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A Sufficient Statistics Approach for Welfare Analysis of Oligopolistic Third-Degree Price Discrimination*

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Abstract

This paper proposes a sufficient statistics approach to welfare analysis of third-degree price discrimination in differentiated oligopoly. Specifically, our sufficient conditions for price discrimination to increase or decrease aggregate output, social welfare, and consumer surplus simply entail a cross-market comparison of multiplications of two or three of the sufficient statistics—*pass-through*, *conduct*, and *profit margin*—that are functions of first-order and second-order elasticities of the firm's demand. Notably, these results are derived under a general class of demand, and can be readily be extended to accommodate heterogeneous firms. These features suggest that our approach has potential for conducting welfare analysis without a full specification of an oligopoly model.

Keywords: Third-Degree Price Discrimination; Oligopoly; Sufficient Statistics.

JEL classification: D43; L11; L13.

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1 Introduction

This paper explores the welfare effects of third-degree price discrimination in oligopoly and its output implications as well as the effects on consumer surplus. Specifically, we consider a fairly general setting, and present the sufficient conditions under which oligopolistic third-degree price discrimination increases or decreases aggregate output, consumer surplus, and Marshallian social welfare (i.e., the sum of the consumer and producer surpluses) when all discriminatory markets are served even in the absence of price discrimination. Our analysis is firstly developed under firm symmetry, and is extended to accommodate heterogeneous firms. Moreover, our analysis permits a moderate degree of cost differences to exist across separate markets. To do all these tasks, we employ the *sufficient statistics approach* as a unifying methodology: a technique often used in public economics (Chetty 2009; Kleven 2021) as well as macroeconomics (Barnichon and Mesters 2021).

Under third-degree price discrimination, consumers are segmented into separate markets and charged different unit prices in accordance with their identifiable characteristics (e.g., age, occupation, location, or time of purchase). In contrast, all consumers are charged the same price if third-degree price discrimination is not practiced (i.e., "uniform pricing"). Without loss of generality, the case of two markets can be considered to understand how price discrimination might change output and welfare in each market. The prevailing price is identical in both markets if all firms are symmetric. In this situation, if a discriminatory price becomes greater than the uniform price in one market, and the unit price decreases in the other market, Robinson (1933) calls the former market a "strong" market (s), and the latter a "weak" market (w). More formally, this situation is expressed by $p_s^* > \overline{p} > p_w^*$, where p_s^* and p_w^* are the equilibrium prices under price discrimination in the strong and the weak markets, respectively, and $\overline{p}$ is the uniform price.[1] Given such a price change, price discrimination increases output and social welfare in the weak market, but decreases them in the strong market. What are the overall effects of the price change?

In the analysis below, we follow Leontieff (1940), Silberberg (1970), Schmalensee (1981), Holmes (1989), and Aguirre, Cowan, and Vickers (2010) to add the constraint $p_s - p_w = t$,

[1] In this paper, price discrimination is present when $p_s > p_w$, i.e., when prices between markets are not uniform. As Clerides (2004, p. 402) states, once cost differentials are allowed, "there is no single, widely accepted definition of price discrimination." To understand this, consider symmetric firms and let mc_s and mc_w be the marginal cost at equilibrium output in markets s and w, respectively (they do not necessarily have to be constants for any output levels). Then, two alternative definitions can be considered. One is the *margin* definition: price discrimination occurs when $p_s - mc_s > p_w - mc_w$. The other one is the *markup* definition as per Stigler (1987): price discrimination occurs when $p_s/mc_s > p_w/mc_w$. Our simpler definition is aligned with the former definition, and employed for its tractability and connectivity to the existing literature on third-degree price discrimination with no cost differentials. Moreover, our definition of price discrimination coincides with what Chen and Schwartz (2015) and Chen, Li and Schwartz (2021) call "differential pricing." As long as cost differentials are sufficiently small, these differences will not significantly alter the results because if $mc_s = mc_w$, these three definitions are equivalent.

where $t \geq 0$ is interpreted as an artificial constraint on the profit maximization problem for oligopolistic firms under symmetry. Then, the regime change, which is discrete in its nature, is now measured by t and is continuously connected between $t = 0$ as uniform pricing and $t^* \equiv p_s^* - p_w^*$ as price discrimination in equilibrium. This formulation enables us to describe social welfare as a function of t, $W(t)$, and characterize $W'(t)$ in terms of economic concepts based on elasticity terms of market demand. In this way, whether social welfare improves or deteriorates by this global change of the regime can be determined. This methodology shares the central idea of the sufficient statistics approach where the welfare consequences of policy changes are derived "in terms of estimable elasticities" (Kleven 2021, p. 516) "rather deep primitives" (Chetty 2009, p. 452). One benefit of focusing on sufficient statistics rather than deep parameters in conducting welfare analysis is that one can focus on the deeper *structure* that is "robust across a broad class of underlying models," (Kleven 2021, p. 535) without a specification of market demand. If we instead start with a particular class of demand, it remains unclear to what extent the welfare analysis is valid under another class of market demand.[2]

Our sufficient conditions for oligopolistic price discrimination to increase or decrease aggregate output, social welfare, and consumer surplus are provided by means of a cross-market comparison of the multiplications of two or three of the following economic concepts: (i) *profit margin*, which is the difference between price and marginal cost ($\mu \geq 0$); (ii) *pass-through*, i.e., how the price responds to a small change in marginal cost ($\rho > 0$); and (iii) *conduct*, which measures the degree of market monopolization ($\theta \in [0, 1]$). These three sufficient statistics are determined by the following two first-order and two second-order elasticities: (a) the own price elasticity of the firm's demand (ε^{own}), (b) the cross price elasticity of the firm's demand (ε^{cross}), (c) the curvature of the firm's demand (α^{own}), and (d) the elasticity of the cross-price effect of the firm's demand (α^{cross}).

Specifically, in a series of propositions, we demonstrate that the product of conduct and pass-through, $\theta\rho$, is an important measurement for determining the *output* effects, whereas the product of all three concepts, $\theta\mu\rho$, provides the sufficient condition for the change in *welfare*. Intuitively, the product of conduct and pass-through measures how output in each individual market changes in response to a marginal change in price. To evaluate a marginal change in welfare, profit margin should be considered because it measures the welfare gain or loss

[2]One may criticize that sufficient statistics are only endogenous variables by holding that a sufficient condition is meaningful only when it consists of exogenous parameters. However, in equilibrium, our sufficient conditions are functions of exogenous parameters for the same reason that in equilibrium, endogenous variables are functions of exogenous variables, as demonstrated in Section 4. However, deep parameters themselves do not always allow economic interpretations in a direct manner; for example, in the case of linear demand, the slope coefficient is not directly to related to demand elasticity. In contrast, sufficient statistics such as elasticities almost always have economic interpretations. This *is* the benefit from the sufficient statistics approach because welfare analysis can be conducted based on economic concepts one-level higher that underlie a plausible class of model specification.

that results from a marginal change in quantity under imperfect competition in which the price exceeds marginal cost. In this way, the welfare implications can be obtained by means of a cross-market comparison of the quantity change multiplied by the profit margin. To determine the effect on *consumer surplus*, the product of profit margin and pass-through, $\mu\rho$, is important because it measures the price change multiplied by the level of output. However, one may wonder if this sufficient statistics approach is valid once firm symmetry is relaxed. Section 6 provides a positive answer: no additional complications are necessary to extend our analysis under firm symmetry.

Existing literature on third-degree price discrimination has a centennial tradition, pioneered by Pigou (1920) and Robinson (1933), with their main focus on whether price discrimination increases or decreases social welfare (see Varian (1989); Armstrong (2006, 2008); and Stole (2007) for comprehensive surveys of this literature). Among others, Schmalensee (1981) and Aguirre, Cowan, and Vickers (hereafter, ACV) (2010) study how demand curvatures relate to output and welfare effects. Third-degree price discrimination necessarily entails allocative inefficiency because some consumers exist who have the same marginal utility but face different prices simply because they belong to different markets. Thus, for third-degree price discrimination to increase social welfare, it must sufficiently expand aggregate output to offset such misallocation across markets. Schmalensee (1981) shows that an increase in aggregate output is a necessary condition for third-degree price discrimination to increase social welfare—a conclusion that is generalized by Varian (1985) and Schwartz (1990)—and ACV (2010) identify a sufficient condition for price discrimination to raise social welfare: inverse demand in the weak market is more convex than that in the strong market at the discriminatory prices. Figure 1 provides a graphical illustration: if uniform pricing is implemented instead, welfare loss in the weak market due to the output reduction that has arisen under price discrimination is sufficiently large (Panel b) as compared to the welfare gain in the strong market (Panel a).

However, in this approach, a change in welfare is not predictable if the demand primitives do not satisfy the conditions that are focused on. In addition, most of these studies analyze *monopolistic* third-degree discrimination: to date, "there are *virtually no predictions* as to how discrimination impacts welfare" (Hendel and Nevo 2013, p. 2723; emphasis added) when *oligopolistic* competition is considered. For example, Holmes (1989) employs the same technique used by Schmalensee (1981) and ACV (see Section 3 for details) to examine the output effects of third-degree price discrimination in a symmetric oligopoly. However, Holmes (1989) provides no welfare predictions (see also Dastidar 2006).[3] In this paper, we contribute to the literature by

[3]In a similar vein, Armstrong and Vickers (2001) consider a model of symmetric duopoly with product differentiation à la Hotelling (1929), and study the consequences of third-degree price discrimination in the competitive limit around zero transportation costs wherein the equilibrium prices are almost equal to marginal cost. Under this setting, Armstrong and Vickers (2001) show that price discrimination decreases social welfare if the weak market has a lower value of price elasticity of demand (Adachi and Matsushima (2014) also derive a similar result by assuming linear demand in a standard model of symmetrically differentiated duopoly). Our

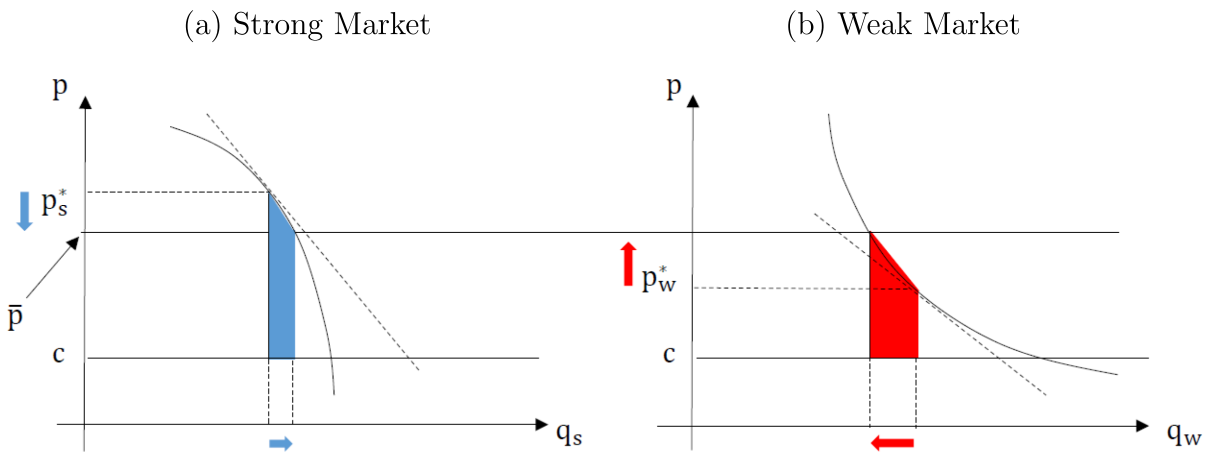

Figure 1: Welfare Comparison in terms of Demand Curvatures (Aguirre, Cowan, and Vickers 2010)

providing fairly general conditions regarding whether oligopolistic price discrimination increases or decreases aggregate output, social welfare, and consumer surplus.

Our study is also in line with Mrázová and Neary (2017) who show the usefulness of the *demand manifold*—the relationship between demand elasticity and convexity which is not ascribed to a function or a correspondence—in comparative statics by suggesting the linkage between these first- and second-order elasticities and sufficient statistics such as markup and pass-through as shown in an empirical study by De Loecker, Goldberg, Khandelwal, and Pavcnik (2016).[4] Mrázová and Neary (2017) point out that one of the advantages of working with the demand manifold instead of the demand function per se is that it is clearer to understand results from comparative statics and counterfactual experiments because demand elasticity and convexity are more closely related to them than demand primitives themselves are. However, Mrázová and Neary (2017) mainly focus on perfect and monopolistic competition: when firm heterogeneity is taken into account, only cost/productivity heterogeneity à la Melitz (2003) is considered. In other words, neither ε^{cross} nor α^{cross} appears in Mrázová and Neary's (2017) analysis because they do not consider product differentiation (firm heterogeneity on the demand side). Therefore, they are able to focus on only two parameters, ε^{own} nor α^{own}. While we do not make use of their method, we explicitly consider imperfect competition based on product differentiation, and further research would be promising to investigate how Mrázová and Neary's (2017) methodology is utilized for issued of imperfect competition such as this

paper aims to fill the gap between monopoly, such as in Schmalensee (1981) and ACV (2010), and Armstrong and Vickers' (2001) competitive limit with respect to welfare implications. We thank Susumu Sato for suggesting this interpretation.

[4] In our context, (i) *profit margin* is determined by the firm-level price elasticity (or the own price elasticity), (ii) *pass-through* is determined mainly by the demand curvature, and (iii) *conduct* is determined by the ratio of the industry-level elasticity to the firm-level elasticity. See the expressions (10) below for the case of price discrimination when market-wise elasticities are defined.

study.

The remainder of this paper is organized as follows. Section 2 presents our base model of oligopolistic pricing with symmetric firms and constant marginal costs. Then, we derive the sufficient statistics implications of aggregate output, social welfare, and consumer surplus in Section 3. Subsequently, Section 4 provide parametric examples of three representative classes of market demand differentiated goods that are often employed in applies studies are linear, CES (constant elasticity of substitution), and logit to discuss how the sufficient statistics approach works if demand primitives (expressed by parameters) demonstrate the results of the effects on consumer surplus, and discuss the case of non-constant marginal costs. ... Section 5... In Section 6, we argue that our differential method is readily extendible to the introduction of firm heterogeneity. Finally, Section 7 concludes.[5]

2 The Model of Oligopolistic Pricing

For ease of exposition, we follow Holmes (1989) and ACV to consider the case of two symmetric firms and two separate markets or consumer groups (hereafter, simply called "markets"): it is straightforward to extend the following analysis to the case of more than two symmetric firms and more than two separate markets.[6] As explained in Introduction, we call one market s (strong), where the equilibrium discriminatory price is higher than the equilibrium uniform price, and the other w (weak), where the opposite is true. Two firms, A and B, have an identical cost structure in each market. Specifically, each firm has an identical cost function, $c_m(q_{jm})$, in market $m = s, w$, where q_{jm} is firm j's output ($j = A$, B). For simplicity of exposition, we assume, with a bit abuse of notation, that firms have a constant marginal cost in each market m, $c_m \geq 0$; here, c_s and c_w can be different. However, as mentioned again in Subsection 2.3 below, it is assumed that the strong market either has a higher marginal cost or only slightly lower marginal cost so that its price still increases with price discrimination. In this sense, this paper does not consider the role of cost differences in differential pricing (see Footnote 1 above).

2.1 Consumers

In market $m = s, w$, given firms A and B's prices p_{Am} and p_{Bm}, the representative consumer purchases $x_{Am} > 0$ and $x_{Bm} > 0$, and her (net) utility (i.e., surplus) is quasi-linear and thus

[5] In this paper, the only policy instrument is an enforcement of uniform pricing. Cowan (2018) studies a model of monopoly to consider a more moderate instrument by which a government regulates the monopolist's profit margins or price-marginal cost ratios across different markets.

[6] We assume that resale between markets is impossible to prevent consumers in the strong market from being better off buying the good at a lower price in the week market (see Boik (2017) for an empirical analysis of oligopolistic third-degree price discrimination when arbitrage may matter).

written as

$$U_m(\mathbf{x}_m) - p_{Am}x_{Am} - p_{Bm}x_{Bm},$$

where $\mathbf{x}_m = (x_{Am}, x_{Bm})$, and U_m is three-times continuously differentiable, $\frac{\partial U_m}{\partial x_{jm}} > 0$, $\frac{\partial^2 U_m}{\partial x_{jm}^2} < 0$, $j = A$, B, and $\frac{\partial^2 U_m}{\partial x_{Am}\partial x_{Bm}} < 0$ (i.e., firms A and B produce substitutable products).

Inverse demands in market m, $p_{jm} = P_{jm}(x_{jm}, x_{-j,m})$, are derived from the representative consumer's utility maximization ($-j = A, B$, $-j \neq j$): $\frac{\partial U_m}{\partial x_{jm}}(x_{jm}, x_{-j,m}) - p_{jm} = 0$, which also implicitly defines firm j's *direct demand* in market m, $x_{jm} = x_{jm}(p_{jm}, p_{-j,m})$. We assume that $x_{jm}(\cdot)$ is twice continuously differentiable. Because of the assumptions regarding the utility, firm j's demand in market m decreases as its own price increases ($\frac{\partial x_{jm}}{\partial p_{jm}} < 0$), and it rises as the rival's price increases ($\frac{\partial x_{jm}}{\partial p_{-j,m}} > 0$; the firms' products are substitutes).[7] We also assume that from a viewpoint of consumers, firms are symmetric: $U_m(x', x'') = U_m(x'', x')$ for any $x' > 0$ and $x'' > 0$. Then, the firms' demands in market m are also symmetric: $x_{Am}(p', p'') = x_{Bm}(p', p'')$ for any $p' > 0$ and $p'' > 0$. Because the firms' technologies are also identical, we focus on symmetric Nash equilibrium until we allow firm heterogeneity in Section 6.

We define the demand in symmetric pricing by $q_m(p) \equiv x_{Am}(p, p)$. Another interpretation of $q_m(p)$ is: both firms take $2q_m(p)$ as the joint demand, "cooperatively" choose the same price (behaving as an "industry"), and divide the joint demand equally to obtain $q_m(p)$. Note here that

$$q_m'(p) = \underbrace{\frac{\partial x_{Am}}{\partial p_A}(p_A, p)\bigg|_{p_A=p}}_{<0 \text{ (ACV's } q_m')} + \underbrace{\frac{\partial x_{Am}}{\partial p_B}(p, p_B)\bigg|_{p_B=p}}_{>0 \text{ (strategic)}}. \tag{1}$$

Thus, for $q_m'(p)$ to be negative, we assume that $\left|\frac{\partial x_{Am}}{\partial p_A}(p, p)\right| > \frac{\partial x_{Am}}{\partial p_B}(p, p)$. Note also that by symmetry, the following relationship also holds (this corresponds to Holmes' (1989) Equation 4):

$$\underbrace{\frac{\partial x_{Am}}{\partial p_A}(p, p)}_{\text{own}} = \underbrace{q_m'(p)}_{\text{industry}} - \underbrace{\frac{\partial x_{Bm}}{\partial p_A}(p, p)}_{\text{strategic effects}}.$$

This exchangeability is key in Holmes' (1989) derivation below. Intuitively, each firm, under symmetry,treats the industry demand $q_m(p)$ as if it is its own demand. Thus, how a firm's pricing behavior affects its own demand as an industry demand has the following two effects: a small decrease in p_A by firm A by deviating from the "coordinated" price p (i) not only raises its own demand by $\frac{\partial x_{Am}}{\partial p_A}$ as the *residual* monopolist (taking the rival's pricing as fixed; *intrinsic effects*), (ii) firm A can now also obtain some of the consumers originally attached to firm B, and this amount is $\frac{\partial x_{Bm}}{\partial p_A}$ (*strategic effects*).

[7] Here, $\frac{\partial^2 x_{jm}}{\partial p_j^2}(p, p)$ can be positive, zero or negative. Following Dastidar's (2006, p. 234) Assumption 2 (iv), we assume that $\frac{\partial^2 x_{jm}}{\partial p_j^2}(p, p) + \frac{\partial^2 x_{jm}}{\partial p_j \partial p_{-j}}(p, p) \leq 0$.

Under symmetric pricing, we are able to define, following Holmes (1989, p. 245), the *price elasticity of the industry's demand* by

$$\varepsilon_m^I(p) \equiv -\frac{pq_m'(p)}{q_m(p)}.$$

This corresponds to η in ACV (p. 1603) as well as ϵ_D in Weyl and Fabinger (2013, p. 542): it should not "be confused with the elasticity of the residual demand that any of the firms faces." Similarly, the *own* and the *cross price elasticities of the firm's demand* are defined by

$$\varepsilon_m^{own}(p) \equiv -\frac{p}{q_m(p)}\frac{\partial x_{Am}}{\partial p_A}(p,p) > 0$$

and by

$$\varepsilon_m^{cross}(p) \equiv \frac{p}{q_m(p)}\frac{\partial x_{Bm}}{\partial p_A}(p,p) > 0,$$

respectively. Then, Holmes (1989) shows that under symmetric pricing, $\varepsilon_m^{own}(p) = \varepsilon_m^I(p) + \varepsilon_m^{cross}(p)$ holds.[8] This implies that the own-price elasticity must be equal to or greater than the industry's elasticity and greater than the cross-price elasticity (i.e., $\varepsilon_m^{own}(p) \geq \varepsilon_m^I(p)$ and $\varepsilon_m^{own}(p) > \varepsilon_m^{cross}(p)$).

2.2 Firms

Firm j's profit in market m is written as

$$\pi_{jm}(\mathbf{p}_m) = (p_{jm} - c_m)x_{jm}(\mathbf{p}_m), \tag{2}$$

where $\mathbf{p}_m = (p_{jm}, p_{-j,m})$. As in Dastidar's (2006, pp. 235-6) Assumptions 3 and 4, for the existence and the global uniqueness of pricing equilibrium under either uniform pricing or price discrimination, we assume that for each firm $j = A, B$, $\frac{\partial^2 \pi_{jm}}{\partial p_{jm}^2} < 0$, $\frac{\partial^2 \pi_{jm}}{\partial p_{jm}\partial p_{-j,m}} > 0$, and $-\frac{\partial^2 \pi_{jm}/(\partial p_{jm}\partial p_{-j,m})}{\partial^2 \pi_{jm}/\partial p_{jm}^2} < 1$ (see Dastidar's (2006) Lemmas 1 and 2 for the existence and the uniqueness). We then define the first-order partial derivative of the profit in market m, evaluated at a symmetric price p, by

$$\begin{aligned} \partial_p \pi_m(p) &\equiv \left.\frac{\partial \pi_{jm}(p_{jm}, p_{-j,m})}{\partial p_{jm}}\right|_{p_{jm}=p_{-j,m}=p} \\ &= q_m(p) + (p - c_m)\frac{\partial x_{Am}}{\partial p_A}(p,p). \end{aligned} \tag{3}$$

[8]In general, when there are $N \geq 2$ firms, this identity still holds if the cross price elasticity is defined by $\varepsilon_m^{cross}(p) \equiv (N-1)\frac{p}{q_m(p)}\frac{\partial x_{Bm}}{\partial p_A}(p,p)$.

Then, under symmetric discriminatory pricing, p_m^* satisfies $\partial_p \pi_m(p_m^*) = 0$ for $m = s, w$. Under symmetric uniform pricing, $\overline{p}$ is a (unique) solution of $\partial_p \pi_s(\overline{p}) + \partial_p \pi_w(\overline{p}) = 0$. Throughout this paper, we consider the situation where the weak market is open under uniform pricing (for which $q_w(p_s^*) > 0$ is a sufficient condition).[9]

2.3 Equilibrium

The equilibrium discriminatory price in market $m = s, w$, p_m^*, satisfies the following Lerner formula:

$$\varepsilon_m^{own}(p_m^*)\frac{p_m^* - c_m}{p_m^*} = 1.$$

This shows that the discriminatory price in market m approaches to the marginal cost as the own-price elasticity for the firm, $\varepsilon_m^{own}(p_m^*)$, becomes large. Because of Holmes' (1989) elasticity formula explained above, $\varepsilon_m^{own}(p_m^*)$ can be large (i) when $\varepsilon_m^{I}(p_m^*)$ is very large even if $\varepsilon_m^{cross}(p_m^*)$ is close to zero, or (ii) when $\varepsilon_m^{cross}(p_m^*)$ is very large even if $\varepsilon_m^{I}(p_m^*)$ is close to zero. Evidently, if there are no cost differentials between markets, which market is strong or weak is solely determined by the difference in the own-price elasticity. As mentioned above, we assume that the marginal cost in the strong market is not sufficiently low to assure that $p_s^* > \overline{p} > p_w^*$ indeed holds.[10,11]

Lastly, let y_m be per-firm (symmetric) *market share* of output in market m, that is, $y_m(p_s, p_w) \equiv \frac{q_m(p_m)}{q_s(p_s)+q_w(p_w)}$. Then, the equilibrium uniform price, $\overline{p} \equiv \overline{p}(c_s, c_w)$, satisfies:

$$\sum_{m=s,w} \overline{y}_m \varepsilon_m^{own}(\overline{p})\frac{\overline{p} - c_m}{\overline{p}} = 1,$$

where $\overline{y}_m \equiv y_m(\overline{p}(c_s, c_w), \overline{p}(c_s, c_w))$ for $m = s, w$.[12] In this way, the equilibrium level of uniform

[9] Note that $q_w(\overline{p}) > q_w(p_s^*)$ because $q_w(\cdot)$ is strictly decreasing and $p_s^* > \overline{p}$. Thus, if $q_w(p_s^*) > 0$, then the weak market is open under uniform pricing, i.e., $q_w(\overline{p}) > 0$. Alternatively, we would be able to show that there exist $\underline{c}_s$ and $\overline{c}_s$, $\underline{c}_s < \overline{c}_s$, such that $p_s^* > p_w^*$ and $q_w(\overline{p}) > 0$ for $c_s \in (\underline{c}_s, \overline{c}_s)$ in a similar spirit of Adachi and Matsushima (2014).

