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An Irreversible Change of Correlations in the US Equities Market and Difficulties in Using the Information*

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Abstract

This study statistically confirms the change of correlations across industries in the United States equities market in the financial crisis of 2007–2008. We use the regime-switching framework to identify the phenomenon and to study whether investors can use information about the structural change effectively in facing another crisis of that magnitude. To capture the irreversible structural change in the financial crisis and to separate it from the recurring boom–recession switches, we introduce two Markov chains and succeed in identifying these two different market movements. Our simulations of asset allocation demonstrate that the informed investor who knows the timing of the structural change can outperform uninformed investors from the viewpoint of mean-variance efficiency. However, our simulations also show that if the investor only assumes the possibility of the structural change and does not know its timing, then the result is not the same, which reveals the difficulty in detecting when the change actually occurs.

Key words: stock market returns; regime-switching models; structural change; correlation risk; EM algorithm; financial crisis.

JEL Classification: C22, G10

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1 Introduction

The global financial crisis in 2008 shows the importance of understanding correlations of financial instruments. During the catastrophic shock, prices of financial assets are reported to have moved together; correlations of asset returns, including those traditionally considered weak, rapidly increased. This phenomenon causes significant impacts on derivatives prices and hence, on risk management. Accordingly, as part of post-crisis study, there has been an increasing body of literature about measuring and managing correlation risk. To name just a few, Driessen, Maenhout and Vilkov (2009) study whether exposure to marketwide correlation shocks affects expected option returns, and Buraschi, Porchia and Trojani (2010) propose a new optimal portfolio choice model by allowing correlation across industries to be stochastic. Driessen et al. (2009) find that assets sensitive to higher marketwide correlations earn negative excess returns. Given the above studies, this study has two aims: the first is to identify the irreversible structural change in correlations across industries' returns in the United States (US) equities market, and the second is to test whether the information regarding a structural change can be effective in an investment decision.

As for the first goal, we note that the abovementioned studies suggest that the correlations among the returns of the financial instruments change after the global financial crisis. Since the global financial crisis started from the US, we focus on the equities market there. To capture the correlation changes, we use the Markov regime-switching model introduced by Hamilton (1989) in financial economics. More specifically, we propose a regime-switching model with two (mutually independent) Markov chains: one reversible and the other irreversible. The latter chain is used in an attempt to separate the possibly irreversible structural change in correlations from the ordinary regime switching induced by bull and bear markets.

Our empirical findings include (1) the reversible chain captures the shifts of the mean and variance of the individual industry indexes, which alone cannot explain the change in the correlation structure among the industries, (2) the irreversible change is estimated to have occurred between August 2007 and October 2007, which roughly coincides with the period when the financial crisis was becoming obviously imminent, and (3) there is clear evidence that the correlation across industries increased after that period. These results confirm our perceptions that the financial crisis in 2007–2008 changed the market structure.

As for the second goal, we test whether investors can use information of the structural change effectively in facing a crisis of comparable magnitude to the collapse in 2007–2008. To answer this question, we conduct Monte Carlo simulations of asset allocations. In our simulations, we set investors to optimize their mean-variance criteria since the real-life measure of investment efficiency is usually based on the mean-variance preference. Our hypothetical investors are characterized by their information levels, depending on which models they believe and how much information they have about regimes and correlations. If their performances are different, then the difference represents the benefits or costs of the information regarding regimes and structure.

By the simulations, we first confirm that the information about market regimes and structure is valuable: the informed investor who knows the true market model achieves better performance than less informed investors. The sample standard deviations of the informed investor's global minimum variance portfolio is the lowest of all the investment strategies. The sample Sharpe ratios of the informed investor's tangency portfolio is the greatest of all the investment strategies. Next, we conduct the test in the setting in which investors do not know the distribution parameters and current regimes and structure. Thus, they need to estimate repeatedly the distribution parameters and current regimes and structure at each time period. In contrast to the first test, the portfolios of the investor who believes in both reversible and irreversible chains underperformed in terms of both the standard deviation of the minimum variance portfolio and the Sharpe ratio of the tangency portfolio. To explain why this occurred, we measure the estimation errors of the timing of the structural change. They show that the investors do not always react correctly to the movement of structure. To investigate further whether the failure of the second test is caused by regime-switching estimation, we conduct Monte Carlo simulations based on the regime-switching models without structural changes. Nevertheless, we find that the uninformed investor achieves the best performance of all investor types. In this respect, Guidolin and Ria (2011) report that from the viewpoint of the Sharpe ratio, investors using the regime-switching models do not always outperform investors using other models. This is consistent with our results.

The rest of this section is devoted to a literature review related to our study. There is a large number of studies on the correlation in international stock markets. For example, Berben and Jansen (2005) investigate the correlations of international stock markets by fitting smooth transition generalized autoregressive heteroskedasticity (GARCH) models to weekly return data. They report correlations among the German, United Kingdom (UK), and US stock markets doubled in the period 1980–2000. However, the correlations between Japan and other markets did not change significantly in this period. Their results indicate that the continuous change in correlations can occur whereas our result shows that a drastic change in correlations occurred in the US equities market. Other studies include Karolyi and Stulz (1996), Ramchand and Susmel (1998), Das and Uppal (2004), and Bekaert, Hodrick and Zhang (2009).

The regime-switching model is introduced by Hamilton (1989) to capture sudden changes in economic time-series data. In the line of empirical studies, the literature using recursive regime-switching models suggests the existence of these two kinds of sources for changes of the market conditions, namely, recursive regime shifts and an irreversible change. Ang and Bekaert (2002) use a regime-switching model to identify recursive regime shifts in the international financial market. They succeed in reproducing the asymmetric correlation patterns reported by Longin and Solnik (2001). Okimoto (2008) develops the model of Ang and Bekaert (2002) and finds non-linear and asymmetric dependence patterns of financial assets in the market. Pettenuzzo and Timmermann (2011) find irreversible structural changes in the US financial market using the irreversible regime-switching model. Our analysis integrates these two types of regime-switching models.

The literature also reports the positive values of regime information. Ang and Bekaert (2002) simulate the economic effects of observable regime switching with the CRRA (constant relative risk aversion) utility. Their study of simulation using empirically estimated parameters indicates that there is a positive economic value when an investor takes into account regime switching in the international equities market. Guidolin and Timmermann (2007) extend Ang and Bekaert (2002) under unobservable regime switching in the US financial market. They report the existence of a positive economic value in the unobservable regime-switching market. Guidolin and Timmermann (2008a) consider the optimization problem of the higher-order preference, such as skewness and kurtosis. Tu (2010) applies the Bayesian approach to the mean-variance optimization problem. Other examples include Guidolin and Timmermann (2008b), Pettenuzzo and Timmermann (2011), and Guidolin and Ria (2011).

The advantage of using the regime-switching model includes that it may capture some moment properties, frequently observed in real markets, such as auto-correlation, volatility clustering, asymmetric correlation, non-normal skewness, and kurtosis (e.g., Timmermann (2000)). To study the effects of these properties in investment, many studies consider dynamic portfolio selections with regime switching. In theoretical approaches, Yin and Zhou (2004) study the discrete-time, finite horizon mean-variance preference optimization problem under the observable regime-switching settings. Honda (2003) considers the continuous-time consumption and investment problem with the power utility under the unobservable regime switching. However, since the settings in the abovementioned theoretical literature do not precisely match the settings in our simulations, so we use the one-period optimal portfolio.

The rest of this paper is organized as follows: Section 2 introduces our regime-switching model. Section 3 presents estimation methodology and results. The latter part consists of the following: first, we test if the reversible Markov chain captures a stationary component of asset returns; then, we estimate whether and when the irreversible structural change occurred. Given the estimated timing of the structural change, we compare means and covariance before and after the structural change. Section 4 shows the results of the asset allocation problem using the Monte Carlo simulation. Section 5 concludes, and some technicalities about the estimation procedure, data details, and model specification are provided in the Appendix.

2 Models

We consider a discrete-time, finite horizon regime-switching model, where t varies from 0 to T with $T > 0$ fixed. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space, which hosts a coupled Markov chain $Z = (Z_t)_{t=0}^T$ that explains regime switch and some stochastic process that drives the log-return process $Y := (Y_t)_{t=0}^T$. The variable Z_t indicates a regime at time t and we assume that an evolution of regime Z is described by a couple of two independent Markov chains $(S_t)_{t=0}^T$ and $(D_t)_{t=0}^T$, so that we denote $Z = (S, D)$.

The first component $(S_t)_{t=0}^T$ captures a reversible transition in the regimes of asset returns. We assume that S is a stationary Markov chain with two regimes $\{0, 1\}$ and the time-invariant transition probability matrix

$$\mathcal{P} \;:=\; \begin{pmatrix} \mathbb{P}(S_{t+1} = 0|S_t = 0) & \mathbb{P}(S_{t+1} = 1|S_t = 0) \\ \mathbb{P}(S_{t+1} = 0|S_t = 1) & \mathbb{P}(S_{t+1} = 1|S_t = 1) \end{pmatrix} = \begin{pmatrix} p_{00} & 1 - p_{00} \\ 1 - p_{11} & p_{11} \end{pmatrix},$$

where p_{00} and p_{11} are constant parameters to be estimated.

The second component $(D_t)_{t=0}^T$ captures an irreversible structural change in the asset returns. In particular, we assume that the structural change can occur only once. This can be modeled via a Markov chain consisting of two regimes $\{0, 1\}$, where $D_t = 0$ represents the regime before a structural change occurs and $D_t = 1$ represents the regime after the structural change. Accordingly, the transition probability matrix of D is time invariant and defined by

$$\mathcal{Q} := \begin{pmatrix} \mathbb{P}(D_{t+1} = 0|D_t = 0) & \mathbb{P}(D_{t+1} = 1|D_t = 0) \\ \mathbb{P}(D_{t+1} = 0|D_t = 1) & \mathbb{P}(D_{t+1} = 1|D_t = 1) \end{pmatrix} = \begin{pmatrix} q_{00} & 1 - q_{00} \\ 0 & 1 \end{pmatrix},$$

where q_{00} is a constant parameter to be estimated. By construction, $D_0 = 0$ and D_t cannot return to regime 0 once D_t moves, at some time t, to regime 1. Furthermore, for comparison, we also consider the model without any structural change, which is done by setting $q_{00} = 1$. Note that in this case, the model reduces to a conventional regime-switching model with Markov chain S. Hereafter, the model constrained with $q_{00} = 1$ is denoted by "Model $S2$" and the model without any constraint is denoted by "Model $S2D2$."

The process Y is a vector of log returns of N-industry stock indexes defined by

$$Y_t = \mu_{Z_t} + \Sigma_{Z_t}^{1/2} e_t,$$

where, for each $t \geq 0$, μ_{Z_t} is the N-dimensional vector, $\Sigma_{Z_t}^{1/2}$ is the $N \times N$ matrix, which satisfies $\Sigma_{Z_t}^{1/2} (\Sigma_{Z_t}^{1/2})^\top = \Sigma_{Z_t}$, and $(e_t)_{t=0}^T$ is an N-dimensional identically and independent distributed (i.i.d.) standard normal vector. The mean and covariance of process $(Y_t)_{t=1}^T$ are affected by the regime described by $(Z_t)_{t=1}^T$ in the following way:

$$\mu_{Z_t} = \sum_{s=0}^{1} \sum_{d=0}^{1} \mathbf{1}\{S_t = s, D_t = d\} \mu_{s,d}, \; \Sigma_{Z_t} = \sum_{s=0}^{1} \sum_{d=0}^{1} \mathbf{1}\{S_t = s, D_t = d\} \Sigma_{s,d}$$

where $\mu_{s,d}$ $(s = 0, 1, d = 0, 1)$ is a constant N-dimensional vector and $\Sigma_{s,d}$ $(s = 0, 1, d = 0, 1)$ is an $N \times N$ constant positive definite symmetric matrix.

By this formulation, the conditional mean and covariance of Y_t given Z_t are

$$E(Y_t|Z_t) = \mu_{Z_t}, \quad \text{Var}(Y_t|Z_t) = \Sigma_{Z_t}.$$

Accordingly, the distribution of Y_t given Z_t is

$$Y_t|Z_t \sim N(\mu_{Z_t}, \Sigma_{Z_t})$$

where $N(A, B)$ is the normal distribution with mean A and covariance B.

The marginal conditional distribution is

$$Y_t^j|Z_t \sim N(\mu_{Z_t}^j, (\sigma_{Z_t}^j)^2), \quad j = 1, \ldots, N$$

where μ_z^j and $(\sigma_z^j)^2$ are the jth component of μ_{Z_t} and the $j \times j$th-element of Σ_{Z_t}, respectively.

The standard assumptions about the dependence structure of the model are as follows:

Assumption 2.1 *Let $\mathcal{F}_t := \sigma\{Y_s : 0 \leq s \leq t\}$ be the σ-algebra generated by the log-return process Y.*

(i) S_u and D_v are independent for any pair of u and v; $0 \leq u, v \leq T$.

(ii) For any $t \geq 0$, given S_t, S_{t+1} is independent of $\mathcal{F}_t$, and given D_t, D_{t+1} is independent of $\mathcal{F}_t$.

We employ these assumptions in estimation; see Section 3 and the Appendix.

3 Estimation

3.1 Data

We estimate the models of Section 2 using monthly industry indexes. All the data are obtained from the "Thomson Reuters Datastream." We use the indexes of Standard & Poor's 500 sector total return indexes classified into 10 industries, following the Global Industry Classification Standard. They are the total return indexes calculated by the data source. Before estimation, we divide them to into six large groups and integrate indexes in each group[1]. How we integrate them is outlined in Appendix 6.2. We pick out the monthly data of the integrated indexes from the daily data and compute their monthly log-returns. Consequently, we obtain the monthly data of six large sector indexes. The monthly log-returns of the integrated indexes covered from February 1995 to December 2011, so it have a total of 203 data points. Table 1 shows the acronyms of the industries and the correspondence of the acronyms to the group of integration.

[1]The estimation using the original 10 indexes is very difficult because the 10-index model has many parameters. Their number is 263 when estimating our main model, whereas the number of parameters in the 6-index model is 111; 263 is much greater than the length of available monthly data. Thus, we integrate the index to reduce the parameters.

