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Abstract

We provide a model to explain hidden profiles, a series of persuasion cascades where players choose not to share their private information with the others and the group therefore fails. In our model, rational players will jointly select a decision. Attributes decide which decision is optimal, but each player privately and imperfectly knows these attributes. Hence, before decision-making, the players meet and sequentially talk. A player benevolently talks based on his limited information. But under communication constraints, the benevolent talk may cause the next player to infer that a suboptimal decision is most likely to be optimal. The next player repeats the previous talk because he is afraid that his private information may misguide the group. In this way, the players persuade one another by withholding private information.

Keyword: Group Decision-Making. Hidden Profiles. Persuasion Cascades.

JEL Codes: D79, D82, D83.

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1 Introduction

We study a model of *persuasion cascades in group decision-making* to demonstrate the impact of *hidden profiles* in group decision-making processes where a group fails to pool individual members' unique information and selects a suboptimal decision (Stasser, 1988). *Hidden profiles* have been argued among psychologists and behavioral scientists (Lightle, Kagel, & Arkes, 2009; Stasser & Stewart, 1992; Stasser & Titus, 1985, 1987; Wittenbaum, 2000). In all of these empirical and experimental studies, players with similar preferences discussed and then jointly selected an action. Players tended to discuss common information (known to everyone before discussion) or already shared information rather than unique information which, therefore, remained hidden. Then, the group chose an action "that was favored by their common knowledge—that is, the information they all knew before discussion. Moreover, this common knowledge solution would likely persist even when their combined or collective knowledge clearly favored another route (action)" (Stasser & Titus, 2003, pp. 304).

In this paper, we examine hidden profiles in persuasion cascades through a game of group decision-making among rational players with common interests. Hence, each player has an incentive to persuade the other players to support an optimal decision. Our players have imperfect and asymmetric information, hence, they meet and talk before decision-making. We consider a sequential talk under communication constraints such as a limited message set. Specifically, we explore how one speaker's talk under communication constraints may lead the next speaker to believe that a suboptimal decision is more likely to be optimal than an optimal decision. This next speaker may then withhold his private information if it is against a decision which, based on the previous speaker's talk and his own private information, seems to him to be most likely to be optimal. Then, this next speaker repeats the previous message because he wants to avoid misguiding the group with his information. In this way, the rational players may try to persuade one another by withholding their private information and repeating already commonly shared information. This cascade of hidden profiles leads to group failure.

Our model can apply to the following example. A university's recruiting committee is going to decide whether to hire some job candidate. Their decision is binary, hiring the candidate or not. Multiple characteristics (attributes) affect the value of the hiring decision such as the candidate's research abilities, his teaching abilities, his abilities to collaborate with the university's faculty members, and his commitment to the position. The characteristics are not necessarily known to everyone. Some members may happen to know the candidate's research and teaching abilities. Other members may only know his research and collaborative abilities. Hence, the committee members meet and talk before they decide.

More formally, in our model, a group of rational players with common interests will jointly select a binary decision 1 or 0 through sincere voting. Three attributes jointly decide which decision is optimal. Each attribute is positive or negative. If two or more attributes are positive, then, decision 1 benefits the players. If two or more attributes are negative, then, decision 1 hurts the players. Decision 0 yields zero profit to the players regardless of the attributes. But information on the attributes is imperfectly and asymmetrically distributed among the players. That is, each player only observes one or two of the attributes among the three, and no one knows what the other players know.

First, we consider an outcome-based talk where each player sends a binary cheap talk message to the others. A player only says 1 or 0 (e.g., "I agree" or "I disagree" on a proposal of hiring the candidate). For example, the meeting is set for half an hour, and there are ten participants. Each participant is given only three minutes to talk. Each participant should talk concisely and effectively. Or, even if the player explains details behind his agreement or disagreement on the proposal, the other players will not absorb such details and only recognize that he said, "I agree" or "I disagree."

We examine an equilibrium in which speakers start to mimic a previous message unless the current speaker has a particularly strong private signal. We call this *persuasion cascade equilibrium*. As a result, hidden profiles—the players withhold their private information—occur with a positive probability. For example, when one attribute is positive and two are negative, the optimal decision is 0. If the first two speakers observe the one positive attribute, even when the other speakers observe one of the two negative attributes (i.e., all the speakers have weak signals), information from the first two speakers will alter the ensuing persuasion cascade. The first two speakers benevolently send message 1 ("I agree"). Then, the third speaker updates his belief and expects that decision 1 is more likely to be optimal than decision 0. Similarly, the following speakers also repeat message 1. The aggregate of all players' information may suffice to support decision 0. However, these persuasion cascades lead all players to believe that decision 1 is more likely to be optimal than decision 0.

This equilibrium also has an advantage when at least one speaker observes both negatives (i.e., one or more people observe a strong, double, signal while other participants receive a weak, single signal). In the previous example, if one speaker observes a strong signal, he overturns repetition of message 1 and guides the group to choose decision 0.

Finally, we consider *an attribute-based talk* where each speaker states which attribute is positive or negative. We use two examples and explain how communication constraints can cause hidden profiles in attribute-based talk examples as well. For simplicity, we include a benevolent decision maker. Now there are three privately informed speakers and one uninformed decision maker. All players have the same preferences over the binary decisions. The three speakers sequentially talk to the decision maker in a public meeting. In the first example, we consider *limited* private information and *constrained* attribute-based talk. That is, each speaker privately observes two attributes with the same sign or one attribute. Each speaker can only state one attribute. In the second example, we consider a *noisy and constrained* attribute-based talk. Even if a speaker says, "the first attribute is positive," he may fail to make every word understood by the others. That is, the other players only hear "positive" with a positive probability. In both examples, the speakers can withhold their private information (hidden profiles) and repeat the previous message (persuasion cascades).

The remainder of the article is organized as follows. Section 2 reviews related literature. Section 3 analyzes a model with outcome-based talk. Section 4 examines attribute-based talk examples. Section 5 concludes. Proofs are in the appendix.

2 Related Literature

Theoretical models of informational cascades—This is related to literature on herd behavior and informational cascades initiated by Banerjee (1992) as well as Bikhchandani, Hirshleifer, and Welch (1992). In their models, when each player decides, he does *not* consider the effect of his action on the other players' actions.[1] On the other hand, in our model, when each player decides, he considers the effect of his talk on the other players' actions (or/and talks). Also, our model allows a speaker who stated his support for a proposal to be affected by the following speakers' messages. He may finally vote against the proposal.

[1] See also Chamley (2004) for the background of rational herding studies.

Lee (1993) showed that there do not occur (action) cascades among rational players given rich-enough action spaces. Our result is consistent with his in the sense that our cascades occur given limited spaces for talks and decisions. Smith and Sørensen (2000) showed that with a rich-enough signal space (i.e., there is a positive probability of very strong signals to be observed by some players), players' behavior converges to an optimal action. Similarly, in our model, incorrect persuasion cascades will be overturned by a talk of a player with strong confidence.

Ellison and Fudenberg (1995) as well as Eyster and Rabin (2010) proposed theoretical models of cascades assuming naivety and irrationality of players (without persuasive motives) given rich-enough action spaces. Our persuasion cascades do not depend on naivety (or irrationality) of players.[2]

On other applications of herd behavior models, Callander and Hörner (2009) theoretically showed the importance of minority's private information for the society when "minority and majorities are the only available information" (p. 1423) using a model of herding (without persuasive motives). We argue the importance of unique information compared to common information using a model of persuasion cascades. Mueller-Frank and Pai (2016) studied a sequential social learning model where players acquire information by costly search of the other players' actions. We do not consider costly information acquisition.

In Ottaviani and Sørensen (2001)'s model of committee decisions, committee members' talks may herd due to their reputational concerns. Each member wished to appear well informed.

In Caillaud and Tirole (2007)'s model of persuasion cascades, there is one proposer (i.e., information provider). There are also multiple committee members with different prior beliefs about the proposal. Hence, each player is a speaker *or* a decision maker. In our paper, each player is a speaker *and* a decision maker.

Bénabou (2013) proposed a theoretical model to explain Groupthink—denial of reality among joint project members—assuming endogenous imperfect recall and anticipated feeling. Unlike us, he considered simultaneous joint efforts among players (i.e., there is no talk).

Group decision-making and experimental evidence in psychology—As Stasser and Titus (2003) summarized, *hidden profiles* have been examined by psychologists since 1985. Stasser and Titus (1985, 1987) first challenged the idea that group decisions are more

[2]See also Eyster and Rabin (2014) for the study of herding among irrational players.

informed than individual decisions. Their experimental findings suggest that decision-making groups are more likely to discuss information which is shared among the members rather than information which is held privately. As a result, they choose an action that is favored by their common knowledge. Stasser (1988) called these patterns of unshared information *hidden profiles*. (On psychologists' experimental study on hidden profiles, see also Rulke & Galaskiewicz, 2000; Stasser & Davis, 1981; Stasser & Stewart, 1992; Stasser, Stewart, & Wittenbaum, 1995; Stasser & Taylor, 1991; Stasser, Taylor, & Hanna, 1989; Stasser, Vaughan, & Stewart, 2000; Stewart & Stasser, 1995; Sunstein & Hastie, 2014; Wittenbaum, 2000; Wittenbaum & Park, 2001.) We propose game theoretic models to explain hidden profiles.

According to experimental studies by Lightle, Kagel, and Arkes (2009), hidden profiles mainly result from *mistakes in recalling information*. Mistakes in recalling common information, they found, are more likely to be corrected than mistakes in recalling in private information. In our basic model, hidden profiles are driven by the influence of imperfectly informed players on the imperfectly informed players. In our extended model, if there is a positive probability that details in each player's message are not perceived (due to exogenous noises), hidden profiles can occur.

Attributes and information aggregation—Like De Clippel and Eliaz (2014), we consider two types of information transmission, the attribute-based talk and the outcome-based talk. The main difference is that De Clippel and Eliaz (2014) studied information aggregation through voting alone while we study information aggregation through group discussion as well as voting. The idea of a multi-attribute object with uncertain qualities was also considered by Klabjan, Olszewski and Wolinsky (2014), but their model is about costly information acquisition rather than hidden profiles.

Last, our model is related to models of information aggregation through voting among players with common preferences (Austen-Smith & Banks, 1996; Feddersen & Pesendorfer 1998; McLennan 1998), in particular when the decision is associated with multiple issues (Ahn & Oliveros 2013). However, voting is not the only way to aggregate information in our model.

3 Model with Outcome-Based Talk

3.1 Setup

There are L players, where $L \geq 3$. The players jointly make a binary decision, denoted:

$$d \in \{1, 0\}.$$

Decision $d = 1$ ($d = 0$) can be interpreted as an action (no action).

The state is

$$\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3).$$

We call elements in the state *attributes*. $\boldsymbol{\theta}$ is drawn from a discrete uniform distribution with support $\{1, -1\}^3$. In other words, for each $i \in \{1, 2, 3\}$, the i-th attribute is either of $\theta_i = 1$ or $\theta_i = -1$, which are equally likely. All attributes are independent and identically distributed (i.i.d.). These distributions are common knowledge.

The players have the same preference over decisions. Each player's payoff is given by:

$$\begin{cases} \theta_1 + \theta_2 + \theta_3 & \text{if } d = 1 \\ 0 & \text{if } d = 0 \end{cases} \tag{1}$$

For example, if $(\theta_1, \theta_2, \theta_3) = (1, 1, -1)$ (if $(\theta_1, \theta_2, \theta_3) = (1, -1, -1)$), $d = 1$ yields a payoff 1 (-1) to every player.

Each player observes a private signal on $\boldsymbol{\theta}$. Given $\boldsymbol{\theta}$, for each player $l \in \{1, 2, ..., L\}$, his private signal σ_l is drawn from a discrete uniform distribution with support $I(\boldsymbol{\theta})$ such that:

$$I(\boldsymbol{\theta}) = \{\theta_1, \varnothing\} \times \{\theta_2, \varnothing\} \times \{\theta_3, \varnothing\} \setminus \{(\theta_1, \theta_2, \theta_3), (\varnothing, \varnothing, \varnothing)\}.$$

The private signals are i.i.d. These distributions are common knowledge.[3]

After observing private signals, all players meet and *sequentially talk*. The order of the players' talks is determined randomly when they meet.

