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Recent Advances in the Theory of Third-Degree Price Discrimination: A Brief Survey

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Recent Advances in the Theory of Third-Degree Price Discrimination: A Brief Survey*

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Abstract

This survey provides a selected review of the recent progress in the theory of third-degree price discrimination. First, I focus on two well-known results in the literature: (i) an increase in aggregate output is necessary for price discrimination to increase social welfare, and (ii) price discrimination leads to a Pareto welfare improvement if one of the two markets is not served under uniform pricing. I argue when these results hold and when they fail to hold. Second, I consider oligopolistic competition and stress that there is no great divide between monopoly and oligopoly because both situations can be treated systematically in terms of an index that governs the intensity of competition.

1. Introduction: A Centennial Tradition

In the real world, market competition is more or less imperfect: the market price of a good may not be equal to the marginal cost of production. Moreover, identical units may be sold at different prices as a result of firms' maneuver due to their market power. Arguably, Arthur C. Pigou, a 20th-century British economist, was one of those who recognized this phenomenon characterizing our daily economy, and categorized it into three types (Pigou 1920, Ch.14):

(1) First-degree price discrimination: each consumer has to pay their willingness-to-pay

* This survey is based on the author's invited talk, "The Past, Present, and Future of the Research on Third-Degree Price Discrimination," delivered on June 27, 2021, at the spring meeting of the Japan Association of Applied Economics. This manuscript is prepared as a draft for a book that the present author edits. I thank Ryo Hashizume for helpful comments in preparing for the talk as well as this manuscript. I also acknowledge a Grant-in-Aid for Scientific Research (C) (21K01440) from the Japan Society for the Promotion of Science. Any remaining errors are mine.

and thus all surplus of trade is extracted by sellers.

(2) Second-degree price discrimination: each consumer voluntarily selects one of many price schedules to offered to all consumers by sellers.

(3) Third-degree price discrimination: each consumer is segmented by their identifiable trait and faces different unit prices according to which group they belong to.

The existing studies of price discrimination is vast,[1] and hence I confine my attention to the research on third-degree discrimination on which I myself have worked in recent years.

A central question about third-degree price discrimination is related to welfare evaluation. Namely, is third-degree price discrimination socially beneficial or harmful? As explained below, it is well known that it is not easy to judge the direction of changes in Marshallian social welfare, i.e., the sum of consumer and producer surplus caused by an introduction of price discrimination. To understand this issue, suppose that there are a group of non-elderly citizens and a group of elderly citizens. Under uniform pricing, the price of a firm's product or service is the same for both groups. However, under price discrimination, senior citizens typically pay a lower price than the uniform price, whereas non-senior citizens face a higher price. Hence, price discrimination raises social welfare for the group of senior citizens, whereas the opposite is true for the group of non-senior citizens. The overall welfare effects are, therefore, ambiguous. This survey explains some of recent efforts that put this centennial question into a new framework.

2. Source of Inefficiency and Welfare Implications

By introducing some more notations, we can understand the issue in a clear manner. For simplicity, suppose that there are two identifiable markets or groups (e.g., elderly and non-elderly citizens) and a monopolistic theater. In the literature on third-degree price discrimination, a market with a higher discriminatory price than the uniform price is called a *strong* market, and the other market a *weak* market (Robinson 1933). We expect that the group of non-elderly citizens is the strong market and that of elderly citizens is the weak market. We then denote the uniform price and the discriminatory prices for the strong and weak market by $\bar{p}$, p_s^*, and p_w^*.

Then, under price discrimination, the discriminatory price in market $m = s, w$ should satisfy:

[1] See Varian (1989) and Stole (2012) for two comprehensive surveys on price discrimination.

$$\frac{p_m^* - c}{p_m^*} = \frac{1}{\epsilon_m(p_m^*)}$$

where $c \geq 0$ is the marginal cost of production that is assumed to be a constant, and $\epsilon_m(\cdot)$ denotes the price elasticity of demand in market m. Similarly, the markup formula for $\bar{p}$ under uniform pricing is expressed in terms of the weighted sum of two elasticities:

$$\frac{\bar{p} - c}{\bar{p}} = \frac{1}{\epsilon_s(\bar{p})\frac{q_s(\bar{p})}{\bar{Q}} + \epsilon_w(\bar{p})\frac{q_w(\bar{p})}{\bar{Q}}}$$

where $q_m(\cdot)$ is market demand in m, and $\bar{Q} \equiv q_s(\bar{p}) + q_w(\bar{p})$ is the aggregate output under uniform pricing. Evidently, under either regime, aggregate output is less than the efficient level because $\bar{p} > c$ and $p_m^* > c$. Moreover, for a given level of aggregate output, price discrimination typically generates inter-consumer misallocations as compared to uniform pricing. This is clearly shown in Figure 1 below that is adapted from Layson's (1988) Figure 1.