3.2 Method

By using the log return process Y of the integrate sector indexes, we estimate the two unknown components of the model. The first component is an evolution of the regimes, in which we can estimate only the probability of being at particular regimes of $Z = (S, D)$. The second component is the mean vector μ_{Z_t} and covariance matrix Σ_{Z_t}, which are distinct for each regime. We jointly estimate these components based on an iteration method. In particular, we use the expectation-maximization (EM) algorithm, as introduced by Dempster, Laird and Rubin (1977), after suitably making it fit our framework. By construction, the initial state is $Z_0 = (0, 0)$ or $Z_0 = (1, 0)$, and we denote by ρ_s the initial marginal probability of being at regime s, that is, $\mathbb{P}(S_0 = s)$.

Recall that $(\mathcal{F}_t)_{t=0}^T$ is the information of observable process $(Y_t)_{t=0}^T$ up to time t and let $\Theta^{(k)}$ be the candidate of parameters $\mathcal{P}$, $\mathcal{Q}$, $\mu_{s,d}$, $(s, d = 0, 1)$, $\Sigma_{s,d}$, $(s.d = 0, 1)$, and ρ_s, $(s = 0, 1)$ in the kth iteration of the EM algorithm. Following Hamilton (1990), the updating formulae for our model parameters in the $(k + 1)$th iteration are

$$\begin{aligned}
\mu_{s,d}^{(k+1)} &= \frac{\sum_{t=0}^T Y_t \mathbb{P}(S_t = s, D_t = d|\mathcal{F}_T; \Theta^{(k)})}{\sum_{t=0}^T \mathbb{P}(S_t = s, D_t = d|\mathcal{F}_T; \Theta^{(k)})} \\
\Sigma_{s,d}^{(k+1)} &= \frac{\sum_{t=0}^T (Y_t - \mu_{s,d}^{(k+1)})(Y_t - \mu_{s,d}^{(k+1)})^\top \mathbb{P}(S_t = s, D_t = d|\mathcal{F}_T; \Theta^{(k)})}{\sum_{t=0}^T \mathbb{P}(S_t = s, D_t = d|\mathcal{F}_T; \Theta^{(k)})} \\
p_{ss}^{(k+1)} &= \frac{\sum_{t=1}^T \mathbb{P}(S_t = s, S_{t-1} = s|\mathcal{F}_T; \Theta^{(k)})}{\sum_{t=1}^T \mathbb{P}(S_{t-1} = s|\mathcal{F}_T; \Theta^{(k)})} \\
q_{00}^{(k+1)} &= \frac{\sum_{t=1}^T \mathbb{P}(D_t = 0, D_{t-1} = 0|\mathcal{F}_T; \Theta^{(k)})}{\sum_{t=1}^T \mathbb{P}(D_{t-1} = 0|\mathcal{F}_T; \Theta^{(k)})} \\
\rho_s^{(k+1)} &= \mathbb{P}(S_0 = s) = \mathbb{P}(S_0 = s|\mathcal{F}_T; \Theta^{(k)}) \quad s = 0, 1,\ d = 0, 1,
\end{aligned}$$

where $\mathbb{P}(\,\cdot \mid \cdot\,, \Theta^{(k)})$ is the probability calculated under the parameter set $\Theta^{(k)}$. Hamilton (1990) shows if we repeat updating the parameters using these formulae, then the sequence of the parameters obtained by this algorithm converges, as $k \to \infty$, to the maximum likelihood estimators. The EM algorithm is based on the following probabilities of being at a particular regime:

$$\begin{aligned}
&\mathbb{P}(S_t = s, D_t = d|\mathcal{F}_T; \Theta^{(k)}) \\
&\mathbb{P}(S_t = s, S_{t-1} = \hat{s}|\mathcal{F}_T; \Theta^{(k)}) \\
&\mathbb{P}(D_t = d, D_{t-1} = \hat{d}|\mathcal{F}_T; \Theta^{(k)}) \\
&\mathbb{P}(S_0 = s|\mathcal{F}_T; \Theta^{(k)}),
\end{aligned}$$

which we estimate by the method Kim (1994) proposed. For the detail of the derivation, see the Appendix.

3.3 Results

The objective of this subsection is to investigate whether and when an irreversible structural change occurred and how it affected a stationary component of asset returns. In particular, we want to see how the irreversible change, if it exists, altered the correlation structure of asset returns.

3.3.1 Identifying the Markov chains (S, D)

We first want to test whether a two-state reversible regime-switching model reasonably captures a stationary component of asset returns.

The first figure in Figure 1 shows both National Bureau of Economic Research (NBER) recession dates (shaded regions) and the probability of being at regime $S_t = 0$ estimated by Model $S2$ (without a structural change). On the other hand, the second figure in Figure 1 replaces the latter probability with the one estimated by Model $S2D2$ (with a structural change) and adds the probability of being at regime $D_t = 0$. Both figures indicate nearly identical probability of the recursive state variable, which supports our assumption that the irreversible structural change in asset returns occurs independently of the stationary and reversible transition in asset returns. Furthermore, we conduct the Carrasco, Hu and Ploberger (2014) test (hereafter the CHP test) used widely for detecting the existence of Markov switching[2]. The result is reported in Table 2, where the test is performed sector by sector. The test statistics exceed 1% critical value except for the energy industry (ENE), so that the data indicate the existence of Markov switching structure. Given the above results, our two-state regime-switching model seems to capture a stationary component of asset returns.

Figure 1 also identifies a relationship between regime S_t and the economic environment in the US. Although regime $S_t = 0$ in Model $S2$ and regime $S_t = 0$ in Model $S2D2$ do not necessarily represent a period of the economic boom, the estimated probabilities of being at these regimes and a boom period in the US economy almost coincide after 2005. This suggests that the asset returns depend on the economic environment. Therefore, for simplicity, we call $S_t = 0$ a *boom regime* and $S_t = 1$ a *recession regime*.

Next, we estimate whether and when the irreversible structural change occurred. Table 3 shows the Akaike information criterion (AIC) statistics for the model with and without the irreversible structural change. These statistics imply that the model with the irreversible structural change fits the data better than the model without the irreversible structural change. Moreover, the AIC of Model $S2D2$ is lowest in our examined models (see Appendix 6.3). Thus, the data suggest the existence of the irreversible structural change.

In terms of the timing of the irreversible structural change, the dashed line in the second figure of Figure 1 reports the estimated probability of being at regime $D_t = 0$. The result implies that the irreversible structural change occurred between August 2007 and October 2007. This roughly corresponds to a period at the start of the financial crisis. Thus, the estimated irreversible structural change corresponds to the financial crisis.

To identify the abovementioned irreversible structural change, it is crucial to estimate the model by jointly using multi-sector returns. To observe this further, we report, in Figure 2, the probability of being at regime $D_t = 0$ obtained by estimating the model consisting of single-sector returns.

Although all the sector returns imply that the probability of being at regime $D_t = 0$ started to decline soon after the beginning of the data, the probability declined gradually. This means that we cannot infer the exact timing of the irreversible structural change via single-sector data. On the contrary, in our multi-sector estimation (see the second figure in Figure 1 again), the probability of being at regime $D_t = 0$ declined dramatically from 1 to 0 in a single period. This confirms that the multi-sector estimation is a key to infer the timing of the irreversible structural change.

[2]The CHP test is developed by Carrasco et al. (2014). The null hypothesis of this test is that a time series is not Markov switching.

3.3.2 Impacts of structural change on means and covariance

Table 4 summarizes means and volatilities of each sector returns for each regime of Models $S2$ and $S2D2$. We confirm that at the boom regime $S_t = 0$, means are relatively higher than the recession regime $S_t = 1$. Moreover, volatilities are higher in the recession regime than the boom regime. These relationships of parameters are consistent with the meanings of S_t explained in the previous subsection, that is, a business cycle.

To confirm this conjecture, we test the null hypothesis that means and volatilities in the boom regime are equal to those in the recession regime. These tests are the standard Wald-type tests for maximum likelihood estimators. As shown in Panel A of Table 5, we reject the null hypotheses at the 1% significant level except for a few tests. These results are statistical evidence that the reversible variable S captures booms and recessions in the US stock market. Similarly, the results of tests among different values of S in Model $S2$ (Panel C) show significant differences, although these results are weaker than in Model $S2D2$.

On the other hand, Panel B in Table 5 shows the results of tests whose hypotheses are that means and volatilities before the irreversible structural change are equal to those after the irreversible structural change. Contrary to the results with equality across different S, the significant difference of parameters across different D are not always found. In particular, the hypotheses that assume equality of means across different values of D in the boom regime ($S = 0$) are not rejected at the 10% significance level. Although the means in the boom regime, except for the consumer goods industry (CG), increase when the structural change occurs (see Table 4); however, these increases are not significant.

By contrast, the irreversible structural change has a stronger effect on the correlation structure across sector returns, which is summarized in Table 8. The full correlation coefficients are reported in Tables 6 and 7.

We test whether the recursive regime and structural change have increased or decreased correlations. In Model $S2D2$ of Table 8, many pairs among the indexes after the structural change are more strongly correlated than before the change in each recursive regime. In the boom regime $S = 0$, 13 pairs (87%) after the change became more strongly correlated than that before, whereas 11 pairs (73%) after the change became more strongly correlated than that before in the recession regime $S = 1$. By contrast, a clear relationship of the correlations between the different S is not found compared to that between the different D. These results imply that the structural change variable D affects the structure of correlations.

Furthermore, we examine the hypothesis testings for whether the correlation structure changes across the two regimes. As with the tests of means and volatilities, these tests are the Wald-type test. First, we consider the hypothesis testings that test the equality of correlations before and after the structural change. In the boom regime, the significant increases of the correlations between before and after the structural change are found in 11 pairs (73%) at the critical level 10%. However, only four pairs (27%) in the recession regime increase their correlations significantly when the structural change occurs, and three pairs (20%) of correlations in the same regime after the change are significantly lower than that before the change. Therefore, the structural change affects correlations in the boom regime more strongly than correlations in the recession regime.

Second, we consider the hypothesis testings that test the equality of correlations between the boom regime and the recession regime. Before the change, the correlations of 4 out of 15 pairs among the indexes in the recession regime are significantly higher than those in the boom regime at the 10% critical level, and no pair in the boom regime is found to be more significantly correlated than in the recession regime. This result implies higher correlation in the recession regime than in the boom regime before the structural change. On the contrary, after the change, 10 pairs (67%) are more strongly correlated in the boom regime than in the recession regime, and the differences in 4 pairs (27%) of them are significant at the 10% level.

However, only five pairs (33%) are more strongly correlated in the recession regime than in the boom regime and the differences in two pairs (13%) of them are significant. This result suggests the indexes are more strongly correlated in the boom regime than in the recession regime after the structural change.

See Tables 9 and 10 for the hypothesis testing results on correlation coefficients before and after the structural change, based on which we created the summary table (i.e., Table 8).

4 Asset allocation test for regime switching

To examine the effects of the information about regime switching and structural change, we consider asset allocation problems in the regime-switching market. We consider the global minimum variance portfolios and tangency portfolios and simulate the performances of these portfolios by Monte Carlo simulations. In the simulations, we consider two different market environments. One is the market in which investors know the market structure, namely, the distribution parameters and values of state variables. Another is the market in which investors do not know the market structure. Then, the investors need to estimate the market structure to invest rationally. In both of the simulations, we generate simulated time-series using the estimated parameters in Section 3. In the first simulation, we conclude that there are statistically significant advantages of the regimes and structure's information. However, the second simulation reveals that these advantages vanish in more realistic settings and indicates that there is a difficult implementation problem of regime-switching information. In subsection 4.1, we report the first test. The results of second simulation are shown in subsection 4.2. Subsection 4.3 checks the robustness of the results in the second subsection.

4.1 Values of information

In this subsection, there are five types of investor and we examine the performance of each investor's portfolio. We assume that the true market model is Model $S2D2$ in Section 2 with the parameters estimated in Section 3. The first type of investor knows the true model and true parameters, which are estimated in Section 3. She knows the true regime and structure of the market at every time and rebalances her portfolio in response to regime switches and structural change optimally. We call this rebalance scheme "$S2D2$." The second type of investor believes that the returns are generated by the regime-switching model based on only the recursive Markov chain, S, that is, Model $S2$ in Section 2. She knows the true state S_t and rebalances her portfolio in response to the regime shifts. To construct her portfolio, she uses the Model $S2$'s distribution parameters estimated in Section 3. We call the second type rebalance scheme "$S2$." The third type of investor believes that the initial regime will not change, but the structural change will occur. Therefore, she thinks the market model is Model $D2$ (see Appendix 6.3). She knows only the current value of the irreversible Markov chain, D, at each time and uses the estimated parameters of Model $D2$ to construct her portfolios. We call the third type rebalance scheme "$D2$." The remaining two types of investors are used as benchmarks. The fourth investor type considers that the regimes and structure will not change, and so, she thinks that the returns of indexes are i.i.d. Therefore, in order to construct her portfolios, she uses the sample mean and variance of the actual data that we used in Section 3. We call her "IID." The fifth type of investor adopts the equally weighted portfolio at every time, so her portfolio is always $\mathbf{1}_6/6$. We call this strategy "EW."

In summary,

1. $S2D2$: The investor knows that the true market model is Model $S2D2$. She knows the current regime and structure at each time and uses the Model $S2D2$'s parameters estimated in Section 3 to construct her portfolios.

2. $S2$: The investor believes that the market model is Model $S2$ (only recursive regime shifts). She knows the current regime at each time and uses the parameters Model $S2$'s parameters estimated in Section 3 to construct her portfolios.

3. $D2$: The investor believes that the market model is Model $D2$ (only irreversible structural change). She knows the current structure at each time and uses the parameters Model $D2$'s parameters estimated in Section 6.3 to construct her portfolios.

4. IID: The investor believes that the returns of vector are i.i.d. and uses the sample mean and variance of the actual data used in Section 3 to construct her portfolios.