In this section, we consider *the outcome-based talk* such that each player l sends a

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[3]If the true state is $\theta = (1, 1, -1)$, then, for any player l's private information is $\sigma_l \in I(\theta)$ where:

$$I(\theta) = \{(1, 1, \varnothing), (1, \varnothing, -1), (\varnothing, 1, -1), (1, \varnothing, \varnothing), (\varnothing, 1, \varnothing), (\varnothing, \varnothing, -1)\}.$$

We expect that given any $I(\boldsymbol{\theta}) \subseteq \{\theta_1, \varnothing\} \times \{\theta_2, \varnothing\} \times \{\theta_3, \varnothing\}$, our main results, which we will show, remain unchanged.

binary message denoted:

$$m_l \in \{1, 0\},$$

and this message is perfectly observed by the others before the next player talks. This is a *cheap talk*—each player can choose either message regardless of his private signal.[4]

After the group talk, all players *simultaneously* cast their votes. Each player l's vote is denoted:

$$v_l \in \{1, 0\}.$$

Then, the group decision is made according to the *random dictatorship rule*—after the players simultaneously vote, one vote is randomly selected from the votes, and the selected vote becomes the group decision.[5]

After the decision is made, all players' payoffs are realized.

In summary, this is a group decision-making game among finite players with imperfect information. The setup of this game is common knowledge. But the attributes are imperfectly known to players, and no player' private information is observed by the other players.

3.2 Timeline

The sequence of events is summarized as follows:

1. Nature decides *attributes* which jointly decide a correct decision. Then, each player observes a private and imperfect signal on these attributes.

2. All players meet and *sequentially talk*.

3. All players *simultaneously vote*.

4. A decision is made according to *the random dictatorship rule*.

5. Finally, payoffs are realized for all players.

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[4]Although this is a cheap talk, we interpret this binary message as agreement on either decision, and hence we call this communication *the outcome-based talk*. Moreover, like De Clippel and Eliaz (2014), we also consider *the attribute-based talk* as well. See Section 4 for further details.

[5]We use *the random dictatorship rule* for simplicity. We expect that as long as sincere voting is expected, our main results still hold.

Under *the random dictatorship rule*, sincere voting is optimal for all players because every player's vote is pivotal, i.e., the player knows that his vote will determine a decision with probability $1/L$. See Gibbard (1973) as well as Börgers and Smith (2014) for the theoretical studies of random dictatorship rule.

3.3 Definitions

We define terminologies.

n_l^+ and n_l^- denote numbers of positive attributes and negative attributes, respectively, included in player l's private signal, σ_l.[6]

h_t denotes a sequence of the first through the t-th messages, where $h_t := (m_1, \cdots, m_t)$ for $t \in \{1, \cdots, L\}$ and $h_0 = \varnothing$ (empty).

Next, we define a symmetric perfect Bayesian equilibrium of this game.

Definition 1 *(Symmetric PBE) A symmetric perfect Bayesian equilibrium of this game (a symmetric PBE) consists of a talking strategy correspondence m^* and a voting strategy correspondence v^* which are used by the all players, as follows:*

(1) *If player l is assigned to be the t-th speaker, the talk strategy correspondence $m^*(\sigma_l, h_{t-1})$ maps his private information σ_l and history h_{t-1} to his talk m_l so that:*

(1-*a*) *Every $m \in m^*(\sigma_l, h_{t-1})$ maximizes his expected payoff.*

(1-*b*) *If $m^*(\sigma_l, h_{t-1}) = \{1, 0\}$, he randomizes $m_l = 1$ and $m_l = 0$ w.p. $\frac{1}{2}$ and $\frac{1}{2}$, respectively.*

(2) *If player l is assigned to be the t-th speaker, the voting strategy correspondence $v^*(\sigma_l, h_L, t)$ maps σ_l, the entire history h_L, and t to his vote v_l such that:*

(2-*a*) *Every $v \in v^*(\sigma_l, h_L, t)$ maximizes his expected payoff.*

(2-*b*) *If $v^*(\sigma_l, h_L, t) = \{1, 0\}$, he randomizes $v_l = 1$ and $v_l = 0$ w.p. $\frac{1}{2}$ and $\frac{1}{2}$, respectively.*

(3) *Moreover, at each point in time, a player updates his belief on attributes using Bayes' rule that takes into account his private information, a sequence of past messages, and the other players' strategies.*

Because this is a cheap talk, there is an equilibrium in which players' messages do not convey any information at all. To eliminate these types of babbling equilibria, we define an informative equilibrium.

Definition 2 *(Informative Equilibrium) Suppose player l is the t-th speaker. In an informative equilibrium it holds that:*

$$\Pr\left(\boldsymbol{\theta} = \theta' | \sigma_s = \sigma, h_{t-1} = h, m_l = 1\right) \neq \Pr\left(\boldsymbol{\theta} = \theta' | \sigma_s = \sigma, h_{t-1} = h, m_l = 0\right).$$

for some $s \neq l$, θ', σ and h.

[6] For example, $\left(n_l^+, n_l^-\right) = (2, 0)$ if $\sigma_l = (1, \varnothing, 1)$, and $\left(n_l^+, n_l^-\right) = (1, 1)$ if $\sigma_l = (-1, 1, \varnothing)$.

In informative equilibrium, player l's message m_l affects some other player (say, player s)'s inference on some attributes (say, $\theta^{'}$).

We also define *a monotonic persuasion equilibrium* of this game such that players' messages affect one another in a monotonic way.

Definition 3 *(Monotonic Persuasion Equilibrium) In a monotonic persuasion equilibrium, every speaker's message is informative. Moreover, for any σ_l and $\widetilde{\sigma}_l$, where n_l^+ and n_l^- correspond to σ_l, $\widetilde{n}_l^+$ and $\widetilde{n}_l^-$ correspond to $\widetilde{\sigma}_l$:*

$$m^*\left(\sigma_l, h_{t-1}\right) = m_{t-1} \Rightarrow m^*\left(\widetilde{\sigma}_l, h_{t-1}\right) = m_{t-1}$$

holds given:

$$\left|n_l^+ - n_l^-\right| \geq \left|\widetilde{n}_l^+ - \widetilde{n}_l^-\right|$$

if $n_l^+ - n_l^-$ and $\widetilde{n}_l^+ - \widetilde{n}_l^-$ are both non-negative (or both non-positive).

We focus on the player's confidence in the optimal decision. In monotone persuasion equilibrium, the less confident the player, the more easily the player is affected by the past messages.

Consider a sequence of past messages, $h_{t-1} = (m_1, \cdots, m_{t-1})$, and suppose that player l is the t-th speaker. Then, we compare different private signal realizations for player l. For example:

σ_l	$n_l^+ - n_l^-$
$\sigma_l = (1, 1, \varnothing)$	2
$\sigma_l = (1, \varnothing, \varnothing)$	1
$\sigma_l = (\varnothing, 1, -1)$	0
$\sigma_l = (-1, \varnothing, \varnothing)$	-1
$\sigma_l = (\varnothing, -1, -1)$	-2

We interpret that player l is more confident given $\sigma_l = (1, \varnothing, \varnothing)$ than given $\sigma_l = (\varnothing, 1, -1)$. Let us compare player l's inferences before communication.

- Given $\sigma_l = (\varnothing, 1, -1)$, he infers that the optimal decision is $d = 1$ and $d = 0$ with probability $1/2$ and $1/2$, respectively, because he infers $\theta = (1, 1, -1)$ and $\theta = (-1, 1, -1)$ with probability $1/2$ and $1/2$, respectively.

- On the other hand, given $\sigma_l = (1, \varnothing, \varnothing)$, he infers that the optimal decision is $d = 1$ and $d = 0$ with probability $3/4$ and $1/4$, respectively, because he infers $\theta = (1, 1, 1)$, $\theta = (1, 1, -1)$, $\theta = (1, -1, 1)$ and $\theta = (1, -1, -1)$ with probability $1/4$, $1/4$, $1/4$ and $1/4$, respectively.

Hence, we interpret that $\sigma_l = (1, \varnothing, \varnothing)$ gives player l greater confidence about which decision is optimal than $\sigma_l = (\varnothing, +1, -1)$ does. Similarly, we say that he is more confident given $\sigma_l = (-1, \varnothing, \varnothing)$ than given $\sigma_l = (\varnothing, 1, -1)$, and so on. We do not directly compare $\sigma_l = (1, \varnothing, \varnothing)$ and $\sigma_l = (-1, \varnothing, \varnothing)$.

In monotone persuasion equilibrium, player l repeats the previous message m_{t-1} given his private information $\sigma_l = (\varnothing, 1, -1)$ if he does so given $\sigma_l = (1, \varnothing, \varnothing)$. Similarly, player l repeats m_{t-1} given $\sigma_l = (\varnothing, 1, -1)$ if he does so given $\sigma_l = (\varnothing, \varnothing. -1)$. We say nothing between $\sigma_l = (1, \varnothing, \varnothing)$ and $\sigma_l = (-1, \varnothing, \varnothing)$.

Finally, we define "hidden profiles" for this game to be outcomes of inefficient information sharing. (See Stasser & Titus, 2003, for their definition.)

Definition 4 *(Hidden Profiles) If a group does not select a decision which is optimal conditional on an aggregate of all players' private information, we call this outcome "hidden profiles."*

A group may select a suboptimal decision due to inefficient information sharing—the group does not effectively pool players' private information, as discussed in this paper. A group may also select the suboptimal decision due to lack of information in the group—some attributes are not observed by any player at all. The latter outcome will be examined in a future paper.

3.4 Result

We study *a symmetric PBE*, which we will simply call *an equilibrium* from now on. All players' strategies are determined before the game starts.

Without loss of generality, we call the t-th speaker "player t" for $\forall t \in \{1, 2, ..., L\}$. Hence, $m^* (\sigma_t, h_{t-1})$ denotes a player's talking strategy when he is the t-th speaker (he is player l). For a voting strategy, we simplify the notation and let $v^* (\sigma_l, h_L)$ denote a player's voting strategy when he is the t-th speaker (he is player l).

Assumption 1 *(Beliefs off the equilibrium path) For every player, if some other player's message is not consistent with his private information, he ignores the message in his inference* *on attributes. If multiple messages are not consistent with one another, a player ignores the oldest message in his inference. If there still remains inconsistency, he ignores the second oldest message, and so on.*

This assumption implies the following. Suppose that $h_3 = (m_1, m_2, m_3) = (1, 1, 0)$ indicates $\left(n_3^+, n_3^-\right) = (0, 2)$ (i.e., player 3 observed two negative attributes) in equilibrium. But player 4 has private information $\sigma_4 = (\varnothing, 1, 1)$, which is not consistent with m_3. In this case, player 4 ignores m_3.

Suppose that $h_4 = (m_1, m_2, m_3, m_4) = (1, 1, 0, 1)$ indicates $\left(n_3^+, n_3^-\right) = (0, 2)$ and $\left(n_4^+, n_4^-\right) = (2, 0)$ (i.e., player 3 observed two negative attributes while player 4 observed two positive attributes) in equilibrium. Two messages, m_3 and m_4, are not consistent with each other, but both messages can be consistent with $\sigma_5 = (\varnothing, 1, -1)$. In this case, player 5 ignores m_3.

We will show the existence of an equilibrium in which *hidden profiles*, decision-making processes that are driven by imperfectly informed players' motives to persuade the other imperfectly informed players, occur as a result of *persuasion cascades in group decision making*.

Proposition 1 *There is a monotonic persuasion equilibrium in which hidden profiles occur with a positive probability.*

Proof. There is an *equilibrium* which consists of a player's talk strategy and voting strategy such that, if he is assigned as the t-th speaker for any $t \in \{1, ..., L\}$, he chooses his message as follows:

$$m^*(\sigma_1, h_0) = \begin{cases} 1 \text{ if } \left(n_1^+, n_1^-\right) \in \{(2,0), (1,0)\} \\ 0 \text{ if } \left(n_1^+, n_1^-\right) \in \{(0,2), (0,1)\} \\ 1 \text{ and } 0 \text{ w.p. } \frac{1}{2} \text{ and } \frac{1}{2}\text{, respectively, otherwise.} \end{cases}$$

$$m^*(\sigma_2, h_1) = \begin{cases} 1 \text{ if } \left(n_2^+, n_2^-\right) \in \{(2,0), (1,0)\} \\ 0 \text{ if } \left(n_2^+, n_2^-\right) \in \{(0,2), (0,1)\} \\ m_1 \text{ otherwise.} \end{cases}$$

$$m^*(\sigma_t, h_{t-1}) = \begin{cases} 1 \text{ if } \left(n_t^+, n_t^-\right) = (2,0) \\ 0 \text{ if } \left(n_t^+, n_t^-\right) = (0,2) \quad \text{for } t \in \{3, ..., L\}, \\ m_{t-1} \text{ otherwise.} \end{cases}$$

and he chooses his vote as follows:

$$v^*(\sigma_t, h_L) = \begin{cases} 1 \text{ if } (n_t^+, n_t^-) = (2,0) \\ 0 \text{ if } (n_t^+, n_t^-) = (0,2) \\ m_L \text{ otherwise.} \end{cases} \quad \text{for } t \in \{1, ..., L-1\}.$$

$$v^*(\sigma_L, h_L) = m_L.$$

This is a *monotone persuasion equilibrium*. See the appendix for the proof of existence of this equilibrium. ■

Player 1's message affects player 2's message only if player 2 is indifferent between the two decisions based on his private information, i.e., $(n_2^+, n_2^-) = (1,1)$. And player 2's message affects player 3's message if player 3 has uncertainty about the optimal decision, i.e., $(n_3^+, n_3^-) \in \{(1,1), (0,1), (0,1)\}$. This continues: player l's message affects player $(l+1)$'s message if player $(l+1)$ has uncertainty about the optimal decision. Finally, the last player's message affects voting decisions of all players who have uncertainty about the optimal decision. In addition, even if he is not the last speaker, each player's message can indirectly affect other players' voting decisions.