[10] See Nahata, Ostaszewski, and Sahoo (1990) for an example of all discriminatory prices being lower than the uniform price with a plausible demand structure under monopoly. In the case of oligopoly, Corts (1998) show that best-response asymmetry, in which firms differ in ranking strong and weak markets, is necessary for all discriminatory prices to be lower than the uniform price ("all-out price competition"). As long as symmetric firms are considered, this case never arises.

[11] When price discrimination is allowed, each firm may not price discriminate even if it is allowed to do so because it is still able to set a uniform price (i.e., it is not forced to price discriminate). We assume that $\pi_{jm}(\cdot, p_{-j,m}^*)$ is strictly increasing (decreasing) at $p_{jm} = \overline{p}$ in market $m = s$ (m) and thus firm j has an incentive to deviate from the equilibrium uniform price if the other firm chooses $p_{-j,s}^*$ and $p_{-j,w}^*$, and that $\pi_{jm}(\cdot, p_{-j,m}^*)$ attains the global optimum at $p_{jm} = p_{jm}^*$.

[12] If there are no cost differentials, i.e., $c_s = c_w$ $(\equiv c)$, then the formula is simpler:

$$\frac{\overline{p} - c}{\overline{p}} = \frac{1}{\sum_{m=s,w} \overline{y}_m \varepsilon_m^{own}(\overline{p})}$$

price is determined by the market-share weighted average of the own price elasticities, whereas the equilibrium level of discriminatory price solely depends on the firm's own price elasticity in that market. In the rest of the paper, the dependence of the equilibrium price is often implicit when there are no confusions. In particular, the superscript star (the upper bar) denotes price discrimination (uniform pricing). For example, we write $(\varepsilon_m^I)^* \equiv \varepsilon_m^I(p_m^*)$ and $\bar{\varepsilon}_m^I \equiv \varepsilon_m^I(\overline{p})$ as the industry's elasticities in equilibrium.

3 Welfare Effects

As mentioned in Introduction, we add the constraint $p_s - p_w = t$, where $t \geq 0$, to the firms' profit maximization problem.[13] Then, we express social welfare (as well as aggregate output and consumer surplus) as a function of t in $[0, t^*]$, where $t = 0$ corresponds to uniform pricing, and $t = t^* \equiv p_s^* - p_w^*$ to price discrimination. Note that under this constrained problem of profit maximization, p_w satisfies $\partial_p \pi_s(p_w + t) + \partial_p \pi_w(p_w) = 0$. Thus, we write the solution by $p_w(t)$. Then, we define $p_s(t) \equiv p_w(t) + t$. Applying the implicit function theorem to this equation yields to $p_w'(t) = -\frac{\pi_s''}{\pi_s''+\pi_w''} < 0$ and $p_s'(t) = \frac{\pi_w''}{\pi_s''+\pi_w''} > 0$, where

$$\begin{aligned}\pi_m''(p) &\equiv q_m'(p) + \frac{\partial x_{Am}}{\partial p_A}(p,p) + (p - c_m)\frac{d}{dp}\left(\frac{\partial x_{Am}}{\partial p_A}(p,p)\right) \\ &= \underbrace{\partial_p^2 \pi_m(p)}_{\text{ACV's } \pi_m''} + \underbrace{\frac{\partial x_{Am}}{\partial p_B}(p,p) + (p - c_m)\frac{\partial^2 x_{Am}}{\partial p_B \partial p_A}(p,p)}_{\text{strategic}},\end{aligned} \tag{4}$$

and $\partial_p^2 \pi_m(p)$ is defined by

$$\partial_p^2 \pi_m(p) \equiv \left[2 + (p - c_m)\frac{\partial^2 x_{Am}(p,p)/\partial p_A^2}{\partial x_{Am}(p,p)/\partial p_A}\right] \frac{\partial x_{Am}}{\partial p_A}(p,p). \tag{5}$$

The latter corresponds to ACV's (p. 1603) $\pi_m''(p)$, and the second and third term arise due to oligopoly. Here, in each m, $\pi_m''(p)$ is assumed to be negative for all $p \geq 0$.[14]

We define the representative consumer's utility in symmetric pricing by $\widetilde{U}_m(q) = U_m(q,q)$.

as shown by Holmes (1989, p. 247): the markup rate (common to all markets) is equal to the inverse of the average of own-price elasticities weighted by the output shares.

[13] Alternatively, Vickers (2020) analyzes properties of social welfare and consumer surplus as a scalar argument to make a comparison between price discrimination and uniform pricing in monopoly. Vickers (2020) especially focuses on the case where quantity elasticity or inverse demand curvature is constant for all markets. See also Cowan (2017) for an analysis of the role of price elasticity and demand curvature in determining the effects of monopolistic third-degree price discrimination.

[14] ACV's Appendix A discusses the concavity of the profit function.

Then, social welfare under symmetric pricing as a function of t is written as

$$\begin{aligned} W(t) &\equiv \widetilde{U}_s(q_s[p_s(t)]) + \widetilde{U}_w(q_w[p_w(t)]) - 2c_s \cdot q_s[p_s(t)] - 2c_w \cdot q_w[p_w(t)] \\ &= (\widetilde{U}_s' - 2c_s) \cdot q_s' \cdot p_s'(t) + (\widetilde{U}_w' - 2c_w) \cdot q_w' \cdot p_w'(t), \end{aligned}$$

which implies

$$\frac{W'(t)}{2} = [p_s(t) - c_s] \cdot q_s' \cdot p_s'(t) + [p_w(t) - c_w] \cdot q_w' \cdot p_w'(t) \tag{6}$$

because $\widetilde{U}_m' = \frac{\partial U_m}{\partial q_A} + \frac{\partial U_m}{\partial q_B} = 2\frac{\partial U_m}{\partial q_A}$ (by symmetry). On the other hand, aggregate output under symmetric pricing is given by

$$Q(t) = Q_s(t) + Q_w(t) = 2\left\{q_s[p_s(t)] + q_w[p_w(t)]\right\},$$

whereas consumer surplus is defined by replacing c_m in $W(t)$ by $p_m(t)$ to define

$$CS(t) = U_s(q_s[p_s(t)]) + U_w(q_w[p_w(t)]) - 2p_s(t) \cdot q_s[p_s(t)] - 2p_w(t) \cdot q_w[p_w(t)].$$

As argued above, $W(t)$, $Q(t)$, and $CS(t)$ are all functions of $t \in [0, t^*]$. The regime change from uniform pricing to price discrimination is captured by a parameter shift from $t = 0$ to $t = t^*$, and vice versa. However, if these functions are globally concave in this range, then the local sign at $t = 0$ or t^* may predict the sign from the regime change. Specifically, consider a representative function, $F(t)$. If the global concavity of $F(t)$ is assured, then $F(t)$ behaves in either manner:

1. If $F'(0) \leq 0$, then $\frac{F(t)}{2}$ is monotonically decreasing in t, and as a result $\frac{\Delta F}{2} = \frac{F(t^*) - F(0)}{2} < 0$; *price discrimination decreases* F.
2. If $F'(0) > 0$, then $\frac{F(t)}{2}$ either
   (a) is monotonically increasing (if $F'(t^*) > 0$, this is true), and as a result, $\frac{\Delta F}{2} > 0$; *price discrimination increases* F.
   (b) first increases, and then after the reaching the maximum (where $F'(t) = 0$), decreases until $t = t^*$. In this case, *price discrimination may increase or decrease* F: it cannot be determined whether $\frac{\Delta F}{2} < 0$ or $\frac{\Delta F}{2} > 0$ without further functional and/or parametric restrictions.

3.1 Curvatures, Conduct, and Pass-Through

In this subsection, we first introduce second-order elastiticities–demand curvatures–to argue how π_m'' is expressed in terms of the first- and second-order elasticities. We then define two of the three sufficient statistics that play an important role in determining the output and welfare effects of third-degree price discrimination in oligopoly in Subsections 5.1, 3.2, and 5.2.

3.1.1 Curvatures

To proceed further, we define the *curvature of the firm's (direct) demand* in market m by

$$\alpha_m^{own}(p) \equiv -\frac{p}{\partial x_{Am}(p,p)/\partial p_A}\frac{\partial^2 x_{Am}}{\partial p_A^2}(p,p),$$

which measures the convexity/concavity of the firm's direct demand, and corresponds to $\alpha_m(p)$ in Aguirre, Cowan and Vickers 2010, p. 1603). The *elasticity of the cross-price effect* of the firm's direct demand in market m is defined by

$$\alpha_m^{cross}(p) \quad \equiv \quad -\frac{p}{\partial x_{Am}(p,p)/\partial p_A}\frac{\partial^2 x_{Am}}{\partial p_B \partial p_A}(p,p),$$

which does not appear in monopoly. Here, α_m^{own} and α_m^{cross} are positive (resp. negative) if and only if $\frac{\partial^2 x_{Am}}{\partial p_A^2}$ and $\frac{\partial^2 x_{Am}}{\partial p_B \partial p_A}$are positive (resp. negative), respectively. Note also that the sign of α_m^{own} indicates whether the firm's own part of the demand slope under symmetric pricing given the rival's price p, $\frac{\partial x_{Am}}{\partial p_A}(\cdot,p)$, is *convex* ($\alpha_m^{own}$ is positive) or *concave* (α_m^{own} is negative). On the other hand, α_m^{cross} measures to what extent the rival's price level matters to how many of the firm's customers switch to the rival's product when the firm raises its own price ($\frac{\partial x_{Am}}{\partial p_A}$). Thus, a large α_m^{cross} implies that $\frac{\partial x_{Am}}{\partial p_A}$ is very responsive to a change in p_B, and vice versa.

Note here that Equation (5) implies that

$$\begin{aligned}
\partial_p^2 \pi_m(p) &= -\{2 - \underbrace{\frac{p-c_m}{p}}_{=L_m(p)}\underbrace{[-\frac{p}{\frac{\partial x_{Am}}{\partial p_A}(p,p)}\frac{\partial^2 x_{Am}}{\partial p_A^2}(p,p)]}_{=\alpha_m^{own}(p)}\}\underbrace{[-\frac{p}{q_m(p)}\frac{\partial x_{Am}}{\partial p_A}(p,p)]}_{=\varepsilon_m^{own}(p)}\frac{q_m(p)}{p} \\
&= -[2 - L_m(p)\alpha_m^{own}(p)]\varepsilon_m^{own}(p)\frac{q_m(p)}{p},
\end{aligned}$$

where

$$L_m(p) \equiv \frac{p-c_m}{p}$$

is the *markup rate* (i.e., the Lerner index). Therefore, from Equation (4), $\pi_m''(p)$ is expressed in terms of the four elasticities (ε_m^{own}, ε_m^{cross}, α_m^{own}, and α_m^{cross}) as well as $q_m(p)$ and p itself:

$$\pi_m''(p) \quad = \quad -[2 - L_m(p)\alpha_m^{own}(p)]\varepsilon_m^{own}(p)\frac{q_m(p)}{p} + [\underbrace{\frac{p}{q_m(p)}\frac{\partial x_{Am}}{\partial p_B}(p,p)}_{=\varepsilon_m^{cross}(p)}]\frac{q_m(p)}{p}$$

$$
\begin{aligned}
& -\underbrace{\frac{p-c_m}{p}}_{=L_m(p)}\underbrace{[-\frac{p}{\frac{\partial x_{Am}}{\partial p_A}(p,p)}\frac{\partial^2 x_{Am}}{\partial p_B \partial p_A}(p,p)]}_{=\alpha_m^{cross}(p)}\underbrace{[\frac{p}{q_m(p)}\frac{\partial x_{Am}}{\partial p_A}(p,p)]}_{=-\varepsilon_m^{own}(p)}\frac{q_m(p)}{p} \\
= \; & -\{[2-L_m(\alpha_m^{own}+\alpha_m^{cross})]\varepsilon_m^{own}-\varepsilon_m^{cross}\}\frac{q_m}{p}. \qquad (7)
\end{aligned}
$$

3.1.2 Conduct

Next, we define the *conduct parameter*[15] in market m by $\theta_m(p) \equiv 1 - ADR_m(p)$, where $ADR_m(p)$ is the *aggregate diversion ratio* (Shapiro 1996) in market m, defined by

$$ADR_m(p) \equiv -\frac{\partial x_{Bm}(p,p)/\partial p_A}{\partial x_{Am}(p,p)/\partial p_A} = \frac{\varepsilon_m^{cross}(p)}{\varepsilon_m^{own}(p)}.$$

Here, $ADR_m(p)$ measures the intensity of *rivalness*: if $ADR_m(p)$ is close to one, consumers who leave a firm as a response to an increase in its price are mostly switching to its rival's product. In this way, ACV's derivation for the case of monopoly, where Schmalensee's (1981) method is utilized, is connected to Weyl and Fabinger's (2013) condition in the case of symmetric oligopoly. In particular, ACV's (p. 1606) Proposition 2 (a sufficient condition for price discrimination to increase social welfare) is extended to the case of oligopoly in a simpler manner, using the concept of *pass-through* introduced later in this subsection.[16]

As Weyl and Fabinger (2013, p. 544) argue, $\theta_m(p)$ captures the degree of *industry-level brand loyalty or stickiness*[17] in market m. To see this, note that the conduct parameter is also

[15] This term originates from the empirical literature where conduct itself is a target of estimation ("parameter") without an exact specification of strategic interaction (see, e.g., Bresnahan 1989; Genesove and Mullin 1998; and Corts 1999).

[16] Alternatively, Weyl and Fabinger (2013, p. 531) and Adachi and Fabinger (2021) define the conduct parameter in a market (which, in our interest in price discrimination, can be indexed by m) by $\theta_m \equiv L_m \varepsilon_m^I$ (their mc and ε_D are replaced by our c_m and ε_m^I, respectively) as the Lerner index adjusted by the elasticity of the *industry's* demand. If the first-order condition is given for each market (that is, if full price discrimination is allowed), then $\theta_m(p)$ defined as in Weyl and Fabinger (2013) coincides with $1 - ADR_m(p)$ because $\frac{p_m - c_m}{p_m}\varepsilon_m^{own} = 1$ and thus

$$
\begin{aligned}
L_m(p)\varepsilon_m^I(p) &= \frac{1}{\varepsilon_m^F(p)}\left(-\frac{p}{q_m(p)}\right)q_m'(p) \\
&= -\frac{q_m(p)}{p}\frac{1}{\partial x_{Am}(p,p)/\partial p_A}\left(-\frac{p}{q_m(p)}\right)\left(\frac{\partial x_{Am}}{\partial p_A}(p,p)+\frac{\partial x_{Am}}{\partial p_B}(p,p)\right) \\
&= \frac{\partial x_{Am}(p,p)/\partial p_A + \partial x_{Bm}(p,p)/\partial p_A}{\partial x_{Am}(p,p)/\partial p_A} \text{ (by symmetry)} \\
&= 1 - ADR_m(p) \equiv \theta_m(p)
\end{aligned}
$$

is established. It turns out that this alternative definition is more tractable when firm heterogeneity is introduced in Section 6.

[17] Even if firms' products have the same characteristics across different markets (with no product differentiation), brand loyalty may differ across markets, reflecting the differences in market characteristics (as summarized in demand functions).

expressed by

$$\theta_m(p) = \frac{\epsilon_m^I(p)}{\epsilon_m^{own}(p)},$$

where $\epsilon_m^{own}(p) \geq \epsilon_m^I(p)$. If $\epsilon_m^{own}(p) \to \infty$ as in the case of the price-taking assumption, $\theta_m(p)$ is zero. On the other hand, if $\epsilon_m^{own}(p)$ is equal to $\epsilon_m^I(p)$, that is, the own elasticity is nothing but the industry's elasticity, then it is monopoly and $\theta_m(p) = 1$.[18]

Note that the markup rate alone is not appropriate to measure the rivalness within market m because it can be the case that p_m is close to c_m (the markup rate is close to zero) simply because the price elasticity of the industry's demand $\varepsilon_m^I(p_m)$ is very large, whereas the brand rivalness is so weak that the cross-price elasticity, ε_m^{cross}, remains very small (as a result, in total, ε_m^{own} is very large, which is actually the reason for the low markup rate). However, if ε_m^{cross} is close to ε_m^{own} (i.e., almost of all consumers who leave a firm as a response to its price increase are switching to other rivals' products), then θ_m becomes close to zero *irrespective of the value of the markup rate*. Thus, $\theta_m(p)$, which ranges between 0 and 1, better captures the brand stickiness than $L_m(p)$ does.

3.1.3 Pass-Through

Lastly, we define *pass-through* in market m by $\rho_m \equiv \frac{\partial p_m}{\partial c_m}$. It is a function of $t \in [0, t^*]$ when the constrained problem is considered. In particular,

$$\rho_m[p_m(t)] = \begin{cases} \dfrac{\partial x_{Am}/\partial p_A}{\pi_s'' + \pi_w''} & \text{for } t < t^* \\ \\ \dfrac{\partial x_{Am}/\partial p_A}{\pi_m''} \; (\equiv \rho_m^*) & \text{for } t = t^* \end{cases}$$

is obtained by applying the implicit function theorem to $\partial_p \pi_s(p_w + t) + \partial_p \pi_w(p_w) = 0$ for $t < t^*$ and $\partial_p \pi_m(p_m) = 0$ for $t = t^*$ (i.e., under price discrimination).

If the marginal costs are constant, *quantity pass-through* in market m under price discrimination, which is defined by $\frac{dq_m^*}{d\widetilde{q}}$, where $\widetilde{q}$ is an exogenous amount of output with $\pi_{jm}(p_{jm}, p_{-j,m}) = (p_{jm} - c_m)[x_{jm}(p_{jm}, p_{-j,m}) - \widetilde{q}]$, is expressed by

$$\frac{dq_m^*}{d\widetilde{q}} = q_m'(p_m^*) \cdot \frac{dp_m^*}{d\widetilde{q}} = \frac{q_m'}{\frac{\partial x_{Am}}{\partial p_A}} \cdot \frac{\partial x_{Am}}{\partial p_A} \cdot \frac{dp_m^*}{d\widetilde{q}} = \left(\frac{q_m'}{\frac{\partial x_{Am}}{\partial p_A}} \right) \cdot \left(\frac{\frac{\partial x_{Am}}{\partial p_A}}{\pi_m''} \right) = \theta_m^* \cdot \rho_m^*$$

because the first-order condition with $\widetilde{q}$ indicates $\frac{dp_m^*}{d\widetilde{q}} = \frac{1}{\pi_m''}$.[19]

[18]Because $\frac{p_m - c_m}{p_m} \varepsilon_m^{own} = 1$ and $\varepsilon_m^{own} = \varepsilon_m^I + \varepsilon_m^{cross}$, it is verified that $\theta_m + \frac{p_m - c_m}{p_m} \varepsilon_m^{cross} = 1$. Thus, as long as the products are substitutes ($\varepsilon_m^{cross} > 0$), θ_m is less than one.

[19]Note that this is the case where $\frac{dq_m^*}{d\widetilde{q}}$ is evaluated at $\widetilde{q} = 0$: Miklós-Thal and Shaffer (2021a) derive a general

Equation (7) indicates that

$$
\begin{aligned}
\rho_m^* &= \frac{\frac{\partial x_{Am}}{\partial p_A}}{\left\{2-(L_m)^*[(\alpha_m^{own})^*+(\alpha_m^{cross})^*]-\frac{(\varepsilon_m^{cross})^*}{(\varepsilon_m^{own})^*}\right\}\frac{\partial x_{Am}}{\partial p_A}} \\
&= \frac{1}{2-\frac{(\varepsilon_m^{cross})^*+(\alpha_m^{own})^*+(\alpha_m^{cross})^*}{(\varepsilon_m^{own})^*}}
\end{aligned}
$$

because $(L_m)^* = 1/(\varepsilon_m^{own})^*$. Note here that in the case of monopoly ($(\varepsilon_m^{cross})^* = 0$ and $(\alpha_m^{cross})^* = 0$),

$$
\rho_m^* = \frac{1}{2-\frac{(\alpha_m^{own})^*}{(\varepsilon_m^{own})^*}} \tag{8}
$$

and $\frac{(\alpha_m^{own})^*}{(\varepsilon_m^{own})^*}$ corresponds to ACV's (p. 1603) curvature of the inverse demand, σ_m^*.

3.2 Social Welfare

First, Equation (6) implies that

$$
\begin{aligned}
\frac{W'(t)}{2} &= [p_s(t)-\overline{p}+\overline{p}-c_s]q_s'[p_s(t)]p_s'(t) \\
&\quad +[p_w(t)-\overline{p}+\overline{p}-c_w]q_w'[p_w(t)]p_w'(t) \\
&= \underbrace{[p_s(t)-\overline{p}]q_s'[p_s(t)]p_s'(t)}_{<0}+\underbrace{[p_w(t)-\overline{p}]q_w'[p_w(t)]p_w'(t)}_{<0} \\
&\quad +\sum_{m=s,w}(\overline{p}-c_m)\,q_m'[p_m(t)]p_m'(t).
\end{aligned}
$$

This derivation coincides with the case of monopoly as shown in ACV's (p. 1604) Equality (3) if there are no cost differentials (i.e., $c_s = c_w \equiv c$), with two minor modifications: (i) the left hand side is $\frac{W'(t)}{2}$ rather than $W'(t)$ itself, and (ii) the last term of ACV's Equality (3) is replaced by $\frac{Q'(t)}{2}$ rather than $Q'(t)$ because $(\frac{1}{2})\sum_{m=s,w}(\overline{p}-c_m)\,q_m'[p_m(t)]p_m'(t) = (\overline{p}-c)\,(\frac{Q'(t)}{2})$. If cost differentials are permitted, it is observed that an increase in the *weighted* aggregate output, $\sum_{m=s,w}(\overline{p}-c_m)\,q_m'[p_m(t)]p_m'(t) > 0$, is *necessary* for price discrimination to increase social welfare, as in the case of monopoly (see Part A of the Online Appendix for the case of a general number of markets).

Now, we examine the effects of allowing third-degree price discrimination on social welfare

formula for $\widetilde{q} > 0$, correcting Weyl and Fabinger's (2013) arguments. If marginal costs are non-constant (see Part B of the Online Appendix), then $\pi_{jm}(p_{jm}, p_{-j,m}) = p_{jm}\cdot[x_{jm}(p_{jm}, p_{-j,m})-\widetilde{q}] - c_m[x_{jm}(p_{jm}, p_{-j,m})-\widetilde{q}]$ should be considered, where $c_m(\cdot)$ is the cost function, and thus $\theta_m^*\rho_m^*$ is no longer the quantity pass-through under price discrimination (that is, when $\widetilde{q} = 0$). See Weyl and Fabinger (2013, p. 572) for a precise expression of quantity pass-through with non-constant marginal costs.

in detail. Note first that

$$\begin{aligned}\frac{W'(t)}{2} &= \underbrace{\left(-\frac{\pi_s''\pi_w''}{\pi_s''+\pi_w''}\right)}_{>0} \\ &\quad\times\left(\frac{(p_w(t)-c_w)q_w'[p_w(t)]}{\pi_w''}-\frac{(p_s(t)-c_s)q_s'[p_s(t)]}{\pi_s''}\right). \qquad (9)\end{aligned}$$

We then define the *profit margin* in market m by

$$\mu_m(p) \equiv p - c_m,$$

and follow ACV (p. 1605), who define

$$z_m(p) \equiv \frac{\mu_m(p)q_m'(p)}{\pi_m''(p)},$$

which is "the ratio of the marginal effect of a price increase on social welfare to the second derivative of the profit function."[20] However, our q_m' and π_m'' have *strategic effects* as Equations (1) and (4) show.

As in ACV (p. 1605), we can write

$$\frac{W'(t)}{2} = \underbrace{\left(-\frac{\pi_s''\pi_w''}{\pi_s''+\pi_w''}\right)}_{>0}\{z_w[p_w(t)]-z_s[p_s(t)]\},$$

and their lemma also holds in our case of oligopoly if we assume z_m is *increasing* (the increasing ratio condition for social welfare; IRCW).[21] Then, the global concavity of $W(t)$ is attained as in the case of $Q(t)$.

