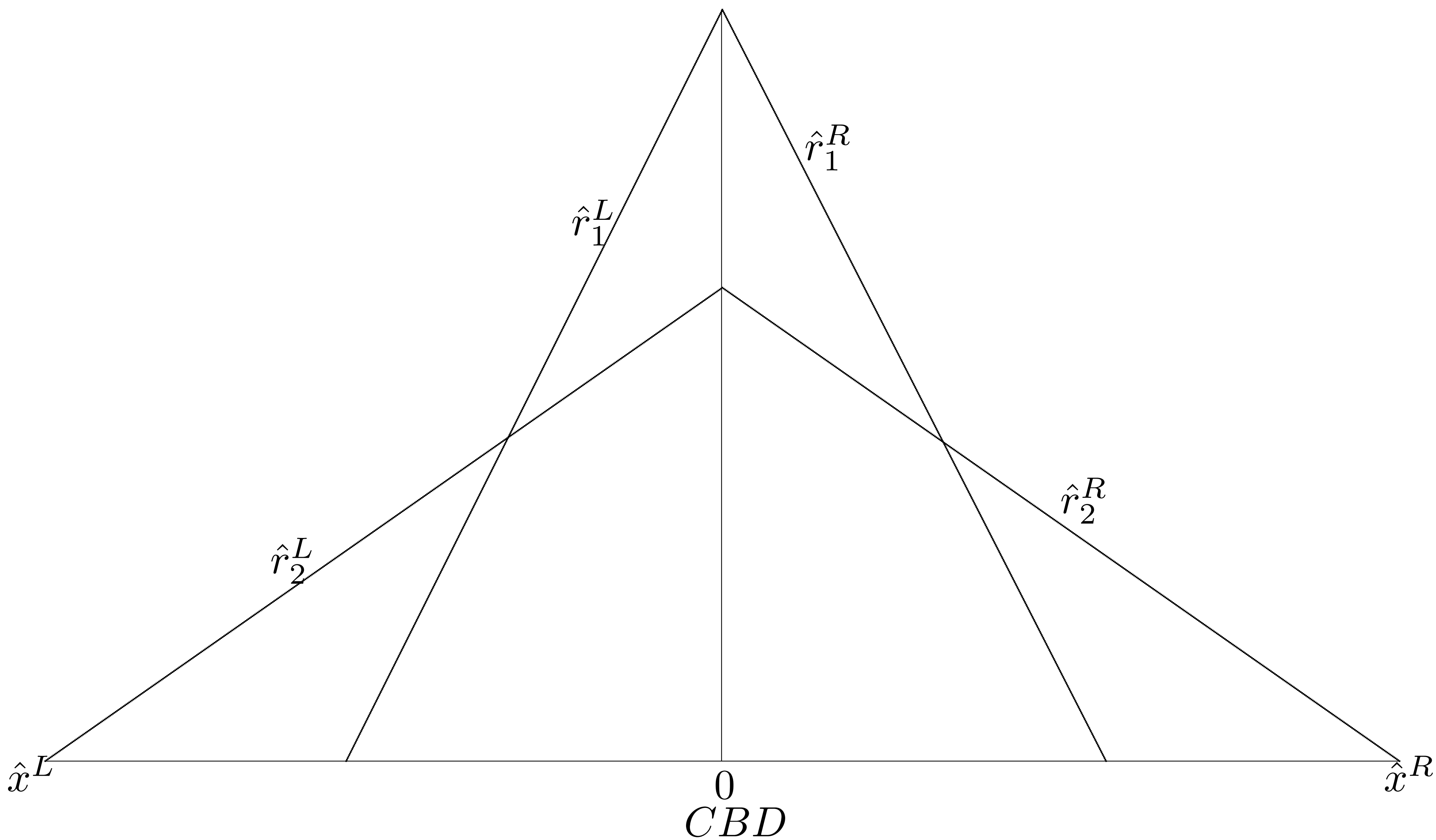
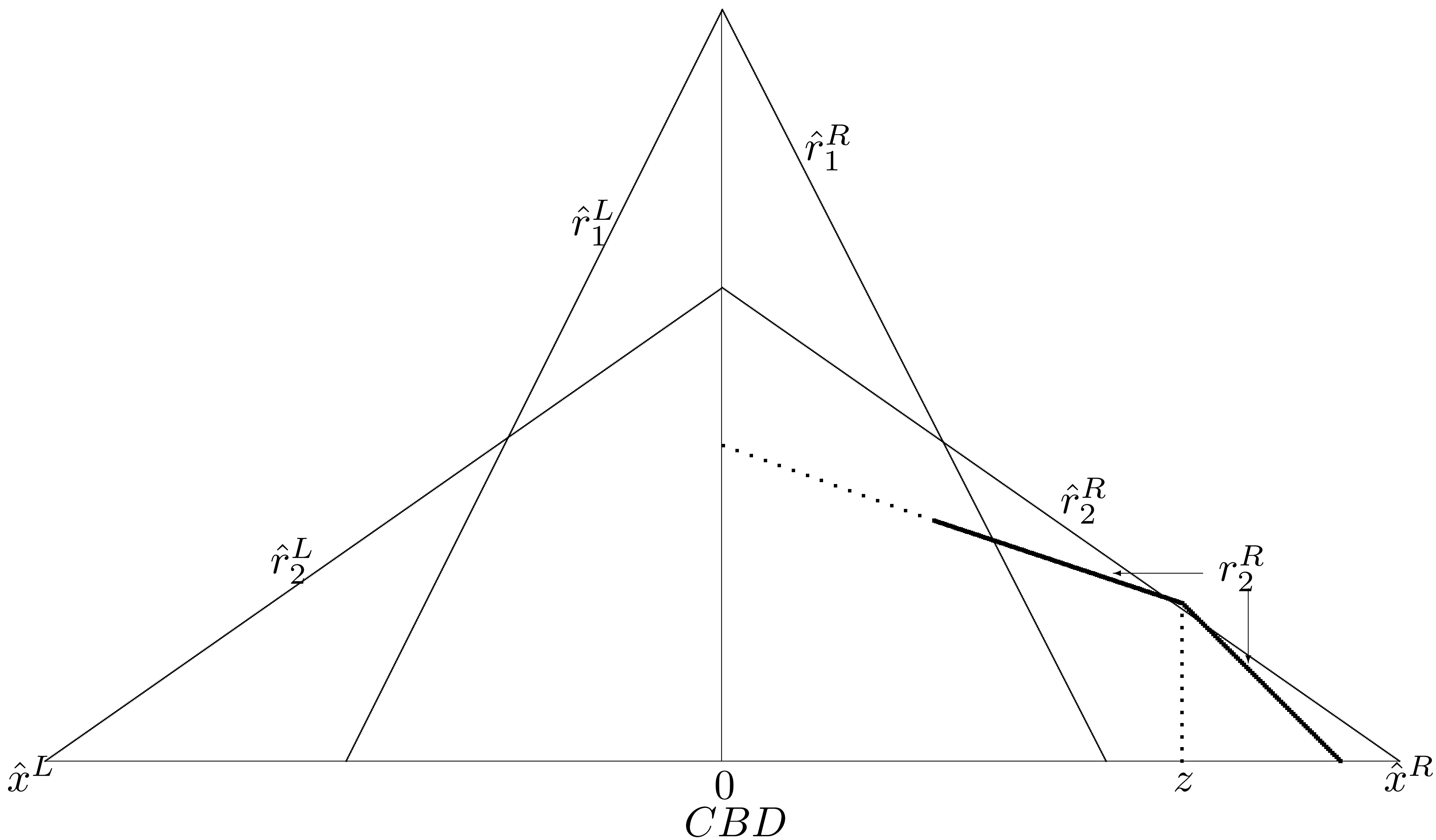