For example, there are three players (i.e., $L = 3$). Consider $\boldsymbol{\theta} = (1, -1, -1)$, i.e., the optimal decision is $d = 0$. The players' private signals are drawn as follows:

$$\begin{cases} \sigma_1 = (1, \varnothing, \varnothing) \\ \sigma_2 = (1, -1, \varnothing) \\ \sigma_3 = (1, \varnothing, -1) \end{cases}$$

Although the aggregate of the players information is complete, no player is 100 % confident about the optimal decision. In this case, player 1's private information favors $d = 1$, and hence his message is $m_1 = 1$. Player 2 is indifferent between two decisions based on his private information. Hence, player 2 is affected by player 1's talk and repeats the talk, i.e., $m_2 = 1$. This is similar for player 3. In summary, the players' talks are:

$$m_1 = 1 \rightarrow m_2 = 1 \ \rightarrow m_3 = 1\ .$$

As a result, all players vote for 1, and hence, the group decision is $d = 1$ (i.e., the group fails) with probability 1. The outcome is summarized as follows:

Private signals	Talks	Voting behavior	Decision
$\begin{cases} \sigma_1 = (1, \varnothing, \varnothing) \\ \sigma_2 = (1, -1, \varnothing) \\ \sigma_3 = (1, \varnothing, -1) \end{cases}$	$m_1 = 1$ $\to m_2 = 1$ $\to m_3 = 1$	$\begin{cases} v_1 = 1 \\ v_2 = 1 \\ v_3 = 1 \end{cases}$	$d = 1$ w.p. 1

The players privately observe one positive attribute (i.e., common information), and they repeatedly talk their support for decision 1. Finally, a decision favored by the common information survives. This outcome fits hidden profiles discussed in Stasser and Titus (2003).

The next example seems more surprising and intuitive than the previous example. There are five players (i.e., $L = 5$). Consider $\boldsymbol{\theta} = (1, -1, -1)$ again. The players' private signals are drawn as follows:

$$\begin{cases} \sigma_1 = (1, \varnothing, \varnothing) \\ \sigma_2 = (1, -1, \varnothing) \\ \sigma_3 = (\varnothing, \varnothing, -1) \\ \sigma_4 = (\varnothing, \varnothing, -1) \\ \sigma_5 = (\varnothing, -1, \varnothing) \end{cases}$$

Although the aggregate of the players information is complete, no player is 100 % confident about the optimal decision. In this case, player 1's private information favors $d = 1$, and hence his message is $m_1 = 1$. Player 2 is indifferent between two decisions based on his private information. Hence, player 2 is affected by player 1's talk and repeats the talk, i.e., $m_2 = 1$. Player 3's private information favors $d = 0$, but he is not sufficiently confident about either decision. Hence, player 3 is affected by earlier speakers' talks and selects $m_3 = 1$. This is similar for players 4 and 5. In summary, the players' talks are:

$$m_1 = 1 \to m_2 = 1 \ \to m_3 = 1 \ \to m_4 = 1 \ \to m_5 = 1.$$

As a result, all players vote for 1, and hence, the group decision is $d = 1$ (i.e., the group fails) with probability 1. The outcome is summarized as follows:

Talks	Voting behavior	Decision
$m_1 = 1$ $\to m_2 = 1$ $\to m_3 = 1$ $\to m_4 = 1$ $\to m_5 = 1$	$\left\{\begin{array}{l} v_1 = 1 \\ v_2 = 1 \\ v_3 = 1 \\ v_4 = 1 \\ v_5 = 1 \end{array}\right.$	$d = 1$ w.p. 1

Without a group discussion, at least players 3 through 5 vote for 0, and the group avoids the loss with the probability more than 3/5.

Player 3 does not select $m_3 = 0$ although his private information favors $d = 0$. Why? The reason is as follows. Player 3 computes trade-offs of selecting $m_3 = 0$. The talk $m_3 = 0$ may give an additional information to the other players. But if players 4 and 5 are not confident (e.g., their private information is $(\varnothing, -1, \varnothing)$ or $(1, -1, \varnothing)$ or $(1, \varnothing, \varnothing)$), they only repeat his talk, i.e., $m_4 = m_5 = 0$, which may lead the other unconfident players to vote for $d = 0$. On the other hand, the previous messages $m_1 = 1$ and $m_2 = 1$ lead player 3 to infer that $d = 1$ is more likely to be optimal than $d = 0$. Hence, the cost of selecting $m_3 = 0$ outweighs the benefit in this case.

Persuasion Cascades—In summary, *hidden profiles result from imperfectly informed players' motives to persuade the other imperfectly informed players.* Every player wants other *imperfectly informed players* to vote for an optimal decision. If the player perfectly knows the optimal decision, he tends to share his private information with the other players. However, if the player only has imperfect information, he may infer that the suboptimal decision is more likely to be optimal than the optimal decision. Then, he hides his private information and misleads the other players. Furthermore, one player's misleading talk may result in the next player's misleading talk: and players repeat the same talk. We interpret this outcome as *persuasion cascades*.

Comparison with the standard informational cascade models—The above mentioned examples also emphasize the difference of our model from the standard information cascade models. Group discussions can change speakers' minds and guide them to the optimal direction. Again, consider a model with five players. Consider the players private information:

$$\begin{cases} \sigma_1 = (1, \varnothing, \varnothing) \\ \sigma_2 = (1, \varnothing, \varnothing) \\ \sigma_3 = (\varnothing, \varnothing, -1) \\ \sigma_4 = (\varnothing, -1, -1) \\ \sigma_5 = (1, \varnothing, \varnothing) . \end{cases}$$

Then, the equilibrium outcome is:

Talks	Voting behavior	Decision
$m_1 = 1$ $\rightarrow m_2 = 1$ $\rightarrow m_3 = 1$ $\rightarrow m_4 = 0$ $\rightarrow m_5 = 0$	$\begin{cases} v_1 = 0 \\ v_2 = 0 \\ v_3 = 0 \\ v_4 = 0 \\ v_5 = 0 \end{cases}$	$d = 0$ w.p. 1

In this case, speaker 1 initiates incorrect persuasion cascades by claiming his support for $d = 1$. However, speaker 4 is confident about the optimal decision, $d = 0$. He claims his support for $d = 0$ and overturns incorrect persuasion cascades. As a result, all speakers are persuaded to vote for $d = 0$. This is an example of a successful group discussion. But if the talk order is changed, a suboptimal persuasion cascade occurs again.

Private Signals	Talks	Voting behavior	Decision
$\begin{cases} \sigma_1 = (\varnothing, -1, -1) \\ \sigma_2 = (1, \varnothing, \varnothing) \\ \sigma_3 = (\varnothing, \varnothing, -1) \\ \sigma_4 = (1, \varnothing, \varnothing) \\ \sigma_5 = (1, \varnothing, \varnothing) . \end{cases}$	$m_1 = 0$ $\rightarrow m_2 = 1$ $\rightarrow m_3 = 1$ $\rightarrow m_4 = 1$ $\rightarrow m_5 = 1$	$\begin{cases} v_1 = 0 \\ v_2 = 1 \\ v_3 = 1 \\ v_4 = 1 \\ v_5 = 1 \end{cases}$	$d = 1$ w.p. $4/5$

In addition, as discussed in Section 2, our result is contrasted with a herding model by Banerjee (1992) as well as a model of informational cascades by Bikhchandani, Hirshleifer, and Welch (1992). Unlike their models, our model involves players' persuasion incentives. Contrary to Ellison and Fudenberg (1995) and Eyster and Rabin (2010), our players are rational. Unlike a talk cascade model by Ottaviani and

Sørensen (2001), our players pursue the goal of making an optimal decision without concerning their own reputation.

4 Attribute-based Talk

4.1 Purpose of This Extension

This section considers *an attribute-based talk*—each player talks one of the attributes. A model of attribute-based talk may be more consistent with *empirical and experimental evidence in psychology* (Stasser & Titus, 1985 & 2003). However, an attribute-based model requires very complicated computations of players' inferences. Hence, for simplicity, we include a benevolent decision maker (DM), and we let three speakers sequentially talk to the decision maker in a public place meeting. The decision maker has the same preference over decisions as the speakers.

We provide informal arguments on two examples.[7] In the first example, the speakers observe two attributes with the same sign or one attribute. Even if a player observes two positive attributes, he can only talk one attribute. In other words, he can reveal his information and support some decision, but he cannot tell how strongly his information supports the decision. In the second example, we consider a *noisy* attribute-based talk. In this case, even if a speaker says, "the first attribute is positive," the other players may fail to hear the entire message. In both examples, the communication constraints can lead the speakers to withhold their private information.

De Clippel & Eliaz (2014) also studied premise-based [attribute-based] and outcome-based information aggregation. However, they considered information aggregation through voting while we focus on information aggregation through a discussion.[8]

4.2 Attribute-based Talk Example 1

There is a decision maker (DM) and three informed players (speakers 1, 2 and 3). The speakers' payoff structure remains unchanged from the one in Section 3. DM's payoff structure is the same as the speakers' structure.

[7]We thank Shinsuke Kanbe for suggestions and comments on the examples in this section.

[8]Moreover, their main question is "Should the group reach a decision by voting whether each premise is true or false, or should they simply vote on the outcome?" (de Clippel & Eliaz, 2014, p. 34). This question is beyond the scope of this paper.

Example 1 assumes a smaller attribute set and a smaller private signal set than the model in Section 3. $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)$ is drawn from a discrete uniform distribution with support:

$$\{1, -1\}^3 \setminus \{(1, 1, 1), (-1, -1, -1)\}.$$

Then, players observe two attributes with the same sign or one attribute. For example, given $\boldsymbol{\theta} = (1, -1, 1)$, each player's signal is i.i.d. and drawn from a discrete uniform distribution with support:

$$\{(1, \varnothing, 1), (1, \varnothing, \varnothing), (\varnothing, -1, \varnothing), (\varnothing, \varnothing, 1)\}.$$

Neither $(1, -1, \varnothing)$ nor $(\varnothing, -1, 1)$ is observed.

After observing private signals, the speakers sequentially talk to DM in a public meeting. Each speaker's message is observable to the other speakers as well as to the DM.

Here we consider an *attribute-based* talk: speaker t (i.e., the t-th speaker) sends a cheap talk message m_t such that:

$$m_t \in \{1, 2, 3\} \times \{+, -\}.$$

After the meeting, DM chooses a binary decision. Finally, the payoffs of all players are realized. Then, the game ends.

Claim 1 *Example 1 has a PBE in which hidden profiles occur with a positive probability.*

We can construct a PBE in which speakers 1 and 2 reveal their private information. However, speaker 3 withholds his information and repeats the previous talk under some conditions.

The following is an informal description of the equilibrium strategies:

1. Speaker 1's talk:

   (a) If he observes two attributes, he randomly selects one attribute to be talked. E.g., he mixes between $m_1 = (1, +)$ and $m_1 = (3, +)$ if $\sigma_1 = (1, \varnothing, 1)$.

   (b) If he observes one attribute, he talks it. E.g., $m_1 = (1, +)$ if $\sigma_1 = (1, \varnothing, \varnothing)$.

2. Speaker 2's talk:

(a) Suppose he observes two attributes.

i. If neither was yet talked, he randomly selects one attribute to be talked. E.g., he mixes between $m_2 = (1, +)$ and $m_2 = (3, +)$ if $\sigma_1 = (1, \varnothing, 1)$ and $m_1 \notin \{(1, +), (3, +)\}$.

ii. If either was talked, he talks the remaining attribute. E.g., he sends $m_2 = (1, +)$ if $\sigma_1 = (1, \varnothing, 1)$ and $m_1 = (3, +)$.

iii. If either was talked wrongly, he talks the attribute (i.e., he corrects the past wrong message). E.g., he sends $m_2 = (1, +)$ if $\sigma_1 = (1, \varnothing, 1)$ and $m_1 = (1, -)$.

