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A General Analysis

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Pass-Through and the Welfare Effects of Taxation under Imperfect Competition: A General Analysis*

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Abstract

This paper provides a comprehensive analysis of welfare effects of taxation under imperfect competition. Specifically, in relation to tax pass-through, we provide "sufficient statistics" formulas for two welfare measures under a fairly general class of demand, production cost, and market competition. The measures are (i) marginal value of public funds (i.e., the marginal loss of social welfare due to an increase in government revenue), and (ii) incidence (i.e., the ratio of a marginal change in consumer surplus to a marginal change in producer surplus). We begin with the case of symmetric firms facing both unit and ad valorem taxes to derive a simple and empirically relevant set of formulas. Then, we provide a substantial generalization of these results to encompass firm heterogeneity by using the idea of tax revenue specified as a general function parameterized by a vector of tax parameters.

Keywords: Imperfect Competition; Pass-through; Marginal Value of Public Funds; Incidence; Sufficient Statistics

JEL classification numbers: D43, H22, L13

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1 Introduction

In thinking of market intervention such as taxation, it is essential to understand how such a policy change distorts economic welfare. Policymakers might also be concerned about how the tax burden is borne by consumers, or in general individuals, and firms subject to such a change in tax policy. A convenient framework for studying this question in the context of commodity taxation is presented by Weyl and Fabinger (2013) who find an important role of *pass-through*, the impact of an infinitesimal change in the marginal cost of production on the equilibrium price, or of an infinitesimal change a change in *unit* tax.[1] Extending that framework, this paper provides a substantial generalization of Weyl and Fabinger's (2013) model to include *ad valorem* tax—another important tax instrument—under general forms of market demand, production cost, and, in particular, imperfect competition. Notably, our framework is readily extendible to the case of heterogeneous firms.[2] In addition, we allow for pre-existing (i.e., non-zero) taxes of either type. These two features imply that there is little gap between our theoretical model and empirically relevant settings of interest, although how pass-through and imperfect competition matter is more easily understood under the assumption of firm symmetry and zero initial taxes as shown in the first part of Section 2. We also argue (in Appendix C) that our analysis of two-dimensional taxation opens up a methodology to encompass more general cases of multiple interventions such as combinations of taxation and other market regulations.

Specifically, we generalize Anderson, de Palma, and Kreider's (2001a) and Häckner and Herzing's (2016) analyses of specific and ad valorem taxation under imperfect competition to derive "sufficient statistics" formulas expressed in terms of observable and estimable variables such as elasticities. These

[1]The importance of (uni-dimensional) pass-through has been long recognized in empirical studies of homogeneous or differentiated product markets in oligopoly, see, e.g., Kim and Cotterill (2008); Bonnet and Réquillart (2013); Jametti, Redonda, Sen (2013); Shrestha and Markowitz (2016); Duso and Szücs (2017); Miller, Osborne, and Sheu (2017); Griffith, Nesheim, and O'Connell (2018); Stolper (2018); Campos-Vázquez and Medina-Cortina (2019); Conlon and Rao (2019, 2020); Ganapati, Shapiro, and Walker (2020); Muehlegger and Sweeney (2019); and Genakos and Pagliero (2021).

[2]From the viewpoint of optimal taxation without entry/exit, it is well known that ad valorem taxes are more efficient (i.e., less welfare distorting) than unit taxes in raising the same amount of tax revenue (see Wicksell 1896; Suits and Musgrave 1953; and Delipalla and Keen 1992 for earlier studies), which implies that no unit taxes should be used (however, in the presence of negative externalities such as pollution, see unit taxes can be superior to ad valorem taxes; see Pirttilä 2002). In our generalized framework, this is also verified from Proposition 3 under the setting of firm heterogeneity. However, this is not always the case once cost heterogeneity between firms is allowed, as pointed out firstly by Anderson, de Palma, and Kreider (2001b). It is also verified in Section 4 below. Moreover, in reality, specific and ad valorem taxes are often used together for commodity taxation (e.g., gasoline, alcohols, tobaccos, sodas, etc), which suggests the relevance of studying both taxes in a unified framework.

formulas relate pass-through of the taxes to (i) *marginal value of public funds* (MVPF) and (ii) *incidence*, i.e., the ratio of a marginal change in consumer surplus to a marginal change in producer surplus.[3] We also generalize Weyl and Fabinger's (2013) analysis in this dimension because they do not focus on MVPF. Here, MVPF means a simple benefit/cost ratio that measures individuals' willingness-to-pay for a change of tax rate per additional government revenue (Marsha 1990; Slemrod and Yitzhaki 2001; Kleven and Kreiner 2006; Hendren 2016; and Hendren and Sprung-Keyser 2020).[4] In addition, we complement Weyl and Fabinger's (2013) analysis by providing graphical illustrations to facilitate an intuitive understanding of the welfare properties of commodity taxation in a broader setting of imperfect competition.

The welfare properties of taxation have been extensively studied since, at least, Pigou (1928). A majority of existing studies simply assume perfect competition (and zero pre-existing taxes).[5] As is widely known, unit and ad valorem taxes are equivalent in achieving the same level of revenue under this situation, and whether consumers or producers bear more is determined by the relative elasticities of demand and supply (Weyl and Fabinger, 2013, p. 534). Relaxing the assumption of perfect competition was initially attempted by the studies of homogeneous-product oligopoly under quantity competition, i.e., Cournot oligopoly. Notably, Delipalla and Keen (1992), Skeath and Trandel (1994), Hamilton (1999), and Anderson, de Palma, and Kreider (2001b) compare unit and ad valorem taxes in such a setting.[6] Anderson, de Palma, and Kreider (2001a), then, extend these results to the case of differentiated

[3] In this sense, our framework is aligned with the "sufficient statistics" approach (Chetty 2009; Kleven 2021) to connecting structural and reduced-form methods. For example, in the study by Atkin and Donaldson (2016), the pass-through rate provides a sufficient statistic for welfare implications of intra-national trade costs in low-income countries, without the need for a full demand estimation. See also Ritz (2018) and references therein for theoretical studies on pass-through and pricing under imperfect competition, including monopolistic competition. In the context of third-degree price discrimination under imperfect competition, Adachi and Fabinger (2021) also share the same spirit as these two studies in that they also provide welfare formulas based on sufficient statistics, including pass-through, under fairly general conditions.

[4] This concept is in contrast to such traditional measures as the marginal cost of public funds (MCPF) and the marginal excess burden (MEB), which require accounting for welfare contributions from public spending and the effects of redistribution in measuring the welfare costs of raising tax revenue (see, e.g., Dahlby 2008). In this paper, the MVPF is a more appropriate measure because we do not explicitly consider public goods provision (see, e.g., Lockwood (2003) for such an analysis) and redistribution that may have an additional effect on welfare. Moreover, the MVPF focuses directly on causal, not on compensated, effects of public policy, and hence it is widely applicable in guiding cost-benefit analysis in a more systematic manner. We thank Nathan Hendren for making us realize this point.

[5] The early studies include Vickrey (1963), Buchanan and Tullock (1965), Johnson and Pauly (1969), and Browning (1976). See also Auerbach and Hines (2002) and Fullerton and Metcalf (2002) for comprehensive surveys of this field.

[6] More specifically, Delipalla and Keen (1992) firstly showed that ad valorem taxes are welfare superior to unit taxes with symmetric quantity-setting firms. Skeath and Trandel (1994) further strengthen Delipalla and Keen's (1992) results by showing the Pareto dominance: for a given level of unit tax under monopoly, there always exists an ad valorem tax that yields higher levels of all of consumer surplus, firm profits, and tax revenue. Under Cournot oligopoly, Skeath and Trandel (1994)

oligopoly under price competition. Specifically, they find that whether the after-tax price for firms and their profits rise by a change in ad valorem tax depends importantly on the ratio of the curvature of the firm's own demand to the elasticity of market demand.

In contrast to these previous studies, one appealing feature of our framework is that – as in Weyl and Fabinger (2013) and Kroft, Laliberté, Leal-Vizcaíno, and Notowidigdo (2020), among others – we use the *conduct index,* by which we mean *conduct parameter* that is not necessarily constant across the level of output. The conduct index measures the degree of market monopolization and hence nesting a variety of market structures. It allows us to work with a fairly general mode of market competition and to capture its complicated nature in reality: both from a theoretical and an empirical standpoint, it is desirable to understand the welfare properties of oligopolistic markets for a fairly general class of competition.[7] In real-world situations, firms' conduct might not simply be categorized into either idealized price competition or idealized quantity competition and includes the possibility of collusive behavior. An additional benefit of our general framework is that one does not necessarily have to assume constant marginal costs in conducting a welfare assessment. Miravete, Seim, and Thurk (2018) also stress the importance of imperfect competition in considering policy recommendations: They find empirical relevance of firms' strategic responses in pricing when evaluating the effect of taxation, implying the necessity of considering imperfect competition for policy evaluation. Whereas the existing literature that uses sufficient statistics in the spirit of Chetty (2009) and Kleven (2021) to study optimal taxation "typically abstract from any market power effects" (Miravete, Seim, and Thurk 2018, p. 1652), we are able to provide sufficient statistics formulas for the welfare measures that are useful for empirical study because we also accommodate firm heterogeneity, which cannot be neglected in almost any data. When firm heterogeneity is considered in Section 4, we introduce the *pricing strength index* that is firm-specific and measures the degree of the firm's market power. It is the related to the concept of conduct index, but is much better to work with when the firms are not identical. It turns out that our characterization of the

shows the same result holds if the required amount of tax revenue is sufficiently large, and this requirement depends on the demand curve and the number of firms in the market.

[7]The interested reader should refer to Weyl and Fabinger (2013) for examples of market structures that are nested by the conduct index approach. The concept of conduct parameter has been developed mainly in the empirical industrial organization literature (see, e.g., Bresnahan 1989 and Delipalla and O'Donnell 2001), and has also been successfully applied to such issues as selection markets (Mahoney and Weyl 2017), supply chains (Gaudin 2018; Adachi 2020) and two-sided markets (Adachi and Tremblay 2020). See Footnote 12 below for more details.

two welfare measures discussed above is readily extendible to the case of firm heterogeneity.

In this sense, we seek to respond to a commonly held view, particularly in the field of public finance, exemplified by the following quotations from two representative textbooks (emphasis added):

> "Unfortunately, there is *no well-developed theory of tax incidence in oligopoly*. [...] As economic behavior under oligopoly becomes better understood, improved models of incidence will be developed" (Rosen and Gayer 2014, pp. 310-311).
>
> "There is no widely accepted theory of firm behavior in oligopoly, so it is *impossible to make any definite predictions about the incidence of taxation in this case*" (Stiglitz and Rosengard 2015, p. 556).

In a similar vein, Kroft, Laliberté, Leal-Vizcaíno, and Notowidigdo (2020) also consider a comparison of ad valorem and unit taxes and derive a sufficient statistics formula for the welfare burden of commodity taxation as well as its incidence under imperfect competition, especially in consideration of the possibility of "behavioral" consumers having misconceptions about whether the price is tax inclusive. Specifically, they parameterize the degree of how accurately consumers attribute a change in consumer price to the change in tax behind and calibrate the marginal excess burden of commodity taxation by maintaining firm symmetry.[8] In contrast, we aim to provide general formulas for welfare measures that allow for firm heterogeneity as well. In this sense, their study and ours are complementary in providing structural frameworks that are useful for welfare evaluation in consideration of a variety of important policy issues under imperfect competition.

The remainder of this paper is organized as follows. In the next section, we construct our model of taxation under symmetric imperfect competition and present general formulas for marginal value of public funds and incidence in relation to unit tax and ad valorem tax pass-through and the elasticity of industry demand. In Section 3, we conduct a numerical analysis for these formulas. Then, Section 4

[8] In their study of tax revenue in the legalized cannabis market, Hollenbeck and Uetake (2021) also use the idea of sufficient statistics to estimate tax incidence and social cost of tax, based on the estimation of the conduct parameter as well as cost pass-through under firm symmetry and find that there is significant room for climbing up the Laffer curve from the left side, i.e., for raising a higher amount of tax revenue by an increase in the tax rate. Alternatively, it is also effective to strengthen the intensity of competition by deregulating the license cap for a higher amount of tax revenue. In a different vein, Montag, Sagimuldina, and Schnitzer (2021) propose a search model of heterogeneous consumers, where different consumers incur different costs of searching sellers and their prices and find that the tax pass-through is higher if the search cost becomes lower.

further generalizes our formulas to include heterogeneous firms. Finally, Section 5 concludes the paper. Note that some detailed arguments are delegated to the appendices. In particular, Appendix C provides a more general framework, which Section 4 is based on, to accommodate multi-dimensional interventions than simply (two-dimensional) specific and ad valorem taxes. We illustrate some applications of interest other than taxation such as a sales restriction due to, for instance, the outbreak of a pandemic, and tax evasion in Online Appendix C.

2 Specific and Ad Valorem Taxation under Symmetric Imperfect Competition

In this section, we study symmetric oligopoly. Before we start, let us point out that the formulas we derive are not much longer than the corresponding formulas for the special case of monopoly. We keep our derivations explicit to emphasize the logical flow, which generalizes beyond specific and ad valorem taxes and beyond symmetric firm oligopoly. We use figures as visual anchors to help the reader clearly understand the many welfare component changes and many forces that play a role in the discussion.

This section generalizes the results of Anderson, de Palma, and Kreider's (2001a) (APK) in several important directions. First, we consider a fairly general class of market competition, captured by the *conduct index* (see below), including both quantity and price competition. Second, we provide a complete characterization of welfare measures that enables one to quantitatively compare consumers' burden with producers' burden, whereas APK focus only on the effective prices for consumers and producers' profits. Third, while APK assume constant marginal cost, we permit non-constant marginal cost and show how this generalization makes a difference in our generalized formulas. Fourth, we further generalize the initial tax level. When they analyze the effects of a unit tax, APK assume that ad valorem tax is zero, and vice versa. In contrast, we allow non-zero initial taxes in both dimensions. Overall, it turns out that generalizing APK results of the two-dimensional tax problem is suggestive in studying a much wider range of interventions/taxes to characterize welfare measures in terms of sufficient statistics.

Below, we employ the standard assumption that the representative consumer has quasi-linear utility, $U(\mathbf{q}, y) = u(\mathbf{q}) + y$, where $\mathbf{q} \equiv (q_1, ..., q_n)$ is their consumption bundle from n single-product firms in the industry, and $y > 0$ is a numeraire outside good with no taxes. In effect, we assume that all markets outside this industry are perfectly competitive to isolate this particular market from such feedback effects as income effects that may arise in a general-equilibrium framework. A full-fledged analysis of "imperfect competition in general equilibrium" awaits further research in this direction (see, e.g., d'Aspremont and Dos Santos Ferreira 2021). We hereafter use t for specific taxes (unit taxes) and v for ad valorem taxes. In most applications, these would be non-negative.[9]

Then, following Kroft, Laliberté, Leal-Vizcaíno, and Notowidigdo (2020) and many others, we define social welfare W as $W = CS + PS + R$, where CS, PS, and R denote consumer surplus, producer surplus (corporate profit), and tax revenue, respectively. The main task of this paper is to characterize two important measures for the welfare effects of commodity taxation: (i) the marginal value of public funds $MVPF_T \equiv -\frac{\partial W/\partial T}{\partial R/\partial T}$, and (ii) the incidence $I_T \equiv \frac{\partial CS/\partial T}{\partial PS/\partial T}$ for $T \in \{t, v\}$.

2.1 Setup

Here we study an oligopolistic market with n symmetric firms and a general mode of competition, and consider the resulting symmetric equilibria. Formally, the demand for firm i's product $q_i = q_i(p_1, ..., p_n) \equiv q_j(\mathbf{p})$ depends on the vector of prices, $\mathbf{p} \equiv (p_1, ..., p_n)$, charged by the individual firms. The demand system is symmetric and the cost function $c(q_i)$ is the same for all firms. We assume that $q_i(\cdot)$ and $c(\cdot)$ are twice differentiable and the conditions for the uniqueness of equilibrium as well as the associated second-order conditions are satisfied. The marginal cost of production is defined by $mc(q) \equiv c'(q)$.

We denote by $q(p)$ per-firm industry demand under symmetric prices: $q(p) \equiv q_i(p, ..., p)$. The elasticity of this function, defined as $\varepsilon(p) \equiv -pq'(p)/q(p) > 0$ and referred to as the *price elasticity of industry demand,* should not be confused with the elasticity of the residual demand that any of these firms faces.[10] We also define by $\eta(q) \equiv 1/\varepsilon(p)|_{q(p)=q}$ the reciprocal of this elasticity as a function of

[9]One may wonder if the welfare distortion in this market can be eliminated if the unit tax is not constrained to be non-negative. This is because, starting from any combination of taxes t and v, it is possible to keep the same level of government revenue but unambiguously lower the deadweight loss by raising v just enough to generate a marginal unit of revenue, and simultaneously lowering t just enough give back that marginal unit of revenue. Extending this reasoning, Myles (1999) finds that the optimal combination entails a positive ad valorem tax and a negative unit tax, although in reality the feasibility of this method would be very limited.

[10]The elasticity ε here corresponds to ε_D in Weyl and Fabinger (2013, p.542). Note that $q'(p) = \partial q_i(\mathbf{p})/\partial p_i + (n-1)\partial q_i(\mathbf{p})/\partial p_j|_{\mathbf{p}=(p,...,p)}$ for any two distinct indices i and j. We define the firm's elasticity and other related concepts in Appendix B.

q. When we do not need to specify explicitly their dependence on either q or p in the following analysis, we use η interchangeably with $1/\varepsilon$. In addition, we define the *industry inverse demand function* $p(q)$ as the inverse of $q(p)$, which satisfies $\eta(q) = -qp'(q)/p(p)$.[11]

As mentioned above, we introduce two types of taxation: a specific tax (unit tax) t and an ad valorem tax v, with firm i's profit being $\pi_i = (1-v)p_i(\mathbf{q})q_i - tq_i - c(q_i)$. At symmetric output q, the government tax revenue per firm is $R(q) \equiv tq + vp(q)q$, which we can separate into the specific tax part and the ad valorem part: $R(q) = R_t(q) + R_v(q), R_t(q) = tq, R_v(q) = vp(q)q$. We denote by $\tau(q)$ the fraction of firm's pre-tax revenue that is collected by the government in the form of taxes: $\tau(q) \equiv R(q)/pq = v + t/p(q)$, as this notation makes many expressions simpler.

In the special case of monopoly, the first-order condition for the equilibrium would be $(1-v)mr(q) - t = mc(q)$ with $mr(q) = p(q) + qp'(q) = p(q) - \eta(q)p(q)$ and $mc(q) = c'(q)$. This condition can be rearranged as $\frac{1}{\eta(q)p(q)}(p(q) - \frac{t+mc(q)}{1-v}) = 1$. Intuitively, the left-hand side measures a degree of departure from competitive pricing, which would have $p(q) - \frac{t+mc(q)}{1-v} = 0$. We use this intuition to write a more general form of the first order condition that applies to oligopoly.

For oligopoly, we introduce the *conduct index* $\theta(q)$, which measures the degree of market monopolization and is determined *independently* of the cost side. The conduct index $\theta(q)$ is defined by the requirement that the symmetric equilibrium condition takes the form

$$\frac{1}{\eta(q)p(q)}\left(p(q) - \frac{t+mc(q)}{1-v}\right) = \theta(q), \tag{1}$$

where $mc(q) \equiv c'(q)$ is the marginal cost of production.[12] Perfect competition corresponds to $\theta(q) = 0$ and monopoly to $\theta(q) = 1$.[13] With a little abuse of notation, we denote the equilibrium price by p, and assume that any equilibrium is symmetric. We further impose a condition on the functions in Equation

[11] In the case of monopoly, there is no distinction between the industry demand and the demand for the monopolist's good. Then $q(p)$ is the monopolist's demand curve, ε is its elasticity, and η is the reciprocal of the elasticity.

[12] As already noted in Footnote 7 above, $\theta(q)$ is a generalization of conduct parameter in the sense that it is a function of q rather than a constant for any q. Hence, Equation (1) *should not be interpreted as an equation that defines* $\theta(q)$. For our analysis, we can just introduce $\theta(q)$ in an implicit manner: $\theta(q)$ is a function independent of the cost side of the problem, in which Equation (1) is the symmetric first-order condition of the equilibrium. Note that $\theta(q) > 1$ is not necessarily excluded, although in most interesting cases, it lies in $[0,1]$.

[13] Symmetric Cournot oligopoly also corresponds to a constant conduct index, which in this case takes the value of $1/n$, where n is the number of firms. But more generally, $\theta(q)$ depends on q.

(1) to ensure that any equilibrium is necessarily unique.[14]

We denote by θ the functional value of $\theta(q)$ at the equilibrium quantity. We can think of it as an *elasticity-adjusted Lerner index*. The Lerner index $[p-(t+mc)/(1-v)]/p$ multiplied by the industry demand elasticity $\varepsilon = 1/\eta$ equals θ. Here the Learner index is based on an effective (perceived) marginal cost $(t+mc)/(1-v)$.[15] We emphasize that once the conduct index is introduced, it becomes possible to describe oligopoly in a unified manner, without specifying whether it is price or quantity setting, or whether it exhibits strategic substitutability or complementarity.[16]

Finally, we define the specific tax pass-through rate ρ_t and the ad valorem pass-through semi-elasticity ρ_v as

$$\rho_t = \frac{\partial p}{\partial t}, \qquad \rho_v = \frac{1}{p}\frac{\partial p}{\partial v}.$$

where the equilibrium price p is considered as a function of the tax levels. Both ρ_t and ρ_v are dimensionless. The reason for considering semi-elasticity for the ad valorem tax becomes clear in the next subsection, where several results take the same form for both taxes and differ just by the presence of ρ_t or ρ_v.[17] They are also non-negative because otherwise second-order conditions for the equilibrium would be violated.[18]

2.2 Welfare components

The welfare characteristics we study are related to four welfare components: producer surplus per firm $PS = (1-v)\,pq - tq$, specific tax revenue per firm $R_t = tq$, ad valorem tax revenue per firm $R_v = vpq$, and consumer surplus per firm $CS = \int_0^q p(\tilde{q})\,d\tilde{q} - pq$. These are pictured in Figure 1.[19] The points

[14]The condition is as follows. Equation 1 may be rearranged as $(1-\eta(q)\,\theta(q))\,p(q) - \frac{1}{1-v}(t+mc(q)) = 0$. We require that the left-hand size be a decreasing function q. For constant marginal cost, this translates to the requirement that $(1-\eta(q)\,\theta(q))\,p(q)$ be a decreasing function of q. In the special case of monopoly, $\theta(q) = 1$, this reduces to the requirement of decreasing marginal revenue.

[15]The tax-adjusted Lerner rule $(p - \frac{t+mc}{1-v})/p = \eta\theta$ implies the restriction on θ, namely $\theta \leq \varepsilon$.

[16]Of course, it is possible to build oligopoly models with even more complicated interactions between firms that would be outside of the scope of the present analysis.

[17]Note that Häckner and Herzing (2016) use the symbol ρ_v for the ad valorem tax pass-through rate $\partial p/\partial v$, which corresponds to $p\rho_v$ in our notation.

[18]This follows using the requirement in Footnote 14 and by totally differentiating $(1-\eta(q)\,\theta(q))\,p(q) - \frac{1}{1-v}(t+mc(q)) = 0$.

[19]In the discussion that follows, we will not say "per firm" explicitly, although we will continue to think about welfare on a per-firm basis. Also, we assume that the producer surplus is finite.

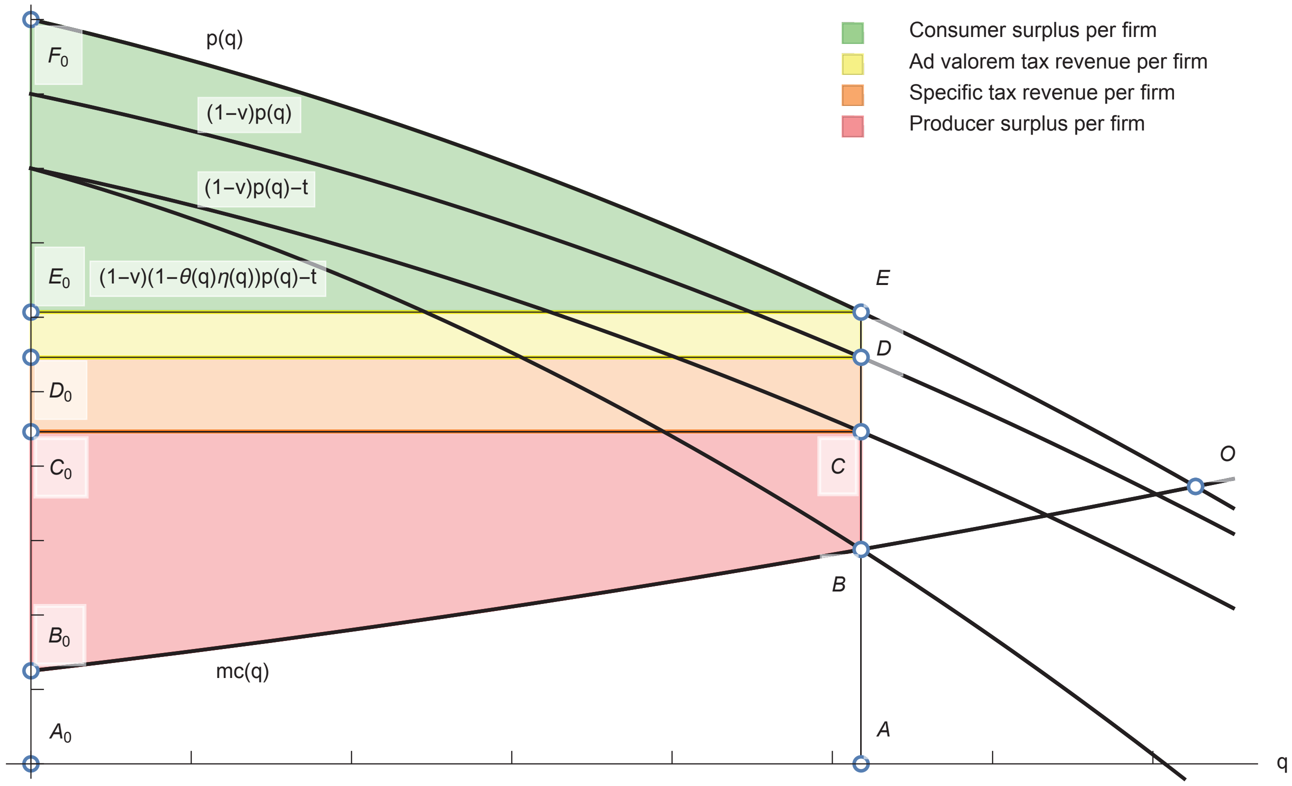

Figure 1: Welfare components at tax levels $t = 0.1$ and $v = 0.1$ for a chosen case of oligopoly.

$A_0, B_0, C_0, D_0, E_0, F_0$ are at $q = 0$ and the points A, B, C, D, E are at the equilibrium quantity for a given value of the taxes t and v. Total cost (per firm) $c(q) = \int_0^q mc(\tilde{q})\, d\tilde{q}$ corresponds to B_0BAA_0, producer surplus to C_0CBB_0, specific tax revenue to D_0DCC_0, ad valorem tax revenue to E_0EDD_0, and consumer surplus to the area F_0EE_0. The total (per firm) welfare $W = PS + R_t + R_v + CS$ is represented by the area F_0EBB_0. The point O is at the socially optimal quantity, and the area EOB represents the deadweight loss.

The figure shows five generally non-linear functions: $mc(q)$, $(1-v)(1-\theta(q)\eta(q))p(q)-t$, $(1-v)p(q)-t$, $(1-v)p(t)$, and $p(q)$ that determine the boundaries of the regions. In the special case of monopoly, the figure would look almost the same, except that $(1-v)(1-\theta(q)\eta(q))p(q)-t$ would be replaced by $(1-v)(1-\eta(q))p(q)-t$.

Figures 2 and 3 show how the diagram would changes if we increase the specific tax and the ad valorem tax, respectively. This graphical illustration is helpful for thinking in a simple way about changes to the welfare components if we infinitesimally change the taxes, although, of course, the changes shown in the figures are non-infinitesimal.