Figure 3: The bid rent pattern in the two-group model ($\hat{x} \leq z \leq \overline{x}^R$)



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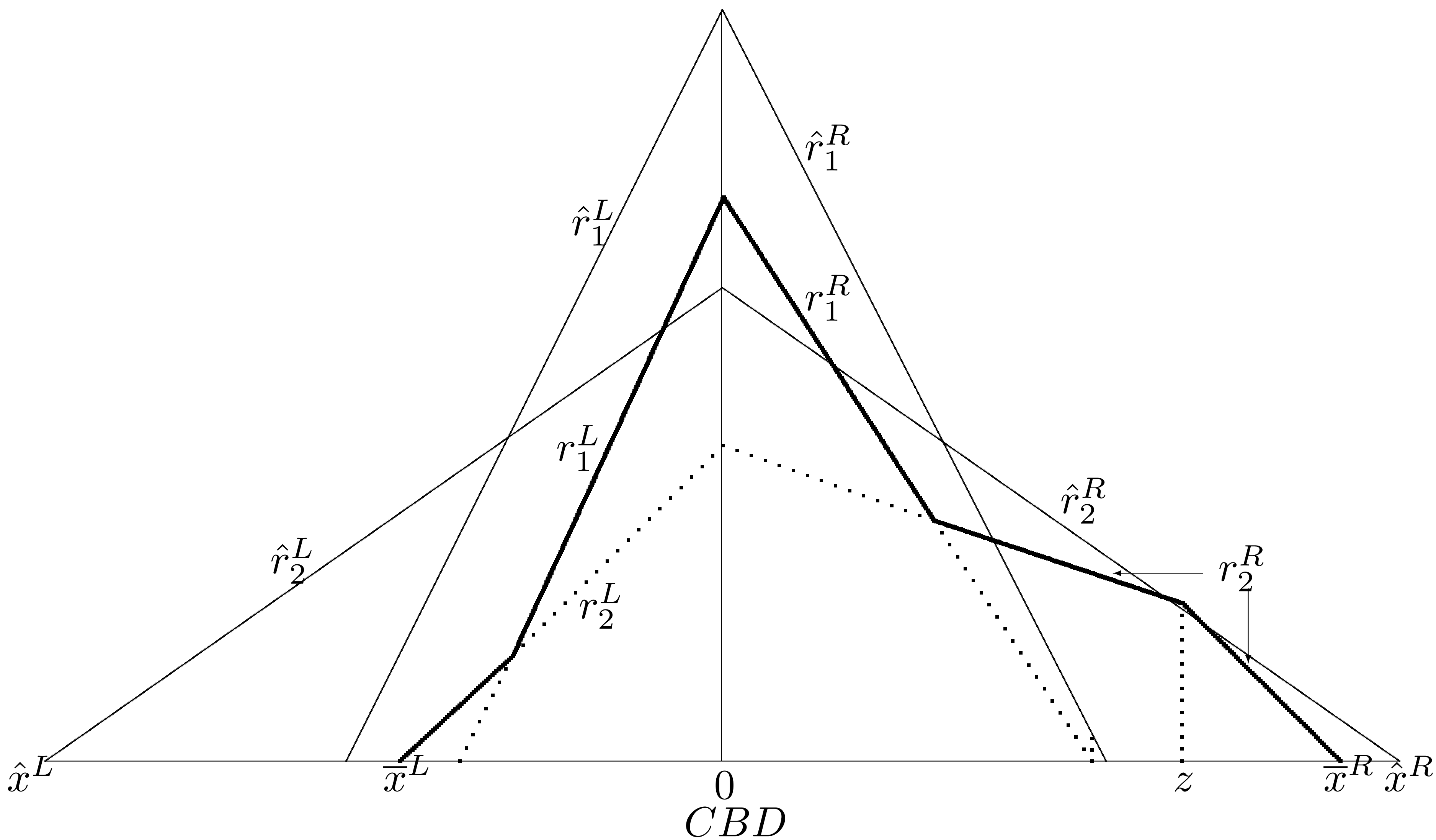