(b) If he observes one attribute, he talks it. E.g., $m_2 = (1, +)$ if $\sigma_2 = (1, \varnothing, \varnothing)$.

3. Speaker 3's talk:

(a) If he observes two attributes, his message choice is similar to 2.a. (See the appendix for details.)

(b) If he observes one attribute:

i. If a different attribute was repeatedly talked by the previous two speakers, he repeats the same talk. E.g., $m_3 = (1, +)$ if $\sigma_3 = (\varnothing, \varnothing, -1)$ and $m_1 = m_2 = (1, +)$.

ii. Otherwise, he talks this attribute. E.g., $m_3 = (1, -)$ if $\sigma_3 = (\varnothing, \varnothing, -1)$ and $m_1 \neq m_2$.

4. DM's beliefs off the equilibrium path:

(a) If multiple messages are not consistent with one another, he ignores the oldest message in his inference. If there still remains inconsistency, he ignores the second oldest message, and so on.

See Appendix B.1 for details of the speakers' strategy and DM's strategy.

Next, let us consider two cases for private signal realization given $\boldsymbol{\theta} = (1, -1, -1)$. In the first case:

Private signals	Talks	Decision
$\left\{\begin{array}{l}\sigma_1 = (1, \varnothing, \varnothing) \\ \sigma_2 = (1, \varnothing, \varnothing) \\ \sigma_3 = (\varnothing, \varnothing, -1)\end{array}\right.$	$m_1 = (1, +)$ $\rightarrow m_2 = (1, +)$ $\rightarrow m_3 = (1, +)$	$d = 1$

Given $m_1 = m_2 = (1, +)$, speaker 3 expects $\sigma_1 = (1, 1, \varnothing)$, $(1, \varnothing, \varnothing)$ or $(\varnothing, 1, \varnothing)$, and he also expects $\sigma_2 = (1, \varnothing, \varnothing)$. Hence, speaker 3 infers $\boldsymbol{\theta} = (1, 1, -1)$ and $\boldsymbol{\theta} = (1, -1, -1)$ with positive probabilities, respectively, and he puts more weight on $\boldsymbol{\theta} = (1, 1, -1)$. Speaker 3 also knows that $m_3 = (3, -)$ would lead DM to select $d = 0$. Hence, in order to induce $d = 1$, speaker 3 withholds his private information.

On the other hands, given $m_1 = m_2 = m_3 = (1, +)$, DM recognizes the possibility of $\sigma_3 = (\varnothing, \varnothing, -1)$ or $\sigma_3 = (\varnothing, -1, \varnothing)$. Then, he infers $\boldsymbol{\theta} = (1, 1, -1)$, $\boldsymbol{\theta} = (1, -1, 1)$ and $\boldsymbol{\theta} = (1, -1, -1)$ with positive probabilities, respectively. But he puts a lot of weight on $\boldsymbol{\theta} = (1, 1, -1)$ and $\boldsymbol{\theta} = (1, -1, 1)$. Then, he chooses $d = 1$.

In the next case, the information is fully aggregated by DM:

Private signals	Talks	Decision
$\left\{\begin{array}{l} \sigma_1 = (1, \varnothing, \varnothing) \\ \sigma_2 = (1, \varnothing, \varnothing) \\ \sigma_3 = (\varnothing, -1, -1) \end{array}\right.$	$m_1 = (1, +)$ $\to m_2 = (1, +)$ $\to m_3 = (2, -)$	$d = 0$

Speaker 3 overturns persuasion cascades $m_1 = m_2 = (1, +)$. Hence, DM infers that speaker 3 observed $\sigma_3 = (\varnothing, -1, -1)$, and DM selects $d = 0$.

We can construct an equilibrium in which no speaker withholds his private information. If this is an equilibrium, given $m_1 = m_2 = (1, +)$ and $m_3 = (i, -)$ for $i \in \{2, 3\}$, DM selects $d = 1$ on the equilibrium path. Moreover, given $m_1 = m_2 = (1, +)$ and $m_3 = (1, -)$ (i.e., m_3 is not consistent with m_1 and m_2), DM should ignore m_3 and select $d = 1$. This does not seem plausible for the following reason.

Fix $m_1 = m_2 = (1, +)$. Given $\sigma_3 = (\varnothing, \varnothing, -1)$, speaker 3 prefers $d = 1$, and he also knows $m_3 = (3, -)$ results in $d = 1$. Hence, speaker 3 has no reason to send an inconsistent message, $m_3 = (1, -)$. On the other hand, given $\sigma_3 = (\varnothing, -1, -1)$, speaker 3 wants to avoid $d = 1$. He knows he cannot do anything by sharing his private information. He may have a reason to send $m_3 = (1, -)$ like an emergency alarm. Hence, given $m_1 = m_2 = (1, +)$ and $m_3 = (1, -)$, it seems plausible that DM takes m_3 seriously and selects $d = 0$.

4.3 Attribute-based Talk Example 2

Example 2 is different from Example 1 in two ways. First, the attribute and private signal sets remain unchanged from the ones in Section 3.

Second, we consider a *noisy* attribute-based cheap talk. Let m_t^s denote a message sent by the t-th speaker (i.e., player t), where:

$$m_t^s \in \{1, 2, 3, \varnothing\} \times \{+, -\}.$$

Let m_t^r denote how m_t^s is received by the other players. For simplicity, m_t^r is publicly known (i.e., every player knows that player t's message is received as m_t^r by all the other players).

For any t, the relation between m_t^s and m_t^r is:

$$\begin{aligned} &\text{If } m_t^s = (i, +1), \quad \text{then } m_t^r = \begin{cases} (i, +) \text{ w.p. } 1-\epsilon \\ (\varnothing, +) \text{ w.p. } \epsilon \end{cases} \quad \text{for } i \in \{1, 2, 3\} \\ &\text{If } m_t^s = (\varnothing, +1), \quad \text{then } m_t^r = (\varnothing, +) \text{ w.p. } 1. \end{aligned}$$

where $\epsilon \in [0, 1]$. The corresponding relations given $m_t^s = (i, -)$ or $m_t^s = (\varnothing, -)$ are defined similarly.

Claim 2 *Example 2 has a PBE in which hidden profiles occur with a positive probability.*

We can construct an equilibrium which induces the same probability distribution of d as the equilibrium given the outcome-based talk. Let players actually use two messages. For example, each player's message at each point in time is drawn from a discrete uniform distribution with supports either $\{1, 2, 3, \varnothing\} \times \{+\}$ or $\{1, 2, 3, \varnothing\} \times \{-\}$.

We also expect to find a more informative equilibrium. For example, the players' private signals are drawn as follows:

$$\begin{cases} \sigma_1 = (1, \varnothing, \varnothing) \\ \sigma_2 = (1, -1, \varnothing) \\ \sigma_3 = (1, \varnothing, -1). \end{cases}$$

In this case, player 1's private information favors $d = 1$, and hence he talks $m_1^s = (1, +)$. If his message is perceived by the other players *with* noise, i.e., $m_1^r = (\varnothing, +)$, his message affects talks by players 2 and 3, who are indifferent between two decisions given their private information.

Private signals	Talks	Decision
$\begin{cases} \sigma_1 = (1, \varnothing, \varnothing) \\ \sigma_2 = (1, -1, \varnothing) \\ \sigma_3 = (1, \varnothing, -1) \end{cases}$	$m_1^s = (1, +) \to m_1^r = (\varnothing, +)$ $\to m_2^s = (1, +) \to m_2^r$ $\to m_3^s = (1, +) \to m_3^r$	$d = 1$

For this outcome to realize, m_2^r (m_3^r) can be either of $(1, +)$ and $(\varnothing, +)$.

However, if player 1's message $m_1^s = (1, +)$ is received by the other players *without* noise, i.e., $m_1^r = (1, +)$, this message does not significantly affect the inference of players 2 and 3.

Private signals	Talks	Decision
$\begin{cases} \sigma_1 = (1, \varnothing, \varnothing) \\ \sigma_2 = (1, -1, \varnothing) \\ \sigma_3 = (1, \varnothing, -1) \end{cases}$	$m_1^s = (1, +) \to m_1^r = (1, +)$ $\to m_2^s = (\varnothing, -) \to m_2^r = (\varnothing, -)$ $\to m_3^s = (\varnothing, -) \to m_3^r = (\varnothing, -)$	$d = 0$

Appendix B.2 presents the speaker's strategy which can lead to the above mentioned outcome.

5 Conclusion

We provide a model of *persuasion cascades in group decision-making*. Our model explains *hidden profiles* where players fail to share their private information with the others even though it is easy to assume that rational players with common interests will jointly select an optimal decision. In our examples, useful information is distributed among players, therefore, the players meet and sequentially talk before making a decision. We examine the communication constraints that might lead the players to repeat the previous speakers' talks and withhold private information. As a result, the group may fail to select an optimal decision, which would probably be chosen if they aggregate all available information. Our model is simple enough to be testable.

Our result explains the role of hidden profiles and persuasion cascades in the failure of conforming (or homogeneous) decision-making groups and suggests possible benefits of a more heterogeneous group for decision-making processes. Investigation the latter point is one of avenues for future research.

Our future studies also include examination of other equilibria. We focused on one type of equilibrium which fits hidden profiles. However, there must be many equilibria in these sequential talk models. The most critical issue is whether the information problems in our equilibria are "accidental and rare" or should be seen as a common occurrence. We can use the methods developed herein to project the most efficient equilibria. The information problems projected in the most efficient equilibria can also be compared to the hidden profiles discussed in this paper.

Investigation of the more efficient decision making mechanism can address new questions. For example, what if all players speak or vote simultaneously? In the study of binary voting procedures, Dekel and Piccione (2000) showed that sequential voting did not improve on simultaneous voting with respect to information aggregation. But in our model of information aggregation via cheap talk, the conclusion can be different. Consideration of conflicts of interests among the players rather than the existence of one shared point of view should also be examined to provide a more "real world" projection of persuasion cascades.

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Appendix A (Outcome-based Talk)

A.0 Definitions

$\Theta^{(i,j)}$ denotes a set of θ which consists of i positive (i.e., 1) attributes and j negative (i.e., -1) attributes. For example, $\Theta^{(2,1)} = \{(1,1,-1),(1,-1,1),(-1,1,1)\}$, and $\Theta^{(3,0)} = \{(1,1,1)\}$.

$\Sigma^{(i,j)}$ denotes a set of private signals which include i positive (i.e., 1) attributes and j negative (i.e., -1) attributes. For example, $\Sigma^{(2,0)} = \{(1,1,0), (1,0,1), (0,1,1)\}$ and $\Sigma^{(1,1)} = \{(1,-1,0), (1,0,-1), (0,1,-1)\}$.

For any l and t,

$$p\left(\theta'|h,\sigma\right) = \Pr[\boldsymbol{\theta} = \theta'|h_t = h, \sigma_l = \sigma].$$

And

$$P_{\Theta^{(i,j)}}\left(h,\sigma\right) = \sum_{\boldsymbol{\theta}\in\Theta^{(i,j)}} p\left(\boldsymbol{\theta}|h,\sigma\right).$$

For $\forall t$:

$$\begin{aligned}
& \text{Expected payoff of } m_t = 1 \text{ given } \sigma_t \text{ and } h_{t-1} \\
:= & \sum_{i=0}^{N} \underbrace{(2i-N)}_{\text{Expected payoff of } m_t=1 \text{ given } \boldsymbol{\theta}\in\Theta^{(i,n-i)}} \cdot \sum_{\boldsymbol{\theta}\in\Theta^{(i,n-i)}} p\left(\boldsymbol{\theta}|h_{t-1},\sigma_t\right) \\
& \underbrace{\times \prod_{u=t+1}^{L} \Pr[m_u = m_{u-1} \text{ for} \forall m_{u-1}|\boldsymbol{\theta}]}_{=:\Pr[m_t \text{ affects } m_n|\boldsymbol{\theta}]} \\
& \underbrace{\times \frac{1}{L}\left(\sum_{u=1}^{t-1} \Pr[v_u = m_L \text{ for} \forall m_L|\boldsymbol{\theta}, h_{t-1}] + \Pr[v_t = m_L \text{ for} \forall m_L|\sigma_t] + (L-t)\right)}_{=:\Pr[m_t \text{ affects } d\ |m_t \text{ affects } m_n.\ \boldsymbol{\theta},h_{t-1},\sigma_t]}
\end{aligned}$$

where:

$$\prod_{u=t+1}^{L} \Pr[m_u = m_{u-1} \text{ for } \forall m_{u-1}|\boldsymbol{\theta}] \quad \text{strictly increase with } t$$

because $\Pr[m_u{=}m_{u-1} \text{ for } \forall m_{u-1}|\boldsymbol{\theta}] < 1$. Also, note that:

$$\begin{aligned}
& \Pr[m_t \text{ affects } d|m_t \text{ affects } m_L.\ \boldsymbol{\theta}, h_{t-1}, \sigma_t] \\
= & \ \tfrac{1}{L}\sum_{u=0}^{L} \Pr[v_u = m_L|m_u = m_{u-1} \text{ for} \forall m_{u-1} \text{ for} \forall u \in \{t+1, ..., L\}, \boldsymbol{\theta}, h_{t-1}, \sigma_t] \\
= & \ \tfrac{1}{L}\left(\sum_{u=1}^{t-1} \Pr[v_u = m_n \text{ for} \forall m_L|\boldsymbol{\theta}, h_{t-1}] + \Pr[v_t = m_L \text{ for} \forall m_L|\sigma_t] + (L-t)\right).
\end{aligned}$$

It suffices to check player t (the t-th speaker)'s optimal m_t given h_{t-1} and his optimal v_t given h_L for each $\sigma_t \in \{(1,-1,\varnothing),(1,\varnothing,\varnothing)\}$ for any t.