Now, using conduct, profit margin, and pass-through, we obtain the following sufficient conditions for price discrimination to increase or decrease social welfare.

[20]Here, $\mu_m(p)q_m'(p)$ can be interpreted as the marginal effect of a price increase on social welfare in market m because:

$$\frac{\mathrm{d}[\overbrace{\tfrac{1}{2}U_m[q_m(p)]-c_mq_m(p)}^{\text{per-firm (normalized)}}]}{\mathrm{d}p} = \mu_m(p)q_m'(p).$$

[21]Note that

$$z_m'(p) = \frac{[\mu_m(p)q_m''(p)+q_m'(p)]\pi_m''(p)-\mu_m(p)q_m'(p)\pi_m'''(p)}{[\pi_m''(p)]^2}$$

and thus, the IRCW is equivalent to

$$[\mu_m(p)q_m''(p)+q_m'(p)]\pi_m''(p) > \mu_m(p)q_m'(p)\pi_m'''(p).$$

Appendix B of ACV discusses sufficient conditions for the IRCW to hold in the case of monopoly.

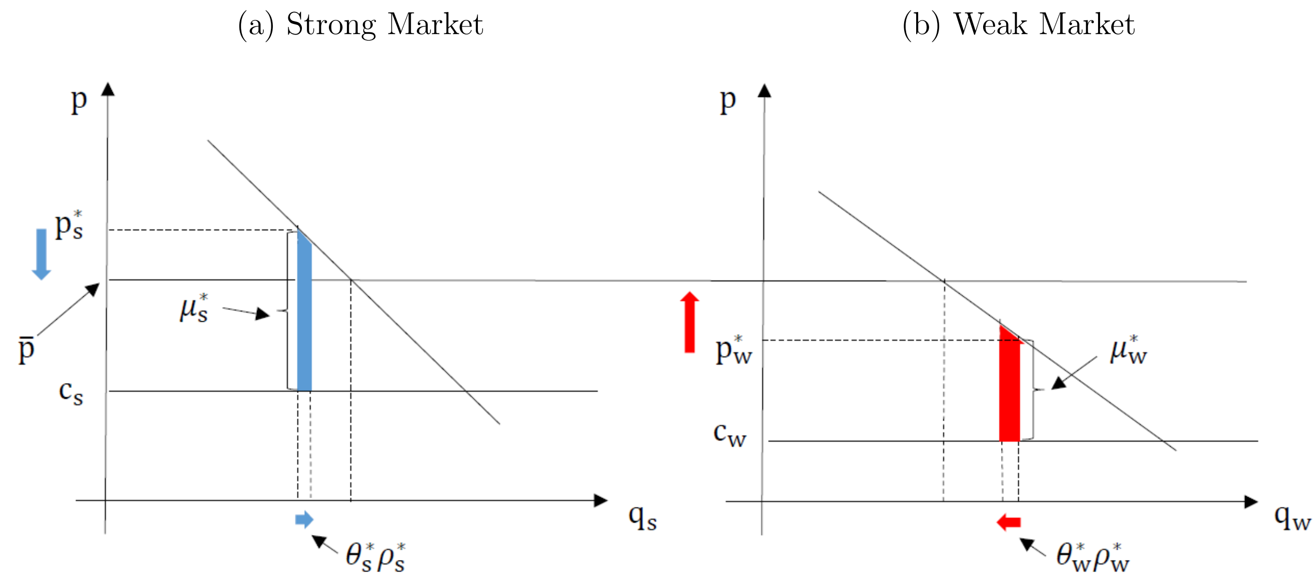

Figure 2: Welfare Comparison in terms of Sufficient Statistics

Proposition 1. *Given the IRCW, if $\mu_w^*\theta_w^*\rho_w^* > \mu_s^*\theta_s^*\rho_s^*$ holds, then price discrimination increases social welfare. Conversely, if*

$$\frac{\overline{\mu}_w\overline{\theta}_w\overline{\rho}_w}{\overline{\mu}_s\overline{\theta}_s\overline{\rho}_s} \leq \frac{\pi_w''(\overline{p})}{\pi_s''(\overline{p})}$$

holds, then price discrimination decreases social welfare.

Proof. See Appendix, Part A. □

Roughly speaking, if either (i) *conduct* (θ), (ii) *profit margin* (μ), or (iii) *pass-through* (ρ) is sufficiently *small* in the *strong* market, then social welfare is likely to be higher under price discrimination. In particular, if these three measures are calculated (or estimated) in each separate market, then it would assist one to judge whether price discrimination is desirable from a society's viewpoint. As explained above, if the marginal costs are constant, $\theta_m^*\rho_m^*$ is interpreted as *quantity* pass-through: $\mu_m^* \times \theta_m^*\rho_m^*$ approximates the trapezoid generated by a small deviation from (perfect) price discrimination that captures the marginal welfare gain in the strong market and the marginal welfare loss in the weak market (see Figure 2). If the latter is larger than the former, such a deviation lowers social welfare, and owing to the IRCW, this argument extends globally so that the regime switch to uniform pricing definitely decreases social welfare. Note that this comparison will be a little bit more involved when starting at uniform pricing for the same reason as explained after Proposition 3.

This proposition also has the following attractive feature. Suppose that price discrimination is being conducted. Then, to evaluate it from a viewpoint of social welfare, one only needs the local information: first, θ_m^*, μ_m^* and ρ_m^* for each $m = s, w$, are computed, and if the sufficient condition above is satisfied, then the ongoing price discrimination is justified. In addition, to compute θ_m^*, μ_m^* and ρ_m^* in equilibrium, *information on marginal cost is unnecessary*: once a specific form of demand function, $q_{jm} = x_{jm}(p_{jm}, p_{-j,m})$, is provided (and if the IRC is satisfied), then the three variables are computed in the following manner:[22]

$$\begin{cases} \theta_m^* = 1 - \dfrac{(\varepsilon_m^{cross})^*}{(\varepsilon_m^{own})^*} \\ \rho_m^* = \dfrac{1}{2 - \frac{(\varepsilon_m^{cross})^* + (\alpha_m^{own})^* + (\alpha_m^{cross})^*}{(\varepsilon_m^{own})^*}} \\ \mu_m^* = \dfrac{p_m^*}{(\varepsilon_m^{own})^*}. \end{cases} \tag{10}$$

Thus, if the firm's demand for each market m is estimated and the discriminatory price p_m^* is observed, then one can easily compute θ_m^*, μ_m^*, and ρ_m^*, using up to second-order demand characteristics.[23]

Why is the adjustment term,

$$\pi_m''(\overline{p}) = -\{[2 - \overline{L}_m(\overline{\alpha}_m^{own} + \bar{\alpha}_m^{cross})]\bar{\varepsilon}_m^{own} - \bar{\varepsilon}_m^{cross}\}\frac{q_m(\overline{p})}{\overline{p}},$$

necessary for the deviation from uniform pricing? As stated above, $\theta_m^*\rho_m^*$ is interpreted as quantity pass-through under price discrimination in market m if marginal cost is constant. Thus, the first part of the proposition simply claims that aggregate output is raised by price discrimination if the marginal reduction in quantity caused by a small deviation from price discrimination in the market where price discrimination increases output (i.e., the weak market) is larger than the marginal increase in quantity in the strong market. The second part describes the opposite case, although it does not permit a direct comparison because pass-through is not defined market-wise unless the pricing regime is "perfect" or "full" price discrimination (i.e., $t = t^*$), where the first-order conditions are given market-wise. Note that if $|\pi_m''|$ is small, then π_m is "flat," and thus the price shift $|\Delta p_m|$ in response to some change would be large. Hence, the role of $\frac{\pi_w''}{\pi_s''}$ is to adjust measurement units for $\frac{\rho_w}{\rho_s}$. For example, if $|\pi_w''|$ is very small, then ρ_w is "over represented," and thus it should be "penalized" so that the right hand side of the inequality in the proposition becomes small. The same argument also applies to the analysis of social welfare and consumer surplus below.

Even if there are no cost differentials (i.e., $c_s = c_w$), this expression cannot be further simplified. In other words, this expression is already robust to the inclusion of cost differentials. Now, if we further assume that there are no strategic effects (i.e., $\theta_m = 1$), then the condition

[22] An alternative expression for μ_m^* is $\mu_m^* = \frac{c_m}{(\varepsilon_m^{own})^* - 1}$ if the cost information is used.

[23] It should be emphasized that the second-order supply property, i.e., the derivative of marginal cost, would be necessary if non-constant marginal cost is allowed, as suggested by Adachi and Fabinger (2021) in the context of general "taxation" (pure taxation and other additional costs from external changes).

$\mu_w^* \theta_w^* \rho_w^* \geq \mu_s^* \theta_s^* \rho_s^*$ becomes $\frac{p_s^* - c}{p_w^* - c} \leq \frac{1/\rho_s^*}{1/\rho_w^*}$, which coincides with

$$\frac{p_w^* - c}{2 - \sigma_w^*} \geq \frac{p_s^* - c}{2 - \sigma_s^*}$$

in Proposition 2 of ACV (p. 1606), where σ_m^* is what they call the curvature of the inverse demand function (under price discrimination), because of $\sigma_m^* = \frac{(\alpha_m^{own})^*}{(\varepsilon_m^{own})^*}$ (see the last part of Subsubsection 3.1.3 above) and Equation (8). Thus, price discrimination increases social welfare "if the discriminatory prices are not far apart and the inverse demand function in the weak market is locally more convex than that in the strong market" (ACV, p. 1602). As compared to Figure 1, Figure 2 shows the usefulness of the sufficient statistics in welfare evaluation.

4 Parametric Examples of Market Demand

To consider the following parametric examples of market demand, we assume that there are two markets (strong and weak), and two symmetric firms. These demand functions are among the commonly-used demand systems (Quint 2014). Note that to save notation, the same β_m is repeatedly used in the following three examples, but with different meanings (similarly, ω_m appears twice in the first and the third examples).

Recall that Proposition 1 provides a sufficient condition for price discrimination to increase social welfare. By noting that

$$\text{sign}[W'(0)] = sign[\mu_w(\overline{p}) \frac{q_w'(\overline{p})}{\pi_w''(\overline{p})} - \mu_s(\overline{p}) \frac{q_s'(\overline{p})}{\pi_s''(\overline{p})}]$$

from Equation (6), we are also able to provide another sufficient condition for price discrimination to decrease social welfare.

Proposition 2. *Given the IRCW, if the profit margin in strong market relative to the weak market at the uniform price $\overline{p}$ is sufficiently large, i.e.,*

$$\overline{\mu}_s \frac{\overline{q}_s'}{\overline{\pi}_s''} \geq \overline{\mu}_w \frac{\overline{q}_w'}{\overline{\pi}_w''}, \tag{11}$$

then price discrimination decreases social welfare.

If there are *no strategic effects* (i.e., $\partial x_{Bm}/\partial p_A = 0$ or $\theta_m(p) = 1$), then $\frac{\pi_m''(\overline{p})}{q_m'(\overline{p})} = 2 - L_m(\overline{p})\overline{\alpha}_m^{own}$, and Inequality (11) above reduces to $\frac{\overline{\mu}_s}{\overline{\mu}_w} \geq \frac{2 - L_s(\overline{p})\overline{\alpha}_s^{own}}{2 - L_w(\overline{p})\overline{\alpha}_w^{own}}$. On the other hand, if there are *no cost differentials* (i.e., $c_s = c_w \equiv c$), then Inequality (11) above reduces to $\frac{\pi_w''(\overline{p})}{q_w'(\overline{p})} \geq \frac{\pi_s''(\overline{p})}{q_s'(\overline{p})}$ because the profit margins are the same in the two markets. Thus, if there are no strategic effects and no cost differentials, then inequality (11) coincides with ACV's (p. 1605) Proposition 1 ($\overline{\alpha}_s^{own} \geq \overline{\alpha}_w^{own}$ in our notation; in their notation, $\alpha_s(\overline{p}) \geq \alpha_w(\overline{p})$) because $L_s(\overline{p}) = L_w(\overline{p})$. That

			Linear	CES	Logit
Conduct		θ_m^*	$1-\delta_m$	$\dfrac{2(1-\beta_m)}{2-\beta_m-\beta_m\eta_m}$	$\dfrac{1-2q_m^*}{1-q_m^*}$
Profit margin		μ_m^*	$\dfrac{(1-\delta_m)(\omega_m-c_m)}{2-\delta_m}$	$\dfrac{2(1-\beta_m)(1-\beta_m\eta_m)}{\beta_m[1+(1-2\beta_m)\eta_m]}c_m$	$\dfrac{1}{\beta_m(1-q_m^*)}$
Pass-through		ρ_m^*	$\dfrac{1}{2-\delta_m}$	$\dfrac{2-\beta_m-\beta_m\eta_m}{\beta_m[1+(1-2\beta_m)\eta_m]}$	$\dfrac{1}{1-\frac{q_m^*(1-2q_m^*)}{(1-q_m^*)^2}}$
Elasticities					
First-order	(own)	ε_m^{own}	$\dfrac{(1-\delta_m)\omega_m+c_m}{(1-\delta_m)(\omega_m-c_m)}$	$\dfrac{2-\beta_m-\beta_m\eta_m}{2(1-\beta_m)(1-\beta_m\eta_m)}$	$\beta_m p_m(1-q_m)$
	(cross)	ε_m^{cross}	$\dfrac{[(1-\delta_m)\omega_m+c_m]\delta_m}{(1-\delta_m)(\omega_m-c_m)}$	$\dfrac{\beta_m(1-\eta_m)}{2(1-\beta_m)(1-\beta_m\eta_m)}$	$\beta_m p_m q_m$
Second-order (Curvatures)	(own)	α_m^{own}	0	$\dfrac{(2-\beta_m\eta_m)(4-\beta_m-3\beta_m\eta_m)}{2(1-\beta_m\eta_m)(2-\beta_m-\beta_m\eta_m)}$	$\beta_m p_m(1-2q_m)$
	(cross)	α_m^{cross}	0	$-\dfrac{\beta_m(1-\eta_m)(2-\beta_m\eta_m)}{2(1-\beta_m\eta_m)(2-\beta_m-\beta_m\eta_m)}$	$\dfrac{\beta_m p_m q_m(1-2q_m)}{1-q_m}$

Table 1: Sufficient statistics under price discrimination (for duopoly). See below for the notations.

is, the firm's "direct demand function in the strong market is at least as convex as that in the weak market at the nondiscriminatory price" (ACV, p. 1602).

Let the set of related parameters be denoted by Θ. If

$$G(\Theta) \equiv \frac{\overline{\mu}_s}{\overline{\mu}_w} - \frac{\overline{\pi}''_s/\overline{q}'_s}{\overline{\pi}''_w/\overline{q}'_w}$$

and

$$H(\Theta) \equiv \frac{\mu_s^*}{\mu_w^*} - \frac{1/(\theta_s^*\rho_s^*)}{1/(\theta_w^*\rho_w^*)}$$

are defined, then $G(\Theta) \geq 0$ implies $\Delta W < 0$ and $H(\Theta) < 0$ implies $\Delta W > 0$. Table 1 shows the sufficient statistics as well as the first- and second-order elasticities under price discrimination for the case of two symmetric firms (see Part C of the Online Appendix for details).

4.1 Linear Demand

Linear demand is derived from the quadratic utility of the representative consumer in market m under symmetric product differentiation:

$$U_m(\mathbf{x}_m) = \omega_m \cdot (x_{Am} + x_{Bm}) - \frac{1}{2}\left(\beta_m [x_{Am}]^2 + 2\gamma_m x_{Am} x_{Bm} + \beta_m [x_{Bm}]^2\right),$$

which yields linear inverse demand, $P_{jm}(x_{jm}, x_{-j,m}) = \omega_m - \beta_m x_{jm} - \gamma_m x_{-j,m}$, and the corresponding direct demand in market m is

$$x_{jm}(p_{jm}, p_{-j,m}; \omega_m, \beta_m, \gamma_m) = \frac{1}{(1-\delta_m^2)\beta_m}\left[\omega_m(1-\delta_m) - p_{jm} + \delta_m p_{-j,m}\right]$$

for firm j, where $\delta_m \equiv \frac{\gamma_m}{\beta_m} \in [0,1)$ is the *strength of substitutability*: if δ_m is close to one, market m is approximated by perfect competition, whereas if δ_m is equal to zero, each firm behaves as a monopolist.[24]

Now, consider

$$G(\boldsymbol{c}, \boldsymbol{\delta}, \boldsymbol{\beta}, \boldsymbol{\omega}) = \frac{\overline{p} - c_s}{\overline{p} - c_w} - \frac{(2-\delta_s)(1-\delta_w)}{(1-\delta_s)(2-\delta_w)} \geq 0$$

as well as

$$H(\boldsymbol{c}, \boldsymbol{\delta}, \boldsymbol{\beta}, \boldsymbol{\omega}) = \frac{(\omega_s - c_s)(1-\delta_s)(2-\delta_w)}{(\omega_w - c_w)(1-\delta_w)(2-\delta_s)} - \frac{(2-\delta_s)(1-\delta_w)}{(1-\delta_s)(2-\delta_w)} < 0.$$

If there is no product differentiation (i.e., monopoly: $\gamma_m = 0$ and hence $\delta_m = 0$), then

$$G(\boldsymbol{c}, \boldsymbol{\delta}, \boldsymbol{\beta}, \boldsymbol{\omega}) = \frac{\overline{p} - c_s}{\overline{p} - c_w} - 1$$

so that $G(\boldsymbol{c}, \boldsymbol{\delta}, \boldsymbol{\beta}, \boldsymbol{\omega}) \geq 0 \Leftrightarrow c_w \geq c_s$. Similarly,

$$H(\boldsymbol{c}, \boldsymbol{\delta}, \boldsymbol{\beta}, \boldsymbol{\omega}) = \frac{p_s^* - c_s}{p_w^* - c_w} - 1$$

[24] Note that the demand system here can be interpreted as the Levitan-Shubik demand system (Shubik and Levitan 1980), in which the representative consumer's utility in our two-firm case is given by

$$\begin{aligned} U_m(\mathbf{x}_m) &= \omega_m \cdot (x_{Am} + x_{Bm}) - \frac{1}{2\lambda_m}\left(\left[\sigma_m + \frac{1-\sigma_m}{1/2}\right][x_{Am}]^2 + 2\sigma_m x_{Am} x_{Bm} + \left[\sigma_m + \frac{1-\sigma_m}{1/2}\right][x_{Bm}]^2\right) \\ &= \omega_m \cdot (x_{Am} + x_{Bm}) - \frac{1}{2}\left(\frac{2-\sigma_m}{\lambda_m}[x_{Am}]^2 + 2\left[\frac{\sigma_m}{\lambda_m}\right] x_{Am} x_{Bm} + \frac{2-\sigma_m}{\lambda_m}[x_{Bm}]^2\right), \end{aligned}$$

where $\lambda_m > 0$ and $0 \leq \sigma_m < 1$ (see Choné and Linnemer's (2020, p. 3) Equation 1). Then, our γ_m is expressed as $\frac{\sigma_m/\lambda_m}{(2-\sigma_m)/\lambda_m} = \frac{\sigma_m}{2-\sigma_m}$, and this range is $[0,1)$ for $\sigma_m \in [0,1)$.

so that $H(\boldsymbol{c},\boldsymbol{\delta},\boldsymbol{\beta},\boldsymbol{\omega}) \leq 0 \Leftrightarrow \omega_s - \omega_w \leq c_s - c_w$. In this case of monopoly under linear demand, Chen and Schwartz (2015, p. 454) obtain a *necessary and sufficient* for price discrimination to increase social welfare: in our notation, it is $\omega_s - \omega_w \leq 3(c_s - c_w)$ (with $\gamma_s = 0$ and $\gamma_w = 0$). As Panel (a) in Figure 3 shows, our sufficient condition for price discrimination to decrease social welfare in the case of monopoly (i.e., $c_s \geq c_w$) is weaker than Chen and Schwartz' (2015, p. 454) necessary and sufficient condition. Under oligopoly, however, the (c_s, c_w) region for sufficiency for $\Delta W \geq 0$, i.e, the region of $H(c_s, c_w; \boldsymbol{\delta},\boldsymbol{\beta},\boldsymbol{\omega}) \leq 0$ is now smaller, whereas the region for sufficiency for $\Delta W \leq 0$, i.e, the region of $G(c_s, c_w; \boldsymbol{\delta},\boldsymbol{\beta},\boldsymbol{\omega}) \geq 0$ becomes larger (Panel (b) in Figure 3).

In this numerical example, the line of $c_w = c_s$ is included in the region of $G(c_s, c_w; \boldsymbol{\delta},\boldsymbol{\beta},\boldsymbol{\omega}) \geq 0$; price discrimination decreases social welfare when the marginal costs are common across markets. However, it is possible that social welfare is higher under price discrimination in this case, more specifically, if $\delta_s \equiv \gamma_s/\beta_s$ is sufficiently higher than $\delta_w \equiv \gamma_w/\beta_w$, as shown in Figure 4, where $c_s = c_w = 0.2$ and G and H are interpreted as $G(\delta_s, \delta_w; \boldsymbol{c},\boldsymbol{\beta},\boldsymbol{\omega})$ and $H(\delta_s, \delta_w; \boldsymbol{c},\boldsymbol{\beta},\boldsymbol{\omega})$, respectively. This example is consistent with Adachi and Matsushima's (2014, p.1239) Proposition 1 (and their Figures 4 and 5) with an additional result: here, the sufficient condition for $\Delta W \leq 0$ is also included, whereas their Proposition 1 establishes a necessary and sufficient condition for $\Delta W > 0$ in the case of linear demands.[25]

4.2 CES (Constant Elasticity of Substitution) Demand

Suppose that the representative consumer's utility in market m is given by:

$$U_m(\mathbf{x}_m) = \left(x_{Am}^{\beta_m} + x_{Bm}^{\beta_m}\right)^{\eta_m},$$

where $\beta_m \eta_m \in (0,1)$ is the *degree of homogeneity*, and $0 < \eta_m < 1$ and $0 < \beta_m \leq 1$ are also assumed (Vives 1999, pp. 147-8).[26] The elasticity of substitution between the two goods is *constant*, $1/(1-\beta_m)$, and the direct demand function for good j is given by

$$x_{jm}(p_{jm}, p_{-j,m}; \beta_m, \eta_m) = (\beta_m \eta_m)^{\frac{1}{1-\beta_m\eta_m}} \frac{p_{jm}^{\frac{-1}{1-\beta_m}}}{\left(p_{jm}^{\frac{-\beta_m}{1-\beta_m}} + p_{-jm}^{\frac{-\beta_m}{1-\beta_m}}\right)^{\frac{1-\eta_m}{1-\beta_m\eta_m}}}.$$

[25] Appendix B of Adachi and Matsushima (2014) discusses the parametric restriction for the weak market to be open. Here, in Figures 3 and 4, we directly verify that the discriminatory price in the weak market is actually lower than that in the strong market.

[26] Anderson, de Palma, and Thisse (1992, pp. 85-90) discuss how this demand system can be microfounded by discrete choice modeling.