As the taxes infinitesimally change, $t \to t+dt, v \to v+dv$, the areas corresponding to a welfare component change due to a horizontal movement of the regions' right borders (points A, B, C, D, E) and due to a vertical movement of the top and bottom borders of the regions. We will call these "quantity effects" ($\leftrightarrow$) and "value effects" ($\updownarrow$), respectively. For example, the specific tax revenue is tq, and the corresponding infinitesimal change $d(tq) = t\,dq + q\,dt$, consists of a quantity effect $t\,dq$ and a value effect $q\,dt$ because the right border of the region shifts by dq and the vertical height of the region changes by dt. We introduce the following notation for infinitesimal changes in welfare components:

$$\begin{cases} dPS = dPS_{\leftrightarrow} + dPS_{\updownarrow} \\ dR_t = dR_{t\leftrightarrow} + dR_{t\updownarrow} \\ dR_v = dR_{v\leftrightarrow} + dR_{v\updownarrow} \\ dCS = dCS_{\leftrightarrow} + dCS_{\updownarrow} \\ dW = dW_{\leftrightarrow} + dW_{\updownarrow}. \end{cases}$$

For the change in producer surplus $PS = (1-v)\,q\,p(q) - tq - c(q)$,

$$dPS = dPS_{\leftrightarrow} + dPS_{\updownarrow}, \tag{2}$$

the contributions are

$$dPS_{\leftrightarrow} = ((1-v)\,p - t - mc)\,dq, \qquad dPS_{\updownarrow} = (1-v)\,q\,dp - q\,dt - pq\,dv, \tag{3}$$

or alternatively, after substituting for mc from Equation (1),

$$dPS_{\leftrightarrow} = (1-v)p\eta\theta\;dq, \qquad dPS_{\updownarrow} = (1-v)\,q\,dp - q\,dt - pq\,dv. \tag{4}$$

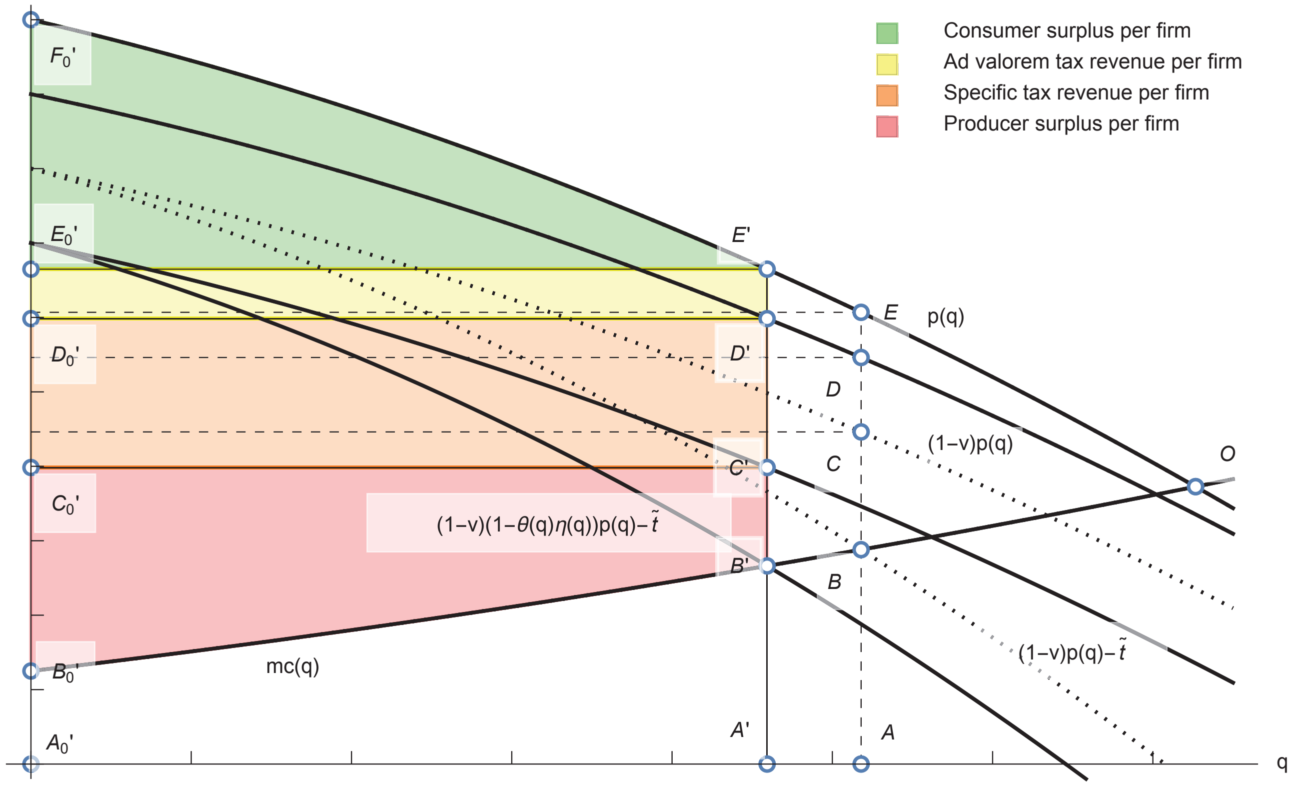

Figure 2: Visualization of oligopoly welfare components after an increase of the specific tax from $t = 0.1$ to $\tilde{t} = 0.2$, with $v = 0.1$ and $p(0) = 1$, starting from the situation in Figure 1. In this figure, PS, R_v, and CS decrease, whereas R_t increases. For a general understanding of the possible signs of the changes, see Appendix A.1; an extended caption for this figure.

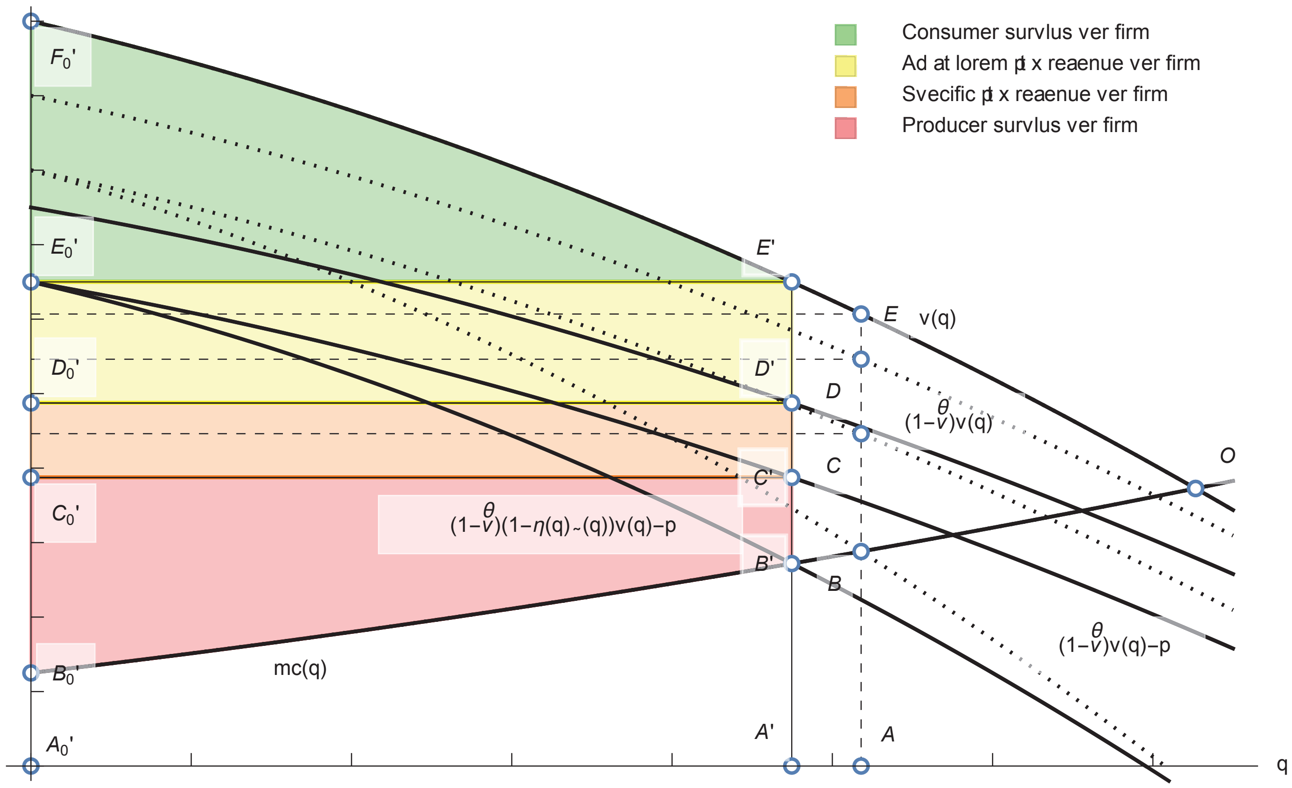

Figure 3: Visualization of oligopoly welfare components after an increase of the ad valorem tax from $v = 0.1$ to $\tilde{v} = 0.2$, with $t = 0.1$ and $p(0) = 1$, starting from the situation in Figure 1. In this figure, PS, R_t, and CS decrease, whereas R_v increases. For a general understanding of the possible signs of the changes, see Appendix A.2; an extended caption for this figure.

Next, the change in tax revenue $R = tq + vpq$ is

$$dR = dR_t + dR_v = dR_{t\leftrightarrow} + dR_{v\leftrightarrow} + dR_{t\updownarrow} + dR_{v\updownarrow} \tag{5}$$

with

$$dR_{t\leftrightarrow} = t\,dq, \qquad dR_{v\leftrightarrow} = vp\,dq, \qquad dR_{t\updownarrow} = q\,dt, \qquad dR_{v\updownarrow} = qv\,dp + qp\,dv. \tag{6}$$

For consumer surplus CS, the quantity effect is zero, $dCS_{\leftrightarrow} = 0$, and the value effect is $dCS_{\updownarrow} = -q\,dp$, so

$$dCS = -q\,dp, \tag{7}$$

which matches common formulas.

Finally, for welfare W, the value effect is zero, $dW_{\updownarrow} = 0$, because the curves $mc(q)$ and $p(q)$ do not move in response to a tax change. The quantity effect is $dW_{\leftrightarrow} = (p - mc)\,dq$, so $dW = (p - mc)\,dq$, which of course matches common formulas for welfare. Substituting for mc using Equation (1) gives $dW = (t + vp + (1-v)p\eta\theta)\,dq$, or using our definition $\tau = v + t/p$,

$$dW = ((1-v)\eta\theta + \tau)\,p\,dq. \tag{8}$$

2.3 Changes in equilibrium prices and quantities

It is useful to express infinitesimal price changes and tax changes in terms of infinitesimal quantity changes. In the case of a change in specific tax dt, the price changes by $dp = \rho_t dt$, and the quantity changes by $dq = -q\varepsilon\,dp/p$. These relationships imply

$$dp = -\frac{\eta p}{q}dq, \qquad dt = -\frac{\eta p}{q\rho_t}dq. \tag{9}$$

In the case of a change in ad valorem tax dv, the price changes by $dp = \rho_v p dv$, while the quantity changes by $dq = -q\varepsilon\, dp/p$. Therefore

$$dp = -\frac{\eta p}{q} dq, \qquad dv = -\frac{\eta}{q\rho_v} dq. \tag{10}$$

2.4 Marginal value of public funds

We define the *marginal value of public funds $MVPF_t$* of the specific tax t and the *marginal value of public funds $MVPF_v$* of the ad valorem tax v as the ratio of (a) the change in social welfare induced by an infinitesimal increase the corresponding tax, and (b) the associated change in tax revenue, i.e.:

$$MVPF_t \equiv -\left(\frac{\partial R}{\partial t}\right)^{-1} \frac{\partial W}{\partial t}, \qquad MVPF_v \equiv -\left(\frac{\partial R}{\partial v}\right)^{-1} \frac{\partial W}{\partial v}.$$

First let us consider the marginal value of public funds $MVPF_t$ for changes in the specific tax, $dt \neq 0, dv = 0$. Using Equations (5), (6), and (8), we have

$$MVPF_t = -\frac{dW}{dR} = -\frac{((1-v)\eta\theta + \tau)\, p\, dq}{t\, dq + vp\, dq + q\, dt + qv\, dp}.$$

In order to cancel the infinitesimal changes on the right-hand side, we substitute for dp and dt in terms of dq using Equations (9),

$$MVPF_t = -\frac{((1-v)\eta\theta + \tau)\, p\, dq}{t\, dq + vp\, dq + q\left(-\frac{\eta p}{q\rho_t} dq\right) + qv\left(-\frac{\eta p}{q} dq\right)} = -\frac{((1-v)\eta\theta + \tau)\, p}{t + vp - \frac{\eta p}{\rho_t} - v\eta p}.$$

Dividing the numerator and denominator by ηp gives

$$MVPF_t = \frac{(1-v)\eta\theta + \tau}{\left(\frac{1}{\rho_t} + v\right)\eta - \tau}.$$

We proceed in a similar fashion for changes in the ad valorem tax, $dv \neq 0, dt = 0$. The marginal value of public funds $MVPF_v$ is

$$MVPF_v = -\frac{dW}{dR} = -\frac{((1-v)\eta\theta + \tau)\, p\, dq}{t\, dq + vp\, dq + qv\, dp + qp\, dv}.$$

We substitute for dp and dv in terms of dq using Equations (10),

$$MVPF_v = -\frac{((1-v)\eta\theta+\tau)\,p\,dq}{t\,dq+vp\,dq+qv\left(-\frac{\eta p}{q}dq\right)+qp\left(-\frac{\eta}{q\rho_v}dq\right)} = -\frac{((1-v)\eta\theta+\tau)\,p}{t+vp-v\eta p-\frac{\eta p}{\rho_v}}.$$

Dividing the numerator and denominator by ηp gives

$$MVPF_v = \frac{(1-v)\eta\theta+\tau}{\left(\frac{1}{\rho_v}+v\right)\eta-\tau}.$$

We summarize these findings in the following proposition.

Proposition 1. *Under symmetric oligopoly with a possibly non-constant marginal cost, the marginal value of public funds (MVPF) associated with a change in the specific tax t and the ad valorem tax v is characterized by:*

$$MVPF_t = \frac{(1-v)\,\eta\,\theta+\tau}{\left(\frac{1}{\rho_t}+v\right)\eta-\tau}, \qquad MVPF_v = \frac{(1-v)\,\eta\,\theta+\tau}{\left(\frac{1}{\rho_v}+v\right)\eta-\tau},$$

respectively, where $\eta = 1/\varepsilon$.

The result for $MVPF_t$ has some intuitive properties. It can be rewritten as

$$MVPF_t = \frac{(1-v)\,\theta+\left(v+\frac{t}{p}\right)\varepsilon}{\left(\frac{1}{\rho_t}+v\right)-\left(v+\frac{t}{p}\right)\varepsilon}.$$

If we think of $MVPF_t$ as a function of t, keeping all other variables in the expression fixed, we see that it is an increasing function of t. That is intuitive: The tax is more distortionary on the margin if the initial tax level is already high. Since t in the expression is multiplied by ε/p, the dependence of $MVPF_t$ on t will be stronger if ε/p is large. This is also intuitive: (a) for a low price p, t is sizable relative to the price, and (b) for a large elasticity ε of the industry demand, an increase in t may have a larger effect on the quantity supplied. In both cases we would expect the initial tax level t to have a strong influence on how distortionary the tax is on the margin.

If we think of $MVPF_t$ as a function of ρ_t, keeping all other variables in the expression fixed, we find that it is an increasing function of ρ_t, the pass-through rate. This is intuitive, as the tax will be more distortionary on the margin if the tax is strongly passed through to the prices.

Similarly, if we think of $MVPF_t$ as a function of θ, keeping all other variables in the expression fixed, we see that it is an increasing function of θ, the conduct index. This is consistent with the intuition that when the market is very competitive, with a small θ, the tax should not be as distortionary on the margin as when the market is non-competitive.

For $MVPF_v$, the expression is the same, except that ρ_t is replaced by ρ_v. The intuition regarding the pass-through and market competitiveness applies for $MVPF_v$ as well. The dependence on v is more complicated, though, than the dependence on t.

2.5 Incidence

We define the *incidence* I_t of the specific tax t and the *incidence* I_v of the ad valorem tax v as the ratio of (a) the change in consumer surplus induced by an infinitesimal increase the corresponding tax, and (b) the associated change in producer surplus, i.e.:[20]

$$I_t \equiv \left(\frac{\partial PS}{\partial t}\right)^{-1}\frac{\partial CS}{\partial t}, \qquad I_v \equiv \left(\frac{\partial PS}{\partial v}\right)^{-1}\frac{\partial CS}{\partial v}.$$

For a specific tax change $dt \neq 0, dv = 0$, we get, using Equations (7), (2) and (4),

$$I_t = \frac{dCS}{dPS} = \frac{-q\,dp}{(1-v)p\eta\theta\,dq + (1-v)\,q\,dp - q\,dt} = \frac{-q\left(-\frac{\eta p}{q}dq\right)}{(1-v)p\eta\theta\,dq + (1-v)\,q\left(-\frac{\eta p}{q}dq\right) - q\left(-\frac{\eta p}{q\rho_t}dq\right)},$$

where we eliminated dp and dt using Equations (9). After a simplification,

$$I_t = \frac{1}{\frac{1}{\rho_t} - (1-v)(1-\theta)}.$$

[20]One can also define *social incidence* by $SI_T \equiv dW/dPS$ in association with a small change in $T \in \{t, v\}$ (see Weyl and Fabinger 2013, p. 538). In this paper, we focus on $MVPF_T$ as a measure of welfare burden in society, and I_T as a measure of loss in consumer welfare because once $MVPF_T \equiv -dW/dR$ and $I_T \equiv dCS/dPS$ are obtained, $SI_T = (dCS + dPS + dR)/dPS = (1 + I_T)/(1 + 1/MVPF_T)$ can be readily calculated.

For an ad valorem tax change, $dv \neq 0, dt = 0$, we obtain, again using Equations (7), (2) and (4),

$$I_v = \frac{dCS}{dPS} = \frac{-q\,dp}{(1-v)p\eta\theta\,dq + (1-v)\,q\,dp - pq\,dv} = \frac{-q\left(-\frac{\eta p}{q}dq\right)}{(1-v)p\eta\theta\,dq + (1-v)\,q\left(-\frac{\eta p}{q}dq\right) - pq\left(-\frac{\eta}{q\rho_v}dq\right)},$$

where we substituted for dp and dv from Equations (10). This simplifies to

$$I_v = \frac{1}{\frac{1}{\rho_v} + (1-v)(1-\theta)}.$$

We summarize these findings in the following proposition.

Proposition 2. *Under symmetric oligopoly with a general type of competition and with a possibly non-constant marginal cost, the incidence of the specific tax t and the ad valorem tax v is characterized by:*

$$\frac{1}{I_t} = \frac{1}{\rho_t} - (1-v)(1-\theta), \qquad \frac{1}{I_v} = \frac{1}{\rho_v} - (1-v)(1-\theta),$$

respectively.

Note that in the case of zero ad valorem tax, the expression for I_t reduces to Weyl and Fabinger's (2013, p. 548) Principle of Incidence 3, that states $\frac{1}{I_t} = \frac{1}{\rho_t} - (1-\theta)$. In this way, we are able to generalize Weyl and Fabinger's (2013) formula for incidence, and respond to the statements by Rosen and Gayer (2014) and Stiglitz and Rosengard (2015) mentioned in the Introduction.

Next, we show how ρ_t and ρ_v are related in the following proposition.

Proposition 3. *Under symmetric oligopoly with a possibly non-constant marginal cost, the pass-through semi-elasticity ρ_v of an ad valorem tax may be expressed in terms of the unit tax pass-through rate ρ_t, the conduct index θ, and the industry demand elasticity ε as*

$$\rho_v = \left(1 - \frac{\theta}{\varepsilon}\right)\rho_t. \tag{11}$$

The proposition is proven Appendix A.3. Combined with Proposition 1, it is consistent with the well-known result that unit tax and ad valorem tax are equivalent in the welfare effects under perfect competition: if $\theta = 0$, then $\rho_t = \rho_v$, and under imperfect competition, $\rho_t > \rho_v$, and $MVPF_t > MVPF_v$. This provides another look of Anderson, de Palma, and Kreider's (2001b) result that unit taxes are welfare-inferior to ad valorem taxes.[21]

To understand this Proposition 3 intuitively, note that to keep prices and quantities constant, Δt and Δv must satisfy:

$$\frac{t+\Delta t+mc}{1-(v+\Delta v)}=\frac{t+mc}{1-v}.$$

Thus, the relative Δt that must be offset by a reduction $-\Delta v$ is equal to $(t+mc)/(1-v)$: $\Delta t=-(t+mc)\Delta v/(1-v)$, which, along with $\rho_t dt+\rho_v p\,dv=0$, leads to $(t+mc)\rho_t/[(1-v)p]=\rho_v$. Now, recall the Lerner rule:

$$1-\frac{t+mc}{(1-v)p}=\eta\theta,$$

which implies that $(1-\eta\theta)\rho_t=\rho_v$, as Proposition 3 claims. Here, $\theta/\varepsilon=1-\rho_v/\rho_t$ implies that $\rho_v\leq\rho_t\leq(1-1/\varepsilon)\,\rho_v$.

Next, by combining Propositions 1 and 3, we find that $MVPF_t$ and $MVPF_v$ can be expressed without the conduct index θ.

Proposition 4. *Under symmetric oligopoly with a possibly non-constant marginal cost, the unit pass-through rate ρ_t, the ad valorem pass-through semi-elasticity ρ_v, and the elasticity of industry demand ε (along with the tax rates and the fraction τ of the firm's pre-tax revenue collected by the government in the form of taxes) serve as sufficient statistics for the marginal changes in deadweight loss both with respect to unit taxes and ad valorem taxes. Specifically,*

$$MVPF_t=\frac{(1-v+\tau)\rho_t-(1-v)\rho_v}{1+(v-\varepsilon\tau)\rho_t}\,\varepsilon,\qquad MVPF_v=\frac{(1-v+\tau)\,\rho_t-(1-v)\rho_v}{1+(v-\varepsilon\tau)\,\rho_v}\,\frac{\rho_v}{\rho_t}\varepsilon.$$

The proof is simple: Proposition 3 allows us to express the conduct index θ as $\theta=(1-\rho_v/\rho_t)\varepsilon$. Substituting this into the relationships in Proposition 1 then gives the desired result.

[21] Under Cournot competition, Equation (6.13) of Auerbach and Hines (2002) coincides with Equation (11) above. Proposition 3 implies that their equation holds more generally. We thank Germain Gaudin for pointing this out.

To gain a further understanding of Proposition 4, recall from Proposition 1 that

$$MVPF_t = \frac{(1-v)\eta\theta + \tau}{\left(\frac{1}{\rho_t} + v\right)\eta - \tau}.$$

Now, Proposition 4 states that it is also understood as

$$MVPF_t = \frac{(1-v)\left(1 - \frac{\rho_v}{\rho_t}\right) + \tau}{\left(\frac{1}{\rho_t} + v\right)\eta - \tau}.$$

Of course, it is true that θ is expressed by the empirical measures such as $\theta = (1 - \rho_v/\rho_t)\varepsilon$. For example, in the case of the assumption of Cournot competition, researchers often may observe the number of firms, n, and conclude that the value of conduct index is $\theta = 1/n$. However, even in the case of homogeneous products, the true conduct θ may be higher than $1/n$ due to reasons such as collusion. Proposition 4 above circumvents this difficulty in calibrating $MVPF_t$ and $MVPF_v$.[22] Conversely, one would be able to estimate θ using the proposition above once ε, ρ_t, and ρ_v are calibrated.

2.6 Tax pass-through

As the last result presented in this section, the following proposition shows how the two forms of pass-through are characterized.

Proposition 5. *Under symmetric oligopoly with a general mode of competition and a possibly non-constant marginal cost, the unit tax pass-through is characterized by:*

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{\left[1 + \frac{1-\tau}{1-v}\varepsilon\chi\right] - (\eta + \chi)\theta + \varepsilon q (\theta\eta)'},$$

where the derivative is taken with respect to q and $\chi \equiv mc' q/mc$ is the elasticity of the marginal cost

[22] Similarly, the incidence of a unit tax is expressed as

$$\frac{1}{I_t} = \frac{1}{\rho_t} - (1-v)\left[(1-\varepsilon) + \frac{\rho_v}{\rho_t}\varepsilon\right],$$

and analogously for the case of an ad valorem tax.

with respect to quantity. Similarly, the ad valorem tax pass-through is characterized by:

$$\rho_v = \frac{\varepsilon-\theta}{(1-v)\,\varepsilon}\cdot\frac{1}{\left[1+\frac{1-\tau}{1-v}\varepsilon\chi\right]-(\eta+\chi)\,\theta+\varepsilon q\,(\theta\eta)'}.$$

The proof is in Appendix A.4. Further, in Online Appendix A, we discuss its relationship with Weyl and Fabinger's (2013) result for the case of a specific tax only.

Let us provide a brief discussion of these results. In the case of prefect competition and zero initial taxes, the pass through is given by $\rho_t = 1/(1+\varepsilon\chi)$ (see Weyl and Fabinger 2013, p. 534) and $\rho_v = 1/(1+\varepsilon\chi)$. With non-zero initial taxes, there are adjustment factors, but the nature of the formulas is similar.

With imperfect competition, the term in the denominator $-\eta\theta$ is negative and leads to higher pass-through. This is intuitive because in less competitive markets, firms have the ability to reflect higher costs in their prices to a larger extent. The term in the denominator $-\chi\theta$ has a sign opposite to that of $\chi = mc'q/mc$. For increasing marginal costs, χ is positive and $-\chi\theta$ negative, which leads to higher pass-through, especially if θ is high.

Further, with imperfect competition, the term in the denominator $\varepsilon q\,(\theta\eta)'$ may be split into two parts: $\varepsilon q\,(\theta\eta)' = q\theta' + q\varepsilon\theta\eta'$. If at lower quantities the market is less competitive, then $\theta' < 0$ and $q\theta' < 0$, which leads to higher pass-through. Intuitively, in such situations, increasing taxes decreases the quantity provided, which in turn makes the market less competitive, leading to an even larger increase in prices than in the case of $\theta' = 0$. Similarly, if at lower quantities the industry demand elasticity (ε) is lower, then $\eta' < 0$ and $q\varepsilon\theta\eta' < 0$, which leads to higher pass-through. Intuitively, in such situations, increasing taxes decreases the quantity provided, which in turn makes the industry demand more inelastic, leading to an even larger increase in prices than in the case of $\eta' = 0$. This effect is larger for larger θ, which is consistent with the fact that in these situations the firms are more sensitive to the properties of the overall industry demand.

We extended these results on pass-through in several directions. In Online Appendix B, we show how our framework applies to the case *multi-product* firms if intra-firm symmetry is guaranteed. In Online Appendix C, we present generalizations that go beyond the case taxation and include other market changes.[23]

3 Numerical Analysis if Parametric Examples

Although our formulas are presented in a general form, it would be illustrative to work through some parametric examples. Below we consider three demand specifications with n symmetric firms and constant marginal cost: $\chi = 0$. We define the *own-price elasticity* $\varepsilon_{own}(p)$ *of the firm's direct demand* and the *own quantity elasticity* $\eta_{own}(q)$ *of the firm's inverse demand* by

$$\varepsilon_{own}(p) \equiv -\frac{p}{q(p)} \cdot \frac{\partial q_i(\mathbf{p})}{\partial p_i}|_{\mathbf{p}=(p,\ldots,p)}$$

and

$$\eta_{own}(q) \equiv -\frac{q}{p(q)} \cdot \frac{\partial p_i(\mathbf{q})}{\partial q_i}|_{\mathbf{q}=(q,\ldots,q)},$$

respectively. Similarly, the *curvature of the industry's direct demand* $\alpha(p)$ and the *curvature of the industry's inverse demand* $\sigma(q)$ are defined as follows:

$$\alpha(p) \equiv \frac{-pq''(p)}{q'(p)}$$

and

$$\sigma(q) \equiv \frac{-qp''(q)}{p'(q)}.$$

Then, the results derived in Appendix B indicate that in this case, the pass-through expressions become

$$\rho_t = \frac{1}{(1-v)\left[1+\left(1-\frac{\alpha}{\varepsilon_{own}}\right)\theta\right]}, \qquad \rho_v = \frac{\varepsilon_{own}-1}{\varepsilon_{own}\left\{(1-v)\left[1+\left(1-\frac{\alpha}{\varepsilon_{own}}\right)\theta\right]\right\}}$$

[23]While many issues in public economics entail small changes such as a shift in tax rate, it would also be interesting to consider expressions for global changes in the surplus measures: see Online Appendix D. Furthermore, free entry is analyzed in Online Appendix E as an additional extension. In addition, Online Appendix F discusses the relationship with the concept of aggregative games.

under price competition, where $\theta = \varepsilon/\varepsilon_{own}$, and

$$\rho_t = \frac{1}{(1-v)\left[1+\left(1-\frac{\sigma}{\theta}\right)\theta\right]}, \qquad \rho_v = \frac{1-\eta_{own}}{(1-v)\left[1+\left(1-\frac{\sigma}{\theta}\right)\theta\right]}$$

under quantity competition, where $\theta = \eta_{own}/\eta$.