A.1.1 Player 1's talk and inferences

The optimal talk m_1 given each $\sigma_1 = \sigma_1'$ is:

$$m_1 = \begin{cases} \begin{array}{|l|l|} \hline 1 & \text{if } \sigma_1' = (1,\varnothing,\varnothing) \\ \hline \text{Indifferent} & \text{if } \sigma_1' = (1,\varnothing,-1) \\ \hline \end{array} \end{cases}$$

The optimal talk is computed in two steps.

[Step 1] Find $P_{\Theta'}(h_0,\sigma_1')$ given each σ_1', where $h_0 = \varnothing$. To save space, let $P_{\Theta'} = P_{\Theta'}(h_0,\sigma_1')$ in the following table:

	$P_{\Theta(3,0)}$	$P_{\Theta(2,1)}$	$P_{\Theta(1,2)}$	$P_{\Theta(0,3)}$
$\sigma_1' = (1,\varnothing,\varnothing)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	0
$\sigma_1' = (1,\varnothing,-1)$	0	$\frac{1}{2}$	$\frac{1}{2}$	0

[Step 2] Compute the conditional expected payoff:

$$\begin{aligned} &\left[\text{Expected payoff of } m_1 = 1 | \sigma_1 = \sigma_1'\right] \\ = &\begin{cases} 3 \cdot \frac{1}{4}\left(\frac{1}{2}\right)^{L-1} + \frac{2}{4}\left(\frac{1}{3}\right)\left(\frac{5}{6}\right)^{L-2} - \frac{1}{4}\left(\frac{1}{3}\right)\left(\frac{5}{6}\right)^{L-2} > 0 & \text{if } \sigma_1' = (1,\varnothing,\varnothing) \\ \frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{5}{6}\right)^{L-2}(2) + \frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{5}{6}\right)^{L-2}(-2) = 0 & \text{if } \sigma_1' = (1,\varnothing,-1) \end{cases} \end{aligned}$$

To compute the above mentioned expected payoffs, we use the results in Step 1. Also, find a probability that m_1 affects d given each θ.

Fix any $\sigma_1 = \sigma_1'$. Given $\theta' = (1,1,1)$, m_1 affects d w.p. $\left(\frac{1}{2}\right)^{L-1}$ as follows: m_1 affects m_L w.p. $\left(\frac{1}{2}\right)^{L-1}$, i.e., $\sigma_t \in \Sigma^{(1,0)}$ for $\forall\ t \in \{2,...,L\}$; hence, if m_1 affects m_L, m_1 affects v_t w.p. 1 for $\forall\ t$.

Given $\theta' \in \{(1,1,-1),(1,-1,-1)\}$, m_1 affects d w.p. $\frac{1}{3}\left(\frac{5}{6}\right)^{L-2}$ as follows: m_1 affects m_L w.p. $\frac{1}{3}\left(\frac{5}{6}\right)^{L-2}$, i.e., $\sigma_2 \in \Sigma^{(1,1)}$ and $\sigma_t \in \Sigma^{(0,1)} \cup \Sigma^{(1,1)} \cup \Sigma^{(1,0)}$ for $\forall t \in \{3,...,L\}$; hence, if m_1 affects m_L, m_1 affects v_t w.p. 1 for $\forall\ t$.

A.1.2 Player 2's talk and inferences

The optimal talk m_2 given $h_1 = m_1$ and $\sigma_1 = \sigma_1'$ is:

	$\sigma_2' = (1, \varnothing, \varnothing)$	$\sigma_2' = (1, \varnothing, -1)$
$m_1 = 1$	$m_2 = 1$	$m_2 = m_1$
$m_1 = 0$	$m_2 = 0$	$m_2 = m_1$

The optimal talk is computed in two steps.

[Step 1] Find $P_{\Theta'}(h_1', \sigma_2')$ given σ_2' for each $h_1' = m_1$. Let $P_{\Theta'} = P_{\Theta'}(h_1', \sigma_2')$ in two tables below. Given $\sigma_2' = (1, \varnothing, \varnothing)$:

	$P_{\Theta(3,0)}$	$P_{\Theta(2,1)}$	$P_{\Theta(1,2)}$	$P_{\Theta(0,3)}$
$m_1 = 1$	$\frac{1}{1+2\left(\frac{2}{3}\right)+\frac{1}{3}} = \frac{3}{8}$	$\frac{2\left(\frac{2}{3}\right)}{1+2\left(\frac{2}{3}\right)+\frac{1}{3}} = \frac{4}{8}$	$\frac{\frac{1}{3}}{1+2\left(\frac{2}{3}\right)+\frac{1}{3}} = \frac{1}{8}$	0
$m_1 = 0$	0	$\frac{2\left(\frac{1}{3}\right)}{2\left(\frac{1}{3}\right)+\frac{2}{3}} = \frac{1}{2}$	$\frac{\frac{2}{3}}{2\left(\frac{1}{3}\right)+\frac{2}{3}} = \frac{1}{2}$	0

Given $\sigma_2' = (1, \varnothing, -1)$:

	$P_{\Theta(3,0)}$	$P_{\Theta(2,1)}$	$P_{\Theta(1,2)}$	$P_{\Theta(0,3)}$
$m_1 = 1$	0	$\frac{\frac{2}{3}}{\frac{2}{3}+\frac{1}{3}} = \frac{2}{3}$	$\frac{\frac{1}{3}}{\frac{2}{3}+\frac{1}{3}} = \frac{1}{3}$	0
$m_1 = 0$	0	$\frac{\frac{1}{3}}{\frac{1}{3}+\frac{2}{3}} = \frac{1}{3}$	$\frac{\frac{2}{3}}{\frac{1}{3}+\frac{2}{3}} = \frac{2}{3}$	0

[Step 2] Compute the conditional expected payoff:

$$\left[\text{Expected payoff of } m_2 = 1 | \sigma_2 = \sigma_2', h_1 = m_1\right]$$
$$\begin{cases} \begin{array}{c} 3 \cdot \frac{3}{8}\left(\frac{1}{2^{L-2}}\right)\left(1-\frac{1}{2L}\right)+\frac{4}{8}\left(\frac{5}{6}\right)^{L-2}\left(1-\frac{1}{4L}\right) \\ -\frac{1}{8}\left(\frac{5}{6}\right)^{L-2} > 0 \end{array} & \text{if } m_1 = 1 \& \ \sigma_2 = (1, \varnothing, \varnothing) \\ \frac{1}{2}\left(\frac{5}{6}\right)^{L-2} - \frac{1}{2}\left(\frac{5}{6}\right)^{L-2}\left(1-\frac{1}{4L}\right) > 0 & \text{if } m_1 = 0 \ \& \ \sigma_2 = (1, \varnothing, \varnothing) \\ \left(\frac{2}{3}\right)\left(\frac{5}{6}\right)^{L-2}\left(1-\frac{1}{4L}\right) - \left(\frac{1}{3}\right)\left(\frac{5}{6}\right)^{L-2} > 0 & \text{if } m_1 = 1 \ \& \ \sigma_2 = (1, \varnothing, -1) \\ \frac{1}{3}\left(\frac{5}{6}\right)^{L-2} - \frac{2}{3}\left(\frac{5}{6}\right)^{L-2}\left(1\text{-}\frac{1}{4L}\right) < 0 & \text{if } m_1 = 0 \ \& \ \sigma_2 = (1, \varnothing, -1) \end{cases}$$

To compute the above mentioned expected payoffs, we use the results in Step 1. Also, find a probability that m_2 affects d given each θ fixing m_1. The probability remains unchanged for $\sigma_2 \in \{(1, \varnothing, \varnothing), (1, \varnothing, -1)\}$.

(i) Fix $m_1 = 1$ and any σ_2. Given $\theta' = (1,1,1)$, m_2 affects d w.p. $\frac{1}{2^{L-2}}\left(1-\frac{1}{2L}\right)$ as follows: m_2 affects m_L w.p. $\frac{1}{2^{L-2}}$, i.e. $\sigma^t \in \Sigma^{(1,0)}$ for $\forall\ t \in \{3,...,L\}$. $m_1 = 1$ implies $\sigma_1 \in \Sigma^{(1,0)} \cup \Sigma^{(2,0)}$; hence, if m_2 affects m_L, m_2 affects v_1 and v_t for $\forall\ t \in \{2,...,L\}$ w.p. $\frac{1}{2}$ and 1, respectively.

Given $\theta' = (1,1,-1)$, m_2 affects d w.p. $\left(\frac{5}{6}\right)^{L-2}\left(1-\frac{1}{4L}\right)$ as follows: m_2 affects m_L w.p.$\left(\frac{5}{6}\right)^{L-2}$, i.e. $\sigma^t \in \Sigma^{(0,1)} \cup \Sigma^{(1,1)} \cup \Sigma^{(1,0)}$ for $\forall\ t \in \{3,...,L\}$. $m_1 = 1$ implies $\sigma_1 \in \Sigma^{(1,1)} \cup \Sigma^{(1,0)} \cup \Sigma^{(2,0)}$; hence, if m_2 affects m_L, m_2 affects v_1 and v_t for $\forall\ t \in \{2,...,L\}$ w.p. $\frac{3}{4}$ and 1, respectively.

Given $\theta' = (1,-1,-1)$, m_2 affects d w.p. $\left(\frac{5}{6}\right)^{L-2}$as follows: m_2 affects m_L w.p. $\left(\frac{5}{6}\right)^{L-2}$, i.e., $\sigma^t \in \Sigma^{(0,1)} \cup \Sigma^{(1,1)} \cup \Sigma^{(1,0)}$ for $\forall\ t \in \{3,...,L\}$. m_1=1 implies $\sigma_1 \in \Sigma^{(1,1)} \cup \Sigma^{(1,0)}$; hence, if m_2 affects m_L, m_2 affects $v_{\prime}$ w.p. 1 for $\forall\ t$.

(ii) Now, fix $m_1 = 0$. Given $\theta' = (1,1,-1)$, m_2 affects d w.p. $\left(\frac{5}{6}\right)^{L-2}$as follows: m_2 affects m_L w.p. $\left(\frac{5}{6}\right)^{L-2}$, i.e., $\sigma^t \in \Sigma^{(0,1)} \cup \Sigma^{(1,1)} \cup \Sigma^{(1,0)}$ for $\forall\ t \in \{3,...,L\}$. $m_1 = 0$ implies $\sigma_1 \in \Sigma^{(0,1)} \cup \Sigma^{(1,1)}$. Hence, if m_2 affects m_L, m_2 affects $v_{\prime}$ w.p. 1 for $\forall\ t$.

Given $\theta' = (1,-1,-1)$, m_2 affects d w.p. $\left(\frac{5}{6}\right)^{L-2}\left(1-\frac{1}{4L}\right)$ as follows: m_2 affects m_L w.p. $\left(\frac{5}{6}\right)^{L-2}$, i.e., $\sigma^t \in \Sigma^{(0,1)} \cup \Sigma^{(1,1)} \cup \Sigma^{(1,0)}$ for $\forall\ t \in \{3,...,L\}$. $m_1 = 0$ implies $\sigma_1 \in \Sigma^{(1,1)} \cup \Sigma^{(0,1)} \cup \Sigma^{(0,2)}$. Hence, if m_2 affects m_L, m_2 affects v_1 and v_t for $\forall\ t \in \{2,...,L\}$ w.p. $\frac{3}{4}$ and 1, respectively.