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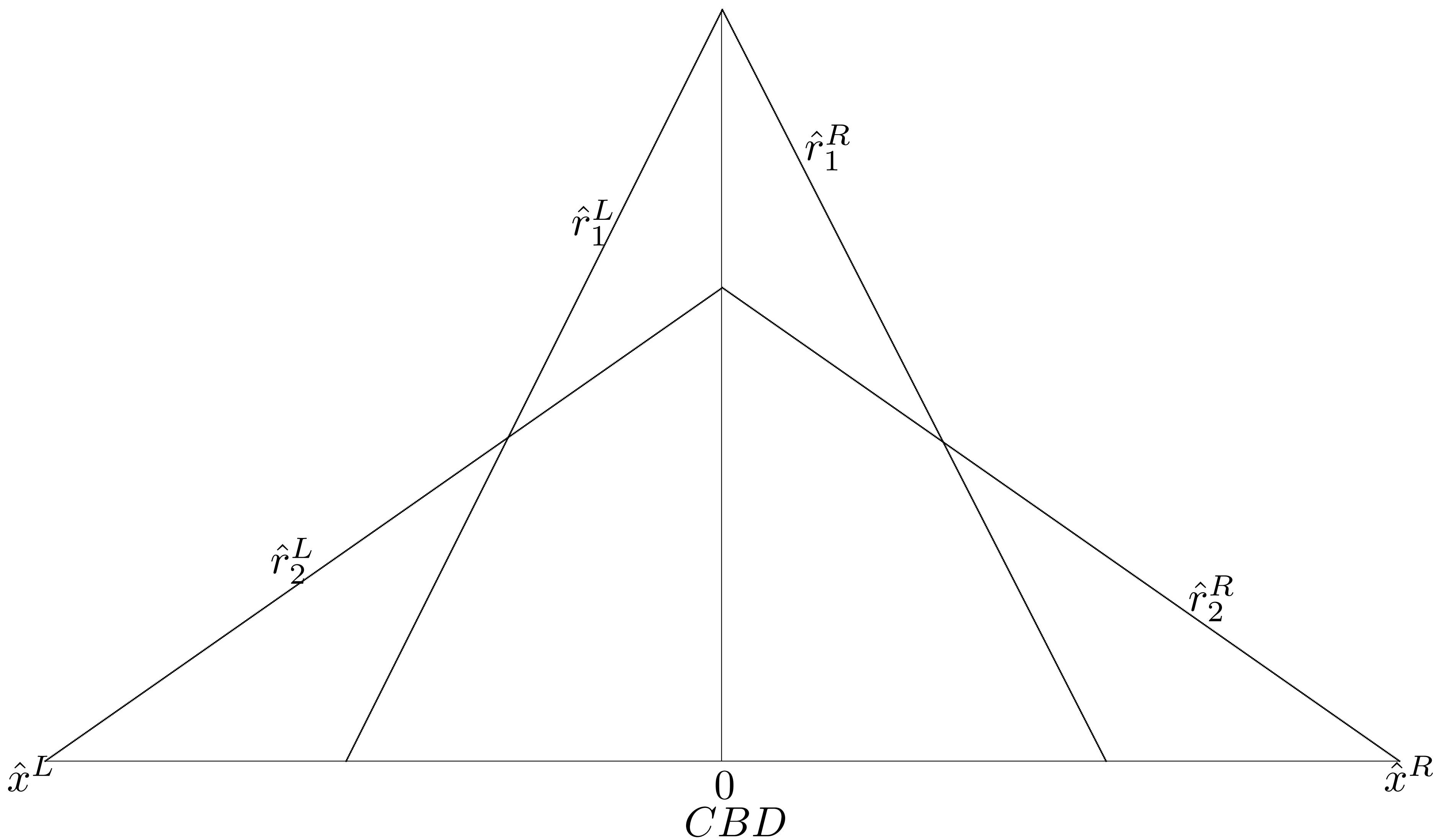
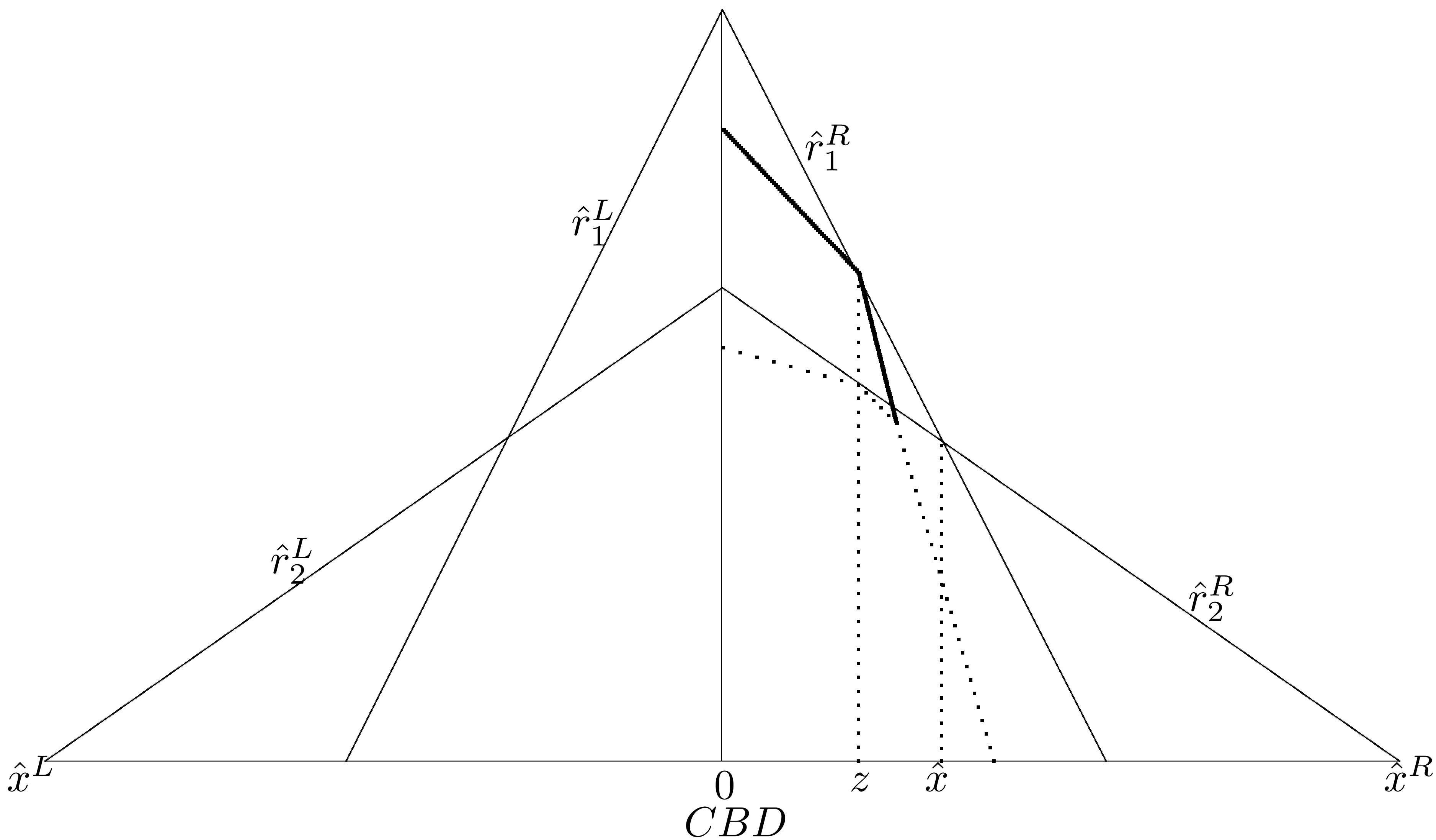