5. EW: The investor always adopts the equally weighted portfolio.

Next, we consider the construction of the investors' portfolios. We should consider the dynamic portfolio selections based on the mean-variance criteria; however, they cannot be implemented easily[3]. Therefore, we assume that in every time, the investors optimize their 1-month objectives as well as Markowitz (1952) and Merton (1972). We denote the return vector of the type I investor at time t by R^I_t. Now, we consider the five types of investor $I = S2D2, S2, D2$,IID, and EW; however, the EW portfolio is $\mathbf{1}_6/6$, and so, we consider only the cases of $S2D2, S2, D2$, and IID.

Let $\phi_{I,t} = (\phi_{I,t,1}, \cdots, \phi_{I,t,6})^\top$ be a portfolio of the type I investor at time t. Then, the return of the portfolio ϕ^I_t is

$$R^I_{t+1} = \sum_{i=1}^{6} \phi_{I,t,i} \left(\exp\left\{ Y^{(i)}_{t+1} \right\} - 1 \right),$$

where $Y^{(i)}_{t+1}$ is the ith component of the log-return vector Y_{t+1}. The investors' information differs. We denote by $\mathcal{F}^I_t$ the information of the type I investor. The investor chooses the two portfolios that minimize the variance of return and that maximize the Sharpe ratio at each time with a short-selling constraint.

The portfolio minimizing the variance at time t is the solution of the following minimization problem.

$$\begin{aligned} &\min_{\phi_{I,t}} \mathrm{Var}(R^I_{t+1} | \mathcal{F}^I_t) \\ &\text{subject to } \sum_{i=1}^{6} \phi_{I,t,i} = 1, \quad \phi_{I,t,i} \geq 0 \text{ for all } i = 1, \ldots, 6. \end{aligned}$$

We call the solution the "global minimum variance portfolio," according to the literature. Furthermore, the solution of the abovementioned problem depends only on the values of S_t and D_t, does not depend on their past values before time $t-1$, and so, it is sufficient to compute the four portfolios with four different values of S and D at most in order to consider the dynamic portfolio selection.

The portfolio maximizing the Sharpe ratio at time t is the solution of the following maximization problem.

$$\begin{aligned} &\max_{\phi_{I,t}} \frac{E(R^I_{t+1} | \mathcal{F}^I_t)}{\sqrt{\mathrm{Var}(R^I_{t+1} | \mathcal{F}^I_t)}} \\ &\text{subject to } \sum_{i=1}^{6} \phi_{I,t,i} = 1, \quad \phi_{I,t,i} \geq 0 \text{ for all } i = 1, \ldots, 6, \end{aligned}$$

[3] A straightforward multi-period, mean-variance optimization problem is time inconsistent in the sense that the dynamic programming principle does not hold. See Li and Ng (2000).

where we assume that the risk-free rate is 0. Similarly to the global minimum variance portfolio, the solution of the abovementioned problem depends only on the value of S_t and D_t. Therefore, we need to compute the four portfolios maximizing the Sharpe ratios the most. We call the solution of the abovementioned problem the "tangency portfolio."

Since the prices of indexes have the mixed log-normal distributions, the conditional expectation value of return R^{S2D2}_{t+1} of the type $S2D2$ investor given by $S_t = s, D_t = d$ is,

$$E(R^{S2D2}_{t+1}|S_t = s, D_t = d) = \phi^\top_{S2D2,t}\Big(g_{s,d}(\mu, \Sigma) - \mathbf{1}_6\Big),$$

where $\mathbf{1}_6$ is the six-dimensional vector whose all elements are 1, and where $g_{s,d}$ is a $\mathbb{R}^6$-valued function defined as follows,

$$\begin{aligned} g_{s,d}(\mu, \Sigma) &:= \sum_{s',d'} P^{s,d}_{s',d'} \begin{pmatrix} \exp\{\mu^1_{s',d'} + \sigma^1_{s',d'}/2\} \\ \vdots \\ \exp\{\mu^6_{s',d'} + \sigma^6_{s',d'}/2\} \end{pmatrix}, \\ P^{s,d}_{s',d'} &:= \mathbb{P}(S_1 = s', D_1 = d' \mid S_0 = s, D_0 = d). \end{aligned}$$

$\mu^i_{s',d'}$ is the ith component of the vector $\mu_{s',d'}$ and $\sigma^i_{s',d'}$ is the $i \times i$th element of the matrix $\Sigma_{s',d'}$. $\mu_{s,d}$ and $\Sigma_{s,d}$ are defined in Section 2. The conditional variance of her return R^{S2D2}_{t+1} given by $S_t = s, D_t = d$ is

$$\mathrm{Var}(R^{S2D2}_{t+1}|S_t = s, D_t = d) = \phi^\top_{S2D2,t} H_{s,d}(\mu, \Sigma)\phi_{S2D2,t},$$

where $H_{s,d}$ is the $\mathbb{R}^{6\times 6}$-valued function defined as follows,

$$\begin{aligned} [H_{s,d}(\mu, \Sigma)]_{i,j} &= \sum_{s',d'} P^{s,d}_{s',d'} \exp\left\{\mu^i_{s',d'} + \mu^j_{s',d'} + \frac{1}{2}(\sigma^i_{s',d'} + \sigma^j_{s',d'} + \sigma^{i,j}_{s',d'})\right\} \\ &\quad - \left(\sum_{s',d'} P^{s,d}_{s',d'} \exp\left\{\mu^i_{s',d'} + \frac{1}{2}\sigma^i_{s',d'}\right\}\right)\left(\sum_{s',d'} P^{s,d}_{s',d'} \exp\left\{\mu^j_{s',d'} + \frac{1}{2}\sigma^j_{s',d'}\right\}\right), \\ &\quad i, j = 1, \ldots, 6. \end{aligned}$$

$[H_{s,d}(\mu, \Sigma)]_{i,j}$ is the $i \times j$th element of the matrix $H_{s,d}(\mu, \Sigma)$ and $\sigma^{i,j}_{s',d'}$ is the $i \times j$th element of the matrix $\Sigma_{s',d'}$.

The partially informed investors also consider that similar moments of returns have conditioned their information. The type $S2$ investor uses the conditional mean $E[R^{S2}_{t+1}|S_t = s]$ and variance $\mathrm{Var}(R^{S2}_{t+1}|S_t = s)$ to solve the optimization problem. These moments are computed using the estimated parameters of Model $S2$ in Section 3. Similarly, the type $D2$ investor uses the conditional mean $E[R^{D2}_{t+1}|D_t = d]$ and variance $\mathrm{Var}(R^{D2}_{t+1}|D_t = d)$, which are computed using the estimated parameter of Model $D2$. The type IID investor uses the sample mean and variance of the actual data as the one-step-ahead mean and variance in order to solve her optimization problems.

Now, we estimate the performances of these portfolios. We generate the log-return processes of Model $S2D2$ by the Monte Carlo method using the parameters estimated in Section 3. The rebalancing interval of the investors' portfolios is 1 month. The investment horizons are 140 months. We examine the simulations in the initial state $S_0 = 0$ or 1 and the initial structure $D_0 = 0$ or 1. The number of trials is 10,000. Therefore, we simulate 10,000 independent data series with the 140 months. In each trial, nine sequences of the returns are computed—the global minimum variance portfolios and tangency portfolios of $S2D2$, $S2$, $D2$, and IID, and the equally weighted portfolio (EW). We compute the sample means, standard deviations, skewness, kurtosis, and Sharpe ratios of those nine return sequences during 140 months in each simulation trial and measure the performances of the investment strategies by the means of these statistics over the trials.

Tables 12 and 13 show the simulation results. The performances of the type $S2D2$ investor are the best of the investors in all the initial regimes and structures. The mean of sample standard deviations of the global minimum variance portfolio of $S2D2$ at each initial regime and structure is the smallest of all the strategies and the mean of the sample Sharpe ratios of the tangency portfolio of $S2D2$ is the largest of them. In each initial regime and structure, the 90% credible interval of the Sharpe ratio of the $S2D2$'s tangency portfolio does not overlap the 90% credible intervals of the Sharpe ratios of other portfolios. This implies that investing in the tangency portfolio of $S2D2$ is statistically significantly efficient. Furthermore, when the initial regime and structure start at $S = 0$ and $D = 1$, the mean of the Sharpe ratios of the $S2D2$'s tangency portfolios is positive, however the means of the Sharpe ratios of other portfolios are negative. When the initial regime and structure start at $S = 1$ and $D = 1$, all of the means of the Sharpe ratios of the tangency portfolios are negative, and so, these Sharpe ratios may not represent the efficiency of the investment performances. However, since the mean of $S2D2$'s tangency portfolio is the highest and the standard deviation is the smallest of the tangency portfolios, we can conclude that the tangency portfolio of $S2D2$ is the most effective portfolio among them. These results indicate that the knowledge of the actual market model and the values of state variables brings a positive effect in investment.

Next, we consider whether the regimes or the structures are important for investment. Tables 12 and 13 report that the investment strategies based on $D2$ are more effective than those based on $S2$. In all the initial regimes and structures, the means of the standard deviations of the $D2$'s global minimum variance portfolios are smaller than those of the $S2$'s portfolios and the means of the Sharpe ratios of the $D2$'s tangency portfolios are larger than those of the $S2$'s portfolios. For instance, in the case in which the initial regime is 0 and the initial structure is 0, the mean of the standard deviations of the $D2$'s global minimum variance portfolio is 3.519, whereas the mean of the standard deviations of the $S2$'s global minimum variance portfolio is 3.550. The mean of the Sharpe ratios of the $D2$'s tangency portfolio is 0.235, however the mean of the Sharpe ratios of the $S2$'s portfolios is 0.201. Similar results appear in the other initial regimes and structures. These results suggest that the structural information is more important than the regime information. In addition, the tangency portfolios of $S2$ and $D2$ underperform the IID tangency portfolios. For instance, in the case of the initial regime 1 and structure 1, the difference of the means of the tangency between $S2$ and IID is 0.731, and this is statistically significant in the sense that their 99% credible intervals do not overlap each other. Moreover, the mean of $S2$ (-1.857) is smaller than the IID (-1.126) and the standard deviation of $S2$ (6.178) is larger than the IID (5.187). Thus, it is clear that the tangency portfolio of $S2$ is less efficient than the IID in the mean-variance sense when the initial regime is 1 and the initial structure is 1. $D2$'s tangency portfolio also is less efficient than IID in the mean-variance sense. These results suggest that the IID investor's tangency portfolio is more mean-variance efficient than the partially informed investors' tangency portfolios.

Figure 3 shows the means of the standard deviations of global minimum variance portfolios and the Sharpe ratios of tangency portfolios. The horizontal line represents the length of months investing portfolios. In all the initial regimes and structures, the standard deviation of the global minimum variance portfolios of $S2D2$ is the smallest of the other portfolios in the whole of the investment horizon. Similarly, the Sharpe ratio of the tangency portfolios of $S2D2$ is always the highest of the other portfolios in all the initial conditions. These imply that the advantages of full information exist at various investment horizons.

Table 14 shows the weights of the global minimum variance portfolios and tangency portfolios. The weight of the consumer goods industry (CG) of the type $S2D2$ investor's global minimum variance portfolio decreases after the structural change, but the type $D2$ investor increases the weight of CG after the structural change and this weight is very high (more than 60%). On the other hand, the type $S2D2$ and $S2$ investors have similar tangency portfolios before the structural change. In the boom regime, they invest their wealth mainly in the indexes of the energy industry (ENE), CG, and the utilities industry (U). In the recession regime, they invest almost all their wealth in ENE. However, after the structural change, the type $S2D2$ investor decreases the weight of ENE and increases CG in each regime. These portfolio differences among the investors confirm that the partially informed investors (type $S2$ and $D2$ investors) cannot react to the change of the market condition correctly.

4.2 Rolling estimation: limitation of the regime-switching model with the structural change

Next, we simulate the performances of the investors in the market in which they cannot access the distribution parameters and the information of the market regimes and structure. However, the advantages of the knowledge of the market conditions that we observed in subsection 4.1 vanish in the test of this subsection.

As well as subsection 4.1, we consider the four investor types, namely, type $S2D2, S2$, IID, and EW. However, in contrast to the settings of subsection 4.1, they do not know the distribution parameters and the movement of the state variables, S and D. Therefore, they apply the assumed models to the observed data repeatedly in their investment horizon to construct their portfolios. The estimation methods are the same as those in this study (see Section 3 and Appendix). The IID investor constructs her portfolios using the sample means and variances of the past simulated returns. The EW investor always invests in the equally weighted portfolio, $\mathbf{1}_6/6$. The type $S2D2, S2$, and IID investors construct the two portfolios, the global minimum variance portfolio and the tangency portfolio. In subsection 4.1, the portfolios depend only on the values of state variables; however, the portfolios in this simulation vary through time, since the estimated parameters change when the investors observe the new return data. Finally, we consider seven investment strategies, the global minimum variance portfolios and tangency portfolios of $S2D2, S2$, and IID, and the equally weighted portfolio (EW).

In summary,

1. $S2D2$: The investor knows that the true market model is Model $S2D2$. She repeatedly estimates the Model $S2D2$'s parameters to construct her portfolios.

2. $S2$: The investor believes that the market model is Model $S2$ (only recursive regime shifts). She repeatedly estimates the parameters of Model $S2$ to construct her portfolios.

3. IID: The investor believes that the returns of vector are i.i.d. and repeatedly computes the sample mean and variance of the simulated sample to construct her portfolios.

4. EW: The investor always adopts the equally weighted portfolio.

Unlike the simulation in subsection 4.1, we do not consider the investor who believes Model $D2$ in this simulation. This is because it is numerically difficult to compute the optimal portfolios of Model $D2$. More specifically, we fail to compute the optimal portfolios of Model $D2$ in many trials because of the singularities of the estimated variance matrixes. One explanation is that it is rare to capture changes of market conditions in early periods since Model $D2$ needs the long-run data. Furthermore, the purpose of our analysis is to investigate the values of the information of the structural change. In this respect, the difference between

$S2D2$ and $S2$ is more important than the difference between $S2D2$ and $D2$ since the difference between $S2D2$ and $S2$ represents the effect of the irreversible Markov chain D. Therefore, we omit the simulations of $D2$.

