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Cournot Platform Competition with Mixed-Homing*

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Abstract

Firms in traditional markets often compete in output *à la* Cournot. In this paper, we consider Cournot platform competition in two-sided markets with single-, multi-, and mixed-homing allocations and find that the markup and markdown terms are distorted toward zero for (i) greater levels of platform competition and (ii) greater levels of single-homing. Furthermore, we develop side specific conduct parameters that depend on the underlying platform conduct as well as the homing allocation; these effectively extend the monopoly platform Lerner indices from Armstrong (2006) and Weyl (2010) to the general case of platform competition. Finally, we show that, in utter contrast to the welfare results in traditional Cournot markets, greater Cournot platform competition often decreases welfare across all feasible homing allocations.

Keywords: Two-sided markets, conduct parameter, network externality, Lerner index, single-homing, multi-homing, mixed-homing.
JEL Classifications: D40, L10, L20, L40.

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1 Introduction

The study of how firms compete was first introduced in the seminal work by Cournot (1838) and Bertrand (1883). Since then, both models remain fundamental to the study of competition with several generalizations between the two. One of these generalizations is the conduct parameter approach which provides a single parameter that characterizes the level of competition between firms (see Corts (1999), d'Aspremont and Dos Santos Ferreira (2009) and Weyl and Fabinger (2013)). This allows for the consideration of many competition structures in a simple framework.[1]

Given the importance of comparing outcomes across a variety of competition structures in traditional markets, it is natural to consider such a comparison for platforms in two-sided markets. While two-sided markets have received considerable attention (Rochet and Tirole (2003), Caillaud and Jullien (2003), Armstrong (2006), Hagiu (2006) and Weyl (2010)), a direct comparison across different types of competition structures remains largely absent in the platform literature. This is because a substantial amount of the literature considers horizontally differentiated platforms, a model pioneered by Armstrong (2006), where two differentiated platforms compete in prices and market structure is effectively fixed due to tractability concerns.

Given the extent to which competition structures vary across platform industries, the consideration of alternative forms of platform conduct is important. In terms of Cournot competition amongst platforms, the studies are few. Gabszewicz and Wauthy (2014) use Cournot competition to consider vertical differentiation between platforms where agents single-home. White and Weyl (2016) show that the Cournot game can be interpreted as a special case of an insulated tariff game between platforms. However, both models consider competition between two platforms and neither investigates how changes to platform market structure impact equilibrium pricing and welfare. One paper that considers changes to

[1] For example, the conduct parameter approach can describe symmetric Cournot or Bertrand competition of either the homogeneous or differentiated type.

platform market structure is Correia-da Silva et al. (2019). They consider a model of Cournot competition between more than two platforms and they determine the cases for which a platform merger harms consumers and sellers. Unfortunately, their model assumes that all agents single-home on both sides of the market. Instead, we allow for any homing allocation to better understand how both the number of platforms and agent homing decisions impacts platform conduct on each side of the market.

To better understand the impact of platform mergers, several studies consider platform competition between more than two platforms. Indeed, Gautier and Lamesch (2021) highlight the importance of studying platform mergers by categorizing how Google, Amazon, Facebook, Amazon and Microsoft have acquired (as horizontal or vertical mergers) more than 175 companies from 2015-2017. Reisinger et al. (2009), Alexandrov et al. (2011), Anderson et al. (2012) and Anderson et al. (2019) consider N platforms using a Salop circle approach; however, these models consider media or search platforms where indirect network externalities do not exist between the two sides of the market. In contrast, Baranes et al. (2019) consider how four platforms on the Salop circle compete, or merge, when network externalities are present on both sides.

In a different vein, Sato (2021) uses an aggregative-games approach to study a merger of multi-product firms in the presence of direct and indirect network externalities, and finds that in the contexts of two-sided markets, the relative market size matters in evaluation of a merger. Tan and Zhou (2021) also study the effects of platform entry (expressed as an increase in the number of symmetric platforms) on welfare. In a similary manner, platform asymmetry is allowed in Belleflamme et al. (2020). However, these studies assume all agents single-home. Lastly, Liu et al. (2019) allow multi-homing on the consumer side in two-sided markets and study the effects of platform competition on the prices on the consumer side as well as on the seller side, assuming symmetric platforms and single-homing firms. They show that results are crucially affected by whether consumers multi-home or not (conditional on all firms single-homing).[2]

One model that considers platform competition across homing allocations is that of Jeitschko and Tremblay (2020). They consider price competition between homogeneous platforms in a two-sided market where every agent, on each side of the market, makes endogenous homing decisions. The main focus of Jeitschko and Tremblay's (2020) study is endogenous homing decisions and the resulting equilibrium allocations that arise when platforms compete in prices.[3] In terms of pricing, they find that the Bertrand Paradox occurs with platforms setting prices on each side of the market equal to the corresponding marginal costs. However, they only focus on the extreme case of homogeneous price competition between platforms and they do not consider Cournot competition or a conduct parameter approach to platform competition.[4]

We generalize the model of Jeitschko and Tremblay (2020) to propose a new framework to study general implications of platform competition and homing decisions. This allow us to make comparisons across platform conduct for any feasible homing allocation.[5] For the competitive bottleneck allocation (the commonly analyzed allocation where consumers single-home and sellers multi-home) with Cournot competition, we find that competition reduces the markup in the consumer price as well as the markdown in the seller price but

[2]Similar to platform mergers, Lefouili and Pinho (2020) consider platform collusion in a duopoly setting; however, they only consider allocations where each side fully single-homes or fully multi-homes so that mixed-homing allocations and comparative statics on homing are not considered.

[3]For example, they show that the competitive bottleneck allocation where all consumers single-home and all firms multi-home is always an equilibrium. In addition, they show that mixed-homing allocations where single-homing and multi-homing consumers as well as single-homing and multi-homing firms can also exist in equilibrium.

[4]Another paper that allows for mixed-homing on both sides of the market is Bakos and Halaburda (2020) who study platform competition à la Hotelling (1929) (see, e.g., Doganoglu and Wright (2006), Bryan and Gans (2019), and Belleflamme and Peitz (2019) for additional studies on multi-homing). They show that the effects of cross-subsidization are diminished when multi-homing takes place on both sides of the market. This is because when multi-homing is allowed, aggressive pricing on one side (i.e., below-marginal-cost pricing, giving subsidies to agents on this side) to lure more agents on the other side through cross-subsidization has a limited role. Thus, aggressive pricing on one side does not pay to increase profits on the other side.

[5]Multi-homing on both sides was not considered in the seminal studies of platform competition such as Rochet and Tirole (2003) and Armstrong (2006). Typically, it is assumed that all agents on both sides single-home or that some agents may multi-home only on one side (the competitive bottleneck). This was justified in the following manner: If all agents on one side multi-home, then any agent on the other side would not gain by multi-homing.

the markup to sellers and the markdown to consumers are consistent with the monopoly platform pricing strategy. In the limit, this implies that greater platform competition results in consumers being subsidized while sellers face markups. This pricing result coincides with optimal pricing in competitive bottleneck models where the single-homing side of the market faces a more competitive price than the multi-homing side (see Belleflamme and Peitz (2019) for details).

This phenomenon, pricing distortions that depend on homing decisions, generalizes to any feasible homing allocation. In particular, we find that a greater proportion of single-homing consumers or sellers increases the extent to which competition distorts markups and markdowns toward zero. This implies that with mixed-homing on both sides of the market, greater platform competition results in both platform prices converging to their respective marginal costs (a result that is consistent with the traditional Cournot model). Comparing the mixed-homing and competitive bottleneck results we see that the assumption of an entire side multi-homing generates a discontinuity in the optimal pricing strategy. This suggests that caution should be taken we considering such an assumption.

To gain a better understanding of how platform competition and agent homing decisions impact the extent of platform monopolization and cross-subsidization, we generalize our model of Cournot platform competition to a model that utilizes the conduct parameter approach to platforms. To the best of our knowledge, we are the first to attempt such an exercise. We show that both platform conduct and agent homing allocations generate side specific conduct parameters, and we find that greater single-homing on a particular side decreases platform monopolization over that side while greater platform market power increases monopolization over that side. We also show that, similar to the traditional market, the side specific conduct parameters correlate to elasticity-adjusted Lerner indices and this highlights the tractability of the conduct parameter approach to platform competition.

Finally, by investigating the welfare effects from changes in the number of competing platforms we find that the similarities end between traditional and two-sided Cournot markets. More specifically, we find that welfare is decreasing in the number of platforms across a variety of allocations. This result is largely driven by platforms that shrink in size (and thus their network effect surpluses), regardless of the underlying homing allocations, as the number of competing platforms increases. Thus, even for those allocations where total participation increases, greater platform competition can still be detrimental to welfare under Cournot competing platforms.

