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Takanori Adachi, Susumu Sato, and Mark J. Tremblay

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Graduate School of Economics
Kyoto University
Yoshida-Hommachi, Sakyo-ku
Kyoto City, 606-8501, Japan

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Platform Oligopoly with Endogenous Homing: Implications for Mergers and Free Entry*

Takanori Adachi
Graduate School of Management and
Graduate School of Economics, Kyoto University
adachi.takanori.8m@kyoto-u.ac.jp

Susumu Sato
Institute of Economic Research
Hitotsubashi University
susumusato.econ@gmail.com

Mark J. Tremblay
Farmer School of Business
Miami University
tremblmj@miamioh.edu

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Abstract

Consumer multi-homing is considered to be critical for competition policy regarding digital platforms. To assess the role of consumer multi-homing in competition policy, we embed consumer multi-homing into a model of oligopolistic competition between two-sided platforms and apply it to mergers and free entry. We find that a required level of merger-specific cost reduction is larger if consumers benefit more from multi-homing and that the equilibrium level of platform entry can be insufficient in the presence of consumer multi-homing. We also show that reductions to sellers' benefit from multi-homing reduces entry (i.e., is an effective barrier to entry). These results contrast the popular belief that multi-homing mitigates the need for stricter competition policy.

Keywords: Two-sided markets; Indict network externalities; Multi-homing; Platform entry; Platform mergers.
JEL Classifications: D40, L10, L20, L40.

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1 Introduction

It is well recognized in two-sided markets that some agents will single-home while others will multi-home. Furthermore, consumer multi-homing is considered to be critical for competition policy surrounding digital platforms because multi-homing mitigates the tendency of market tipping (e.g. Crémer et al., 2019). However, less is understood about how the extent of multi-homing impacts oligopoly platforms (e.g., platform merger policies in the short run, and platform entry in the long run). To gain a better understanding of these effects, we develop a model of endogenous homing decisions in two-sided markets with multiple platforms. We also include parameters that impact the benefits from multi-homing for consumers and sellers and this allows our model to account for differences in homing preferences across platform industries. Utilizing the incremental-value principle (Anderson et al., 2019) for the seller side and Cournot competition on the consumer side, we develop a new framework of oligopolistic platform competition.

This modeling approach allows us to conduct a merger evaluation and examine the welfare effects of platform entry using the familiar techniques. As the benefits from consumer multi-homing change, equilibrium platform behavior adjusts in two ways. First, with greater benefits from multi-homing, more consumers multi-home and overlapping consumer participation between different platforms increases; as a result, the seller price lowers as in the case of multi-homing in media markets. Second, because a lower consumer price induces a higher level of consumer multi-homing (which in turn lowers the seller price), platforms have less incentive to lower the consumer price when consumers earn greater benefits from multi-homing. These two effects play an important role in our analysis of platform mergers and free entry.

Regarding platform mergers, we show that the required level of merger-specific cost reduction that is necessary for consumer surplus to increase post-merger is larger when there exists a higher ratio of multi-homing consumers. To see this, note that when the number of platforms is reduced by a merger, each platform is more willing to reduce consumer multi-homing, resulting in a reduction of aggregate output. This adverse effect is stronger when the benefit from consumer multi-homing is larger. Our result stands in sharp contrast to the existing literature on multi-homing (Ambrus et al., 2016, Anderson et al., 2019) which argues that when consumers multi-home, mergers become less harmful to consumers and more harmful to advertisers instead. However, such a reasoning may be misleading because the associated increase in the level of consumer multi-homing can reduce the platforms' incentive to expand their outputs which harms consumers.

For platform entry, we show that the presence of consumer multi-homing is necessary for inefficient entry to arise. This inefficiency becomes severe if the fraction of multi-homing consumers becomes larger. Intuitively, a greater level of consumer multi-homing makes it more difficult for platforms to capture the surplus from sellers because the presence of overlapping membership lowers the sellers' incremental value of each platform. This effect discourages a potential platform to enter despite the presence of the welfare gains for sellers. When this gap is large, the equilibrium level of platform entry can be insufficient, which stands in sharp contrast to the standard excessive entry result for traditional markets ((Mankiw and Whinston, 1986, Suzumura and Kiyono, 1987)).

Naturally, our results provide implications to competition policy. Although facilitating consumer multi-homing itself might be pro-competitive, our work suggests that competition policy should scrutinize platform markets more when consumers and sellers have greater benefits from multi-homing. For example, our analyses suggest that merger policy should be more stringent and that promoting platform entry is more important with larger amounts of multi-homing consumers. This implication is in line with the discussion of Athey and Scott Morton (2022) that, in a multi-homing environment, competition authorities should be cautious about the platforms' tactics that hinder multi-homing. By incorporating endogenous homing into a model of oligopolistic platform competition, we are able to analyze the importance of endogenous homing decisions with respect to competition policy.

Several existing studies point out the importance of homing decisions in platform markets

(Armstrong and Wright, 2007, Belleflamme and Peitz, 2019, Bakos and Halaburda, 2020, Jeitschko and Tremblay, 2020). Our work extends these studies by allowing for more than two platforms and by parameterizing multi-homing benefits that allow for an analysis that targets standard competition policy concerns in two-sided markets. One paper that is similar to us is Liu et al. (2021) who also provide a model of platform oligopoly with consumer multi-homing. Our study differs from Liu et al. (2021) in that we consider the endogenous choice of consumer multi-homing, whereas they compare exogenously given homing patterns.[1]

We also contribute to the growing platform literature that considers a Cournot framework approach to competition in two-sided markets. In particular, Correia-da Silva et al. (2019) study the effects of a merger between two platforms and show that mergers tend to reduce the user surplus if the sum of the cross-group externalities is sufficiently small, whereas it raises user surplus on both sides if the sum is sufficiently large. Their model, however, assumes that all agents single-home on both sides of the market.

The rest of this paper is organized as follows. After introducing our model of platform competition in the next section, we develop consumer and seller demand for the platform and determine equilibrium pricing in Section 3. We then, in Section 4, apply our framework to two important issues in competition policy: entry and a platform mergers. Finally, we also tie our results to existing work on particular industries in Section 5 before concluding in Section 6.

2 The Model

In a two-sided market, two groups of agents (consumers and sellers) benefit from the indirect network externalities that exist between them, and these two groups are connected by a platform. For example, gamers benefit from greater video game availability, game developers

[1]Several studies consider platform oligopoly using a Salop circle approach: see Reisinger et al. (2009), Alexandrov et al. (2011), Anderson et al. (2012, 2019), and Baranes et al. (2019). Platform entry and platform mergers are also studied by Tan and Zhou (2021) and Sato (2020), respectively. However, both studies assume that all agents single-home.

benefit from greater console ownership, and video game platforms connect these two groups. Let the consumer side be denoted as Side C and the seller side as Side S, and suppose that there are N competing platforms.

2.1 Consumers and Sellers

First consider the consumer side of the market: Side C. Consumers benefit from interaction with the seller side of the market, and some consumers benefit more from sellers than others. More formally, let consumer types be denoted by $\tau \in [0, \overline{\tau}]$ and let consumers be distributed uniformly. The utility from joining platform X, $X = 1, 2, ..., N$, for a consumer of type τ is given by:

$$U_C^X(\tau) = \alpha_C(\tau) n_S^X - p_C^X, \tag{1}$$

where $\alpha_C(\cdot)$ denotes the indirect network benefit function for consumers, n_S^X denotes the number of sellers on platform X, and p_C^X denotes the consumer price of platform X. Without loss of generality, suppose that consumers with lower τ types have greater indirect network benefits than consumers with higher τ types. This implies that $\alpha_C(\cdot)$ is decreasing.[2]

Now consider the seller side of the market: Side S. Sellers benefit from greater consumer participation on a platform. We assume there are a unit mass of sellers and the utility from joining platform X, $X = 1, 2, ..., N$, for a seller is given by:

$$U_S^X = \pi \cdot n_C^X - p_S^X, \tag{2}$$

where $\pi > 0$ denotes the indirect network benefit for sellers, n_C^X denotes the number of consumers on platform X, and p_S^X denotes the seller price of platform X.