Below, we consider three classes of demand specification: linear, constant elasticity of substitution (CES), and logit, and we assume that the marginal cost is constant.

3.1 Linear demand

The first one is the case wherein each firm faces the following *linear demand*, $q_i(\mathbf{p}) = b - \lambda p_i + \mu \sum_{i' \neq i} p_{i'}$, where $\lambda > (n-1)\mu$ and $0 \leq mc < b/[\lambda - (n-1)\mu]$, implying that all firms produce substitutes and μ measures the degree of *substitutability* (firms are effectively monopolists when $\mu = 0$).[24,25] Under symmetric pricing, the industry's demand is thus given by $q(p) = b - [\lambda - (n-1)\mu]p$. The inverse demand system is given by

$$p_i(\mathbf{q}) = \frac{\lambda - (n-2)\mu}{(\lambda+\mu)[\lambda-(n-1)\mu]}(b-q_j) + \frac{\mu}{(\lambda+\mu)[\lambda-(n-1)\mu]}\left[\sum_{i' \neq i}(b-q_{i'})\right],$$

implying that $p(q) = (b-q)/[\lambda - (n-1)\mu]$ under symmetric production. Obviously, both the direct and the indirect demand curvatures are zero: $\alpha = 0, \sigma = 0$. Under price competition, the pass-through expressions are

$$\rho_t = \frac{1}{(1-v)(1+\theta)}, \qquad \rho_v = \frac{\varepsilon_{own}-1}{\varepsilon_{own}(1-v)(1+\theta)},$$

[24]This linear demand is derived by maximizing the representative consumer's net utility, $U(q_1,...,q_n) - \sum_{i=1}^{n} pq_i$, with respect to $q_1,...,$ and q_n. See Vives (1999, pp. 145-6) for details.

[25]In our notation below, the demand in symmetric equilibrium is given by $q_i(p_i, p_{-i}) = b - \lambda p_i + \mu(n-1)p_{-i}$, whereas it is written as

$$q_i(p_i,p_{-i}) = \frac{\alpha}{1+\gamma(n-1)} - \frac{1+\gamma(n-2)}{(1-\gamma)[1+\gamma(n-1)]}p_i + \frac{\gamma(n-1)}{(1-\gamma)[1+\gamma(n-1)]}p_{-i}$$

in Häckner and Herzing's (2016) notation, in which $\gamma \in [0,1]$ is the parameter that measures substitutability between (symmetric) products. Thus, if our (b,λ,μ) is determined by $b = \alpha/[1+\gamma(n-1)]$, $\lambda = [1+\gamma(n-2)]/\{(1-\gamma)[1+\gamma(n-1)]\}$, and $\mu = \gamma/\{(1-\gamma)[1+\gamma(n-1)]\}$, given Häckner and Herzing's (2016) (α,γ), then our results below can be expressed by Häckner and Herzing's (2016) notation as well. Note here that our formulation is more flexible in the sense that the number of the parameters is three. This is because the coefficient for the own price is normalized to one: $p_i(q_i,q_{-i}) = \alpha - q_i - \gamma(n-1)q_{-i}$, which is analytically innocuous, and Häckner and Herzing's (2016) γ is the normalized parameter.

where $\theta = [\lambda - (n-1)\mu]/\lambda$, and $\varepsilon_{own} = \lambda(p/q)$. Under quantity competition,

$$\rho_t = \frac{1}{(1-v)(1+\theta)}, \qquad \rho_v = \frac{1-\eta_{own}}{(1-v)(1+\theta)},$$

where $\theta = [\lambda - (n-2)\mu]/(\lambda+\mu)$ and $\eta_{own} = \{[\lambda - (n-2)\mu](q/p)\}/\{(\lambda+\mu)[\lambda-(n-1)\mu]\}$.

Under price competition, the marginal value of public funds and the incidence, discussed in Propositions 1 and 2, respectively, are given by

$$MVPF_t = \frac{(1-v)\theta + \varepsilon\tau}{1+(1-v)\theta - \varepsilon\tau}, \qquad MVPF_v = \frac{(1-v)\theta + \varepsilon\tau}{\frac{(1-v)(1+\theta)}{\varepsilon_{own}-1} + v - \varepsilon\tau},$$

$$I_t = \frac{1}{2(1-v)[1-(n-1)(\mu/\lambda)]}, \qquad I_v = \frac{\varepsilon_{own}-1}{(1-v)[2-\varepsilon_{own}(1-\theta)]},$$

with $\varepsilon = [\lambda - (n-1)\mu](p/q)$. Under quantity competition,

$$MVPF_t = \frac{(1-v)\theta + \frac{1}{\eta}\tau}{1+(1-v)\theta - \frac{1}{\eta}\tau}, \qquad MVPF_v = \frac{(1-v)\theta + \frac{1}{\eta}\tau}{\frac{(1-v)(1+\theta)}{1-\eta_{own}} + v - \frac{1}{\eta}\tau},$$

$$I_t = \frac{\lambda+\mu}{2(1-v)[\lambda-(n-2)\mu]}, \qquad I_v = \frac{1-\eta_{own}}{(1-v)[\eta_{own} + (2-\eta_{own})\theta]},$$

with $1/\eta = [\lambda - (n-1)\mu](p/q)$. Thus, in both cases, it suffices to solve for the equilibrium price and output to compute the pass-through and the marginal value of public funds.

Table 1 (a) summarizes the key variables that determine these values for the case of linear demand. It is verified that under both price and quantity competition, θ is a decreasing function of n and μ. To focus on the role of these two parameters, n and μ, which directly affect the intensity of competition, we employ the following simplification to compute the ratio p/q in equilibrium: $b = 1$, $mc = 0$, and $\lambda = 1$. (See Online Appendix I for the expressions of the equilibrium prices and output levels under price and quantity competition).

The top two panels in Figure 4 illustrate how ρ_t and ρ_v behave as we increase the number of firms (n, the left side) or the sustainability parameter (μ, the right side). The initial tax levels are $t = 0.05$ and $v = 0.05$. We distinguish price setting and quantity setting by superscripts P and Q, respectively.

Table 1: Elasticities, Conduct Indices, and Curvatures

(a) Linear Demand	
Price setting	Quantity setting
$\varepsilon = [\lambda - (n-1)\mu]\left(\frac{p}{q}\right)$	$\eta = \frac{1}{\lambda-(n-1)\mu}\left(\frac{q}{p}\right)$
$\varepsilon_{own} = \lambda\left(\frac{p}{q}\right)$	$\eta_{own} = \frac{\lambda-(n-2)\mu}{(\lambda+\mu)[\lambda-(n-1)\mu]}\left(\frac{q}{p}\right)$
$\theta = \varepsilon/\varepsilon_{own} = 1-(n-1)\left(\frac{\mu}{\lambda}\right)$	$\theta = \eta_{own}/\eta = \frac{\lambda-(n-2)\mu}{\lambda+\mu}$
$\alpha = 0$	$\sigma = 0$

(b) CES Demand	
Price setting	Quantity setting
$\varepsilon = \frac{1}{1-\gamma\xi}$	$\eta = 1-\gamma\xi$
$\varepsilon_{own} = \frac{n-\gamma-(n-1)\gamma\xi}{n(1-\gamma)(1-\gamma\xi)}$	$\eta_{own} = \frac{\gamma(1-\xi)+(1-\gamma)n}{n}$
$\theta = \varepsilon/\varepsilon_{own} = \frac{n(1-\gamma)}{n-\gamma-(n-1)\gamma\xi}$	$\theta = \eta_{own}/\eta = \frac{\gamma(1-\xi)+(1-\gamma)n}{n(1-\gamma\xi)}$
$\alpha = \frac{2-\gamma\xi}{1-\gamma\xi}$	$\sigma = 2-\gamma\xi$

(c) Logit Demand	
Price setting	Quantity setting
$\varepsilon = \beta(1-ns)p$	$\eta = \frac{1}{\beta(1-ns)p}$
$\varepsilon_{own} = \beta(1-s)p$	$\eta_{own} = \frac{1-(n-1)s}{\beta(1-ns)p}$
$\theta = \varepsilon/\varepsilon_{own} = \frac{1-ns}{1-s}$	$\theta = \eta_{own}/\eta = 1-(n-1)s$
$\alpha = \frac{(2ns-3)ns}{1-ns}p$	$\sigma = \frac{1-2ns}{1-ns}$

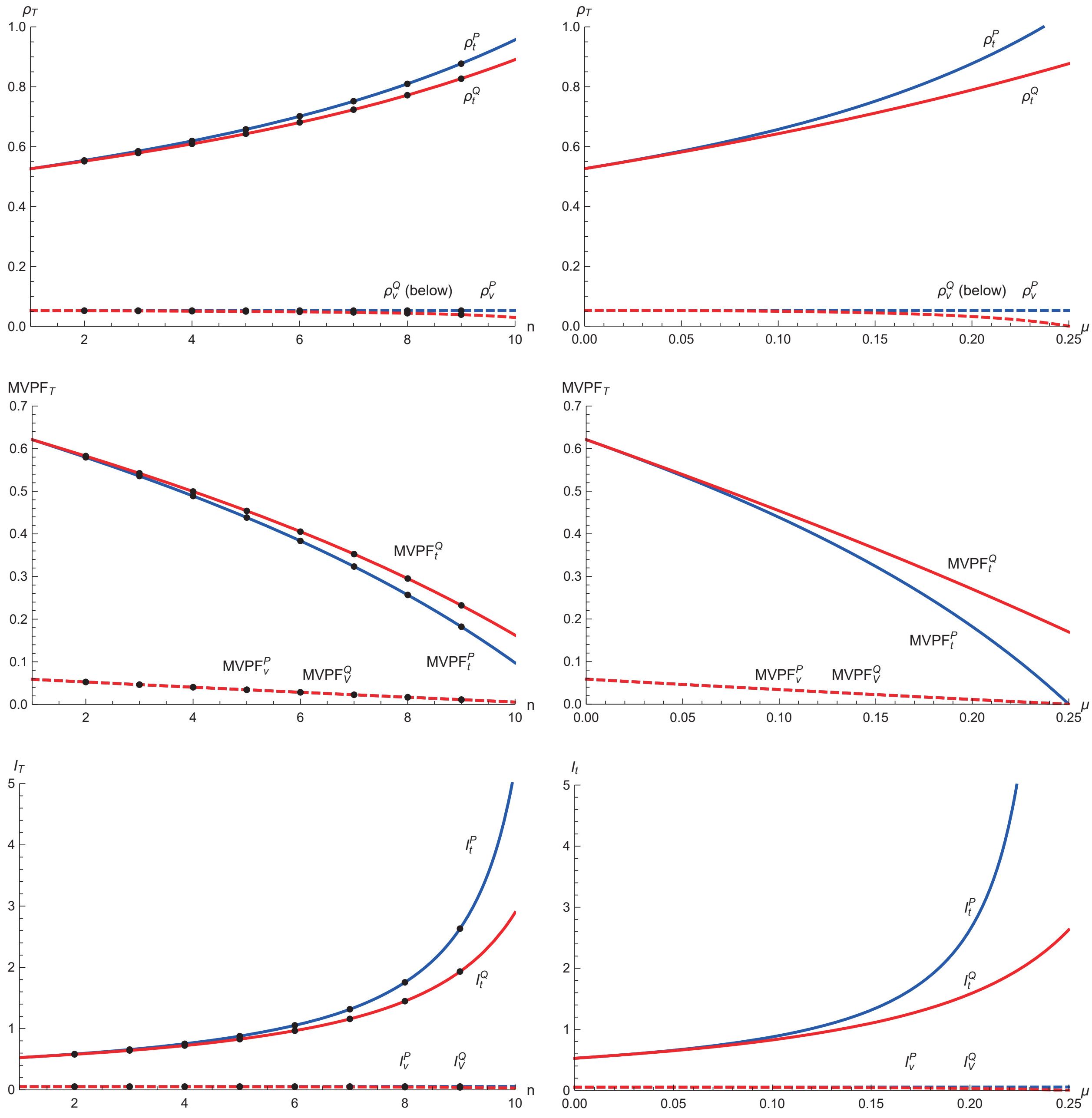

Figure 4: Pass-through (top), marginal value of public funds (middle), and incidence (bottom) with linear demand. The horizontal axes on the left and the right panels correspond to the number of firms (n) with $\mu = 0.1$, and the substitutability parameter (μ) with $n = 5$, respectively, with the initial tax level, $(t, v) = (0.05, 0.05)$.

The middle panels show $MVPF_t$ and $MVPF_v$, while the bottom panels depict I_t and I_v. We observe that the ad valorem tax pass-through is close to zero because in this case both ε_{own} and η_{own} are close to 1. As competition becomes more intense, both ρ_t^P and ρ_t^Q become larger, and their difference also becomes larger. In the case of linear demand, the difference in the mode of competition does not yield a substantial difference in the three measures. As is verified by Anderson, de Palma, and Kreider (2001b), the ad valorem tax is more efficient on the margin than the specific tax: the dashed lines in the two middle panels lie below the solid lines. This ranking is related inversely to pass-through and incidence: as pass-through or incidence increases, the marginal value of public funds decreases.

3.2 Constant elasticity of substitution (CES) demand

We next consider the market demand with constant elasticity of substitution given by

$$q_i(\mathbf{p}) = (\gamma\xi)^{\frac{1}{1-\gamma\xi}} \frac{p_i^{\frac{-1}{1-\gamma}}}{\left(\sum_{i'=1}^{n} p_{i'}^{\frac{-\gamma}{1-\gamma}}\right)^{\frac{1-\xi}{1-\gamma\xi}}},$$

where $0 < \gamma < 1$ and $0 < \xi < 1$.[26] Hence the direct demand under symmetric pricing is $q(p) = (\gamma\xi)^{\frac{1}{1-\gamma\xi}} n^{\frac{-(1-\xi)}{1-\gamma\xi}} p^{\frac{-1}{1-\gamma\xi}}$. The elasticity of substitution, $1/(1-\gamma)$, is constant. Table 1 (b) shows the price elasticity of industry demand (ε), the own-price elasticity of a firm's demand (ε_{own}), the conduct index (θ), and the curvature of the industry's direct demand (α) are all independent of the equilibrium price.[27] This feature is in contrast to the linear demand above or the logit demand below.

Similarly, the inverse demand is given by

$$p_i(\mathbf{q}) = (\gamma\xi)\left(\sum_{i'=1}^{n} q_{i'}^{\gamma}\right)^{-(1-\xi)} q_i^{-(1-\gamma)}.$$

Hence the inverse demand under symmetric pricing is $p(q) = (\gamma\xi)n^{-(1-\xi)}q^{-(1-\gamma\xi)}$. Table 1 (b) indicates

[26] This CES demand is derived from $U(q_1,\ldots,q_n) = \left(\sum_{i=1}^{n} q_i^{\gamma}\right)^{\xi}$ as the representative consumer's utility (Vives 1999, pp. 147-8), where the elasticity of substitution between the firms is given by $1/(1-\gamma)$.

[27] We use the first-order derivative of $q(p)$, $q'(p) = -[n^{\frac{-(1-\xi)}{1-\gamma\xi}}(\gamma\xi)^{\frac{1}{1-\gamma\xi}}/(1-\gamma\xi)]p^{\frac{-(2-\gamma\xi)}{1-\gamma\xi}}$, and its second-order derivative, $q''(p) = [n^{\frac{-(1-\xi)}{1-\gamma\xi}}(\gamma\xi)^{\frac{1}{1-\gamma\xi}}(2-\gamma\xi)/(1-\gamma\xi)^2]p^{\frac{-(3-2\gamma\xi)}{1-\gamma\xi}}$ for these derivations.

that for the case of quantity setting, η, η_{own}, θ, and σ are also independent of the equilibrium output or price.[28]

Note that for each tax $T \in \{t, v\}$, only ρ_T and θ, as well as the initial value of ad valorem tax v, are necessary to compute I_T, whereas the equilibrium price is necessary to compute $\tau = v + t/p$. With CES demand and a constant marginal cost mc, the equilibrium price under price competition is analytically solved as

$$p = \frac{n(1-\gamma\xi) - \gamma(1-\xi)}{\gamma n(1-\gamma\xi) - \gamma(1-\xi)} mc > mc,$$

and the equilibrium price under quantity competition is given by

$$p = \frac{n}{\gamma[n-(1-\xi)]} mc > mc.$$

More details on the equilibria are included in Online Appendix I.

Figure 5 depicts the differences across the competition-tax pairs regarding the pass-through value (top), the marginal value of public funds (middle), and the incidence (bottom) when $mc = 1$, $\xi = 0.9$, and $(t, v) = (0.05, 0.05)$. The left panel shows how ρ, $MVPF$, and I change in response to changes in the number of firms, and the right panel shows such changes in response to changes in γ.[29]

3.3 Logit demand

The last parametric example is the *logit demand*. Each firm $i = 1, \ldots, n$ faces the following demand: $s_i(\mathbf{p}) = \exp(\delta - \beta p_i)/[1 + \sum_{i'=1,\ldots,n} \exp(\delta - \beta p_{i'})] \in (0,1)$, where δ is the (symmetric) product-specific utility and $\beta > 0$ is the *responsiveness* to the price.[30] We define $s_0 = 1 - \sum_{i=1,\ldots,n} s_i < 1$ as the share of all outside goods. Table 1 (c) summarizes the key variables that determine the pass-through, the

[28] Here, we use the first-order derivative of $p(q)$, $p'(q) = -(1-\gamma\xi)(\gamma\xi)n^{-(1-\xi)}q^{-(2-\gamma\xi)}$, and its second-order derivative, $p''(q) = (2-\gamma\xi)(1-\gamma\xi)(\gamma\xi)n^{-(1-\xi)}q^{-(3-\gamma\xi)}$, for these derivations.

[29] Here we focus only on the intermediate values of γ (i.e., $\gamma \in [0.3, 0.7]$) to ensure that the elasticity of substitution is not close to zero or one.

[30] Here, $q_i(p_1, \ldots, p_n)$ is derived by aggregating over individuals who choose product i (the total number of individuals is normalized to one): an individual's net utility from consuming i is given by $u_i = \delta - \beta p_i + \tilde{\varepsilon}_i$, whereas $u_0 = \tilde{\varepsilon}_0$ is the net utility from consuming nothing, and $\tilde{\varepsilon}_0, \tilde{\varepsilon}_1, \ldots, \tilde{\varepsilon}_n$ are independently and identically distributed according to the Type I extreme value distribution for all individuals. See Anderson, de Palma, and Thisse (1992, pp. 39-45) for details. We work in terms of market share variables s_i and s, instead of q_i and q, which is consistent with the standard notation in the industrial organization literature.

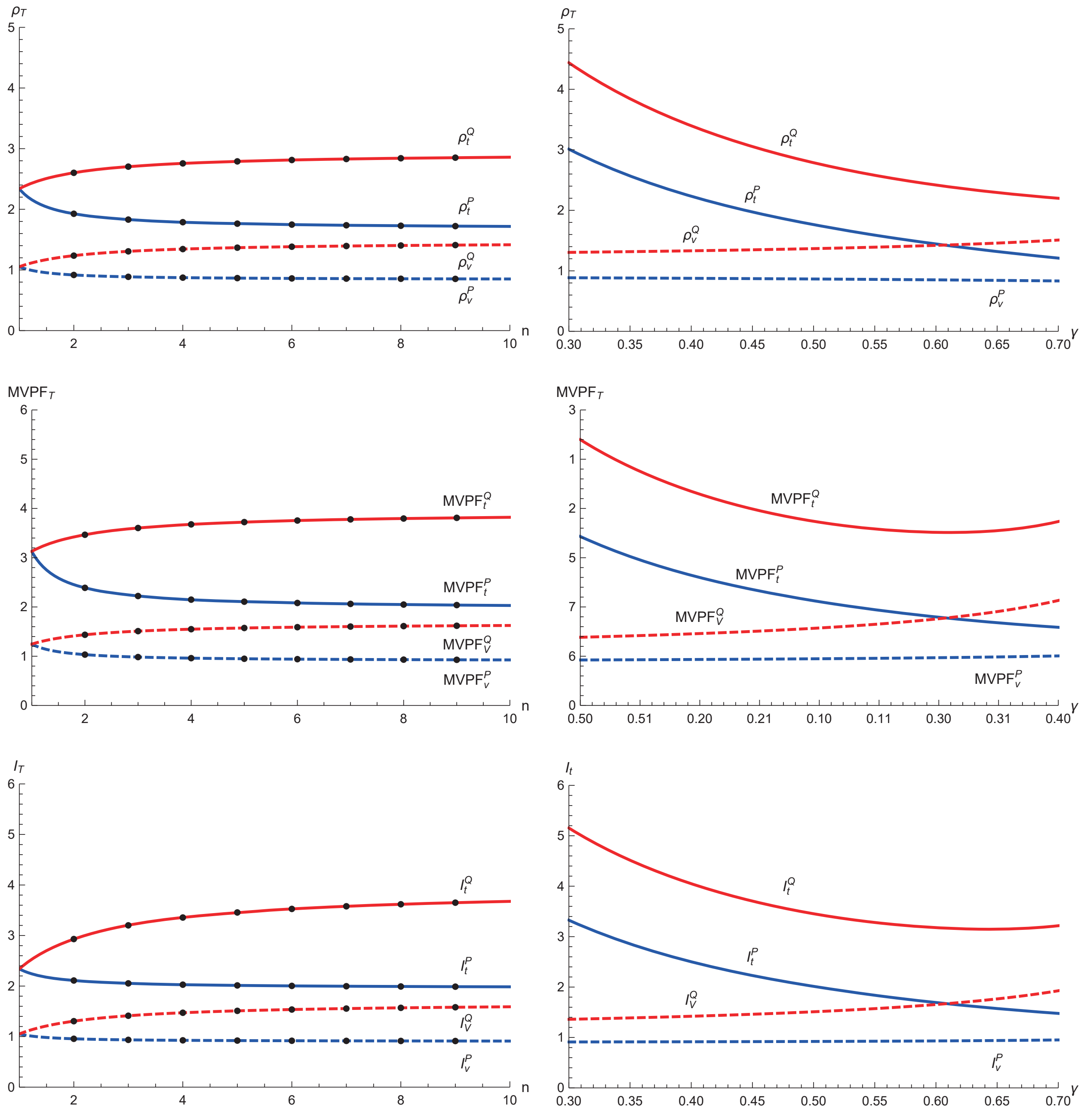

Figure 5: Pass-through (top), marginal value of public funds (middle), and incidence (bottom) with constant elasticity of substitution (CES) demand. The horizontal axes on the left and the right panels are the number of firms (n) with $\gamma = 0.5$, and the substitution parameter (γ) with$n = 5$, respectively (with the initial tax level, $(t, v) = (0.05, 0.05)$).

marginal value of public funds, and the incidence. We need to numerically solve for the equilibrium price and market share under both settings to compute these values for all four cases. To focus on the two parameters, β and n, we assume that $\delta = 1$ and $mc = 0$. Because $\partial s_i(\mathbf{p})/\partial p_i|_{\mathbf{p}=(p,\ldots,p)} = -\beta s(1-s)$, the first-order conditions for the symmetric equilibrium price and the market share satisfy $p - t/(1-v) = 1/[\beta(1-s)]$ and $s = \exp(1-\beta p)/[1+n\cdot\exp(1-\beta p)]$. If p and s are solved numerically, then ε, ε_{own}, θ and α can also be numerically computed.[31]

Next, we consider the inverse demands under quantity competition. Then, as in Berry (1994), firm i's inverse demand is given by $p_i(\mathbf{s}) = [\delta - \log(s_i/s_0)]/\beta$, where $\mathbf{s} = (s_1,\ldots,s_n)$, which implies that $\partial p_i(\mathbf{s})/\partial s_i|_{\mathbf{s}=(s,\ldots,s)} = -[1-(n-1)s]/[\beta s(1-ns)]$. Thus, the first-order conditions for the symmetric equilibrium price and the market share satisfy $p - t/(1-v) = [1-(n-1)s]/[\beta(1-ns)]$ and $p = [1 - \log(s/[1-ns])]/\beta$. Then, as above, η, η_{own}, θ and σ are computed by numerically solving the first-order conditions for p and s. Interestingly, it is verified that in symmetric equilibrium under quantity setting, $\partial p/\partial n = 0$: the equilibrium price is the same irrespective of the number of firms, whereas the individual market share is decreasing in the number of firms: $\partial s/\partial n < 0$. On the other hand, both the equilibrium price and market share are decreasing in the price coefficient, β.

Figure 6 illustrates the pass-through, the marginal value of public funds, and the incidence, in analogy with Figures 4 and 5. The right panels now show the variables' dependence on the price coefficient β. Overall, as in the case of the linear demand and the CES demand, an increase in the ad valorem tax has a small impact on these measures for each of n and β, whereas an increase in the unit tax has a large effect.

However, there are two important differences between linear and logit demands. First, the unit tax pass-through under quantity competition ρ_t^Q is *decreasing* in the number of firms. To understand this, compare the difference in the denominators of $\rho_t^P = 1/\{(1-v)[1+(1-\alpha/\varepsilon_{own})\theta]\}$ and $\rho_t^Q = (1-v)[1+\theta-\sigma]$. As θ decreases (i.e., as competition becomes fiercer), the second term in the denominator of ρ_t^P decreases, and thereby ρ_t^P *increases* as n increases. However, $\theta - \sigma$ increases as θ decreases, and thus ρ_t^Q *decreases*. This difference in the denominators is also reflected in the fact that I_t^Q is decreasing

[31] It can be verified that $s_i(\cdot;\mathbf{p}_{-i})$ is convex as long as $s_i < 1/2$ because $\partial^2 s_i/\partial p_i^2 = -\beta(\partial s_i/\partial p_i)(1-2s_i) > 0$. However, the second-order condition is always satisfied because $\partial^2\pi_i/\partial p_i^2 = -\beta s_i < 0$. In symmetric equilibrium with $\delta = 1$ and $mc = 0$, the largest market share is attained as $1/(n+1)$ when the equilibrium price is zero, which implies that the market share of the outside goods s_0 is no less than each firm's market share: $s_0 > s$.

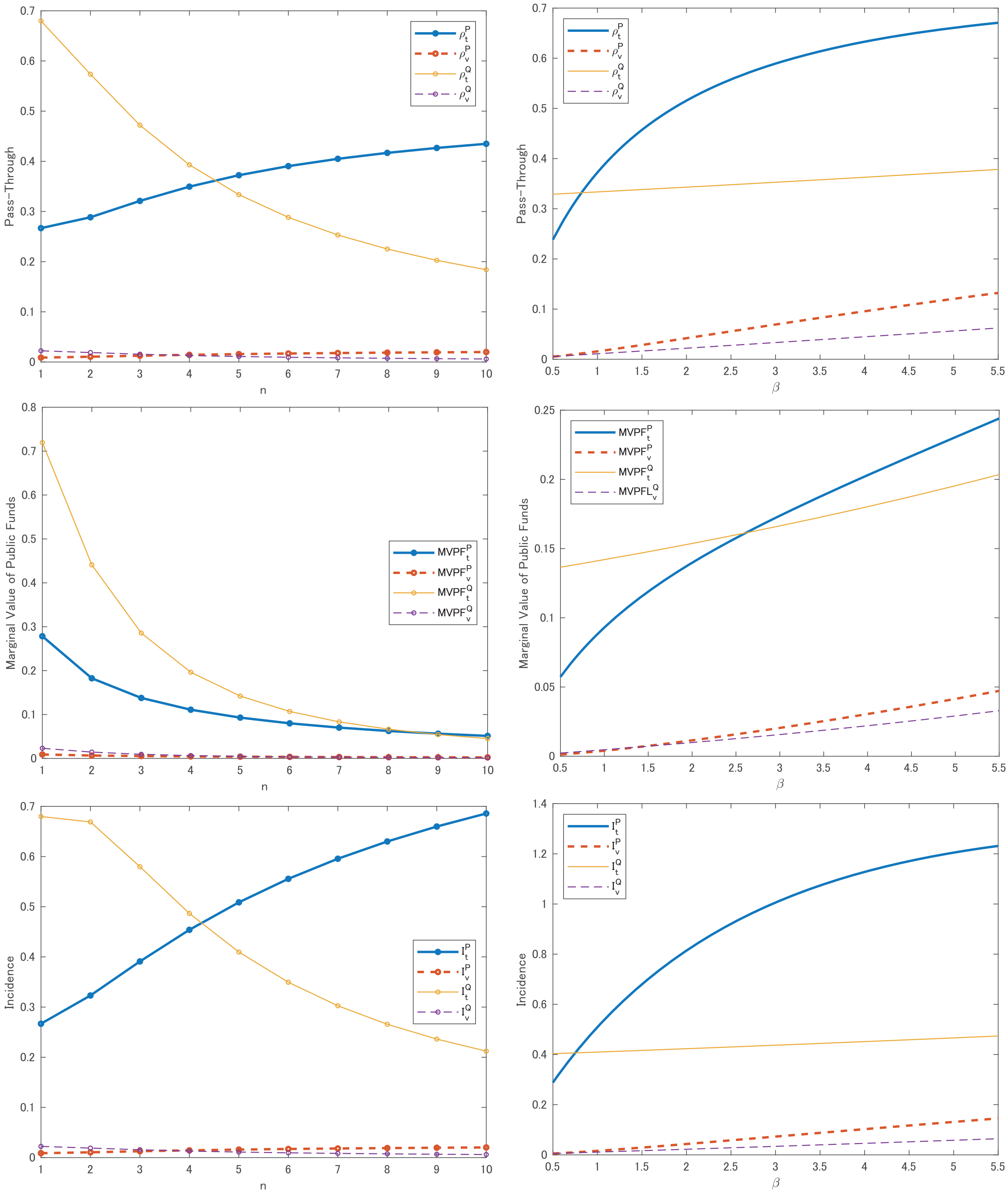

Figure 6: Pass-through (top), marginal value of public funds (middle), and incidence (bottom) with logit demand. The horizontal axes on the left and the right panels are the number of firms (n) with $\beta = 1.0$, and the price coefficient (β) with$n = 5$, respectively (with the initial tax level, $(t, v) = (0.05, 0.05)$).

in n as well. Naturally, $MVPF_t^Q$ is decreasing in n as in the case of linear demand because $1/\rho_t^Q$ becomes larger (see the formulas in Proposition 1). Second, while the pass-through and the incidence increase as β increases, the marginal value of public funds is also increasing in contrast to the case of linear demands. The reason is that the effect on $MVPF$ of decreases in θ is weaker than the effect of the increase in ε: the industry's demand becomes elastic quickly as consumers become more sensitive to a price increase.