2 A Refresher on Traditional Markets

Consider Cournot competition in a traditional market. Suppose that there are N competing firms, denoted by $X = 1, 2, ..., N$, that sell homogeneous goods. Each firm chooses their output q^X for $X = 1, 2, ..., N$. Inverse demand for the product is given by: $p = p(Q)$, where p is the price, $Q = \sum q^X$ is the total quantity, $p(Q)$ is the downward sloping inverse demand function, and $\epsilon = \frac{\partial Q}{\partial p} \cdot \frac{p}{Q} < 0$ is the price elasticity of demand. Each firm has profit given by $\pi^X = [p(Q) - c] \cdot q^X$, where c denotes the marginal cost. Maximizing profit with respect to firm X's output yields a first-order condition given by $\frac{\partial \pi^X}{\partial q^X} = p'(Q) \cdot q^X + [p(Q) - c] = 0$. Symmetry implies that $q^X = q = \frac{Q}{N}$ for all $X = 1, 2, ..., N$, and so we have

$$\frac{1}{N} = \frac{p-c}{p} \cdot (-\epsilon) \text{ or } p = c\ + \frac{1}{N} \cdot \frac{p}{-\epsilon}. \tag{1}$$

The conduct parameter approach offers a more general model of symmetric imperfect competition where, instead of explicitly defining interaction between firms, we require that the Lerner index be equal to the elasticity normalized conduct parameter:

$$L := \frac{p-c}{p} = \frac{\theta}{-\epsilon} \in [0, 1] \text{ or } p = c\ + \theta \cdot \frac{p}{-\epsilon}. \tag{2}$$

Notice that conduct (i) equals zero under perfect competition so that $p = c$, (ii) equals one under monopoly or full collusion, and (iii) equals $\frac{1}{N}$ when N firms compete in output.

3 A Two-Sided Market

In a two-sided market, two groups of agents (consumers and sellers)[6] benefit from indirect network externalities that exist across the two groups. For example, gamers benefit from greater video game availability and game developers benefit from greater console ownership (a.k.a. potential customers). Let the consumer side be denoted as Side 1 and the seller side as Side 2, and suppose that there are N (symmetric) competing platforms.

3.1 Consumers and Sellers

First consider the consumer side of the market. Consumers benefit from interaction with the seller side of the market, and some consumers benefit more from sellers than others. For example, some consumers benefit more from apps (teens) than others (parents).[7] Let consumer types be denoted by $\tau_1 \in [0, \overline{\tau}_1]$ and be distributed uniformly, and suppose that consumers with lower τ_1 types have greater network benefits than consumers with higher τ_1 types. Specifically, the utility from joining platform X, $X = 1, 2, ..., N$, exclusively for a consumer of type τ_1 is given by:

$$u_1^X(\tau_1) = \alpha_1(\tau_1) \cdot q_2^X - p_1^X, \tag{3}$$

where q_2^X denotes the number of sellers on platform X, p_1^X denotes the consumer price of platform X, and $\alpha_1(\cdot)$ is a decreasing function that denotes the network externality received by consumers.[8]

Now consider the seller side of the market. Sellers benefit from greater consumer participation on a platform. Like consumers, sellers are heterogeneous in their network gains. That is, we allow some sellers to be more successful than others (e.g., game developers differ

[6] Depending on the industry, a seller can be a content provider, game developer, book publisher, etc.

[7] Similar arguments can be made for consumer heterogeneity in game content for the gaming industry, in video content for the video streaming industry, etc.

[8] Furthermore, suppose that $\alpha_1(\cdot)$ is twice continuously differentiable.

in terms of sales and profitability). Let seller types be denoted by $\tau_2 \in [0, \overline{\tau}_2]$ and let sellers be distributed uniformly. The utility from joining platform X, $X = 1, 2, ..., N$, exclusively for a seller of type τ_2 is given by:

$$u_2^X(\tau_2) = \alpha_2(\tau_2) \cdot q_1^X - p_2^X, \tag{4}$$

where q_1^X denotes the number of consumers on platform X, p_2^X denotes the seller price of platform X, and $\alpha_2(\cdot)$ is a decreasing function that denotes the network externality received by sellers.[9]

Consumers and sellers might have the option to multi-home (join more than one platform). There are several issues that arise when modeling multi-homing. The most important, and most difficult, is how to deal with duplicated interactions between the same buyer and the same seller across multiple platforms.[10] For example, if a video game is available on the Xbox, then a multi-homing gamer might not benefit if that game is also available on the Playstation but the gamer may benefit if the same game is on the Nintendo Switch because the Switch is portable. Similarly, some advertisers might benefit from the same consumer seeing their ad on Facebook and Instagram while other advertisers may not. To allow for multi-homing, we assume that multi-homing agents do not benefit from duplication so that an agent on Side i that multi-homes on a set of platforms, $\mathcal{X} \subset \{1, 2, ..., N\}$, earns utility

$$u_i^{\mathcal{X}}(\tau_i) = \alpha_i(\tau_i) \cdot \tau_j^{\mathcal{X}} - \sum_{X \in \mathcal{X}} p_i^X,$$

where $\tau_j^{\mathcal{X}}$ denotes the unique number of agents on Side j that participate across the $\mathcal{X}$ platforms.[11] In addition to this multi-homing utility specification, we detail further structure on agent homing and equilibrium selection in Subsections 3.3 and 3.4.

[9]Furthermore, suppose that $\alpha_2(\cdot)$ is twice continuously differentiable.

[10]Bakos and Halaburda (2020) arguably do the best job of investigating this issue.

[11]This is known as "no double counting" in Bakos and Halaburda (2020) and is the most common assumption in the limited literature on this issue.

3.2 The Platforms

Each platform X, $X = 1, 2, ..., N$, obtains profit from both sides of the market so that platform profits are expressed as:

$$\Pi^X = [p_1^X - c_1] \cdot q_1^X + [p_2^X - c_2] \cdot q_2^X, \tag{5}$$

where $c_i \geq 0$ is the marginal cost to the platform for an additional Side i agent.

3.3 Platform Strategies and Market Clearing Conditions

Much like in traditional Cournot competition, we assume that each platform commits to output choices for both sides of the market, taking the output choices of their competitors as given. More specifically, platform X chooses q_1^X and q_2^X to maximize profits, taking the q_1^Y and q_2^Y for $Y \neq X$ as given. To utilize this approach, we require market clearing conditions that satisfy demands on each of the two sides. Thus we impose the following conditions:

1. Given the quantity selections by all platforms, the Side 1 and Side 2 prices for each platform are such that the marginal participating agent on each side of the market earns zero utility from every platform.

2. Given the quantity selections and market clearing prices, each unit of output on each side of the market is distributed to single-homing and multi-homing agents based on the following rationing rule: (i) given that the lowest output on Side i is denoted by $q_i^L = \min\{q_i^1, ..., q_i^N\}$, an amount of $(1 - \phi_i)q_i^L$ is allocated to multi-homing agents for all the platforms where $\phi_i \in [0, 1]$ denotes the extent of single-homing activity, and (ii) the remaining output is allocated randomly to single-homing agents.

It is helpful to breakdown each of these conditions. For Condition 1, note that the "marginal participating agent" will always be an agent that single-homes. That is, from Equations (3) and (4) and since $u_i^{\mathcal{X}} \leq \sum_{X \in \mathcal{X}} u_i^X$ for $\mathcal{X} \subset \{1, 2, ..., N\}$ and for $i = 1, 2$, we see that for every τ_i' such that $u_i^{\mathcal{X}}(\tau_i') \geq 0$ we have that $u_i^X(\tau_i') \geq 0$ for some $X \in \mathcal{X}$. This implies the marginal participating agents will be given by setting Equations (3) and (4) equal to zero.[12] More specifically, let τ_i^* be the marginal participating agent on Side i, where τ_i^* is given by

$$u_i^X(\tau_i^*) = 0 \text{ or } p_i^X = \alpha_i(\tau_i^*) \cdot q_j^X, \tag{6}$$

for all $X = 1, ..., N$. In this case, the $\tau_i \in [0, \tau_i^*]$ are interested in participating in the market and the τ_i^* agent earns zero utility from every platform available. Lastly, Condition 1 requires that the entire market output on Side i, denoted by $Q_i = \sum_{X=1}^{N} q_i^X$, be sold to the $\tau_i \in [0, \tau_i^*]$ participating agents. Naturally, homing allocations will play an important role in establishing the relationship between τ_i^* and Q_i (e.g., if all agents on Side i single-home, then $\tau_i^* = Q_i$), and we derive this relationship explicitly in the following subsection.

Given the market clearing prices, Condition 2 implies that outputs are first distributed to some fraction of multi-homing agents and then distributed randomly to single-homing agents. An assumption on output allocation, not uniform randomization specifically, is necessary since some platforms might be more desirable than others. For example, if there are two platforms (A and B) and Platform A has lots of games relative to Platform B ($q_2^A > q_2^B$), then market clearing prices will imply that $p_1^A = \alpha_1(\tau_1^*)q_2^A > \alpha_1(\tau_1^*)q_2^B = p_2^B$; in this case, the marginal consumer earns zero utility from either platform but all other participating consumers, the $\tau_1 \in [0, \tau_1^*)$, prefer Platform A over Platform B. Thus, Condition 2 offers one of many possible output distribution mechanisms that ensures that the output clears.[13] Our formulation is motivated by the fact that many desirable platforms face output distributional problems in practice. For example, the Nintendo Switch, Playstation 5, and Xbox Series X have had output shortages due to high demand (corresponding to Platform A in the discussion above).[14]

[12] We assume that agents outside option from not joining a platform is valued at zero.

[13] A Pareto allocation where the low τ_1s get Platform A (which maximizes consumer surplus) is another output distribution specification that clears the market. In addition, such a Pareto allocation of outputs would eliminate a black market that might be brought on by randomization

[14] Similarly, iPhone shortages have left some consumers with inferior cell phones in the past.

Conditions 1 and 2 highlight the many ways in which the "homogeneous" Cournot model for platform markets is much more complicated than for traditional ones. In particular, the two-sided nature of platform competition can create differentiation between platforms (e.g., platforms that differ in size also differ in attractiveness to agents). As a result, we employ these market clearing conditions to consider platform competition that captures the spirit of traditional Cournot competition.