Note that Equations (1) and (2) imply that consumers and sellers see platforms as homogenous, unless platforms differ in size in which case they are effectively seen as vertically differentiated. We make this assumption to simplify our analysis so that we can remain fairly

[2]Furthermore, suppose that $\alpha_C(\cdot)$ is twice continuously differentiable.

general when it comes to agent homing decisions.

Many platform industries exhibit allocations where some agents on each side of the market multi-home. For example, some consumers own a single video game console while others own multiple consoles; at the same time, some video games are available across all consoles while other games are only produced for a single console. Similarly, some consumers subscribe to a single video streaming service (Amazon Video, HBO Go, Hulu, or Netflix) while others join multiple; simultaneously, some video content is exclusive to a streaming platform while other content is available across several platforms.

Given this, we allow for multi-homing by assuming that consumers derive a fraction $\beta \in [0,1]$ of network benefits when they meet with the second seller and that they do not derive extra network benefits from the third, fourth, ... transactions. In this way, consumers have no incentive to join more than two platforms. Similarly, we assume that sellers derive a fraction $\delta \in [0,1]$ of network benefits by multi-homing on two platforms, whereas no extra network benefits accrue if they multi-home on more than two platforms.[3]

2.2 Platforms

Platforms generate revenues from each side of the market so that profits for platform X, $X = 1, 2, ..., N$, are given by:

$$\Pi^X = [p_C^X - c^X] \cdot n_C^X + p_S^X \cdot n_S^X,$$

where $c^X \geq 0$ denotes the marginal cost to platform X for an additional consumer. We assume that the marginal cost to the platform for an additional seller is zero.[4]

[3]While the assumption of zero utility for the third, fourth, ... duplications may appear ad hoc, it is hard to imagine scenarios where these duplications generate significant utility.

[4]This should be simply understood as a normalization. As long as the cost of serving sellers is moderately small so that platforms serve sellers in equilibrium, the value of the cost does not affect any equilibrium properties.

2.3 Timing

In terms of timing, the game proceeds as follows. First the platforms choose the output on the consumer side: each platform X chooses q^X. Then, consumer prices are determined in a manner that ensures consistency with the output choices of the platforms so that $n_C^X = q^X$ for all X (we discuss this further in the next section). As a tie-breaking assumption, we assume that if a consumer is indifferent between multi-homing on different combinations of platforms, then they uniformly choose one alternative — multi-homing consumers will only join two platforms (recall that network benefits depreciate to zero after the second platform). Finally, platforms choose their price for sellers, p_S^X, and the sellers choose the portfolio of the platforms to join.

The timing of the game is made to model platform competition in a Cournot-style fashion. As we will see below, this timing structure leads to a market clearing price structure so that prices are consistent with the pre-determined output choices. We focus on Cournot competition on the consumer side because this is often the side of the market where output competition aligns with traditional products; for example, smartphones and video game consoles must be produced and physically purchased by a consumer while software developers (the seller side) can join a platform digitally so that capacity constraints may not exist on the seller side of the market.

3 Equilibrium

We model how the N platforms compete *a la* Cournot along with competing for sellers in prices. We first consider consumer demand given a specific seller allocation and then turn to seller demand to complete the demand analysis.

3.1 Demand for the Platform

Consumer Demand: For a moment, suppose that all sellers multi-home (i.e., $n_S^X = 1$ for any X) and outputs chosen by each platform on the consumer side (q^X), the market clearing price must satisfy the single-homing margin so that $\alpha_C(\tau_S) - p_C = 0$ or

$$p_C = \alpha_C(\tau_S).$$

On the other hand, the multi-homing margin is given by the type τ_M consumer that is indifferent between single-homing and multi-homing. Thus, the multi-homing margin implies that

$$\beta\alpha_C(\tau_M) - \alpha_C(\tau_S) = 0 \text{ or } p_C = \beta \cdot \alpha_C(\tau_M), \tag{3}$$

which gives the multi-homing type τ_M as a function of τ_S, $\tau_M = \tau_M(\tau_S)$ so that, by the implicit function theorem, we have that $\tau_M'(\tau_S) = \frac{\alpha_C'(\tau_S)}{\beta\alpha_C'(\tau_M)} > 0$ so that greater consumer participation implies greater consumer multi-homing. Altogether we have that the $\tau \in [0, \tau_M]$ join two platforms and those with $\tau \in [\tau_M, \tau_S]$ join one platform. Figure 1 illustrates how heterogeneous consumers make their homing decisions.

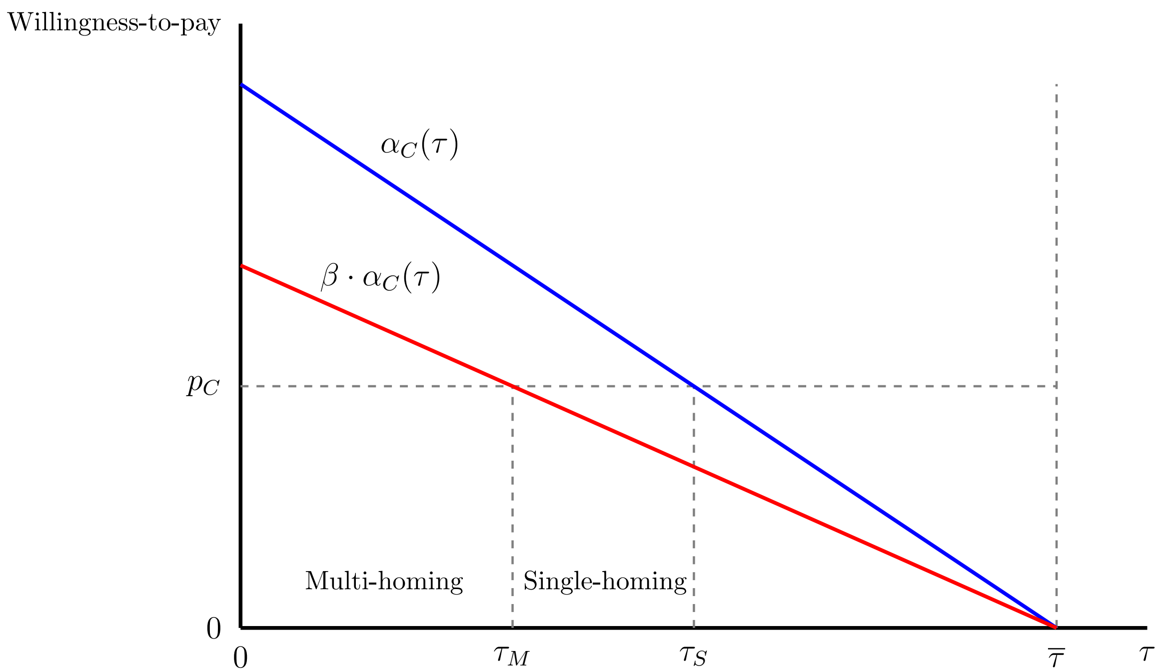
Note that the total output $Q = \sum_{X=1}^{N} q^X$ must be equal to $2\tau_M + (\tau_S - \tau_M)$ to ensure that the τ_M consumer join two platforms and that the $\tau_S - \tau_M$ consumers join one platform. Therefore, the single-homing margin is implicitly given by:

$$Q = \underbrace{2\tau_M}_{\text{multi-homing}} + \underbrace{(\tau_S - \tau_M)}_{\text{single-homing}} = \tau_M(\tau_S) + \tau_S, \tag{4}$$

That is, Equation (4) implicitly defines τ_S as a function of Q: $\tau_S = \tau_S(Q)$, and therefore also defines the market clearing price since $p_C = \alpha_C(\tau_S)$.