4 Firm Heterogeneity

In this section, we extend our results to the case of n heterogeneous firms, where each firm i controls a strategic variable σ_i, which would be, for example, the price or quantity of its product. Appendix C presents the general version of multi-dimensional interventions and establishes some results on pass-through and welfare measures. In the following, p_i is the price of firm i's product, q_i is the quantity of the product sold by firm i.

Under firm heterogeneity, Equation (1) is generalized as

$$\left[\left(1-\frac{t}{p_i(\mathbf{q})}-v\right)-\psi_i(\mathbf{q})(1-v)\right]p_i(\mathbf{q})=mc_i(q_i), \tag{12}$$

for $i=1,2,...,n$, where we call $\psi_i(\mathbf{q})$ firm i's *pricing strength index*. In the case of symmetric firms, the pricing strength index is related to the conduct index $\theta(q)$ by $\theta=\varepsilon\psi$.[32]

[32]For clarity of intuition, suppose that $(t,v)=(0,0)$. Then, Equation (12) implies $p_i(\mathbf{q})=mc_i(q_i)/[1-\psi_i(\mathbf{q})]$. If it was the case that $\psi_i(\mathbf{q})=0$ for any $\mathbf{q}$ and i, all firms would adopt marginal cost pricing. If ψ_i is sufficiently large, p_i can be substantially above the marginal cost. We find that with heterogeneous firms, it is significantly more convenient to use the pricing strength index than to use the conduct index when we characterize the marginal value of public funds and the incidence. Appendix D discusses the relationship between these two concepts.

4.1 Pass-through

The pass-through matrix for the two-dimensional taxation (t and v) is defined as

$$\tilde{\rho} \equiv \begin{pmatrix} \dfrac{\partial p_1}{\partial t} & \dfrac{\partial p_1}{\partial v} \\ \vdots & \vdots \\ \dfrac{\partial p_n}{\partial t} & \dfrac{\partial p_n}{\partial v} \end{pmatrix}.$$

Using the results from Proposition 10 in Appendix C, the pass-through matrix is characterized as follows.

Proposition 6. *For heterogeneous firms with specific and ad valorem taxation, the pass-through matrix equals*

$$\tilde{\rho} = \mathbf{b}^{-1} \begin{pmatrix} 1 & p_1 \cdot (1-\psi_1) \\ \vdots & \vdots \\ 1 & p_n \cdot (1-\psi_n) \end{pmatrix},$$

where the (i, j) element of the $\mathbf{b}$ matrix is given by

$$b_{ij} = (1-v)\left[(1-\psi_i)\delta_{ij} - \psi_i \Psi_{ij}\right] + \left[(1-\tau_i) - (1-v)\psi_i\right] \chi_i \varepsilon_{ij},$$

where δ_{ij} is the Kronecker delta,[33] $\varepsilon_{ij} \equiv -\frac{p_i}{q_i}\frac{\partial q_i(\mathbf{p})}{\partial p_j}$, $\Psi_{ij} \equiv \frac{p_i}{\psi_i}\frac{\partial \psi_i(\mathbf{q}(\mathbf{p}))}{\partial p_j}$, *and* $\tau_i = \frac{t}{p_i} + v$.

Note that if all firms have constant marginal cost ($\chi_i = 0$ for all i), the expression for b_{ij} simplifies to $b_{ij} = (1-v)\left[(1-\psi_i)\delta_{ij} - \psi_i \Psi_{ij}\right]$.

4.2 Characterization of the two welfare measures

Similarly, by using the results in Appendix C, we obtain the following proposition that characterizes the marginal value of public funds and the incidence for the case of heterogeneous firms, where we define ε_i, an n-dimensional row vector with its j-th component equal to ε_{ij} for each i, by $\varepsilon_i = (\varepsilon_{i1}, ..., \varepsilon_{ij}, ...\varepsilon_{in})$.

Proposition 7. *Let* $\varepsilon_{iT}^{\rho} \equiv \varepsilon_i \tilde{\rho}_T / \tilde{\rho}_{iT} = \varepsilon_i \rho_T / \rho_{iT}$ *for* $T \in \{t, v\}$. *Then, the marginal value of public funds*

[33] As usual, the Kronecker delta δ_{ij} is defined to be equal to 1 if its two indices are the same and zero otherwise.

Table 2: Summary of the Expressions for the Two Welfare Measures for $T \in \{t, v\}$ under Imperfect Competition

	Symmetric firms: No pre-existing taxes	Symmetric firms: With pre-existing taxes	Heterogeneous firms
Marginal Value of Public Funds	$\theta\rho_T$	$\frac{(1-v)(\theta/\varepsilon)+\tau}{\frac{\frac{1}{\rho_T}+v}{\varepsilon}-\tau}$	$\frac{(1-v)\,\psi_i+\tau_i}{\frac{\frac{1}{\rho_{iT}}+v}{\varepsilon_{iT}^{\rho}}-\tau_i}$
Incidence	$\frac{1}{\frac{1}{\rho_T}-(1-\theta)}$	$\frac{1}{\frac{1}{\rho_T}-(1-v)(1-\theta)}$	$\frac{1}{\frac{1}{\rho_{iT}}-(1-v)(1-\psi_i\,\varepsilon_{iT}^{\rho})}$

Note: See the main text for the notations.

associated with intervention T, $MVPF_{iT} = (\nabla W_i)_T \,/\, (\nabla R_i)_T$, is characterized by:

$$MVPF_{iT} = \frac{(1-v)\;\psi_i + \tau_i}{\frac{1}{\varepsilon_{iT}^{\rho}}\left(\frac{1}{\rho_{iT}} + v\right) - \tau_i},$$

and the incidence of this intervention, $I_{iT} = (\nabla CS_i)_T \,/\, (\nabla PS_i)_T$, is characterized by:

$$I_{iT} = \frac{1}{\frac{1}{\rho_{iT}} - (1-v)\,(1-\psi_i\,\varepsilon_{iT}^{\rho})}.$$

Table 2 summarizes our characterization at each stage of generality. The ratios of the corresponding total welfare changes will be weighted averages of these firm-specific ratios. The weights correspond to the sizes of the denominators times q_i. For example, $MVPF_T$ will lie between $\min_i MVPF_{iT}$ and $\max_i MVPF_{iT}$. The same reasoning also holds for I_T.

4.3 Cost heterogeneity

To understand how firm heterogeneity is related to the welfare implications of taxation, we consider an example where two firms are symmetrically differentiated—hence facing an identical demand—but have different marginal costs. Specifically, firm $i = 1,2$ faces the linear demand, $q_i(p_1, p_2) = b - \lambda p_i + \mu p_j$, $j \neq i$, $j = 1,2$. Suppose that either firm's marginal cost of production is constant, $mc_i \geq 0$, and Firm 1 is a low-cost firm: $mc_1 < mc_2$.

4.3.1 Price competition

The first-order conditions for firm i in this pricing game is expressed as:

$$\left[\left(1-\frac{t}{p_i}-v\right)-\frac{q_i}{p_i\cdot\left(-\frac{\partial q_i}{\partial p_i}\right)}(1-v)\right]p_i = mc_i$$

in accordance with Equation (13), where $-\frac{\partial q_i}{\partial p_i}=\lambda$. To compute the welfare characteristics

$$MVPF_{iT} = \frac{(1-v)\;\psi_i+\tau_i}{\dfrac{1}{\varepsilon_{iT}^{\rho}}\left(\dfrac{1}{\rho_{iT}}+v\right)-\tau_i}, \qquad I_{iT} = \frac{1}{\dfrac{1}{\rho_{iT}}-(1-v)\,(1-\psi_i\,\varepsilon_{iT}^{\rho})},$$

we need the values for ψ_i (firm i's pricing strength index), ρ_{iT} (firm i's pass-through), and ε_{iT}^{ρ}, as well as v (ad valorem tax) and $\tau_i \equiv v+\dfrac{t}{p_i}$ (the government tax revenue divided by firm i's gross revenue). See Online Appendix H for these calculations.

As in Section 3, Figure 7 depicts how the pass-through (top), the marginal value of public funds (middle), and the incidence (bottom) vary differently across the two firms (the left side is for Firm 1 and the right for Firm 2), assuming $b = 1$, $(mc_1, mc_2) = (0, 0.5)$, $\mu = 1.25$ and $(t, v) = (0.05, 0.05)$.[34] A noticeable fact is that for Firm 2, the marginal value of public funds associated with an increase in ad valorem tax v ($MVPF_{2v}$) is *larger* than that with unit tax t ($MVPF_{2t}$), meaning that ad valorem taxes are not necessarily welfare superior to unit taxes once firm heterogeneity is introduced. This result is consistent with the previous finding by Anderson, de Palma, and Kreider (2001b) because with cost asymmetries, ad valorem taxes exacerbate the absolute differences in marginal costs across firms" (p. 249).

[34] Here, we consider the restriction, $\mu < \lambda < \frac{b}{c_2}+\mu$, for the range of λ. In Figure 7, we highlight $\lambda \in [1.75, 2.25]$.

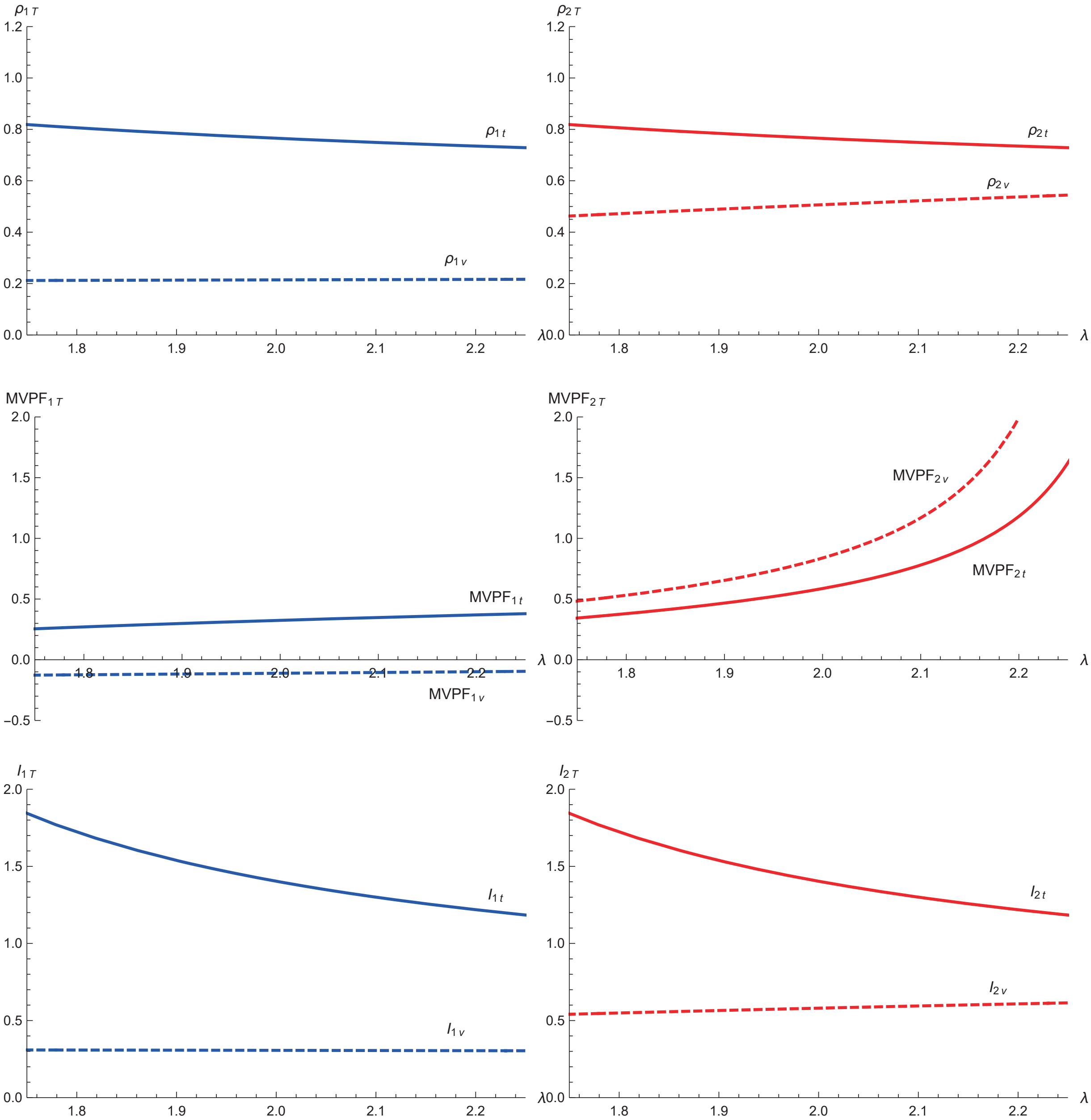

Figure 7: Pass-through (top), marginal value of public funds (middle), and incidence (bottom) when Firm 1 (left) and Firm 2 (right) face an identical demand (linear) but have different marginal costs: The case of price competition.

4.3.2 Quantity competition

Similarly, the first-order conditions for firm i under quantity competition is given by:

$$\left[\left(1-\frac{t}{p_i}-v\right)-\frac{q_i}{p_i\cdot\left(-\frac{1}{\partial p_i/\partial q_i}\right)}(1-v)\right]p_i = mc_i,$$

in accordance with Equation (13), where $-\frac{1}{\partial p_i/\partial q_i}=\frac{(\lambda+\mu)(\lambda-\mu)}{\lambda}$.

Figure 8 exhibits the similarity to the case of price competition, although it appears that $MVPF_{2v}$ is lower and thus closer to $MVPF_{2v}$. Once firm heterogeneity is allowed, the welfare superiority of ad valorem tax over unit tax can break down under either price or quantity competition.

5 Concluding Remarks

In this paper, we characterize the welfare measures of taxation under general specifications of market demand, production cost, and imperfect competition. For symmetric oligopoly, we first derive formulas for measuring marginal welfare losses resulting from unit and ad valorem taxation, $MVPF_t$ and $MVPF_v$, respectively, using the unit tax pass-through rate ρ_t and the ad valorem tax pass-through semi-elasticity ρ_v (Proposition 1) as well as the formulas for tax incidence, I_t and I_v (Proposition 2). We then demonstrate that ρ_v can be related to ρ_t (i.e., Proposition 3). These relationships are used to derive sufficient statistics for $MVPF_t$ and $MVPF_v$ (Proposition 4). The pass-through is also characterized, generalizing Weyl and Fabinger's (2013) formula (i.e., Proposition 5). Section 3 computes these welfare measures using the representative classes of market demand.

We then introduce heterogeneous firms in Section 4 to generalize these formulas that can be understood as a natural extension of those obtained under firm symmetry (Proposition 7). Our derivation is based on a general framework, illustrated in Appendix C, which uses the idea of tax revenue as a function parameterized by a vector of tax parameters and thus can allow multi-dimensional pass-through: Proposition 6) on the specific and the ad valorem tax pass-through is the result tailored to the case of two-dimensional government intervention. Using a specific example of two differentiated firms facing an identical linear demand but heterogeneous marginal costs, we find that the marginal value of public

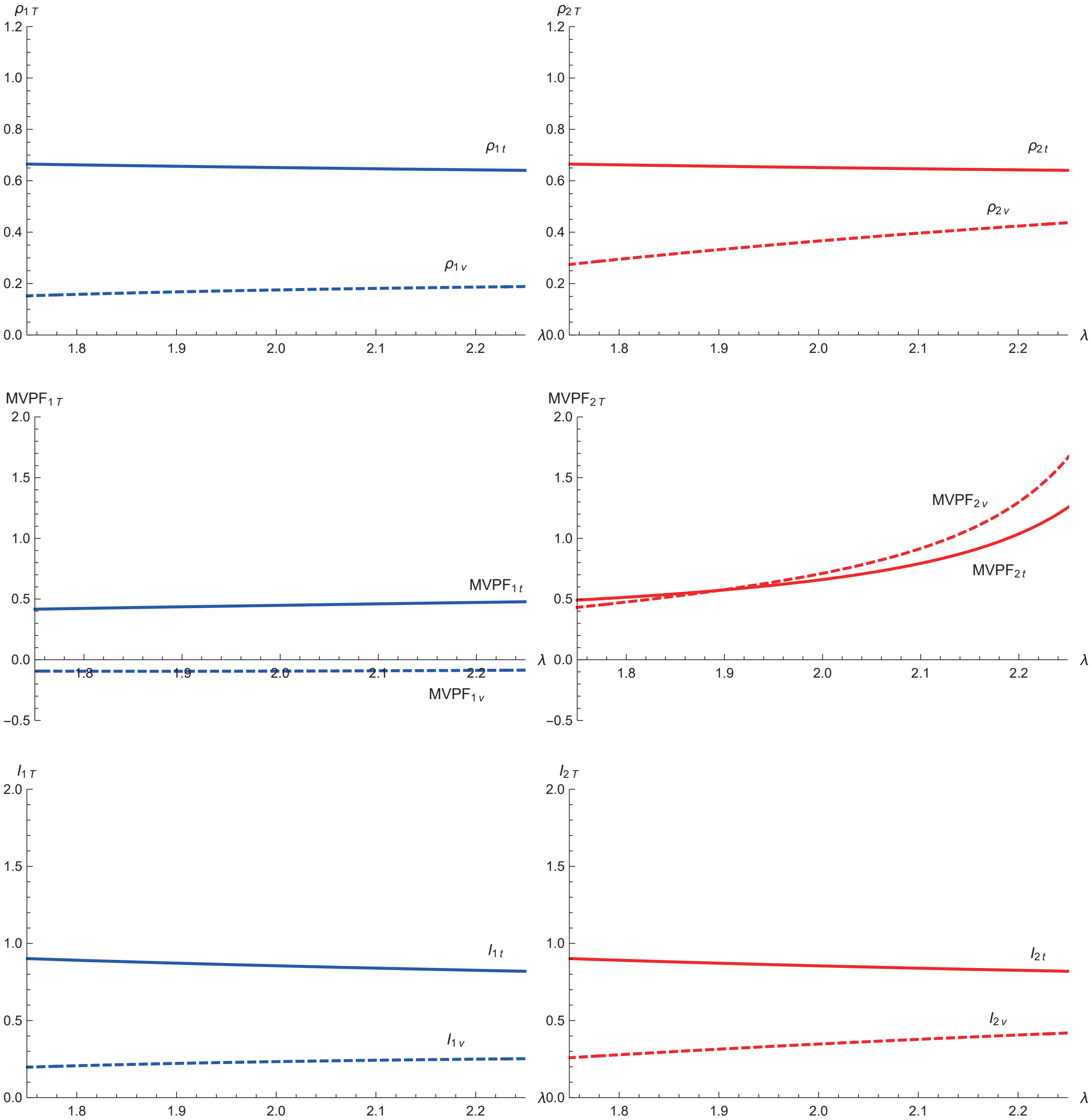

Figure 8: Pass-through (top), marginal value of public funds (middle), and incidence (bottom) when Firm 1 (left) and Firm 2 (right) face an identical demand (linear) but have different marginal costs: The case of quantity competition.

funds for ad valorem tax that is attributed to the high-cost firm can be higher than that for unit tax. In this way, we have provided a comprehensive framework for welfare evaluation of taxation under imperfect competition, which can also allow many applications in a variety of contexts other than taxation.

Appendix A Proofs and further discussion for Section 2

A.1 Discussion of signs of changes in welfare components for a specific tax increase

Figure 2 shows the effect of a specific tax increase in one case. Here we discuss the signs of welfare component changes in generality. It is helpful to work at the infinitesimal level, where such a tax change would correspond to $dt > 0$ and $dv = 0$. For the producer surplus, the contributions the quantity effect is negative $dPS_{\leftrightarrow} = (1-v)p\eta\theta\, dq < 0$, and the value effect $dPS_{\updownarrow} = (1-v)q\, dp - q\, dt = -q(1-(1-v)\rho_t)\, dt$ is negative for $\rho_t < \frac{1}{1-v}$ and positive for $\rho_t > \frac{1}{1-v}$. The overall change is $dPS = (1-v)p\eta\theta\, dq - q(1-(1-v)\rho_t)\, dt = (\frac{1}{\rho_t} - (1-v)(1-\theta))\eta p\, dq$, which is negative for $\frac{1}{\rho_t} > (1-v)(1-\theta)$ and positive for $\frac{1}{\rho_t} < (1-v)(1-\theta)$. For a sufficiently small value of pass-through, the firms' profit will decrease when t is increased. For the specific tax revenue, the quantity effect and the value effect have opposite signs: $dR_{t\leftrightarrow} = t\, dq < 0$, $dR_{t\updownarrow} = q\, dt > 0$. The overall change $dR_t = t\, dq + q\, dt = (t - \frac{\eta p}{\rho_t})dq$ is positive for $t < \frac{\eta p}{\rho_t}$ and negative for $t > \frac{\eta p}{\rho_t}$. For the ad valorem tax revenue, the quantity effect and the value effect again have opposite signs: $dR_{v\leftrightarrow} = vp\, dq < 0$, $dR_{v\updownarrow} = qv\, dp > 0$. The overall change $dR = dR_{v\leftrightarrow} + dR_{v\updownarrow} = (1-\eta)\, vp\, dq$ is negative, if assume $\eta < 1$, as we typically do. The consumer surplus decreases, as $dCS_{\leftrightarrow}$ is zero, and $dCS_{\updownarrow} = -q\, dp$ is unambiguously negative for $dt > 0$.

A.2 Discussion of signs of changes in welfare components for an ad valorem tax increase

Figure 3 show the effect of a specific tax increase in one case. Here we discuss the signs of welfare component changes in generality. It is helpful to work at the infinitesimal level where such a tax change would correspond to $dv > 0$ and $dt > 0$. For the producer surplus, the contributions the quantity effect is negative $dPS_{\leftrightarrow} = (1-v)p\eta\theta\, dq < 0$, and the value effect $dPS_{\updownarrow} = (1-v)\,q\, dp - pq\, dv = -\left(1-(1-v)\,\rho_v\right) pq\, dv$ is negative for $\rho_v < \frac{1}{1-v}$ and positive for $\rho_v > \frac{1}{1-v}$. The overall change is $dPS = (1-v)p\eta\theta\, dq - \left(1-(1-v)\,\rho_v\right) pq\, dv = (\frac{1}{\rho_v} - (1-v)\,(1-\theta))\eta p\, dq$, which is negative for $\frac{1}{\rho_v} > (1-v)\,(1-\theta)$ and positive for $\frac{1}{\rho_v} < (1-v)\,(1-\theta)$. For a sufficiently small value of pass-through, the firms' profit will decrease when v is increased. For the specific tax revenue, the quantity effect $dR_{t\leftrightarrow} = t\, dq$ is negative, while the value effect $dR_{t\updownarrow}$ is zero as the specific tax rate is unchanged. The overall change $dR_t = dR_{t\leftrightarrow} = t\, dq$ is therefore negative. For the ad valorem tax revenue, the quantity effect and the value effect again have opposite signs: $dR_{v\leftrightarrow} = vp\, dq < 0$, $dR_{v\updownarrow} = qv\, dp + qp\, dv > 0$. The overall change $dR = dR_{v\leftrightarrow} + dR_{v\updownarrow} = (1-\eta)\, vp\, dq$ is negative, if assume $\eta < 1$, as we typically do. The consumer surplus decreases, as $dCS_{\leftrightarrow}$ is zero, and $dCS_{\updownarrow} = -q\, dp$ is unambiguously negative for $dt > 0$.

A.3 Proof of Proposition 3

Let us consider a simultaneous infinitesimal change dt and dv in the taxes t and v that leaves the equilibrium price (and quantity) unchanged, which requires the "perceived" marginal cost $(t+mc)\,/\,(1-v)$ in Equation (1) to remain the same. This implies the following comparative statics relationship:

$$\frac{\partial}{\partial t}\left(\frac{t+mc}{1-v}\right)dt + \frac{\partial}{\partial v}\left(\frac{t+mc}{1-v}\right)dv = 0 \Rightarrow \frac{dt}{1-v} + \frac{t+mc}{(1-v)^2}dv = 0 \Rightarrow dt = -\frac{t+mc}{1-v}dv.$$

Note here that we do not need to take derivatives of mc even though it depends on q, simply because by assumption the quantity is unchanged. The total induced change in price, which is generally expressed as $dp = \rho_t dt + \rho_v p\, dv$, must equal zero in this case, implying the desired result:

$$\rho_t dt + \rho_v p \cdot dv = 0 \Rightarrow -\frac{t+mc}{1-v}\rho_t dv + \rho_v p \cdot dv = 0 \Rightarrow \rho_v = (1-\eta\theta)\,\rho_t \Rightarrow \rho_v = \frac{\varepsilon-\theta}{\varepsilon}\rho_t.$$

A.4 Proof of Proposition 5

Consider the comparative statics with respect to a small change dt in the per-unit tax t. Following Weyl and Fabinger (2013, p. 538), we define $ms \equiv -p'q$ as the negative of marginal consumer surplus. Then, the Learner condition becomes:

$$\underbrace{p-\frac{t+mc}{1-v}}_{\text{markup}}=\theta\cdot ms.$$

Then, in equilibrium,

$$dp-\frac{dt+dmc}{1-v}=d(\theta\cdot ms) \qquad \Leftrightarrow \qquad \underbrace{(1-v)[\underbrace{dp}_{>0}-\underbrace{d(\theta\cdot ms)}_{<0}]}_{\text{change in marginal benefit}}= \qquad \underbrace{\underbrace{dt}_{>0}+\underbrace{dmc}_{<0}}_{\text{change in virtual marginal cost}},$$

and thus, using $dt=dp/\rho_t$, the equation is rewritten as

$$\rho_t=\frac{1}{\underbrace{(1-v)\,[dp+(-d\,(\theta\cdot ms))]}_{(1)>0:\text{ revenue increase}}+\underbrace{(-dmc)}_{(2)>0:\text{ cost savings}}}dp.$$

Now, consider term (1). Note first $d(\theta\cdot ms)=(\theta\cdot ms)'dq$ so that $d(\theta\cdot ms)=-q\varepsilon(\theta\cdot ms)'(dp/p)$, because by definition $dq=-q\varepsilon\cdot(dp/p)$. Here, for a small increase $dt>0$,

$$\underbrace{d(\theta\cdot ms)}_{<0}=\underbrace{-q\varepsilon}_{>0}(\theta\cdot ms)'\underbrace{\frac{dp}{p}}_{>0}$$

so that $(\theta\cdot ms)'>0$. By definition, $ms\equiv-p'q=\eta p$. Thus, $d(\theta\cdot ms)=-q\varepsilon(\theta\eta p)'(dp/p)$. Now, note that $(\theta\eta p)'=(\theta\eta)'p+(\theta\eta)p'$. Hence,

$$d(\theta\cdot ms)=-q\varepsilon\left[(\theta\eta)'p+(\theta\eta)p'\right]\frac{dp}{p}$$

$$\Leftrightarrow d(\theta\cdot ms)=-q\varepsilon(\theta\eta)'dp+[-q\varepsilon(\theta\eta)p'\cdot(dp/p)]=[\theta\eta-q\varepsilon(\theta\eta)']dp>0.$$

Next, consider term (2). A change in marginal cost, dmc, is expressed in terms of dp by $dmc=-[(1-v)\,\theta\eta+1-\tau]\chi\varepsilon\cdot dp<0$. To see this, note first that $dmc=\chi mc\cdot(dq/q)=-(\chi\varepsilon\cdot mc)(dp/p)$. Then, mc in this expression can be eliminated by rewriting $p-\theta\cdot ms=(mc+t)\,/\,(1-v)\Rightarrow mc=(1-v)\,(p+\theta qp')-t=(1-v)\,(1-\theta\eta)\,p-t$, which implies that $dmc=-[(1-v)\,(1+\theta\eta)-t/p]\chi\varepsilon\cdot dp$. Then, in terms of the per-unit revenue burden, $\tau\equiv v+t/p$, that is, $dmc=-[(1-v)\,(1-\theta\eta)-\tau+$

$v]\chi\varepsilon\, dp = -[-(1-v)\,\theta\eta + 1 - \tau]\chi\varepsilon\, dp$. Finally, using the expressions for dmc and $d(\theta \cdot ms)$,

$$\rho_t = \frac{dp}{(1-v)\,[dp - d\,(\theta \cdot ms)] - dmc}$$

$$= \frac{1}{\underbrace{(1-v)\,[(1-\theta\eta) + (\theta\eta)'\,\varepsilon q]}_{\text{revenue increase}} + \underbrace{(1-\tau)\,\varepsilon\chi - (1-v)\,\theta\chi}_{\text{cost savings}}},$$

$$\Leftrightarrow \rho_t = \frac{1}{1-v} \cdot \frac{1}{\underbrace{[(1-\theta\eta) + (\theta\eta)'\,\varepsilon q]}_{\text{revenue increase}} + \underbrace{\left[-\theta + \frac{1-\tau}{1-v}\varepsilon\right]\chi}_{\text{cost savings}}}.$$

Finally, ρ_v is obtained from this expression and Equation (11).