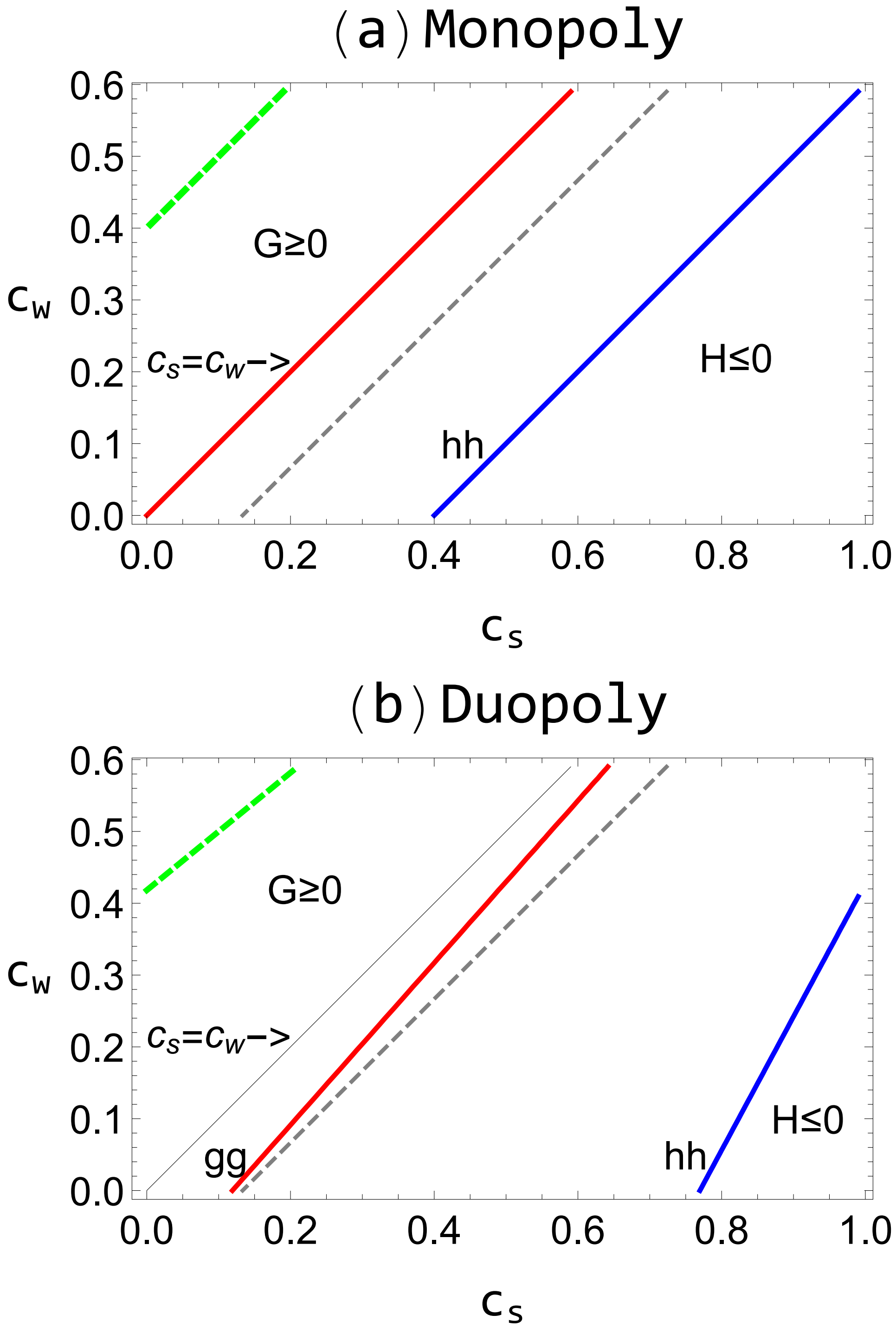

Figure 3: Linear demand with $\omega_s = 1.2$, $\omega_w = 0.8$, $\beta_s = 1.2$, and $\beta_w = 1.4$. For (a), $\gamma_s = \gamma_w = 0$ (monopoly), whereas for (b), $\gamma_s = 0.2$ and $\gamma_w = 0.7$ (duopoly). For p_w^* to be actually lower than p_s^*, c_w, relative to c_s, must be sufficiently small. Specifically, the region of (c_s, c_w) is restricted to the area below the dashed thick line in the upper left. In each panel, the dashed line corresponds to Chen and Schwarz' (2015) threshold for the necessity and sufficiency for $\Delta W \geq 0$ in the case of monopoly. The region for $H \leq 0$ is the area below line hh. In panel (a), the region for $G \geq 0$ is the area between the dashed thick line and line $c_s = c_w$, whereas it is the area between the dashed thick line and line gg in panel (b).

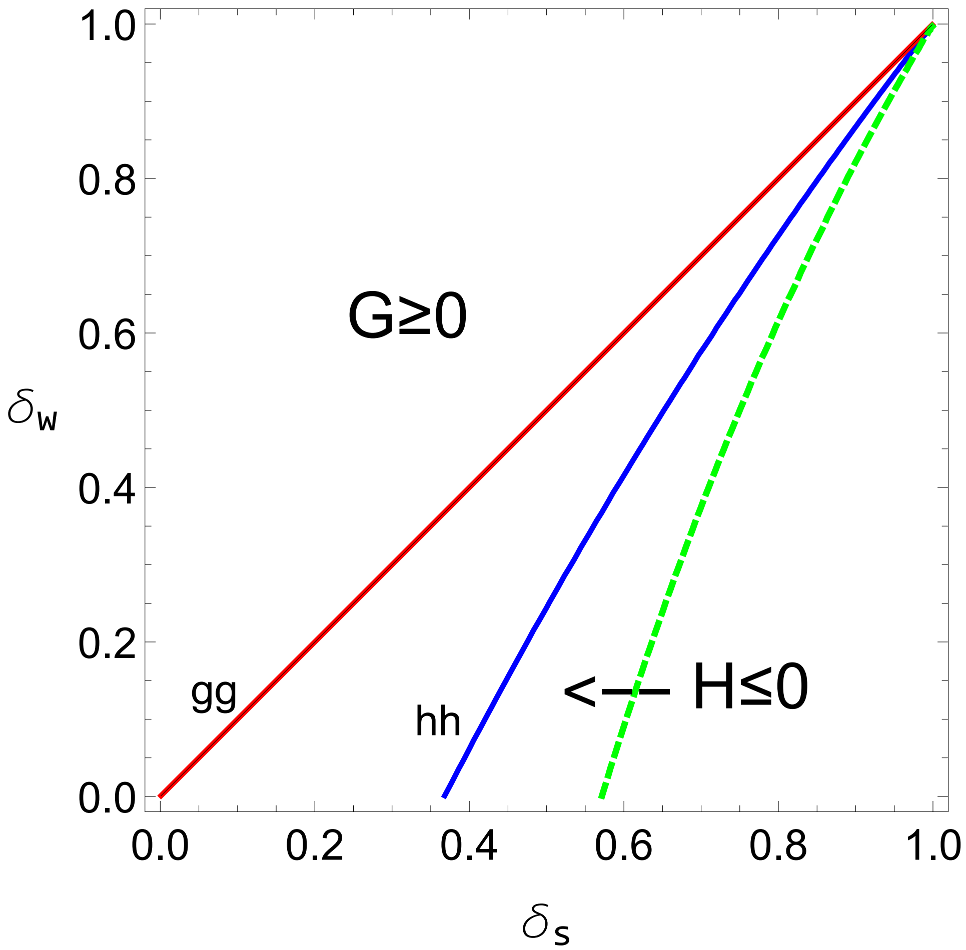

Figure 4: Linear demands with $\omega_s = 1.2$, $\omega_w = 0.8$, $\beta_s = 1.2$, and $\beta_w = 1.4$. It assumed that $c_s = c_w = 0.2$. The region of (δ_s, δ_w) is restricted to the area above the dashed thick line in the lower right for p_s^* to be actually higher than p_w^*. The region for $H \leq 0$ is the area between the dashed thick line and line hh, whereas the region for $G \geq 0$ is the area above line gg.

Thus, each firm's demand under symmetric pricing is given by:

$$q_m(p) \quad = \quad 2^{-\frac{1-\eta_m}{1-\beta_m\eta_m}} (\beta_m\eta_m)^{\frac{1}{1-\beta_m\eta_m}} \cdot p^{\frac{-1}{1-\beta_m\eta_m}},$$

which implies that

$$q_m'(p) = -\frac{2^{-\frac{1-\eta_m}{1-\beta_m\eta_m}} (\beta_m\eta_m)^{\frac{1}{1-\beta_m\eta_m}}}{1-\beta_m\eta_m} \cdot p^{-\frac{2-\beta_m\eta_m}{1-\beta_m\eta_m}}.$$

First, it is verified that

$$H(\boldsymbol{c},\boldsymbol{\beta},\boldsymbol{\eta}) \quad = \quad \frac{\beta_w[1+(1-2\beta_w)\,\eta_w](1-\beta_s)(1-\beta_s\eta_s)c_s}{\beta_s[1+(1-2\beta_s)\,\eta_s](1-\beta_w)(1-\beta_w\eta_w)c_w} - \frac{\beta_s[1+(1-2\beta_s)\eta_s](1-\beta_w)}{\beta_w[1+(1-2\beta_w)\eta_w](1-\beta_s)}.$$

Then, consider the uniform price. In symmetric equilibrium,

$$\bar{y}_m\bar{\varepsilon}_m^{own} \quad = \quad \frac{q_m(\overline{p})}{q_s(\overline{p})+q_w(\overline{p})} \cdot \frac{2-\beta_m-\beta_m\eta_m}{2(1-\beta_m)(1-\beta_m\eta_m)},$$

and hence the equilibrium uniform price satisfies:

$$\sum_{m=s,w} q_m(\overline{p}) \left[\frac{(2-\beta_m-\beta_m\eta_m)(\overline{p}-c_m)}{2(1-\beta_m)(1-\beta_m\eta_m)} - \overline{p}\right] = 0,$$

which should be numerically solved. Hence, it is verified that

$$G(\boldsymbol{c},\boldsymbol{\beta},\boldsymbol{\eta}) \quad = \quad \frac{\overline{p}-c_s}{\overline{p}-c_w} - \frac{(1-\beta_w)(1-\beta_w\eta_w)}{(1-\beta_s)(1-\beta_s\eta_s)} \times \frac{(4-3\beta_s-\beta_s\eta_s)(1-\beta_s\eta_s)-(2-\beta_s-\beta_s\eta_s)(2-\beta_s\eta_s)\cdot\dfrac{\overline{p}-c_s}{\overline{p}}}{(4-3\beta_w-\beta_w\eta_w)(1-\beta_w\eta_w)-(2-\beta_w-\beta_w\eta_w)(2-\beta_w\eta_w)\cdot\dfrac{\overline{p}-c_w}{\overline{p}}}.$$

Figure 5 shows the region of (β_s, β_w), assuming that $\eta_s = 0.6$, $\eta_w = 0.4$, and $c_s = c_w = 0.2$. In this case, the region for $H \leq 0$ does not exist in duopoly.

4.3 Multinomial Logit Demand with Outside Opition

In each market $m = s, w$, firm j faces the following market share/demand function:

$$x_{jm}(p_{jm}, p_{-j,m}; \omega_m, \beta_m) = \frac{\exp(\omega_m - \beta_m p_{jm})}{1+\sum_{j'=A,B}\exp(\omega_m - \beta_m p_{j'm})} \in (0,1),$$

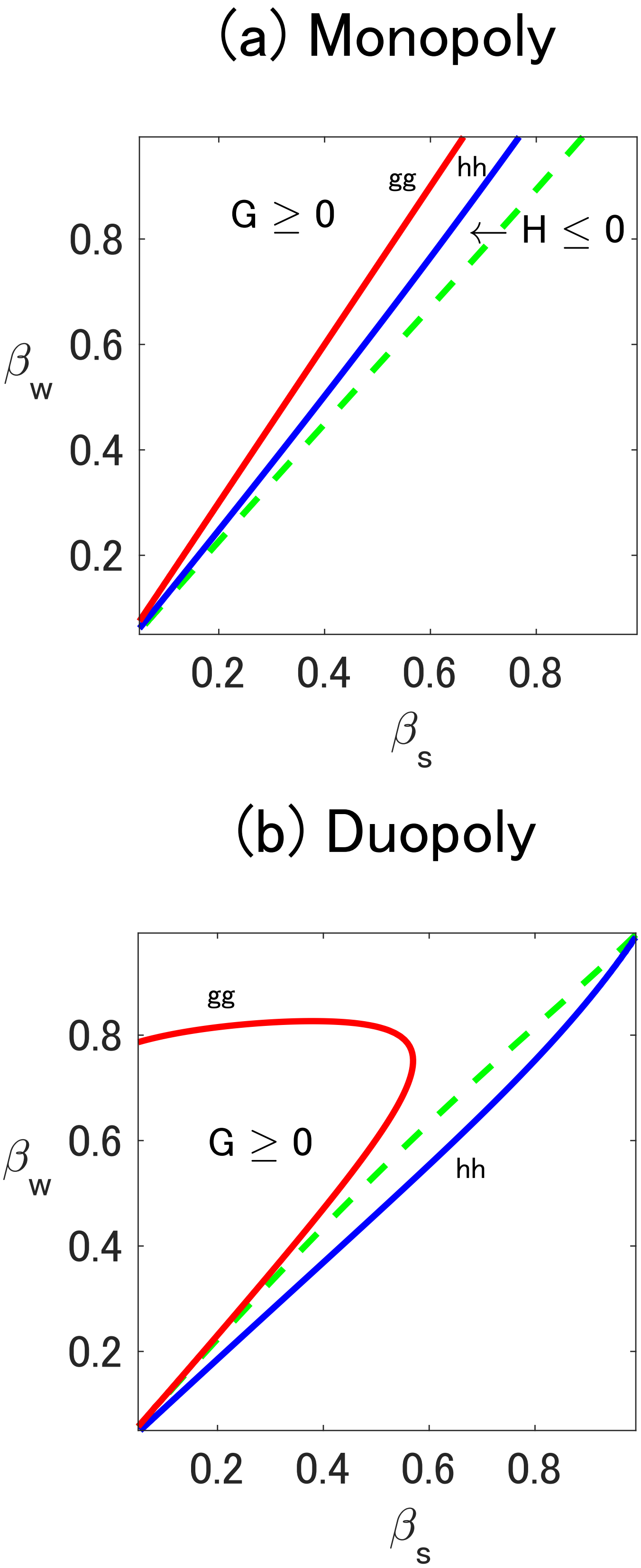

Figure 5: CES demand with $\eta_s = 0.6$, $\eta_w = 0.4$, and $c_s = c_w = 0.2$. Panels (a) and (b) are monopoly and duopoly, respectively. As in Figure 4, the region of (β_s, β_w) is restricted to the area above the dashed thick curve in the lower right for p_s^* to be actually higher than p_w^*.

where $\omega_m > 0$ is now the product-specific utility and $\beta_m > 0$ is the *responsiveness* of the representative consumer in market m to the price.[27] Then, under symmetric pricing, each firm's share is

$$q_m(p) = \frac{\exp(\omega_m - \beta_m p)}{1 + 2\exp(\omega_m - \beta_m p)}$$

and the symmetric discriminatory equilibrium price $p_m^* = p_m^*(c_m, \omega_m, \beta_m)$ satisfies:

$$\underbrace{p_m^* - c_m}_{=\mu_m^*} - \frac{1}{\beta_m(1 - q_m^*)} = 0$$

and∉

$$q_m^* \equiv q_m(p_m^*) = \frac{\exp(\omega_m - \beta_m p_m^*)}{1 + 2\exp(\omega_m - \beta_m p_m^*)}.$$

Both p_m^* and q_m^* should be jointly solved numerically, and it is shown that

$$H(\boldsymbol{c}, \boldsymbol{\beta}, \boldsymbol{\omega}) = \left(\frac{1 - q_w^*}{1 - q_s^*}\right)\left(\frac{\beta_w}{\beta_s} - \frac{1 - 2q_w^*}{1 - 2q_s^*} \cdot \frac{1 - q_s^* - [q_s^*]^2}{1 - q_w^* - [q_w^*]^2}\right).$$

The equilibrium uniform price $\overline{p} = \overline{p}(\mathbf{c}, \boldsymbol{\omega}, \boldsymbol{\beta})$ satisfies

$$\sum_{m=s,w} q_m(\overline{p})\left\{1 - \beta_m(\overline{p} - c_m)[1 - q_m(\overline{p})]\right\} = 0,$$

which should also be numerically solved. It is also verified that

$$G(\boldsymbol{c}, \boldsymbol{\beta}, \boldsymbol{\omega}) = \frac{\overline{p} - c_s}{\overline{p} - c_w} - \frac{\overline{\pi}_s''/\overline{q}_s'}{\overline{\pi}_w''/\overline{q}_w'},$$

where,

$$\begin{aligned}
\frac{\overline{\pi}_m''}{\overline{q}_m'} &= \frac{2 - L_m(\overline{p}) \cdot [\ \underbrace{\alpha_m^{own}(\overline{p})}_{=\beta_m \overline{p}(1-2\overline{q}_m)} + \underbrace{\alpha_m^{cross}(\overline{p})}_{=\frac{\beta_m \overline{p} \cdot \overline{q}_m(1-2\overline{q}_m)}{1-\overline{q}_m}}\ ]}{\underbrace{\theta_m(\overline{p})}_{=\frac{1-2\overline{q}_m}{1-\overline{q}_m}}} - \underbrace{\frac{1 - \theta_m(\overline{p})}{\theta_m(\overline{p})}}_{=\frac{\overline{q}_m}{1-2\overline{q}_m}} \\
&= \frac{2 - 3\overline{q}_m}{1 - 2\overline{q}_m} - \beta_m(\overline{p} - c_m).
\end{aligned}$$

[27] Anderson, de Palma, and Thisse (1987) argue that the indirect utility of the representative consumer in market m is given by

$$V_m(\mathbf{p}_m) = \frac{\ln\left[\exp\left(\omega_m - \beta_m p_{Am}\right) + \exp\left(\omega_m - \beta_m p_{Bm}\right)\right]}{\beta_m}.$$

This demand form can also be microfounded by the random utility model (see, e.g., Anderson, de Palma, and Thisse 1992, Ch. 2).

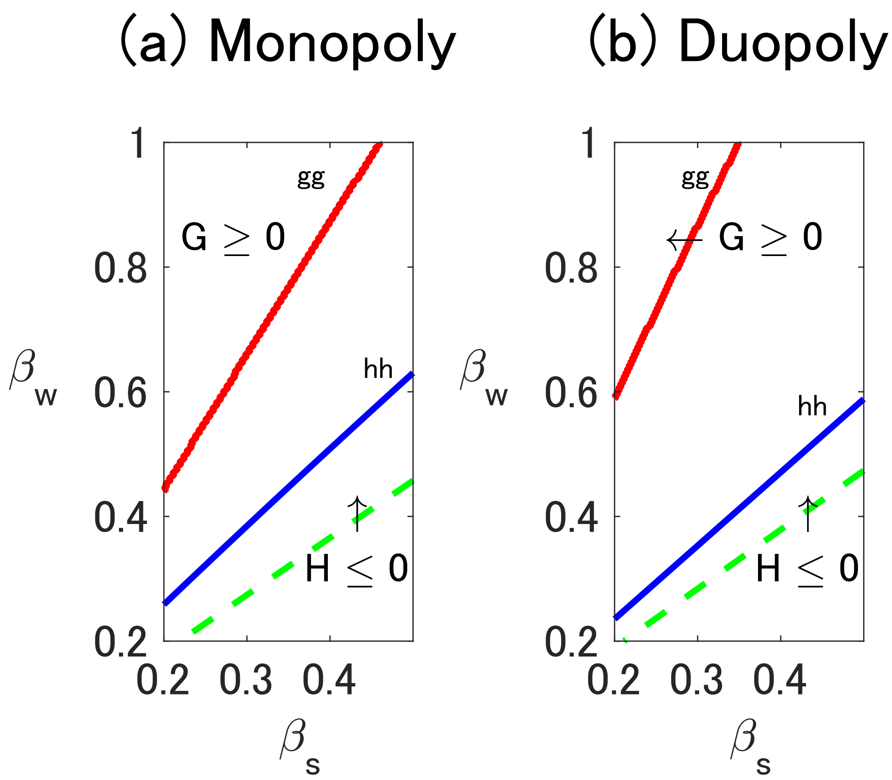

Figure 6: Multinomial logit demand with $\omega_s = 1.2$, $\omega_w = 0.8$, and $c_s = c_w = 0.2$. Panels (a) and (b) are monopoly and duopoly, respectively. As in Figures 4, and 5, the region of (β_s, β_w) is restricted to the area above the dashed thick curve in the lower right for p_s^* to be actually higher than p_w^*. The region for $G \geq 0$ is the area above curve gg, whereas the region for $H \leq 0$ is the area between the dashed thick curve and curve hh.

Here, to see the role of the demand curvatures, we consider the region of (β_s, β_w), with a fixed value of $c_s = c_w = 0.2$. Figure 6 shows that a higher value of β_w, relative to β_s, that is, a higher degree of convexity in the weak market, is associated with a *negative* change in social welfare by price discrimination. This result firstly appears not to be consistent with ACV (2010), who emphasize that as the demand in the weak market becomes more convex, it is more likely that price discrimination increases social welfare because a larger increase in output in the weak market offsets the misallocation effect caused by price discrimination. However, ACV's (2010) result holds if the demand in the strong market is concave. Here, the demand in the strong market is also convex. In this case, the uniform price is kept relatively low; thus an introduction of price discrimination again highlights the misallocation effect.

5 Aggregate Output and Consumer Surplus

In this section, we extend our sufficient statistics approach to analysis of aggregate output as well as consumer surplus.

5.1 Output Effects

First, we define $h_m(p) \equiv \frac{q_m'(p)}{\pi_m''(p)} > 0$ so that

$$\frac{Q'(t)}{2} = \underbrace{\left(-\frac{\pi_s''\pi_w''}{\pi_s'' + \pi_w''}\right)}_{>0} \{h_w[p_w(t)] - h_s[p_s(t)]\}. \tag{12}$$

and assume that this h_m is *increasing* (and call it the increasing ratio condition for quantity; IRCQ). This condition is equivalent to $\varsigma_m^I(p) > \alpha_m^I(p)$, where $\varsigma_m^I(p) \equiv -\frac{d\pi_m''}{dp}\frac{p}{\pi_m''} = -\frac{p\pi_m'''}{\pi_m''}$ is the *industry-level price elasticity of* π_m'' and $\alpha_m^I(p) \equiv -\frac{pq_m''}{q_m'}$ is the *industry-level demand curvature.*[28] It is also expressed by $\nu_m^I(p) > 0$, where $\nu_m^I(p) \equiv -\frac{ph_m'}{h_m}$ is the *industry-level price elasticity of*

[28] To see this, note that $h_m' < 0$ is equivalent to $\pi_m''' > (\frac{\pi_m''}{q_m'})q_m''$, where $\frac{\pi_m''}{q_m'} > 0$, because $h_m'(p) = \frac{\pi_m'''q_m' - \pi_m''q_m''}{(q_m')^2}$, which implies that:

$$h_m' < 0 \Leftrightarrow -\frac{p\pi_m'''}{\pi_m''} > -\frac{pq_m''}{q_m'} \Leftrightarrow \varsigma_m^I > \alpha_m^I.$$

Essentially, the IRCQ states that the profit function, starting from the zero price, increases quickly, attaining the optimal price, and then decreases slowly as p becomes larger and larger beyond the optimum. In this way, the optimal price is reached "close" enough to the zero price, rather than "still climbing up" even far away from it. To see this, if $q_m'' > 0$, then it is necessary for π_m''' to be positive. This means that π_m'', which is negative, should be larger (i.e., the negative slope of π_m'' should be gentler) as p increases. If $q_m'' \leq 0$, then π_m''' should be, whether it is positive or negative, sufficiently large. In either case, as p increases, π_m increases quickly below the optimum, and decreases slowly beyond it.

h_m.[29] Now, it is shown that

$$\frac{Q''(t)}{2} = \left(-\frac{q_s' q_w'}{\pi_s'' + \pi_w''}\right)[h_s' p_s' - h_w' p_w'] + [h_s - h_w]\frac{d}{dr}\left(-\frac{q_s' q_w'}{\pi_s'' + \pi_w''}\right),$$

so that there exists $\widehat{t}$ such that $Q'(\widehat{t}) = 0$ and $\frac{Q''(\widehat{t})}{2} = -\frac{q_s' q_w'}{\pi_s''+\pi_w''}(h_s' p_s' - h_w' p_w') < 0$ because $h_s' p_s' < 0$ and $h_w' p_w' > 0$, implying the the global concavity of $Q(t)$ is attained. Based on these results, the following proposition is obtained.

Proposition 3. *Given the IRCQ, if $\theta_w^* \rho_w^* \geq \theta_s^* \rho_s^*$ holds, the price discrimination increases aggregate output. Conversely, if*

$$\frac{\overline{\theta}_w \overline{\rho}_w}{\overline{\theta}_s \overline{\rho}_s} \leq \frac{\pi_w''(\overline{p})}{\pi_s''(\overline{p})}$$

holds, then price discrimination decreases aggregate output.

Proof. See Appendix, Part B. □

Part C of the Appendix discusses the relationship between Holmes' (1989) expression of the output effect and ours, and interprets his result in terms of the sufficient statistics.

In a similar vein, Miklós-Thal and Shaffer (2021a) provide another sufficient condition for price discrimination to increase aggregate output: $\theta_w \rho_w > \theta_s \rho_s$ and $\rho_w(1+\theta_w) > \rho_s(1+\theta_s)$ over the relevant range. Obviously, if Miklós-Thal and Shaffer's (2021a) sufficient condition holds, our sufficient condition $\theta_w^* \rho_w^* \geq \theta_s^* \rho_s^*$ automatically holds because ours is a special case of theirs. However, Miklós-Thal and Shaffer's (2021a) sufficient condition must hold globally, whereas, by imposing the IRCQ, we only require the condition to hold locally. Our sufficient condition provides a prediction for when a change from the current regime decreases aggregate output by using only the information available under the current regime: if price discrimination is currently conducted and if $\theta_w^* \rho_w^* \geq \theta_s^* \rho_s^*$ holds, then banning price discrimination *unambiguously* decreases aggregate output. Similarly, if the current regime is uniform pricing and $\frac{\overline{\theta}_w \overline{\rho}_w}{\overline{\theta}_s \overline{\rho}_s} \leq \frac{\pi_w''(\overline{p})}{\pi_s''(\overline{p})}$ holds, then allowing price discrimination *unambiguously* decreases aggregate output.[30]

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[29] This is because it is verified that

$$\frac{h_m'}{h_m} = \underbrace{\frac{\pi_m''' q_m' - \pi_m'' q_m''}{[q_m']^2}}_{<0} \cdot \underbrace{\frac{q_m'}{\pi_m''}}_{>0} = \frac{\pi_m''' q_m' - \pi_m'' q_m''}{q_m' \pi_m''},$$

and thus,

$$\nu_m^I = \left(-\frac{p\pi_m'''}{\pi_m''}\right) - \left(-\frac{p q_m''}{q_m'}\right) = \varsigma_m^I - \alpha_m^I,$$

which implies that $h_m' < 0 \Leftrightarrow \nu_m^I > 0$.