In this consumer price analysis, it is important to note that we assume that the consumer marginal costs, c^X, do not differ too drastically across platforms. If there is a significant level of cost asymmetry, then $\tau_M > q^X$ might hold for some X. In this case, the market

Figure 1: Consumer Homing Decisions



clearing price, p_C^X, would not be equal to $\alpha_C(\tau_S)$ and our consumer demand analysis above will unravel.

Seller Demand: In the consumer demand analysis, we have assumed that all sellers join all the platforms. We show that this actually holds true in the equilibrium of the subgame given any output profile $(q^X)_{X=1,\ldots,N}$. To see this, suppose that platforms set outputs $(q^X)_{X=1,\ldots,N}$ and τ^M consumers multi-home on two platforms uniformly. Then, the incremental value that a seller enjoys from joining a platform is given by

$$\Delta\pi_X = \pi \cdot \left[\left(q^X - \frac{2}{N}\tau_M \right) + \frac{2}{N}\delta\tau_M \right]. \tag{5}$$

The first term of the bracket represents the value of interacting with single-homing consumers, and the second represents the value of interacting with multi-homing consumers. Because the sellers are homogeneous, each platform can attract all the sellers by setting $p_S^X \leq \Delta\pi_X$, whereas it loses all the sellers by setting $p_X^S > \Delta\pi_X$. Thus, in any subgame, the platforms set the price that induces all the sellers to join.

3.2 The Platform Equilibrium

Solving the game backwards, we first consider the seller side of the market. Given consumer and seller demands for the platform, the following lemma formally shows that the price to sellers is derived from the incremental value and that all sellers join all platforms:

Lemma 1. *For any given profile* $(q^X)_{X=1,\ldots,N}$*, we have that* $\tau^S - \tau^M$ *consumers single-home,* τ^M *consumers multi-home two platforms uniformly, and all sellers multi-home on all platforms. In addition, each platform* X *sets a seller price given by Equation* (5).

All proofs are in the appendix.

Lemma 1 is particularly interesting for two reasons. First note that sellers join all platforms endogenously (this homing allocations is not exogenously assumed as is common in the literature). Second, note that sellers choose to multi-home on *all* platforms even though they only receive duplication benefits from interacting with consumers twice. The reasoning here is that sellers want to gain access to all consumers (and not all consumers will multi-home and those that do will only join two platforms) and that the platform offers a low enough price to allow them to do so. This lemma is a version of incremental-value pricing principle, as shown in analysis of the media market such as Anderson et al. (2019), where all sellers join all the platforms and any increase in one platform's price leads to the zero participation on that platform.

Turning to the consumer side of the market, note that while the inverse demand systems developed in the previous section capture the two-sidedness of the market, the platform profit function can be simplified to a variant of a one-sided market Cournot model. This simplification exists because all sellers participate and multi-home, and while this eliminates a seller extensive margin, there still exists an intensive margin of platform competition for sellers through the incremental-value pricing principle on the seller side (where more single-homing consumers on a platform generates a higher seller side price for that platform). Altogether, our model preserves two-sided features that are important for studying platforms and allows us to conduct an equilibrium analysis that can tractably consider antitrust concerns.

Given the demand structure in the previous section, platform X maximizes its profit, $\Pi^X = [\alpha_C(\tau_S) - c^X]q^X + \pi\left[q^X - \frac{2}{N}(1-\delta)\tau_M(\tau_S)\right]$, with respect to its own capacity, q^X. The first-order condition is then given by $[\alpha_C(\tau_S) - c^X] + \alpha'_C \cdot \tau'_S \cdot q^X + \pi\left(1 - \frac{2(1-\delta)}{N} \cdot \tau'_M \cdot \tau'_S\right) = 0$ which reduces to

$$p_C - c^X + \frac{dp_C}{dQ}q^X + \pi\left(1 - \frac{2(1-\delta)}{N}\frac{d\tau_M}{dQ}\right) = 0,$$

where $p_C(Q) \equiv \alpha_C[\tau_S(Q)]$ and $\tau_M(Q) \equiv \tau_M[\tau_S(Q)]$, which is further rearranged to

$$\frac{s^X p_C}{\epsilon} + p_C - c^X + \pi\left(1 - \frac{2(1-\delta)}{N}\frac{d\tau_M}{dQ}\right) = 0, \tag{6}$$

where $s^X \equiv \frac{q^X}{Q}$ is platform X's market share on the consumer side and $\epsilon = \frac{dQ}{dp_C} \cdot \frac{p_C}{Q}$ is the elasticity of consumer demand. Note that Equation (4) implies that $\frac{d\tau_M}{dQ} = \frac{\tau'_M(\tau_S)}{\tau'_M(\tau_S)+1}$.[5] Let $r(Q, c^X)$ be the solution to Equation (6). Finally, the equilibrium total output, Q^*, is obtained by solving:

$$\frac{p_C}{\epsilon} + Np_C - \sum_{X=1}^{N} c^X + N\pi\left(1 - \frac{2(1-\delta)}{N}\frac{1}{\frac{1}{\tau'_M[\tau_S(Q)]} + 1}\right) = 0, \tag{7}$$

where $p_C = \alpha_C[\tau_S(Q)]$, and hence, the equilibrium number of unique consumers who join any platform $\tau_S^* = \tau_S(Q^*)$. The following proposition summarizes the above discussion.

Proposition 1. *The equilibrium output of each firm is given by* $q^X = r(Q^*, c^X)$*, where* Q^* *is the solution to Equation* (7)*. In particular, when* $c^X = c$ *for all* X*, the symmetric equilibrium*

[5] This is because $\frac{d\tau_M}{dQ} = \frac{d\tau_M}{d\tau_S} \cdot \frac{d\tau_S}{dQ} = \tau'_M \cdot \frac{1}{\tau'_M + 1}$.

pricing strategy for each platform is implicitly given by

$$p_C^* = c + \underbrace{\frac{1}{N} \cdot \frac{p_C^*}{-\epsilon}}_{markup} - \underbrace{\pi \cdot \left[1 - \frac{2(1-\delta)}{N} \frac{1}{\frac{1}{\tau_M'(\tau_S(Q^*))} + 1}\right]}_{markdown}. \tag{8}$$

On the consumer side, the equilibrium price induced by Cournot platform competition resembles a combination of both (i) the traditional Cournot pricing where $p^* = c + \frac{1}{N} \cdot \frac{p^*}{-\epsilon}$ and (ii) the monopoly platform pricing strategy where $p_C = c + \frac{p_C}{-\epsilon} - \pi n_S$. First, notice that markup term in Equation (8) follows that of the traditional Cournot market. This is precisely the distortion we would expect from Cournot platform competition. Second, the markdown term gets larger in absolute terms as the number of competing platforms increases.[6] This effect stems from the incremental pricing strategy on the seller side: More platforms will increase competition on the seller side of the market so that each platform attempts to attract more consumers through a larger markdown term in the consumer price.

Unlike much of the previous literature, one benefit to our approach is that we can investigate how changes in the extensive margin of platform competition, N, impact welfare much like the approaches considered in the traditional Cournot setting. This allows us to analyze mergers using the traditional techniques. At the same time, the parameters characterizing network effects and consumer multi-homing are embedded into the shape of platforms' profit functions so that we can also examine the impacts of those parameters on the competitive effects of mergers. As a summary, our framework provides a parsimonious way to examine the importance of two-sidedness and multi-homing in competition among platforms.