Appendix B Specifying the mode of imperfect competition under firm symmetry

In this appendix, we demonstrate that for a static game of price or quantity competition with no anti-competitive conduct, our general formulas of the marginal value of public funds and the pass-through derive the expressions in terms of demand primitives such as the *elasticities*, the *curvatures*, and the *marginal cost elasticity* χ.[35] Throughout this appendix, we assume that firms' conduct is simply described by one-shot Nash behavior, without any other further possibilities such as tacit collusion. As seen below, this assumption enables one to express the conduct index in terms of demand and inverse demand elasticities, using Equation (1) directly (see Subsection B.2 below). Online Appendix G further investigates the relationship between elasticities and curvatures.

[35] The question of whether quantity- or price-setting firms are more appropriate depends on the nature of competition. As Riordan (2008, p. 176) argues, quantity competition is a more appropriate model if one depicts a situation where firms determine the necessary production capacity. However, price-setting firms are more suitable if firms in the industry of focus can quickly adjust to demand by changing their prices.

B.1 Elasticities and curvatures of the demand system

B.1.1 Direct demand

We additionally define the *cross-price elasticity* $\varepsilon_{cross}(p)$ *of the firm's direct demand* by

$$\varepsilon_{cross}(p) \equiv \frac{(n-1)p}{q(p)} \cdot \frac{\partial q_{i'}(\mathbf{p})}{\partial p_i}|_{\mathbf{p}=(p,\ldots,p)},$$

where i and i' is an arbitrary pair of distinct indices. It is related to the industry demand elasticity $\varepsilon(p)$ by $\varepsilon_{own} = \varepsilon + \varepsilon_{cross}$.[36] Next, we define the *own curvature* $\alpha_{own}(p)$ *of the firm's direct demand* and the *cross curvature* $\alpha_{cross}(p)$ *of the firm's direct demand* by:[37]

$$\alpha_{own}(p) \equiv -p \cdot \left(\frac{\partial q_i(\mathbf{p})}{\partial p_i}\right)^{-1} \cdot \frac{\partial^2 q_i(\mathbf{p})}{\partial p_i^2},$$

and

$$\alpha_{cross}(p) \equiv -(n-1)p \cdot \left(\frac{\partial q_i(\mathbf{p})}{\partial p_i}\right)^{-1} \cdot \frac{\partial^2 q_i(\mathbf{p})}{\partial p_i \partial p_{i'}},$$

respectively, where again the derivatives are evaluated at $\mathbf{p} = (p, ..., p)$, and i and i' is an arbitrary pair of distinct indices. These curvatures satisfy $\alpha = (\alpha_{own} + \alpha_{cross})\varepsilon_{own}/\varepsilon$ and are related to the elasticity of $\varepsilon_{own}(p)$ by $p\,\varepsilon'_{own}(p)/\varepsilon_{own}(p) = 1 + \varepsilon(p) - \alpha_{own}(p) - \alpha_{cross}(p)$ (see 5 below for the derivation and a related discussion).

[36]Holmes (1989) shows this for two symmetric firms, but it is straightforward to verify this relation more generally. See the equation in Footnote 10 above. Note that the equation $\varepsilon_{own} = \varepsilon + \varepsilon_{cross}$ simply means that the percentage of consumers who cease to purchase firm i's product in response to its price increase is decomposed into (i) those who no longer purchase from any of the firms (ε) and (ii) those who switch to (any of) the other firms' products (ε_{cross}). Thus, ε_{own} measures the *firm's own competitiveness*, which is expressed in terms of the industry elasticity and the intensity of rivalry. In this sense, these three price elasticities characterize the "first-order" competitiveness, which determines whether the equilibrium price is high or low, but one of them is not independently determined from the other two elasticities.

[37]The curvature $\alpha_{own}(p)$ here corresponds to $\alpha(p)$ of Aguirre, Cowan, and Vickers (2010, p. 1603).

B.1.2 Inverse demand

We introduce analogous definitions for inverse demand. First, we define the *cross quantity elasticity* $\eta_{cross}(q)$ *of the firm's inverse demand* as

$$\eta_{cross}(q) \equiv (n-1)\frac{q}{p(q)} \cdot \frac{\partial p_{i'}(\mathbf{q})}{\partial q_i}|_{\mathbf{q}=(q,\ldots,q)}$$

for arbitrary distinct i and i'. It is verified that $\eta_{own} = \eta + \eta_{cross}$.[38] We furthermore define the *own curvature* $\sigma_{own}(q)$ *of the firm's inverse demand* and the *cross curvature* $\sigma_{cross}(q)$ *of the firm's inverse demand* by:

$$\sigma_{own}(q) \equiv -q \cdot \left(\frac{\partial p_i(\mathbf{q})}{\partial q_i}\right)^{-1} \cdot \frac{\partial^2 p_i(\mathbf{q})}{\partial q_i^2}$$

and

$$\sigma_{cross}(q) \equiv -(n-1)q \cdot \left(\frac{\partial p_i(\mathbf{q})}{\partial q_i}\right)^{-1} \cdot \frac{\partial^2 p_i(\mathbf{q})}{\partial q_i \partial q_{i'}},$$

respectively, where again the derivatives are evaluated at $\mathbf{q} = (q,\ldots,q)$ and the indices i and i' are distinct. These curvatures represent an oligopoly counterpart of monopoly $\sigma(q)$ of Aguirre, Cowan, and Vickers (2010, p. 1603). They satisfy the relationship $\sigma = (\sigma_{own} + \sigma_{cross})(\eta_{own}/\eta)$ and are related to the elasticity of $\eta_{own}(q)$ by $q\eta'_{own}(q)/\eta_{own}(q) = 1 + \eta(q) - \sigma_{own}(q) - \sigma_{cross}(q)$ (see 5 below for the derivation and a related discussion).

B.2 Expressions for pass-through and the conduct index

B.2.1 Price competition

In the case of price competition, the conduct index θ is $\theta = \varepsilon/\varepsilon_{own} = 1/(\eta\varepsilon_{own})$, which is verified by comparing the firm's first-order condition with Equation (1). The marginal change in deadweight loss

[38] The identity $\eta_{own} = \eta + \eta_{cross}$ means that as a response to firm i's increase in its output, the industry as a whole reacts by lowering firm i's price (η). However, each firm (other than i) reacts to this firm i's output increase by reducing its own output. This counteracts the initial change in the price ($\eta_{cross} < 0$), and thus a percentage reduction in the price for firm i (η_{own}) is smaller than η, which does not take into account strategic reactions. Note here that $1/\eta_{own}$, not η_{own}, measures the industry's competitiveness. Thus, as in the case of price competition, these three quantity elasticities characterize "first-order" competitiveness, which determines whether the equilibrium quantity is high or low.

and the incidence are obtained by substituting these expressions into those of Propositions 1 and 2.

Proposition 8. *Under symmetric oligopoly with price competition and with a possibly non-constant marginal cost, the unit tax pass-through and the ad valorem tax pass-through are characterized by*

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{1 + \frac{(1-\alpha/\varepsilon_{own})\varepsilon}{\varepsilon_{own}} + \left(\frac{1-\tau}{1-v} - \frac{1}{\varepsilon_{own}}\right)\varepsilon\chi}$$

and

$$\rho_v = \frac{1}{1-v} \cdot \frac{1}{\frac{1}{1-1/\varepsilon_{own}} + \frac{(1-\alpha/\varepsilon_{own})\varepsilon}{\varepsilon_{own}-1} + \left(\frac{1-\tau}{1-v} \cdot \frac{\varepsilon_{own}}{\varepsilon_{own}-1} - \frac{1}{\varepsilon_{own}-1}\right)\varepsilon\chi},$$

respectively.

Proof. Since in the case of price setting $\theta = \varepsilon/\varepsilon_{own} = 1/(\eta\varepsilon_{own})$, we have $(\eta+\chi)\theta = (1+\varepsilon\chi)/\varepsilon_{own}$ and $(\theta\eta)'\varepsilon q = \varepsilon q \frac{d}{dq}(\theta\eta) = \varepsilon q \frac{d}{dq}(\varepsilon_{own}^{-1}) = -\varepsilon_{own}^{-2}\varepsilon q \frac{d}{dq}\varepsilon_{own} = \varepsilon_{own}^{-2} p \frac{d}{dp}\varepsilon_{own} = (1+\varepsilon-\alpha\varepsilon/\varepsilon_{own})/\varepsilon_{own}$, where in the last equality we utilize the expression for the elasticity of $\varepsilon_{own}(p)$ and $\alpha_{own}+\alpha_{cross} = \alpha\varepsilon/\varepsilon_{own}$ from 5 above. Substituting these into the expression for ρ_t in Proposition 5 gives

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{1 - \frac{1}{\varepsilon_{own}}(1+\varepsilon\chi) + \frac{1}{\varepsilon_{own}}\left(1+\varepsilon - \frac{\alpha\varepsilon}{\varepsilon_{own}}\right) + \frac{1-\tau}{1-v}\varepsilon\chi},$$

which is equivalent to the expression for ρ_t in the proposition. Since for price setting $\theta = \varepsilon/\varepsilon_{own}$, the relationship in Proposition 3 implies $\rho_v = (\varepsilon-\theta)\rho_t/\varepsilon = (\varepsilon_{own}-1)\rho_t/\varepsilon_{own}$, which leads to the desired expression for ρ_v. □

To understand this proposition, first recall from Proposition 5 that

$$\rho_t = \frac{1}{1-v} \frac{1}{\underbrace{[(1-\theta\eta)+(\theta\eta)'\varepsilon q]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v}\varepsilon - \theta\right]\chi}_{\text{cost savings}}}.$$

Then, with $\theta = \varepsilon/\varepsilon_{own}$, $1-\theta\eta = 1-1/\varepsilon_{own}$, $(\theta\eta)'\varepsilon q = (1+\varepsilon-\alpha\varepsilon/\varepsilon_{own})/\varepsilon_{own}$, the equality above is rewritten as

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{\underbrace{\left[\left(1-\frac{1}{\varepsilon_{own}}\right)+\frac{1+\varepsilon-\alpha\varepsilon/\varepsilon_{own}}{\varepsilon_{own}}\right]}_{\text{revenue increase}}+\underbrace{\left[\frac{1-\tau}{1-v}-\frac{1}{\varepsilon_{own}}\right]\varepsilon\chi}_{\text{cost savings}}}$$

$$= \frac{1}{1-v} \cdot \frac{1}{\underbrace{\left[1+\frac{(1-\alpha/\varepsilon_{own})\varepsilon}{\varepsilon_{own}}\right]}_{\text{revenue increase}}+\underbrace{\left[\frac{1-\tau}{1-v}-\frac{1}{\varepsilon_{own}}\right]\varepsilon\chi}_{\text{cost savings}}}.$$

To further facilitate the understanding of the connection of this result to Proposition 5, consider the case of zero initial taxes ($t = v = \tau = 0$). Then, Proposition 5 claims that

$$\rho_t = \frac{1}{1+\varepsilon\chi-\theta\chi+[-\eta\theta+\varepsilon q(\theta\eta)']},$$

whereas Proposition 8 shows that

$$\rho_t = \frac{1}{1+\varepsilon\chi-\theta\chi+[-\frac{1}{\varepsilon}\cdot\frac{\varepsilon}{\varepsilon_{own}}+\frac{1+(1-\alpha/\varepsilon_{own})\varepsilon}{\varepsilon_{own}}]} = \frac{1}{1+\varepsilon\chi-\theta\chi+\left(1-\frac{\alpha}{\varepsilon_{own}}\right)\theta},$$

because $\theta = \varepsilon/\varepsilon_{own}$. Here, the direct effect from $-\eta\theta$ is canceled out by the part of the indirect effect from $\varepsilon q(\theta\eta)'$. The new term, which appears as the fourth term in the denominator, shows *how the industry's curvature affects the pass-through*: as the demand curvature becomes larger (i.e., as the industry's demand becomes more convex), then the pass-through becomes higher, although this effect is mitigated by the intensity of competition, θ.

B.2.2 Quantity competition

Next, in the case of quantity competition, the conduct index θ is given by $\theta = \eta_{own}/\eta$, which is, again, verified by comparing the firm's first-order condition with Equation (1). Again, the marginal change in deadweight loss and the incidence are obtained by substituting these expressions into those of Propositions 1 and 2.

Proposition 9. *Under symmetric oligopoly with quantity competition and with a possibly non-constant marginal cost, the unit tax pass-through and the ad valorem tax pass-through are characterized by*

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{1+\frac{\eta_{own}}{\eta} - \sigma + \left(\frac{1-\tau}{1-v} - \eta_{own}\right)\frac{\chi}{\eta}}$$

and

$$\rho_v = \frac{1}{1-v} \cdot \frac{(1-\eta_F)}{1+\frac{\eta_{own}}{\eta} - \sigma + \left(\frac{1-\tau}{1-v} - \eta_{own}\right)\frac{\chi}{\eta}},$$

respectively.

Proof. In the case of quantity setting, $\theta = \eta_{own}/\eta$, so $(\eta+\chi)\theta = (1+\chi/\eta)\eta_{own}$ and $(\theta\eta)'\varepsilon q = q(\eta_{own})'/\eta = (1+\eta-\sigma\eta/\eta_{own})\eta_{own}/\eta$, where in the last equality we utilize the expression for the elasticity of $\eta_{own}(q)$ and $\sigma_{own}+\sigma_{cross} = \sigma\eta/\eta_{own}$ from 5 above. Substituting these into the expression for ρ_t in Proposition 5 gives

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{1-(1+\frac{1}{\eta}\chi)\eta_{own} + \frac{1}{\eta}\left(1+\eta-\frac{\sigma\eta}{\eta_{own}}\right)\eta_{own} + \frac{1-\tau}{1-v}\frac{1}{\eta}\chi},$$

which is equivalent to the expression for ρ_t in the proposition. Since $\theta = \eta_{own}/\eta$, Proposition 3 implies $\rho_v = (\varepsilon-\theta)\rho_t/\varepsilon = (1/\eta - \eta_{own}/\eta)\rho_t\eta = (1-\eta_{own})\rho_t$, which can be used to verify the expression for ρ_v. $\square$

This proposition is similar to Proposition 8 above. Recall again that

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{\underbrace{[(1-\theta\eta)+(\theta\eta)'\varepsilon q]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v}\varepsilon - \theta\right]\chi}_{\text{cost savings}}}.$$

Then, $\theta = \eta_{own}/\eta$ implies $(1/\varepsilon_S - \eta)\theta = [(1/\varepsilon_S\eta) - 1]\eta_{own}$ and $(\theta\eta)'(q/\eta) = q(\eta_{own})'/\eta$

$= (1+\eta-\sigma_{own}-\sigma_{cros})\,(\eta_{own}/\eta)$. Thus, the equality above is rewritten as

$$\rho_t = \frac{1}{1-v}\cdot\frac{1}{\underbrace{\left[(1-\eta_{own})+\frac{1+\eta-\sigma\eta/\eta_{own}}{\eta}\eta_{own}\right]}_{\text{revenue increase}}+\underbrace{\left[\frac{1-\tau}{1-v}\cdot\frac{1}{\varepsilon_S\eta}-\frac{\eta_{own}}{\varepsilon_S\eta}\right]}_{\text{cost savings}}}$$

$$= \frac{1}{1-v}\cdot\frac{1}{\underbrace{\left[1+\frac{\eta_{own}-\sigma\eta}{\eta}\right]}_{\text{revenue increase}}+\underbrace{\left[\frac{1-\tau}{1-v}-\eta_F\right]\frac{1}{\varepsilon_S\eta}}_{\text{cost savings}}}.$$

To further facilitate the understanding of the connection of this result for to Proposition 5, consider the case of zero initial taxes ($t = v = \tau = 0$) again. Then, Proposition 9 shows that

$$\rho_t = \frac{1}{1+\varepsilon\chi-\theta\chi+[-\eta\cdot\frac{\eta_{own}}{\eta}+\left(1+\frac{1}{\eta}-\frac{\sigma}{\eta_{own}}\right)\eta_{own}]} = \frac{1}{1+\varepsilon\chi-\theta\chi+\left(1-\frac{\sigma}{\theta}\right)\theta}$$

because $\theta = \eta_{own}/\eta$. Here, the term $(1-\sigma/\theta)\,\theta$ demonstrates the *effects of the industry's inverse demand curvature, σ, on the pass-through*: as the inverse demand curvature becomes larger (i.e., as the industry's inverse demand becomes more convex), the pass-through becomes higher. Interestingly, in contrast to the case of price competition, this effect is not mitigated by the intensity of competition, θ.

Appendix C Multi-dimensional pass-through framework under firm heterogeneity

As shown below, it turns out that it is useful to consider a general version of multi-dimensional interventions because specific and ad valorem taxation can be deemed as a special case of a two-dimensional intervention. A key concept is *multi-dimensional pass-through*, which is defined as the impact of infinitesimal changes in interventions $\mathbf{T} \equiv (T_1,...,T_d)$—a d-dimensional vector of tax instruments—on the equilibrium price p_i for firm $i = 1,...,n$. Multi-dimensional pass-through corresponds to a matrix in the case of heterogeneous firms, which can be simplified as a vector under symmetric oligopoly. We argue that multi-dimensional pass-through is an important determinant of the welfare effects of various kinds of government intervention and external changes, not limited to the two-dimensional taxation.

C.1 Price sensitivity and quantity sensitivity of taxes

Consider a tax structure under which firm i's tax payment is expressed as $\phi_i(p_i, q_i, \mathbf{T})$, so that the firm's profit is written as $\pi_i = p_i q_i - c_i(q_i) - \phi_i(p_i, q_i, \mathbf{T})$.[39] Note that the production cost, and hence, the marginal cost $mc_i(q_i)$ of firm i is also allowed to depend on the identity of the firm, and we denote its elasticity by $\chi_i(q_i) \equiv mc_i'(q_i)\, q_i / mc_i(q_i)$. In the special case of a unit tax t and an ad valorem tax v, $\phi_i(p_i, q_i, \mathbf{T}) = t q_i + v p_i q_i$, where $\mathbf{T} = (t, v)$. Below, we argue how to generalize our previous framework with two policy instruments by defining analogs of t and v even for general interventions that may include multiple instruments, not just two.

We aim to express a decomposition of $\phi_i(p_i, q_i, \mathbf{T})$ analogous to $\phi_i(p_i, q_i, \mathbf{T}) = t q_i + v p_i q_i$. Specifically, we argue that it is possible to write $\phi_i(p_i, q_i, \mathbf{T}) = \bar{t} q_i + \bar{\nu} p_i q_i$, where $\bar{t}$ and $\bar{\nu}$ are the averages of appropriately defined functions t and ν over the ranges $(0, q_i)$ and $(0, p_i q_i)$. In the special case of specific and ad valorem taxes, these functions should reduce to constants t and v. We verify this property by decomposing $\phi_i(p_i, q_i, \mathbf{T})$ into infinitesimal contributions, each of which resembles specific and ad valorem taxes, respectively. If we set the tax burden at zero quantities and prices: $\phi_i(0, 0, \mathbf{T}) = 0$, we can write the desired relationship $\phi_i(p_i, q_i, \mathbf{T}) = \bar{t} q_i + \bar{\nu} p_i q_i$ as $\phi_i(p_i, q_i, \mathbf{T}) = \int_0^{q_i} t(\tilde{p}, \tilde{q}, \mathbf{T})\, d\tilde{q} + \int_0^{p_i q_i} \nu(\tilde{p}, \tilde{q}, \mathbf{T})\, d(\tilde{p}\tilde{q})$, or alternatively as

$$\phi_i(p_i, q_i, \mathbf{T}) = \int_0^1 \left[\left(\frac{t(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T})}{\tilde{p}_i} + \nu(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}) \right) \tilde{p}_i \frac{d\tilde{q}_i}{ds} + \nu(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T})\, \tilde{q}_i \frac{d\tilde{p}_i}{ds} \right] ds,$$

where the integration is over an auxiliary parameter s that parameterizes a path $(\tilde{p}_i(s), \tilde{q}_i(s))$ in the price-quantity plane such that $(\tilde{p}_i(0), \tilde{q}_i(0)) = (0, 0)$ and $(\tilde{p}_i(1), \tilde{q}_i(1)) = (p_i, q_i)$.

At the same time, $\phi_i(p_i, q_i, \mathbf{T})$ can be expressed by an integral of its total differential:

$$\phi(p_i, q_i, \mathbf{T}) = \int_0^1 \left[\phi_{\tilde{q}_i}(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}) \frac{d\tilde{q}_i}{ds} + \phi_{\tilde{p}_i}(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}) \frac{d\tilde{p}_i}{ds} \right] ds,$$

[39] To be precise, $\phi(p, q, \mathbf{T})$ represents a simplified notation for a function $\phi(p, q, T_1, \ldots, T_d)$ with $d + 2$ arguments.

where a subscript notation is used for partial derivatives. We observe that if we identify

$$\begin{cases} \left(\dfrac{t\left(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}\right)}{\tilde{p}_i} + \nu\left(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}\right) \right) \tilde{p}_i = \phi_{\tilde{q}_i}(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}) \\ \nu\left(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}\right) \tilde{q}_i = \phi_{\tilde{p}_i}(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}), \end{cases}$$

then the desired relationship $\phi_i(p_i, q_i, \mathbf{T}) = \bar{t} q_i + \bar{\nu} p_i q_i$ is satisfied.

Now, we define the (*first-order*) *price sensitivity* of the (per-firm) tax revenue by

$$\nu_i(p_i, q_i, \mathbf{T}) \equiv \frac{1}{q_i} \frac{\partial}{\partial p_i} \phi_i(p_i, q_i, \mathbf{T}),$$

and the (*first-order*) *quantity sensitivity* by

$$\tau_i(p_i, q_i, \mathbf{T}) \equiv \frac{1}{p_i} \frac{\partial}{\partial q_i} \phi_i(p_i, q_i, \mathbf{T})$$

so that $t_i(p_i, q_i, \mathbf{T}) = \tau_i(p_i, q_i, \mathbf{T}) p_i + \nu_i(p_i, q_i, \mathbf{T})$. Note that both the first-order and second-order sensitivities are dimensionless.

C.2 Pricing strength index

We now introduce the *pricing strength index* $\psi_i(\mathbf{q})$ of firm i as a function of $\mathbf{q}$—but independent of the cost side—such that the first-order condition for firm i is:

$$\{1 - \tau_i\left(p_i(\mathbf{q}), q_i, \mathbf{T}\right) - \psi_i(\mathbf{q})\left[1 - \nu_i\left(p_i(\mathbf{q}), q_i, \mathbf{T}\right)\right]\} p_i(\mathbf{q}) = mc_i(q_i). \tag{13}$$

In the special case of symmetric firms, this pricing strength index is expressed by $\psi_i = \eta \cdot \theta$ for all i. Because of this simplicity, analyzing oligopoly in terms of the pricing strength index does not differ from analyzing it in terms of the conduct index. However, these two approaches would differ for heterogeneous firms. An innovation of this paper is to provide an oligopoly analysis in terms of the pricing strength index.

Note here that in the case of specific and ad valorem taxation, it is verified that

$$\tau_i(p_i,q_i,\mathbf{T}) \equiv \frac{1}{p_i}\frac{\partial \phi_i}{\partial q_i}(p_i,q_i,\mathbf{T}) = \frac{t}{p_i} + v$$

and

$$\nu_i(p_i,q_i,\mathbf{T}) \equiv \frac{1}{q_i}\frac{\partial \phi_i}{\partial p_i}(p_i,q_i,\mathbf{T}) = v$$

so that Equation (13) becomes

$$\left[\left(1-\frac{t}{p_i(\mathbf{q})}-v\right)-\psi_i(\mathbf{q})(1-v)\right]p_i(\mathbf{q}) = mc_i(q_i),$$

as appeared in the main text.

C.3 Pass-through

We express the pass-through rate matrix in terms of these pricing strength indices. Specifically, the pass-through rate is an $n \times d$ matrix $\tilde{\rho}$ whose (i, T_ℓ) element is $\tilde{\rho}_{iT_\ell} = \partial p_i/\partial T_\ell$. First, we define the following functions: $\kappa_i(p_i,q_i,\mathbf{T}) \equiv \frac{\partial^2\phi_i(p_i,q_i,\mathbf{T})}{\partial p_i \partial q_i}$, $\nu_{(2),i}(p_i,q_i,\mathbf{T}) \equiv \frac{p_i}{q_i}\frac{\partial^2\phi_i(p_i,q_i,\mathbf{T})}{\partial p_i^2}$, $\tau_{(2),i}(p_i,q_i,\mathbf{T}) \equiv \frac{q_i}{p_i}\frac{\partial^2\phi_i(p_i,q_i,\mathbf{T})}{\partial q_i^2}$, $\varepsilon_{ij} \equiv -\frac{p_i}{q_i}\frac{\partial q_i(\mathbf{p})}{\partial p_j}$, and $\Psi_{ij} \equiv \frac{p_i}{\psi_i}\frac{\partial \psi_i(\mathbf{q}(\mathbf{p}))}{\partial p_j}$. Then, the following proposition is obtained.