Finally note that we assume that the homing distribution (ϕ_1, ϕ_2) is known to the platforms prior to choosing their output. While this assumption may appear strong, there are many industries where platforms might know the distribution of agent homing prior to production. For example, when gaming platforms like Microsoft, Nintendo, and Sony produce a new video game console, they likely have an idea of the homing distribution based on the homing distribution for the previous console generation.[15]

3.4 Agent Homing

A common feature in the literature on platform competition is to exogenously assume particular homing allocations between the two sides of the market.[16] The most prevalent is the competitive bottleneck allocation where all consumers are assumed to single-home and all sellers are assumed to multi-home. To better understand how different homing allocations impact our main results, we consider the scenario where the amount of single-homing on each side of the market is defined by a parameter, ϕ_i, for $i = 1, 2$. This allows for extreme allocations such as the competitive bottleneck allocation as well as more general allocations of mixed-homing that remain feasible.

Specifically, suppose that there exist critical values $\hat{\tau}_i \in [0, \tau_i^*)$ for Side $i = 1, 2$ such that only the agents with type $\tau_i \leq \hat{\tau}_i$ are willing to multi-home. As motivation for the direction of the $\tau_i \leq \hat{\tau}_i$ inequality, note that if a multi-homing agent on Side i can choose to join all

[15]Similarly, smartphone platforms and video streaming platforms likely know the amount of consumer and content multi-homing.

[16]See Belleflamme and Peitz (2019), Bakos and Halaburda (2020), Jeitschko and Tremblay (2020) and Adachi et al. (2021) for exceptions.

the platforms, then they obtain the utility

$$U_i^M(\tau_i) = \alpha_i(\tau_i)\tau_j^* - \sum_{X=1}^{N} p_i^X = \alpha_i(\tau_i)\tau_j^* - \alpha_i(\tau_i^*)Q_j.$$

Similarly, if the agent single-homes, then the expected utility is given by

$$U_i^S(\tau_i) = \alpha_i(\tau_i)q_j^e - p_i^e = \alpha_i(\tau_i)q_j^e - \alpha_i(\tau_i^*)q_j^e,$$

where p_i^e is the expected membership fee, and $q_j^e \in [\min_{X=1,...,N} q_j^X, \max_{X=1,...,N} q_j^X]$ is the expected number of Side j agents a single-homing Side i agent interact with, which depends on the rationing rule draw. Altogether, this implies that there exists a unique value $\hat{\tau}_i$ such that Side i agents with $\tau_i \leq \hat{\tau}_i$ are willing to multi-home.

To ensure that our homing allocations follow those described by Condition 2, we assume that the rationing rule that assigns multi-homing agents to platforms is Pareto efficient so that the agents with the lowest type τ_i are assigned as the multi-homing agents (this is the Pareto efficient rationing rule since agents with the lowest type earn the largest network benefits); in this case, the $\tau_i \in [0, \tau_i^M]$ multi-home, where $\tau_i^M = (1-\phi_i)q_i^L$ with $q_i^L = \min\{q_i^1, ..., q_i^N\}$.[17] Naturally, the micro-foundations within our multi-homing rationing rule are valid so long as $\hat{\tau}_i > (1-\phi_i)q_i^L$ occurs in equilibrium. We acknowledge that these assumptions on the ability to multi-home and the resulting allocation rule have a number of ad-hoc features that deserve criticism. While this approach is far from ideal, it still generalizes from the common approach of specifying a particular homing allocation for the entirety of an analysis (e.g., the competitive bottleneck allocation) and it allows for comparative statics on homing allocations to a certain extent — as we discuss in the next section (Condition (9)), not all homing allocations will be feasible in equilibrium under Cournot platform

[17]Our motivation for using the Pareto efficient rationing rule for multi-homing is to make our welfare computations more tractable in Section 6. Another possible multi-homing rationing rule is where the $(1-\phi_i)q_i^L$ assignments are given randomly to the agents $\tau_i \leq \hat{\tau}_i$ that are willing to multi-home.

competition.[18]

Altogether, we have that $\phi_i \in [0,1]$ denotes the fraction of output that are sold to single-homing agents on Side i, as defined by the platform with the lowest output on Side i ($q_i^L = \min\{q_i^1, ..., q_i^N\}$), so that a $(1-\phi_i)q_i^L$ amount of output for each platform is allocated to agents that choose to join all the platforms, and the remaining $q_i^X - (1-\phi_i)q_i^L$ portion is allocated to single-homing agents. This implies that the number of unique agents who multi-home is given by $(1-\phi)q_i^L$, and that the number of unique agents on Side i that join at least one platform is given by

$$\tau_i^* = \underbrace{(1-\phi_i)q_i^L}_{\text{Multi-homing Agents}} + \underbrace{\sum_{X=1}^{N}[q_i^X - (1-\phi_i)q_i^L]}_{\text{Single-homing Agents}}, \tag{7}$$

Note that if outputs are equal across platforms, then (i) a ϕ_i portion of each platform's output goes toward single-homers on Side i and the $(1-\phi_i)$ portion goes toward multi-homers on Side i and (ii) we see that $(\phi_1, \phi_2) = (1, 0)$ corresponds to the competitive bottleneck allocation.

4 The Two-Sided Market Cournot Equilibrium

Combined, Equations (6) and (7) effectively capture the aggregate inverse demands for Sides 1 and 2 that the platforms use to maximize profit. Given the extent of single-homing, (ϕ_1, ϕ_2), the Cournot platform equilibrium prices are given by:

[18] In many ways, our ad hoc assumptions stem from the issue that we previously discuss in Subsection 3.1: How to deal with duplicated interactions between consumers and sellers that multi-home across platforms? From a micro-founded perspective, the most desirable approach would be to develop a model with duplication parameters on each side of the market that endogenously determine the (ϕ_1, ϕ_2) parameters that we exogenously define. With such an approach one would expect that larger (smaller) benefits from duplication would make multi-homing more (less) attractive. In this case a duplication parameter on Side i that is close to one would correspond to a ϕ_i close to one. Unfortunately, existing studies by Bakos and Halaburda (2020) and Jeitschko and Tremblay (2020) highlight how such an approach will produce multiple equilibrium allocations since the homing decisions on one side of the market still impact the desirability of multi-homing for the other side. For this reason, we move forward under the more exogenous structure of homing allocations that we outline above.

Proposition 1. *If N platforms compete in output, then equilibrium prices in the symmetric equilibrium are given by:*

$$p_i^G = c_i + \frac{1}{1+(N-1)\phi_i} \cdot \frac{p_i^G}{-\epsilon_i} - \frac{1}{1+(N-1)\phi_j} \cdot \alpha_j(\tau_j^G)\tau_j^G, \tag{8}$$

where the superscript G denotes the equilibrium across general allocations, (ϕ_1, ϕ_2), and $\epsilon_i = \frac{\partial Q_i}{\partial p_i} \cdot \frac{p_i}{Q_i} < 0$ is the elasticity of demand on Side i.

First notice that each price equals the sum of three terms: (i) marginal cost, (ii) a markup term, (iii) a markdown term that incorporates the network benefit to the other side. Second, note that features from traditional Cournot pricing (Section 2) and monopoly platform pricing are present in Equation (8). Specifically, traditional Cournot competition results in markup distortions similar to those above; although now we see that homing allocations impact this distortion in addition to the number of platforms. In addition, the markdown term that is present in monopoly platform pricing now faces a similar distortion of its own (one that depends on the number of competing platforms and homing allocations). Altogether we see that greater platform competition reduces both markup and markdown terms so that in the limit, and just like with traditional firms competing *à* la Cournot, the platform pricing strategy approaches marginal cost pricing so long as at least some agents single-home on *both* sides of the market: $\phi_1, \phi_2 > 0$. We present this result more formally as a corollary:

Corollary 1. *If at least some agents single-home on each side of the market ($\phi_1, \phi_2 > 0$), then platform prices approach marginal costs as the number of platforms increase: $N \to \infty$ implies that $p_i^G \to c_i$.*

In many ways, this result highlights how multi-homing effectively gives platforms market power. In the extreme, if all agents multi-home so that $\phi_1 = \phi_2 = 0$, then each platform pursues the monopoly platform pricing strategy:[19]

[19]It is important to note that such an extreme result is largely driven by our simplifying assumption

Corollary 2. *If all agents multi-home on both sides of the market ($\phi_1 = \phi_2 = 0$), then every platform offers monopoly platform prices:* $p_i^G = c_i + \frac{p_i^G}{-\epsilon_i} - \alpha_j(\tau_j^G)\tau_j^G$.