A.1.3 Player t's talk and inferences, 2<t<L

Fix any $t \geq 3$. The optimal talk m_t given each $\sigma_t = \sigma'_t$ and $h'_{t-1} = (m_1,...,m_{t-1})$ is:

	$\sigma'_t = (1,\varnothing,\varnothing)$	$\sigma'_t = (1,\varnothing,-1)$
$m_1 = m_2 = m_{t-1} = 1$	$m_t = m_{t-1}$	$m_t = m_{t-1}$
$m_1 = 0$ & $m_2 = m_{t-1} = 1$	$m_t = m_{t-1}$	$m_t = m_{t-1}$
$m_2 = 0$ & $m_{t-1} = 1$	$m_t = m_{t-1}$	$m_t = m_{t-1}$
$m_1 = m_2 = m_{t-1} = 0$	$m_t = m_{t-1}$	$m_t = m_{t-1}$
$m_1 = 1$ & $m_2 = m_{t-1} = 0$	$m_t = m_{t-1}$	$m_t = m_{t-1}$
$m_2 = 1$ & $m_{t-1} = 0$	$m_t = m_{t-1}$	$m_t = m_{t-1}$

[Step 1] Find $P_{\Theta'}\left(h'_{t-1},\sigma'_t\right)$ given σ'_t for each h'_{t-1}. Let $P_{\Theta'} = P_{\Theta'}\left(h'_{t-1},\sigma'_t\right)$ in tables below. For $\sigma'_t = (1, \varnothing, \varnothing)$:

h_{t-1}	$P_{\Theta(3,0)}$	$P_{\Theta(2,1)}$	$P_{\Theta(1,2)}$	$P_{\Theta(0,3)}$
$m_1 = m_2 = m_{t-1} = 1$	$\frac{1}{1+\frac{10}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{t-3}}$	$\frac{\frac{10}{9}}{1+\frac{10}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{t-3}}$	$\frac{\frac{1}{6}\left(\frac{5}{6}\right)^{t-3}}{1+\frac{10}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{t-3}}$	0
$m_1 = 0$ & $m_2 = m_{t-1} = 1$	0	$\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{9}\left(\frac{5}{6}\right)^{t-3}}$	$\frac{\frac{1}{9}\left(\frac{5}{6}\right)^{t-3}}{\frac{1}{3}+\frac{1}{9}\left(\frac{5}{6}\right)^{t-3}}$	0
$m_2 = 0$ & $m_{t-1} = 1$	0	1	0	0
$m_1 = m_2 = m_{t-1} = 0$	0	$\frac{\frac{1}{3}\left(\frac{5}{6}\right)^{t-3}}{\frac{1}{3}\left(\frac{5}{6}\right)^{t-3}+\frac{5}{9}}$	$\frac{\frac{5}{9}}{\frac{1}{3}\left(\frac{5}{6}\right)^{t-3}+\frac{5}{9}}$	0
$m_1 = 1$ & $m_2 = m_{t-1} = 0$	0	$\frac{\frac{2}{9}\left(\frac{5}{6}\right)^{t-3}}{\frac{2}{9}\left(\frac{5}{6}\right)^{t-3}+\frac{5}{18}}$	$\frac{\frac{5}{18}}{\frac{2}{9}\left(\frac{5}{6}\right)^{t-3}+\frac{5}{18}}$	0
$m_2 = 1$ & $m_{t-1} = 0$	0	0	1	0

For $\sigma'_t = (1, \varnothing, -1)$:

h_{t-1}	$P_{\Theta(3,0)}$	$P_{\Theta(2,1)}$	$P_{\Theta(1,2)}$	$P_{\Theta(0,3)}$
$m_1 = m_2 = m_{t-1} = 1$	0	$\frac{\frac{5}{9}}{\frac{5}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{t-3}}$	$\frac{\frac{1}{6}\left(\frac{5}{6}\right)^{t-3}}{\frac{5}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{t-3}}$	0
$m_1 = 0$ & $m_2 = m_{t-1} = 1$	0	$\frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{9}\left(\frac{5}{6}\right)^{t-3}}$	$\frac{\frac{1}{9}\left(\frac{5}{6}\right)^{t-3}}{\frac{1}{6}+\frac{1}{9}\left(\frac{5}{6}\right)^{t-3}}$	0
$m_2 = 0$ & $m_{t-1} = 1$	0	1	0	0
$m_1 = m_2 = m_{t-1} = 0$	0	$P_{\Theta(2,1)}$	$P_{\Theta(1,2)}$	0
$m_1 = 1$ & $m_2 = m_{t-1} = 0$	0	$P_{\Theta(2,1)}$	$P_{\Theta(1,2)}$	0
$m_2 = 1$ & $m_{t-1} = 0$	0	0	1	0

[Step 2] Compute the conditional expected payoff as follows.

$$
\begin{aligned}
&\left[\text{Expected payoff of } m_t = 1 | \sigma_t = \sigma_t', h_{t-1} = (m_1, ..., m_{t-1})\right] \\
&= \begin{cases}
\begin{array}{l} 3 \cdot \frac{1}{1+\frac{10}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{t-3}} \left(\frac{1}{2}\right)^{L-t} \left(1 - \frac{t-1}{2L}\right) \\ + \frac{\frac{10}{9}}{1+\frac{10}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{t-3}} \left(\frac{5}{6}\right)^{L-t} \left(1 - \frac{10t-3}{60L}\right) \\ - \frac{\frac{1}{6}\left(\frac{5}{6}\right)^{t-3}}{1+\frac{10}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{t-3}} \left(\frac{5}{6}\right)^{L-t} > 0 \end{array} & \text{if } \begin{array}{l} m_1{=}m_2{=}m_{t-1}{=}1 \\ \sigma_t'{=}\,(1, \varnothing, \varnothing) \end{array} \\
\frac{\frac{5}{9}}{\frac{5}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{t-3}} \left(\frac{5}{6}\right)^{L-t} \left(1 - \frac{10t-3}{60L}\right) - \frac{\frac{1}{6}\left(\frac{5}{6}\right)^{t-3}}{\frac{5}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{t-3}} \left(\frac{5}{6}\right)^{L-t} > 0 & \text{if } \begin{array}{l} m_1{=}m_2{=}m_{t-1}{=}1 \\ \sigma_t'{=}\,(1, \varnothing, -1) \end{array} \\
\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{9}\left(\frac{5}{6}\right)^{t-3}} \left(\frac{5}{6}\right)^{L-t} \left(1\text{-}\frac{t}{6L}\right) - \frac{\frac{1}{9}\left(\frac{5}{6}\right)^{t-3}}{\frac{1}{3}+\frac{1}{9}\left(\frac{5}{6}\right)^{t-3}} \left(\frac{5}{6}\right)^{L-t} \left(1 - \frac{1}{4L}\right) > 0 & \text{if } \begin{array}{l} m_1{=}0{,}m_2{=}m_{t-1}{=}1 \\ \sigma_t'{=}\,(1, \varnothing, \varnothing) \end{array} \\
\frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{9}\left(\frac{5}{6}\right)^{t-3}} \left(\frac{5}{6}\right)^{L-t} \left(1 - \frac{t}{6L}\right) - \frac{\frac{1}{9}\left(\frac{5}{6}\right)^{t-3}}{\frac{1}{6}+\frac{1}{9}\left(\frac{5}{6}\right)^{t-3}} \left(\frac{5}{6}\right)^{L-t} \left(1 - \frac{1}{4L}\right) > 0 & \text{if } \begin{array}{l} m_1{=}\text{-}{,}m_2{=}m_{t-1}{=}1 \\ \sigma_t'{=}\,(1, \varnothing, -1) \end{array} \\
\frac{\frac{1}{3}\left(\frac{5}{6}\right)^{t-3}}{\frac{1}{3}\left(\frac{5}{6}\right)^{t-3}+\frac{5}{9}} \left(\frac{5}{6}\right)^{L-t} - \frac{\frac{5}{9}}{\frac{1}{3}\left(\frac{5}{6}\right)^{t-3}+\frac{5}{9}} \left(\frac{5}{6}\right)^{L-t} \left(1 - \frac{10t-3}{60L}\right) < 0 & \text{if } \begin{array}{l} m_1{=}m_2{=}m_{t-1}{=}0 \\ \sigma_t'{=}\,(1, \varnothing, \varnothing) \end{array} \\
\begin{array}{l} \frac{\frac{2}{9}\left(\frac{5}{6}\right)^{t-3}}{\frac{2}{9}\left(\frac{5}{6}\right)^{t-3}+\frac{5}{18}} \left(\frac{5}{6}\right)^{L-t} \left(1 - \frac{1}{4L}\right) - \frac{\frac{5}{18}}{\frac{2}{9}\left(\frac{5}{6}\right)^{t-3}+\frac{5}{18}} \left(\frac{5}{6}\right)^{L-t} \left(1 - \frac{10t-3}{60L}\right) \\ < 0 \end{array} & \text{if } \begin{array}{l} m_1{=}1,\, m_2{=}m_{t-1}{=}0 \\ \sigma_t'{=}\,(1, \varnothing, \varnothing) \end{array}
\end{cases}
\end{aligned}
$$

Given $\sigma_t' = (1, \varnothing, -1)$, "$m_1 = m_2 = m_{t-1} = 1$" and "$m_1 = m_2 = m_{t-1} = 0$" ("$m_1 = 0$ & $m_2 = m_{t-1} = 1$" and "$m_1 = 1$ & $m_2 = m_{t-1} = 0$") are symmetry to each other.

To compute the above mentioned expected payoffs, we use the results in Step 1. Also, find a probability that m_t affects d given each θ fixing h_{t-1}, which remains unchanged for $\sigma_t \in \{(1, \varnothing, \varnothing), (1, \varnothing, -1)\}$.

(i) So, fix any σ_t and $m_1 = m_2 = m_{t-1} = 1$. Given $\theta' = (1, 1, 1)$, m_t affects d w.p.

$$
\tfrac{1}{2^{L-t}} \tfrac{1}{L} \left(L - t + 1 + \tfrac{t-1}{2}\right) \;\; = \tfrac{1}{2^{L-t}} \left(1 - \tfrac{t-1}{2L}\right)
$$

as follows: m_t affects m_L w.p. $\frac{1}{2^{L-t}}$, i.e., $\sigma^t \in \Sigma^{(1,0)}$ for $\forall\ t \in \{t+1, ..., L\}$. h_{t-1} implies $\sigma_t \in \Sigma^{(1,0)} \cup \Sigma^{(2,0)}$ for $\forall\ t \in \{1, ..., L\} \setminus \{t\}$; hence, if m_t affects m_L, m_t affects v_t for $\forall$ $t \in \{1, 2, ..., t-1\}$ and $v_{t'}$ for $\forall\ t' \in \{t, t+1, ..., L\}$ w.p. $\frac{1}{2}$ and 1, respectively.

Given $\theta' = (1, 1, -1)$, m_t affects d w.p.

$$
\left(\tfrac{5}{6}\right)^{L-t} \tfrac{1}{L} \left(L - t + 1 + \tfrac{3}{4} + \tfrac{4}{5} + \tfrac{5(t-3)}{6}\right) \;\; = \left(\tfrac{5}{6}\right)^{L-t} \left(1 - \tfrac{10t-3}{60L}\right)
$$

as follows: m_t affects m_L w.p. $\left(\frac{5}{6}\right)^{L-t}$, i.e., $\sigma_t \in \Sigma^{(0,1)} \cup \Sigma^{(1,1)} \cup \Sigma^{(1,0)}$ for $\forall\ t \in \{t+1, ..., L\}$. h_{t-1} implies:

$$\sigma_t \in \Sigma^{(1,1)} \cup \Sigma^{(1,0)} \cup \Sigma^{(2,0)} \text{ for } t \in \{1, 2\}.$$
$$\sigma_{t'} \in \Sigma^{(0,1)} \cup \Sigma^{(1,1)} \cup \Sigma^{(1,0)} \cup \Sigma^{(2,0)} \text{ for } t' \in \{3, 4, ..., t-1\}.$$

Hence, if m_t affects m_L, m_t affects v_1, v_2, v_t for $\forall\ t \in \{3, 4, ..., t-1\}$ and $v_{t'}$ for $\forall\ t' \in \{t, t+1, ..., L\}$ w.p. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$ and 1, respectively.

Given $\theta' = (1, -1, -1)$, m_t affects d w.p. $\left(\frac{5}{6}\right)^{L-t}$as follows: m_t affects m_L w.p. $\left(\frac{5}{6}\right)^{L-t}$, i.e.,

$$\sigma_t \in \Sigma^{(0,1)} \cup \Sigma^{(1,1)} \cup \Sigma^{(1,0)} \text{ for} \forall t \in \{t+1, ..., L\}.$$

h_{t-1} implies:

$$\sigma_t \in \Sigma^{(1,1)} \cup \Sigma^{(1,0)} \text{ for } t \in \{1, 2\}.$$
$$\sigma_t \in \Sigma^{(0,1)} \cup \Sigma^{(1,1)} \cup \Sigma^{(1,0)} \text{ for } t \in \{3, 4, ..., t-1\}.$$

Hence, if m_t affects m_L, m_t affects v_t for $\forall\ t$ w.p. 1.