4 Platform Mergers and Entry

Our baseline analysis provides the tractable framework to evaluate the importance of consumer multi-homing and platform market structure on competition policy. In this section,

[6] Since $\tau_M'(\tau_S(Q^*)) > 0$ by Equation (3).

we consider platform free entry and mergers. Contrary to the popular belief that consumer multi-homing is a factor that mitigates market power concerns in two-sided markets, we find that the presence of consumer multi-homing requires stricter competition policies.

In order to consider welfare explicitly, we must specify the functional form for the heterogenous network externalities amongst consumers (the $\alpha_C(\tau)$). To ease our exposition, we assume that network effects take on a constant-elasticity specification:

$$\alpha_C(\tau) = \tau^{-\frac{1}{\eta}},$$

where $\eta > 1$ captures the network effect elasticity.[7] With this specification, the equilibrium condition (Equation (6)) is given by

$$-\frac{s^X p_C}{\eta} + p_C - c^X + \pi\left(1 - \frac{2(1-\delta)\theta_M}{N}\right) = 0,$$

where $\theta_M \equiv \frac{\tau_M}{Q} = \frac{\beta^\eta}{1+\beta^\eta}$ denotes the fraction of multi-homing consumers relative to total output (see Appendix for this derivation).[8] Hence, the equilibrium total output, Q^*, is given by $-\frac{p_C}{\eta} + Np_C - \sum_{X=1}^{N} c^X + N\pi\left(1 - \frac{2(1-\delta)\theta_M}{N}\right) = 0$. This gives the an explicit solution for Q^*:

$$\begin{aligned} Q^* &= (1+\beta^\eta)p_C^{-\eta} \\ &= (1+\beta^\eta)\left(\frac{\sum_{X=1}^{N} c^X - \pi\left[N - 2(1-\delta)\theta_M\right]}{N - \frac{1}{\eta}}\right)^{-\eta}, \end{aligned}$$

using $p_C = \alpha_C[\tau_S(Q)] = (\frac{Q}{1+\beta^\eta})^{-\frac{1}{\eta}}$. We can also derive consumer surplus (CS), seller surplus

[7] It should be noted that this network effect externality is distinguished from the elasticity of consumer demand, ϵ.

[8] This is because under this specification, $\tau_M(\tau_S) = \beta^\eta \tau_S$, and $\tau_S(Q) = \frac{Q}{1+\beta^\eta}$, which implies that (i) $\frac{d\tau_M}{dQ} = \beta^\eta \cdot \frac{1}{1+\beta^\eta}$ and (ii) $\frac{\tau_M}{\beta^\eta} = \tau_S = \frac{Q}{1+\beta^\eta}$. Hence, $\frac{d\tau_M}{dQ} = \frac{\tau_M}{Q} \equiv \theta_M$.

(SS), and platform profit under symmetric costs:

$$CS = \underbrace{\int_0^{\tau_S} [\alpha_C(\tau) - \alpha_C(\tau_S)] d\tau}_{\text{single-homing surplus}} + \underbrace{\int_0^{\tau_M} [\beta \alpha_C(\tau) - \alpha_C(\tau_S)] d\tau}_{\text{multi-homing surplus}} = \frac{(1+\beta^\eta)^{\frac{1}{\eta}}}{\eta - 1} (Q^*)^{\frac{\eta-1}{\eta}},$$

$$SS = \pi \cdot [\tau_S + (1+\delta)\tau_M] - Np_S = \frac{(2-\delta)\beta^\eta}{1+\beta^\eta} Q^*,$$

$$\Pi^* = \frac{Q^*}{N} [p_C - c + \pi - 2(1-\delta)\theta_M \pi].$$

4.1 A Merger Analysis

We analyze platform mergers as the first application where consumer multi-homing affects the welfare properties of competition. There is a burgeoning literature on mergers between two-sided platforms. However, except for the studies that focus solely on media mergers such as Ambrus et al. (2016) and Anderson et al. (2019), there are few studies of platform mergers under consumer multi-homing. We fill this gap in the literature in the analysis that follows.

We follow the approach taken by Farrell and Shapiro (1990), Nocke and Whinston (2010), and Nocke and Whinston (2013) who characterize the conditions for which the mergers improve consumer surplus. We model a merger between platforms X and Y as the transformation into the new firm M that has a new marginal cost which may be smaller than pre-merger marginal costs of merging platforms because of some merger-specific synergies which we denote by Δc^M. To evaluate whether a platform merger improves consumer surplus or seller surplus, it suffices to examine whether the merger increases the total output (since both CS and SS increase in Q). Taking this approach, we find that the benefits from multi-homing on each side of the market have meaningful effects on the cost synergy that is necessary to improve consumer and seller surpluses:

Proposition 2. *The level of merger-specific synergy required for consumer and seller sur-*

pluses to increase from the merger increases with the level of consumer multi-homing $\left(\frac{\partial \Delta c^M}{\partial \beta} > 0\right)$*, decreases with the level of seller multi-homing* $\left(\frac{\partial \Delta c^M}{\partial \delta} < 0\right)$*, and decreases with the number of platforms in the market* $\left(\frac{\partial \Delta c^M}{\partial N} < 0\right)$*.*

Not surprisingly, lesser platform competition (smaller N) implies that a merger will need greater cost synergies for consumers and sellers to be better-off. This aligns with traditional markets. More interestingly, Proposition 2 shows the impacts of consumer and seller multi-homing on the competitive effects of a platform merger. First, the level of merger-specific synergy Δc^M increases with β; that is, greater consumer multi-homing leads mergers to require greater synergies to improve consumer surplus. The intuition is that a reduction in the number of platforms (due to a merger) increases the overlap between platforms among multi-homing consumers which weakens platforms' motivation to on-board consumers as a means of extracting revenues from sellers. Thus, greater multi-homing reduces a platform's incentive to on-board consumers and this makes mergers even more costly to consumers, suggesting that greater cost synergies are required in markets with greater multi-homing.

On the contrary, Δc^M decreases with δ; that is, an increase in the seller benefits from multi-homing leads mergers to require smaller cost synergies to improve consumer surplus. This is because the adverse effects mentioned above become weaker as the sellers' willingness to pay for interaction with overlapped consumers increases. Hence, greater seller benefits from multi-homing will increase the platforms desire to obtain consumers and this relaxes the cost synergy requirements for a merger to benefit consumers.

Proposition 2 has some novel implication for competition policy. Most of multi-homing literature such as Ambrus et al. (2016) and Anderson et al. (2019) argue that when consumers multi-home, mergers become less harmful to consumers and more harmful to advertisers instead. Competition authorities often justify mergers on the ground that consumers multi-home because the substitution between platforms might be low. However, our results suggest that such reasoning may be misleading because an increase in consumer multi-homing can hurt consumers by reducing the platforms' incentive to expand their outputs post merger.

4.2 Social Efficiency of Entry

As another important application, we discuss the social efficiency of free entry under platform competition. In the traditional model of Cournot competition, the number of firms in free entry equilibrium is socially excessive (Mankiw and Whinston, 1986). In two-sided markets it has been shown that insufficient entry arises due to the presence of indirect network externalities or the nonappropriability of the surplus. For example, Tan and Zhou (2021) show that there is a case where the equilibrium entry is insufficient in multi-sided markets with single-homing consumers. In the competitive-bottleneck setting of media markets, Choi (2006) shows that when free ad-sponsored media outlets compete, the equilibrium level of entry may also be insufficient. We contribute to this literature on the welfare properties of free entry by showing the combination of two-sidedness and consumer multi-homing makes the equilibrium level of entry insufficient. In particular, we show that when the fraction of multi-homing consumers is large and the seller network benefit is large, then the equilibrium number of platforms becomes insufficient.