[30] If h_m is decreasing, as we assume throughout, then z_m is increasing because

$$z_m'(p) = \frac{1 - z_m(p) h_m'(p)}{h_m(p)}$$

5.2 Consumer Surplus

First, note that

$$\begin{aligned}
\frac{CS'(t)}{2} &= p_s(r) \cdot {q'}_s \cdot p_s'(t) + p_w(t) \cdot q_w' \cdot p_w'(t) \\
&\quad -p_s'(t)[p_s(t) \cdot q_s' + q_s] - p_w'(t)[p_w(t) \cdot q_w' + q_w] \\
&= -[p_s'(t)q_s + p_w'(t)q_w] \\
&= \underbrace{\left(-\frac{\pi_s''\pi_w''}{\pi_s'' + \pi_w''}\right)}_{>0} \{g_s[p_s(t)] - g_w[p_w(t)]\}\,, \qquad (13)
\end{aligned}$$

where $g_m(p) \equiv \frac{q_m(p)}{\pi_m''(p)}$. If g_m is assumed to be *decreasing*, then one can use a similar argument. We call this the decreasing marginal consumer loss condition (DMCLC), which is equivalent to $\varsigma_m^I(p) > \varepsilon_m^I(p)$.[31] Then, the global concavity of $CS(t)$ is attained. Thus, we can determine the sign of $CS'(0)$: it follows that $\text{sign}[CS'(0)] = \text{sign}[\frac{q_s(\overline{p})}{\pi_s''(\overline{p})} - \frac{q_w(\overline{p})}{\pi_w''(\overline{p})}]$, and thus, the following proposition is obtained.

Proposition 4. *Given the DMCLC, if the output in the weak market at the uniform price $\overline{p}$ is*

so that z_m' is positive if h_m' is negative. That is, the IRCQ is a *sufficient* condition for the IRCW to hold. Thus, our IRCQ is a sophistication of ACV's monotonicity condition (i.e., their IRC); it is a sophistication that requires "not too convex" demand functions to a stricter degree.

[31] Recall that $\varsigma_m^I(p) \equiv -\frac{d\pi_m''}{dp}\frac{p}{\pi_m''} = -\frac{p\pi_m'''}{\pi_m''}$ was defined as the *industry-level price elasticity of* π_m'' in Subsection 5.1 above. To see this relationship, note first that $g_m = \frac{q_m'}{\pi_m''} \cdot \frac{q_m}{q_m'} = \frac{1}{h_m} \times \frac{q_m}{q_m'}$. Thus,

$$\begin{aligned}
g_m' &= -\frac{h_m'}{[h_m]^2} \times \frac{q_m}{q_m'} + \frac{1}{h_m}(1 - \sigma_m^I) \\
&= \frac{1}{h_m}[-\underbrace{\left(-\frac{ph_m'}{h_m}\right)}_{=\nu_m^I}\underbrace{\left(-\frac{q_m}{pq_m'}\right)}_{=\frac{1}{\epsilon_m^I}} + (1 - \sigma_m^I)],
\end{aligned}$$

where $\sigma_m^I(p) \equiv \frac{q_m q_m''}{[q_m']^2}$ corresponds to what ACV (p. 1603) call the *curvature of the the inverse demand.*, and it is also written as:

$$\sigma_m^I = \underbrace{\left(\frac{-pq_m''}{q_m'}\right)}_{=\alpha_m^I}\underbrace{\left(-\frac{q_m}{pq_m'}\right)}_{=\frac{1}{\epsilon_m^I}} = \frac{\alpha_m^I}{\epsilon_m^I}.$$

Hence, $g_m' = \frac{1}{h_m}\left[1 - \frac{\nu_m^I + \alpha_m^I}{\epsilon_m^I}\right]$, which implies that $g_m' < 0 \Leftrightarrow \nu_m^I(p) + \alpha_m^I(p) > \epsilon_m^I(p) \Leftrightarrow \varsigma_m^I(p) > \varepsilon_m^I(p)$. it is also verified that $g_m' < 0$ is equivalent to $\pi_m''' > \frac{q_m'\pi_m''}{q_m}/$, where the right hand side is positive, because $g_m' = \frac{q_m'\pi_m'' - q_m\pi_m'''}{[\pi_m'']^2}$. Now, recall that the IRCQ is equivalent to $\pi_m''' > \frac{q_m''\pi_m''}{q_m'}$: if $\frac{q_m'\pi_m''}{q_m} > \frac{q_m''\pi_m''}{q_m'} / \Leftrightarrow q_m''(p) < \frac{[q_m'(p)]^2}{q_m(p)}$, that is $q_m(p)$ is not "too convex," then the DMCLC is a *sufficient* condition for the IRCQ to hold. Thus, under this "not too convex" assumption, the relationship, "DMCLC $\Rightarrow$ IRCQ $\Rightarrow$ IRCW," holds if $\sigma_m^I(p) < 1$ is additionally imposed. The details are explained in Part E of the Online Appendix.

sufficiently large, i.e.,

$$\frac{q_w(\overline{p})}{\pi''_w(\overline{p})} \geq \frac{q_s(\overline{p})}{\pi''_s(\overline{p})},$$

then price discrimination decreases consumer surplus.

Then, using profit margin and pass-through, we can rewrite Equality (13) as

$$\frac{CS'(t)}{2} = \underbrace{(-\pi''_s\pi''_w)}_{<0}\left(\frac{\mu_w(t)\rho_w(t)}{\pi''_w} - \frac{\mu_s(t)\rho_s(t)}{\pi''_s}\right)$$

for $t < t^*$, and

$$\frac{CS'(t^*)}{2} = \underbrace{\left(-\frac{\pi''_s\pi''_w}{\pi''_s + \pi''_w}\right)}_{>0}(\mu^*_w\rho^*_w - \mu^*_s\rho^*_s)$$

for $t = t^*$ which immediately leads to the following proposition.

Proposition 5. *Given the DMCLC, if $\mu^*_w\rho^*_w \geq \mu^*_s\rho^*_s$ holds, then price discrimination increases consumer surplus. Conversely, if*

$$\frac{\overline{\mu}_w\overline{\rho}_w}{\overline{\mu}_s\overline{\rho}_s} \leq \frac{\pi''_w(\overline{p})}{\pi''_s(\overline{p})}$$

holds, then price discrimination decreases consumer surplus.

Part F of the Online Appendix discusses whether the DMCLC holds in each of the three parametric examples in Section 4 above.

6 Firm Heterogeneity

In this section, we argue that the main thrusts under firm symmetry also hold when heterogeneous firms are introduced. Without loss of generality, we keep considering one strong market and one weak market. We also assume Corts' (1998, p. 315) *best response symmetry*: all firms agree on which market is strong and which market is weak. The case of best response *asymmetry* is studied by Corts (1998) (see also Footnote 11 above).

The number of firms is N (≥ 2),[32] and each firm $j = 1, 2, ..., N$ has the constraint, $p_{js} - p_{jw} \leq t_j$. Then, as above, firm j's price in the weak market under all of these constraints is written

[32] Here, all N firms are assumed to be present in both markets. This aspect of symmetry might be relaxed: the difference in the intensity of competition across the markets can also depend on the difference in the number of active firms across them. For example, Aguirre (2019) shows that price discrimination increases aggregate output under linear demand either with Cournot competition or product differentiation if the number of firms in the strong market is larger than that in the weak market. This counters the well-know result that in monopoly price discrimination never changes aggregate output under linear demand (see, e.g., Robinson 1933; Schmalensee 1981; Varian 1989). A similar finding is also obtained by Miklós-Thal and Shaffer (2021b) in the context of intermediate price discrimination. We thank Iñaki Aguirre for pointing this out to us.

as $p_{jw}(\mathbf{t})$ as a function of $\mathbf{t} = (t_1, t_2, ..., t_N)^{\mathrm{T}}$, where T denotes transposing. Accordingly, firm j's price in the strong market is written as $p_{js}(\mathbf{t}) = p_{jw}(\mathbf{t}) + t_j$. Therefore, the firms' price pair in market $m = w, s$ is written as $\mathbf{p}_m(\mathbf{t}) = (p_{1m}(\mathbf{t}), p_{2m}(\mathbf{t}), ..., p_{Nm}(\mathbf{t}))^{\mathrm{T}}$. Then, social welfare is defined as a function of $\mathbf{t}$:

$$W(\mathbf{t}) \equiv U_s(\mathbf{x}_s[\mathbf{p}_s(\mathbf{t})]) + U_w(\mathbf{x}_w[\mathbf{p}_w(\mathbf{t})]) - \mathbf{c}_s^{\mathrm{T}} \cdot \mathbf{x}_s[\mathbf{p}_s(\mathbf{t})] - \mathbf{c}_w^{\mathrm{T}} \cdot \mathbf{x}_w[\mathbf{p}_w(\mathbf{t})],$$

where $\mathbf{x}_m[\mathbf{p}_m(\mathbf{t})] = (x_{1m}[\mathbf{p}_m(\mathbf{t})], x_{2m}[\mathbf{p}_m(\mathbf{t})], ..., x_{Nm}[\mathbf{p}_m(\mathbf{t})])^{\mathrm{T}}$ and $\mathbf{c}_m = (c_{1m}, c_{2m}, ..., c_{Nm})^{\mathrm{T}}$.[33]

Now, let $t_j^* \equiv p_{js}^* - p_{jw}^*$ for each j so that $\mathbf{t}^* \equiv (t_1^*, t_2^*, ..., t_N^*)^{\mathrm{T}}$. Then, each firm's constraint is written as $0 \leq t_j = \lambda t_j^* \leq t_j^*$, with $\lambda \in [0, 1]$. Using this, we re-define the functions of $\mathbf{t}$ as functions of *one-dimensional* variable, λ. In particular, the social welfare is written as:

$$W(\lambda) = U_s(\mathbf{x}_s[\mathbf{p}_s(\lambda)]) + U_w(\mathbf{x}_w[\mathbf{p}_w(\lambda)]) - \mathbf{c}_s^{\mathrm{T}} \cdot \mathbf{x}_s[\mathbf{p}_s(\lambda)] - \mathbf{c}_w^{\mathrm{T}} \cdot \mathbf{x}_w[\mathbf{p}_w(\lambda)],$$

where $\mathbf{p}_m$ can also be interpreted as a function of λ. Hence, the equilibrium uniform price is written as $\overline{\mathbf{p}} \equiv \mathbf{p}_s(0) = \mathbf{p}_w(0)$, whereas the equilibrium discriminatory prices are $\mathbf{p}_s^* \equiv \mathbf{p}_s(1)$ and $\mathbf{p}_w^* \equiv \mathbf{p}_w(1)$.

We then use $\boldsymbol{\partial}_{\mathbf{x}} U_m = \mathbf{p}_m$ from the representative consumer's utility maximization problem in each market m, where $\boldsymbol{\partial}_{\mathbf{x}} U_m \equiv \left(\frac{\partial U_m}{\partial x_{1m}}, \frac{\partial U_m}{\partial x_{2m}}, ..., \frac{\partial U_m}{\partial x_{Nm}}\right)$, to derive

$$\underbrace{W'(\lambda)}_{1\times 1} = \sum_{m=s,w} [\underbrace{\boldsymbol{\mu}_m(\mathbf{p}_m)^{\mathrm{T}}}_{1\times N} \cdot (\underbrace{\boldsymbol{\partial}_{\mathbf{p}_m}\mathbf{x}_m}_{N\times N} \cdot \underbrace{\mathbf{p}'_m}_{N\times 1})],$$

where $\boldsymbol{\mu}_m(\mathbf{p}_m) \equiv \mathbf{p}_m - \mathbf{c}_m$ is the profit margin vector,

$$\boldsymbol{\partial}_{\mathbf{p}_m}\mathbf{x}_m \equiv \left(\underbrace{\begin{pmatrix} \frac{\partial x_{1m}}{\partial p_{1m}} \\ \vdots \\ \frac{\partial x_{Nm}}{\partial p_{1m}} \end{pmatrix}}_{\equiv \boldsymbol{\partial}_{p_{1m}}\mathbf{x}_m} \cdots \underbrace{\begin{pmatrix} \frac{\partial x_{1m}}{\partial p_{Nm}} \\ \vdots \\ \frac{\partial x_{Nm}}{\partial p_{Nm}} \end{pmatrix}}_{\equiv \boldsymbol{\partial}_{p_{Nm}}\mathbf{x}_m} \right)$$

is the Jacobian for market demands, and $\mathbf{p}'_{\mathrm{m}} \equiv ({p'}_{1m}(\lambda), {p'}_{2m}(\lambda), ..., {p'}_{Nm}(\lambda))^{\mathrm{T}}$.

[33]By allowing cost differences across firms and markets, Dertwinkel-Kalt and Wey (2020) study oligopolistic third-degree price discrimination under the demand system proposed by Somaini and Einiv (2013), in which demand in each separate market is covered by all firms and thus no consumers are opting out. Under this demand system, Dertwinkel-Kalt and Wey (2020) show that each firm's profit margin (i.e., the Lerner index) under uniform pricing is expressed as the weighted harmonic mean of its market-specific Lerner indices under price discrimination. This result indicates that the profit margin is strictly lower the weighted arithmetic mean of the market-specific margins. In this sense, the market power measured by the Lerner concept is always lower under uniform pricing, and consumer surplus is strictly greater than under price discrimination. Note, however, that a change in social welfare is *not* an issue under this demand system because each firm's output remains the same for both regimes.

Firm j's profit function in market $m = s, w$ is given by Equation (2), where $\mathbf{p}_m$ now consists of N firms' prices as above, and

$$\partial_{p_{jm}} \pi_{jm}(\mathbf{p}_m) \equiv x_{jm}(\mathbf{p}_m) + (p_{jm} - c_{jm}) \frac{\partial x_{jm}}{\partial p_{jm}}(\mathbf{p}_m)$$

is defined. Now, we apply the implicit function theorem to $\mathbf{f}(\mathbf{p_w}, \lambda) = \mathbf{0}$, where

$$\mathbf{f}(\underbrace{\mathbf{p_w}}_{N\times 1}, \underbrace{\lambda}_{1\times 1}) \equiv \begin{pmatrix} \partial_{p_{1s}} \pi_{1s}(\mathbf{p}_w + \lambda \mathbf{t}^*) + \partial_{p_{1w}} \pi_{1w}(\mathbf{p}_w) \\ \vdots \\ \partial_{p_{js}} \pi_{js}(\mathbf{p}_w + \lambda \mathbf{t}^*) + \partial_{p_{jw}} \pi_{jw}(\mathbf{p}_w) \\ \vdots \\ \partial_{p_{Ns}} \pi_{Ns}(\mathbf{p}_w + \lambda \mathbf{t}^*) + \partial_{p_{Nw}} \pi_{Nw}(\mathbf{p}_w) \end{pmatrix},$$

is a collection of all firms' first-order conditions for profit maximization under regime λ, to obtain $\mathbf{p}'_w(\lambda) = -[\mathbf{D}_{\mathbf{p}_w}\mathbf{f}]^{-1}[\mathbf{D}_\lambda \mathbf{f}]$, where

$$\mathbf{D}_{\mathbf{p}_w}\mathbf{f} \equiv \underbrace{\begin{pmatrix} \frac{\partial^2 \pi_{1s}}{\partial p_{1s}^2} + \frac{\partial^2 \pi_{1w}}{\partial p_{1w}^2} & \cdots & \frac{\partial^2 \pi_{1s}}{\partial p_{Ns} \partial p_{1s}} + \frac{\partial^2 \pi_{1w}}{\partial p_{Nw} \partial p_{1w}} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 \pi_{Ns}}{\partial p_{1s} \partial p_{Ns}} + \frac{\partial^2 \pi_{Nw}}{\partial p_{1w} \partial p_{Nw}} & \cdots & \frac{\partial^2 \pi_{Ns}}{\partial p_{Ns}^2} + \frac{\partial^2 \pi_{Nw}}{\partial p_{Nw}^2} \end{pmatrix}}_{\equiv \mathbf{K}}$$

$$= \underbrace{\begin{pmatrix} \frac{\partial^2 \pi_{1s}}{\partial p_{1s}^2} & \cdots & \frac{\partial^2 \pi_{1s}}{\partial p_{Ns} \partial p_{1s}} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 \pi_{Ns}}{\partial p_{1s} \partial p_{Ns}} & \cdots & \frac{\partial^2 \pi_{Ns}}{\partial p_{Ns}^2} \end{pmatrix}}_{\equiv \mathbf{H}_s} + \underbrace{\begin{pmatrix} \frac{\partial^2 \pi_{1w}}{\partial p_{1w}^2} & \cdots & \frac{\partial^2 \pi_{1w}}{\partial p_{Nw} \partial p_{1w}} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 \pi_{Nw}}{\partial p_{1w} \partial p_{Nw}} & \cdots & \frac{\partial^2 \pi_{Nw}}{\partial p_{Nw}^2} \end{pmatrix}}_{\equiv \mathbf{H}_w}$$

and $\mathbf{D}_\lambda \mathbf{f} = \mathbf{H}_s \mathbf{t}^*$.

Here, the elasticity matrix and the curvature matrix can be defined by

$$\boldsymbol{\varepsilon}_m = \begin{pmatrix} \varepsilon_{11,m} & \varepsilon_{21,m} & \cdots & \varepsilon_{N1,m} \\ \varepsilon_{12,m} & \varepsilon_{22,m} & & \vdots \\ \vdots & & \ddots & \vdots \\ \varepsilon_{1N,m} & \varepsilon_{2N,m} & \cdots & \varepsilon_{NN,m} \end{pmatrix}$$

$$\equiv \begin{pmatrix} -\frac{p_{1m}}{x_{1m}}\frac{\partial x_{1m}}{\partial p_{1m}} & \frac{p_{1m}}{x_{1m}}\frac{\partial x_{2m}}{\partial p_{1m}} & \cdots & \frac{p_{1m}}{x_{Nm}}\frac{\partial x_{Nm}}{\partial p_{1m}} \\ \frac{p_{2m}}{x_{1m}}\frac{\partial x_{1m}}{\partial p_{2m}} & -\frac{p_{2m}}{x_{2m}}\frac{\partial x_{2m}}{\partial p_{2m}} & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{p_{Nm}}{x_{1m}}\frac{\partial x_{1m}}{\partial p_{Nm}} & \cdots & \cdots & -\frac{p_{Nm}}{x_{Nm}}\frac{\partial x_{Nm}}{\partial p_{Nm}} \end{pmatrix}$$

and

$$\begin{aligned} \boldsymbol{\alpha}_m &= \begin{pmatrix} \alpha_{11,m} & \alpha_{21,m} & \cdots & \alpha_{N1,m} \\ \alpha_{12,m} & \alpha_{22,m} & & \vdots \\ \vdots & & \ddots & \vdots \\ \alpha_{1N,m} & \cdots & \cdots & \alpha_{NN,m} \end{pmatrix} \\ &\equiv \begin{pmatrix} -\frac{p_{1m}}{\frac{\partial x_{1m}}{\partial p_{1m}}}\frac{\partial^2 x_{1m}}{\partial p_{1m}^2} & -\frac{p_{2m}}{\frac{\partial x_{2m}}{\partial p_{2m}}}\frac{\partial^2 x_{2m}}{\partial p_{2m}\partial p_{1m}} & \cdots & -\frac{p_{Nm}}{\frac{\partial x_{Nm}}{\partial p_{Nm}}}\frac{\partial^2 x_{Nm}}{\partial p_{Nm}\partial p_{1m}} \\ -\frac{p_{1m}}{\frac{\partial x_{1m}}{\partial p_{1m}}}\frac{\partial^2 x_{1m}}{\partial p_{1m}\partial p_{2m}} & -\frac{p_{2m}}{\frac{\partial x_{2m}}{\partial p_{2m}}}\frac{\partial^2 x_{2m}}{\partial p_{2m}^2} & & \vdots \\ \vdots & & \ddots & \vdots \\ -\frac{p_{1m}}{\frac{\partial x_{1m}}{\partial p_{1m}}}\frac{\partial^2 x_{1m}}{\partial p_{1m}\partial p_{Nm}} & \cdots & \cdots & -\frac{p_{Nm}}{\frac{\partial x_{Nm}}{\partial p_{Nm}}}\frac{\partial^2 x_{Nm}}{\partial p_{Nm}^2} \end{pmatrix}, \end{aligned}$$

respectively. Note also that for $j = 1, 2, ..., N$,

$$\begin{aligned} \frac{\partial^2 \pi_{jm}(\mathbf{p}_m)}{\partial p_{jm}^2} &= 2\frac{\partial x_{jm}}{\partial p_{jm}}(\mathbf{p}_m) + (p_{jm} - c_{jm})\frac{\partial^2 x_{jm}}{\partial p_{jm}^2}(\mathbf{p}_m) \\ &= -\frac{x_{jm}}{p_{jm}}\left(-\frac{p_{jm}}{x_{jm}}\frac{\partial x_{jm}}{\partial p_{jm}}\right)\left[2 - \frac{p_{jm} - c_{jm}}{p_{jm}}\left(-\frac{p_{jm}}{\frac{\partial x_{jm}}{\partial p_{jm}}}\frac{\partial^2 x_{jm}}{\partial p_{jm}^2}\right)\right] \\ &= -\left[2 - L_{jm}(p_{jm})\alpha_{jj,m}\right]\varepsilon_{jj,m}\frac{x_{jm}}{p_{jm}} \end{aligned}$$

and for $j, k = 1, 2, ..., N$, $j \neq k$,

$$\begin{aligned} \frac{\partial^2 \pi_{jm}(\mathbf{p}_m)}{\partial p_{jm}\partial p_{km}} &= \frac{\partial x_{jm}}{\partial p_{km}}(\mathbf{p}_m) + (p_{jm} - c_{jm})\frac{\partial^2 x_{jm}}{\partial p_{jm}\partial p_{km}}(\mathbf{p}_m) \\ &= \frac{x_{jm}}{p_{km}}\left(\frac{p_{km}}{x_{jm}}\frac{\partial x_{jm}}{\partial p_{km}}\right)\left[1 + \left(\frac{p_{km}}{p_{jm}}\right)\left(\frac{p_{jm} - c_{jm}}{p_{jm}}\right)\frac{\left(-\frac{p_{jm}}{x_{jm}}\frac{\partial x_{jm}}{\partial p_{jm}}\right)}{\left(\frac{p_{km}}{x_{jm}}\frac{\partial x_{jm}}{\partial p_{km}}\right)}\left(-\frac{p_{jm}}{\frac{\partial x_{jm}}{\partial p_{jm}}}\frac{\partial^2 x_{jm}}{\partial p_{jm}\partial p_{km}}\right)\right] \\ &= \left[1 + \left(\frac{p_{km}}{p_{jm}}\right)L_{jm}(p_{jm})\frac{\varepsilon_{jj,m}}{\varepsilon_{jk,m}}\alpha_{jk,m}\right]\varepsilon_{jk,m}\frac{x_{jm}}{p_{km}}, \end{aligned}$$

which implies that $\mathbf{H}_s$ and $\mathbf{H}_w$ are expressed in terms of the sufficient statistics.