We simulate samples of the returns based on Model $S2D2$ using the parameters estimated in Section 3. In each simulation trial, the investors do not invest in the first 60 months and they start to invest after these months passed. However, the data in the first 60 months are used for each investor to estimate parameters. The length of the total investment horizons is 140 months, and so, we simulate the 200-month data series of the indexes in each trial. The investors have to estimate the market models and compute the portfolio repeatedly, and so, this simulation is computationally intensive. Therefore, we decrease the number of simulation trials to 2,000. As a result of the abovementioned simulation plan, we simulate 2,000 time series of 200 monthly return data. We compute the sample statistics of the portfolios' returns—means, standard deviations, skewness, kurtosis, and the Sharpe ratios in each simulation trial. Therefore, we obtain the 2,000 statistics of the portfolios' returns during 140 months and take means of these statistics over the trials. The initial regime of S is generated randomly with its steady-state probabilities. The initial structure is $D = 0$.

Panel A in Table 15 shows the results of the Model $S2D2$ simulation. Unlike the simulation in the case in which investors can use the information of values of the state variables, the performance of type $S2D2$ is the worst of the strategies except for the EW portfolio. The mean of the standard deviations of $S2D2$'s global minimum portfolios is the largest of the global minimum variance portfolios and the mean of the Sharpe ratios of $S2D2$'s tangency portfolios is the lowest of the tangency portfolios. Notable differences appear in their variances. In both the global minimum variance portfolios and tangency portfolios, the differences of the means of the standard deviation over the trials between $S2D2$ and the other models are statistically significant in the sense that the 90% credible intervals do not overlap each other. These results imply that there are not any advantages of knowledge of the market model in the optimization of the mean-variance criterion, even though Ang and Bekaert (2002) and Guidolin and Timmermann (2007) find positive economic values of knowledge in the optimization of expected utilities.

In this simulation, the major change from subsection 4.1 is that each informed investor estimates the parameters and states repeatedly, whereas the investors in subsection 4.1 believe certain parameters a priori and know the current state at each time. Therefore, the estimation errors of the parameters and the states movement exist in this simulation. It is possible that the estimation errors cannot be ignored in the asset allocation with rolling estimation. Indeed, Panel A in Table 15 shows that the type $S2$ investor who needs to estimate the moderately complicated model underperforms the type IID investor while she outperforms the type $S2D2$ investor who needs to estimate the most complicated model. Another explanation of the disadvantage of knowledge is that the regime-switching framework will not work under the mean-variance criterion while it works under the expected utility criterion (see Ang and Bekaert (2002) and Guidolin and Timmermann (2007)). The rest of this subsection measures the estimation errors and the next subsection tests whether the regime-switching frameworks works.

To measure the degree of the estimation errors, we compute the gaps between the actual and estimated structural changing time in the Model $S2D2$ simulation. As mentioned earlier in this subsection, we fail to compute the Model $D2$'s portfolios in this simulation. Therefore, it is natural to focus on the estimation error of the timing of the structural change. The type $S2D2$ investor estimates the parameters and smoothed probabilities at each time. In the bth trial, the estimated smoothed probability at time T with which the structural change had not occurred before the time $t \leq T$ is denoted by $p^b_{t,T}$, that is,

$$p^b_{t,T} = \mathbb{P}(D_t = 0 \mid \mathcal{F}^b_T),$$

where $\mathcal{F}_T^b$ is the type $S2D2$ investor's information at time T in the bth trial. Let τ^b be the structural change time in bth trial. We define the mean of the smoothed probabilities around the structural change as

$$\overline{p}_{i,T} = \sum_{\{b \mid \tau^b < \min\{200,T\}\}} \frac{p^b_{\tau^b+i,T}}{\text{The number of elements of } \{b \mid \tau^b < \min\{200,T\}\}}.$$

If the type $S2D2$ investor estimates the market model correctly, it can be expected that $\overline{p}_{i,T}$ decreases as i increases and a large gap between $\overline{p}_{-1,T}$ and $\overline{p}_{0,T}$ exists.

Figure 4 displays the means of the smoothed probabilities around the structural change in the Model $S2D2$ simulation. As well as our expectation, $\overline{p}_{i,T}$ is decreasing at i and there is a large gap between $\overline{p}_{-1,T}$ and $\overline{p}_{0,T}$ for all $T = 100, 150, 199$. However, its quantity remains more than 0.5 after the structural change, that is, $\overline{p}_{i,T} > 0.5$ holds for $i > 0$. This means that the estimation procedure detects the structural change after the actual structural change occurs in most cases and that there is a limit to what the type $S2D2$ investor estimates as the time of the structural change.

However, it is possible that the early structural change causes the limitation. Since the time of structural change has a geometric distribution, the structural change tends to occur at the early time. Thus, in most of the trials, it is possible that the $S2D2$ investor cannot identify the structural change. Therefore, we condition the means of the smoothed probabilities around the structural change to the time of the structural change. Computing them, we use only the trials in which the structural change occurs during the periods from $t = 51$ to $t = 150$, that is, we compute

$$\overline{p}^c_{i,T} := \sum_{\{b \mid 51 \le \tau^b \le 150\}} \frac{p^b_{\tau^b+i,T}}{\text{The number of elements of } \{b \mid 51 \le \tau^b \le 150\}}.$$

Under the condition of the timing of the structural change, the type $S2D2$ investor observes many returns' data before and after the structural change. In addition, Figure 4 shows the changes of $\overline{p}^c_{i,T}$ at $T = 199$. As with the unconditional case, $\overline{p}^c_{i,T}$ is decreasing at i and a large gap between $i = -1$ and 0 appears. Moreover, the gap of $\overline{p}^c_{i,T}$ is larger than that of the unconditional means. However, its quantity is also maintained at more than 0.5 after the structural change. Thus, limits of estimation still exist.

4.3 Rolling estimation: limitation of regime-switching models without a structural change

To study the other explanation, we conduct three asset allocation tests of regime-switching models with a recursive Markov chain only. In subsection 4.2, we find that the informed investor does not have an advantage compared to the uninformed investor from the views of mean-variance efficiency and that it is difficult to identify the timing of the structural change. However, it is possible that the mean-variance optimization will not work, even if only recursive Markov chains change the market environment. Thus, we focus on the regime-switching models with a recursive Markov chain only. We first conduct the simulation in which the true market model is Model $S2$, that is, the market obeys movement of one recursive Markov chain. To confirm robustness, we conduct two simulations under the settings of the regime-switching model literature.

We conduct the simulation in which the simulated market model is Model $S2$. In the simulation of Model $S2$, we simulate the sample of return vectors using the estimated parameters of Model $S2$, but the other settings do not change from the simulation of Model $S2D2$. In the simulation of Model $S2$, the type $S2$ investor correctly understands the true market model.

Panel B in Table 16 reports the results of the simulation of Model $S2$. Similar to subsection 4.2, the type $S2$ investor underperforms the IID investor in both the global minimum variance portfolios and tangency portfolios. The mean of the standard deviations of $S2$'s global minimum variance portfolios is greater than the mean of the standard deviations of IID's global minimum variance portfolios. Moreover, the mean of the Sharpe ratios of $S2$'s tangency portfolios is lower than the mean of the Sharpe ratios of IID's tangency portfolios. Furthermore, the 90% credible interval of the Sharpe ratios of $S2$'s tangency portfolios does not overlap the 90% credible interval of the Sharpe ratios of IID's tangency portfolios. It follows that the difference of the Sharpe ratios between $S2$ and IID's tangency portfolios is statistically significant at the 10% significance level. However, the type $S2$ investor outperforms the type $S2D2$ investor in this simulation.

To confirm the robustness of these results, we conduct two additional simulations using the parameters and settings of the other literature. Using the estimation results of the regime-switching models in Ang and Bekaert (2002) and Guidolin and Timmermann (2007), we simulate the samples of the data process that they report. Ang and Bekaert (2002) estimate US, UK, and German equity indexes in various models. We choose the regime-switching model with two recursive states and simulate it. On the other hand, Guidolin and Timmermann (2007) identify three assets in the US—a small stock index, a large stock index, and 10-year T-bonds—as the four recursive states' regime-switching model, and so, we simulate the time-series using their results. In both simulations, we assume there are four types of investors, recursive 4 states, and recursive 2 states, IID and EW. The type of 4 states (resp. 2 states) investor believes that the number of market regimes is 4 (resp. 2). The IID investor believes that the returns of assets are i.i.d. The EW investor always invests in the equally weighted portfolio $\mathbf{1}_6/6$. As well as the former simulation, they do not know the distribution parameters and movement of the regimes, and they estimate their believed models repeatedly. The initial running periods are 200 months in the simulation of Ang and Bekaert (2002) and 288 months in the simulation of Guidolin and Timmermann (2007). The investment periods are 300 months in the simulation of Ang and Bekaert (2002) and 312 months in the simulation of Guidolin and Timmermann (2007). To compute the tangency portfolios and the sample Sharpe ratios, we use a positive risk-free rate in both simulations. In the simulation of Ang and Bekaert (2002), the monthly risk-free rate is 0.0041, and the monthly risk-free rate in the simulation of Guidolin and Timmermann (2007) is 0.0044. These risk-free rates are based on these studies. In both of the simulations, there are 2,000 trials and the initial regime in each trial is generated randomly with the steady probabilities.

Table.16 reports the results of the simulation based on the literature. In the case of Ang and Bekaert (2002), the investment strategy of the global minimum variance portfolios based on the true model (2 states) achieves the smallest mean of the standard deviation among all the strategies, but the difference is not statistically significant. The mean of the Sharpe ratios when the investor invests in the tangency is lower than that of the IID. In the case of Guidolin and Timmermann (2007), both of the standard deviations of the global minimum variance portfolios and the Sharpe ratios of the tangency portfolios are worse than the other strategies. In both simulations, the portfolios based on the market models do not outperform the IID portfolios in most cases.

5 Concluding remarks

In this study, we identify the change in correlation structure across industries in the US market in the financial crisis of 2007–2008 and find limitations of the use of regime-switching information. To capture the change in the global financial crisis, we use the regime-switching model with both a reversible Markov chain and an irreversible Markov chain, and succeed in separating the two dynamics in the market, namely, the recursive regime shifts and the irreversible structural change.

The recursive regime shifts represent the dynamics of the marginal distribution parameters of the returns, that is, means and variances. These two recursive regimes are the boom and recession regimes. In the boom regime, the indexes have high (conditionally) expected returns and low volatilities, whereas they have low expected returns and high volatility in the recession regime. Furthermore, the probability with which the market is in the recession regime remains at a high degree at the NBER announced recession periods. By contrast, the irreversible structural change represents the dynamics of the joint distribution parameters, that is, correlations. Most pairs of correlations among the industries increase in each regime after the change. The irreversible change is not found in the estimation of the time-series of the single-industry index. Moreover, the timing of the structural change almost coincides with the financial crisis in 2007–2008.

In the simulations of asset allocation, we find positive values for use of the regime-switching information if the investor knows the true parameters and current states. The global minimum variance portfolios of the informed investor achieve the smallest variance in the investors and their tangency portfolio also achieves the highest Sharpe ratio. Moreover, these advantages appear in the various investment horizons and initial regimes and structures. However, the investors cannot exploit the structural change from the viewpoints of mean-variance efficiency if they only know the kind of the true market model. The performances of the global minimum portfolios and tangency portfolios of the informed investor are worse than those of the uninformed investors.

6 Appendix

6.1 The EM algorithm

Based on Hamilton (1989) (1990) and Kim (1994), we explain the EM algorithm that we use in this study. For notational simplicity, we consider the case of $q_{00} = 1$, so Model $S2$, since the extension to the general case is straightforward. First, we introduce the *full-information* likelihood function, which means we hypothesize that we can observe unobserved variables. For the random vectors, $Y = (Y_0, \cdots, Y_T)$ and $S = (S_0, \cdots, S_T)$, the full-information likelihood function is

$$\begin{aligned} f(Y, S; \Theta) &= \prod_{t=0}^{T} \left(\sum_{s=0}^{1} \mathbf{1}\{S_t = s\} f(Y_t | \mathcal{F}_{t-1}, S_t = s; \Theta) \right) \\ &\times \prod_{t=1}^{T} \left(\sum_{s=0}^{1} \sum_{\widehat{s}=0}^{1} \mathbf{1}\{S_t = s, S_{t-1} = \widehat{s}\} \mathbb{P}(S_t = s | S_{t-1} = \widehat{s}) \right) \\ &\times \sum_{s=0}^{1} \mathbf{1}\{S_1 = s\} \mathbb{P}(S_1 = s) \end{aligned}$$

where $\mathcal{F}_t$ is the σ-algebra generated by Y up to time t and Θ is the distribution parameters and the conditional density $f(Y_t | \mathcal{F}_{t-1}, S_t = s; \Theta)$ is

$$f(Y_t | \mathcal{F}_{t-1}, S_t = s; \Theta) = \frac{1}{\sqrt{(2\pi)^N \det \Sigma_s}} \exp \left\{ -\frac{1}{2} (Y_t - \mu_s)^\top \Sigma_s^{-1} (Y_t - \mu_s) \right\}.$$

For the "expectation" step, we start with the initial parameter set $\Theta^{(0)}$ to compute

$$\begin{aligned} Q(\Theta;\Theta^{(0)}) &= E^{\Theta^{(0)}}[\log f(Y,S;\Theta)|\mathcal{F}_T] \\ &= \sum_{t=0}^{T}\sum_{s=0}^{1}\mathbb{P}(S_t=s|\mathcal{F}_T;\Theta^{(0)})\log f(Y_t|\mathcal{F}_{t-1},S_t=s;\Theta) \\ &+ \sum_{t=1}^{T}\sum_{s=0}^{1}\sum_{\widehat{s}=0}^{1}\mathbb{P}(S_t=s,S_{t-1}=\widehat{s}|\mathcal{F}_T;\Theta^{(0)})\log\mathbb{P}(S_t=s|S_{t-1}=\widehat{s}) \\ &+ \sum_{s=0}^{1}\mathbb{P}(S_1=s|\mathcal{F}_T;\Theta^{(0)})\log\mathbb{P}(S_1=s). \end{aligned}$$

Next, we search the parameters maximizing $Q(\Theta;\Theta^{(0)})$ function,

$$\Theta^{(1)} = \arg\max_{\Theta\in\overline{\Theta}} Q(\Theta;\Theta^{(0)})$$

where $\overline{\Theta}$ is the parameter space of this model. This step is the "maximization" step. We then continue these steps from $k = 1, 2, \ldots$.