Consider the case where there is an infinite number of potential entrant platforms with marginal cost c and entry cost $K > 0$. In this scenario, platforms first choose whether to enter the market and upon entry, they play a Cournot platform competition. Recall that we consider the case where $\alpha_C(\tau) = \tau^{-\frac{1}{\eta}}$. Furthermore, assume that all the platforms are symmetric so that $c^X = c$ for all X. Thus, the equilibrium total output given the number of platforms N is

$$Q^*(N) = (1 + \beta^\eta) \left(\frac{Nc - \pi[N - 2(1-\delta)\theta_M]}{N - \frac{1}{\eta}} \right)^{-\eta}.$$

To guarantee the existence of the positive equilibrium output, we assume that $c > \left(1 - \frac{2(1-\delta)}{N}\right) \pi$. The equilibrium profit of each platform given the number of platforms N is

$$\Pi^*(N) = \frac{Q^*(N)}{N} [p_C - c + \pi - 2(1-\delta)\theta_M \pi] - K.$$

Therefore, in the free-entry equilibrium, the number of platforms N^* is given by $\Pi^*(N^*) = 0$.

The following proposition characterizes the effect of homing patterns and network externalities on equilibrium number of platforms.

Proposition 3. *The equilibrium number of platforms increases with the sellers' benefit from multi-homing* $\left(\frac{dN^*}{d\delta} > 0\right)$*. However, we find an ambiguous effect on the equilibrium number of platforms with respect to increases in consumers' benefit from multi-homing (β) or sellers network benefit (π).*

An increase in the benefit to sellers from multi-homing increases the incremental value of each platform for sellers. This is unambiguously beneficial for platforms and results in greater platform entry. The intuition behind the ambiguous effects are as follows: An increase in the benefit of consumers from multi-homing (β) increases the demand for the platforms which is beneficial for platforms; At the same time, it increases the overlapped membership, which decreases the incremental value of each platform for sellers and reduces platform profitability. Similarly, an increase in seller network benefits (π) has two effects: First, an increase in the seller network benefit directly increases platform profits from sellers; second, it induces platforms to expand their output to consumers which lowers consumer prices and increases the fraction of multi-homing consumers, thereby lowering the prices charged to sellers. The latter negative effect may overturn the former positive effect by suppressing the margin of each platform. Therefore, the impact of π on the profitability of the platforms is ambiguous.

We also consider the welfare effects from free entry. In particular, we find that the platform entry has two welfare effects. An entry of new platform leads to an expansion of total outputs and benefits consumers and sellers. At the same time, the entry reduces the profits of existing platforms and increases the amount of fixed costs associated with entry which hurts platforms and welfare. The balance between these two effects determines the welfare property of entry. To determine whether or not platform entry is excessive or insufficient in equilibrium, we first derive the social welfare as a function of the number of platforms:

$$\begin{aligned} W(N) &= CS + SS + N\Pi^*(N) \\ &= \frac{\eta}{\eta - 1}(1+\beta^\eta)^{\frac{1}{\eta}}[Q^*(N)]^{\frac{\eta-1}{\eta}} - [c - \pi(1+\theta_M\delta)]\, Q^*(N) - NK \end{aligned}$$

In the context of traditional markets, it is shown that the equilibrium level of entry is excessive (Mankiw and Whinston, 1986, Suzumura and Kiyono, 1987). We follow the literature on entry where we say that the equilibrium number of platforms is insufficient (excessive) if $W'(N^*) = \frac{dW}{dN}\big|_{N^*} > 0$ (resp. $W'(N^*) = \frac{dW}{dN}\big|_{N^*} < 0$). Unlike traditional markets where equilibrium entry is excessive, we find that equilibrium entry in two-sided markets is often insufficient:

Proposition 4. *Consider a free entry equilibrium and let N^* denote the equilibrium number of platforms. If $c \in \left((1 - \frac{2(1-\delta)}{N}\theta_M)\pi, \pi\right]$, then the equilibrium number of platforms is always insufficient in terms of social welfare. If $c > \pi$, then there exists $\hat{\omega}(N^*) > 0$ such that the equilibrium number of platforms is insufficient if and only if*

$$\frac{\theta_M \pi}{c - \pi} > \hat{\omega}(N^*),$$

where, recall, $\theta_M \equiv \frac{\tau_M}{Q} = \frac{\beta^\eta}{1+\beta^\eta}$ is the fraction of multi-homing consumers relative to total output.

There are several remarks to make at this point. First, insufficient entry takes place only if $\theta_M > 0$ and $\pi > 0$ holds. Thus both the presence of consumer multi-homing and indirect network externalities are necessary for the insufficient entry result (generating the difference in entry results with one-sided markets). Furthermore, the higher θ_M and π are, the more likely it is that insufficient entry takes place. This property is driven by the fact that, when consumers multi-home, platforms cannot extract the surplus from sellers because the presence of overlapping membership lowers the incremental value of each platform for sellers. As a result, the profit each platform obtains from sellers becomes lower than the surplus that sellers obtain from platform entry. This creates the source of insufficient entry.

Proposition 4 suggests that contrary to the standard excessive entry result under Cournot competition of Mankiw and Whinston (1986), the presence of consumer multi-homing in two-sided markets tends platform entry to be insufficient. This result provides the following policy implication. There is a popular discussion that consumer multi-homing increases the entry of new platforms so that entry barriers are of less importance in markets with consumer multi-homing. However, our insufficient entry result suggests that from the welfare perspective, policymakers should be more cautious about entry barriers in markets with high levels of consumer multi-homing.

5 Practical Implications

Up until this point, our findings were largely applied to work on competition policy. However, it is important to note that there is a variety of platform research and industries where our results can provide a richer understanding. We outline these applications in this section.

Online travel is one industry where numerous mergers have occurred (e.g., Expedia and Orbitz are under the same company as are Booking and Priceline), and this setting arguably best fits into our model since the majority of the seller side (hotels, rental cars, etc.) multi-homes while consumers typically single-home. Mapping this industry into our findings from Proposition 2, we see that the cost synergy for consumers and sellers to benefit from such mergers is not necessarily very large since this industry likely experiences a small β, large δ, and large N. Hence, our results suggest that mergers in online travel have likely improved welfare.

Another interesting application to our work stems from a merger between the two largest dog sitting platforms. Farronato et al. (2021) find limited multi-homing on either side of the dog sitting market, but they also show that multi-homers typically transact more than single-homers. Farronato et al. (2021) also show that this merger between the two platforms is neutral toward consumer and host surplus which, based on Proposition 2, suggests that the cost synergies were substantial.[9]

There are additional platform industries where homing preference make things interesting in the context of our model. In the market for daily deals, Li and Zhu (2021) document reasonable amounts of consumer multi-homing (10-20%) and considerable merchant multi-homing (30-50%). They show that the incumbent platform's decision to make seller multi-homing more difficult, reducing δ in the context of our model, made platform entry more difficult for the entrant. This aligns with Proposition 3, where $\frac{dN^*}{d\delta} > 0$, so that any platform behavior that reduces seller benefits from multi-homing (reducing δ) will reduce platform entry — in practice and in theory. While the idea that platforms may restrict multi-homing to reducing platform competition was informally conceived by Athey and Scott Morton (2022), we are the first to validate this formally in a model of endogenous homing.

In media markets, Park et al. (2021) and Affeldt et al. (2021) document considerable multi-homing by consumers and advertisers. Park et al. (2021) also highlight how the advent of television resulted in media entry against newspaper incumbents. With considerable multi-homing by consumers and advertisers across newspaper and television, we expect that the television innovation would increase δ (advertisers can now advertise their products in a video format). As Proposition 3 predicts, media entry occurs during this time period.