An important caveat to the results above (and all of our results that follow), is that the equilibrium prices (p_1^G, p_2^G) and participation on the two sides (τ_1^G, τ_2^G) should satisfy the condition that all the multi-homing agents have an incentive to do so. That is, whenever $\phi_i < 1$, we must have

$$U_i^M((1-\phi_i)q_i^G) = \alpha_i((1-\phi_i)q_i^G)\tau_j^G - Np_i^G \geq \alpha[(1-\phi_i)q_i^G]q_j^G - p_i^G = u_i^S((1-\phi_i)q_i^G), \quad (9)$$

where $\tau_i^M = (1-\phi_i)q_i^G$ represents that last agent type to multi-home. Note that if there exists a $\phi_i' \in (0,1)$ such that Condition (9) holds with equality, then Condition (9) fails for all $\phi_i \in [0, \phi_i']$. This implies that it is possible that allocations with substantial multi-homing (with both ϕ_1 and ϕ_2 close to zero) will not exist. To this extent, our micro-foundations on homing imply that not all mixed-homing allocations — in particular, those with greater multi-homing on both sides of the market — may be feasible in a symmetric Cournot platform equilibrium.[20]

Another important takeaway from Corollaries 1 and 2 is that pricing behavior is discontinuous at the point where an entire side multi-homes. This is particularly important for models that assume that an entire side multi-homes (as is commonly done in the competitive bottleneck model). Given that an entire side always includes at least some single-homers in practice (e.g., even in the smartphone market where nearly all app developers are on both iOS and Android, some only develop for one of the two platforms), our findings suggest that

that a multi-homing agent will join every platform. At the same time, however, this result highlights how multi-homing improves platform market power in the most extreme scenario. In addition, a more mild approach still offers the same result: greater multi-homing (single-homing) results in weaker (stronger) platform competition. To see this, note that comparative statics on ϕ_i reveal that greater single-homing on Side i will (a) reduce the Side i markup term and (b) reduce the Side j markdown term. Thus, greater single-homing on Side i reduces platform competition through both the markup and investment channels.

[20]To see Condition (9) impacting equilibrium existence in practice, note that in Figure 1 and in the Proof of Proposition (6) we show that there are linear $\alpha_i(\cdot)$ such that some allocations do not occur as Cournot platform equilibrium.

caution should be taken when modeling such allocations. To see this discontinuity explicitly for the competitive bottleneck allocation and to observe how differences in homing allocations impact optimal pricing strategies, consider the case where all consumers single-home ($\phi_1 = 1$) and all sellers multi-home ($\phi_2 = 0$):

Corollary 3 (The Competitive Bottleneck)**.** *The Cournot platform pricing strategies under the competitive bottleneck allocation are given by:*

$$p_1^{CB} = c_1 + \frac{1}{N} \cdot \frac{p_1^{CB}}{-\epsilon_1} - \alpha_2(\tau_2^{CB})\tau_2^{CB}, \tag{10}$$

$$p_2^{CB} = c_2 + \frac{p_2^{CB}}{-\epsilon_2} - \frac{1}{N} \cdot \alpha_1(\tau_1^{CB})\tau_1^{CB}. \tag{11}$$

where superscript CB denotes the competitive bottleneck equilibrium.

In this case we see that the asymmetric homing decisions distort prices across the two sides. On the consumer side where single-homing occurs, greater competition reduces the markup term but does not impact the markdown term. In contrast, on the seller side where multi-homing occurs, greater competition reduces the markdown term but does not impact the markup term. Thus, as competition increases, prices diverge so that the consumer price includes no markup and the seller price includes no markdown.[21] In this case, Cournot competition results in a straddle pricing equilibrium (where $p_1 > c_1$ and $p_2 < c_2$) which Jeitschko and Tremblay (2020) show to be the only pricing strategy where platforms that compete in price a la Bertrand can earn profit. Thus, even in the limit, N Cournot competing platforms might earn profit under the competitive bottleneck allocation.

[21]Such pricing distortions across homing decisions is consistent with the previous literature where platforms compete for single-homers and have market power over the multi-homers (e.g., Rochet and Tirole (2003), Armstrong (2006) and Hagiu (2006)).

5 The Conduct Parameter Approach to Platforms

As shown in Section 2, there is an isomorphic relationship between the conduct parameter approach and Cournot competition in traditional markets. More explicitly, we see that the traditional market equilibrium prices given by Equations (1) and (2) coincide when $\theta = \frac{1}{N}$. Given that the conduct parameter captures the extent of monopolization, our results from Section 4 suggest that homing decisions will impact conduct across the two sides of the market. To investigate the relationship between conduct and homing, we maintain consistency with the traditional market and suppose that platform level conduct, θ, is given by $\theta = \frac{1}{N}$. Thus, our results from Proposition 1 imply that

Corollary 4 (Conduct Parameter Prices)**.** *If $\theta = \frac{1}{N}$ captures the platform conduct parameter, then platform pricing strategies are characterized by*

$$p_i^G = c_i + \theta_i \cdot \frac{p_i^G}{-\epsilon_i} - \theta_j \cdot \alpha_j(\tau_j^G)\tau_j^G, \tag{12}$$

where $i, j = 1, 2$, $j \neq i$, and $\theta_i := \frac{1}{1+\left(\frac{1}{\theta}-1\right)\phi_i} \in [0, 1]$ represents the Side i specific conduct parameter.

By formulating platform pricing strategies using the conduct parameter approach, we see that side specific conduct parameters emerge: $\theta_i = \frac{1}{1+\left(\frac{1}{\theta}-1\right)\phi_i}$ for Side $i = 1, 2$. In particular, note that $\theta_i \in [0, 1]$ and it depends on both Side i's homing allocation (ϕ_i) and the underlying platform conduct (θ). Furthermore, a $\theta_i \to 0$ matches the case where platforms have no market power on Side i, since the markup term goes to zero, and $\theta_i \to 1$ matches the case where platforms have monopoly power on Side i, since the markup term approaches the monopoly markup. To see how homing and platform conduct impact side specific conduct, note that $\frac{\partial \theta_i}{\partial \phi_i} < 0$ implies that greater single-homing on Side i increases Side i market power and $\frac{\partial \theta_i}{\partial \theta} > 0$ implies that greater platform market power (through fewer competing platforms) increase Side i market power. Both of these effects align with our intuition.[22]

The side specific conduct parameters that naturally arise in this setting also allow us to better compare the approach across the two models (traditional and two-sided markets). That is, by comparing Equations (2) and (12) we see that the conduct approach to platform competition offers a natural extension to traditional markets by including the across the market markdown term that is normalized by the corresponding side specific parameter: $p^* = c + \theta \cdot \frac{p^*}{-\epsilon}$ for traditional markets corresponds to $p_i^G = c_i + \theta_i \cdot \frac{p_i^G}{-\epsilon_i} - \theta_j \cdot \alpha_j(\tau_j^G)\tau_j^G$ for two-sided markets, where $\theta_i = \frac{1}{1+\left(\frac{1}{\theta}-1\right)\phi_i} \in [0, 1]$ represents the Side i specific conduct parameter. In many ways, this highlights (1) the similarities between traditional and two-sided market pricing and (2) the consistency of the monopoly platform pricing strategy in generalizing to other forms of oligopoly platform pricing.

One of the main purposes of the conduct parameter in traditional markets is its connection to the Lerner index: $L \equiv \frac{p-c}{p} = \frac{\theta}{-\epsilon}$ as shown in Equation (2). Naturally, we would like to derive a similar relationship for the two-sided market case. To do so, note that Armstrong (2006) and Weyl (2010) derive the modified version of the Lerner index for two-sided markets with a monopoly platform. In the context of our model, they show that a monopoly platform sets prices so that $L_i^{AW} \equiv \frac{p_i - c_i + \alpha_j(\tau_j)\tau_j}{p_i} = \frac{1}{-\epsilon_i}$ for $i, j = 1, 2$ and $i \neq j$. This ensures the traditional Lerner relationship, where $L = \frac{1}{-\epsilon}$, for a monopoly platform. Adapting our pricing results from Corollary 4, we generalize the Lerner indices proposed by Armstrong (2006) and Weyl (2010) to include platform competition so that

$$L_i^{2SM} := \frac{p_i - c_i + \theta_j \cdot \alpha_j(\tau_j)\tau_j}{p_i} = \frac{\theta_i}{-\epsilon_i}. \tag{13}$$

Notice that Equation (13) bears resemblance to both the traditional Lerner index formula with conduct $\left(L \equiv \frac{p-c}{p} = \frac{\theta}{-\epsilon}\right)$ and the two-sided market Lerner index formula for a monopoly

[22] As a followup on Corollary 3 (the competitive bottleneck allocation), note that $\phi_1 = 1$ and $\phi_2 = 0$ imply that $\theta_1 = \theta$ and $\theta_2 = 0$. In this case, competitive bottleneck prices reduce to $p_1^{CB} = c_1 + \theta \cdot \frac{p_1^{CB}}{-\epsilon_1} - \alpha_2(\tau_2^{CB})\tau_2^{CB}$ and $p_2^{CB} = c_2 + \frac{p_2^{CB}}{-\epsilon_2} - \theta \cdot \alpha_1(\tau_1^{CB})\tau_1^{CB}$ which clearly mirror Equations (10) and (11).

platform derived by Armstrong (2006) and Weyl (2010) $\left(L_i^{AW} \equiv \frac{p_i - c_i + \alpha_j(\tau_j)\tau_j}{p_i} = \frac{1}{-\epsilon_i}\right)$. Comparing to the traditional market formula, we see that the two-sided market Lerner definition results in the same elasticity formula as the elasticity formula for the traditional market: $\frac{\theta_i}{-\epsilon_i}$ mirrors $\frac{\theta}{-\epsilon}$. At the same time, the two-sided market Lerner definition includes the across network markdown term as in Armstrong (2006) and Weyl (2010), but with the inclusion of the side specific conduct parameter that impacts the network markdown term. Thus, we generalize the existing Lerner indices to consider two-sided markets with platform competition. Furthermore, by allowing for platform competition, we see that the two-sided market Lerner index formula depends on the side specific conduct parameters (which depend on the underlying platform conduct and the homing allocation). This implies that both platform competition and homing allocations are important considerations when formulating Lerner indices in platform industries.