Proposition 10. *The pass-through rate equals*

$$\underbrace{\tilde{\rho}_{T_\ell}}_{n\times 1} = \underbrace{\mathbf{b}^{-1}}_{n\times n}\underbrace{\iota_{T_\ell}}_{n\times 1}, \tag{14}$$

where $\mathbf{b}$ *is an* $n \times n$ *matrix, independent of the choice of* T_ℓ*, with the* (i, j) *element being:*

$$\begin{aligned} b_{ij} &= [1-\kappa_i-(1-\nu_i-\nu_{(2)i})\psi_i]\,\delta_{ij}-(1-\nu_i)\psi_i\Psi_{ij} \\ &+ \{\tau_{(2)i}+(\nu_i-\kappa_i)\psi_i+[1-\tau_i-(1-\nu_i)\,\psi_i]\chi_i\}\,\varepsilon_{ij}, \end{aligned}$$

where δ_{ij} *is the Kronecker delta, and for each tax* T_ℓ*,* ι_{T_ℓ} *is an n-dimensional vector with i-th element* *being:*

$$\iota_{iT_\ell} \equiv p_i \cdot \left(\frac{\partial \tau_i (p_i, q_i, \mathbf{T})}{\partial T_\ell} - \psi_i \frac{\partial \nu_i (p_i, q_i, \mathbf{T})}{\partial T_\ell} \right).$$

Proof. Equation (13) indicates that

$$\left[p_i \cdot \left(\frac{\partial \tau_i}{\partial p_i} - \psi_i \cdot \frac{\partial \nu_i}{\partial p_i} \right) + (1-\nu_i)\psi_i - (1-\tau_i) \right] dp_i + \left[p_i \cdot \left(\frac{\partial \tau_i}{\partial q_i} - \psi_i \cdot \frac{\partial \nu_i}{\partial q_i} \right) + mc_i' \right] dq_i$$
$$+ p_i \cdot \left(\frac{\partial \tau_i}{\partial T_\ell} - \psi_i \cdot \frac{\partial \nu_i}{\partial T_\ell} \right) dT_\ell + p_i (1-\nu_i) d\psi_i = 0,$$

implying that

$$\iota_{iT_\ell} dT_\ell = \left[1 - \kappa_i - \left(1 - \nu_i - \nu_{(2),i}\right) \psi_i \right] dp_i - (1-\nu_i)\psi_i \left(\sum_{j=1}^{n} \Psi_{ij} dp_j \right)$$
$$+ \left\{ \tau_{(2),i} + (\nu_i - \kappa_i)\psi_i + [1 - \tau_i - (1-\nu_i)\psi_i]\chi_i \right\} \left(\sum_{j=1}^{n} \varepsilon_{i,j} dp_j \right),$$

where $dq_i = -\frac{q_i}{p_i} \sum_{j=1}^{n} \varepsilon_{i,j} dp_j$, $d\psi_i = \frac{\psi_i}{p_i} \sum_{j=1}^{n} \Psi_{ij} dp_j$ and $mc_i' = \frac{\chi_i mc_i}{q_i} = \frac{p_i}{q_i}[1 - \tau_i - (1-\nu_i)\psi_i]\chi_i$ are used.[40] Hence

$$\underbrace{\iota_{T_\ell}}_{n\times 1} = \underbrace{\begin{pmatrix} 1-\kappa_1 - \left(1-\nu_1-\nu_{(2),1}\right)\psi_1 \\ \vdots \\ 1-\kappa_j - \left(1-\nu_j-\nu_{(2),j}\right)\psi_j \\ \vdots \\ 1-\kappa_n - \left(1-\nu_n-\nu_{(2),n}\right)\psi_n \end{pmatrix}}_{n\times 1} \circ \underbrace{\tilde{\rho}_{T_\ell}}_{n\times 1} - \underbrace{\begin{pmatrix} \ddots & & \\ & (1-\nu_i)\psi_i\Psi_{ij} & \\ & & \ddots \end{pmatrix}}_{n\times n} \tilde{\rho}_{T_\ell}$$
$$+ \underbrace{\begin{pmatrix} \ddots & & \\ & \left\{ \tau_{(2),i} + (\nu_i - \kappa_i)\psi_i + [1-\tau_i - (1-\nu_i)\psi_i]\chi_i \right\} \varepsilon_{i,j} & \\ & & \ddots \end{pmatrix}}_{n\times n} \tilde{\rho}_{T_\ell},$$

and thus, assuming that **b** is invertible, Equation (14) holds. □

[40]Note that $\frac{\partial \nu_i}{\partial p_i} = \frac{\nu_{(2),i}}{p_i}$, $\frac{\partial \nu_i}{\partial q_i} = \frac{\kappa_i - \nu_i}{q_i}$, $\frac{\partial \tau_i}{\partial p_i} = \frac{\kappa_i - \tau_i}{p_i}$ and $\frac{\partial \tau_i}{\partial q_i} = \frac{\tau_{(2),i}}{q_i}$ are also used.

Under the two-dimensional taxation, it is verified that $\nu_{(2)i}(p,q,\mathbf{T})=0$, $\tau_{(2)i}(p,q,\mathbf{T})=0$, and $\kappa_i(p,q,\mathbf{T})=v$. In addition, $\iota_{it}=1$ and $\iota_{iv}=p_i\cdot(1-\psi_i)$ because $\frac{\partial\tau_i}{\partial t}=\frac{1}{p_i}$, $\frac{\partial\nu_i}{\partial t}=0$, $\frac{\partial\tau_i}{\partial v}=1$, and $\frac{\partial\nu_i}{\partial v}=1$.

C.4 Welfare changes

So far, we have introduced $\phi_i(p_i,q_i,\mathbf{T})$ as an additional cost in the firm's profit function: $\pi_i=p_iq_i-c_i(q_i)-\phi_i(p_i,q_i,\mathbf{T})$. Here $\mathbf{T}$ is a vector of interventions (in governmental and other external circumstances), which may or may not include traditional taxes. To evaluate welfare changes, we also need to know what part of this cost is collected by the government in the form of taxes. We now introduce the notation $\widehat{\phi}_i(p_i,q_i,\mathbf{T})$ for the tax bill of the firm. The difference $\phi_i(p_i,q_i,\mathbf{T})-\widehat{\phi}_i(p_i,q_i,\mathbf{T})$ corresponds to additional non-tax costs the firm faces. In the case of pure taxation, $\widehat{\phi}_i(p_i,q_i,\mathbf{T})=\phi_i(p_i,q_i,\mathbf{T})$.[41] Then, for each firm i, we define $\widehat{\nu}_i(p_i,q_i,\mathbf{T})\equiv\frac{1}{q_i}\frac{\partial}{\partial p_i}\widehat{\phi}_i(p_i,q_i,\mathbf{T})$, and $\widehat{\tau}_i(p_i,q_i,\mathbf{T})\equiv\frac{1}{p_i}\frac{\partial}{\partial q_i}\widehat{\phi}_i(p_i,q_i,\mathbf{T})$. We also write $\mathbf{f}_i\equiv\frac{1}{q_i}\nabla\phi_i(p_i,q_i,\mathbf{T})$, where $\nabla\phi_i$'s components are $\phi_{iT_\ell}(p,q,\mathbf{T})\equiv\frac{\partial\phi_i(p_i,q_i,\mathbf{T})}{\partial T_\ell}$, and $\widehat{\mathbf{f}}_i\equiv\frac{1}{q_i}\nabla\widehat{\phi}_i(p_i,q_i,\mathbf{T})$ is also defined analogously.[42]

Let ε_i be an n-dimensional row vector with its j-th component equal to ε_{ij} for each i: $\varepsilon_i=(\varepsilon_{i1},...,\varepsilon_{ij},...\varepsilon_{in})$. For convenience, we also define $\mathbf{e}_i$ to be an indicator vector with the i-th component equal to 1 and other components zero: $\mathbf{e}_i=(0,...,\underbrace{1}_{\text{i-th}},...,0)$. Then, the following proposition is obtained.

Proposition 11. *The intervention gradients of consumer surplus, producer surplus, tax revenue, and social welfare with respect to the taxes are*

$$\frac{1}{q_i}\nabla CS_i=-\mathbf{e}_i\tilde{\rho},$$

[41] If all of the additional cost to the firm comes from the production side, we have $\widehat{\phi}_i(p_i,q_i,\mathbf{T})=0$.

[42] For the two-dimensional taxation,

$$\mathbf{f}_i\equiv\frac{1}{q_i}\nabla\phi_i(p_i,q_i,\mathbf{T})=\frac{1}{q_i}\begin{pmatrix}\frac{\partial\phi_i(p_i,q_i,\mathbf{T})}{\partial t}\\ \frac{\partial\phi_i(p_i,q_i,\mathbf{T})}{\partial v}\end{pmatrix}=\begin{pmatrix}1\\ p_i\end{pmatrix}$$

because

$$\begin{cases}\phi_{i,t}(p_i,q_i,\mathbf{T})\equiv\dfrac{\partial\phi_i(p_i,q_i,\mathbf{T})}{\partial t}=q_i\\ \phi_{i,v}(p_i,q_i,\mathbf{T})\equiv\dfrac{\partial\phi_i(p_i,q_i,\mathbf{T})}{\partial v}=p_iq_i.\end{cases}$$

$$\frac{1}{q_i}\nabla PS_i = (1-\nu_i)\left(\mathbf{e}_i - \psi_i\,\varepsilon_i\right)\tilde{\rho} - \mathbf{f}_i,$$

$$\frac{1}{q_i}\nabla R_i = \left(\widehat{\nu}_i\,\mathbf{e}_i - \hat{\tau}_i\,\varepsilon_i\right)\tilde{\rho} + \widehat{\mathbf{f}}_i,$$

$$\frac{1}{q_i}\nabla W_i = -\left[\widehat{\tau}_i + \psi_i\,(1-\nu_i)\right]\varepsilon_i\tilde{\rho} + \left(\widehat{\nu}_i - \nu_i\right)\mathbf{e}_i\tilde{\rho} + \widehat{\mathbf{f}}_i - \mathbf{f}_i,$$

respectively.

Proof. The result for $\frac{1}{q_i}\nabla CS_i$ is straightforward. It suffices to provide expressions for $\frac{1}{q_i}\nabla PS_i$ and $\frac{1}{q_i}\nabla R_i$ since $\frac{1}{q_i}\nabla W_i$ equals the sum of the other three expressions. Note first that in response to a change $T_\ell \to T_\ell + dT_\ell$, we have $dPS_i = d(p_i q_i - c_i(q_i) - \phi_i(p_i, q_i, \mathbf{T}))$ and $d\phi_i(p_i, q_i, \mathbf{T}) = p_i\tau_i(p_i, q_i, \mathbf{T})dq_i + q_i\nu_i(p_i, q_i, \mathbf{T})dp_i + \frac{\partial \phi_i}{\partial T_\ell}dT_\ell$. Then by using Equation (13), one can rewrite:

$$\begin{aligned}
dPS_i &= \left[-\left(\nu_i\psi_i - \tau_i - \psi_i + 1\right)p_i - p_i\tau_i + p_i\right]dq_i + (q_i - \nu_i q_i)dp_i - \frac{\partial \phi_i}{\partial T_\ell}dT_\ell \\
&= (1-\nu_i)\left[p_i\psi_i \cdot \left(\sum_{j=1}^{n}\frac{\partial q_i}{\partial p_j}dp_j\right) + q_i dp_i\right] - \frac{\partial \phi_i}{\partial T_\ell}dT_\ell \\
&= (1-\nu_i)\left[p_i\psi_i \cdot \left(\sum_{j=1}^{n}\frac{\partial q_i}{\partial p_j}\tilde{\rho}_{jT_\ell}\right) + q_i\tilde{\rho}_{iT_\ell}\right]dT_\ell - \frac{\partial \phi_i}{\partial T_\ell}dT_\ell \\
&= (1-\nu_i)q_i\left[\psi_i \cdot \left(\sum_{j=1}^{n}\frac{p_i}{q_j}\frac{\partial q_i}{\partial p_j}\tilde{\rho}_{jT_\ell}\right) + \tilde{\rho}_{iT_\ell}\right]dT_\ell - \frac{\partial \phi_i}{\partial T_\ell}dT_\ell,
\end{aligned}$$

which indicates that

$$\begin{aligned}
\frac{1}{q_i}\nabla PS_i &= (1-\nu_i)\left[\tilde{\rho}_{iT_\ell} - \psi_i \cdot \left(\sum_{j=1}^{n}\varepsilon_{ij}\tilde{\rho}_{jT_\ell}\right)\right] - \mathbf{f}_i \\
&= (1-\nu_i)\left(\mathbf{e}_i - \psi_i\,\varepsilon_i\right)\tilde{\rho} - \mathbf{f}_i.
\end{aligned}$$

Next, note first that $dR_i = [\partial_p\widehat{\phi}_i]^{\mathrm{T}}\mathbf{dp} + [\partial_q\widehat{\phi}_i]^{\mathrm{T}}\mathbf{dq} + \frac{\partial\widehat{\phi}_i}{\partial T_\ell}dT_\ell$, where $\mathbf{dp} = \left(\frac{\partial p_1}{\partial T_\ell}\dots\frac{\partial p_n}{\partial T_\ell}\right)^{\mathrm{T}}$ and $\partial_p\widehat{\phi}_i = \left(\frac{\partial\widehat{\phi}_i}{\partial T_\ell}\dots\frac{\partial\widehat{\phi}_i}{\partial T_\ell}\right)^{\mathrm{T}}$, and analogously for $\mathbf{q}$. Here, $dq_i = \sum_{j=1}^{n}\frac{\partial q_i}{\partial p_j}dp_j$. By using $\widetilde{\nu}_i = \frac{1}{q_i}\widehat{\phi}_{i,p_i}$, $\widetilde{\tau}_i = \frac{1}{p_i}\widehat{\phi}_{i,q_i}$, one can rewrite: $dR_i = \widehat{\nu}_i q_i dp_i + \widehat{\tau}_i p_i dq_i + \frac{\partial\widehat{\phi}_i}{\partial T_\ell}dT_\ell$. Then, by using $\tilde{\rho}_{iT_\ell} = \partial p_i/\partial T_\ell$, one can further proceed:

$$\frac{dR_i}{dT_l} = \widehat{\nu}_i q_i\tilde{\rho}_{iT_\ell} + \widehat{\tau}_i p_i \cdot \left(\sum_{j=1}^{n}\frac{\partial q_i}{\partial p_j}\tilde{\rho}_{iT_\ell}\right) + \frac{\partial\widehat{\phi}_i}{\partial T_\ell}$$

$$= \quad \widehat{\nu}_i q_i \tilde{\rho}_{iT_\ell} + \widehat{\tau}_i p_i \cdot \left(-\sum_{j=1}^{n} \frac{q_i}{p_i} \varepsilon_{ij} \tilde{\rho}_{iT_\ell} \right) + \frac{\partial \widehat{\phi}_i}{\partial T_\ell}$$
$$= \quad q_i \widehat{\nu}_i \tilde{\rho}_{iT_\ell} - q_i \widehat{\tau}_i \cdot \left(\sum_{j=1}^{n} \varepsilon_{ij} \tilde{\rho}_{iT_\ell} \right) + \frac{\partial \widehat{\phi}_i}{\partial T_\ell},$$

which indicates that $= \frac{1}{q_i} \nabla R_i = \widehat{\nu}_i \tilde{\rho}_{iT_\ell} - \hat{\tau}_i \cdot \left(\sum_{j=1}^{n} \varepsilon_{ij} \tilde{\rho}_{iT_\ell} \right) + \widehat{\mathbf{f}}_i = (\widehat{\nu}_i \mathbf{e}_i - \hat{\tau}_i \varepsilon_i) \tilde{\rho} + \hat{\mathbf{f}}_i$, completing the proof. □

The corresponding gradients of total welfare components are then obtained by adding up contributions from individual firms. For example, $\nabla CS = \sum_{i=1}^{n} \nabla CS_i$. Denoting the total quantity as $Q \equiv \sum_{i=1}^{n} q_i$, this means that $\frac{1}{Q} \nabla CS$ is a weighted average of $-\mathbf{e}_i.\tilde{\rho}$, with the weights proportional to q_i.

Now, we define the *pass-through quasi-elasticity matrix* ρ as an $n \times d$ matrix with elements: $\rho_{iT_\ell} = \frac{1}{f_{iT_\ell}(p_i, q_i, \mathbf{T})} \tilde{\rho}_{iT_\ell}$, and with rows denoted ρ_{T_ℓ}.[43] We also define, for each firm i, $g_{iT_\ell} \equiv \hat{f}_{iT_\ell} / f_{iT_\ell} = \left(\widehat{\mathbf{f}}_i \right)_{T_\ell} / (\mathbf{f}_i)_{T_\ell}$. Then, for the firm-specific welfare change ratios, we obtain the following proposition by using the results of Proposition 11.

Proposition 12. *Let $\varepsilon^{\rho}_{iT_\ell} \equiv \varepsilon_i \tilde{\rho}_{T_\ell} / \tilde{\rho}_{iT_\ell} = \varepsilon_i \rho_{T_\ell} / \rho_{iT_\ell}$. Then, the marginal value of public funds associated with intervention T_ℓ, $MVPF_{iT_\ell} = (\nabla W_i)_{T_\ell} / (\nabla R_i)_{T_\ell}$, is characterized by:*

$$MVPF_{iT_\ell} = \frac{\frac{(1-\nu_i)\psi_i \varepsilon^{\rho}_{iT_\ell}}{\varepsilon^{\rho}_{iT_\ell}} + \hat{\tau}_i + \frac{1}{\varepsilon^{\rho}_{iT_\ell}} \left(\frac{1 - g_{iT_\ell}}{\rho_{iT_\ell}} + \nu_i - \widehat{\nu}_i \right)}{\frac{\frac{g_{iT_\ell}}{\rho_{iT_\ell}} + \widehat{\nu}_i}{\varepsilon^{\rho}_{iT_\ell}} - \widehat{\tau}_i},$$

and the incidence of this intervention, $I_{iT_\ell} = (\nabla CS_i)_{T_\ell} / (\nabla PS_i)_{T_\ell}$, is characterized by:

$$I_{iT_\ell} = \frac{1}{\frac{1}{\rho_{iT_\ell}} - (1-\nu_i)(1 - \psi_i \varepsilon^{\rho}_{iT_\ell})}.$$

[43] For the two-dimensional taxation, it is easily verified that $\rho_{it} = \frac{1}{f_{it}} \tilde{\rho}_{it} = \tilde{\rho}_{it}$ and $\rho_{iv} = \frac{1}{f_{iv}} \tilde{\rho}_{iv} = \frac{\tilde{\rho}_{iv}}{p_i}$.

Appendix D Conduct index and welfare changes

For heterogeneous firms, we can also consider the conduct index of firm i, instead of the pricing strength index, so that

$$\theta_i = -\frac{\sum_{j=1}^{n}\left\{p_j\left[1-\tau_j\left(p_j,q_j,\mathbf{T}\right)\right]-mc_j\left(q_j\right)\right\}\frac{dq_j}{d\sigma_i}}{\sum_{j=1}^{n}\left[1-\nu_j\left(p_j,q_j,\mathbf{T}\right)\right]q_j\frac{dp_j}{d\sigma_i}}$$

holds. In the special case of only unit taxation being present, this definition reduces to Weyl and Fabinger's (2013, p. 552) Equation (4). In the special case of symmetric firms, the definition reduces to $[1-\tau-(1-\nu)\,\eta\,\theta]\,p = mc$ with $\theta_i = \theta$.

The conduct index θ_i is closely connected to the marker power index ψ_i, but not as closely as it would be in the case of symmetric oligopoly. Using the definitions of the indices, it is shown that

$$\theta_i = -\frac{\sum_{j=1}^{n}\left(1-\nu_j\right)\psi_j\,p_j\frac{dq_j}{d\sigma_i}}{\sum_{j=1}^{n}\left(1-\nu_j\right)q_j\frac{dp_j}{d\sigma_i}}.$$

For symmetric oligopoly, this equation reduces simply to $\theta = \varepsilon\psi$.

The conduct index is used to express welfare component changes in response to infinitesimal changes in taxes. The relationships are a bit more complicated than when the pricing strength index is alternatively used. To see this, we define the price response to an infinitesimal change in the strategic variable σ_k of firm j by $\zeta_{ij} \equiv \frac{dp_i}{d\sigma_j}$. Since the vectors ζ_{i1}, ζ_{i2}, ... , ζ_{in} form a basis in the n-dimensional vector space to which $\tilde{\rho}_{iT_\ell}$ for a given ℓ belongs, we can write $\tilde{\rho}_{iT_\ell}$ as a linear combination of them for some coefficients λ_{iT_ℓ}: $\tilde{\rho}_{iT_\ell} = \sum_{j=1}^{n}\lambda_{jT_\ell}\zeta_{ij}$. For changes in consumer and producer surplus, we obtain:

$$\frac{dCS}{dT_\ell} = -\sum_{i=1}^{n} q_i\tilde{\rho}_{iT_\ell} = -\sum_{j=1}^{n}\left(\sum_{i=1}^{n} q_i\zeta_{ij}\right)\lambda_{jT_\ell},$$

$$\frac{dPS}{dT_\ell} = -\sum_{i=1}^{n} f_{iT_\ell}\left(p_i,q_i,\mathbf{T}\right) - \sum_{j=1}^{n}\hat{\zeta}_j\left(1-\theta_j\right)\lambda_{jT_\ell},$$

where we use the notation $\hat{\zeta}_j \equiv \sum_{i=1}^{n}\left[1-\nu_i\left(p_i,q_i,\mathbf{T}\right)\right]q_i\,\zeta_{ij}$.

These surplus change expressions represent a generalization of the surplus expressions in Weyl and Fabinger's (2013) Section 5. Note, however, that the results in the previous subsections are significantly more straightforward and applicable than the ones in this subsection.

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“Pass-Through and the Welfare Effects of Taxation under Imperfect Competition: A General Analysis”

Online Appendix A Relationship to Weyl and Fabinger (2013)

It can be verified that our formula for ρ_t above is a generalization of Weyl and Fabinger’s (2013, p. 548) Equation (2):

$$\rho = \frac{1}{1+\frac{\varepsilon_D-\theta}{\varepsilon_S}+\frac{\theta}{\varepsilon_\theta}+\frac{\theta}{\varepsilon_{ms}}},$$

where $\varepsilon_\theta \equiv \theta/[q\cdot(\theta)']$, $\varepsilon_{ms} \equiv ms/[ms'q]$ ($ms \equiv -p'q$ is defined in the proof of Proposition 5 just above), and ε_D and ε_S here are our ε and $1/\chi$, respectively. First, the denominator in our formula is rewritten as:

$$1-(\eta+\chi)\,\theta+\varepsilon q\,(\theta\eta)'+\frac{1-\tau}{1-v}\varepsilon\chi = 1+\frac{\frac{1-\tau}{1-v}\varepsilon_D-\theta}{\varepsilon_S}+\frac{\theta}{\varepsilon_\theta}+\theta\cdot\left(-\frac{1}{\varepsilon_D}+\eta'\varepsilon_D q\right)$$

because

$$(\theta\eta)'\varepsilon q = (\theta'\eta+\theta\eta')\varepsilon q = \left[\frac{\theta}{q\varepsilon_\theta}\eta+\theta\eta'\right]\varepsilon q = \frac{\theta}{\varepsilon_\theta}+\theta\eta'\varepsilon q.$$

Next, since $\eta = -qp'/p$, it is verified that $\eta' = -\{p'p+qpp''-q[p']^2\}/p^2$, implying that

$$\eta'\varepsilon_D q = \frac{p'p+qpp''-q[p']^2}{p^2}\cdot\frac{p}{p'q}\cdot q = \frac{1}{\varepsilon_D}+\left(1+\frac{p''}{p'}q\right),$$

where $1+p''q/p$ is replaced by $1/\varepsilon_{ms}$ because $ms \equiv -p'q$ and thus $ms' = -(p''q+p')$. Then, it is readily verified that

$$1-(\eta+\chi)\,\theta+\varepsilon q\,(\theta\eta)'+\frac{1-\tau}{1-v}\varepsilon\chi = 1+\frac{\frac{1-\tau}{1-v}\varepsilon_D-\theta}{\varepsilon_S}+\frac{\theta}{\varepsilon_\theta}+\frac{\theta}{\varepsilon_{ms}}.$$

In summary, Weyl and Fabinger’s (2013, p. 548) original Equation (2) is generalized to

$$\rho = \frac{1}{1-v}\cdot\frac{1}{1+\frac{\frac{1-\tau}{1-v}\varepsilon_D-\theta}{\varepsilon_S}+\frac{\theta}{\varepsilon_\theta}+\frac{\theta}{\varepsilon_{ms}}}$$

with non-zero initial ad valorem tax, which is equivalent to our formula for ρ_t:

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{1+\frac{1-\tau}{1-v}\varepsilon\chi - (\eta+\chi)\theta + \varepsilon q(\theta\eta)'},$$

and from Proposition 3, it is readily observed that ρ_v can also be written in terms of Weyl and Fabinger's (2013) notation:

$$\rho_v = \frac{\varepsilon_D - \theta}{(1-v)\varepsilon_D} \cdot \frac{1}{1+\frac{\frac{1-\tau}{1-v}\varepsilon_D-\theta}{\varepsilon_S} + \frac{\theta}{\varepsilon_\theta} + \frac{\theta}{\varepsilon_{ms}}}.$$

Online Appendix B Oligopoly with multi-product firms

Here, we argue that the results obtained in Section 2 can be extended to the case of multi-product firms just by a reinterpretation of the same formulas (without modifying them).[44] Assume that there are n_p product categories, and the demand for firm i's k-th product is given by $q_{ik} = q_{ik}(\mathbf{p}_1, \mathbf{p}_2, .., \mathbf{p}_n)$, where $\mathbf{p}_i = (p_{i1}, ..., p_{ik}, ..., p_{iK})$ for each $i = 1, 2, ..., n$.[45] The firms are symmetric, and for each firm, the product it produces are also symmetric. The firm's profit per product is:

$$\pi_i = \frac{1}{n_p} \sum_{k=1}^{n_p} \left((1-v)\,p_{ik}q_{ik} - tq_{ik} - c(q_{ik})\right).$$

We work with an equilibrium in which any firm i sets a uniform price p_i for all of its products: $p_{ik} = p_i$, and consequently sells an amount q_i of each of them: $q_{ik} = q_i$.[46] In this case, the profit per product equals $\pi_i = (1-v)\,p_iq_i - tq_i - c(q_i)$, which is formally the same as for single-product firms. For this reason, we can identify the prices p_i and quantities q_i of Section 2 with the prices p_i and quantities q_i introduced here in this paragraph. The discussion in Section 2 was general and applies to this case of symmetric oligopoly with multi-product firms as well. We can use the same definitions for the variables of interest, including the industry demand elasticity ε and the conduct index θ.

It may be useful to translate some of the most important variables of that discussion into product-level

[44] Lapan and Hennessy (2011) study unit and ad valorem taxes in multi-product Cournot oligopoly. Alexandrov and Bedre-Defolie (2017) also study cost pass-through of multi-product firms in relation to the Le Chatelier–Samuelson principle.

[45] See, e.g., Armstrong and Vickers (2018) and Nocke and Schutz (2018) for recent studies of multi-product oligopoly.

[46] For brevity, we do not explicitly discuss the standard conditions for the existence and uniqueness of non-cooperative Nash equilibria of the different underlying oligopoly games.

variables. For derivatives of the direct demand system, we introduce the notation:

$$\xi_1 \equiv \frac{\partial q_{ik}}{\partial p_{ik}}, \qquad \xi_{0,1} \equiv \frac{\partial q_{ik}}{\partial p_{ik'}},$$
$$\xi_2 \equiv \frac{\partial q_{ik}}{\partial p_{ik}^2}, \quad \xi_{1,1} \equiv \frac{\partial q_{ik}}{\partial p_{ik}\partial p_{ik'}}, \quad \xi_{0,2} \equiv \frac{\partial q_{ik}}{\partial p_{ik'}^2}, \quad \xi_{0,1,1} \equiv \frac{\partial q_{ik}}{\partial p_{ik'}\partial p_{ik''}},$$
$$\tilde{\xi}_2 \equiv \frac{\partial q_{ik}}{\partial p_{ik}\partial p_{i'k}} \quad \tilde{\xi}_{1,1} \equiv \frac{\partial q_{ik}}{\partial p_{ik}\partial p_{i'k'}}, \quad \tilde{\xi}_{0,2} \equiv \frac{\partial q_{ik}}{\partial p_{ik'}\partial p_{i'k'}}, \quad \tilde{\xi}_{0,1,1} \equiv \frac{\partial q_{ik}}{\partial p_{ik'}\partial p_{i'k''}},$$

where the derivatives are evaluated at the fully symmetric point, where any p_{ik} equals the common value p.[47] For specific choices of the demand system, these derivatives can be closely related. For example, if the substitution pattern between two goods produced by two different firms does not depend on the identity of the goods, then $\tilde{\xi}_2 = \tilde{\xi}_{0,2} = \tilde{\xi}_{1,1} = \tilde{\xi}_{0,1,1}$. Similarly, the analogous definitions for the inverse demand system are also obtained:

$$\zeta_1 \equiv \frac{\partial q_{ik}}{\partial p_{ik}}, \qquad \zeta_{0,1} \equiv \frac{\partial q_{ik}}{\partial p_{ik'}},$$
$$\zeta_2 \equiv \frac{\partial q_{ik}}{\partial p_{ik}^2}, \quad \zeta_{1,1} \equiv \frac{\partial q_{ik}}{\partial p_{ik}\partial p_{ik'}}, \quad \zeta_{0,2} \equiv \frac{\partial q_{ik}}{\partial p_{ik'}^2}, \quad \zeta_{0,1,1} \equiv \frac{\partial q_{ik}}{\partial p_{ik'}\partial p_{ik''}},$$
$$\tilde{\zeta}_2 \equiv \frac{\partial q_{ik}}{\partial p_{ik}\partial p_{i'k}} \quad \tilde{\zeta}_{1,1} \equiv \frac{\partial q_{ik}}{\partial p_{ik}\partial p_{i'k'}}, \quad \tilde{\zeta}_{0,2} \equiv \frac{\partial q_{ik}}{\partial p_{ik'}\partial p_{i'k'}}, \quad \tilde{\zeta}_{0,1,1} \equiv \frac{\partial q_{ik}}{\partial p_{ik'}\partial p_{i'k''}}.$$

Online Appendix C Other applications than taxation

Our general formulation presented in Appendix C allows us to consider many applications beyond public finance since policy interventions and non-governmental external changes can be specified in a very flexible manner. In the following, we provide five such examples that can potentially be investigated in a thorough manner: the last two examples are particularly related to public economics.