Figure 4: The land rent pattern in the two-group model ($0 \leq z \leq \hat{x}$)



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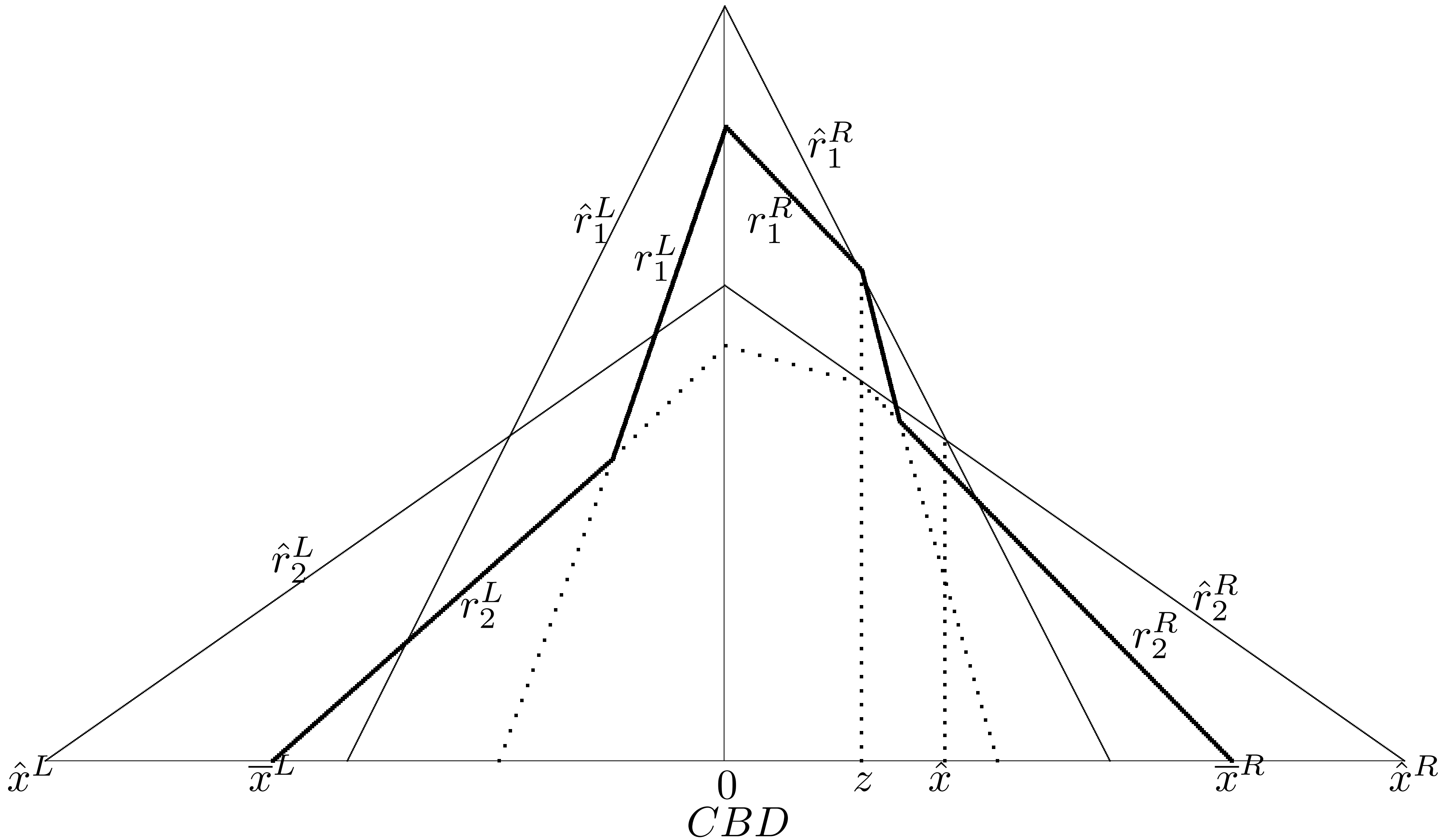

Figure 4: The land rent pattern in the two-group model ($0 \leq z \leq \hat{x}$)

2.2.1 Market equilibrium analysis

First: When the vendor locates in the rich area

$$p^* = \frac{y_2}{2}, \tag{27}$$

$$z^* = \frac{y_2}{2t} = \overline{x}_2^{R*}. \tag{28}$$

$$ATC^* = \frac{y_2(t+k)}{4t}, \tag{29}$$

$$\pi^* = \frac{y_2^2}{2(t+k)}, \tag{30}$$

$$N^* = \overline{x}^{R*} - \overline{x}^{L*} = \frac{y_2}{t+k}. \tag{31}$$

Second: When the vendor locates in the poor area

$$z' = 0$$

$$p' = \frac{2y_2\alpha + t + k}{4\alpha} \stackrel{\text{def}}{=} \underline{y}_1$$

Since $p' < y_1$ (so as some poor residents) and

$$\pi(z', p') - \pi(z^*, p^*) = \frac{t + k - 8\alpha y_1 + 4\alpha y_2}{8\alpha^2} \geq 0$$

$$\Longrightarrow y_1 \leq \frac{4\alpha y_2 + t + k}{8\alpha} \stackrel{\text{def}}{=} \bar{y_1}$$

But

$$\underline{y}_1 - \bar{y_1} = \frac{24\alpha^2 y_1 + (1-\alpha)(t+k)}{4\alpha(1+3\alpha)} > 0$$