6 Merger and Welfare Implications

The approach that we have selected enables us to consider comparative statics relating to platform mergers. In particular, we are interested in how changes in the number of platforms, N, impacts equilibrium pricing and welfare. It has been well established that a welfare tradeoff occurs with greater platform competition: more platforms result in (1) lowers prices which improves welfare and (2) reduces an individual platform's size which decreases the accumulation of network effects and harms welfare. To definitively sign comparative statics and to make welfare comparisons, we require closed form solutions. Thus we make simplify assumptions on functional forms so that network externalities are linear in agent type (so that $\alpha_i''(\cdot) = 0$) and we also assume marginal costs are zero ($c_1 = c_2 = 0$). These simplifying assumptions are not uncommon in the literature (as in for example, the base models of Katz and Shapiro (1985), Cabral (2011), Jullien (2011), Adachi and Tremblay (2020), Bakos and Halaburda (2020), and Halaburda et al. (2020)).

Based on our results so far, it is natural to think that agent homing allocations will impact the welfare tradeoff. As result, we start by considering the simple case of platform competition under the competitive bottleneck allocation and then we turn to the case of general homing allocations.

6.1 The Competitive Bottleneck Allocation

For the competitive bottleneck allocation, we have the following result:

Proposition 2. *In the competitive bottleneck equilibrium, equilibrium consumer participation increases and seller participation decreases as the number of platforms increases:* $\frac{\partial \tau_1^{CB}}{\partial N} > 0$ *and* $\frac{\partial \tau_2^{CB}}{\partial N} < 0$*. In addition, the price to consumers decreases while the price to sellers is ambiguous with an increase in platforms:* $\frac{\partial p_1^{CB}}{\partial N} < 0$ *and* $\frac{\partial p_2^{CB}}{\partial N} > 0$*.*

Our pricing results from Corollary 3 show that platforms have an incentive to price more competitively to consumers while extracting rents from sellers under the competitive bottleneck allocation. We see that this pricing incentive carries through to increases in platform competition: greater platform competition decreases the consumer price and increases the seller price. Furthermore, we see that participation levels coincide with the pricing effects so that more consumers and fewer sellers participate when platform competition increases.

Given that the changes in participation are negatively correlated across the two sides, we must investigate welfare explicitly to determine how greater platform competition impacts total welfare in two-sided markets under the competitive bottleneck allocation. In the competitive bottleneck, note that every participating consumer interacts with every participating seller and vice versa. Thus, total welfare is given by:

$$W^{CB} = \int_0^{\tau_1^{CB}} \alpha_1(\tau_1)\tau_2^{CB}\, d\tau_1 + \int_0^{\tau_2^{CB}} \alpha_2(\tau_2)\tau_1^{CB}\, d\tau_2. \tag{14}$$

Differentiating total welfare with respect to the number of platforms generates the following result:

Proposition 3. *Under the competitive bottleneck allocation, welfare is decreasing in the number of platforms:* $\frac{\partial W^{CB}}{\partial N} < 0$ *for all* $N \geq 2$*.*

There is an important caveat to the result in Proposition 3 that is worth mentioning.[23] Note that we focus on interior solutions to the platform competition game when considering welfare effects in this section. We have implicitly made this assumption in all our main results and this restriction is made for two important reasons.

First, focusing on interior solutions eases exposition by eliminating discrete changes in the type of equilibrium (interior or corner) upon differentiation. However, this does restrict the network effect magnitudes required to ensure that the interior solution occurs. In particular, the magnitudes of the network effects cannot be too different between the two sides, and we derive these restrictions explicitly in our proofs.[24] Second, it makes sense to focus on welfare when the extensive margins of participation exist on both sides of the market. That is, if N changes, then we want to determine the resulting welfare effects when participation on both sides the market are allowed to increase or decrease. Such a setting implies a focus on interior solutions.

Turning to the result in Proposition 3, many have speculated welfare comparisons across competition structures for the competitive bottleneck allocation. To the best of our knowledge, this is the first *formal* result for how welfare changes with the number of platforms in the competitive bottleneck equilibrium. And, in terms of potential mergers, our results suggest that platform mergers will improve welfare in platform industries where the competitive bottleneck allocation occurs.

While platform mergers under the competitive bottleneck allocation are uncommon in practice, we have observed several industries where the number of platforms has changed due to entry and exit.[25] For example, Google entered the smartphone market (and Microsoft

[23]This caveat also applies to the welfare results derived in Propositions 5 and 6.

[24]If, for example, $\alpha_1(\cdot)$ is large and $\alpha_2(\cdot)$ is very small, then a corner solution occurs where all sellers participate. In this case, no extensive margin exists on the seller side when differentiating with respect to N, unless such a differentiation moves to an interior solution equilibrium.

[25]One potential example of a merge to monopoly is Facebook's acquisition of Instagram in 2012 since

followed suit) that was originally developed by Apple; however, Microsoft began to exit the market in 2016.[26] Our findings from Proposition 3 suggest that these entries would actually harm welfare given that these industries adhere to the competitive bottleneck allocation. The smartphone market is the most likely to fit the competitive bottleneck bill as most consumers own a single smartphone while app providers make their apps available across platforms. This suggests that Microsoft's exit from the smartphone market has improved welfare in the smartphone industry.

6.2 General Homing Allocations

The competitive bottleneck allocation offers an excellent starting point for our welfare analysis as it offers unambiguous comparative statics. However, it is important to consider other feasible homing allocations and determine how robust those results, and their corresponding policy recommendations, actually are. By considering general homing allocations, we also determine the extent for which single- and multi-homing distributions impact our findings; something that is often overlooked in the literature.

Recall that in the general homing setup, ϕ_i captures the extent of single-homing on Side i so that ϕ_i closer to one (zero) corresponds to the case where the majority of Side i agents single-homing (multi-home). This implies that the competitive bottleneck allocation corresponds to the special case where $\phi_1 = 1$ and $\phi_2 = 0$. In terms of prices and participation, we expect that the comparative statics might hinge on agent homing decisions. We find that this is indeed the case:

Proposition 4. *For general homing allocations characterized by $\phi_1, \phi_2 \in [0, 1]$, equilibrium consumer (seller) participation increases in the number of platforms if and only if $2\phi_1 > \phi_2$ ($2\phi_2 > \phi_1$): $\frac{\partial \tau_1^G}{\partial N} > 0$ if and only if $2\phi_1 > \phi_2$ and $\frac{\partial \tau_2^G}{\partial N} > 0$ if and only if $2\phi_2 > \phi_1$. In addition, individual output is decreasing in the number of platforms across all allocations with at least*

Snapchat was in its infancy at the time. However, Instagram was not selling ads at the time and so it is difficult to argue that this was indeed a two-sided platform.

[26]See Microsoft's Exit From Smartphone Business Moves Into High Gear; Bloomberg 2016.

some single-homing: $\frac{\partial q_i^G}{\partial N} < 0$ *for* $i = 1, 2$ *so long as* $\phi_1 > 0$ *or* $\phi_2 > 0$.

First investigating total participation, if the extent of single-homing on Side i is at least half the extent of single-homing on Side j ($\phi_i > 0.5\phi_j$), then greater platform competition results in greater equilibrium participation on Side i. However, if there is too little single-homing (relative to the other side), then participation decreases with greater platform competition. These results echo our results from the competitive bottleneck allocation where greater platform competition increases participation on the single-homing side but decreases participation on the multi-homing side.

One important conclusion from our findings in Proposition 4 is that there is a large mass of homing allocations where participation on *both* sides increases, a result that is not possible when focusing on the competitive bottleneck allocation. This occurs when the $\phi_1, \phi_2 \in [0, 1]$ are such that $2\phi_1 > \phi_2 > 0.5\phi_1$. However, even though the total number of participating agents can increase on both sides of the market, we see that individual platforms get smaller as the number of platforms increases, and this may reducs the amount of network surplus generated in the market which has important implications for welfare.

Total welfare in the general homing case is given as

$$W^G = \sum_{i=1}^{2} \left[\int_0^{(1-\phi_i)q_i^G} \alpha_i(\tau_i)\tau_j^G d\tau_i + \int_{(1-\phi_i)q_i^G}^{\tau_i^G} \alpha_i(\tau_i) q_j^G d\tau_i \right]. \tag{15}$$

The first integral is all surplus captured by multi-homers and the second term captures the surplus from all single-homers.

As an extension of the competitive bottleneck welfare result in Proposition 3, we first consider the case where multi-homing inverses across the two sides so that $\phi_1 = \phi \in [0, 1]$ and $\phi_2 = 1 - \phi$. In this case, the competitive bottleneck is given by $\phi = 1$ so that $\phi_1 = 1$ (all consumers single-home) and $\phi_2 = 0$ (all sellers multi-home). There are many reasons why the inverse homing allocation is worth investigating. First, greater multi-homing on one side generates an incentive to single-home on the other side (and vice a versa). Second, many industries appear to have this inverse relationship (e.g., both the consumer and game developer sides of the video game industry experience single- and multi-homers, but the game developer side has considerably more multi-homing than on the consumer side). As we show in the following proposition, our results from Proposition 3 generalize to the case of the inverse allocation:

Proposition 5. *If homing inverses across sides so that $\phi_1 = 1 - \phi_2$, then welfare is decreasing in the number of platforms: $\frac{\partial W^G}{\partial N} < 0$ for all $N \geq 2$.*

To the extent that a platform industry has multi-homers concentrated on one side of the market (making single-homers concentrated on the other side of the market), this result implies that platform mergers would increase welfare.