Now, we further proceed to obtain:

$$\begin{cases} \mathbf{p}'_w(\lambda) = [-\underbrace{\mathbf{K}^{-1}}_{N\times N}\underbrace{\mathbf{H}_s}_{N\times N}]\underbrace{\mathbf{t}^*}_{N\times 1} \\ \mathbf{p}'_s(\lambda) = -\mathbf{K}^{-1}\mathbf{H}_s\mathbf{t}^* + \mathbf{t}^* = \underbrace{[\mathbf{I} - \mathbf{K}^{-1}\mathbf{H}_s]}_{N\times N}\underbrace{\mathbf{t}^*}_{N\times 1}, \end{cases}$$

so that

$$\begin{aligned} W'(\lambda) &= [\boldsymbol{\mu}_s^{\mathrm{T}}\boldsymbol{\partial}_{\mathbf{p}_s}\mathbf{x}_s][\mathbf{I} - \mathbf{K}^{-1}\mathbf{H}_s]\mathbf{t}^* - [\boldsymbol{\mu}_w^{\mathrm{T}}\boldsymbol{\partial}_{\mathbf{p}_w}\mathbf{x}_w][\mathbf{K}^{-1}\mathbf{H}_s]\mathbf{t}^* \\ &= \{[\boldsymbol{\mu}_s^{\mathrm{T}}\boldsymbol{\partial}_{\mathbf{p}_s}\mathbf{x}_s]\mathbf{K}^{-1}[\mathbf{K} - \mathbf{H}_s] - [\boldsymbol{\mu}_w^{\mathrm{T}}\boldsymbol{\partial}_{\mathbf{p}_w}\mathbf{x}_w][\mathbf{K}^{-1}\mathbf{H}_s]\}\mathbf{t}^* \\ &= \big\{\big([\boldsymbol{\mu}_s^{\mathrm{T}}\boldsymbol{\partial}_{\mathbf{p}_s}\mathbf{x}_s]\mathbf{H}_s^{-1}\big)\big(\mathbf{H}_s\mathbf{K}^{-1}[\mathbf{K} - \mathbf{H}_s]\big) \\ &\qquad - \big([\boldsymbol{\mu}_w^{\mathrm{T}}\boldsymbol{\partial}_{\mathbf{p}_w}\mathbf{x}_w]\mathbf{H}_w^{-1}\big)\big(\mathbf{H}_w\mathbf{K}^{-1}\mathbf{H}_s\big)\big\}\,\mathbf{t}^*. \end{aligned}$$

Subsequently, we define

$$\mathbf{Z}_m(\mathbf{p}) \equiv \underbrace{[\boldsymbol{\mu}_m(\mathbf{p})^{\mathrm{T}}\boldsymbol{\partial}_{\mathbf{p}_m}\mathbf{x}_m(\mathbf{p})]}_{1\times N}\underbrace{\mathbf{H}_m^{-1}(\mathbf{p})}_{N\times N}$$

to proceed:

$$\begin{aligned} W'(\lambda) &= \{\underbrace{\mathbf{Z}_w}_{1\times N} - \underbrace{\mathbf{Z}_s}_{1\times N} \cdot \underbrace{(\mathbf{K} - \mathbf{H}_s)\mathbf{H}_w^{-1}}_{N\times N}\}\underbrace{(-\mathbf{H}_w\mathbf{K}^{-1}\mathbf{H}_s)\mathbf{t}^*}_{N\times 1} \\ &= (\mathbf{Z}_w - \mathbf{Z}_s)(\boldsymbol{\Gamma}\mathbf{t}^*), \end{aligned}$$

where $\boldsymbol{\Gamma} \equiv -\mathbf{H}_w\mathbf{K}^{-1}\mathbf{H}_s \gg \mathbf{0}$ is assumed. We also assume that multi-dimensional version of the IRC: for each market m and each firm j, Z_{jm} is increasing in p_l, $l = 1, 2, ..., N$.

We then define the multi-dimensional version of the conduct parameter (Weyl and Fabinger 2013, p. 552; see Footnote 16) by:

$$\boldsymbol{\theta}_m(\mathbf{p})^{\mathrm{T}} \equiv \left(\begin{array}{ccccc} \dfrac{\boldsymbol{\mu}_m(\mathbf{p})^{\mathrm{T}}\boldsymbol{\partial}_{p_{1m}}\mathbf{x}_m(\mathbf{p})}{-x_{1m}(\mathbf{p})} & \dots & \dfrac{\boldsymbol{\mu}_m(\mathbf{p})^{\mathrm{T}}\boldsymbol{\partial}_{p_{jm}}\mathbf{x}_m(\mathbf{p})}{-x_{jm}(\mathbf{p})} & \dots & \dfrac{\boldsymbol{\mu}_m(\mathbf{p})^{\mathrm{T}}\boldsymbol{\partial}_{p_{Nm}}\mathbf{x}_m(\mathbf{p})}{-x_{Nm}(\mathbf{p})} \end{array} \right)$$

as well as the pass-through matrix by:

$$\boldsymbol{\rho}_m(\lambda) = \begin{cases} \begin{pmatrix} \frac{\partial x_{1m}}{\partial p_{1m}}(\mathbf{p}) & 0 & \cdots & 0 \\ 0 & \frac{\partial x_{2m}}{\partial p_{2m}}(\mathbf{p}) & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial x_{Nm}}{\partial p_{Nm}}(\mathbf{p}) \end{pmatrix} \mathbf{K}^{-1}(\mathbf{p}) & \text{for } \lambda < 1 \\ \\ \begin{pmatrix} \frac{\partial x_{1m}}{\partial p_{1m}}(\mathbf{p}^*_m) & 0 & \cdots & 0 \\ 0 & \frac{\partial x_{2m}}{\partial p_{2m}}(\mathbf{p}^*_m) & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial x_{Nm}}{\partial p_{Nm}}(\mathbf{p}^*_m) \end{pmatrix} \mathbf{H}^{-1}_m(\mathbf{p}^*_m) & \text{for } \lambda = 1 \end{cases}$$

as in the case of firm symmetry (recall the definition in Subsubsection 3.1.3).[34]

Hence, the three sufficient statistics under price discrimination are given by $\boldsymbol{\theta}^*_m \equiv \boldsymbol{\theta}_m(\mathbf{p}^*_m)$, $\boldsymbol{\mu}^*_m \equiv \boldsymbol{\mu}_m(\mathbf{p}^*_m)$, and $\boldsymbol{\rho}^*_m \equiv \boldsymbol{\rho}_m(1)$. Similarly, those under uniform pricing are $\overline{\boldsymbol{\theta}}_m \equiv \boldsymbol{\theta}_m(\overline{\mathbf{p}})$ $\overline{\boldsymbol{\mu}}_m \equiv \boldsymbol{\mu}_m(\overline{\mathbf{p}})$, and $\overline{\boldsymbol{\rho}}_m \equiv \boldsymbol{\rho}_m(0)$.

We are now able to generalizes Proposition 1 in the previous section for the case of firm heterogeneity.

Proposition 6. *Given the IRC, if $[[\boldsymbol{\theta}^*_w]^{\mathrm{T}} \circ [\boldsymbol{\mu}^*_w]^{\mathrm{T}}]\boldsymbol{\rho}^*_w > [[\boldsymbol{\theta}^*_s]^{\mathrm{T}} \circ [\boldsymbol{\mu}^*_s]^{\mathrm{T}}]\boldsymbol{\rho}^*_s$ holds, where $\circ$ indicates element-by-element multiplication, then price discrimination increases social welfare. Conversely, if $[[\overline{\boldsymbol{\theta}}_w]^{\mathrm{T}} \circ [\overline{\boldsymbol{\mu}}_w]^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_w < [[\overline{\boldsymbol{\theta}}_s]^{\mathrm{T}} \circ [\overline{\boldsymbol{\mu}}_s]^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_s\overline{\boldsymbol{\Delta}}$, where $\overline{\boldsymbol{\Delta}} \equiv \overline{\mathbf{K}}\,\overline{\mathbf{H}}_s^{-1}\overline{\mathbf{H}}_w\overline{\mathbf{K}}^{-1}$ is defined for adjustment, where $\overline{\mathbf{K}} \equiv \mathbf{K}(\overline{\mathbf{p}})$ and $\overline{\mathbf{H}}_m \equiv \mathbf{H}_m(\overline{\mathbf{p}})$, $m = s, w$, holds, then price discrimination decreases social welfare.*

Proof. See Appendix, Part D. □

Note that for each j, $Z^*_{jm} = \sum_{k=1}^N \theta^*_{km}\mu^*_{km}\rho^*_{jkm}$, which is interpreted as the *weighted sum* of firm j's own pass-through (ρ^*_{jjm}) and the collection of its cross pass-through (ρ^*_{jkm}, $k \neq j$). For aggregate output and consumer surplus, we can readily generalize our previous results to the case of firm heterogeneity in a similar manner by noting that

$$Q(\lambda) = \sum_{j=1}^N x_{js}[\mathbf{p}_s(\lambda)] + \sum_{j=1}^N x_{jw}[\mathbf{p}_w(\lambda)]$$

[34]Given this definition, $\boldsymbol{\theta}_m$ is rewritten as

$$\boldsymbol{\theta}^{\mathrm{T}}_m = \begin{pmatrix} \sum_{k=1}^N L_{km}\varepsilon_{k1,m}\left(-\frac{x_{km}}{x_{1m}}\right) & \ldots & \sum_{k=1}^N L_{km}\varepsilon_{kj,m}\left(-\frac{x_{km}}{x_{jm}}\right) & \ldots & \sum_{k=1}^N L_{km}\varepsilon_{kN,m}\left(-\frac{x_{km}}{x_{Nm}}\right) \end{pmatrix},$$

which implies that it can be expressed in terms of the sufficient statistics. Note also that $\boldsymbol{\rho}_m$ can be expressed in terms of the sufficient statistics as well because $\frac{\partial x_{jm}}{\partial p_{jm}} = -\frac{x_{jm}}{p_{jm}}\varepsilon_{jj,m}$ holds for $j = 1, 2, ..., N$.

(a) Aggregate Output
If $[\boldsymbol{\theta}_w^*]^{\mathrm{T}}\boldsymbol{\rho}_w^* > [\boldsymbol{\theta}_s^*]^{\mathrm{T}}\boldsymbol{\rho}_s^*$, then $Q^* > \overline{Q}$. If $[\overline{\boldsymbol{\theta}}_w]^{\mathrm{T}}\overline{\boldsymbol{\rho}}_w < [\overline{\boldsymbol{\theta}}_s]^{\mathrm{T}}\overline{\boldsymbol{\rho}}_s\overline{\boldsymbol{\Delta}}$, then $Q^* < \overline{Q}$.
(b) Social Welfare
If $[[\boldsymbol{\theta}_w^*]^{\mathrm{T}} \circ [\boldsymbol{\mu}_w^*]^{\mathrm{T}}]\boldsymbol{\rho}_w^* > [[\boldsymbol{\theta}_s^*]^{\mathrm{T}} \circ [\boldsymbol{\mu}_s^*]^{\mathrm{T}}]\boldsymbol{\rho}_s^*$, then $W^* > \overline{W}$. If $[[\overline{\boldsymbol{\theta}}_w]^{\mathrm{T}} \circ [\overline{\boldsymbol{\mu}}_w]^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_w < [[\overline{\boldsymbol{\theta}}_s]^{\mathrm{T}} \circ [\overline{\boldsymbol{\mu}}_s]^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_s\overline{\boldsymbol{\Delta}}$, then $W^* < \overline{W}$.
(c) Consumer Surplus
If $[\boldsymbol{\mu}_w^*]^{\mathrm{T}}\boldsymbol{\rho}_w^* > [\boldsymbol{\mu}_s^*]^{\mathrm{T}}\boldsymbol{\rho}_s^*$, then $CS^* > \overline{CS}$. If $[\overline{\boldsymbol{\mu}}_w]^{\mathrm{T}}\overline{\boldsymbol{\rho}}_w < [\overline{\boldsymbol{\mu}}_s]^{\mathrm{T}}\overline{\boldsymbol{\rho}}_s\overline{\boldsymbol{\Delta}}$, then $CS^* < \overline{CS}$.

Table 2: Summary of the Sufficient Conditions (with N heterogeneous firms, $\boldsymbol{\theta}_m$, $\boldsymbol{\mu}_m$, and $\boldsymbol{\rho}_m$ are the conduct vector ($N \times 1$), the profit margin vector ($N \times 1$), and the pass-through matrix ($N \times N$), respectively, in market $m = s, w$; asterisks and upper bars indicate price discrimination and uniform pricing, respectively; and $\overline{\boldsymbol{\Delta}}$ is a term for adjustment defined in the text).

and

$$CS(\lambda) = U_s(\mathbf{x}_s[\mathbf{p}_s(\lambda)]) + U_w(\mathbf{x}_w[\mathbf{p}_w(\lambda)]) - [\mathbf{p}_s(\lambda)]^{\mathrm{T}} \cdot \mathbf{x}_s[\mathbf{p}_s(\lambda)] - [\mathbf{p}_w(\lambda)]^{\mathrm{T}} \cdot \mathbf{x}_w[\mathbf{p}_w(\lambda)],$$

respectively. Table 2 summarizes our results for heterogeneous firms.

7 Concluding Remarks

This paper presents the theoretical implications of oligopolistic third-degree price discrimination with general non-linear demand, allowing cost differentials to exist across separate markets. In this sense, this paper, with the help of the methodology proposed by Weyl and Fabinger (2013), synthesizes Aguirre, Cowan, and Vickers' (2010) analysis of monopolistic third-degree price discrimination with general demands and Chen and Schwartz' (2015) analysis of monopolistic differential pricing and extends them to the case of differentiated oligopoly.

Our theoretical analysis, which accommodates firm heterogeneity, can also be utilized to empirically assess the welfare effects of third-degree price discrimination under oligopoly. In particular, in line with the "sufficient statistics" approach (Chetty 2009), our predictions regarding the welfare effects do *not* rely on functional specifications, and are thus considered to be fairly robust, although these sufficient statistics can take different values, depending on functional specifications. However, once the numerical values of sufficient statistics are obtained, there should be no disagreement regarding welfare assessment.

As a promising direction, it would be interesting to apply our methodology to the analysis of the welfare effects of *wholesale/input* third-degree price discrimination (Katz 1987; DeGraba 1990; Yoshida 2000; Inderst and Valletti 2009; Villas-Boas 2009; Arya and Mittenforf 2010; Li 2014; O'Brien 2014; Gaudin and Lestage 2019; and Miklós-Thal and Shaffer 2021b).[35] To do so, one would need to properly define the sufficient statistics at each stage of a vertical relationship. Another important issue to consider is the case of *multi-product* oligopolistic firms (Armstrong and Vickers 2018; and Nocke and Schutz 2018). What happens if price discrimination is allowed for some products, whereas uniform pricing is enforced for others? These and other important issues related to third-degree price discrimination await further research.

Appendix

A. Proof of Proposition 1

Note first that

$$\begin{aligned} z_m(p_m) &= -\frac{(p_m - c_m)\varepsilon_m^I(p_m)q_m(p_m)}{p_m\pi_m''(p_m)} \\ &= -\theta_m(p_m)\frac{q_m(p_m)}{\pi_m''(p_m)} \end{aligned}$$

holds. Now, define

$$F(p_m, c_m) = \frac{q_m(p_m)}{\frac{\partial x_{Am}}{\partial p_A}(p_m, p_m)} + p_m - c_m$$

so that $F(p_m^*, c_m) = 0$ for $m = s, w$. Then, from the implicit function theorem, it is verified that

$$\begin{aligned} \rho_m &= \frac{1}{1 + \dfrac{q_m'}{\partial x_{Am}/\partial p_A} - q_m\dfrac{d\left(\partial x_{Am}/\partial p_A\right)/dp_m}{\left(\partial x_{Am}/\partial p_A\right)^2}} \\ &= \frac{\partial x_{Am}/\partial p_A}{\dfrac{\partial x_{Am}}{\partial p_A} + q_m' - \dfrac{q_m}{\partial x_{Am}/\partial p_A}\dfrac{d}{dp_m}\left(\dfrac{\partial x_{Am}}{\partial p^A}\right)}, \end{aligned}$$

and under the equilibrium discriminatory prices,

$$\rho_m(p_m^*) = \frac{\partial x_{Am}(p_m^*, p_m^*)/\partial p_A}{q_m'(p_m^*) + \dfrac{\partial x_{Am}}{\partial p_A}(p_m^*, p_m^*) + (p_m^* - c_m)\dfrac{d}{dp_m}\left(\dfrac{\partial x_{Am}}{\partial p_A}(p_m^*, p_m^*)\right)}$$

[35] See Jaffe and Weyl (2013) for such an attempt.

$$= \frac{\partial x_{Am}(p_m^*, p_m^*)/\partial p_A}{\pi_m''(p_m^*, c_m)},$$

which implies that

$$\begin{aligned} z_m(p_m^*) &= \left(\frac{-q_m(p_m^*)}{\frac{\partial x_{Am}}{\partial p_A}(p_m^*, p_m^*)}\right)\theta_m(p_m^*)\rho_m(p_m^*) \\ &= \mu_m(p_m^*)\theta_m(p_m^*)\rho_m(p_m^*) \end{aligned}$$

and thus

$$\frac{W'(t^*)}{2} = \left(-\frac{\pi_s''\pi_w''}{\pi_s'' + \pi_w''}\right)(\mu_w^*\theta_w^*\rho_w^* - \mu_s^*\theta_s^*\rho_s^*) > 0$$

if $\mu_w^*\theta_w^*\rho_w^* > \mu_s^*\theta_s^*\rho_s^*$ holds. Given the IRC, this means that $W(t)$ is strictly increasing in $[0, t^*]$. This completes the proof for the first part.

For $t < t^*$, note that

$$\begin{aligned} z_m(p_m) &= \theta_m\left(\frac{-q_m}{\frac{\partial x_{Am}}{\partial p_A}}\right)\left(\frac{\frac{\partial x_{Am}}{\partial p_A}}{\pi_m''}\right) \\ &= \mu_m\theta_m\underbrace{\left(-\frac{\frac{\partial x_{Am}}{\partial p_A}}{\pi_s'' + \pi_w''}\right)}_{=\rho_m}\left(\frac{\pi_s'' + \pi_w''}{\pi_m''}\right). \end{aligned}$$

Thus, it is verified that

$$\frac{W'(t)}{2} = \underbrace{(-\pi_s''\pi_w'')}_{<0}\left(\frac{\mu_w(t)\theta_w(t)\rho_w(t)}{\pi_w''} - \frac{\mu_s(t)\theta_s(t)\rho_s(t)}{\pi_s''}\right),$$

which implies that given the IRC, $W(t)$ is strictly decreasing in $[0, t^*]$ if $W'(0) \leq 0$. This completes the proof for the second part.

B. Proof of Proposition 2

First, note that $\frac{q_m'}{\pi_m''} = \theta_m^* \cdot \rho_m^*$. Then, this implies that from equality (12),

$$\frac{Q'(\bar{t})}{2} = \left(-\frac{\pi_s''\pi_w''}{\pi_s'' + \pi_w''}\right)(\theta_w^*\rho_w^* - \theta_s^*\rho_s^*).$$

Conversely, for $t < \bar{t}$, it can be verified that:

$$\frac{Q'(t)}{2} = \left(\frac{q_w'}{\frac{\partial x_{A,w}}{\partial p_A}}\right)\left(\frac{\partial x_{A,w}}{\partial p_A}\right)p_w' + \left(\frac{q_s'}{\frac{\partial x_{A,s}}{\partial p_A}}\right)\left(\frac{\partial x_{A,s}}{\partial p_A}\right)p_s'$$

$$= \theta_w \left(\frac{\frac{\partial x_{A,w}}{\partial p_A}}{\pi_s'' + \pi_w''} \right) (\pi_s'' + \pi_w'') p_w' + \theta_s \left(\frac{\frac{\partial x_{A,s}}{\partial p_A}}{\pi_s'' + \pi_w''} \right) (\pi_s'' + \pi_w'') p_s'$$

$$= \underbrace{(-\pi_s'' \pi_w'')}_{<0} \left(\frac{\theta_w(r)\rho_w(t)}{\pi_w''} - \frac{\theta_s(r)\rho_s(t)}{\pi_s''} \right).$$

C. Reinterpretation of Holmes' (1989) Result on the Output Effects in Terms of Sufficient Statistics

The following lemma holds when cost differentials are permitted.

Lemma. $Q'(t) > 0$ *if and only if (suppressing the dependence on* $p_m(t)$*)*

$$\underbrace{L_w \cdot \frac{\alpha_w^{own} + \alpha_w^{cross}}{\theta_w} - L_s \cdot \frac{\alpha_s^{own} + \alpha_s^{cross}}{\theta_s}}_{\textit{Adjusted-concavity}} + \underbrace{\frac{1}{\theta_s} - \frac{1}{\theta_w}}_{\textit{Elasticity-ratio}} > 0, \tag{14}$$

where, following Holmes (1989), we call the first and the second terms in the left hand side of inequality the adjusted-concavity part, *and the third and the fourth terms the* elasticity-ratio part.

Proof. It is immediate to see that $Q'(r)/2$ is also given by

$$\begin{aligned} \frac{Q'(r)}{2} &= q_w' \cdot p_w' + q_s' \cdot p_s' \\ &= -\frac{\pi_s'' q_w'}{\pi_s'' + \pi_w''} + \frac{\pi_w'' q_s'}{\pi_s'' + \pi_w''} \\ &= \underbrace{\left(-\frac{q_s' q_w'}{\pi_s'' + \pi_w''} \right)}_{>0} \left(\frac{\pi_s''}{q_s'} - \frac{\pi_w''}{q_w'} \right), \end{aligned}$$

where (see Equation 7)

$$\begin{aligned} \frac{\pi_m''(p)}{q_m'(p)} &= \{2 - L_m(p)[\alpha_m^{own}(p) + \alpha_m^{cross}(p)]\} \underbrace{\frac{\varepsilon_m^{own}(p)}{\varepsilon_m^I(p)}}_{=\frac{1}{\theta_m(p)}} - \underbrace{\frac{\varepsilon_m^{cross}(p)}{\varepsilon_m^I(p)}}_{=\frac{1-\theta_m(p)}{\theta_m(p)}} \\ &= \frac{2 - L_m \cdot (\alpha_m^{own} + \alpha_m^{cross}) - (1 - \theta_m)}{\theta_m}, \end{aligned}$$

Hence,

$$\frac{Q'(r)}{2} = \underbrace{\left(-\frac{q_s' q_w'}{\pi_s'' + \pi_w''} \right)}_{>0} \left[\left(L_w \cdot \frac{\alpha_w^{own} + \alpha_w^{cross}}{\theta_w} - \frac{1}{\theta_w} \right) - \left(L_s \cdot \frac{\alpha_s^{own} + \alpha_s^{cross}}{\theta_s} - \frac{1}{\theta_s} \right) \right],$$

which completes the proof. This is also interpreted as a generalization of ACV's (2010, p. 1608) Equation (6).

□

Consider the adjusted-concavity part. A larger α_w^{own} and/or α_w^{cross} make a positive $Q'(r)$ more likely. A larger α_w^{own} means that the firm's own part of the demand in the weak market ($\frac{\partial x_{A,w}}{\partial p_A}$) is more convex ("the output expansion effect"). Similarly, a larger α_w^{cross} means that how many of the firm's customers switch to the rival's product as a response to the firm's price increase is not so much affected by the current price level ("the countervailing effect"). In this sense, the strategic concerns in the firm's pricing are small. Thus, both a larger α_w^{own} and a larger α_w^{cross} indicate that the weak market is competitive. Even if $\frac{\partial x_{A,w}}{\partial p_A}$ is not so convex, a larger α_w^{cross} can substitute it. Here, the intensity of market competition, $1/\theta_w$, magnifies both effects, resulting in $\frac{\alpha_w^{own}}{\theta_w}$ and $\frac{\alpha_w^{cross}}{\theta_w}$. A similar argument also holds for α_s^{own} and α_s^{cross}. In Part D of the Online Appendix, we show that Holmes' (1989) expression for $Q'(r)$ (expression (9) in Holmes 1989, p. 247) is equivalent to the left hand side of inequality (14) above.

D. Proof of Proposition 6

Using the definition of $\boldsymbol{\theta}_m(\mathbf{p})$, we can rewrite:

$$\mathbf{Z}_m[\mathbf{p}_m(\lambda)] = \underbrace{[[\boldsymbol{\theta}_m[\mathbf{p}_m(\lambda)]]^{\mathrm{T}}}_{1\times N} \circ \underbrace{(-x_{1m}[\mathbf{p}_m(\lambda)], ..., -x_{jm}[\mathbf{p}_m(\lambda)], ..., -x_{Nm}[\mathbf{p}_m(\lambda)])]}_{1\times N}\underbrace{\mathbf{H}_m^{-1}[\mathbf{p}_m(\lambda)]}_{N\times N}$$

$$= [[\boldsymbol{\theta}_m[\mathbf{p}_m(\lambda)]]^{\mathrm{T}} \circ [\boldsymbol{\mu}_m[\mathbf{p}_m(\lambda)]]^{\mathrm{T}}] \begin{pmatrix} \frac{\partial x_{1m}}{\partial p_{1m}}[\mathbf{p}_m(\lambda)] & 0 & \cdots & 0 \\ 0 & \frac{\partial x_{2m}}{\partial p_{2m}}[\mathbf{p}_m(\lambda)] & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial x_{Nm}}{\partial p_{Nm}}[\mathbf{p}_m(\lambda)] \end{pmatrix} \mathbf{H}_m^{-1}[\mathbf{p}_m(\lambda)],$$

where

$$\boldsymbol{\mu}_m(\mathbf{p})^{\mathrm{T}} = \begin{pmatrix} \frac{-x_{1m}(\mathbf{p})}{\frac{\partial x_{1m}}{\partial p_{1m}}(\mathbf{p})} & \cdots & \frac{-x_{jm}(\mathbf{p})}{\frac{\partial x_{jm}}{\partial p_{jm}}(\mathbf{p})} & \cdots & \frac{-x_{Nm}(\mathbf{p})}{\frac{\partial x_{Nm}}{\partial p_{Nm}}(\mathbf{p})} \end{pmatrix}$$

is used.