Zhu and Iansiti (2012) show how strong network externalities in the video game industry promoted a successful entry by Microsoft's Xbox. In addition, Lee (2013) and Derdenger (2014) find that the entry of Xbox increased welfare in the video game industry. While our results in Proposition 3 on how network externalities impact equilibrium platform entry are ambiguous, combining their assessment of the video game industry with our results from Proposition 4 suggests that entry was insufficient prior to the entry of Xbox and that entry benefited aggregate surplus within the industry.

[9] Indeed, the purchased platform was eventually closed in an effort to reduce costs.

6 Conclusion

In this paper, we have provided a tractable framework that considers a platform oligopoly and endogenizes the observed pattern of multi-homing in many contexts: sellers tend to transact with as many platforms as possible to reach many consumers whereas consumers buy from one or two platforms. The prices for consumers and sellers follow traditional frameworks for each of the two groups: Cournot competition over consumers and incremental-value pricing over sellers. By considering platform mergers and entry, we show how greater consumer multi-homing (due to increased benefits from multi-homing) increases the required level of merger-specific cost reduction that is necessary for consumer and seller surpluses to increase with a platform merger. These results imply that the basis of the popular belief that multi-homing mitigates the need for a strict implementation of competition policy is not warranted.

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Appendix of Proofs

Proof of Lemma 1: Let $\mathcal{N} = \{1, 2, \dots, N\}$ and fix any profile $\{q^X\}_{X=1,\dots,N}$ of the outputs of platforms. We show that (i) when τ^M consumers multi-home on two platforms uniformly, all the sellers join all the platforms. To see this, consider the total network benefit of a seller from joining the set $\mathcal{X}$ of platforms (i.e., all platforms in $\mathcal{X}$), which is given by

$$\widetilde{\pi}_S^{\mathcal{X}} = \sum_{X \in \mathcal{X}} \left(q^X - \frac{2}{N}\tau^M \right)\pi + \left[\frac{|\mathcal{X}|(|\mathcal{X}|-1)}{N(N-1)}(1+\delta)\pi + \frac{2|\mathcal{X}|(N-|\mathcal{X}|)}{N(N-1)}\pi \right]\tau^M.$$

Note first that for each platform X, $q^X - \frac{2}{N}\tau^M$ consumers single-home. This explains the first term in the right hand side of the equation above. Next, there are τ^M multi-homing consumers who join two platforms. When sellers join the set $\mathcal{X}$ of platforms, there are two relevant cases: (i) where the consumers join two platforms in $\mathcal{X}$ and (ii) where consumers join one platform in $\mathcal{X}$ and one platform in $\mathcal{N} \setminus \mathcal{X}$. In the former case, the seller obtains network benefit $\pi(1+\delta)$, and in the latter case, π. Then, the number of combination of two platforms in $\mathcal{X}$ that multi-homing consumers join is given by $|\mathcal{X}|(|\mathcal{X}|-1)/2$, whereas the number of combination of one platforms in $\mathcal{X}$ and one platform in $\mathcal{N} \setminus \mathcal{X}$ is $|\mathcal{X}|(N-|\mathcal{X}|)$. This explains the second term in the right hand size of the equation above because the total number of combination of two platforms that multi-homing consumers join is given by $N(N-1)/2$.

Thus, an decrease in the network benefit from subtracting one platform Y from $\mathcal{X}$ is given by

$$\begin{aligned}
\Delta\widetilde{\pi}_S^{\mathcal{X},Y} &= \left[q^X - \frac{2}{N}\tau^M + \frac{2[N-1-(N-1)(1-\delta)]}{N(N-1)}\tau^M \right]\pi \\
&= [q^X + (1-\delta)\tau^M]\pi,
\end{aligned}$$

which is decreasing in $|\mathcal{X}|$ and has value $\frac{2}{N}\delta$ when $|\mathcal{X}| = N$.

Given a price profile $P_S = (p_S^1, p_S^2, \dots, p_S^N)$, a seller chooses the portfolio $\mathcal{X}$ of platforms to join to maximize the following utility

$$U_S^{\mathcal{X}}(P_S) = \widetilde{\pi}_S^{\mathcal{X}} - \sum_{X\in\mathcal{X}} p_S^X.$$

From the fact that $\Delta\widetilde{\pi}_S^{\mathcal{X},Y}$ is decreasing in $|\mathcal{X}|$, for any $\mathcal{T} \subset \mathcal{X} \setminus \{X\}$, we have

$$\Delta\widetilde{\pi}_S^{\mathcal{X}\cup\{X\},X} \leq \Delta\widetilde{\pi}_S^{\mathcal{T}\cup\{X\},X}.$$

This implies that each seller joins the set $\mathcal{X}$ of the platforms only if

$$p_S^X \leq \Delta\widetilde{\pi}_S^{\mathcal{X},X}.$$

for all $X \in \mathcal{X}$, and

$$p_S^Y > \Delta\widetilde{\pi}_S^{\mathcal{X}\cup\{Y\},Y}$$

for all $Y \in \mathcal{N} \setminus \mathcal{X}$.

Suppose that for any given price profile $P_S = (p_S^X)_{X\in\mathcal{N}}$, sellers join the set $\mathcal{X}(P_S)$ of platforms. We show that the equilibrium prices for sellers should satisfy $\mathcal{X}(P_S) = \mathcal{N}$. Suppose to the contrary that there exists $Y \in \mathcal{N} \setminus \{\mathcal{X}(P_S)\}$. Then, by deviating to the price $p_S^Y = \tilde{p}_S^Y = \Delta\widetilde{\pi}_S^{\mathcal{X}\cup\{Y\},Y}$, the platform Y can increase its profit. To see this, let $P_S^Y = (p_S^1, \ldots, \tilde{p}_S^Y, \ldots, p_S^N)$

$$\mathcal{T} = \{T \subset \mathcal{N} | Y \notin T\}$$

and

$$\mathcal{T}' = \{T \subset \mathcal{N} | Y \in T\}$$

Then we have

$$\max_{T\in\mathcal{T}} U_S^{\mathcal{X}(P_S)}(P_S^Y) \leq U_S^{\mathcal{X}(P_S)\cup\{Y\}}(P_S^Y) \leq \max_{T'\in\mathcal{T}'} U_S^{T'}(P_S^Y),$$

which implies that the seller chooses to join platform Y when platform j deviates to the price $\tilde{p}_S^Y$. Then, platform Y's profit from the seller increases from 0 to $\tilde{p}_S^Y$. Thus, all the platforms set prices to be joined so that $\mathcal{X}(P_S) = \mathcal{N}$ holds.

Finally, the maximal price each platform i can charge to attain $X \in \mathcal{X}(P_S)$ is given by

$$p_S^X = \Delta\widetilde{\pi}_S^{\mathcal{N},X} = \left[q^X - \frac{2(1-\delta)}{N}\tau^M\right]\pi,$$

which proves that given that τ^M consumers multi-home on two platforms uniformly, then all sellers join all the platforms.