Moving to the case of entirely general homing allocations, it appears that fewer platforms is welfare improving even if ϕ_1 and ϕ_2 are not restricted to the case of inverse homing decisions. To see this explicitly, note that Figure 1 depicts the regions of $(\phi_1, \phi_2) \in [0,1] \times [0,1]$ for when a reduction in the number of platforms improves social welfare — importantly, note that the black regions in Figure 1 are where at least one of these four inequalities associated with Condition (9) fails (there are two sides for each N).[27] Without the black regions in Figure 1 from Condition (9) there are allocations close to the origin where n platforms generate more surplus than $n-1$ platforms for all four subgraphs. Hence, the feasibility conditions are an important feature in determining welfare results. We present this more formally with the following result:[28]

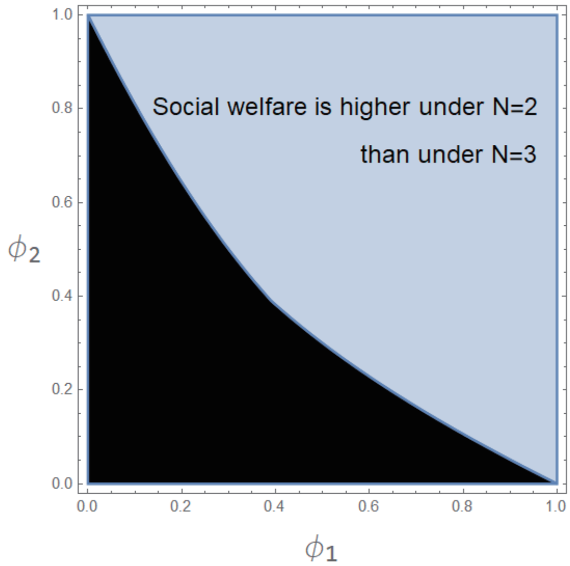
Proposition 6. *If the $\alpha_i(\cdot)$ are linear and symmetric so that $\alpha_1(\tau) = \alpha_2(\tau)$, then $\frac{\partial W^G}{\partial N} < 0$ for all $N \geq 2$ under the (ϕ_1, ϕ_2) that are feasible in the Cournot platform equilibrium.*[29]

[27]Note that the 45 degree line from (0,1) to (1,0) remains outside the black area in Figure 1 so that inverse homing allocations are feasible.

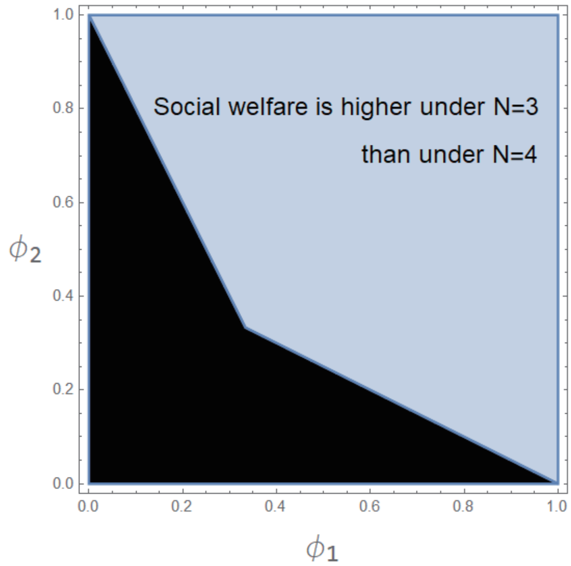
[28]Figure 1 considers the symmetric case where $\alpha_i(\tau_i) = 1 - \tau_i$. It is important to note that, as we show explicitly in Proposition 6, the qualitative results behind the welfare ranks remain for symmetric network benefit functions that are linear.

[29]We find similar findings by focusing exclusively on the surplus generated by non-platform agents (consumers and firms).

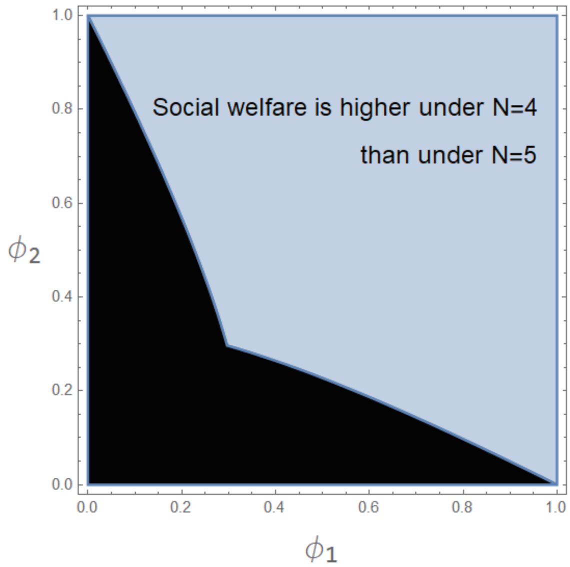
Figure 1: Region for Social Welfare to be Higher



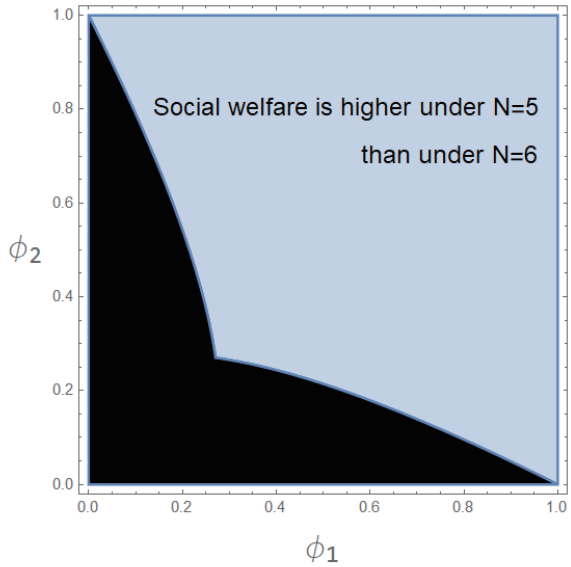
(a) $N = 2, 3$



(b) $N = 3, 4$



(c) $N = 4, 5$



(d) $N = 5, 6$

While our results on welfare suggest that platform competition is harmful (so that platform mergers should be allowed), there are several caveats to our analysis that should be considered when evaluating platform mergers. First, we assume symmetric network benefits for analytical purposes; with significant asymmetries, the welfare effects become ambiguous.[30] Second, platforms are often differentiated in practice; naturally this increases surplus and makes a platform merger more detrimental than in our setting of homogeneous platforms. Finally, we abstract away from any standalone value that is generated by the platform (e.g., a smartphone generates considerable value outside of apps). How these standalone values change across homing decisions complicates the analysis considerably (see Jeitschko and Tremblay (2020) for details), but they may also impact the welfare effects from different levels of competition.

7 Conclusion

The study of platform competition often presents researchers with difficulties that do not arise in the study of competition amongst traditional firms. As a result, there is limited variety in the models that consider platform competition (especially for the case of more than two platforms). In particular, and, to the best of our knowledge, we are the first to integrate the conduct parameter approach with Cournot platform competition.

By allowing for general homing allocations on both sides of the market, we find that greater single-homing increases the extent to which competition distorts markups and markdowns toward zero. This implies that with mixed-homing on both sides of the market, greater platform competition results in both platform prices converging to their respective marginal cost (a result that is consistent with a traditional Cournot market).

In terms of results specific to conduct, we show that both platform conduct and agent homing allocations generate side specific conduct parameters, and we find that greater single-homing on a particular side decreases platform monopolization over that side while greater

[30]However, note that symmetric network benefits was not assumed in Proposition 5.

platform market power increases monopolization over that side. We also show that, similar to the traditional market, the side specific conduct parameters can be derived from elasticity-adjusted Lerner indices and this highlights the tractability of the conduct parameter approach to platform competition.

Lastly, we verify that platform competition decreases welfare. This is a result that holds for any inverse allocation, including the competitive bottleneck, and for more general homing allocations that are feasible in our model. These results stress that, unlike traditional Cournot markets, mergers between symmetric homogenous platforms competing *à la* Cournot should be allowed (even under a variety of homing allocations).

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Appendix of Proofs

Proof of Proposition 1: Equation (6) provides consumer and seller inverse demands for each platform for the general homing case. Substituting into the platform X's profit function yields:

$$\Pi^X = [\alpha_1(\tau_1^*) \cdot q_2^X - c_1] \cdot q_1^X + [\alpha_2(\tau_2^*) \cdot q_1^X - c_2] \cdot q_2^X.$$

Using the output functions for τ_i^* given by Equation (7) we see that there are two possible first-order conditions for each output variable q_i^X (one for $q_i^X \geq q_i^{\mathcal{L}} := \min_{Y \neq X}^N \{q_i^Y\}$ and another for $q_i^X < q_i^{\mathcal{L}}$). These first-order conditions are

$$0 = \frac{\partial \Pi^X}{\partial q_i^X} = \alpha_i'(\tau_i) \cdot q_j^X \cdot q_i^X + [\alpha_i(\tau_i) \cdot q_j^X - c_i] + \alpha_j(\tau_j) \cdot q_j^X,$$

for $q_i^X \geq q_i^{\mathcal{L}}$, and

$$0 = \frac{\partial \Pi^X}{\partial q_i^X} = \phi_i \cdot \alpha_i'(\tau_i) \cdot q_j^X \cdot q_i^X + [\alpha_i(\tau_i) \cdot q_j^X - c_i] + \alpha_j(\tau_j) \cdot q_j^X,$$

for $q_i^X < q_i^{\mathcal{L}}$. Focusing on the symmetric equilibrium, we have that $q_i^X = q_i$ for all $X = 1, ..., N$ and so the first-order condition with $q_i^X \geq q_i^{\mathcal{L}}$ provides the equilibrium. Furthermore, symmetry and Equation (7) imply that the unique number of Side i agents is given by $\tau_i^G = [1 + (N-1)\phi_i]q_i$ so that $q_i = \frac{\tau_i^G}{[1+(N-1)\phi_i]}$. Substituting q_i^X and q_j^X for $q_i^X = q_i = \frac{\tau_i^G}{[1+(N-1)\phi_i]}$ and $q_j^X = q_j = \frac{\tau_j^G}{[1+(N-1)\phi_j]}$ into the first-order condition above implicitly defines the solutions for (τ_1^B, τ_2^G).