Hamilton (1990) shows if we repeat these steps to infinity, then the sequence of the parameters obtained by this algorithm converges to the maximum likelihood estimators. According to Hamilton (1990), the updating formulae of the parameters in the $(k+1)$th iteration are

$$\begin{aligned} \mu_s^{(k+1)} &= \frac{\sum_{t=0}^{T} Y_t\mathbb{P}(S_t=s|\mathcal{F}_T;\Theta^{(k)})}{\sum_{t=0}^{T}\mathbb{P}(S_t=s|\mathcal{F}_T;\Theta^{(k)})} \quad s=0,1 \\ \Sigma_s^{(k+1)} &= \frac{\sum_{t=0}^{T}(Y_t-\mu_s^{(k+1)})(Y_t-\mu_s^{(k+1)})^\top\mathbb{P}(S_t=s|\mathcal{F}_T;\Theta^{(k)})}{\sum_{t=0}^{T}\mathbb{P}(S_t=s|\mathcal{F}_T;\Theta^{(k)})} \quad s=0,1 \\ p_{ss}^{(k+1)} &:= \mathbb{P}(S_{t+1}=s|S_t=s;\Theta^{(k+1)}) = \frac{\sum_{t=1}^{T}\mathbb{P}(S_t=s,S_{t-1}=s|\mathcal{F}_T;\Theta^{(k)})}{\sum_{t=1}^{T}\mathbb{P}(S_{t-1}=s|\mathcal{F}_T;\Theta^{(k)})} \quad s=0,1 \qquad (6.1) \\ \rho_s^{(k+1)} &:= \mathbb{P}(S_0=s;\Theta^{k+1}) = \mathbb{P}(S_1=s|\mathcal{F}_T;\Theta^{(k)}) \quad s=0,1. \end{aligned}$$

These updating formulae are obtained by the first-order conditions of the maximization of $Q(\Theta;\Theta^{(0)})$ with respect to Θ.

By considering the formulae (6.1), we observe that we need to compute

(a) $\mathbb{P}(S_t=s|\mathcal{F}_T;\Theta^{(k)}), t=0,\cdots,T$ and

(b) $\mathbb{P}(S_t=s,S_{t-1}=s|\mathcal{F}_T;\Theta^{(k)}), t=1,\cdots,T$,

which are called *smoothed probabilities*.

Let us suppose that we have estimated up to the kth iteration and explain how to update to the $(k+1)$th estimates. Now, we perform the following:

Forward calculation: Assume further that we have obtained $\mathbb{P}(S_{t-1}=s|\mathcal{F}_{t-1};\Theta^{(k)})$, then we have, for the next time step t,

$$\mathbb{P}(S_t=s|\mathcal{F}_{t-1};\Theta^{(k)}) = \sum_{\widehat{s}=0}^{1}\mathbb{P}(S_t=s|S_{t-1}=\widehat{s})\mathbb{P}(S_{t-1}=\widehat{s}|\mathcal{F}_{t-1};\Theta^{(k)}), \quad s=0,1. \qquad (6.2)$$

By using this, we compute $\mathbb{P}(S_t = s|\mathcal{F}_t;\Theta^{(k)}), s = 0,1$ in the following way: By Bayes' rule, we obtain

$$
\begin{aligned}
(6.3)\qquad \mathbb{P}(S_t = s|\mathcal{F}_t;\Theta^{(k)}) &= \mathbb{P}(S_t = s|Y_t,\mathcal{F}_{t-1};\Theta^{(k)}) \\
&= \frac{\mathbb{P}(Y_t \in \mathrm{d}y|S_t = s,\mathcal{F}_{t-1};\Theta^{(k)})}{\mathbb{P}(Y_t \in \mathrm{d}y|\mathcal{F}_{t-1};\Theta^{(k)})}\mathbb{P}(S_t = s|\mathcal{F}_{t-1};\Theta^{(k)}) \\
&= \frac{f(Y_t|S_t = s,\mathcal{F}_{t-1};\Theta^{(k)})\mathbb{P}(S_t = s|\mathcal{F}_{t-1};\Theta^{(k)})}{\sum_{\widehat{s}=0}^{1} f(Y_t|S_t = \widehat{s},\mathcal{F}_{t-1};\Theta^{(k)})\mathbb{P}(S_t = \widehat{s}|\mathcal{F}_{t-1};\Theta^{(k)})}, \quad s = 0,1
\end{aligned}
$$

where $\mathbb{P}(S_t = s|\mathcal{F}_t;\Theta^{(k)})$ is called the *filtered probability.* As a proxy for the filtered probability at $t = 0$, we use $\mathbb{P}(S_0 = s;\Theta^{(k)})$. We then repeat this procedure *forwards* up to time T. In other words, we obtain the whole set of probabilities (6.2) and (6.3) for $t = 0,\dots,T$. Recall k is still fixed.

<u>Backward calculations:</u> For computing (a) and (b), first we use Bayes' rule to write

$$
\begin{aligned}
&\mathbb{P}(S_t = s, S_{t+1} = \widehat{s}|\mathcal{F}_T;\Theta^{(k)}) \\
&\quad = \mathbb{P}(S_t = s|S_{t+1} = \widehat{s},\mathcal{F}_T;\Theta^{(k)})\mathbb{P}(S_{t+1} = \widehat{s}|\mathcal{F}_T;\Theta^{(k)}) \\
&\quad \approx \mathbb{P}(S_t = s|S_{t+1} = \widehat{s},\mathcal{F}_t;\Theta^{(k)})\mathbb{P}(S_{t+1} = \widehat{s}|\mathcal{F}_T;\Theta^{(k)}) \\
&\quad = \frac{\mathbb{P}(S_{t+1} = \widehat{s}|S_t = s,\mathcal{F}_t;\Theta^{(k)})\mathbb{P}(S_t = s|\mathcal{F}_t;\Theta^{(k)})}{\mathbb{P}(S_{t+1} = \widehat{s}|\mathcal{F}_t;\Theta^{(k)})}\mathbb{P}(S_{t+1} = \widehat{s}|\mathcal{F}_T;\Theta^{(k)}) \\
(6.4)\qquad &\quad = \frac{\mathbb{P}(S_{t+1} = \widehat{s}|S_t = s;\Theta^{(k)})\mathbb{P}(S_t = s|\mathcal{F}_t;\Theta^{(k)})}{\mathbb{P}(S_{t+1} = \widehat{s}|\mathcal{F}_t;\Theta^{(k)})}\mathbb{P}(S_{t+1} = \widehat{s}|\mathcal{F}_T;\Theta^{(k)})
\end{aligned}
$$

where in the last line we use Assumption 2.1-(ii). We compute *backwards* starting with $t = T-1$ down to $t = 0$. All the probabilities in the last line are known[4] and hence, we obtain

$$
(6.5)\qquad \mathbb{P}(S_t = s|\mathcal{F}_T;\Theta^{(k)}) = \sum_{\widehat{s}=0}^{1} \mathbb{P}(S_t = s, S_{t+1} = \widehat{s}|\mathcal{F}_T;\Theta^{(k)}).
$$

for all $t = T,\cdots,1$. The resulting probabilities in (6.5) and (6.4) for $t = 0,\cdots,T$ are the smoothed probabilities (a) and (b), respectively. Plugging (a) and (b) into the recursive formulae for parameter estimation above, we have updated for the $(k+1)$th iteration. Note that the smoothed probabilities, which are used in the updating formulae for parameter estimation, have rich information since they estimate the probabilities of being in a certain regime at time t by using the full observations.

6.2 Data details and integration

There are two steps to compute the integrated indexes: the first step is to compute price-based integrated indexes, the second step is to compute total integrated indexes, that is, the indexes, including aggregate daily dividends.

[4]Except for the last one, we already have the whole set of probabilities ($t = 0\cdots T$) in (6.4): Namely, $\mathbb{P}(S_{t+1} = \widehat{s}|S_t = s;\Theta^{(k)})$ is $p_{ss}^{(k)}$ obtained in the kth iteration, and $\mathbb{P}(S_t = s|\mathcal{F}_t;\Theta^{(k)})$ and $\mathbb{P}(S_{t+1} = \widehat{s}|\mathcal{F}_t;\Theta^{(k)})$ are from (6.2) and (6.3), respectively. Finally, for the last one, if we set $t = T-1$, then $\mathbb{P}(S_T = \widehat{s}|\mathcal{F}_T;\Theta^{(k)})$ is known again by (6.2). With all these, we obtain (6.5) at $t = T-1$, that is, $\mathbb{P}(S_{T-1} = s|\mathcal{F}_T;\Theta^{(k)})$. This is then plugged for the next time step $t = T-2$ into (6.4).

The first step is as follows. $\mathbb{J}$ denotes the set of some industries, which is the group we want to integrate. To summarize the indexes of $\mathbb{J}$ into one index, we compute a capitalization-weighted average index of $\mathbb{J}$, denoted by $P_t^{\mathbb{J}}$:

$$P_t^{\mathbb{J}} = \frac{\sum_{j\in\mathbb{J}} M_t^j}{\sum_{j\in\mathbb{J}} AM_{t-1}^j} P_{t-1}^{\mathbb{J}}, \quad t \geq 1, \quad P_0^{\mathbb{J}} = 100,$$

where M_t^j and AM_t^j are the usual and adjusted market value of j-industry index at time t; "adjusted" means that its variable takes into account capital actions of firms. By the data source, the industry index P_t^j is constructed as follows,

$$P_t^j = \frac{\sum_k n_{k,t}^j p_{k,t}^j}{\sum_k n_{k,t}^j p_{k,t-1}^j Adj_{k,t}^j} P_{t-1}^j, \quad t \geq 1, \quad P_0^j = 100$$

where $n_{k,t}^j$ and $p_{k,t}^j$ are the number of shares in issue and unadjusted price of firm k, which constitutes the j industry's index at time t. $Adj_{k,t}^j$ is the adjusted factor of firm k at time t. This adjusts the capital action of firm k at time t. We define the adjusted market value of j industry's index as

$$AM_{t-1}^j := \frac{P_{t-1}^j}{P_t^j} M_t^j = \sum_k n_{k,t}^j p_{k,t-1}^j Adj_{k,t}^j.$$

Then, AM_{t-1}^j is the adjusted market value taking capital actions of firms into consideration. In this manner, we compute the integrated indexes.

In the second step, we need to compute aggregate daily dividends of the industries' indexes. The Thomson Reuters Datastream provides a total return index in each industry. The total return index of industry j, denoted by RI^j, is defined as

$$RI_t^j = RI_{t-1}^j \frac{P_t^j}{P_{t-1}^j} \left(1 + \frac{DY_t^j}{100\Delta}\right), \quad t \geq 1, \quad RI_0^j = 100,$$

where DY_t^j is the aggregate dividend yield of industry i at time t (expressed as a percentage) and where Δ is a certain number of days in a financial year (normally 260). By the definition of the total index, we compute the aggregate dividend yield (expressed as a real number),

$$\frac{DY_t^j}{100} = \Delta \left(\frac{RI_t^j}{RI_{t-1}^j} \frac{P_{t-1}^j}{P_t^j} - 1 \right).$$

The dividend yield is defined as

$$\frac{DY_t^j}{100} = \frac{\sum_k n_{k,t}^j d_{k,t}^j}{\sum_k n_{k,t}^j p_{k,t}^j} = \frac{\sum_k n_{k,t}^j d_{k,t}^j}{M_t^j},$$

where $d_{k,t}^j$ is the dividend per share of firm k at time t. Of course, firm k constitutes j industry's index. Therefore, the aggregate dividend of industry j at time t is

$$\frac{Div_t^j}{100} = M_t^j \frac{DY_t^j}{100} = M_t^j \Delta \left(\frac{RI_t^j}{RI_{t-1}^j} \frac{P_{t-1}^j}{P_t^j} - 1 \right) = \sum_k n_{k,t}^j d_{k,t}^j.$$

The dividend yield of the industry group $\mathbb{J}$ at time t, denoted by $DY_t^{\mathbb{J}}$, is

$$\frac{DY_t^{\mathbb{J}}}{100} = \frac{\sum_{j\in\mathbb{J}} Div_t^j/100}{\sum_{j\in\mathbb{J}} M_t^j} = \frac{\sum_{j\in\mathbb{J}} \sum_k n_{k,t}^j d_{k,t}^j}{\sum_{j\in\mathbb{J}} M_t^j}.$$

Finally, the total integrated index of the industry group $\mathbb{J}$ is

$$RI_t^{\mathbb{J}} = RI_{t-1}^{\mathbb{J}} \frac{P_t^{\mathbb{J}}}{P_{t-1}^{\mathbb{J}}} \left(1 + \frac{DY_t^{\mathbb{J}}}{100\Delta}\right), \quad t \geq 1, \quad RI_0^{\mathbb{J}} = 100.$$

To confirm the validity of the abovementioned integration method, we compute the S&P 500 indexes integrating the all-sector indexes. Following the above method, we can compute the S&P 500 index when the all-industry indexes are in one group. Our integrated indexes are adjusted as if the initial values are the same values as the S&P 500 indexes on the initial day (January 23, 1995). The root mean square of the difference between our integrated price index and the S&P 500 price index is 0.366. For the total return index, the root mean square is 0.409. This indicates that the abovementioned method is a valid method to integrate different industries' indexes.

We compute the daily data of the integrated six indexes using the original 10 daily indexes and extract the monthly data from these daily data. The daily data covered the period from January 23, 1995 to December 30, 2011. Thus, the monthly data covered the period from January 1995 to December 2011.