C.1 Exchange rate changes

First, let us point out that the *exchange rate pass-through* can be included naturally in our framework of Appendix C.[48] Suppose that domestic firms in a country of interest use some imported inputs for

[47] In this notation, the first subscript counts the derivatives with respect to the relevant price with index k, the second subscript counts the derivatives with respect to the price with index k' distinct from k, and the third subscript counts derivatives respect to the price with index k'' distinct from both k and k'. Further, ξ corresponds to derivatives with respect to prices charged by the same firm i, while $\tilde{\xi}$ corresponds to derivatives with respect to prices charged by firm i and some other firm i'.

[48] See, e.g., Feenstra (1989); Feenstra, Gagnon, and Knetter (1996); Yang (1997); Campa and Goldberg (2005); Hellerstein (2008); Gopinath, Itskhoki, and Rigobon (2010); Goldberg and Hellerstein (2013); Auer and Schoenle (2016); and Chen and

production. For concreteness, let us specify the profit function of firm i as $\pi_i = [(1-v)p_i - t]q_i - (1 + a \cdot e)c(q_i)$, where the constant coefficient a measures the importance imported inputs and $e > 0$ is the exchange rate. Notice that the firm's profit is rewritten as:

$$\pi_i = (1+ae)\left[\left(\frac{1-v}{1+ae}p_i - \frac{t}{1+ae}\right)q_i - c(q_i)\right].$$

Since the first factor on the right-hand side is constant, the firm will behave as if its profit function was simply $\tilde{\pi}_i = [(1-\tilde{v})p_i - \tilde{t}]\,q_i - c(q_i)$, with $\tilde{v} \equiv (v+ae)/(1+ae)$ and $\tilde{t} \equiv t/(1+ae)$. By utilizing the explicit expressions for the derivatives $\frac{\partial \tilde{v}}{\partial e} = (a-v)/(1+ae)^2$ and $\frac{\partial \tilde{t}}{\partial e} = -at/(1+ae)^2$, one can analyze the effect of a change in the exchange rate e on social welfare. Note that this is simply interpreted as the *cost pass-through* as well. It would be interesting to incorporate uncertainty into this framework.

C.2 Exogenous competition

Weyl and Fabinger's (2013) results under symmetric oligopoly can be interpreted as special cases of our results here. In particular, Weyl and Fabinger's (2013) analysis considers either unit taxes or exogenous competition (an exogenous quantity supplied to the market). The case of unit taxes is clearly included in the present results. At the same time, the case of exogenous competition can be included as well. The reasoning is as follows.

Consider a tax $T_1 = \tilde{q}$ of the form: $\phi\,(p,q,\tilde{q}) = \tilde{q}p + c(q-\tilde{q}) - c(q)$. Then, the firm's profit is given by $pq - c\,(q) - \phi\,(p,q,\tilde{q}) = p\,(q-\tilde{q}) + c(q-\tilde{q})$. The firm, therefore, has the same profit function as in the case of exogenous competition $\tilde{q}$ in Weyl and Fabinger (2013). Then, Proposition 10 above (if specialized to constant marginal cost and zero initial $\tilde{q}$) implies the social incidence result in Weyl and Fabinger's (2013, p. 548) Principle of Incidence 3.[49]

Similarly, the pass-throughs of unit tax and exogenous competition are implied by Proposition 10

Juvenal (2016) for empirical studies of exchange rate pass-through.

[49]Miklós-Thal and Shaffer (2021) point out that the way that Weyl and Fabinger (2013) use exogenous competition to extend their Principle of Incidence 3 for global changes (see their argument on page 541) has some technical flaws. However, their Pricinple of Incidenec 3 itself is still correct if exogeneous competition is interpreted in a way that it consists of a part of market demand. Note also that this is also trule when we discuss global changes in surplus measures in Online Appendix D, hence our arguments there are not affected by Miklós-Thal and Shaffer's (2021) correction, either.

above with the tax specification, $T_1 = t$, $T_2 = \tilde{q}$, and $\phi(p,q,t,\tilde{q}) = tq + \tilde{q}p + c(q-\tilde{q}) - c(q)$. More generally, ϕ is extended as $\phi = c(q-\tilde{q}) + v(q-\tilde{q})\,p(q) + (1-v)\tilde{q}p(q) + t(q-\tilde{q}) - c(q)$, where an ad valorem tax is also considered. As an example, one can think of a government which procures goods from abroad and supplies them to the market in order to lower domestic prices.

C.3 Depreciating licenses

In the special case of a monopolist with constant marginal cost, another interesting interpretation can also be considered: it is isomorphic to the case of "depreciating licenses" in Weyl and Zhang (2021).[50] Depreciating licenses correspond to a tax scheme where the owner of an asset announces a reservation price at which she is willing to sell it and gets taxed a fixed fraction of that prices. Another agent in the economy may buy the asset at the announced price. The owner then faces a tradeoff between announcing a low price for a low tax payment and announcing a high price in order to be able to keep the asset to derive utility from it. The optimization problem then leads to exactly the same mathematical form as the problem of a monopolist with constant marginal cost facing exogenous competition. Here we discuss the relationship of exogenous competition to depreciating licenses in Weyl and Zhang (2021).

In the setup of Weyl and Zhang (2021), there are two agents, S and B ("seller" and "buyer"). Agent S holds an asset and declares a reservation value p, which influences the tax ("license fee") $\tilde{q}p$ that the agent needs to pay to the government. Here $\tilde{q}$ is the license tax rate (denoted by τ in their paper). Then, agent B may purchase the asset at that price p from agent S. The value for agent S is $\eta + \gamma_S$, and for agent B it is $\eta + \gamma_B$, for some common value component η.[51] Here, γ_B is a random variable with a cumulative distribution function $F(\gamma_B)$ representing heterogeneity in agent B's value, which is not observed by S. As Weyl and Zhang (2021) show, the sales probability q is then determined as the solution of $P(q) = p$, where $P(q) \equiv F^{-1}(1-q) + \eta$. Up to a constant, agent S's expected profit function (utility function) is $(P(q) - \eta - \gamma_S)(q - \tilde{q})$ or $P(q)(q-\tilde{q}) - (q-\tilde{q})mc$, where $mc \equiv \eta + \gamma_S$.[52] We recognize that this is exactly of the same form as the profit function in the case of monopoly with constant marginal cost mc

[50] We thank Glen Weyl for suggesting this relationship between Weyl and Zhang (2021) and our analysis.

[51] Weyl and Zhang (2021) consider η as being determined by agent S at the very beginning. Here we focus on the subgame after η has been determined.

[52] In Weyl and Zhang's (2021) analysis, the profit function is written as $(M(q) - \gamma_S)(q-\tau) + (\eta + \gamma_S)(1-\tau) - c(\eta)$. The last two terms are constant. The $M(q)$ in the first term corresponds to our $P(q) - \eta$.

subject to exogenous competition $\tilde{q}$ and inverse demand function $P(q)$.

C.4 Sales restrictions

Governments often regulate when, where, and to whom products may be sold. For example, business hours may get regulated due to a pandemic. A simple way of modeling this situation is to assume that a firm loses a fixed proportion of its customers because of the restriction. In the absence of taxation and the regulation, the profit function is $p(q)q - c(q)$. The new profit function will be $(1-v)\,p([1+\kappa]q)q - tq - c(q)$, where $1 - 1/(1+\kappa)$ is the fraction of customers lost. The only change is in the argument of the inverse demand function: for the firm to sell quantity q, each remaining customer needs to buy $(1+\kappa)$ times more than in the absence of the regulation, and correspondingly, the price would have to be lower. This change can be described as $\phi(p,q,\mathbf{T}) = [1-(1-v)\,h(q,\kappa)]p(q)q + tq$, where $\mathbf{T} = (t,v,\kappa)$ is a three-dimensional vector of policy parameters, and $h(q,\kappa) \equiv \frac{p(q)-p[(1+\kappa)q]}{p(q)}$. As a specification for constant elasticity demand, ε, $h(q,\kappa) = 1-(1+\kappa)^{-1/\varepsilon}$, independently of q, can be used.

C.5 Tax evasion/avoidance and concealment costs

Tax evasion is clearly a very important problem in many situations since economic agents do not always strictly follow the law (Choi, Furusawa, and Ishikawa 2020). For simplicity, consider a firm that has to pay an ad valorem tax $v\tilde{p}q$, where $\tilde{p}$ is the price reported to the government and may differ from the true price p. We capture the cost associated with deceiving the government by introducing a concealment cost, $c_c(\tilde{p},p,q) \equiv \frac{1}{4\lambda}p^{-\zeta}(\tilde{p}-p)^2 q^{1-\xi}$. The parameter $\lambda > 0$ could be set by the government: to choose a definite value of λ, the government may need to pay additional enforcement cost inversely related to λ. Of course, this extra enforcement cost would need to be remembered in welfare analysis.

The firm then chooses the reported price $\tilde{p}$ to minimize the sum of these two additional costs: $v\tilde{p}q + c_c(\tilde{p},p,q) = v\tilde{p}q + \frac{1}{4\lambda}p^{-\zeta}q^{1-\xi}(\tilde{p}-p)^2$. The corresponding first-order condition implies $\tilde{p} = p - 2\lambda v p^{\zeta} q^{\xi}$, which gives an additional cost, $(v\tilde{p}q + c_c(\tilde{p},p,q))_{\tilde{p}=p-2\lambda v p^{\zeta} q^{\xi}} = pqv - \lambda v^2 p^{\zeta} q^{1+\xi}$. Then the firm's total additional cost, which may include a unit tax tq, is given by $\phi(p,q,\mathbf{T}) = tq + pqv - \lambda v^2 p^{\zeta} q^{1+\xi}$, where $\mathbf{T} = (t,v,\lambda)$, of which the government receives $\tilde{\phi}(p,q,\mathbf{T}) = tq + pqv - 2\lambda v^2 p^{\zeta} q^{1+\xi}$.[53]

[53] See Häckner and Herzing (2017) for a related study of environmental regulations in oligopoly.

Online Appendix D Global changes in surplus measures

In the main text, we discuss local, i.e., infinitesimal, changes in surplus measures (CS, PS, R, W). To consider a larger change in some intervention T (such as a tax or a technology parameter), it is desirable to have a conceptual understanding of global changes in these surplus measures. We present a counterpart of the methodology used in Weyl and Fabinger (2013) to consider non-infinitesimal ("global") changes, as reflected in their propositions ("Principle of Incidence 5" on pages 536, 541, and 551), whose applications are discussed in detail in their Section 4.

Consider surplus measures A and B. Their finite changes $\Delta A = \int_{T_1}^{T_2} \frac{dA(T)}{dT} dT$ and $\Delta B = \int_{T_1}^{T_2} \frac{dB(T)}{dT} dT$ induced by a change from $T = T_1$ to $T = T_2$ are related to their incidence ratios $\Theta_{AB} \equiv \frac{dA(T)}{dT} / \frac{dB(T)}{dT}$. In particular, $\Delta A / \Delta B$ is a weighted average of Θ_{AB} over the interval (T_1, T_2):

$$\frac{\Delta A}{\Delta B} = \int_{T \in (T_1, T_2)} \Theta_{AB} \, dw_B^{(T_1,T_2)}(T) \, ,$$

where $dw_B^{(T_1,T_2)}(T) \equiv \frac{dB(T)}{dT} dT / \int_{T_1}^{T_2} \frac{dB(T')}{dT'} dT'$ is a weight, normalized to unity, on the corresponding interval: $\int_{T_1}^{T_2} dw_B^{(T_1,T_2)}(T) = 1$. The weight is positive as long as $\frac{dB(T)}{dT}$ has the same sign as $\int_{T_1}^{T_2} \frac{dB(T)}{dT} dT$, which is typically satisfied in interesting applications. In many useful cases, A and B are zero at infinite T (e.g., with an infinite tax, the market participants gain no surplus). Then

$$\frac{A(T_1)}{B(T_1)} = \int_{T \in (T_1, \infty)} \Theta_{AB} \, dw_B^{(T_1,\infty)}(T) \, .$$

Specifically, the change in deadweight loss and the associated incidence are

$$\frac{W(T_1)}{R(T_1)} = \int_{T \in (T_1, \infty)} MVPF_T \, dw_R^{(T_1,\infty)}(T) \, , \qquad \frac{CS(T_1)}{PS(T_1)} = \int_{T \in (T_1, \infty)} I_T \, dw_{PS}^{(T_1,\infty)}(T) \, ,$$

respectively. In the case of a per-unit tax, for example, we obtain

$$\frac{CS(t_1)}{PS(t_1)} = \frac{\int_{t_1}^{\infty} I_T \, q \, dt}{\int_{t_1}^{\infty} q \, dt} = \int_{t_1}^{\infty} I_T(t) \, \frac{q(t)}{\int_{t_1}^{\infty} q(\tilde{t}) \, d\tilde{t}} dt,$$

which means that the surplus ratio is a weighted average of the incidence I_T over the relevant range, with weight $q(t) / \int_{t_1}^{\infty} q(\tilde{t})\, d\tilde{t}$.[54]

Online Appendix E Free entry

If decisions to enter the industry are made before the firms start competing in the market, our results would be unchanged if we set the number of firms n equal to the number of firms that is endogenously determined given the entry game specification, except at points when a policy change would trigger a change the number of firms. However, besides using the previous results, we may also consider a limit in which the number of firms is large and may be treated as continuous, after an appropriate rescaling. Here we briefly discuss that situation. The newly defined number of firms n, or more precisely the mass of firms n, is a real number in $(0, \infty)$.

E.1 General description

Given the equilibrium price function, $p(q,n)$, which is determined by the demand side of the model,[55] a change in the number of firms, treated as a continuous variable, is

$$\frac{dn}{dT_\ell} = \frac{1}{\frac{\partial}{\partial n} p(q,n)} \cdot \frac{dp}{dT_\ell} - \frac{\frac{\partial}{\partial q} p(q,n)}{\frac{\partial}{\partial n} p(q,n)} \cdot \frac{dq}{dT_\ell}.$$

The equilibrium is determined by the same firms' first-order condition as before and by the zero profit condition: $-\phi(p(q,n), q, \mathbf{T}) + q\, p(q,n) - c(q) - c_e(n) = 0$, where we allow for a dependence of the entry cost c_e on the equilibrium number of firms n. Totally differentiating the first-order condition gives

$$\frac{dp}{dT_\ell} = \frac{f(p,q,\mathbf{T})}{q(1-\nu(p,q,\mathbf{T}))} + \frac{p\,\tau(p,q,\mathbf{T}) + mc(q) - p}{q(1-\nu(p,q,\mathbf{T}))} \cdot \frac{dq}{dT_\ell}.$$

[54] Note that in this case $dw_{PS}^{(t,\infty)} = q\,dt/PS = q\,dt / \int_{t_1}^{\infty} q\,dt$.

[55] It would be more precise to denote the equilibrium price function as $p^{\star}(q,n)$. We use the simpler notation $p(q,n)$ because it is better suited for comparative statics calculations.

This means that if for a given dT_ℓ we find dq, we will also know dn and dp. To find dq we use the comparative statics of the firm's first-order condition, which leads to the relationship

$$\frac{dq}{dT_\ell} = \frac{\frac{1}{1-\nu}\left\{\frac{f}{p}\left[\frac{1}{1-\nu}\varepsilon_{\mathrm{p,n}}(\kappa-1-\nu_{(2)}\psi-\nu\psi+\psi)-\psi\varepsilon_{\psi,\mathrm{n}}\right]+\varepsilon_{\mathrm{p,n}}(\tau_T-\psi\nu_T)\right\}q}{\varepsilon_{\mathrm{p,n}}\left[\frac{1}{1-\nu}\{-\tau_{(2)}+\psi[(\nu-1)\varepsilon_\psi-\psi\nu_{(2)}]+\psi(2\kappa-\nu-1)+(\tau-1)\chi\}+\psi(\chi+\psi)\right]+\psi(\eta-\psi)\varepsilon_{\psi,\mathrm{n}}},$$

where

$$\varepsilon_{\mathrm{p,n}} \equiv -\frac{n\,p^{(0,1)}}{p}, \qquad \varepsilon_\psi \equiv \frac{q\psi^{(1,0)}}{\psi}, \qquad \varepsilon_{\psi,\mathrm{n}} \equiv \frac{n\psi^{(0,1)}}{\psi}.$$

These expressions make it possible to find the value of the pass-through rate $\tilde{\rho}_{T_\ell} \equiv dp/dT_\ell$.

The marginal value of public funds, $MVPF_{T_\ell}$, is simply $-1-dCS/dR$, since firms earn zero profits. We obtain

$$MVPF_{T_\ell} = -1 - \frac{\theta\rho_{T_\ell}\varepsilon_{pn} - (\frac{1}{1-\nu}-(1+\theta)\rho_{T_\ell})\alpha_{cs} - \frac{1}{1-\nu}\rho_{T_\ell}\varepsilon_{c_e}\alpha_{c_e}}{\left[\theta+\frac{\tau\varepsilon}{1-\nu}+(\theta\nu-\tau\varepsilon)\rho_{T_\ell}\right]\varepsilon_{pn}+\left[\frac{1}{1-\nu}-(1+\theta)\rho_{T_\ell}\right]\alpha_\phi - \frac{1+\rho_{T_\ell}(\nu+\tau\varepsilon)}{1-\nu}\varepsilon_{c_e}\alpha_{c_e}},$$

where cs is the consumer surplus per firm. Under specific assumptions on the function $p(q,n)$ these expressions may be manipulated further.

Another possible specification of the free entry game is to assume that firm $i \in [0,n]$ faces an entry cost $c_e(i)$, rather than $c_e(n)$. In this case, producer surplus, $PS(n) = \int_0^n (c_e(i) - c_e(n))\,di$, can be positive. Its derivative with respect to n is $nc_e'(n)$, which also contributes positively to social welfare. In this case, the marginal change in deadweight loss is

$$MVPF_{T_\ell} = -1 - \frac{\theta\rho_{T_\ell}\varepsilon_{pn} - (\frac{1}{1-\nu}-(1+\theta)\rho_{T_\ell})\,\alpha_{cs} - \frac{1}{1-\nu}(1+\rho_{T_\ell})\varepsilon_{c_e}\alpha_{c_e}}{\left[\theta+\frac{\tau\varepsilon}{1-\nu}+(\theta\nu-\tau\varepsilon)\rho_{T_\ell}\right]\varepsilon_{pn}+\left[\frac{1}{1-\nu}-(1+\theta)\rho_{T_\ell}\right]\alpha_\phi - \frac{1+\rho_{T_\ell}(\nu+\tau\varepsilon)}{1-\nu}\varepsilon_{c_e}\alpha_{c_e}},$$

which differs from the previous expression in the last term in the numerator.

E.2 The case of unit and ad valorem taxes

Here, we provide an analysis in the case of unit and ad valorem taxes, and it is essentially a generalization of Besley's (1989) study on unit tax in Cournot oligopoly with free entry. Let denote the equilibrium inverse demand $p(q;n)$. Each firm has a fixed cost of entry, $K > 0$. The zero-profit condition is $(1-$

$v)pq-tq-c(q)-K=0$, whereas the first-order condition is rewritten as $[(1-v)pq-tq-c(q)]+[c(q)-mc(q)q]=(1-v)\eta\theta pq$. Thus, the equilibrium q and n satisfy:

$$\begin{cases} (1-v)[1-\eta(q)\theta(q)]p(q;n)-t-mc(q)=0 \\ (1-v)\eta(q)\theta(q)p(q;n)-[ac(q)-mc(q)]=0, \end{cases}$$

where $ac(q)=[c(q)+K]/q$ is the average cost in equilibrium. Then,

$$\begin{bmatrix} -(\eta\theta)'p+(1-\eta\theta)\dfrac{\partial p}{\partial q}-\dfrac{mc'}{1-v} & (1-\eta\theta)\dfrac{\partial p}{\partial n} \\ (\eta\theta)'p+(\eta\theta)\dfrac{\partial p}{\partial q}-\dfrac{ac'-mc'}{1-v} & \eta\theta\dfrac{\partial p}{\partial n} \end{bmatrix}\begin{bmatrix} \dfrac{dq}{dt} \\ \dfrac{dn}{dt} \end{bmatrix}=\begin{bmatrix} \dfrac{1}{1-v} \\ 0 \end{bmatrix}.$$

Thus,

$$\begin{bmatrix} \dfrac{dq}{dt} \\ \dfrac{dn}{dt} \end{bmatrix}=\frac{1}{1-v}\cdot\frac{1}{D}\begin{bmatrix} \eta\theta\dfrac{\partial p}{\partial n} \\ -(\eta\theta)'p-(\eta\theta)\dfrac{\partial p}{\partial q}+\dfrac{ac'-mc'}{1-v} \end{bmatrix},$$

where

$$D=\left[(\eta\theta)'p-\frac{(1-\eta\theta)ac'-mc'}{1-v}\right]\left(-\frac{\partial p}{\partial n}\right),$$

which implies that

$$\frac{dq}{dt}=\frac{1}{1-v}\cdot\frac{-\eta\theta}{(\eta\theta)'p-\frac{(1-\eta\theta)ac'-mc'}{1-v}}.$$

Thus, the unit-tax pass-through rate is:

$$\rho_t=-\frac{p\eta}{q}\cdot\frac{dq}{dt}=\frac{1}{1-v}\cdot\frac{1}{\left(\frac{q}{p\eta}\right)\left[\frac{(\eta\theta)'p}{\eta\theta}-\frac{(1-\eta\theta)ac'-mc'}{(1-v)\eta\theta}\right]}.$$

However, for a change in v,

$$\begin{bmatrix} -(\eta\theta)'p+(1-\eta\theta)\dfrac{\partial p}{\partial q}-\dfrac{mc'}{1-v} & (1-\eta\theta)\dfrac{\partial p}{\partial n} \\ (\eta\theta)'p+(\eta\theta)\dfrac{\partial p}{\partial q}-\dfrac{ac'-mc'}{1-v} & \eta\theta\dfrac{\partial p}{\partial n} \end{bmatrix}\begin{bmatrix} \dfrac{dq}{dv} \\ \dfrac{dn}{dv} \end{bmatrix}=\begin{bmatrix} \dfrac{(1-\eta\theta)p}{1-v} \\ \dfrac{\eta\theta p}{1-v} \end{bmatrix}.$$

implies that

$$\begin{bmatrix} \dfrac{dq}{dv} \\ \dfrac{dn}{dv} \end{bmatrix} = \frac{1}{1-v}\cdot\frac{1}{D}\begin{bmatrix} \eta\theta\dfrac{\partial p}{\partial n} & -(1-\eta\theta)\dfrac{\partial p}{\partial n} \\ -\{(\eta\theta)'p+(\eta\theta)\dfrac{\partial p}{\partial q}-\dfrac{ac'-mc'}{1-v}\} & -(\eta\theta)'p+(1-\eta\theta)\dfrac{\partial p}{\partial q}-\dfrac{mc'}{1-v} \end{bmatrix} \times \begin{bmatrix} (1-\eta\theta)p \\ \eta\theta p \end{bmatrix},$$

and thus $\frac{dq}{dv}=0$.

Online Appendix F Aggregative games

Finally, we argue that it is possible to further manipulate the above formulas for the conduct index and the pricing strength index expressed as an *aggregative game*, in which each firm's profit is a function of its own action and a *single aggregating variable* of all the other firms' actions.[56] We identify the firm's strategic variable σ_i with an action $a_i \equiv \sigma_i$ that the firm can take, which contributes to an aggregator, $A=\sum_{i=1}^{n} a_i$. The prices and quantities are functions of just two arguments: $p_i(A,a_i)$ and $q_i(A,a_i)$. Their derivatives that take into account the dependence of A on the action of firm i are $\frac{dq_j}{d\sigma_i}=\frac{\partial q_j}{\partial a_i}+\frac{\partial q_j}{\partial A}$ and $\frac{dp_j}{d\sigma_i}=\frac{\partial p_j}{\partial a_i}+\frac{\partial p_j}{\partial A}$. Thus, the firm's first-order condition is given by:

$$0=\left(\frac{\partial p_j}{\partial a_i}(A,a_i)+\frac{\partial p_j}{\partial A}(A,a_i)\right)q_i(A,a_i)\left(\nu_i\left(p_i(A,a_i),q_i(A,a_i),\mathbf{T}\right)-1\right)+$$

$$\left(\frac{\partial q_j}{\partial a_i}(A,a_i)+\frac{\partial q_j}{\partial A}(A,a_i)\right)\left(mc\left(q_i(A,a_i)\right)+p_i(A,a_i)\left(\tau_i\left(p_i(A,a_i),q_i(A,a_i),\mathbf{T}\right)-1\right)\right),$$

which gives us a relatively simple expression for the market power index:

$$\psi_i(A,a_i)=-\frac{q_i(A,a_i)}{p_i(A,a_i)}\cdot\frac{\frac{\partial p_j}{\partial a_i}(A,a_i)+\frac{\partial p_j}{\partial A}(A,a_i)}{\frac{\partial q_j}{\partial a_i}(A,a_i)+\frac{\partial q_j}{\partial A}(A,a_i)}.$$

[56]Here, we consider a setup in Anderson, Erkal, and Piccinin (2020). See also Nocke and Schutz (2018) for a related formulation.

The expression for the conduct index also simplifies:

$$\theta_i = \sum_{j=1}^{n} w_j \cdot \frac{\gamma_j(A, a_i)}{\gamma_j(A, a_j)},$$

where $w_i \equiv \frac{\tilde{w}_i}{\sum_{j=1}^{n} \tilde{w}_j}$ is a normalized version of unnormalized "weights,"

$$\tilde{w}_j \equiv (1 - \nu_j)\, q_j(A, a_j) \cdot \left(\frac{\partial p_j}{\partial a_i}(A, a_i) + \frac{\partial p_j}{\partial A}(A, a_i) \right),$$

and $\gamma_j(A, a_i) \equiv \frac{\partial q_j}{\partial a_i}(A, a_i) + \frac{\partial q_j}{\partial A}(A, a_i)$. These simplified formulas would be used for further analysis of pass-through and welfare in aggregative oligopoly games.

Online Appendix G Relationship between elasticities and curvatures

G.1 Under the direct demand system

This relationship, $p\varepsilon'_{own}(p)/\varepsilon_{own}(p) = 1 + \varepsilon(p) - \alpha_{own}(p) - \alpha_{cross}(p)$, in Appendix B.1 can be verified as follows. The elasticity of the function $\varepsilon_{own}(p)$ equals the sum of the elasticities of the three factors it is composed of:

$$\frac{1}{\varepsilon_{own}(p)} p \frac{d}{dp} \varepsilon_{own}(p) = \frac{1}{p} p \frac{d}{dp} p + q(p)\, p \frac{d}{dp} \frac{1}{q(p)} + \left(\frac{\partial q_j(\mathbf{p})}{\partial p_j} \right)^{-1} |_{\mathbf{p}=(p,\ldots,p)} p \frac{d}{dp} \left(\frac{\partial q_j(\mathbf{p})}{\partial p_j} |_{\mathbf{p}=(p,\ldots,p)} \right).$$

The first elasticity on the right-hand side equals 1, the second elasticity equals $\varepsilon(p)$, and the third elasticity equals $-\alpha_{own}(p) - \alpha_{cross}(p)$, since

$$p \frac{d}{dp} \left(\frac{\partial q_j(\mathbf{p})}{\partial p_j} |_{\mathbf{p}=(p,\ldots,p)} \right) = p \frac{\partial^2 q_j(\mathbf{p})}{\partial p_j^2} |_{\mathbf{p}=(p,\ldots,p)} + (n-1)\, p \frac{\partial^2 q_j(\mathbf{p})}{\partial p_j \partial p_{j'}} |_{\mathbf{p}=(p,\ldots,p)}.$$

Note that α is *weakly positive* (*weakly negative*) *if the industry demand is convex* (*concave*), and α_{own} is weakly positive (weakly negative) if the demand as a function of firm i's own price is convex (concave). Hence, both α and α_{own} measure the degree of convexity in the demand function for an industry-wide price change and for an individual firm's price change, respectively.