Also note that the total output on Side i can be written as $Q_i = N \cdot q_i = \frac{N \cdot \tau_i^G}{[1+(N-1)\phi_i]}$. As a result, we can write the classic markup term given by $\frac{-p_i}{\epsilon_i}$, where the elasticity of demand on Side i is given by $\epsilon_i = \frac{\partial Q_i}{\partial p_i} \cdot \frac{p_i}{Q_i} < 0$, as the following:

$$\begin{aligned} \frac{p_i}{\epsilon_i} &= \frac{\partial p_i}{\partial Q_i} \cdot Q_i = \left[\alpha_i'(\tau_i^G) \cdot q_j \cdot \frac{[1+(N-1)\phi_i]}{N}\right] \cdot \frac{N \cdot \tau_i^G}{[1+(N-1)\phi_i]} = \alpha_i'(\tau_i^G) \cdot q_j \tau_i^G \\ &= \alpha_i'(\tau_i^G) \cdot q_j \cdot [1+(N-1)\phi_i] q_i \end{aligned}$$

Substituting into the first-order condition reduces to

$$0 = \frac{1}{1+(N-1)\phi_i} \cdot \frac{p_i}{\epsilon_i} + [p_i^G - c_i] + \alpha_j(\tau_j^G) \cdot \frac{\tau_j^G}{1+(N-1)\phi_j}. \tag{16}$$

Rearranging terms implies that

$$p_i^G = c_i + \frac{1}{1+(N-1)\phi_i} \cdot \frac{p_i}{-\epsilon_i} - \frac{1}{1+(N-1)\phi_j} \cdot \alpha_j(\tau_j^G)\tau_j^G.$$

□

Proof of Proposition 2: We know from the proof of Proposition 1 that $\frac{p_i^{CB}}{-\epsilon_i} = -\alpha_i'(\tau_i^{CB}) \cdot q_j^{CB}\tau_i^{CB}$, $p_i^{CB} = \alpha_i(\tau_i^{CB}) \cdot q_j^{CB}$, and $q_i = \frac{\tau_i}{1+(N-1)\phi_i}$. Thus, Equation (8) with $\phi_1 = 1$, $\phi_2 = 0$, and $c_i = 0$ becomes

$$\alpha_1(\tau_1^{CB})\tau_2^{CB} = 0 - \frac{1}{N} \cdot \alpha_1'(\tau_1^{CB})\tau_1^{CB}\tau_2^{CB} - \alpha_2(\tau_2^{CB})\tau_2^{CB},$$

$$\alpha_2(\tau_2^{CB})\frac{\tau_1^{CB}}{N} = 0 - \alpha_2'(\tau_2^{CB})\tau_2^{CB}\frac{\tau_1^{CB}}{N} - \frac{1}{N} \cdot \alpha_1(\tau_1^{CB})\tau_1^{CB}.$$

With cancelations and by rearranging terms we have that

$$0 = \frac{1}{N} \cdot \alpha_1'(\tau_1^{CB})\tau_1^{CB} + \alpha_1(\tau_1^{CB}) + \alpha_2(\tau_2^{CB}),$$

$$0 = \alpha_2'(\tau_2^{CB})\tau_2^{CB} + \alpha_2(\tau_2^{CB}) + \alpha_1(\tau_1^{CB}).$$

Totally differentiating each equation when the $\alpha_i''(\cdot) = 0$ implies that

$$0 = \left(1 + \frac{1}{N}\right)\alpha_1'(Q_1^{CB})\partial Q_1^{CB} + \alpha_2'(Q_2^{CB})\partial Q_2^{CB} - \alpha_1'(Q_1^{CB}) \cdot \frac{Q_1^{CB}}{N^2}\partial N,$$

$$0 = 2\alpha_2'(Q_2^{CB})\partial Q_2^{CB} + \alpha_1'(Q_1^{CB})\partial Q_1^{CB}.$$

After some algebra, we have that $\frac{\partial \tau_1^{CB}}{\partial N} = \frac{2\tau_1^{CB}}{N(N+2)} > 0$ and $\frac{\partial \tau_2^{CB}}{\partial N} = -\frac{\alpha_1'(\tau_1^{CB})}{\alpha_2'(\tau_2^{CB})}\frac{\tau_1^{CB}}{N(N+2)} < 0$.

In terms of prices, we know that $p_1^{CB} = \alpha_1(\tau_1^{CB}) \cdot \tau_2^{CB}$ and $p_2^{CB} = \alpha_2(\tau_2^{CB}) \cdot \frac{\tau_1^{CB}}{N}$. Differentiating with respect to N implies that

$$\frac{\partial p_1^{CB}}{\partial N} = \alpha_1'(\tau_1^{CB})\tau_2^{CB}\frac{\partial \tau_1^{CB}}{\partial N} + \alpha_1(\tau_1^{CB})\frac{\partial \tau_2^{CB}}{\partial N},$$

$$\frac{\partial p_2^{CB}}{\partial N} = \alpha_2'(\tau_2^{CB})\frac{\tau_1^{CB}}{N}\frac{\partial \tau_2^{CB}}{\partial N} + \alpha_2(\tau_2^{CB})\frac{1}{N}\frac{\partial \tau_1^{CB}}{\partial N} - \alpha_2(\tau_2^{CB})\frac{\tau_1^{CB}}{N^2}.$$

The first equation is less than zero since $\alpha_1'(\cdot), \frac{\partial \tau_2^{CB}}{\partial N} < 0$ and $\frac{\partial \tau_1^{CB}}{\partial N} > 0$. The second equation is ambiguous at first glance. Here, however, the consideration for interior solutions might play a role. To see this, note that with differentiable $\alpha_i(\cdot)$, the assumption that $\alpha_i''(\cdot) = 0$ implies that the $\alpha_i(\cdot)$ are linear. Thus, we impose a linear structure so that $\alpha_i(\tau_i) = a_i - b_i\tau_i$ for $i = 1, 2$. This linearization implies that the first-order conditions for consumers and sellers that reduce to:

$$0 = a_1 - \left(1 + \frac{1}{N}\right) b_1 \cdot \tau_1^{CB} + a_2 - b_2 \cdot \tau_2^{CB},$$

$$0 = a_2 - 2b_2 \cdot \tau_2^{CB} + a_1 - b_1 \cdot \tau_1^{CB}.$$

As a result, we have that

$$\tau_1^{CB} = \frac{(a_1 + a_2)N}{b_1(N+2)} \text{ and } \tau_2^{CB} = \frac{a_1 + a_2}{b_2(N+2)}.$$

This implies that $p_2^{CB} = \alpha_2(\tau_2^{CB})\frac{\tau_1^{CB}}{N}$ becomes

$$p_2^{CB} = \frac{[(N+1)a_2 - a_1] \cdot (a_1 + a_2)}{b_1 b_2 (N+2)^2}.$$

Differentiating implies that $\frac{\partial p_2^{CB}}{\partial N} > 0$ if and only if $2a_1 > Na_2$. In the end, an interior solution requires that $\tau_i^{CB} < \overline{\tau}_i := \frac{a_i}{b_i}$. Given the τ_1^{CB} and τ_2^{CB} above, we require that $2a_1 > Na_2$ and $(N+1)a_1 > a_2$ (which is satisfied to by the former). Thus, $\frac{\partial p_2^{CB}}{\partial N} > 0$. □

Proof of Proposition 3: As noted at the start of Section 6, we assume that the $\alpha_i''(\cdot) = 0$ and that the $c_i = 0$ for $i = 1, 2$. Given that the $\alpha_i(\cdot)$ are differentiable, $\alpha_i''(\cdot) = 0$ implies that the $\alpha_i(\cdot)$ are linear. Thus, as discussed in the Proof of Proposition 2, we impose a linear structure so that $\alpha_i(\tau_i) = a_i - b_i\tau_i$ for $i = 1, 2$. From the Proof of Proposition 2 we have that

$$\tau_1^{CB} = \frac{(a_1 + a_2)N}{b_1(N+2)} \;\&\; \tau_2^{CB} = \frac{a_1 + a_2}{b_2(N+2)}.$$

An interior solution occurs whenever $\tau_i^{CB} \leq \frac{a_i}{b_i}$ (so that $\alpha_i(\cdot) \geq 0$ for all τ_i). This restriction implies that we require $\frac{N}{2} \cdot a_2 \leq a_1 \leq (N+1)a_2$ so that the network effect magnitudes cannot differ too drastically. Differentiating implies that

$$\frac{\partial \tau_1^{CB}}{\partial N} = \frac{2(a_1 + a_2)}{b_1(N+2)^2} \;\&\; \frac{\partial \tau_2^{CB}}{\partial N} = -\frac{a_1 + a_2}{b_2(N+2)^2}.$$