(ii) Fix $m_1 = 0$ and $m_2 = m_{t-1} = 1$. Given $\theta' = (1, 1, -1)$, m_t affects d w.p. $\left(\frac{5}{6}\right)^{L-t}\left(1 - \frac{t}{6L}\right)$ as follows: h_{t-1} implies:

$$\sigma_1 \in \Sigma^{(1,1)} \cup \Sigma^{(0,1)} \ \& \ \sigma_2 \in \Sigma^{(1,0)} \cup \Sigma^{(2,0)},$$
$$\sigma_t \in \Sigma^{(1,1)} \cup \Sigma^{(0,1)} \cup \Sigma^{(1,0)} \cup \Sigma^{(2,0)} \text{ for } t \in \{3, 4, ..., t-1\},$$

Hence, if m_t affects m_L, m_t affects v_1, v_2 and v_t for $\forall\ t \in \{3, 4, ..., t-1\}$ and $v_{t'}$ for $\forall\ t' \in \{t, t+1, ..., L\}$ w.p. 1 , $\frac{1}{2}$, $\frac{5}{6}$ and 1, respectively.

Given $\theta' = (1, -1, -1)$, m_t affects d w.p.

$$\left(\frac{5}{6}\right)^{L-t} \frac{1}{L}\left(L - 1 + \frac{3}{4}\right) \ = \left(\frac{5}{6}\right)^{L-t}\left(1 - \frac{1}{4L}\right)$$

as follows: h_{t-1} implies:

$$\sigma_1 \in \Sigma^{(1,1)} \cup \Sigma^{(0,1)} \cup \Sigma^{(0,2)} \ \& \ \sigma_2 \in \Sigma^{(1,0)},$$
$$\sigma_t \in \Sigma^{(0,1)} \cup \Sigma^{(1,1)} \cup \Sigma^{(1,0)} \text{ for } t \in \{3, 4, ..., t-1\},$$

Hence, if m_t affects m_L, m_t affects v_1 and v_t for $t \in \{2, ..., L\}$ w.p. $\frac{3}{4}$ and 1, respectively.

(iii) Fix $m_1 = m_2 = m_{t-1} = 0$. Given $\theta' = (1, 1, -1)$, m_t affects d w.p. $\left(\frac{5}{6}\right)^{L-t}$as follows: h_{t-1} implies:

$$\sigma_t \in \Sigma^{(1,1)} \cup \Sigma^{(0,1)} \text{ for } t \in \{1,2\}\,,$$
$$\sigma_t \in \Sigma^{(1,0)} \cup \Sigma^{(1,1)} \cup \Sigma^{(0,1)} \text{ for } t \in \{3,4,...,t-1\}\,.$$

Hence, if m_t affects m_L, m_t affects affects v_t w.p. 1 for $\forall\ t$.

Given $\theta' = (1,-1,-1)$, m_t affects d w.p.

$$\frac{1}{L}\left(\frac{5}{6}\right)^{L-t}\left(L-t+1+\frac{3}{4}+\frac{4}{5}+\frac{5(t-3)}{6}\right) \ = \left(\frac{5}{6}\right)^{L-t}\left(1\text{-}\frac{10t-3}{60L}\right)$$

as follows: h_{t-1} implies:

$$\sigma_t \in \Sigma^{(1,1)} \cup \Sigma^{(0,1)} \cup \Sigma^{(0,2)} \text{ for } t \in \{1,2\}\,,$$
$$\sigma_t \in \Sigma^{(1,0)} \cup \Sigma^{(1,1)} \cup \Sigma^{(0,1)} \cup \Sigma^{(0,2)} \text{ for } t \in \{3,4,...,t-1\}\,,$$

Hence, if m_t affects m_L, m_t affects v_1, v_2, v_t for $\forall\ t \in \{3,4,...,t-1\}$ and $v_{t'}$ for $\forall\ t' \in \{t,t+1,...,L\}$ w.p. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$ and 1, respectively.

(iv) Fix $m_1 = 1$ and $m_2 = \cdots = m_{t-1} = 0$. Given $\theta' = (1,1,-1)$, m_t affects d w.p. $\left(\frac{5}{6}\right)^{L-t}\left(1-\frac{1}{4L}\right)$ as follows: h_{t-1} implies:

$$\sigma_1 \in \Sigma^{(2,0)} \cup \Sigma^{(1,0)} \cup \Sigma^{(1,1)}\ \&\ \sigma_2 \in \Sigma^{(0,1)},$$
$$\sigma_t \in \Sigma^{(1,0)} \cup \Sigma^{(1,1)} \cup \Sigma^{(0,1)} \text{ for } t \in \{3,4,...,t-1\}\,,$$

Hence, if m_t affects m_L, m_t affects v_1 and v_t for $\forall\ t \in \{2,...,L\}$ w.p. $\frac{3}{4}$ and 1, respectively.

Given $\theta' = (1,-1,-1)$, m_t affects d w.p. $\left(\frac{5}{6}\right)^{L-t}\left(1-\frac{t+2}{6L}\right)$ as follows: h_{t-1} implies:

$$\sigma_1 \in \Sigma^{(1,0)} \cup \Sigma^{(1,1)}\ \&\ \sigma_2 \in \Sigma^{(0,2)} \cup \Sigma^{(0,1)},$$
$$\sigma_t \in \Sigma^{(1,0)} \cup \Sigma^{(1,1)} \cup \Sigma^{(0,1)} \cup \Sigma^{(0,2)} \text{ for } t \in \{3,4,...,t-1\}\,.$$

Hence, if m_t affects m_L, m_t affects v_1, v_2, v_t for $\forall\ t \in \{3,4,...,t-1\}$ and $v_{t'}$ for $\forall\ t' \in \{t,t+1,...,L\}$ w.p. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$ and 1, respectively.

A.1.4 Player j's vote and inferences on attributes, j<l

Fix $j < L$. The optimal vote v_j given each $\sigma_j = \sigma'_j$ and $h'_L = (m_1, ..., m_L)$ is:

$h_L \backslash \sigma_t$	$\sigma_t = (1, \varnothing, \varnothing)$	$\sigma_t = (1, \varnothing, -1)$
$m_1 = m_2 = m_L = 1$	$v_j = m_L$	$v_j = m_L$
$m_1 = 0$ & $m_2 = m_L = 1$	$v_j = m_L$	$v_j = m_L$
$m_2 = 0$ & $m_L = 1$	$v_j = m_L$	$v_j = m_L$
$m_1 = m_2 = m_L = 0$	$v_j = m_L$	$v_j = m_L$
$m_1 = +$ & $m_2 = m_L = 0$	$v_j = m_L$	$v_j = m_L$
$m_2 = +$ & $m_L = 0$	$v_j = m_L$	$v_j = m_L$

The optimal vote is computed in two steps.

[Step 1] Find $P_{\Theta'}\left(h'_L, \sigma'_j\right)$ given σ'_j for each h'_L. Let $P_{\Theta'} = P_{\Theta'}\left(h'_L, \sigma'_j\right)$ in tables below. Given $\sigma'_j = (1, \varnothing, \varnothing)$:

h_L	$P_{\Theta(3,0)}$	$P_{\Theta(2,1)}$	$P_{\Theta(1,2)}$	$P_{\Theta(0,3)}$
$m_1 = m_2 = m_L = 1$	$\frac{1}{1+\frac{10}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{L-2}}$	$\frac{\frac{10}{9}}{1+\frac{10}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{L-2}}$	$\frac{\frac{1}{6}\left(\frac{5}{6}\right)^{L-2}}{1+\frac{20}{18}+\frac{1}{6}\left(\frac{5}{6}\right)^{L-2}}$	0
m_1=0 & m_2=m_L=1	0	$\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{9}\left(\frac{5}{6}\right)^{L-2}}$	$\frac{\frac{1}{9}\left(\frac{5}{6}\right)^{L-2}}{\frac{1}{3}+\frac{1}{9}\left(\frac{5}{6}\right)^{L-2}}$	0
$m_2 = 0$ & $m_L = 1$	0	1	0	0
$m_1 = m_2 = m_L = 0$	0	$\frac{\frac{1}{3}\left(\frac{5}{6}\right)^{L-2}}{\frac{1}{3}\left(\frac{5}{6}\right)^{L-2}+\frac{5}{9}}$	$\frac{\frac{5}{9}}{\frac{1}{3}\left(\frac{5}{6}\right)^{L-2}+\frac{5}{9}}$	0
$m_1 = 1$ & $m_2 = m_L = 0$	0	$\frac{\frac{2}{9}\left(\frac{5}{6}\right)^{L-2}}{\frac{2}{9}\left(\frac{5}{6}\right)^{L-2}+\frac{5}{18}}$	$\frac{\frac{5}{18}}{\frac{2}{9}\left(\frac{5}{6}\right)^{L-2}+\frac{5}{18}}$	0
$m_2 = 1$ & $m_L = 0$	0	0	1	0

For $\sigma'_j = (+, \varnothing, -)$:

h_L	$P_{\Theta(3,0)}$	$P_{\Theta(2,1)}$	$P_{\Theta(1,2)}$	$P_{\Theta(0,3)}$
$m_1 = m_2 = m_L = 1$	0	$\frac{\frac{5}{9}}{\frac{5}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{L-2}}$ $(*1)$	$\frac{\frac{1}{6}\left(\frac{5}{6}\right)^{L-2}}{\frac{5}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{L-2}}$ $(*2)$	0
$m_1 = 0$ & $m_2 = m_L = 1$	0	$\frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{9}\left(\frac{5}{6}\right)^{L-2}}$ $(*3)$	$\frac{\frac{1}{9}\left(\frac{5}{6}\right)^{L-2}}{\frac{1}{6}+\frac{1}{9}\left(\frac{5}{6}\right)^{L-2}}$ $(*4)$	0
$m_2 = 0$ & $m_L = 1$	0	1	0	0
$m_1 = m_2 = m_L = 0$	0	(*2)	(*1)	0
$m_1 = 1$ & $m_2 = m_L = 0$	0	(*4)	(*3)	0
$m_2 = 1$ & $m_L = 0$	0	0	1	0

Given $\sigma'_j = (1, \varnothing, -1)$, "$m_1 = m_2 = m_{L-1} = 1$" and "$m_1 = m_2 = m_{L-1} = 0$" ("$m_1 = 0$ & $m_2 = m_{L-1} = 1$" and "$m_1 = 1$ & $m_2 = m_{L-1} = 0$") are symmetry to each other.

[Step 2] Compute "the expected payoff of selecting $v_j = 1$" given each σ'_j and h'_L.

A.1.5 Player l's talk & vote and inferences

The optimal talk m_L given $h_{L-1} = (m_1, ..., m_{L-1})$ and $\sigma_L = \sigma'_L$ is:

	$\sigma'_L = (1, \varnothing, \varnothing)$	$\sigma'_L = (1, \varnothing, -1)$
$m_1 = m_2 = m_{L-1} = 1$	$m_L = m_{L-1}$	$m_L = m_{L-1}$
$m_1 = 0$ & $m_2 = m_{L-1} = 1$	$m_L = m_{L-1}$	$m_L = m_{L-1}$
$m_2 = 0$ & $m_{L-1} = 1$	$m_L = m_{L-1}$	$m_L = m_{L-1}$
$m_1 = m_2 = m_{L-1} = 0$	$m_L = m_{L-1}$	$m_L = m_{L-1}$
$m_1 = 1$ & $m_2 = m_{L-1} = 0$	$m_L = m_{L-1}$	$m_L = m_{L-1}$
$m_2 = 1$ & $m_{L-1} = 0$	$m_L = m_{L-1}$	$m_L = m_{L-1}$

The optimal talk is computed in two steps.