Note also that $\partial(\partial q_i/\partial p_i)/\partial p_{i'}$ in α_{cross} measures the effects of firm i's price change on how many consumers rival i' loses if it raises its price. If this is negative (positive), then firm i' loses more (less) consumers by its own price increase for a higher value of p_i. Thus, because $\partial q_i/\partial p_{i'}$ is *positive* in the expression for α_{cross}, a higher α_{cross} also indicates more competitiveness in the industry.

It is also expected that the industry is more competitive if α and α_{own} are higher. In effect, the equilibrium price is characterized by ε_{own}. However, a policy change around equilibrium is also affected by the curvatures, which measure "second-order" competitiveness around the equilibrium. Proposition 8 shows that α is the only curvature that determines the pass-through.

G.2 Under the inverse demand system

In analogy with Online Appendix C.1 above, the elasticity of the function $\eta_{own}(q)$ is the sum of 1, $\eta(q)$, and $-\sigma_{own}(q) - \sigma_{cross}(q)$.

Now, σ is *weakly positive* (*weakly negative*) if the industry's inverse demand is *convex* (*concave*), and σ_{own} is weakly positive (weakly negative) if the inverse demand as a function of firm i's own output is convex (concave). Here, concavity, not convexity, is related to a sharp reduction in price in response to an increase in firm i's output. Thus, $-\sigma$ and $-\sigma_{own}$ measure "second-order" competitiveness of the industry, which characterizes the responsiveness of the equilibrium output when a policy is changed.

Note also that $\partial(\partial p_i/\partial q_i)/\partial q_{i'}$ in σ_{cross} measures the effects of firm i's output increase on the extent of rival (i')'s price drop if it increases its output. If this is negative (positive), then firm i' expects a large (little) drop in its price by increasing its output for a higher value of q_i. Because $\partial p_i/\partial q_{i'}$ is negative in the expression for σ_{cross}, a *lower* σ_{cross} or a higher $-\sigma_{cross}$ indicates more competitiveness in the industry.

In sum, while $1/\eta_{own}$ characterizes competitiveness that determines the level of the equilibrium quantity, $-\sigma$, $-\sigma_{own}$, and $-\sigma_{cross}$ determine competitiveness that characterizes the responsiveness of the equilibrium output by a policy change. However, similar to price competition, Proposition 9 above shows that σ is the only curvature that determines the pass-through.

Online Appendix H Calculating the sufficient statistics when two symmetrically differentiated firms face different marginal costs

In this appendix, we compute the sufficient statistics necessary for evaluating $MVPF_{iT}$ and I_{iT} under the setting in Subsection 4.3. See Online Appendix I for the equilibrium prices and outputs.

H.1 Price competition

Here, firm i's pricing strength index in equilibrium is given by

$$\psi_i = \frac{q_i(p_1, p_2)}{\lambda p_i},$$

for $i = 1, 2$, which implies that[57]

$$\begin{pmatrix} \dfrac{\partial \psi_1}{\partial p_1} & \dfrac{\partial \psi_1}{\partial p_2} \\ \dfrac{\partial \psi_2}{\partial p_1} & \dfrac{\partial \psi_2}{\partial p_2} \end{pmatrix} = \begin{pmatrix} -\dfrac{\lambda p_1 + q_1}{\lambda p_1^2} & \dfrac{\mu}{\lambda p_1} \\ \dfrac{\mu}{\lambda p_2} & -\dfrac{\lambda p_2 + q_2}{\lambda p_2^2} \end{pmatrix}.$$

First, it is verified that

$$\begin{cases} \psi_1 = \dfrac{(2\lambda + \mu)[(1-v)b - t(\lambda - \mu)] - \left(2\lambda^2 - \mu^2\right) c_1 + \lambda \mu c_2}{(2\lambda + \mu)[(1-v)b + \lambda t] + \lambda\,(2\lambda c_1 + \mu c_2)} \\ \psi_2 = \dfrac{(2\lambda + \mu)[(1-v)b - t(\lambda - \mu)] + \lambda \mu c_1 - \left(2\lambda^2 - \mu^2\right) c_2}{(2\lambda + \mu)[(1-v)b + \lambda t] + \lambda\,(\mu c_1 + 2\lambda c_2)} \end{cases}$$

[57] Recall that Appendix C (C.3) defines

$$\Psi_{ij} = \frac{p_i}{\psi_i} \frac{\partial \psi_i[q_1(p_1, p_2), q_2(p_1, p_2)]}{\partial p_j},$$

for $i, j = 1, 2$, where

$$\psi_i[q_1(p_1, p_2), q_2(p_1, p_2)] = \frac{q_i(p_1, p_2)}{p_i\left(-\frac{\partial q_i}{\partial p_i}(p_1, p_2)\right)}.$$

in equilibrium. It is also verified that

$$
\begin{aligned}
\begin{pmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{pmatrix} &= \begin{pmatrix} -\dfrac{p_1}{\psi_1}\cdot\dfrac{\lambda p_1+q_1}{\lambda p_1^2} & \dfrac{p_1}{\psi_1}\dfrac{\mu}{\lambda p_1} \\ \dfrac{p_2}{\psi_2}\dfrac{\mu}{\lambda p_2} & -\dfrac{p_2}{\psi_2}\cdot\dfrac{\lambda p_2+q_2}{\lambda p_2^2} \end{pmatrix} \\
&= \begin{pmatrix} -\dfrac{\lambda+(q_1/p_1)}{\lambda\psi_1} & \dfrac{\mu}{\lambda\psi_1} \\ \dfrac{\mu}{\lambda\psi_2} & -\dfrac{\lambda+(q_2/p_2)}{\lambda\psi_2} \end{pmatrix} \\
&= \begin{pmatrix} -\left(\dfrac{1}{\psi_1}+\dfrac{(q_1/p_1)}{\lambda\psi_1}\right) & \left(\dfrac{\mu}{\lambda}\right)\dfrac{1}{\psi_1} \\ \left(\dfrac{\mu}{\lambda}\right)\dfrac{1}{\psi_2} & -\left(\dfrac{1}{\psi_2}+\dfrac{(q_2/p_2)}{\lambda\psi_2}\right) \end{pmatrix},
\end{aligned}
$$

which implies that

$$
\begin{cases}
\Psi_{11} = -\dfrac{(2\lambda+\mu)[2(1-v)b+\mu t]+\mu^2 c_1+2\lambda\mu c_2}{(2\lambda+\mu)[(1-v)b-t(\lambda-\mu)]-\left(2\lambda^2-\mu^2\right)c_1+\lambda\mu c_2} \\
\Psi_{12} = \left(\dfrac{\mu}{\lambda}\right)\dfrac{(2\lambda+\mu)[b(1-v)+\lambda t]+\lambda\left(2\lambda c_1+\mu c_2\right)}{(2\lambda+\mu)[(1-v)b-t(\lambda-\mu)]-\left(2\lambda^2-\mu^2\right)c_1+\lambda\mu c_2} \\
\Psi_{21} = \left(\dfrac{\mu}{\lambda}\right)\dfrac{(2\lambda+\mu)[b(1-v)+\lambda t]+\lambda\left(\mu c_1+2\lambda c_2\right)}{(2\lambda+\mu)[(1-v)b-t(\lambda-\mu)]+\lambda\mu c_1-\left(2\lambda^2-\mu^2\right)c_2} \\
\Psi_{22} = -\dfrac{(2\lambda+\mu)[2(1-v)b+\mu t]+2\lambda\mu c_1+\mu^2 c_2}{(2\lambda+\mu)[(1-v)b-t(\lambda-\mu)]+\lambda\mu c_1-\left(2\lambda^2-\mu^2\right)c_2}.
\end{cases}
$$

Next, recall that

$$
\begin{aligned}
\mathbf{b} &= (1-v)\begin{pmatrix} (1-\psi_1)-\psi_1\Psi_{11} & -\psi_1\Psi_{12} \\ -\psi_2\Psi_{21} & (1-\psi_2)-\psi_2\Psi_{22} \end{pmatrix} \\
&= (1-v)\begin{pmatrix} (1-\psi_1)+\left(1+\dfrac{q_1}{\lambda p_1}\right) & -\dfrac{\mu}{\lambda} \\ -\dfrac{\mu}{\lambda} & (1-\psi_2)+\left(1+\dfrac{q_2}{\lambda p_2}\right) \end{pmatrix} \\
&= (1-v)\begin{pmatrix} 2 & -\dfrac{\mu}{\lambda} \\ -\dfrac{\mu}{\lambda} & 2 \end{pmatrix}
\end{aligned}
$$

because $\chi_1 = 0$ and $\chi_2 = 0$.

Here, recall that

$$\begin{pmatrix} \tilde{\rho}_{1,t} & \tilde{\rho}_{1,v} \\ \tilde{\rho}_{2,t} & \tilde{\rho}_{2,v} \end{pmatrix} = \mathbf{b}^{-1} \begin{pmatrix} 1 & p_1 \cdot (1-\psi_1) \\ 1 & p_2 \cdot (1-\psi_2) \end{pmatrix}.$$

Therefore,

$$\begin{aligned}
\begin{pmatrix} \rho_{1,t} & \rho_{1,v} \\ \rho_{2,t} & \rho_{2,v} \end{pmatrix} &= \begin{pmatrix} \tilde{\rho}_{1,t} & \frac{\tilde{\rho}_{1,v}}{p_1} \\ \tilde{\rho}_{2,t} & \frac{\tilde{\rho}_{2,v}}{p_2} \end{pmatrix} \\
&= \mathbf{b}^{-1} \begin{pmatrix} 1 & 1-\psi_1 \\ 1 & 1-\psi_2 \end{pmatrix} \\
&= \frac{1}{(1-v)\left[4-\left(\frac{\mu}{\lambda}\right)^2\right]} \begin{pmatrix} 2 & \frac{\mu}{\lambda} \\ \frac{\mu}{\lambda} & 2 \end{pmatrix} \begin{pmatrix} 1 & 1-\psi_1 \\ 1 & 1-\psi_2 \end{pmatrix} \\
&= \frac{1}{(1-v)\left[4-\left(\frac{\mu}{\lambda}\right)^2\right]} \begin{pmatrix} 2+\frac{\mu}{\lambda} & 2(1-\psi_1)+\frac{\mu}{\lambda}(1-\psi_2) \\ 2+\frac{\mu}{\lambda} & \frac{\mu}{\lambda}(1-\psi_1)+2(1-\psi_2) \end{pmatrix}.
\end{aligned}$$

Finally, recall that $\varepsilon_1 = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} \end{pmatrix}$ and $\varepsilon_2 = \begin{pmatrix} \varepsilon_{21} & \varepsilon_{22} \end{pmatrix}$. Hence,

$$\begin{cases} \varepsilon_{1T}^{\rho} = \dfrac{\varepsilon_{11}\rho_{1,T} + \varepsilon_{12}\rho_{2,T}}{\rho_{1,T}} \\ \varepsilon_{2T}^{\rho} = \dfrac{\varepsilon_{21}\rho_{1,T} + \varepsilon_{22}\rho_{2,T}}{\rho_{2,T}} \end{cases}$$

for $T \in \{t, v\}$, where

$$\begin{aligned}
\begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{pmatrix} &= \begin{pmatrix} -\frac{p_1}{q_1}\frac{\partial q_1(\mathbf{p})}{\partial p_1} & -\frac{p_1}{q_1}\frac{\partial q_1(\mathbf{p})}{\partial p_2} \\ -\frac{p_2}{q_2}\frac{\partial q_2(\mathbf{p})}{\partial p_1} & -\frac{p_2}{q_2}\frac{\partial q_2(\mathbf{p})}{\partial p_2} \end{pmatrix}. \\
&= \begin{pmatrix} \lambda\frac{p_1}{q_1} & -\mu\frac{p_1}{q_1} \\ -\mu\frac{p_2}{q_2} & \lambda\frac{p_2}{q_2} \end{pmatrix}.
\end{aligned}$$

Therefore,

$$\begin{cases} \varepsilon^{\rho}_{1T} = \dfrac{\lambda\left(\dfrac{p_1}{q_1}\right)\rho_{1,T} - \mu\left(\dfrac{p_1}{q_1}\right)\rho_{2,T}}{\rho_{1,T}} \\ \varepsilon^{\rho}_{2T} = \dfrac{-\mu\left(\dfrac{p_2}{q_2}\right)\rho_{1,T} + \lambda\left(\dfrac{p_2}{q_2}\right)\rho_{2,T}}{\rho_{2,T}} \end{cases}$$

for $T \in \{t, v\}$.

H.2 Quantity competition

Here, firm i's pricing strength index in equilibrium is given by

$$\psi_i = \frac{\lambda q_i(p_1, p_2)}{(\lambda+\mu)(\lambda-\mu)p_i},$$

for $i = 1, 2$, which implies that

$$\begin{pmatrix} \dfrac{\partial \psi_1}{\partial p_1} & \dfrac{\partial \psi_1}{\partial p_2} \\ \dfrac{\partial \psi_2}{\partial p_1} & \dfrac{\partial \psi_2}{\partial p_2} \end{pmatrix} = \begin{pmatrix} -\dfrac{\lambda(\lambda p_1 + q_1)}{(\lambda+\mu)(\lambda-\mu)p_1^2} & \dfrac{\lambda\mu}{(\lambda+\mu)(\lambda-\mu)p_1} \\ \dfrac{\lambda\mu}{(\lambda+\mu)(\lambda-\mu)p_2} & -\dfrac{\lambda(\lambda p_2 + q_2)}{(\lambda+\mu)(\lambda-\mu)p_2^2} \end{pmatrix}.$$

First, in equilibrium, it is verified

$$\begin{cases} \psi_1 = \dfrac{\lambda\{(2\lambda-\mu)[(1-v)b - t(\lambda-\mu)] - (\lambda-\mu)(2\lambda c_1 - \mu c_2)\}}{(2\lambda-\mu)[\lambda(1-v)b + t(\lambda^2-\mu^2)] + (\lambda-\mu)[(2\lambda^2-\mu^2)c_1 + \lambda\mu c_2]} \\ \psi_2 = \dfrac{\lambda\{(2\lambda-\mu)[(1-v)b - t(\lambda-\mu)] - (\lambda-\mu)(-\mu c_1 + 2\lambda c_2)\}}{(2\lambda-\mu)[\lambda(1-v)b + t(\lambda^2-\mu^2)] + (\lambda-\mu)[\lambda\mu c_1 + (2\lambda^2-\mu^2)c_2]} \end{cases}.$$

It is also verified that

$$\begin{pmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{pmatrix} = \begin{pmatrix} -\dfrac{\lambda(\lambda + q_1/p_1)}{(\lambda+\mu)(\lambda-\mu)\psi_1} & \dfrac{\lambda\mu}{(\lambda+\mu)(\lambda-\mu)\psi_1} \\ \dfrac{\lambda\mu}{(\lambda+\mu)(\lambda-\mu)\psi_2} & -\dfrac{\lambda(\lambda + q_2/p_2)}{(\lambda+\mu)(\lambda-\mu)\psi_2} \end{pmatrix},$$

which implies that

$$\begin{cases}\Psi_{11}=-\dfrac{(2\lambda-\mu)[\mu t(\lambda-\mu)(\lambda+\mu)+b(1-v)(2\lambda^2-\mu^2)]+\mu(\lambda-\mu)[\lambda\mu c_1+(2\lambda^2-\mu^2)c_2]}{(\lambda-\mu)(\lambda+\mu)\{(2\lambda-\mu)[(1-v)b-t(\lambda-\mu)]-(\lambda-\mu)(2\lambda c_1-\mu c_2)\}}\\ \Psi_{12}=\dfrac{\mu\{(2\lambda-\mu)[\lambda(1-v)b+t\left(\lambda^2-\mu^2\right)]+(\lambda-\mu)[(2\lambda^2-\mu^2)c_1+\lambda\mu c_2]\}}{(\lambda-\mu)(\lambda+\mu)\{(2\lambda-\mu)[(1-v)b-t(\lambda-\mu)]-(\lambda-\mu)(2\lambda c_1-\mu c_2)\}}\\ \Psi_{21}=\dfrac{\mu\{(2\lambda-\mu)[\lambda(1-v)b+t\left(\lambda^2-\mu^2\right)]+(\lambda-\mu)[\lambda\mu c_1+(2\lambda^2-\mu^2)c_2]\}}{(\lambda-\mu)(\lambda+\mu)\{(2\lambda-\mu)[(1-v)b-t(\lambda-\mu)]-(\lambda-\mu)(-\mu c_1+2\lambda c_2)\}}\\ \Psi_{22}=-\dfrac{(2\lambda-\mu)[\mu t(\lambda-\mu)(\lambda+\mu)+b(1-v)(2\lambda^2-\mu^2)]+\mu(\lambda-\mu)[(2\lambda^2-\mu^2)c_1+\lambda\mu c_2]}{(\lambda-\mu)(\lambda+\mu)\{(2\lambda-\mu)[(1-v)b-t(\lambda-\mu)]-(\lambda-\mu)(-\mu c_1+2\lambda c_2)\}}.\end{cases}$$

Now, it is observed that

$$\begin{aligned}\mathbf{b} &= (1-v)\begin{pmatrix}(1-\psi_1)-\psi_1\Psi_{11} & -\psi_1\Psi_{12}\\ -\psi_2\Psi_{21} & (1-\psi_2)-\psi_2\Psi_{22}\end{pmatrix}\\ &= (1-v)\begin{pmatrix}1-\dfrac{\lambda q_1/p_1}{(\lambda+\mu)(\lambda-\mu)}+\dfrac{\lambda(\lambda+q_1/p_1)}{(\lambda+\mu)(\lambda-\mu)} & -\dfrac{\lambda\mu}{(\lambda+\mu)(\lambda-\mu)}\\ -\dfrac{\lambda\mu}{(\lambda+\mu)(\lambda-\mu)} & 1-\dfrac{\lambda q_2/p_2}{(\lambda+\mu)(\lambda-\mu)}+\dfrac{\lambda(\lambda+q_2/p_2)}{(\lambda+\mu)(\lambda-\mu)}\end{pmatrix}\\ &= \frac{1-v}{\lambda^2-\mu^2}\begin{pmatrix}2\lambda^2-\mu^2 & -\lambda\mu\\ -\lambda\mu & 2\lambda^2-\mu^2\end{pmatrix}.\end{aligned}$$

Here, recall that

$$\begin{pmatrix}\tilde\rho_{1,t} & \tilde\rho_{1,v}\\ \tilde\rho_{2,t} & \tilde\rho_{2,v}\end{pmatrix}=\mathbf{b}^{-1}\begin{pmatrix}1 & p_1\cdot(1-\psi_1)\\ 1 & p_2\cdot(1-\psi_2)\end{pmatrix}.$$

Therefore,

$$\begin{aligned}\begin{pmatrix}\rho_{1,t} & \rho_{1,v}\\ \rho_{2,t} & \rho_{2,v}\end{pmatrix} &= \begin{pmatrix}\tilde\rho_{1,t} & \frac{\tilde\rho_{1,v}}{p_1}\\ \tilde\rho_{2,t} & \frac{\tilde\rho_{2,v}}{p_2}\end{pmatrix}\\ &= \mathbf{b}^{-1}\begin{pmatrix}1 & 1-\psi_1\\ 1 & 1-\psi_2\end{pmatrix}\\ &= \frac{\lambda^2-\mu^2}{(1-v)[4\lambda^4-5\lambda^2\mu^2+\mu^4]}\begin{pmatrix}2\lambda^2-\mu^2 & \lambda\mu\\ \lambda\mu & 2\lambda^2-\mu^2\end{pmatrix}\begin{pmatrix}1 & 1-\psi_1\\ 1 & 1-\psi_2\end{pmatrix}\end{aligned}$$

$$= \quad \frac{\lambda^2-\mu^2}{(1-v)[4\lambda^4-5\lambda^2\mu^2+\mu^4]} \begin{pmatrix} 2\lambda^2-\mu^2+\lambda\mu & (2\lambda^2-\mu^2)(1-\psi_1)+\lambda\mu(1-\psi_2) \\ 2\lambda^2-\mu^2+\lambda\mu & \lambda\mu(1-\psi_1)+(2\lambda^2-\mu^2)(1-\psi_2) \end{pmatrix}.$$

Finally, as in the case of price competition,

$$\begin{cases} \varepsilon^{\rho}_{1T} = \dfrac{\lambda\left(\dfrac{p_1}{q_1}\right)\rho_{1,T} - \mu\left(\dfrac{p_1}{q_1}\right)\rho_{2,T}}{\rho_{1,T}} \\ \varepsilon^{\rho}_{2T} = \dfrac{-\mu\left(\dfrac{p_2}{q_2}\right)\rho_{1,T} + \lambda\left(\dfrac{p_2}{q_2}\right)\rho_{2,T}}{\rho_{2,T}} \end{cases}$$

for $T \in \{t, v\}$.

Online Appendix I Equilibrium prices and outputs in the parametric examples

I.1 Symmetric imperfect competition

I.1.1 Linear demand

If $b = 1$ and $mc = 0$ are additionally imposed, the equilibrium price and output under price competition are obtained as

$$p = \frac{1+\frac{t}{1-v}}{2-(n-1)\mu}, \qquad q = \frac{1-[1-(n-1)\mu]\frac{t}{1-v}}{2-(n-1)\mu},$$

and thus

$$\frac{p}{q} = \frac{1}{1-[1-(n-1)\mu]\frac{t}{1-v}}\left(1+\frac{t}{1-v}\right),$$

implying that

$$\varepsilon = \frac{[1-(n-1)\mu]\left(1+\frac{t}{1-v}\right)}{1-[1-(n-1)\mu]\frac{t}{1-v}}, \qquad \varepsilon_F = \frac{1+\frac{t}{1-v}}{1-[1-(n-1)\mu]\frac{t}{1-v}}.$$

Similarly, the equilibrium price and output under quantity competition are given by

$$p = \frac{\frac{1-(n-2)\mu}{1-(n-1)\mu} + (1+\mu)\frac{t}{1-v}}{2-(n-3)\mu}, \qquad q = (1+\mu)\frac{1-[1-(n-1)\mu]\frac{t}{1-v}}{2-(n-3)\mu},$$

and thus

$$\frac{p}{q} = \frac{1}{1-[1-(n-1)\mu]\frac{t}{1-v}}\left(\frac{1-(n-2)\mu}{(1+\mu)[1-(n-1)\mu]} + \frac{t}{1-v}\right),$$

implying that

$$\eta = \frac{1-[1-(n-1)\mu]\frac{t}{1-v}}{\frac{1-(n-2)\mu}{1+\mu} + [1-(n-1)\mu]\frac{t}{1-v}}, \qquad \eta_F = \frac{1-[1-(n-1)\mu]\frac{t}{1-v}}{1+\frac{(1+\mu)[1-(n-1)\mu]}{1-(n-2)\mu}\frac{t}{1-v}}.$$

I.1.2 CES demand

We assume that the marginal cost mc is constant and positive. The equilibrium price under price competition is

$$p = \frac{n(1-\gamma\xi)-\gamma(1-\xi)}{\gamma n(1-\gamma\xi)-\gamma(1-\xi)}mc,$$

and the associated output is

$$q = (\gamma\xi)^{\frac{1}{1-\gamma\xi}} n^{\frac{-(1-\xi)}{1-\gamma\xi}} \left(\frac{n(1-\gamma\xi)-\gamma(1-\xi)}{\gamma n(1-\gamma\xi)-\gamma(1-\xi)}mc\right)^{\frac{-1}{1-\gamma\xi}}.$$

Similarly, the equilibrium output per firm under quantity competition is

$$q = \left(\frac{n^{2-\xi}}{(\gamma^2\xi)[n-(1-\xi)]}mc\right)^{\frac{-1}{1-\gamma\xi}},$$

and the associated price is

$$p = \frac{n}{\gamma[n-(1-\xi)]}mc.$$

I.2 The case of two firms facing the linear demands and heterogeneous costs

I.2.1 Price competition

Firm i's profit function under quantity competition is:

$$\begin{aligned}\pi_i &= [(1-v)p_i - t - c_i]q_i(p_1, p_2) \\ &= [(1-v)p_i - t - c_i]\left(b - \lambda p_i + \mu p_j\right).\end{aligned}$$

Hence, the equilibrium prices under price competition are obtained as

$$\begin{cases} p_1 = \dfrac{(2\lambda+\mu)[(1-v)b+\lambda t]+\lambda(2\lambda c_1+\mu c_2)}{(1-v)(2\lambda-\mu)(2\lambda+\mu)} \\ p_2 = \dfrac{(2\lambda+\mu)[(1-v)b+\lambda t]+\lambda(\mu c_1+2\lambda c_2)}{(1-v)(2\lambda-\mu)(2\lambda+\mu)}, \end{cases}$$

and the associated outputs are

$$\begin{cases} q_1 = \dfrac{\lambda\{(2\lambda+\mu)[(1-v)b-t(\lambda-\mu)]-(2\lambda^2-\mu^2)c_1+\lambda\mu c_2\}}{(1-v)(2\lambda-\mu)(2\lambda+\mu)} \\ q_2 = \dfrac{\lambda\{(2\lambda+\mu)[(1-v)b-t(\lambda-\mu)]+\lambda\mu c_1-(2\lambda^2-\mu^2)c_2\}}{(1-v)(2\lambda-\mu)(2\lambda+\mu)}, \end{cases}$$

which implies that

$$\begin{cases} \dfrac{q_1}{p_1} = \dfrac{\lambda\{(2\lambda+\mu)[(1-v)b-t(\lambda-\mu)]-\left(2\lambda^2-\mu^2\right)c_1+\lambda\mu c_2\}}{(2\lambda+\mu)[(1-v)b+\lambda t]+\lambda\left(2\lambda c_1+\mu c_2\right)} \\ \dfrac{q_2}{p_2} = \dfrac{\lambda\{(2\lambda+\mu)[(1-v)b-t(\lambda-\mu)]+\lambda\mu c_1-\left(2\lambda^2-\mu^2\right)c_2\}}{(2\lambda+\mu)[(1-v)b+\lambda t]+\lambda\left(\mu c_1+2\lambda c_2\right)}. \end{cases}$$

I.2.2 Quantity competition

Firm i's profit function under quantity competition is:

$$\begin{aligned}\pi_i &= [(1-v)p_i(q_1, q_2) - t - c_i]q_i \\ &= \left[(1-v)\frac{b(\lambda+\mu)-\lambda q_i-\mu q_j}{(\lambda+\mu)(\lambda-\mu)} - t - c_i\right]q_i.\end{aligned}$$

Hence, the equilibrium outputs under quantity competition are obtained as

$$\begin{cases} q_1 = \dfrac{(\lambda+\mu)\{(2\lambda-\mu)[(1-v)b-t(\lambda-\mu)]-(\lambda-\mu)(2\lambda c_1-\mu c_2)\}}{(1-v)(2\lambda-\mu)(2\lambda+\mu)} \\ q_2 = \dfrac{(\lambda+\mu)\{(2\lambda-\mu)[(1-v)b-t(\lambda-\mu)]-(\lambda-\mu)(-\mu c_1+2\lambda c_2)\}}{(1-v)(2\lambda-\mu)(2\lambda+\mu)} \end{cases}$$

and the associated prices are

$$\begin{cases} p_1 = \dfrac{(2\lambda-\mu)[\lambda(1-v)b+t(\lambda^2-\mu^2)]+(\lambda-\mu)[(2\lambda^2-\mu^2)c_1+\lambda\mu c_2]}{(1-v)(\lambda-\mu)(2\lambda-\mu)(2\lambda+\mu)} \\ p_2 = \dfrac{(2\lambda-\mu)[\lambda(1-v)b+t(\lambda^2-\mu^2)]+(\lambda-\mu)[\lambda\mu c_1+(2\lambda^2-\mu^2)c_2]}{(1-v)(\lambda-\mu)(2\lambda-\mu)(2\lambda+\mu)}, \end{cases}$$

which implies that

$$\begin{cases} \dfrac{q_1}{p_1} = \dfrac{(\lambda-\mu)(\lambda+\mu)\{(2\lambda-\mu)[(1-v)b-t(\lambda-\mu)]-(\lambda-\mu)(2\lambda c_1-\mu c_2)\}}{(2\lambda-\mu)[\lambda(1-v)b+t(\lambda^2-\mu^2)]+(\lambda-\mu)[(2\lambda^2-\mu^2)c_1+\lambda\mu c_2]} \\ \dfrac{q_2}{p_2} = \dfrac{(\lambda-\mu)(\lambda+\mu)\{(2\lambda-\mu)[(1-v)b-t(\lambda-\mu)]-(\lambda-\mu)(-\mu c_1+2\lambda c_2)\}}{(2\lambda-\mu)[\lambda(1-v)b+t(\lambda^2-\mu^2)]+(\lambda-\mu)[\lambda\mu c_1+(2\lambda^2-\mu^2)c_2]}. \end{cases}$$

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