Applying Leibniz integral rule to Equation (14) implies that

$$\frac{\partial W^{CB}}{\partial N} = \alpha_1(\tau_1^{CB})\tau_2^{CB} \cdot \frac{\partial \tau_1^{CB}}{\partial N} + \int_0^{\tau_1^{CB}} \alpha_1(\tau_1)\frac{\partial \tau_2^{CB}}{\partial N}\, d\tau_1 + \alpha_2(\tau_2^{CB})\tau_1^{CB} \cdot \frac{\partial \tau_2^{CB}}{\partial N} + \int_0^{\tau_2^{CB}} \alpha_2(\tau_2)\frac{\partial \tau_1^{CB}}{\partial N}\, d\tau_2.$$

Substituting for the τ_i^{CB}, $\frac{\partial \tau_i^{CB}}{\partial N}$, and $\alpha_i(\cdot)$, and after some algebra, we have that

$$\frac{\partial W^{CB}}{\partial N} = \frac{(a_1 + a_2)^3}{2b_1b_2(N+2)^4} \cdot (-N^2 - 2N + 6).$$

Thus, $\frac{\partial W^{CB}}{\partial N} > 0$ if and only if $-N^2 - 2N + 6 > 0$ which never holds for $N \geq 2$. $\square$

Proof of Proposition 4: We know from the proof of Proposition 1 that $\frac{p_i^G}{-\epsilon_i} = -\alpha_i'(\tau_i^G) \cdot q_j^G \tau_i^G$, $p_i^G = \alpha_i(\tau_i^G) \cdot q_j^G$, and $q_i^G = \frac{\tau_i^G}{1+(N-1)\phi_i}$. Thus, Equation (8) with $c_i = 0$ becomes

$$\alpha_i(\tau_i^G) \cdot \frac{\tau_j^G}{1 + (N-1)\phi_j} = 0 - \frac{1}{1 + (N-1)\phi_i} \cdot \alpha_i'(\tau_i^G) \cdot \frac{\tau_j^G}{1 + (N-1)\phi_j}\tau_i^G - \frac{1}{1 + (N-1)\phi_j} \cdot \alpha_j(\tau_j^G)\tau_j^G,$$

which reduces to

$$0 = \alpha_i(\tau_i^G) + \alpha_i'(\tau_i^G)\frac{\tau_i^G}{1+(N-1)\phi_i} + \alpha_j(\tau_j^G),$$

for $i, j = 1, 2$ and $i \neq j$.

To determine an explicit expression for the $\frac{\partial \tau_i^G}{\partial N}$, note that differentiable $\alpha_i(\cdot)$ and the assumption that $\alpha_i^{''}(\cdot) = 0$ implies that the $\alpha_i(\cdot)$ are linear so that $\alpha_i(\tau_i) = a_i - b_i\tau_i$ for $i = 1, 2$. This linearization implies that:

$$0 = a_i - b_i\tau_i^G - \frac{b_i\tau_i^G}{1+(N-1)\phi_i} + a_2 - b_2\tau_2^G,$$

for $i, j = 1, 2$ and $i \neq j$. Solving the system of equations for τ_1^G and τ_2^G we have that

$$\tau_i^G = \frac{(a_1+a_2)[1+(N-1)\phi_i]}{b_i[3+(N-1)\phi_1+(N-1)\phi_2]},$$

which implies that

$$q_i^G = \frac{\tau_i^G}{1+(N-1)\phi_i} = \frac{(a_1+a_2)}{b_i[3+(N-1)\phi_1+(N-1)\phi_2]},$$

for $i = 1, 2$. An interior solution occurs whenever $\tau_i^{CB} \leq \frac{a_i}{b_i}$ (so that $\alpha_i(\cdot) \geq 0$ for all τ_i). This restriction implies that we require $\frac{1+(N-1)\phi_1}{2+(N-1)\phi_2} \cdot a_2 \leq a_1 \leq \frac{2+(N-1)\phi_1}{1+(N-1)\phi_2} \cdot a_2$ so that the network effect magnitudes cannot differ too drastically. Differentiating implies that

$$\frac{\partial \tau_i^G}{\partial N} = \frac{(a_1+a_2)}{b_i[3+(N-1)\phi_1+(N-1)\phi_2]^2} \cdot [\phi_i[3+(N-1)\phi_1+(N-1)\phi_2] - (\phi_1+\phi_2)[1+(N-1)\phi_i]],$$

for $i = 1, 2$. This implies that $\frac{\partial \tau_i^G}{\partial N} > 0$ if and only if $2\phi_i > \phi_j$ for $i, j = 1, 2$ and $i \neq j$. Similarly, we clearly see that $\frac{\partial q_i^G}{\partial N} < 0$ so long as $\phi_1 > 0$ or $\phi_2 > 0$. □

Proof of Proposition 5: As noted at the start of Section 6, we assume that the $\alpha_i^{''}(\cdot) = 0$ and that the $c_i = 0$ for $i = 1, 2$. Given that the $\alpha_i(\cdot)$ are differentiable, $\alpha_i^{''}(\cdot) = 0$ implies that the $\alpha_i(\cdot)$ are linear. Thus, as discussed in the Proof of Proposition 2, we impose a linear structure so that $\alpha_i(\tau_i) = a_i - b_i\tau_i$ for $i = 1, 2$. In this case, Equation (15) simplifies to

$$\begin{aligned} W^G &= \frac{(a_1 + a_2)^3}{2b_1b_2[3 + (N-1)(\phi_1 + \phi_2)]^3} \\ &\quad \times [4 + 5(N-1)(\phi_1 + \phi_2) + (N-1)(4N-6)\phi_1\phi_2 \\ &\quad\quad + (N-1)^2(\phi_1^2 + \phi_2^2) - (N-1)(2N-1)\phi_1\phi_2(\phi_1 + \phi_2)\big] . \end{aligned}$$

Differentiating with respect to N implies that

$$\begin{aligned} \frac{\partial W^G}{\partial N} &= \frac{(a_1 + a_2)^3}{2b_1b_2[3 + (N-1)(\phi_1 + \phi_2)]^4} \\ &\quad \times \big[3(\phi_1 + \phi_2) + 2(2N-5)\phi_1\phi_2 - 4(N-1)(\phi_1^2 + \phi_2^2) \\ &\quad\quad - N(5N-2)\phi_1\phi_2(\phi_1 + \phi_2) - (N-1)^2(\phi_1^3 + \phi_2^3) \\ &\quad\quad + 4N(N-1)\phi_1^2\phi_2^2 + 2N(N-1)\phi_1\phi_2(\phi_1^2 + \phi_2^2)\big] . \end{aligned}$$

Finally, by imposing $\phi_1 = \phi$ and $\phi_2 = 1 - \phi$, this reduces to

$$\begin{aligned} \frac{\partial W^G}{\partial N} &= \underbrace{-\frac{(a_1 + a_2)^2}{2b_1b_2(2 + N)^4}}_{<0} \\ &\quad \times \big[\{N + [1 - 3(1-\phi)\phi]\}^2 - \big\{7 - 21(1-\phi)\phi - 9(1-\phi)^2\phi^2\big\}\big] , \end{aligned}$$

where $1-3(1-\phi)\phi > 0$ for $\phi \in [0, 1]$. Thus, the positive solution for $\{N + [1 - 3(1-\phi)\phi]\}^2 - \{7 - 21(1-\phi)\phi - 9(1-\phi)^2\phi^2\} = 0$ is $N^+ = -1+3\phi(1-\phi)+\sqrt{7 - 21\phi + 30\phi^2 - 18\phi^3 + 9\phi^4}$. It is verified that this $N^+ \in (1, 2)$ for any $\phi \in [0, 1]$. Therefore, $\frac{\partial W^G}{\partial N} < 0$ for all $N \geq 2$. □

Proof of Proposition 6: As noted at the start of Section 6, we assume that the $\alpha_i''(\cdot) = 0$ and that the $c_i = 0$ for $i = 1, 2$. Given that the $\alpha_i(\cdot)$ are differentiable, $\alpha_i''(\cdot) = 0$ implies that the $\alpha_i(\cdot)$ are linear. Thus, as discussed in the Proof of Proposition 2, we impose a linear structure so that $\alpha_i(\tau_i) = a_i - b_i\tau_i$ for $i = 1, 2$. As shown in the proof of Proposition 5, we have that

$$\begin{aligned}\frac{\partial W^G}{\partial N} &= \frac{(a_1+a_2)^{\,3}}{2b_1b_2[3+(N-1)(\phi_1+\phi_2)]^4} \\ &\quad \times \big[3(\phi_1+\phi_2)+2(2N-5)\phi_1\phi_2-4(N-1)(\phi_1^2+\phi_2^2) \\ &\qquad - N(5N-2)\phi_1\phi_2(\phi_1+\phi_2)-(N-1)^2(\phi_1^3+\phi_2^3) \\ &\qquad + 4N(N-1)\phi_1^2\phi_2^2+2N(N-1)\phi_1\phi_2(\phi_1^2+\phi_2^2)\big]\,.\end{aligned}$$

Imposing the constraints from Condition (9), letting $a_1 = a_2 = a$, and letting $b_1 = b_2 = b$ we have that $\frac{\partial W^G}{\partial N} < 0$ for $N \geq 2$ across the homing allocations that exist as Cournot platform equilibria. □