Given that all sellers join all the platforms, consumers with $\tau < \tau_M$ multi-home on two platforms, and those with $\tau \in [\tau_M, \tau_S]$ single-home platforms in the way that market clears. Putting these arguments together proves Lemma 1. □

Proof of Proposition 1: When $c^X = c$ for all X, Equation (7) can be written as

$$\frac{p_C}{\epsilon} + Np_C - Nc + N\pi\left(1 - \frac{2(1-\delta)}{N}\frac{1}{\frac{1}{\tau_M'(\tau_S(Q))} + 1}\right) = 0.$$

Arranging this, we obtain Equation (8). □

Derivation of equilibrium condition with constant-elasticity demand: A calculation shows that

$$\begin{cases} \tau_M(\tau_S) = \beta^\eta \tau_S \\ \tau_S(Q) = \dfrac{Q}{1+\beta^\eta}. \end{cases}$$

Then, it is verified that

$$p_C(Q) = \alpha_C(\tau_S) = \left(\frac{Q}{1+\beta^\eta}\right)^{-\frac{1}{\eta}},$$

which implies that the demand elasticity is given by

$$\epsilon = \frac{Q/p_C}{\partial p_C/\partial Q} = \eta.$$

The first-order condition for the equilibrium output q^X is given by

$$\frac{\partial p_C}{\partial Q}q^X + p_C - c^X + \pi \left[1 - \frac{2(1-\delta)}{N}\frac{\partial \tau_M}{\partial Q}\right] = 0,$$

which can be rearranged to

$$\frac{s^X p_C}{\eta} + p_C - c^X + \pi \left(1 - \frac{2(1-\delta)\theta_M}{N}\right) = 0.$$

Proof of Proposition 2: To evaluate whether a platform merger improves consumer surplus or seller surplus, it suffices to examine whether the merger increases the total output (since both CS and SS increase in Q). In the following, we characterize the level of merger-specific synergies required to increase total outputs. Consider a merger between two platforms X and Y with pre-merger market shares s^X, s^Y and pre-merger total output Q. Then, the marginal cost c^X that is consistent with market share s^X is given by

$$c^X = -\frac{s^X p_C}{\eta} + p_C + \pi \left(1 - \frac{2(1-\delta)\theta_M}{N}\right).$$

Let $c^M = \min\{c^X, c^Y\}$ be the cost without synergies and $\Delta c^M = c^M - \hat{c}^M$ be the size of synergy required to the merger to improve consumer and seller surpluses (that is, increase Q). Then, it is verified that

$$c^M = -\frac{\max\{s^X, s^Y\} p_C}{\eta} + p_C + \pi \left(1 - \frac{2(1-\delta)\theta_M}{N}\right)$$

and

$$\hat{c}^M = -\frac{s^M p_C}{\eta} + p_C + \pi \left[1 - \frac{2(1-\delta)\theta_M}{N-1}\right],$$

where $s^M := s^X + s^Y$, which implies that[10]

$$\begin{aligned}\Delta c^M &= \frac{p_C}{\eta}\left[s^M - \max\{s^X, s^Y\}\right] + 2(1-\delta)\theta_M \pi \left(\frac{1}{N-1} - \frac{1}{N}\right) \\ &= \frac{(1+\beta^\eta)^{\frac{1}{\eta}} Q^{-\frac{1}{\eta}}}{\eta}\left[s^M - \max\{s^X, s^Y\}\right] + 2(1-\delta)\frac{\beta^\eta}{1+\beta^\eta}\pi\left(\frac{1}{N-1} - \frac{1}{N}\right).\end{aligned}$$

Taking a derivative of Δc^M with respect to β, we have

$$\frac{\partial \Delta c^M}{\partial \beta} = \frac{\beta^{\eta-1}(1+\beta^\eta)^{\frac{1+\eta}{\eta}} Q^{-\frac{1}{\eta}}}{\eta}[s^M - \max\{s^X, s^Y\}] + 2(1-\delta)\eta\beta^{\eta-1}\frac{1}{[1+\beta^\eta]^2}\pi\left(\frac{1}{N-1} - \frac{1}{N}\right) > 0.$$

Similarly, taking a derivative of Δc^M with respect to δ, we have

$$\frac{\partial \Delta c^M}{\partial \delta} = -2 \cdot \frac{\beta^\eta}{1+\beta^\eta}\pi\left(\frac{1}{N-1} - \frac{1}{N}\right) < 0.$$

Lastly, taking a derivative of Δc^M with respect to N, we have

$$\frac{\partial \Delta c^M}{\partial N} = 2(1-\delta)\frac{\beta^\eta}{1+\beta^\eta}\pi\left(\frac{-1}{(N-1)2} + \frac{1}{N^2}\right) < 0.$$

□

Proof of Proposition 3:

Each platform's profit for a given N is given by

$$\begin{aligned}\Pi(N) &= \frac{Q^*(N)}{N}\left[p_C - c + \pi - 2(1-\delta)\theta_M \pi\right] - K \\ &= \frac{(1+\widetilde{\beta})}{N(N\eta-1)}\left(\frac{N\eta}{N\eta-1}\right)^{-\eta}\left[c - \pi A\right]^{-\eta} B - K,\end{aligned}$$

where

$$A \equiv 1 - \frac{2(1-\delta)\theta_M}{N},$$

[10] Recall that this is a consumer surplus neutral merger where the aggregate output remains the same. Therefore, p_C remains the same as well.

$$B \equiv c - \pi - 2(1-\delta)\theta_M[\eta(N-1)-1]\pi,$$

and $\widetilde{\beta} = \beta^\eta \leq 1$. Applying the implicit function theorem, we have $\partial N^*/\partial x = -(\partial \Pi/\partial x)/(\partial \Pi/\partial N)$ for any parameter $x \in \{\beta, \delta, \pi\}$. Because $\Pi(N)$ is decreasing in N, if $\Pi(N)$ increases with δ or β, the equilibrium number of platforms increases with it.

First, it is shown that

$$\begin{aligned}\frac{\partial \Pi}{\partial \delta} = &\left(\frac{2\pi\widetilde{\beta}}{N^2(N\eta-1)}\right)\left(\frac{N\eta}{N\eta-1}\right)^{-\eta}[c-\pi A]^{-\eta-1} \\ &\times \left[(c-\pi)(N\eta-1)(N-1)+(\eta-1)B\right],\end{aligned}$$

which must be positive because $B > 0$, implying that N^* increases with δ.

Second, it is also shown that

$$\begin{aligned}\frac{\partial \Pi}{\partial \pi} = &\left(\frac{1}{N(N\eta-1)}\right)\left(\frac{N\eta}{N\eta-1}\right)^{-\eta}[c-\pi A]^{-\eta} \\ &\times \left\{[1-2\delta-2(N-1)(1-\delta)\eta]\,\widetilde{\beta} - 1 + \frac{\eta(1+\widetilde{\beta})AB}{c-\pi A}\right\},\end{aligned}$$

which is positive if and only if

$$f_\pi = \left\{\underbrace{[1-2\eta(N-1)]}_{(-)} + 2\delta\underbrace{[(N-1)\eta-1]}_{(+)}\right\}\widetilde{\beta} + \frac{\eta(1+\widetilde{\beta})AB}{c-\pi A} > 1,$$

where the left hand side can be negative. For example, when $\beta = 0$, we have $A = 1$, and $B = c - \pi$, implying that $f_\pi = \eta > 1$, implying that $\partial\Pi/\partial\pi > 0$. Next, suppose that $\delta = 0$, $\beta > 0$, and $B = \epsilon$ for small $\epsilon > 0$, whereas K is sufficiently small so that $N^* \geq 2$. In this case, for sufficiently small ϵ, we have

$$f_\pi = \widetilde{\beta}[1-2\eta(N-1)] + o(\epsilon) < 1,$$

implying that $\partial\Pi/\partial\pi < 0$.