Then, for the first part of the proposition, it is immediate to see that

$$\mathbf{Z}_m(\mathbf{p}_m^*) = [[\boldsymbol{\theta}_m(\mathbf{p}_m^*)]^{\mathrm{T}} \circ [\boldsymbol{\mu}_m(\mathbf{p}_m^*)]^{\mathrm{T}}]\boldsymbol{\rho}_m(\mathbf{p}_m^*),$$

which is interpreted as a result of applying the implicit function theorem to $\mathbf{g}(\mathbf{p}_m, \mathbf{c}_m) = \mathbf{0}$, where

$$\mathbf{g}(\underbrace{\mathbf{p}_m}_{N\times 1}, \underbrace{\mathbf{c}_m}_{N\times 1}) = \begin{pmatrix} \partial_{p_{1m}}\pi_{1m}(\mathbf{p}_m; c_{1m}) \\ \vdots \\ \partial_{p_{2m}}\pi_{jm}(\mathbf{p}_m; c_{2m}) \\ \vdots \\ \partial_{p_{Nm}}\pi_{Nm}(\mathbf{p}_m; c_{Nm}) \end{pmatrix}$$

so that $\boldsymbol{\rho}_m(\mathbf{p}_m^*) = -[\mathbf{D}_{\mathbf{p}_m}\mathbf{g}]^{-1}[\mathbf{D}_{\mathbf{c}_m}\mathbf{g}]$. Now, note that $W'(1) > 0$ if the inequality in this proposition holds. Thus, given the IRC, $W(\lambda)$ is strictly increasing in $[0,1]$, meaning that social welfare is higher under price discrimination than under uniform pricing. This complete the proof for the first part of the proposition.

For the second part, we proceed:

$$\mathbf{Z}_m(\lambda) = [[\boldsymbol{\theta}_m^{\mathrm{T}} \circ \boldsymbol{\mu}_m^{\mathrm{T}}]\boldsymbol{\rho}_m(\lambda)(\mathbf{K}\mathbf{H}_m^{-1}),$$

for $\lambda < 1$, and thus

$$W'(\lambda) = \left\{[\boldsymbol{\theta}_w^{\mathrm{T}} \circ \boldsymbol{\mu}_w^{\mathrm{T}}]\boldsymbol{\rho}_w[\mathbf{K}\mathbf{H}_w^{-1}\mathbf{K}^{-1}] - [\boldsymbol{\theta}_s^{\mathrm{T}} \circ \boldsymbol{\mu}_s^{\mathrm{T}}]\boldsymbol{\rho}_s[\mathbf{K}\mathbf{H}_s^{-1}\mathbf{K}^{-1}]\right\} \underbrace{(\mathbf{K}\boldsymbol{\Gamma}\mathbf{t}^*)}_{<<0}.$$

Then, it is verified that

$$\begin{aligned} W'(0) < 0 &\Leftrightarrow [\overline{\boldsymbol{\theta}}_w^{\mathrm{T}} \circ \overline{\boldsymbol{\mu}}_w^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_w[\overline{\mathbf{K}}\,\overline{\mathbf{H}}_w^{-1}\overline{\mathbf{K}}^{-1}] > [\overline{\boldsymbol{\theta}}_s^{\mathrm{T}} \circ \overline{\boldsymbol{\mu}}_s^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_s[\overline{\mathbf{K}}\,\overline{\mathbf{H}}_s^{-1}\overline{\mathbf{K}}^{-1}] \\ &\Leftrightarrow [\overline{\boldsymbol{\theta}}_w^{\mathrm{T}} \circ \overline{\boldsymbol{\mu}}_w^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_w < [\overline{\boldsymbol{\theta}}_s^{\mathrm{T}} \circ \overline{\boldsymbol{\mu}}_s^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_s[\overline{\mathbf{K}}\,\overline{\mathbf{H}}_s^{-1}\overline{\mathbf{K}}^{-1}][\overline{\mathbf{K}}\,\overline{\mathbf{H}}_w^{-1}\overline{\mathbf{K}}^{-1}]^{-1} \\ &\Leftrightarrow [\overline{\boldsymbol{\theta}}_w^{\mathrm{T}} \circ \overline{\boldsymbol{\mu}}_w^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_w < [\overline{\boldsymbol{\theta}}_s^{\mathrm{T}} \circ \overline{\boldsymbol{\mu}}_s^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_s[\overline{\mathbf{K}}\,\overline{\mathbf{H}}_s^{-1}\overline{\mathbf{H}}_w\overline{\mathbf{K}}^{-1}], \end{aligned}$$

which completes the proof.

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Online Appendix

A. The Case of a General Number of Markets

Throughout this paper, we assume one strong market and one weak market. More generally, by defining $S \equiv \{m|p_m^* > \overline{p}\}$ and $W \equiv \{m|\overline{p} > p_m^*\}$, we can let $p_s(r)$, $s \in S$, and $p_w(r)$, $w \in S$, consist of the optimal price vector under constraints $|p_m - \overline{p}| \leq r$ for all $m \in S \cup W$, with $r \in [0, \max_m |p_m^* - \overline{p}|]$. Then, for example, social welfare is defined as $W(r; c_s, c_w) \equiv \sum_{s \in S} \widetilde{U}_s(q_s[p_s(r)]) + \sum_{w \in W} \widetilde{U}_w(q_w[p_w(r)]) - 2\sum_{s \in S}(c_s \cdot q_s[p_s(r)]) - 2\sum_{w \in W}(c_w \cdot q_w[p_w(r)])$. Clearly, our two-market analysis does not lose any validity. In this way, an increase in the *weighted* aggregate output, $\sum_{m=s,w} (\overline{p} - c_m)\, q_m'[p_m(r)]p_m'(r) > 0$, can be written as $\mathbb{E}[(\overline{p}-c_m)q_m'p_m'] > 0$ with a general number of markets. For this to hold, $\mathrm{Cov}(\overline{p}-c_m, q_m'p_m') > 0$, that is, on average, the markup under uniform pricing and $q_m'p_m' > 0$ must be positively correlated because $\mathbb{E}[(\overline{p} - c_m)] > 0$ and $\mathbb{E}[q_m'p_m'] > 0$.

B. Non-constant Marginal Costs

Notably, our analysis does not necessitate the assumption of no cost differentials between discriminatory markets. In almost all theoretical studies on price discrimination, this assumption is made mainly to focus on demand differences. However, in many real-world cases of price discrimination, cost differentials are quite often observed, such as in the typical example of freight charges across regional markets with different transportation and storage costs (Phlips 1983, pp. 5-7). In the narrowest definition of price discrimination, this might not be considered price discrimination because they can be regarded as distinct products. However, airlines can be arguably motivated to offer different types of seats because they seek to exploit heterogeneity among consumers. Thus, ideally, a theoretical analysis of price discrimination should accommodate a moderate amount of cost differentials to exist across discriminatory markets.

Even if costs differ across markets, sellers may, in reality, have to engage in uniform pricing due to the universal service requirement, fairness concerns from consumers or for other reasons (Okada 2014; Geruso 2017; DellaVigna and Gentzkow 2019). In the case of monopoly with differentials in marginal costs between markets, Chen and Schwartz (2015) derive sufficient conditions under which the consumer surplus is higher under differential pricing.[1] To ensure that the strong market is indeed strong when cost differentials are allowed, it is sufficient to assume that the marginal cost in the strong market, which is assumed to be a constant and denoted by c_s, is higher than in that in the weak market, c_w: $c_s > c_w$ (although c_s should not be too much higher than c_w). Then, under uniform pricing, the profit margin in the strong

[1] Chen, Li and Schwartz (2021) extend Chen and Schwartz' (2015) analysis to the case of oligopoly. See also Galera and Zaratiegui (2006) and Bertoletti (2009) as studies of conditions under which price discrimination increases social welfare when cost differentials between markets are allowed.

market $\overline{p} - c_s$ is smaller than that in the weak market $\overline{p} - c_w$. Differential pricing enables the monopolist to sell more products in the weak market, which improves efficiency. Chen and Schwartz (2015) discover that while differential pricing with no cost differentials (i.e., third-degree price discrimination in a traditional manner) tends to increase the average price after differential pricing is permitted, differential pricing with cost differentials does not. In contrast, our analysis provides welfare implications more directly. As in Chen and Schwartz (2015), this paper does not make an explicit assumption regarding c_s and c_w as long as the second-order conditions for profit maximization are satisfied and a sufficiently large discrepancy between c_w and c_w does not change the order of discriminatory prices from the one with no cost differentials.[2]

Notice that our results so far do not crucially depend on the assumption of constant marginal costs. The only caveat is the definition of pass-through: to properly define pass-through in accommodation with non-constant marginal costs, we introduce a small amount of unit tax $t_m > 0$ in market m: the firm's first-order derivative of the profit with respect to its own price (Equation 3 in the main text) is now replaced by

$$\partial_p \pi_m(p) = q_m(p) + (p - t_m - mc_m[q_m(p)]) \frac{\partial x_{Am}}{\partial p_A}(p, p),$$

where $mc_m = c_m'[q_m(p)]$ is the marginal cost at $q_m(p)$. Then, pass-through is defined by $\rho_m \equiv \frac{\partial p_m}{\partial t_m}$, and no other changes should be made to derive the results above. In fact, the usefulness of pass-through is that it can easily be accommodated with non-constant marginal costs (Weyl and Fabinger 2013; Fabinger and Weyl 2019; and Adachi and Fabinger 2021). An additional caveat is that $\theta_m^* \rho_m^*$ is no longer interpreted as quantity pass-through under price discrimination (Weyl and Fabinger 2013, p. 572): one needs to take into account the "elasticity of the marginal cost" to approximate the trapezoids of the welfare gain and loss by a deviation from (full) price discrimination.

C. Computing the Sufficient Statistics Using the Parametric Demands

In this subsection, the number of firms is denoted by $N \geq 2$.

[2]In the context of reduced-fare parking as a form of third-degree price discrimination with cost differentials, Flores and Kalashnikov (2017) characterize a sufficient condition for free parking (drivers receive a price discount in the form of complementary parking while pedestrians do not) to be welfare improving.

C.1 Linear Demand

When there are N symmetric firms, firm j's direct demand in market m is given by

$$x_{jm} = \frac{1}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}\left\{\omega_m(1-\delta_m) - [1+(N-2)\delta_m]p_{jm} + \delta_m \sum_{-jm} p_{-j,m}\right\}.$$

In symmetric equilibrium, the firm's demand in market m is $q_m(p) = q_m(p) = \frac{\omega_m - p}{[1+(N-1)\delta_m]\beta_m}$(and thus, $q'_m(p) = -\frac{1}{[1+(N-1)\delta_m]\beta_m}$ and $\varepsilon^I_m(p) = \frac{p}{\omega_m - p}$) and the firm's own price elasticity in market m is

$$\begin{aligned}\varepsilon^{own}_m(p) &= -\frac{[1+(N-1)\delta_m]\beta_m}{\omega_m - p}\cdot p \cdot \left(-\frac{1+(N-2)\delta_m}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}\right)\\ &= \frac{[1+(N-2)\delta_m]p}{(1-\delta_m)(\omega_m - p)},\end{aligned}$$

which implies that the discriminatory price in market m satisfies:

$$\begin{aligned}\frac{p^*_m - c_m}{p^*_m} &= \frac{(1-\delta_m)(\omega_m - p^*_m)}{[1+(N-2)\delta_m]p^*_m}\\ \Leftrightarrow\quad p^*_m = p^*_m(c_m, \omega_m, \delta_m) &\equiv \frac{(1-\delta_m)\omega_m + [1+(N-2)\delta_m]c_m}{2+(N-3)\delta_m}.\end{aligned}$$

and thus $\rho^*_m = \frac{1+(N-2)\delta_m}{2+(N-3)\delta_m}$. Next, consider the uniform price. In symmetric equilibrium,

$$\begin{aligned}\bar{y}_m \bar{\varepsilon}^{own}_m &= \frac{\frac{\omega_m - \overline{p}}{[1+(N-1)\delta_m]\beta_m}}{\sum_{m=s,w}\frac{\omega_m - \overline{p}}{[1+(N-1)\delta_m]\beta_m}}\,\frac{[1+(N-2)\delta_m]\overline{p}}{(1-\delta_m)(\omega_m - \overline{p})}\\ &= \frac{\frac{[1+(N-2)\delta_m]\overline{p}}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}}{\sum_{m=s,w}\frac{\omega_m - \overline{p}}{[1+(N-1)\delta_m]\beta_m}}\end{aligned}$$

for $m = s$, w, which implies that the equilibrium uniform price $\overline{p}$ satisfies

$$\sum_{m=s,w}\frac{[1+(N-2)\delta_m](\overline{p} - c_m)}{[1+(N-1)\delta_m](1-\delta_m)\beta_m} = \sum_{m=s,w}\frac{\omega_m - \overline{p}}{[1+(N-1)\delta_m]\beta_m},$$

leading to the explicit solution:

$$\overline{p} = \overline{p}(\boldsymbol{c}, \boldsymbol{\delta}, \boldsymbol{\beta}, \boldsymbol{\omega}) \equiv \frac{\sum_{m=s,w}\frac{(1-\delta_m)\omega_m + [1+(N-2)\delta_m]c_m}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}}{\sum_{m=s,w}\frac{2+(N-3)\delta_m}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}}.$$

The conduct parameter in the case of linear demand is $\theta_m(p_m) = \frac{p_m - c_m}{\omega_m - p_m}$. In each panel of Figure OA1, the demand curve is depicted so that the ratio of $(p_m - c_m)$ to $(\omega_m - p_m)$ takes

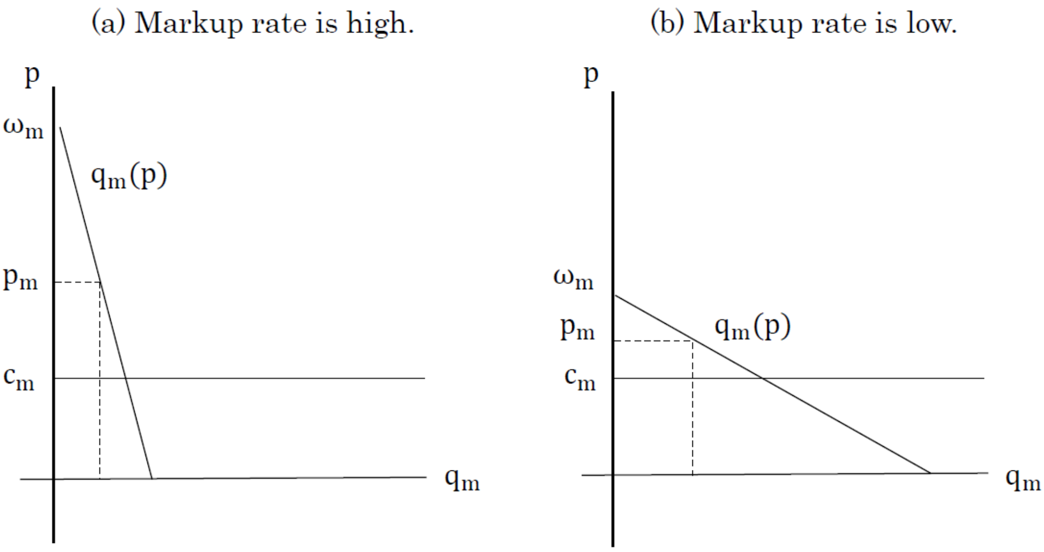

Figure OA1: θ_m with linear demands. In both cases, $(p_m - c_m)/(\omega_m - p_m)$ is almost identical.

almost the same value. However, in the left panel, the markup rate is high. This is mainly due to a low elasticity of the industry's demand rather than a low cross-price elasticity: the rivalness between brands may be sufficiently high. In the right panel, the markup rate is low. However, this does not necessarily mean that brands are in a strong rivalry. Instead,

$$\begin{aligned}
\varepsilon_m^{cross}(p_m) &= \frac{p_m}{q_m(p_m)} \cdot \frac{\partial x_{Bm}}{\partial p_A}(p_m, p_m) \\
&= \frac{[1+(n-1)\delta_m]\beta_m p_m}{\omega_m - p_m} \cdot \frac{\delta_m}{[1+(n-1)\delta_m](1-\delta_m)\beta_m} \\
&= \frac{\delta_m p_m}{(\omega_m - p_m)(1-\delta_m)}
\end{aligned}$$

can be low as long as $\varepsilon_m^I(p_m)$ is sufficiently high (that $\varepsilon_m^{own}(p_m)$ is high, and as a result, the markup rate is low). In both panels, the market is close to monopoly because $\theta_m(p_m)$ is close to one. This graphical argument demonstrates that $\theta_m(p_m)$, rather than $L_m(p_m)$, better captures the competitiveness in market m. Because both the curvature of the firm's direct demand and the elasticity of the cross-price effect are zero ($\alpha_m^{own}(p) = 0$ and $\alpha_m^{cross}(p) = 0$), it is observed that

$$\begin{aligned}
\frac{\pi_m''(p)}{q_m'(p)} &= \frac{2 - L_m(p)\left[\alpha_m^{own}(p) + \alpha_m^{cross}(p)\right] - [1-\theta_m(p)]}{\theta_m(p)} \\
&= \frac{1+\theta_m(p)}{\theta_m(p)} \\
&= \frac{\omega_m - c_m}{p - c_m} \\
&= \frac{2-\delta_m}{1-\delta_m},
\end{aligned}$$

which implies that $Q'(r) > 0$ if and only if $\delta_s > \delta_w$, that is, the firms' products are less differentiated in the strong market. Finally, note that the following two restrictions are inevitably imposed on the sufficient statistics: (i) the two curvatures, α_m^{own} and α_m^{cross}, are necessarily zero, and (ii) pass-through, ρ_m, is necessarily constant, and is always less than one ("cost-absorbing").

Finally, in general (for $N \geq 2$), it is verified that

$$G(\boldsymbol{c}, \boldsymbol{\delta}, \boldsymbol{\beta}, \boldsymbol{\omega}) = \frac{\overline{p} - c_s}{\overline{p} - c_w} - \frac{[2 + (2N-5)\delta_s][1 + (N-3)\delta_w]}{[2 + (2N-5)\delta_w][1 + (N-3)\delta_s]}$$

and

$$H(\boldsymbol{c}, \boldsymbol{\delta}, \boldsymbol{\beta}, \boldsymbol{\omega}) = \frac{(\omega_s - c_s)(1-\delta_s)[2 + (N-3)\delta_w]}{(\omega_w - c_w)(1-\delta_w)[2 + (N-3)\delta_s]} - \frac{[2 + (N-3)\delta_s][1 + (N-3)\delta_w]}{[2 + (N-3)\delta_w][1 + (N-3)\delta_s]}.$$

C.2 CES (Constant Elasticity of Substitution) Demand

The representative consumer's utility in market m when there are N firms is given by

$$U_m(\mathbf{x}_m) = \left(\sum_{j=1}^{N} x_{jm}^{\beta_m} \right)^{\eta_m}.$$

Then, it is shown that

$$\frac{\partial x_{Am}}{\partial p_A}(p,p) = -\frac{N^{-\frac{1-\eta_m}{1-\beta_m\eta_m}} (\beta_m\eta_m)^{\frac{1}{1-\beta_m\eta_m}} \cdot [N - \beta_m - (N-1)\beta_m\eta_m]}{N(1-\beta_m)(1-\beta_m\eta_m)} \cdot p^{-\frac{2-\beta_m\eta_m}{1-\beta_m\eta_m}}$$

and

$$\frac{d}{dp}\left(\frac{\partial x_{Am}}{\partial p_A}(p,p) \right) = \frac{N^{-\frac{1-\eta_m}{1-\beta_m\eta_m}} (\beta_m\eta_m)^{\frac{1}{1-\beta_m\eta_m}} \cdot [N - \beta_m - (N-1)\beta_m\eta_m](2-\beta_m\eta_m)}{N(1-\beta_m)(1-\beta_m\eta_m)^2} \cdot p^{-1-\frac{2-\beta_m\eta_m}{1-\beta_m\eta_m}}.$$

Similarly, it is observed that

$$q_m(p) = N^{-\frac{1-\eta_m}{1-\beta_m\eta_m}} (\beta_m\eta_m)^{\frac{1}{1-\beta_m\eta_m}} \, p^{\frac{-1}{1-\beta_m\eta_m}}$$

and thus

$$q_m'(p) = -\frac{N^{-\frac{1-\eta_m}{1-\beta_m\eta_m}} (\beta_m\eta_m)^{\frac{1}{1-\beta_m\eta_m}}}{1-\beta_m\eta_m} \cdot p^{-\frac{2-\beta_m\eta_m}{1-\beta_m\eta_m}}$$

as well as

$$\begin{aligned} \pi_m''(p) \quad = \quad & -\frac{N^{-\frac{1-\eta_m}{1-\beta_m\eta_m}} (\beta_m\eta_m)^{\frac{1}{1-\beta_m\eta_m}}}{N(1-\beta_m)(1-\beta_m\eta_m)^2} \cdot p^{-\frac{2-\beta_m\eta_m}{1-\beta_m\eta_m}} \\ & \times \{[N(2-\beta_m) - \beta_m - (N-1)\beta_m\eta_m](1-\beta_m\eta_m) \end{aligned}$$

$$-[N-\beta_m-(N-1)\beta_m\eta_m](2-\beta_m\eta_m)\cdot\frac{p-c_m}{p}\Bigg\}.$$

Therefore, the own price elasticity is given by the following *constant*:

$$\begin{aligned}\varepsilon_m^{own}(p) &= -\frac{p}{q_m(p)}\cdot\frac{\partial x_{Am}}{\partial p_A}(p,p)\\ &= \frac{N-\beta_m-(N-1)\beta_m\eta_m}{N(1-\beta_m)(1-\beta_m\eta_m)},\end{aligned}$$

which indicates that the discriminatory price in market m is obtained explicitly by solving:

$$\begin{aligned}&\frac{p_m^*-c_m}{p_m^*}=\frac{N(1-\beta_m)(1-\beta_m\eta_m)}{N-\beta_m-(N-1)\beta_m\eta_m}\\ \Leftrightarrow\quad & p_m^*=p_m^*(c_m,\beta_m,\eta_m)\equiv\frac{[N-\beta_m-(N-1)\beta_m\eta_m]c_m}{\beta_m[(N-1)+(1-N\beta_m)\eta_m]}.\end{aligned}$$

Accordingly, the pass-through and the profit margin are given by:

$$\rho_m^*=\frac{N-\beta_m-(N-1)\beta_m\eta_m}{\beta_m[(N-1)+(1-N\beta_m)\eta_m]}$$

and

$$\mu_m^*=\frac{N(1-\beta_m)(1-\beta_m\eta_m)}{\beta_m[(N-1)+(1-N\beta_m)\eta_m]}c_m$$

respectively, both of which are also constant.

Because it is shown that

$$\frac{\partial x_{Bm}}{\partial p_A}(p,p)=\frac{N^{-\frac{1-\eta_m}{1-\beta_m\eta_m}}(\beta_m\eta_m)^{\frac{1}{1-\beta_m\eta_m}}\cdot\beta_m(1-\eta_m)}{N(1-\beta_m)(1-\beta_m\eta_m)}\cdot p^{-1-\frac{1}{1-\beta_m\eta_m}},$$

the cross price elasticity is given by

$$\begin{aligned}\varepsilon_m^{cross}(p) &= \frac{p}{q_m(p)}\cdot\frac{\partial x_{Bm}}{\partial p_A}(p,p)\\ &= \frac{\beta_m(1-\eta_m)}{N(1-\beta_m)(1-\beta_m\eta_m)}\end{aligned}$$

as a constant, which implies that the conduct is also given by

$$\theta_m^*=1-\frac{(\varepsilon^{cross})^*}{(\varepsilon^{own})^*}=\frac{N-2\beta_m-(N-2)\beta_m\eta_m}{N-\beta_m-(N-1)\beta_m\eta_m}$$

as a constant.

The own and the cross curvatures are also obtained as constants:

$$\alpha_m^{own}(p) = \frac{\beta_m^2(1-\eta_m)(2-\eta_m-\beta_m\eta_m)+(2-\beta_m)N^2(1-\beta_m\eta_m)^2-3N\beta_m(1-\eta_m)(1-\beta_m\eta_m)}{(1-\beta_m)N(1-\beta_m\eta_m)[N(1-\beta_m\eta_m)-\beta_m(1-\eta_m)]}$$

and

$$\alpha_m^{cross}(p) = -\frac{\beta_m(1-\eta_m)[N-2\beta_m-\beta_m\eta_m(N-1-\beta_m)]}{(1-\beta_m)N(1-\beta_m\eta_m)[N-\beta_m-(N-1)\beta_m\eta_m]},$$

respectively.