6.3 Specification of regime number

To determine a number of regimes, we apply some regime-switching models to the integrated indexes. The models are as follows,

1. two recursive regimes without structural change model (Model $S2$),
2. two recursive regimes with once structural change model (Model $S2D2$),
3. three recursive regimes without structural change model (Model $S3$),
4. four recursive regimes without structural change model (Model $S4$),
5. once structural change without recursive regime model (Model $D2$),
6. twice structural changes without recursive regime model (Model $D3$),
7. two recursive state variables that have two regimes without structural change model (Model $S2S2$).

The first two models are those this study is interested in. The seventh model, Model $S2S2$, needs to be explained. It is regarded as the version of Model $S2D2$ in which D is a recursive state variable, that is, this model allows D to change regime 0 from regime 1. Table 17 shows the AICs of these seven models.

The AIC of Model $S2D2$ is lowest among the models, which justifies focus on Model $S2D2$. Comparing Models $S2D2$ and $S2S2$, the difference of AIC between these models is 2, and thus, depends only on the parameter penalties. Indeed, these two models are not different except for numerical errors. Figure 5 shows the smoothed probabilities in each model.

The probabilities of the two models are almost the same in Figure 5. Moreover, the root mean squares of the difference of probabilities between the two models are 1.636×10^{-6} with the probability being $S = 0$ and 8.648×10^{-8} with the probability being $D = 0$. It is interesting that Model $S2S2$ captures the irreversible structural change although all the state variables in the model are reversible. This result is evidence that the irreversible structural change occurs.

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Table. 1: Abbreviations of names of industries

Integrate group	Sector Name
ENE	Energy
MAT/IND	Materials
	Industrials
CG	Consumer Discretionary
	Consumer Staples
	Health Care
FIN	Financials
IT/Tel	Information and Technology
	Telecoms services
U	Utilities

Table. 2: The results of the Carrasco et al. (2014) test (the CHP test). We try 3000 times parametric bootstraps. The null hypothesis is that the mean and variance do not change with the progress of time. The alternative is that they are driven by a recursive Markov chain.

	statistics	10% critical values	5% critical values	1% critical values
ENE	0.015	0.015	0.019	0.028
MAT/IND	0.035	0.015	0.018	0.028
CG	0.035	0.015	0.018	0.028
FIN	0.058	0.015	0.018	0.028
IT/Tel	0.055	0.015	0.019	0.028
U	0.039	0.015	0.019	0.028

Table. 3: The AICs of Model $S2$ and Model $S2D2$. The AICs of the other models are in Table. 17.

	Model $S2$ ($q_{00} = 1$)	Model $S2D2$ ($0 < q_{00} < 1$)
AIC	6684.234	6619.640

Table. 4: The estimated means and standard deviations of each industry returns. The first table is the estimation result of Model $S2D2$. The second is the result of Model $S2$. μ_{sd} and σ_{sd} are the conditional mean and standard deviation of the returns of the individual industry index in the regime s and the structure d, respectively. $p_{ss'}$ is the transition probability of S_t from the regime s to the regime s' and $q_{dd'}$ is also the transition probability of D_t from the structure d to d'. Numbers in parenthesis are standard errors.

Model $S2D2$

regimes	$D=0$				$D=1$			
	$S=0$		$S=1$		$S=0$		$S=1$	
	μ_{00}	σ_{00}	μ_{10}	σ_{10}	μ_{01}	σ_{01}	μ_{11}	σ_{11}
ENE	2.034	4.368	0.067	5.800	1.564	6.327	-7.187	7.496
	(0.452)	(0.395)	(0.140)	(0.620)	(1.018)	(0.690)	(2.566)	(1.127)
MAT/IND	1.577	3.255	-0.229	6.038	1.680	6.162	-9.091	7.754
	(0.335)	(0.226)	(0.453)	(0.547)	(1.039)	(0.631)	(2.540)	(0.140)
CG	1.447	2.792	-0.160	4.476	1.303	3.477	-4.218	6.084
	(0.293)	(0.214)	(0.299)	(0.452)	(0.581)	(0.317)	(1.962)	(0.227)
FIN	1.811	3.860	-0.030	7.181	0.711	7.525	-12.853	12.364
	(0.416)	(0.306)	(0.083)	(0.945)	(1.268)	(0.793)	(3.980)	(1.378)
IT/Tel	1.719	4.159	-0.853	11.019	1.404	5.548	-6.226	6.424
	(0.430)	(0.266)	(1.457)	(0.858)	(0.891)	(0.529)	(2.051)	(0.852)
U	1.492	3.325	-0.534	6.380	1.301	3.177	-5.074	5.770
	(0.345)	(0.252)	(0.702)	(0.542)	(0.487)	(0.366)	(2.038)	(0.727)

probability parameters

p_{00}	$1-p_{00}$	q_{00}	$1-q_{00}$	log likelihood
0.984	0.016	0.993	0.007	
(0.001)		(0.004)		-3198.820
$1-p_{11}$	p_{11}	$1-q_{11}$	q_{11}	
0.034	0.966	0	1	
	(0.004)			

Model $S2$

regimes	$S=0$		$S=1$	
	μ_0	σ_0	μ_1	σ_1
ENE	1.823	4.919	-0.500	6.708
	(0.426)	(0.379)	(0.691)	(0.624)
MAT/IND	1.499	4.194	-0.919	7.089
	(0.373)	(0.298)	(0.837)	(0.656)
CG	1.424	2.985	-0.517	4.829
	(0.278)	(0.183)	(0.579)	(0.417)
FIN	1.384	4.679	-1.291	9.595
	(0.451)	(0.323)	(1.165)	(0.988)
IT/Tel	1.617	4.385	-1.217	10.374
	(0.398)	(0.246)	(1.189)	(0.739)
U	1.487	3.229	-0.963	6.266
	(0.285)	(0.206)	(0.714)	(0.482)

probability parameters

p_{00}	$1-p_{00}$
0.983	0.017
(0.012)	
$1-p_{11}$	p_{11}
0.030	0.970
	(0.020)

log likelihood

-3286.117

Table. 5: The result of the hypothesis testing: the null hypotheses are the equality of means and that of standard deviations across the two regimes. Panel A and B are the results of tests in Model $S2D2$ and Panel C is that in Model $S2$. The hypotheses in Panel A are same value of S but different value of D. On the other hand, the hypotheses in Panel B are same D and different S. Rows of industries are the tests of individual industries. Rows of "All" are the tests of which hypothesis is the equalities of parameters of all the industries. The degrees of freedom in each industries test are 1 and those in all the industries are 15. Numbers in parenthesis are probability values.

Panel A: D is the same among the hypotheses but S is different in the Model $S2D2$.

fixed regime	before structural change ($D = 0$)			after structural change ($D = 1$)		
hypothesis	equal means	equal volatilities	equal both	equal means	equal volatilities	equal both
ENE	17.197	3.537	26.026	9.870	0.753	10.056
	(0.000)	(0.060)	(0.000)	(0.002)	(0.386)	(0.007)
MAT/IND	10.220	21.811	31.722	15.045	6.084	24.347
	(0.001)	(0.000)	(0.000)	(0.000)	(0.014)	(0.000)
CG	15.027	10.938	22.348	7.232	45.327	46.964
	(0.000)	(0.001)	(0.000)	(0.007)	(0.000)	(0.000)
FIN	18.693	11.090	27.653	10.480	9.315	13.924
	(0.000)	(0.001)	(0.000)	(0.001)	(0.002)	(0.001)
IT/Tel	2.827	58.729	59.086	11.284	0.732	11.481
	(0.093)	(0.000)	(0.000)	(0.001)	(0.392)	(0.003)
U	6.750	26.474	34.418	9.197	9.463	23.935
	(0.009)	(0.000)	(0.000)	(0.002)	(0.002)	(0.000)
All	37.346	96.440	138.942	50.815	74.492	140.987
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Panel B: S is the same among the hypotheses but D is different in the Model $S2D2$.

fixed regime	booming regime ($S = 0$)			recession regime ($S = 1$)		
hypothesis	equal means	equal volatilities	equal both	equal means	equal volatilities	equal both
ENE	0.178	6.074	6.087	8.003	1.832	9.597
	(0.673)	(0.014)	(0.048)	(0.005)	(0.176)	(0.008)
MAT/IND	0.009	18.724	18.743	11.746	9.408	26.341
	(0.926)	(0.000)	(0.000)	(0.001)	(0.002)	(0.000)
CG	0.048	3.178	3.181	4.130	10.248	12.522
	(0.826)	(0.075)	(0.204)	(0.042)	(0.001)	(0.002)
FIN	0.669	18.501	24.050	10.282	9.952	13.869
	(0.414)	(0.000)	(0.000)	(0.001)	(0.002)	(0.001)
IT/Tel	0.100	5.429	5.449	4.560	13.943	24.289
	(0.752)	(0.020)	(0.066)	(0.033)	(0.000)	(0.000)
U	0.100	0.113	0.337	4.545	0.501	4.547
	(0.751)	(0.737)	(0.845)	(0.033)	(0.479)	(0.103)
All	4.333	26.082	41.806	51.338	50.097	131.850
	(0.632)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Panel C: $S = 0$ vs. $S = 1$ in the Model $S2$.

hypothesis	equal means	equal volatilities	equal both		equal means	equal volatilities	equal both
ENE	8.066	5.766	12.485	FIN	4.437	22.895	23.366
	(0.005)	(0.016)	(0.002)		(0.035)	(0.000)	(0.000)
MAT/IND	6.772	16.142	19.838	IT/Tel	4.951	59.704	60.887
	(0.009)	(0.000)	(0.000)		(0.026)	(0.000)	(0.000)
CG	8.936	16.238	19.384	U	10.132	33.611	39.973
	(0.003)	(0.000)	(0.000)		(0.001)	(0.000)	(0.000)
All	15.796	114.514	125.268				
	(0.015)	(0.000)	(0.000)				

Table. 6: The estimated correlation coefficients in Model $S2$. Numbers in parenthesis are standard errors. The summary is shown in Table. 8.

booming regime ($S = 0$)

	ENE	MAT/IND	CG	FIN	IT/Tel	U
ENE	1					
	-					
MAT/IND	0.608	1				
	(0.068)	-				
CG	0.420	0.787	1			
	(0.079)	(0.031)	-			
FIN	0.461	0.788	0.791	1		
	(0.086)	(0.036)	(0.032)	-		
IT/Tel	0.445	0.753	0.726	0.662	1	
	(0.072)	(0.036)	(0.042)	(0.050)	-	
U	0.334	0.326	0.491	0.397	0.314	1
	(0.074)	(0.072)	(0.062)	(0.067)	(0.080)	-

recession regime ($S = 1$)

	ENE	MAT/IND	CG	FIN	IT/Tel	U
ENE	1					
	-					
MAT/IND	0.668	1				
	(0.072)	-				
CG	0.524	0.800	1			
	(0.098)	(0.043)	-			
FIN	0.491	0.804	0.805	1		
	(0.101)	(0.040)	(0.044)	-		
IT/Tel	0.365	0.622	0.560	0.471	1	
	(0.088)	(0.061)	(0.076)	(0.068)	-	
U	0.652	0.458	0.361	0.359	0.143	1
	(0.068)	(0.100)	(0.100)	(0.093)	(0.119)	-

Table. 7: The estimated correlation coefficients in Model $S2D2$. Numbers in parenthesis are standard errors. The summary is shown in Table. 8.

before structural change ($D = 0$)

booming regime ($S = 0$)

	ENE	MAT/IND	CG	FIN	IT/Tel	U
ENE	1					
	-					
MAT/IND	0.389	1				
	(0.081)	-				
CG	0.232	0.721	1			
	(0.086)	(0.047)	-			
FIN	0.196	0.685	0.800	1		
	(0.086)	(0.052)	(0.040)	-		
IT/Tel	0.262	0.718	0.670	0.610	1	
	(0.087)	(0.050)	(0.053)	(0.071)	-	
U	0.341	0.240	0.448	0.420	0.186	1
	(0.091)	(0.101)	(0.084)	(0.090)	(0.102)	-

recession regime ($S = 1$)

	ENE	MAT/IND	CG	FIN	IT/Tel	U
ENE	1					
	-					
MAT/IND	0.677	1				
	(0.075)	-				
CG	0.501	0.719	1			
	(0.100)	(0.062)	-			
FIN	0.595	0.740	0.792	1		
	(0.084)	(0.055)	(0.066)	-		
IT/Tel	0.283	0.582	0.498	0.444	1	
	(0.112)	(0.074)	(0.092)	(0.090)	-	
U	0.590	0.369	0.260	0.368	0.025	1
	(0.087)	(0.129)	(0.118)	(0.120)	(0.143)	-

after structural change ($D = 1$)

booming regime ($S = 0$)

	ENE	MAT/IND	CG	FIN	IT/Tel	U
ENE	1					
	-					
MAT/IND	0.754	1				
	(0.066)	-				
CG	0.671	0.885	1			
	(0.084)	(0.029)	-			
FIN	0.599	0.856	0.780	1		
	(0.102)	(0.038)	(0.044)	-		
IT/Tel	0.709	0.825	0.821	0.710	1	
	(0.072)	(0.039)	(0.039)	(0.076)	-	
U	0.391	0.480	0.625	0.377	0.701	1
	(0.142)	(0.096)	(0.067)	(0.102)	(0.074)	-

recession regime ($S = 1$)

	ENE	MAT/IND	CG	FIN	IT/Tel	U
ENE	1					
	-					
MAT/IND	0.438	1				
	(0.126)	-				
CG	0.325	0.973	1			
	(0.163)	(0.013)	-			
FIN	-0.094	0.769	0.835	1		
	(0.142)	(0.070)	(0.083)	-		
IT/Tel	0.408	0.828	0.804	0.518	1	
	(0.200)	(0.117)	(0.127)	(0.165)	-	
U	0.888	0.569	0.439	-0.025	0.427	1
	(0.036)	(0.100)	(0.161)	(0.152)	(0.219)	-

Table. 8: The summary table for changes of correlations at 10% significant level. Numbers in parentheses are percentages of all pairs.