Then $z = 0$ is dominated by $z^* = \bar{x}_2^R$

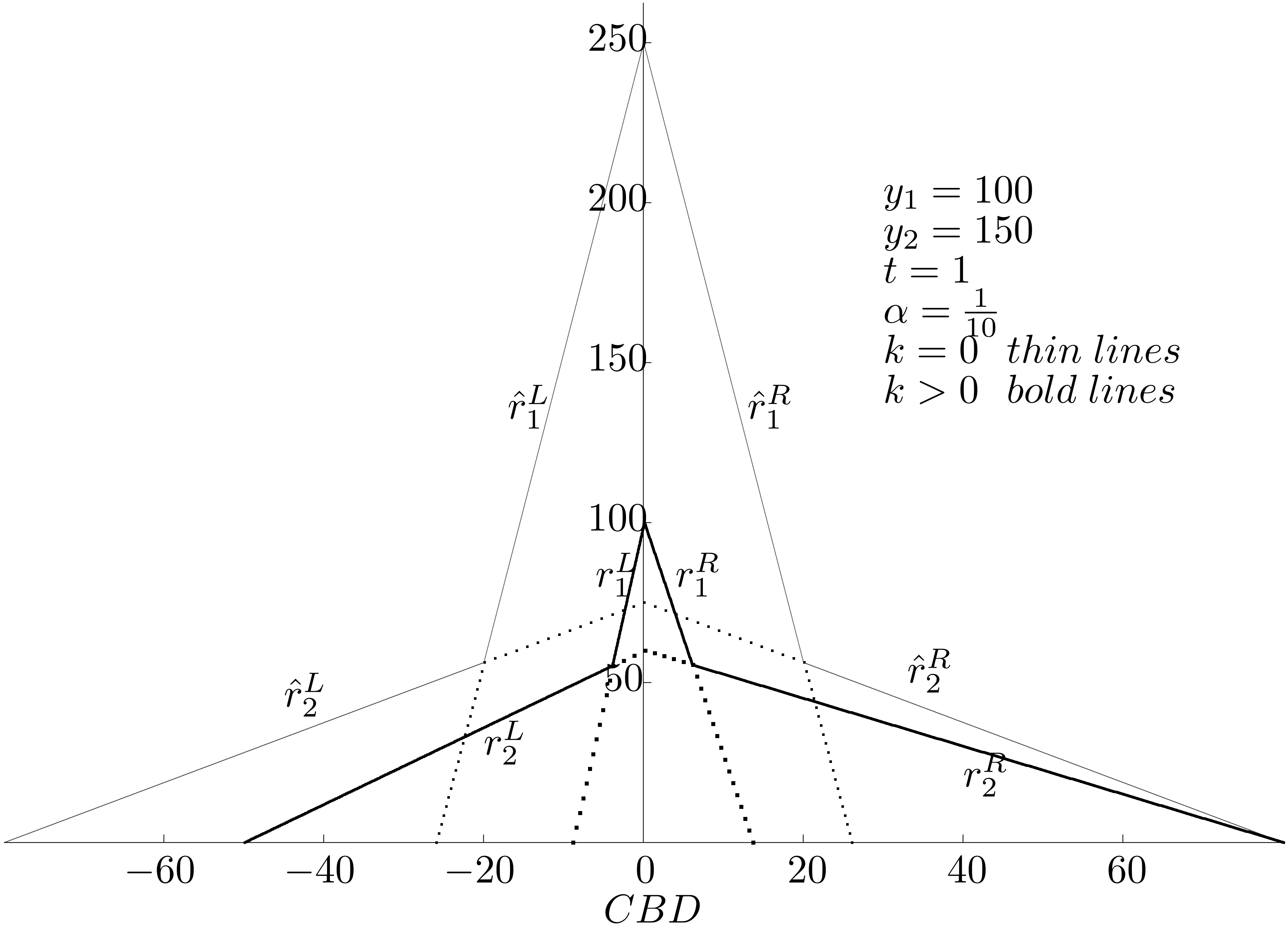

Figure 5: Equilibrium land rent patterns

Proposition 3. *In the two-group model, the equilibrium location of the vendor is at one of the border of the city.*

In AMM model ($k = 0$),

$$\frac{y_1 - p}{\alpha} > (y_2 - p), \tag{40}$$

which means that

$$y_1 > \alpha y_2 + (1 - \alpha)p \stackrel{\text{def}}{=} \underline{y}^0 = \frac{y_2(1 + \alpha)}{2}. \tag{41}$$

When shopping cost be considered ($k > 0$), the existence of the poor must be

$$\frac{y_1 - p - kz}{\alpha} > (y_2 - p - kz), \tag{42}$$

which means that

$$y_1 > \alpha y_2 + (1-\alpha)p + (1-\alpha)kz = \underline{y}^k = \frac{y_2(t+k+\alpha t - \alpha k)}{2t}. \quad (43)$$

As the result in Proposition 2, $z^* = \overline{x}_2^{R^*} > 0$, therefore when $\underline{y}^0 < y_1 < \underline{y}^k$, there are some poor people live in the city in the AMM model, while no poor people live in the city in the current two-group model (reduced to one-group model).