Lastly, it is shown that

$$\begin{aligned}\frac{\partial\Pi}{\partial\beta} &= \left(\frac{\eta\beta^{\eta-1}}{N(N\eta-1)}\right)\left(\frac{N\eta}{N\eta-1}\right)^{-\eta}[c-\pi A]^{-\eta-1} \\ &\times\left\{[c-\pi A]B+\eta\frac{\partial A}{\partial\widetilde{\beta}}\pi B+\frac{\partial B}{\partial\widetilde{\beta}}[c-\pi A]\right\} \\ &= \left(\frac{\eta\beta^{\eta-1}}{N(N\eta-1)}\right)\left(\frac{N\eta}{N\eta-1}\right)^{-\eta}[c-\pi A]^{-\eta-1} \\ &\times\left\{[c-\pi A]B-\eta\frac{2(1-\delta)}{N(1+\widetilde{\beta})^2}\pi B-\frac{2(1-\delta)[\eta(N-1)-1]\pi}{(1+\widetilde{\beta})^2}[c-\pi A]\right\},\end{aligned}$$

which is positive if and only if

$$f_\beta = [c-\pi A]B-\eta\frac{2(1-\delta)}{N(1+\widetilde{\beta})^2}\pi B-\frac{2(1-\delta)[\eta(N-1)-1]\pi}{(1+\widetilde{\beta})^2}[c-\pi A] > 0.$$

When $\pi = 0$, $f_\beta = [c-\pi A]B > 0$, implying that $\partial\Pi/\partial\beta > 0$. Next, suppose that $\beta = 0$, so that $A = 1$, and $B = c-\pi$. Then, we have

$$f_\beta = (c-\pi)\left[c-\pi-\eta\frac{2(1-\delta)\pi}{N}-2(1-\delta)[\eta(N-1)-1]\pi\right].$$

When $\delta = 0$, $N \geq 2$, and $\pi \in (c/3, c)$, we have

$$f_\beta < (c-\pi)(c-3\pi) < 0,$$

implying that $\partial\Pi/\partial\beta < 0$.

□

Proof of Proposition 4: The equilibrium price p_C is given by

$$p_C = \frac{c-\pi\left[1-\frac{2}{N}(1-\delta)\theta_M\right]}{1-\frac{1}{N\eta}}.$$

Furthermore, the equilibrium variables satisfy the following conditions:

$$\frac{dQ^*}{dN} = -\eta Q^* \left(\frac{(c-\pi)}{Nc - \pi[N - 2(1-\delta)\theta_M]} - \frac{1}{N - \frac{1}{\eta}} \right) = \frac{\eta Q^*}{N - \frac{1}{\eta}} \left(1 - \frac{c-\pi}{p_C} \right),$$

and

$$\frac{\partial W}{\partial Q} = p_C - c + \pi(1 + \theta_M \delta).$$

Putting these together, we derivative of the welfare with respect to N, evaluated at $N = N^*$, is given by

$$\begin{aligned} \left.\frac{dW}{dN}\right|_{N=N^*} &= \frac{dQ}{dN}\frac{\partial W}{\partial Q} - K \\ &= \frac{Q^*}{N^*}\left[\frac{\eta}{1 - \frac{1}{N^*\eta}} \left(1 - \frac{c-\pi}{p_C} \right) [p_C - c + \pi(1+\theta_M\delta)] - [p_C - c + \pi - 2(1-\delta)\theta_M\pi] \right]. \end{aligned}$$

where

$$K = \frac{Q^*}{N^*}[p_C - c + \pi - 2(1-\delta)\theta_M\pi],$$

is used. Noting that

$$\begin{cases} p_C - c + \pi(1+\theta_M\delta) = \dfrac{\frac{c-\pi}{N\eta} + [(1 - \frac{1}{N\eta})\delta + \frac{2(1-\delta)}{N}]\theta_M\pi}{1 - \frac{1}{N\eta}} \\ p_C - c + \pi - 2(1-\delta)\theta_M\pi = \dfrac{\frac{c - \pi - 2(1-\delta)[(N-1)\eta - 1]\theta_M\pi}{N\eta}}{1 - \frac{1}{N\eta}} \\ \dfrac{c-\pi}{p_C} = \dfrac{\left(1 - \frac{1}{N\eta}\right)(c-\pi)}{c - \pi[1 - \frac{2}{N}(1-\delta)\theta_M]}, \end{cases}$$

we have

$$\left.\frac{dW}{dN}\right|_{N=N^*} = \frac{Q^*}{N^*} \times \frac{(c-\pi)\Psi}{(1 - \frac{1}{N^*\eta})^2[1 + \frac{2}{N^*}(1-\delta)\omega]},$$

where

$$\begin{aligned}\Psi &= \left[2(1-\delta)\left(1-\frac{1}{N^*\eta}-\frac{1}{N^*}\right)\omega-1\right]\left(1-\frac{1}{N^*\eta}\right)\left[1+\frac{2}{N^*}(1-\delta)\omega\right] \\ &\quad+\left(\frac{1+2\eta(1-\delta)\omega}{N^*}\right)\left(\frac{1+\{2\eta+\delta[\eta(N^*-2)-1]\}}{N^*\eta}\right)\omega.\end{aligned}$$

Thus, for $c < \pi$, entry in insufficient if and only if $\Psi < 0$, and for $c > \pi$, entry is insufficient if and only if $\Psi > 0$.

We first show that when $c \in \left((1-\frac{2(1-\delta)\theta_M}{N^*})\pi, \pi\right]$, $\Psi > 0$, implying that entry is excessive. Note that, $c - \pi \in \left(-\frac{2(1-\delta)\theta_M}{N^*}\pi, 0\right)$ implies that $\omega \in \left(-\infty, -\frac{N^*}{2(1-\delta)}\right)$. Thus, it suffices to see that $\Psi > 0$. Evaluating $\partial\Psi/\partial\omega$ at $\omega = -N^*/(2(1-\delta))$, we have

$$\left.\frac{\partial\Psi}{\partial\omega}\right|_{\omega=-N^*/(2(1-\delta))} = -\frac{1+\{2\eta+\delta[\eta(N^*-2)-1]\}+2(1-\delta)\left(1-\frac{1}{N^*\eta}\right)}{N^*} < 0$$

Because Ψ is convex, $\partial\Psi/\partial\omega < 0$ for all $\omega < -N^*/(2(1-\delta))$, implying that $\Psi|_\omega > \Psi|_{-N^*/(2(1-\delta))}$ for all $\omega < -N^*/(2(1-\delta))$. At $\omega = -N^*/(2(1-\delta))$, we have

$$\Psi\Big|_{\omega=-N^*/(2(1-\delta))} = \frac{N^*\eta-1}{2(1-\delta)}\left(\frac{1+\{2\eta+\delta[\eta(N^*-2)-1]\}}{N^*\eta}\right) > 0$$

Thus, we have $\Psi > 0$ for all $\left(-\infty, -\frac{N^*}{2(1-\delta)}\right)$. Because $\left.\frac{dW}{dN}\right|_{N=N^*}$ is continuous around $c = \pi$ as long as $\theta_M\pi > 0$, we have $\left.\frac{dW}{dN}\right|_{N=N^*} > 0$ at $c = \pi$.

Next, consider the case where $c > \pi$. In this case, we have $\omega > 0$. We also have

$$\Psi\Big|_{\omega=0} = -\left(1-\frac{1}{N^*\eta}\right) < 0.$$

and

$$\left.\frac{\partial\Psi}{\partial\omega}\right|_{\omega=0} = \frac{2(N^*\eta-1)(1-\delta)(N^*\eta-1-2\eta)+\eta+2\eta^2(1-\delta)+\delta\eta(N^*\eta-1)}{(N^*)^2\eta^2}.$$

When $N^*\eta - 1 - 2\eta \geq 0$, this is strictly positive. When $N^*\eta - 1 - 2\eta < 0$, we have $2\eta > N^*\eta - 1$. Then, we have

$$\left.\frac{\partial \Psi}{\partial \omega}\right|_{\omega=0} > \frac{2(N^*\eta - 1)(1-\delta)(N^*\eta - 1 - \eta) + \eta + \delta\eta(N^*\eta - 1)}{(N^*)^2\eta^2} > 0$$

whenever $N^* \geq 2$. Because Ψ is convex in ω, $\partial\Psi/\partial\omega > 0$ for all $\omega > 0$. This completes the proof. □