Model $S2$

	correlation in $S = 0 < S = 1$	correlation in $S = 0 > S = 1$
Number of changes	7 (47%)	8 (53%)
Number of significant changes	3 (20%)	1 (7%)

Model $S2D2$

Fix S.

$S = 0$	correlation in $D = 0 > D = 1$	correlation in $D = 0 < D = 1$
Number of changes	2 (13%)	13 (87%)
Number of significant changes	0 (0%)	11 (73%)

$S = 1$	correlation in $D = 0 > D = 1$	correlation in $D = 0 < D = 1$
Number of changes	4 (27%)	11 (73%)
Number of significant changes	3 (20%)	4 (27%)

Fix D.

$D = 0$	correlation in $S = 0 > S = 1$	correlation in $S = 0 < S = 1$
Number of changes	8 (53%)	7 (47%)
Number of significant changes	0 (0%)	4 (27%)

$D = 1$	correlation in $S = 0 > S = 1$	correlation in $S = 0 < S = 1$
Number of changes	10 (67%)	5 (33%)
Number of significant changes	4 (27%)	2 (13%)

Table. 9: The results of the hypothesis testings in Model $S2D2$: the null hypothesis is the equality of correlations between two regimes $S = 0$ and $S = 1$. Marks $*$, $**$, and $***$ indicate rejecting the null at significance level 10%, 5%, and 1%, respectively. Numbers in parentheses are P-values. The summary is shown in Table. 8.

Null hypothesis: correlations are equal between $S = 0$ and $S = 1$

before structural break ($D = 0$)

	ENE	MAT/IND	CG	FIN	IT/Tel
MAT/IND	4.458				
	0.035				
CG	3.162*	8.015***			
	(0.075)	(0.005)			
FIN	6.114**	0.639	0.267		
	(0.013)	(0.424)	(0.606)		
IT/Tel	2.033	0.001	0.017	1.174	
	(0.154)	(0.978)	(0.897)	(0.279)	
U	11.134***	0.415	1.309	2.743*	1.502
	(0.001)	(0.520)	(0.253)	(0.098)	(0.220)
all correlations are equal (df = 15)			70.494***	(0.000)	

after structural break ($D = 1$)

	ENE	MAT/IND	CG	FIN	IT/Tel
MAT/IND	4.852**				
	(0.028)				
CG	3.217*	0.000			
	(0.073)	(0.987)			
FIN	9.131***	0.465	0.009		
	(0.003)	(0.495)	(0.923)		
IT/Tel	0.008	1.096	1.611	1.072	
	(0.928)	(0.295)	(0.204)	(0.300)	
U	3.249*	0.331	0.905	0.081	0.195
	(0.071)	(0.565)	(0.341)	(0.775)	(0.659)
all correlations are equal (df = 15)			29.976**	(0.012)	

Table. 10: The results of the hypothesis testing in Model $S2D2$: the null hypothesis is the equality of correlations between two regimes $D = 0$ and $D = 1$. Marks $*$, $**$, and $***$ indicate rejecting the null at significance level 10%, 5%, and 1%, respectively. Numbers in parentheses are P-values. The summary is shown in Table. 8.

Null hypothesis: correlations are equal between $D = 0$ and $D = 1$

booming regime ($S = 0$)

	ENE	MAT/IND	CG	FIN	IT/Tel
MAT/IND	12.323***				
	(0.000)				
CG	14.004***	8.852***			
	(0.000)	(0.003)			
FIN	9.152***	6.941***	0.118		
	(0.002)	(0.008)	(0.731)		
IT/Tel	15.775***	2.863*	5.434**	0.888	
	(0.000)	(0.091)	(0.020)	(0.346)	
U	0.090	2.994*	2.785*	0.097	16.936***
	(0.764)	(0.084)	(0.095)	(0.756)	(0.000)
all correlations are equal (df = 15)			54.622***	(0.000)	

recession regime ($S = 1$)

	ENE	MAT/IND	CG	FIN	IT/Tel
MAT/IND	2.729*				
	(0.099)				
CG	0.848	15.766***			
	(0.357)	(0.000)			
FIN	17.21***	0.100	0.161		
	(0.000)	(0.752)	(0.688)		
IT/Tel	0.324	3.504*	3.982**	0.152	
	(0.569)	(0.061)	(0.046)	(0.697)	
U	11.275***	1.224	0.673	3.137*	2.483
	(0.001)	(0.269)	(0.412)	(0.077)	(0.115)
all correlations are equal (df = 15)			123.992***	(0.000)	

Table. 11: The results of the hypothesis testings in Model $S2$: the null hypothesis is the equality of correlations between two regimes $S = 0$ and $S = 1$. Marks $*$, $**$, and $***$ indicate rejecting the null at significance level 10%, 5%, and 1%, respectively. Numbers in parentheses are P-values. The summary is shown in Table. 8.

Null hypothesis: correlations are equal between $S = 0$ and $S = 1$

	ENE	MAT/IND	CG	FIN	IT/Tel
MAT/IND	12.323***				
	(0.000)				
CG	14.004***	8.852***			
	(0.000)	(0.003)			
FIN	9.152***	6.941***	0.118		
	(0.002)	(0.008)	(0.731)		
IT/Tel	15.775***	2.863*	5.434**	0.888	
	(0.000)	(0.091)	(0.020)	(0.346)	
U	0.090	2.994*	2.785*	0.097	16.936***
	(0.764)	(0.084)	(0.095)	(0.756)	(0.000)
all correlations are equal (df = 15)			54.622***	(0.000)	

Table. 12: The simulation results of the performances with various investment strategies. Numbers in parentheses are the standard deviations of them. All of the means and the standard deviations, and their standard errors in this table are multiplied by 100. Number of trials is 10000. Each trial has 140 observations.

	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio
Initial state			$S = 0,\ D = 0$		
Global minimum variance portfolios					
$S2D2$	0.616	3.501	-0.329	3.559	0.237
	(0.010)	(0.008)	(0.004)	(0.009)	(0.003)
$S2$	0.631	3.550	-0.304	3.536	0.233
	(0.010)	(0.008)	(0.004)	(0.008)	(0.003)
$D2$	0.658	3.519	-0.314	3.558	0.245
	(0.010)	(0.008)	(0.004)	(0.009)	(0.003)
IID	0.607	3.557	-0.295	3.514	0.225
	(0.010)	(0.008)	(0.004)	(0.008)	(0.003)
Tangency portfolios					
$S2D2$	0.798	3.962	-0.328	4.115	0.258
	(0.010)	(0.009)	(0.005)	(0.013)	(0.003)
$S2$	0.495	4.386	-0.421	4.378	0.201
	(0.014)	(0.013)	(0.006)	(0.016)	(0.003)
$D2$	0.652	3.857	-0.333	3.651	0.235
	(0.011)	(0.009)	(0.004)	(0.010)	(0.003)
IID	0.681	3.832	-0.241	3.463	0.239
	(0.011)	(0.009)	(0.003)	(0.008)	(0.003)
EW	0.491	4.458	-0.239	3.627	0.184
	(0.014)	(0.012)	(0.004)	(0.009)	(0.003)
Initial state			$S = 1,\ D = 0$		
Global minimum variance portfolios					
$S2D2$	0.313	3.717	-0.302	3.464	0.136
	(0.011)	(0.007)	(0.003)	(0.008)	(0.003)
$S2$	0.335	3.762	-0.280	3.447	0.136
	(0.010)	(0.007)	(0.003)	(0.008)	(0.003)
$D2$	0.363	3.737	-0.287	3.468	0.147
	(0.011)	(0.007)	(0.003)	(0.008)	(0.003)
IID	0.315	3.767	-0.271	3.429	0.130
	(0.011)	(0.007)	(0.003)	(0.007)	(0.003)
Tangency portfolios					
$S2D2$	0.532	4.402	-0.264	4.045	0.163
	(0.011)	(0.009)	(0.004)	(0.011)	(0.003)
$S2$	0.205	4.846	-0.342	4.197	0.111
	(0.015)	(0.012)	(0.005)	(0.013)	(0.003)
$D2$	0.347	4.100	-0.298	3.535	0.141
	(0.012)	(0.009)	(0.004)	(0.008)	(0.003)
IID	0.373	4.035	-0.210	3.368	0.145
	(0.012)	(0.008)	(0.003)	(0.007)	(0.003)
EW	0.145	4.743	-0.197	3.510	0.092
	(0.016)	(0.011)	(0.003)	(0.008)	(0.003)

Table. 13: The simulation results of the performances with various investment strategies. Numbers in parentheses are the standard deviations of them. All of the means and the standard deviations, and their standard errors in this table are multiplied by 100. Number of trials is 10000. Each trial has 140 observations.

	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio
Initial state			$S = 0,\ D = 1$		
Global minimum variance portfolios					
$S2D2$	-0.266	4.195	-0.449	3.456	-0.014
	(0.013)	(0.007)	(0.004)	(0.009)	(0.003)
$S2$	-0.141	4.273	-0.417	3.462	0.011
	(0.012)	(0.007)	(0.003)	(0.008)	(0.003)
$D2$	-0.170	4.227	-0.422	3.458	0.005
	(0.012)	(0.007)	(0.003)	(0.008)	(0.003)
IID	-0.172	4.279	-0.408	3.434	0.004
	(0.013)	(0.007)	(0.003)	(0.008)	(0.003)
Tangency portfolios					
$S2D2$	-0.090	4.404	-0.527	3.844	0.031
	(0.012)	(0.009)	(0.004)	(0.011)	(0.003)
$S2$	-0.752	5.453	-0.665	4.045	-0.069
	(0.018)	(0.013)	(0.005)	(0.015)	(0.003)
$D2$	-0.353	4.712	-0.473	3.530	-0.024
	(0.015)	(0.009)	(0.004)	(0.009)	(0.003)
IID	-0.254	4.882	-0.214	3.129	-0.022
	(0.014)	(0.006)	(0.003)	(0.006)	(0.003)
EW	-0.787	5.857	-0.142	3.012	-0.106
	(0.018)	(0.007)	(0.003)	(0.005)	(0.003)

	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio
Initial state			$S = 1,\ D = 1$		
Global minimum variance portfolios					
$S2D2$	-1.079	4.582	-0.448	3.250	-0.207
	(0.014)	(0.006)	(0.003)	(0.008)	(0.003)
$S2$	-0.903	4.659	-0.423	3.285	-0.166
	(0.013)	(0.006)	(0.003)	(0.007)	(0.003)
$D2$	-0.935	4.609	-0.426	3.276	-0.175
	(0.013)	(0.006)	(0.003)	(0.008)	(0.003)
IID	-0.943	4.658	-0.413	3.257	-0.176
	(0.013)	(0.006)	(0.003)	(0.007)	(0.003)
Tangency portfolios					
$S2D2$	-0.820	4.910	-0.533	3.649	-0.133
	(0.013)	(0.008)	(0.004)	(0.010)	(0.003)
$S2$	-1.857	6.178	-0.656	3.751	-0.261
	(0.019)	(0.010)	(0.004)	(0.013)	(0.003)
$D2$	-1.252	5.167	-0.478	3.329	-0.212
	(0.015)	(0.007)	(0.003)	(0.009)	(0.003)
IID	-1.126	5.187	-0.216	3.007	-0.200
	(0.015)	(0.005)	(0.002)	(0.005)	(0.003)
EW	-1.922	6.181	-0.122	2.896	-0.298
	(0.019)	(0.006)	(0.003)	(0.005)	(0.003)

Table. 14: The global minimum variance portfolios and tangency portfolios. Panel A shows the portfolio weight of global minimum variance portfolio for each investor type. On the other hands, Panel B shows the portfolio weight of tangency portfolio for each investor type.

Panel A: global minimum-variance portfolios

	$S2D2$			
	$S=0, D=0$	$S=1, D=0$	$S=0, D=1$	$S=1, D=1$
ENE	0.128	0.056	0.000	0.000
MAT/IND	0.126	0.000	0.000	0.000
CG	0.472	0.694	0.401	0.357
FIN	0.000	0.000	0.000	0.000
IT/Tel	0.000	0.000	0.000	0.115
U	0.274	0.250	0.599	0.528

	$S2$		$D2$		IID
	$S=0$	$S=1$	$D=0$	$D=1$	
ENE	0.063	0.000	0.159	0.000	0.028
MAT/IND	0.000	0.000	0.000	0.000	0.000
CG	0.547	0.694	0.600	0.635	0.654
FIN	0.000	0.000	0.000	0.000	0.000
IT/Tel	0.000	0.000	0.000	0.000	0.000
U	0.390	0.306	0.241	0.365	0.319

Panel B: tangency portfolios

	$S2D2$			
	$S=0, D=0$	$S=1, D=0$	$S=0, D=1$	$S=1, D=1$
ENE	0.268	0.916	0.025	0.000
MAT/IND	0.073	0.000	0.000	0.000
CG	0.308	0.000	0.396	0.992
FIN	0.066	0.084	0.000	0.000
IT/Tel	0.061	0.000	0.000	0.008
U	0.225	0.000	0.580	0.000

	$S2$		$D2$		IID
	$S=0$	$S=1$	$D=0$	$D=1$	
ENE	0.170	1.000	0.270	0.231	0.318
MAT/IND	0.000	0.000	0.000	0.000	0.000
CG	0.407	0.000	0.631	0.071	0.568
FIN	0.000	0.000	0.099	0.000	0.000
IT/Tel	0.035	0.000	0.000	0.000	0.000
U	0.387	0.000	0.000	0.697	0.114

Table. 15: The simulation results of the performances with various types of investment strategies. 2000 time-series of the indexes with 200 months are simulated. Each type investor starts to invest after 60 months passed from the starting point. The data of the first 60 months are used only for the investors to estimate the distribution parameters in their assumed models. After the first 60 months, the past data-windows for the investors are expanded as time passes. This table reports the means of the statistics. All of the statistics are computed using the data of returns in each trial. Numbers in parentheses are the standard errors. All of the means and standard deviations and their standard errors in this table are multiplied by 100. We assume that the risk-free rate is 0.