Proposition 4. *When* $\underline{y}^0 < y_1 < \underline{y}^k$*, there are some poor residents live in the inner city in the AMM model* $(k = 0)$*, while the poor are forced out of this city when* $(k > 0)$*.*

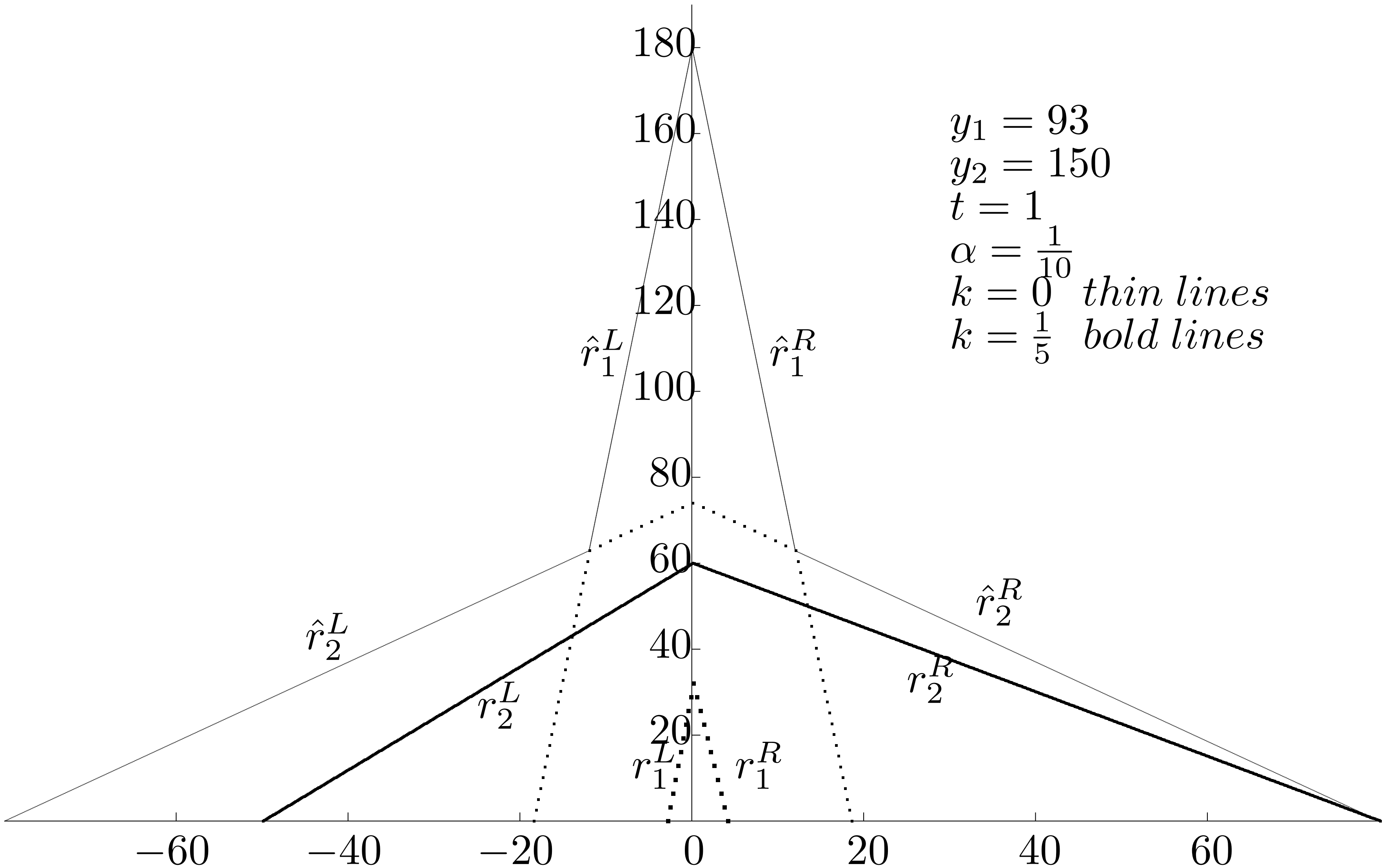

Figure 6: The poor live in the inner city when $k = 0$, while no poor live in the city when $k > 0$

2.2.2 Optimal location regulation

Suppose the government assigns a location (z_g) of a shopping center to minimize ATC in the first stage, and then the monopoly vendor sets up a price to maximize its profit in the second stage. To solve the equilibrium, a backward induction will be employed. In the second stage, the monopolist decides its price given an exogenous variable z_g. There are two possible z_g. That is, $0 \leq z_g \leq \hat{x}$ (in the poor area) and $\hat{x} \leq z_g \leq \overline{x}_2^R$ (in the rich area).