[Step 1] Find $P_{\Theta'}(h'_1, \sigma'_2)$ given σ'_2 for each h'_1=m_1. To save space, let $P_{\Theta'}$=$P_{\Theta'}(h'_1, \sigma'_2)$

in tables below. Given σ'_L=$(1, \varnothing, \varnothing)$:

h_{L-1}	$P_{\Theta(3,0)}$	$P_{\Theta(2,1)}$	$P_{\Theta(1,2)}$	$P_{\Theta(0,3)}$
$m_1 = m_2 = m_{L-1} = 1$	$\frac{1}{1+\frac{10}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{L-3}}$	$\frac{\frac{10}{9}}{1+\frac{10}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{L-3}}$	$\frac{\frac{1}{6}\left(\frac{5}{6}\right)^{L-3}}{1+\frac{10}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{L-3}}$	0
$m_1 = 0$ & $m_2 = m_{L-1} = 1$	0	$\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{9}\left(\frac{5}{6}\right)^{L-3}}$	$\frac{\frac{1}{9}\left(\frac{5}{6}\right)^{L-3}}{\frac{1}{3}+\frac{1}{9}\left(\frac{5}{6}\right)^{L-3}}$	0
$m_2 = 0$ & $m_{L-1} = 1$	0	1	0	0
$m_1 = m_2 = m_{L-1} = 0$	0	$\frac{\frac{1}{3}\left(\frac{5}{6}\right)^{L-3}}{\frac{1}{3}\left(\frac{5}{6}\right)^{L-3}+\frac{5}{9}}$	$\frac{\frac{5}{9}}{\frac{1}{3}\left(\frac{5}{6}\right)^{L-3}+\frac{5}{9}}$	0
$m_1 = 1$ & $m_2 = m_{L-1} = 0$	0	$\frac{\frac{2}{9}\left(\frac{5}{6}\right)^{L-3}}{\frac{2}{9}\left(\frac{5}{6}\right)^{L-3}+\frac{5}{18}}$	$\frac{\frac{5}{18}}{\frac{2}{9}\left(\frac{5}{6}\right)^{L-3}+\frac{5}{18}}$	0
$m_2 = 1$ & $m_{L-1} = 0$	0	0	1	0

Given $\sigma'_L = (1, \varnothing, -1)$:

h_{L-1}	$P_{\Theta(3,0)}$	$P_{\Theta(2,1)}$	$P_{\Theta(1,2)}$	$P_{\Theta(0,3)}$
$m_1 = m_2 = m_{L-1} = 1$	0	$\frac{\frac{5}{9}}{\frac{5}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{L-3}}$	$\frac{\frac{1}{6}\left(\frac{5}{6}\right)^{L-3}}{\frac{5}{9}+\frac{1}{6}\left(\frac{5}{6}\right)^{L-3}}$	0
$m_1 = 0$ & $m_2 = m_{L-1} = 1$	0	$\frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{9}\left(\frac{5}{6}\right)^{L-3}}$	$\frac{\frac{1}{9}\left(\frac{5}{6}\right)^{L-3}}{\frac{1}{6}+\frac{1}{9}\left(\frac{5}{6}\right)^{L-3}}$	0
$m_2 = 0$ & $m_{L-1} = 1$	0	1	0	0
$m_1 = m_2 = m_{L-1} = 0$	0	$P_{\Theta(2,1)}$	$P_{\Theta(1,2)}$	0
$m_1 = 1$ & $m_2 = m_{L-1} = 0$	0	$P_{\Theta(2,1)}$	$P_{\Theta(1,2)}$	0
$m_2 = 1$ & $m_{L-1} = 0$	0	0	1	0

Given $\sigma'_L = (1, \varnothing, -1)$, "$m_1 = m_2 = m_{L-1} = 1$" and "$m_1 = m_2 = m_{L-1} = 0$" ("$m_1 = 0$ & $m_2 = m_{L-1} = 1$" and "$m_1 = 1$ & $m_2 = m_{L-1} = 0$") are symmetry to each other.

The optimal v_L follows. The optimal m_L also follows by repeating the previous analysis for player $j \in \{3, 4, ..., L-1\}$. (Replace j with L.)

Appendix B (Attribute-based Talk)

B.1 Example 1

We describe a PBE which was informally introduced in Section 4.2. WLOG, we focus on each speaker's talk strategy when his private signal is $(1, 1, \varnothing)$ or $(1, \varnothing, \varnothing)$. For any speaker $t \in \{1, 2, 3\}$, his message strategy m_t given $\sigma_t \in \Sigma^{++}$ is symmetric to

m_t given $\sigma_t = (1, 1, \varnothing)$ and $\sigma_t = (1, \varnothing, \varnothing)$, respectively. m_t given any $\sigma_t \in \Sigma^{--}$ and $\sigma_t \in \Sigma^{-}$ is symmetric to m_t given $\sigma_t \in \Sigma^{++}$ and $\sigma_t \in \Sigma^{+}$, respectively.

B.1.1 Speaker 1's talk

Fix $\sigma_1 = (1, 1, \varnothing)$. Then:

$$m_1 = (1, +) \text{ and } m_1 = (2, +) \quad \text{w.p. } \tfrac{1}{2} \text{ and } \tfrac{1}{2}\text{, respectively.}$$

Fix $\sigma_1 = (1, \varnothing, \varnothing)$. Then:

$$m_1 = (1, +)$$

B.1.2 Speaker 2's talk

Fix $\sigma_2 = (1, 1, \varnothing)$. Then:

$$m_2 = \begin{cases} (1, +) \text{ \& } (2, +) \text{ w.p. } \frac{1}{2} \text{ \& } \frac{1}{2}\text{, respectively, if } m_1 \notin \{(1, +), (2, +)\}, \\ (1, +) \text{ if } m_1 = (2, +), \\ (2, +) \text{ if } m_1 = (1, +), \\ (1, +) \text{ if } m_1 = (1, -), \\ (2, +) \text{ if } m_1 = (2, -). \end{cases}$$

Fix $\sigma_2 = (1, \varnothing, \varnothing)$. Then:

$$m_2 = (1, +) \text{ for any } m_1.$$

B.1.3 Speaker 3's talk

Fix $\sigma_3 = (1, 1, \varnothing)$. Then:

$$m_3 = \begin{cases} (1,+) \text{ \& } (2,+) \text{ w.p. } \frac{1}{2} \text{ \& } \frac{1}{2}\text{, respectively, if } m_1, m_2 \notin \{(1,+), (2,+)\} \\ (1,+) \text{ \& } (2,+) \text{ w.p. } \frac{1}{2} \text{ \& } \frac{1}{2}\text{, respectively, if } \{m_1, m_2\} = \{(1,+), (2,+)\} \\ (1,+) \text{ if } m_1 = m_2 = (2,+) \\ (2,+) \text{ if } m_1 = m_2 = (1,+) \\ (1,+) \text{ if } m_2 = (1,-) \\ (2,+) \text{ if } m_2 = (2,-) \\ (1,+) \text{ if } m_1 = (1,-) \text{ \& } m_2 \neq (1,+) \\ (2,+) \text{ if } m_1 = (2,-) \text{ \& } m_2 \neq (2,+) \end{cases}$$

Fix $\sigma_3 = (1, \varnothing, \varnothing)$. Then:

$$m_3 = \begin{cases} m_2 \text{ if } m_1 = m_2 \text{ \& } m_2 \in \{(2,-), (3,-)\} \\ (1,+) \text{ otherwise} \end{cases}$$

B.1.4 DM's Decision

For any $i, j \in \{1, 2, 3\}$ and any $s \in \{1, 2, 3\}$, we say, "m_i and m_j are *not consistent* (or *inconsistent*) with each other," if $m_i = (s, +)$ and $m_j = (s, -)$, and we say "m_i and m_j are *consistent* with each other," otherwise.

Suppose that m_1, m_2 and m_3 are consistent with one another. Then, DM chooses $d = 1$ if one of (1) through (3) holds:

$$\begin{cases} (1)\ m_i = (s,+),\ m_j = (t,+) \text{ and } m_i \neq m_j \text{ for any } i, j \in \{1,2,3\}, \\ (2)\ m_1 = m_2 = m_3 = (s,+), \\ (3)\ m_1 = m_2 = (s,-) \text{ and } m_3 = (t,+), \end{cases}$$

DM chooses $d = 0$ if one of (4) through (6) holds:

$$\begin{cases} (4)\ m_i = (s,-),\ m_j = (t,-) \text{ and } m_i \neq m_j \text{ for any } i, j \in \{1,2,3\}, \\ (5)\ m_1 = m_2 = m_3 = (s,-), \\ (6)\ m_1 = m_2 = (s,+) \text{ and } m_3 = (t,-), \end{cases}$$

and DM chooses $d = 1$ and $d = 0$ w.p. $\frac{1}{2}$ and $\frac{1}{2}$, respectively, if one of (7) through (10) holds:

$$\begin{cases} (7)\ m_1 = m_3 = (s,+) \text{ and } m_2 = (t,-), \\ (8)\ m_1 = m_3 = (s,-) \text{ and } m_2 = (t,+), \\ (9)\ m_1 = (s,+) \text{ and } m_2 = m_3 = (t,-), \\ (10)\ m_1 = (s,-) \text{ and } m_2 = m_3 = (t,+), \end{cases}$$

for any $s, t \in \{1,2,3\}$ in all cases, (1) through (10).

Suppose that m_1 is not consistent with m_2 or/and m_3 while m_2 is consistent with m_3. Then:

$$d = \begin{cases} 1 \text{ if } m_2 = (s,+) \text{ and } m_3 = (t,+) \text{ for any } s,t \in \{1,2,3\}, \\ 0 \text{ if } m_2 = (s,-) \text{ and } m_3 = (t,-) \text{ for any } s,t \in \{1,2,3\}, \\ 1 \text{ \& } 0 \text{ w.p. } \frac{1}{2} \text{ \& } \frac{1}{2}, \text{ respectively, otherwise.} \end{cases}$$

Suppose that m_1 and m_2 are both not consistent with m_3.Then:

$$d = \begin{cases} 1 \text{ if } m_3 = (s,+) \text{ for any } s \in \{1,2,3\}, \\ 0 \text{ if } m_3 = (s,-) \text{ for any } s \in \{1,2,3\}, \end{cases}$$

B.2 Example 2

We only explain each speaker's talk strategy given $\sigma_j \in \{(1,1,\varnothing), (1,\varnothing,\varnothing), (1,\varnothing,-1)\}$ and every possible history.

B.2.1 Speaker 1's talk

Fix $\sigma_1 = (1,1,\varnothing)$. Then:

$$m_1 = (\varnothing,+).$$

Fix $\sigma_1 = (1,\varnothing,\varnothing)$. Then:

$$m_1 = (1,+).$$

Fix $\sigma_1 = (1,\varnothing,-1)$. Then:

$$m_1 = (1,+) \text{ and } (3,-) \quad \text{w.p. } \tfrac{1}{2} \text{ and } \tfrac{1}{2}, \text{ respectively.}$$

B.2.2 Speaker 2's talk

Fix $\sigma_2 = (1, 1, \varnothing)$. Then:

$$m_2 = \begin{cases} (i, +) \text{ if } m_1^r = (j, +) \text{ where } i, j \in \{1, 2\}, i \neq j \\ (\varnothing, +) \text{ otherwise.} \end{cases}$$

Fix $\sigma_2 = (1, \varnothing, \varnothing)$. Then:

$$m_2 = \begin{cases} (\varnothing, -) \text{ if } m_1^r = (\varnothing, -) \\ (1, +) \text{ otherwise.} \end{cases}$$

Fix $\sigma_2 = (1, \varnothing, -1)$. Then:

$$m_2 = \begin{cases} (1, +) \text{ if } m_1^r \in \{(\varnothing, +), (2, +), (3, -)\} \\ (3, -) \text{ otherwise (i.e., } m_1^r \in \{(2, -), (\varnothing, -), (1, +)\}). \end{cases}$$

B.2.3 Speaker 3's talk

Fix $\sigma_3 = (1, 1, \varnothing)$. Then:

$$m_3 = \begin{cases} (2, +) \text{ if } (1, +) \in \{m_1^r, m_2^r\} \\ (1, +) \text{ if } (2, +) \in \{m_1^r, m_2^r\} \text{ but } (1, +) \notin \{m_1^r, m_2^r\} \\ (\varnothing, +) \text{ otherwise.} \end{cases}$$

Fix $\sigma_3 = (1, \varnothing, \varnothing)$. Then:

$$m_3 = \begin{cases} (1, +) \text{ if } m_1^r = m_2^r = (i, -) \text{where } i = 2, 3 \\ (1, +) \text{ if } m_2^r = (\cdot, +) \\ (\cdot, -) \text{ otherwise.} \end{cases}$$

Fix $\sigma_3 = (1, \varnothing, -1)$. Then:

$$m_3 = \begin{cases} (1, +) & \text{if } m_1^r = m_2^r = (3, -) \\ (1, +) & \text{if "} m_2^r = (\cdot, +) \text{ \& Not } [m_1^r = m_2^r = (1, +)]\text{"} \\ (3, -) & \text{if } m_1^r = m_2^r = (1, +) \\ (3, -) & \text{otherwise (i.e., if "} m_2^r = (\cdot, -) \text{ \& Not } [m_1^r = m_2^r = (3, -)]\text{").} \end{cases}$$