Panel A: The actual market model is Model $S2D2$

	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio
Global minimum variance portfolios					
$S2D2$	0.047	4.424	-0.341	3.788	0.077
	(0.034)	(0.022)	(0.010)	(0.027)	(0.007)
$S2$	0.102	3.998	-0.357	3.539	0.096
	(0.032)	(0.020)	(0.009)	(0.021)	(0.008)
IID	0.224	3.882	-0.341	3.484	0.118
	(0.028)	(0.018)	(0.008)	(0.019)	(0.007)
Tangency portfolios					
$S2D2$	0.082	5.269	-0.263	4.464	0.070
	(0.035)	(0.026)	(0.012)	(0.042)	(0.007)
$S2$	0.083	4.920	-0.349	4.167	0.082
	(0.035)	(0.028)	(0.011)	(0.038)	(0.007)
IID	0.172	4.717	-0.235	3.513	0.103
	(0.034)	(0.028)	(0.008)	(0.019)	(0.007)
EW	-0.116	5.032	-0.194	3.389	0.049
	(0.149)	(0.102)	(0.028)	(0.071)	(0.029)

Panel B: The actual market model is Model $S2$

	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio
Global minimum variance portfolios					
$S2D2$	0.835	4.038	-0.137	3.794	0.230
	(0.014)	(0.015)	(0.008)	(0.023)	(0.004)
$S2$	0.813	3.554	-0.192	3.546	0.252
	(0.013)	(0.012)	(0.007)	(0.023)	(0.004)
IID/Myopic	0.801	3.503	-0.184	3.429	0.252
	(0.013)	(0.011)	(0.006)	(0.016)	(0.004)
Tangency portfolios					
$S2D2$	0.852	5.101	-0.086	4.647	0.197
	(0.016)	(0.025)	(0.011)	(0.041)	(0.004)
$S2$	0.855	4.901	-0.146	4.716	0.215
	(0.015)	(0.032)	(0.011)	(0.043)	(0.004)
IID	0.892	4.324	-0.111	3.635	0.241
	(0.014)	(0.026)	(0.007)	(0.021)	(0.004)
EW	0.847	4.375	-0.143	3.568	0.217
	(0.054)	(0.060)	(0.025)	(0.067)	(0.015)

Table. 16: The simulation results of the performances based on the results of the literature. 2000-time series of the indexes are simulated. Each type investor starts to invest after 200 months (resp. 288 months) passed from the starting point in the simulation of Ang and Bekaert (2002) (resp. Guidolin and Timmermann (2007)). The data in the initial runnings periods are used only for the investors to estimate the distribution parameters in their assumed model. After the initial running months, the past data-windows for the investors are expanded as time passes. This table reports the means of the statistics. All of the statistics are computed using the data of returns in each trial. Numbers in parentheses are the standard errors. All of the means and standard deviations and their standard errors in this table are multiplied by 100.

	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio
Simulation of Ang and Bekaert (2002)					
Only 2-state recursive regime (S) model					
The risk-free rate is 0.0041 (monthly rate).					
Global minimum variance portfolios					
4 states	0.618	4.080	-0.106	4.264	0.154
	(0.015)	(0.017)	(0.016)	(0.052)	(0.004)
2 states	0.621	4.040	-0.110	4.182	0.156
	(0.014)	(0.017)	(0.015)	(0.044)	(0.004)
IID	0.623	4.042	-0.109	4.150	0.156
	(0.014)	(0.017)	(0.015)	(0.042)	(0.004)
Tangency portfolios					
4 states	0.602	4.809	-0.034	5.444	0.129
	(0.017)	(0.032)	(0.024)	(0.101)	(0.004)
2 states	0.612	4.654	-0.052	5.260	0.136
	(0.017)	(0.034)	(0.023)	(0.102)	(0.004)
IID	0.621	4.307	-0.072	4.203	0.147
	(0.015)	(0.026)	(0.016)	(0.050)	(0.004)
EW	0.392	4.518	-0.345	4.705	0.089
	(0.016)	(0.022)	(0.017)	(0.052)	(0.004)
Simulation of Guidolin and Timmermann (2007)					
Only 4-state recursive regime (S) model					
The risk-free rate is 0.0044 (monthly rate).					
Global minimum variance portfolios					
4 states	0.357	6.728	0.550	4.710	0.052
	(0.024)	(0.031)	(0.017)	(0.069)	(0.003)
2 states	0.369	6.718	0.525	4.600	0.054
	(0.024)	(0.030)	(0.016)	(0.063)	(0.003)
IID	0.368	6.718	0.512	4.553	0.053
	(0.024)	(0.030)	(0.016)	(0.061)	(0.003)
Tangency portfolios					
4 states	1.853	13.756	0.689	5.085	0.133
	(0.054)	(0.111)	(0.023)	(0.135)	(0.004)
2 states	1.876	13.622	0.643	4.678	0.136
	(0.055)	(0.134)	(0.020)	(0.102)	(0.004)
IID	1.922	13.591	0.629	4.433	0.139
	(0.056)	(0.155)	(0.018)	(0.080)	(0.003)
EW	0.503	9.871	0.070	3.572	0.051
	(0.035)	(0.033)	(0.012)	(0.028)	(0.004)

Table. 17: The AICs of various models. The first row displays the AICs, which are defined as $-2(\text{log likelihood} - \text{\# of parameters})$, where # of parameters means a number of parameters.

Model	$S2$	$S2D2$	$S3$	$S4$	$D2$	$D3$	$S2S2$
AIC	6684.234	6619.640	6662.203	6634.194	6778.565	6717.901	6621.640
# of parameters	56	111	87	120	55	83	112

Figure. 1: The smoothed probabilities and the NBER recession dates. The first figure plots the smoothed probability of Model $S2$. The second figure plots the smoothed probabilities of Model $S2D2$. In each figure, a solid line is the probability of being at $S = 0$ and shadow areas are NBER recession dates. In the second figure, a dashed line is the probability of being at $D = 0$.

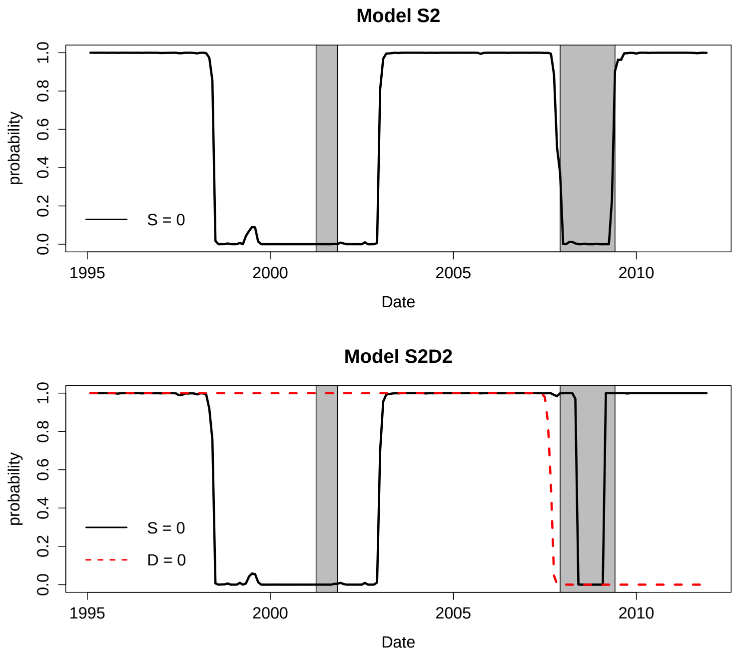

Figure. 2: The smoothed probability of being at $D_t = 0$ computed by using individual industry returns.

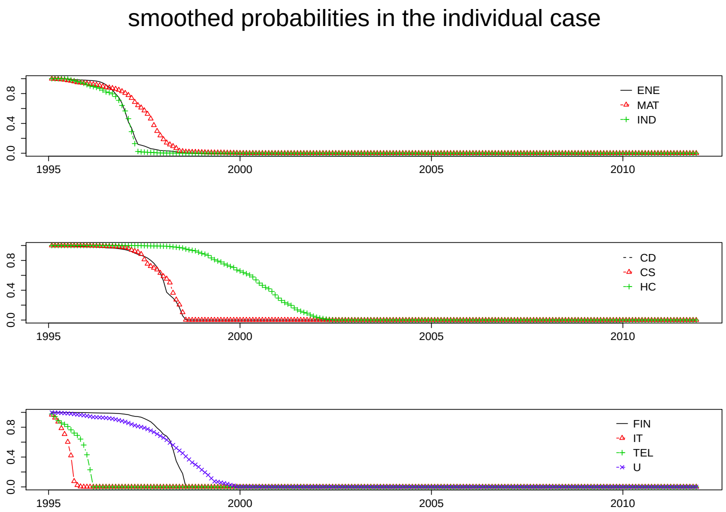

Figure. 3: The standard deviations and Sharpe ratios in simulations. In computing the Sharpe ratios, we assume that the risk-free rate equal 0.

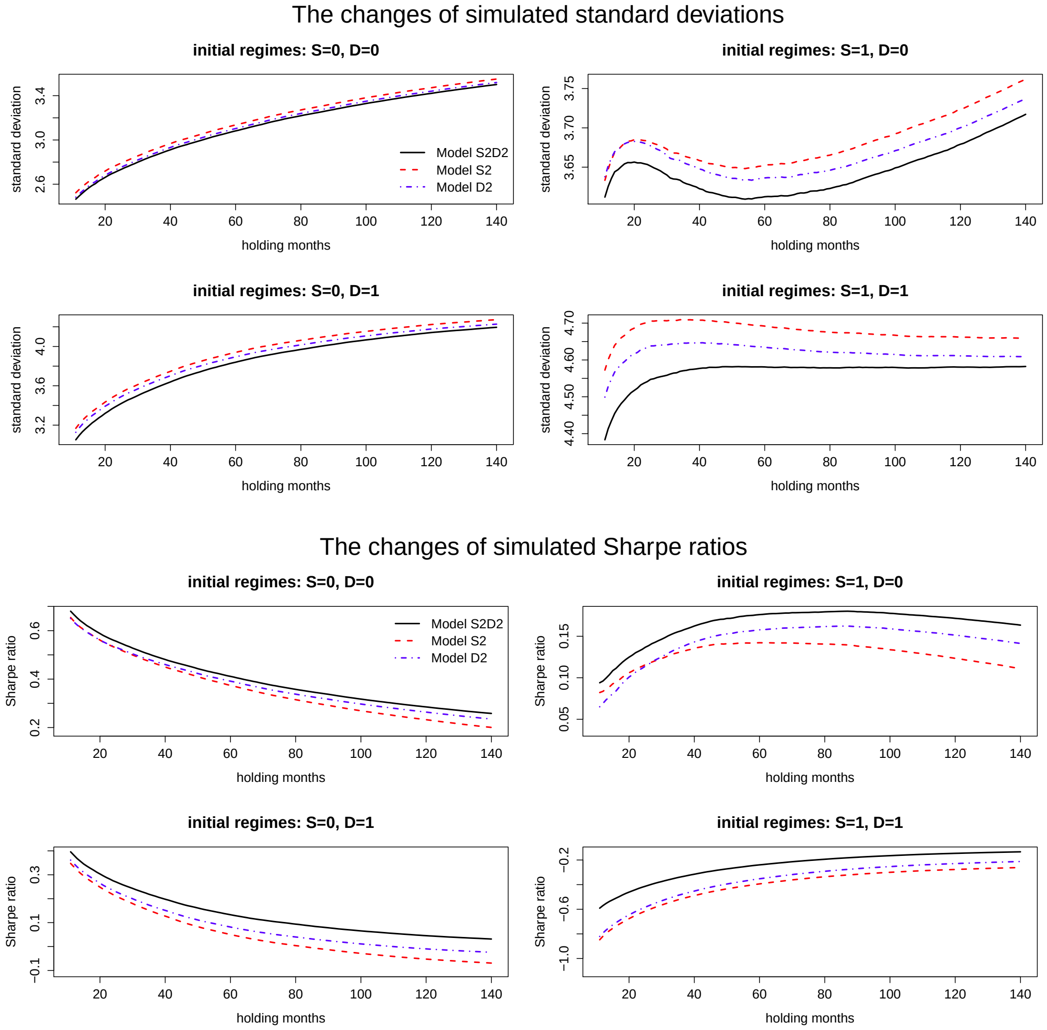

Figure. 4: The means of the smoothed probabilities around the structural change. Lines of the 100th, 150th and 199th month represent the changes of the means of the smoothed probabilities around the structural change for $T = 100, 150$ and 199, respectively. A line of the 199th month conditioned by change time represents the means of the smoothed probabilities around the structural change with the condition that the structural change occurs during the periods from $t = 51$ to $t = 150$.

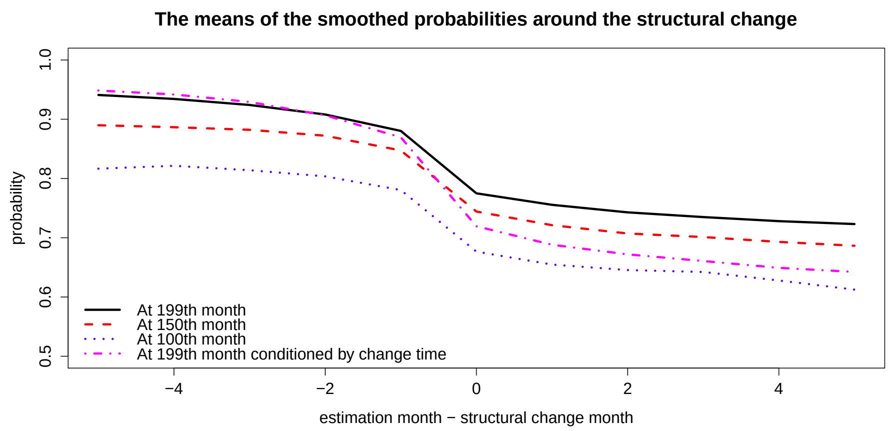

Figure. 5: The smoothed probabilities and the NBER recession dates. The first figure plots the smoothed probabilities of Model $S2S2$. The second figure plots the smoothed probabilities of Model $S2D2$. In each figure, a solid line is the probability of being at $S = 0$, shadow areas are NBER recession dates and a dashed line is the probability of being at $D = 0$.