First, suppose now $0 \leq z_g \leq \hat{x}$ (poor area), then the profit of the vendor is

$$\pi = p(\overline{x}_2^R - \overline{x}_2^L) - \frac{y_1 - p - tz}{\alpha}. \tag{44}$$

Set up Lagrangian as

$$L = \pi + \lambda(y_2 - p - kz + t\overline{x}^L + k\overline{x}^L), \tag{45}$$

where the constraint after λ means that the remotest resident ($\overline{x}^L$) will be served by the vendor. Solving for the optimal price yields

$$p = \frac{2\alpha y_2 + t + k}{4\alpha}. \tag{46}$$

The government minimize ATC by setting a Lagrangian as

$$L = ATC + \lambda(z - \hat{x}), \quad \lambda \geq 0. \tag{47}$$

Solving $\frac{\partial L}{\partial z} \cdot z = 0$ and $\frac{\partial L}{\partial \lambda} \cdot \lambda = 0$ yields an unique solution as

$$z_g^* = 0, \tag{48}$$

and ATC is then

$$ATC_{z_g=0} = \frac{2\alpha y_2 - t - k}{8\alpha}. \tag{49}$$

Second, if $\hat{x} \leq z_g \leq \overline{x}_2^R$ (rich area), then the profit of the firm is

$$\pi = p(\overline{x}_2^R - \overline{x}_2^L) - (y_2 - p - tz). \tag{50}$$

Set up a Lagrangian as

$$L = \pi + \lambda(y_2 - p - kz + t\overline{x}_2^L + k\overline{x}_2^L). \tag{51}$$

Solving $\frac{\partial L}{\partial p} \cdot p = 0$ yields

$$p = \frac{1}{2} y_2 + \frac{1}{4} t + \frac{1}{4} k. \tag{52}$$

Minimizing ATC such that

$$L = ATC + \lambda(\hat{x} - z) + \mu(z - \overline{x}_2^R), \quad \lambda \geq 0. \tag{53}$$

Solving $\frac{\partial L}{\partial z} \cdot z = 0$ and $\frac{\partial L}{\partial \lambda} \cdot \lambda = 0$ yields two solutions as

$$z_{g1} = \frac{2y_2 - t - k}{4t}, \quad z_{g2} = \frac{4y_1 - 2y_2 - t - k - 2\alpha y_2 + t\alpha + k\alpha}{4(1-\alpha)} = \hat{x}. \tag{54}$$

Without loss of generality, assume that $2\alpha y_2 - t - k > 0$ and recall $y_1 > \alpha y_2$ in (43), then

$$ATC_{z_{g2}} - ATC_{z_{g1}} = -\frac{k\,(y_1 - y_2)\,(2\,y_1 - 2\,y_2\,\alpha - t - k + \alpha\,t + \alpha\,k)}{t\,(-1+\alpha)^2\,(-2\,y_2 + t + k)} < 0 \quad (55)$$

Since $ATC_{z=\hat{x}}$ has been excluded in the analysis of $0 \leq z \leq \hat{x}$, the optimal location of the vendor is at $z = 0$ where minimized ATC.

Proposition 5. *The optimal location regulation is at* $z_g^* = 0$*, and then the average transportation cost is minimized as* $ATC_{z_g=0} = (2\alpha y_2 + (\alpha - 1)(t + k)/(8\alpha)$.

Table 1: The results of this paper

Status	1-group		2-group	
	Equilibrium	Optimum	Equilibrium	Optimum
z	$\frac{y}{2t}$	0	$\frac{y_2}{2t}$	0
p	$\frac{y}{2}$	$\frac{y}{2}+\frac{t}{4}+\frac{k}{4}$	$\frac{y_2}{2}$	$\frac{2\alpha y_2+t+k}{4\alpha}$
$\overline{x}^R$	$\frac{y}{2t}$	$\frac{2y-t-k}{4(t+k)}$	$\frac{y_2}{2t}$	$\frac{2\alpha y_2-t-k)}{4\alpha(t+k)}$
$\overline{x}^L$	$-\frac{y(t-k)}{2t(t+k)}$	$-\frac{2y-t-k}{4(t+k)}$	$-\frac{(y_2)(t-k)}{2t(t+k)}$	$-\frac{2\alpha y_2-t-k}{4\alpha(t+k)}$
π	$\frac{y^2}{2(t+k)}$	$\frac{(2y-t-k)^2}{8(t+k)}$	$\frac{(y_2)^2}{2(t+k)}$	$\%$
ATC	$\frac{y(t+k)}{4t}$	$\frac{y}{4}-\frac{t}{8}-\frac{k}{8}$	$\frac{(t+k)y_2}{4t}$	$\frac{2\alpha y_2-t-k)}{8\alpha}$

Note: $\% = \frac{4y_2^2\alpha^2+t^2+2tk+k^2-8y_1\alpha t-8y_1\alpha k+4y_2\alpha t+4y_2\alpha k}{8\alpha^2(t+k)}$.

Conclusions

1. AMM model is a special case of the current model.

2. Equilibrium location of a monopoly vender is at one of the city boundary, while optimal vendor location is at CBD.

3. Equilibrium land lent pattern is asymmetric, while

optimal land rent is symmetric.

4. In some case of y_1 and y_2, some poor people live in central city in AMM model, while they will be forced out of city in the current model.