The equilibrium uniform price satisfies

$$\sum_{m=s,w} q_m(\overline{p}) \cdot \left[\frac{[N-\beta_m-(N-1)\beta_m\eta_m](\overline{p}-c_m)}{n(1-\beta_m)(1-\beta_m\eta_m)} - \overline{p}\right] = 0$$

because

$$\bar{y}_m\bar{\varepsilon}_m^{own} = \frac{q_m(\overline{p})}{q_s(\overline{p})+q_w(\overline{p})} \cdot \frac{N-\beta_m-(N-1)\beta_m\eta_m}{N(1-\beta_m)(1-\beta_m\eta_m)}.$$

It is also verified that

$$\begin{aligned} H(\boldsymbol{c},\boldsymbol{\beta},\boldsymbol{\eta}) &= \frac{\dfrac{N(1-\beta_s)(1-\beta_s\eta_s)c_s}{\beta_s[(N-1)+(1-N\beta_s)\eta_s]}}{\dfrac{N(1-\beta_w)(1-\beta_w\eta_w)c_w}{\beta_w[(N-1)+(1-N\beta_w)\eta_w]}} \\ &\quad - \frac{\dfrac{1}{\dfrac{N-2\beta_s-(N-2)\beta_s\eta_s}{N-\beta_s-(N-1)\beta_s\eta_s}\cdot\dfrac{N-\beta_s-(N-1)\beta_s\eta_s}{\beta_s[(N-1)+(1-N\beta_s)\eta_s]}}}{\dfrac{1}{\dfrac{N-2\beta_w-(N-2)\beta_w\eta_w}{N-\beta_w-(N-1)\beta_w\eta_w}\cdot\dfrac{N-\beta_w-(N-1)\beta_w\eta_w}{\beta_w[(N-1)+(1-N\beta_w)\eta_w]}}} \\ &= \frac{\beta_w[(N-1)+(1-N\beta_w)\eta_w](1-\beta_s)(1-\beta_s\eta_s)c_s}{\beta_s[(N-1)+(1-N\beta_s)\eta_s](1-\beta_w)(1-\beta_w\eta_w)c_w} \\ &\quad - \frac{\beta_s[(N-1)+(1-N\beta_s)\eta_s][N-2\beta_w-(N-2)\beta_w\eta_w]}{\beta_w[(N-1)+(1-N\beta_w)\eta_w][N-2\beta_s-(N-2)\beta_s\eta_s]} \end{aligned}$$

and

$$\begin{aligned} &G(\boldsymbol{c},\boldsymbol{\beta},\boldsymbol{\eta}) \\ &= \frac{\overline{p}-c_s}{\overline{p}-c_w} - \frac{(1-\beta_w)(1-\beta_w\eta_w)}{(1-\beta_s)(1-\beta_s\eta_s)} \\ &\times \frac{[N(2-\beta_s)-\beta_s-(N-1)\beta_s\eta_s](1-\beta_s\eta_s)-[N-\beta_s-(N-1)\beta_s\eta_s](2-\beta_s\eta_s)\cdot\dfrac{\overline{p}-c_s}{\overline{p}}}{[N(2-\beta_w)-\beta_w-(N-1)\beta_w\eta_w](1-\beta_w\eta_w)-[N-\beta_w-(N-1)\beta_w\eta_w](2-\beta_w\eta_w)\cdot\dfrac{\overline{p}-c_w}{\overline{p}}} \end{aligned}$$

because

$$\begin{aligned}\frac{\pi_m''}{q_m'} \quad = \quad & \frac{1}{N(1-\beta_m)(1-\beta_m\eta_m)} \\ & \times \{[N(2-\beta_m)-\beta_m-(N-1)\beta_m\eta_m](1-\beta_m\eta_m) \\ & \quad -[N-\beta_m-(N-1)\beta_m\eta_m](2-\beta_m\eta_m)\cdot\frac{\overline{p}-c_s}{\overline{p}}\bigg\}.\end{aligned}$$

C.3 Multinomial Logit Demand with Outside Option

When there are N symmetric firms, each firm j faces the following market share/demand function:

$$x_{jm} = \frac{\exp(\omega_m - \beta_m p_{jm})}{1 + \sum_{j'=1}^{N} \exp(\omega_m - \beta_m p_{j'm})},$$

and under symmetric pricing, each firm's share is

$$q_m(p) = \frac{\exp(\omega_m - \beta_m p)}{1 + N \cdot \exp(\omega_m - \beta_m p)}.$$

For any $N \geq 2$, the own and cross price elasticities are $\varepsilon_m^{own} = \beta_m p_m(1-q_m)$ and $\varepsilon_m^{cross} = \beta_m p_m q_m$, respectively. Hence, the conduct parameter and the pass-through under price discrimination are

$$\theta_m^* = 1 - \frac{(\varepsilon_m^{cross})^*}{(\varepsilon_m^{own})^*} = \frac{1-2q_m^*}{1-q_m^*},$$

and

$$\rho_m^* = \frac{1}{1 + \dfrac{q_m^*(1-N\cdot q_m^*)}{(1-q_m^*)^2}},$$

respectively. It is thus shown that

$$H(\boldsymbol{c},\boldsymbol{\beta},\boldsymbol{\omega}) = \left(\frac{1-q_w^*}{1-q_s^*}\right)\left(\frac{\beta_w}{\beta_s} - \frac{1-2q_w^*}{1-2q_s^*}\cdot\frac{1-q_s^*-(N-1)[q_s^*]^2}{1-q_w^*-(N-1)[q_w^*]^2}\right).$$

Note here that under logit demand, pass-through, ρ_m^*, is necessarily greater than one ("cost-amplifying").

Similar to the first-order elasticties, the two curvatures are expressed as $\alpha_m^{own} = \beta_m p_m(1-2q_m)$ and $\alpha_m^{cross} = \frac{\beta_m p_m q_m(1-2q_m)}{1-q_m}$ for any $N \geq 2$. Therefore, the expression for $G(\boldsymbol{c},\boldsymbol{\beta},\boldsymbol{\omega})$ cannot become more complicated than that in the main text.

D. Equivalence of Holmes' (1989) and Our Expressions for $Q'(r)$ with No Cost Differentials

Holmes (1989, p. 247), who assumes no cost differentials ($c \equiv c_s = c_w$) as in most of the papers on third-degree price discrimination, also derives a necessary and sufficient condition for $Q'(r) > 0$ under symmetric oligopoly. It is (using our notation) written as:

$$\underbrace{\frac{p_s - c}{q_s'(p_s)} \cdot \frac{d}{dp_s}\left(\frac{\partial x_{A,s}(p_s,p_s)}{\partial p_A}\right) - \frac{p_w - c}{q_w'(p_w)} \cdot \frac{d}{dp_w}\left(\frac{\partial x_{A,w}(p_w,p_w)}{\partial p_A}\right)}_{\text{Adjusted-concavity condition (Robinson 1933)}}$$
$$+ \underbrace{\frac{\varepsilon_s^{cross}(p_s)}{\varepsilon_s^I(p_s)} - \frac{\varepsilon_w^{cross}(p_w)}{\varepsilon_w^I(p_w)}}_{\text{Elasticity-ratio condition (Holmes 1989)}} > 0.$$

Recall that $\frac{1}{\theta_s} - \frac{1}{\theta_w} = \frac{\varepsilon_s^{cross}}{\varepsilon_s^I} - \frac{\varepsilon_w^{cross}}{\varepsilon_w^I}$. The first and the second terms in the left hand side of Holmes' (1989) inequality is rewritten as:

$$\begin{aligned}
& \frac{p_s - c}{q_s'(p_s)} \cdot \frac{d}{dp_s}\left(\frac{\partial x_{A,s}(p_s,p_s)}{\partial p_A}\right) - \frac{p_w - c}{q_w'(p_w)} \cdot \frac{d}{dp_w}\left(\frac{\partial x_{A,w}(p_w,p_w)}{\partial p_A}\right) \\
= \; & L_w(p_w) \cdot \left[\left(-\frac{p_w}{q_w'(p_w)}\right)\frac{d}{dp_w}\left(\frac{\partial x_{A,w}(p_w,p_w)}{\partial p_A}\right)\right] \\
& - L_s(p_s) \cdot \left[\left(-\frac{p_s}{q_s'(p_s)}\right)\frac{d}{dp_s}\left(\frac{\partial x_{A,s}(p_s,p_s)}{\partial p_A}\right)\right].
\end{aligned}$$

Now, it is also observed that

$$\begin{aligned}
\frac{\alpha_m^{own} + \alpha_m^{cross}}{\theta_m} &= \frac{\partial x_{Am}/\partial p_A}{q_m'}\left(-\frac{p_m}{\partial x_{Am}/\partial p_A}\frac{\partial^2 x_{Am}}{\partial p_A^2} - \frac{p_m}{\partial x_{Am}/\partial p_A}\frac{\partial^2 x_{Am}}{\partial p_B \partial p_A}\right) \\
&= -\frac{p_m}{q_m'}\left(\frac{\partial^2 x_{Am}}{\partial p_A^2} + \frac{\partial^2 x_{Am}}{\partial p_B \partial p_A}\right). \\
&= \left(-\frac{p_m}{q_m'(p_m)}\right)\frac{d}{dp_m}\left(\frac{\partial x_{Am}(p_m,p_m)}{\partial p_A}\right).
\end{aligned}$$

Therefore, inequality (14) is another expression for Holmes' (1989, p. 247) inequality (9) because

$$\frac{p_s - c}{q_s'} \cdot \frac{d}{dp_s}\left(\frac{\partial x_{A,s}}{\partial p_A}\right) - \frac{p_w - c}{q_w'(p_w)} \cdot \frac{d}{dp_w}\left(\frac{\partial x_{A,w}}{\partial p_A}\right) = L_w \cdot \frac{\alpha_w^{own} + \alpha_w^{cross}}{\theta_w} - L_s \cdot \frac{\alpha_s^{own} + \alpha_s^{cross}}{\theta_s}.$$

$$\boxed{\begin{array}{c} \text{DMCLC: } g'_m < 0 \\ \Updownarrow \\ \varsigma^I_m > \epsilon^I_m \end{array}}$$

$$\Downarrow \qquad\qquad (\text{If } \sigma^I_m < 1 \Leftrightarrow \epsilon^I_m > \alpha^I_m \text{ is imposed})$$

$$\boxed{\begin{array}{c} \varsigma^I_m > \alpha^I_m \\ \Updownarrow \\ \text{IRCQ: } h'_m < 0 \end{array}}$$

Table OA1: DMCLC implies IRCQ

E. An Alternative Way to Verify "DMCLC $\Rightarrow$ IRCQ $\Rightarrow$ IRCW"

First, recall that

$$g'_m \;=\; -\frac{h'_m}{[h_m]^2} \times \frac{q_m}{q'_m} + \frac{1}{h_m}(1 - \sigma^I_m),$$

where $1 - \sigma^I_m > 0$ is assumed, that is, q_m is not " too convex." Thus, $g'_m < 0$ if and only if

$$h'_m < \underbrace{\left(\frac{q'_m h_m}{q_m}\right)}_{<0} \cdot \underbrace{\left(1 - \frac{q_m q''_m}{[q'_m]^2}\right)}_{>0},$$

indicating that "if $g'_m < 0$ (DMCLC), then $h'_m < 0$ (IRCQ)." However, the converse is not true because $h'_m < 0$ is not sufficient to ensure that $g'_m < 0$. This relationship is illustrated in Table OA1.

Next, recall that $z_m(p) = (p - c_m)/h_m(p)$, which implies that

$$z'_m = \frac{h_m - (p - c_m)h'_m}{[h_m]^2}.$$

Thus, if $h'_m < 0$, then $h_m > 0 > (p - c_m)h'_m$, indicating that "if $h'_m < 0$ (IRCQ), then $z'_m > 0$ (IRCW)." However, the converse is not true because $z'_m > 0$ is not sufficient to ensure that $h'_m < 0$.

F. Whether the DMCLC holds in the Parametric Examples

F.1 Linear Demand

First, it is observed that

$$\pi''_m(p) \;=\; q'_m(p) + \frac{\partial x_{Am}}{\partial p_A}(p,p) + (p - c_m)\frac{d}{dp}\left(\frac{\partial x_{Am}}{\partial p_A}(p,p)\right)$$

$$
\begin{aligned}
&= -\frac{1}{(1+\delta_m)\beta_m} - \frac{1}{\left(1-\delta_m^2\right)\beta_m} \\
&= -\frac{2-\delta_m}{(1-\delta_m^2)\beta_m} < 0
\end{aligned}
$$

because

$$q_m'(p) = -\frac{1}{(1+\delta_m)\beta_m}$$

and

$$\frac{\partial x_{Am}}{\partial p_A}(p,p) = -\frac{1}{\left(1-\delta_m^2\right)\beta_m}.$$

Then, it is verified that

$$
\begin{aligned}
g_m(p) &= \frac{q_m(p)}{\pi_m''(p)} \\
&= \frac{\left(\frac{\omega_m - p}{(1+\delta_m)\beta_m}\right)}{\left(-\frac{2-\delta_m}{(1-\delta_m^2)\beta_m}\right)} \\
&= -\frac{(\omega_m - p)(1-\delta_m)}{2-\delta_m}
\end{aligned}
$$

and thus $g_m'(p) = \frac{1-\delta_m}{2-\delta_m} > 0$; the DMCLC never holds. Moreover, the IRCQ does not hold, either because

$$
\begin{aligned}
h_m(p) &= \frac{1}{\frac{q_m'(p)}{\pi_m''(p)}} \\
&= \frac{\left(-\frac{2-\delta_m}{(1-\delta_m^2)\beta_m}\right)}{\left(-\frac{1}{(1+\delta_m)\beta_m}\right)} \\
&= \frac{2-\delta_m}{1-\delta_m},
\end{aligned}
$$

implying that $h_m'(p) = 0$.

However, the IRCW does hold. This is because

$$
\begin{aligned}
z_m(p) &= \frac{(p-c_m)q_m'(p)}{\pi_m''(p)} \\
&= \frac{(p-c_m)\left(-\frac{1}{(1+\delta_m)\beta_m}\right)}{\left(-\frac{2-\delta_m}{(1-\delta_m^2)\beta_m}\right)} \\
&= (p-c_m)\frac{(1-\delta_m)}{(2-\delta_m)},
\end{aligned}
$$

which implies that $z_m'(p) = \frac{1-\delta_m}{2-\delta_m} > 0$.

F.2 CES (Constant Elasticity of Substitution) Demand

First, it is verified that

$$\begin{aligned} \pi_m''(p) &= -\frac{2^{-\frac{1-\eta_m}{1-\beta_m\eta_m}}(\beta_m\eta_m)^{\frac{1}{1-\beta_m\eta_m}}}{2(1-\beta_m)(1-\beta_m\eta_m)^2} \cdot p^{-\frac{2-\beta_m\eta_m}{1-\beta_m\eta_m}} \\ &\quad \times \left[(4-3\beta_m-\beta_m\eta_m)(1-\beta_m\eta_m) - (2-\beta_m-\beta_m\eta_m)(2-\beta_m\eta_m) \cdot \frac{p-c_m}{p}\right]. \end{aligned}$$

Then, it is observed that

$$\begin{aligned} g_m(p) &= \frac{q_m(p)}{\pi_m''(p)} \\ &= -2(1-\beta_m)(1-\beta_m\eta_m)^2 \\ &\quad \times [(4-3\beta_m-\beta_m\eta_m)(1-\beta_m\eta_m)p - (2-\beta_m-\beta_m\eta_m)(2-\beta_m\eta_m)(p-c_m)] \end{aligned}$$

and thus

$$g_m'(p) = -2\beta_m(1-\beta_m)(1-\beta_m\eta_m)^2[(3-\beta_m)\eta_m - \beta_m\eta_m^2 - 1],$$

which implies that g_m is decreasing (the DMCLC) if and only if $(3-\beta_m)\eta_m - \beta_m\eta_m^2 - 1 < 0$. Figure A1 illustrates where the DMCLC holds in the region of (β_m, η_m). However, the IRCQ always holds because

$$\begin{aligned} h_m(p) &= \frac{1}{\frac{q_m'(p)}{\pi_m''(p)}} \\ &= \frac{1}{2(1-\beta_m)(1-\beta_m\eta_m)} \\ &\quad \times \left[(4-3\beta_m-\beta_m\eta_m)(1-\beta_m\eta_m) - (2-\beta_m-\beta_m\eta_m)(2-\beta_m\eta_m) \cdot \frac{p-c_m}{p}\right] \end{aligned}$$

and thus

$$h_m'(p) = -\frac{(2-\beta_m-\beta_m\eta_m)(2-\beta_m\eta_m)c_m}{2(1-\beta_m)(1-\beta_m\eta_m)p^2} < 0.$$

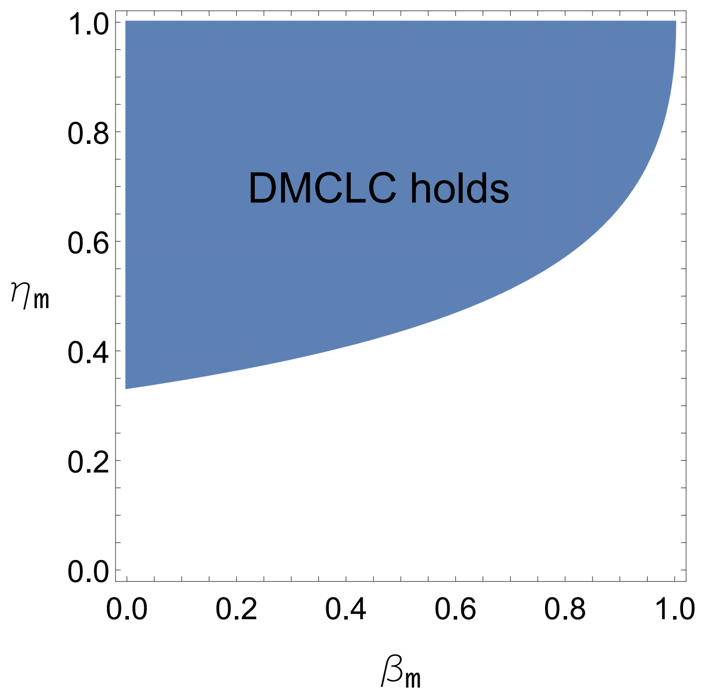

Figure A1: The region for (β_m, η_m) where the DMCLC holds under the CES demand.

F.3 Multinomial Logit Demand with Outside Option

Unfortunately, the DMCLC does not always hold for logit demand, though $\sigma^I_m(p) < 1$ is satisfied.[3] To see this, first, recall that $g'_m < 0$ if and only if $\varsigma^I_m > \epsilon^I_m$ ($\Leftrightarrow -\frac{p\pi'''_m}{\pi''_m} > -\frac{pq'_m}{q_m}$), that is,

$$-\frac{\pi'''_m(p)}{\pi''_m(p)} > \beta_m \cdot [1 - 2q_m(p)] \Leftrightarrow \pi'''_m(p) > -\beta_m \cdot [1 - 2q_m(p)]\pi''_m(p),$$

where we derive (from Equations 2 and 3)

$$\begin{aligned}
\pi''_m(p) &= 2\frac{\partial x_{Am}}{\partial p_{Am}}(p,p) + \frac{\partial x_{Am}}{\partial p_{Bm}}(p,p) + (p - c_m)\left[\frac{\partial^2 x_{Am}}{\partial p^2_{Am}}(p,p) + \frac{\partial^2 x_{Am}}{\partial p_{Bm}\partial p_{Am}}(p,p)\right] \\
&= -\beta_m q_m(p)\left\{2 - 3q_m(p) - \beta_m(p - c_m)[1 - 2q_m(p)]^2\right\}
\end{aligned}$$

[3] To see this, recall that the logit demand under symmetric pricing is given by:

$$q_m(p) = \frac{\exp(\omega_m - \beta_m p)}{1 + 2\exp(\omega_m - \beta_m p)},$$

which implies that $q'_m(p) = -\beta_m q_m(p)[1 - 2q_m(p)]$ and $q''_m(p) = -\beta_m q'_m(p)[1 - 4q_m(p)]$. Hence,

$$q''_m(p) < \frac{[q'_m(p)]^2}{q_m(p)} \Leftrightarrow -\beta_m[1 - 4q_m(p)] > \frac{q'_m(p)}{q_m(p)} \Leftrightarrow 1 - 4q_m(p) < 1 - 2q_m(p),$$

which must be true.

by using

$$\begin{cases} \frac{\partial x_{Am}}{\partial p_{Am}}(p,p) = -\beta_m q_m(p)[1-q_m(p)] \\ \frac{\partial x_{Am}}{\partial p_{Bm}}(p,p) = \beta_m [q_m(p)]^2 \\ \frac{\partial^2 x_{Am}}{\partial p_{Am}^2}(p,p) = \beta_m^2 q_m(p)[1-q_m(p)][1-2q_m(p)] \\ \frac{\partial^2 x_{Am}}{\partial p_{Bm}\partial p_{Am}}(p,p) = -\beta_m^2 [q_m(p)]^2[1-2q_m(p)] \end{cases}$$

and

$$\begin{aligned} \pi_m'''(p) &= 2\frac{\partial^2 x_{Am}}{\partial p_{Am}^2}(p,p) + 3\frac{\partial^2 x_{Am}}{\partial p_{Am}\partial p_{Bm}}(p,p) + \frac{\partial^2 x_{Am}}{\partial p_{Bm}^2}(p,p) \\ &\quad +(p-c_m)\left[\frac{\partial^3 x_{Am}}{\partial p_{Am}^3}(p,p) + 2\frac{\partial^3 x_{Am}}{\partial p_{Am}^2\partial p_{Bm}}(p,p) + \frac{\partial^3 x_{Am}}{\partial p_{Am}\partial p_{Bm}^2}(p,p)\right] \\ &= \beta_m^2 q_m(p)[1-2q_m(p)]\{2[1-3q_m(p)] \\ &\qquad -\beta_m(p-c_m)[1-2q_m(p)][1-6q_m(p)]\} \end{aligned}$$

by additionally using

$$\begin{cases} \frac{\partial^2 x_{Am}}{\partial p_{Bm}^2}(p,p) = -\beta_m^2 [q_m(p)]^2[1-2q_m(p)] \\ \frac{\partial^3 x_{Am}}{\partial p_{Am}^3}(p,p) = -\beta_m^3 q_m(p)[1-q_m(p)]\{1-6q_m(p)+6[q_m(p)]^2\} \\ \frac{\partial^3 x_{Am}}{\partial p_{Am}^2\partial p_{Bm}}(p,p) = \beta_m^3 [q_m(p)]^2\{1-6q_m(p)+6[q_m(p)]^2\} \\ \frac{\partial^3 x_{Am}}{\partial p_{Am}\partial p_{Bm}^2}(p,p) = \beta_m^3 [q_m(p)]^2\{1-4q_m(p)+6[q_m(p)]^2\}. \end{cases}$$

Then, it is observed that

$$\begin{aligned} -\beta_m[1-2q_m(p)]\pi_m''(p) &= \beta_m^2 q_m(p) \\ &\quad \times[1-2q_m(p)]\left\{2-3q_m(p)-\beta_m(p-c_m)[1-2q_m(p)]^2\right\}, \end{aligned}$$

which implies that

$$\begin{aligned} &\pi_m'''(p) > -\beta_m \cdot [1-2q_m(p)]\pi_m''(p) \\ &\Leftrightarrow 2[1-3q_m(p)] - \beta_m(p-c_m)[1-2q_m(p)][1-6q_m(p)] \\ &\qquad > 2-3q_m(p)-\beta_m(p-c_m)[1-2q_m(p)]^2. \end{aligned}$$

This is equivalent to $\beta_m(p-c_m)[1-2q_m(p)] > 3/4$, and is not true for all p. In other words, the DMCLC holds only when $\beta_m(p-c_m)[1-2q_m(p)] > 3/4$.

However, logit demand does satisfy the IRCQ (and hence the IRCW). This is because $h_m^{'}$

is negative if and only if $\varsigma_m^I > \alpha_m^I$ ($\Leftrightarrow -\frac{p\pi_m'''}{\pi_m''} > -\frac{pq_m''}{q_m'}$), that is

$$\begin{aligned}
&\pi_m'''(p) > -\beta_m \cdot [1-4q_m(p)]\pi_m''(p)\\
&\Leftrightarrow [1-2q_m(p)]\left\{2[1-3q_m(p)] - \beta_m(p-c_m)[1-2q_m(p)][1-6q_m(p)]\right\}\\
&\qquad > [1-4q_m(p)]\left\{2-3q_m(p) - \beta_m(p-c_m)[1-2q_m(p)]^2\right\},
\end{aligned}$$

which is further rewritten as $\beta_m(p-c_m)[1-2q_m(p)]^2 > -1/2$, and this is obviously true.

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