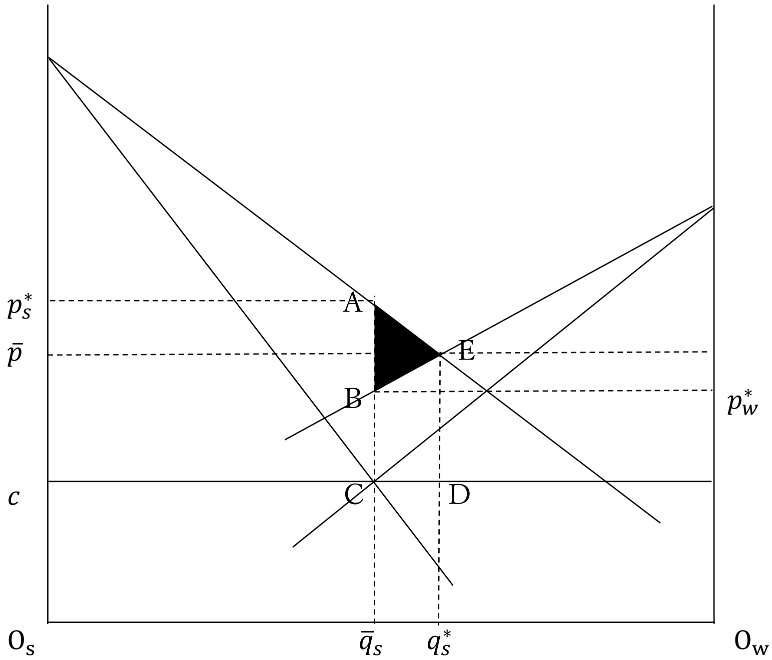

Figure 1. Welfare loss due to inter-consumer misallocations

Here, this figure utilizes the fact that price discrimination does not change aggregate output under linear market demands: output in the strong market is measured from 0_s whereas that in the weak market is measured from 0_w. In the strong market, price discrimination results in welfare loss expressed as area ACDE because output decreases from $q_s^* \equiv q_s(p_s^*)$ to $\bar{q}_s \equiv q_s(\bar{p})$. On the other hand, the weak market experiences welfare improvement captured by area BCDE. Obviously, there is aggregate welfare loss expressed as the black triangle, ABE because those with willingness-to-pay corresponding to line segment BE in the weak market consumes the good, whereas consumers in the strong market who have higher willingness-to-pay with line segment AE do not consume the good. This welfare loss results from this inter-consumer misallocation. This argument can be generalized to include non-linear market demands and is summarized as follows:

Claim 1. *An increase in aggregate output is necessary for social welfare to improve.*

This result has been considered a hallmark of the research on third-degree price discrimination, and has been verified by a series of such studies as Pigou (1920), Robinson (1933), Schmalensee (1981), Varian (1985), Schwartz (1990), and Bertoletti (2004). In fact, Pigou (1920, Part 2, Ch. 16, Sects. 11-15) was arguably the first economist who noticed this relationship, though, in an implicit way. Although he asserts that due to price discrimination, "the resources invested in the industry fall into a number of different parts, in each of which the value of the marginal social net product is different" (Section 11), he just compares this inefficiency of under-supply with the perfectly competitive supply. So, he does not necessarily uncover the relationship clearly. Robinson (1933, Book 5, Ch. 15 "Price Discrimination") does not clearly mention the relationship, neither. In fact, she shows geometrically a part of statement 2 above, i.e., "if demands in two markets under constant costs are linear, total output is unchanged by introduction of price discrimination." And, by focusing on shapes of demands curves, she just proposes a test that can determine whether total output increases or decreases.

About half a century later, Schmalensee (1981) confirms this conjecture assuming that demand in one market is independent from prices in other markets and that marginal cost is constant with no fixed costs.[2] Successively, Varian (1985) and Schwartz (1990) take a further step to prove the conjecture: Varian (1985) shows that the conjecture holds even if demand in any market is dependent on prices in other markets and marginal cost is constant or increasing. Furthermore, he proves that without assuming that the monopolist maximizing her profit both under uniform pricing and under price discrimination. Schwartz's (1990) revealed-preference argument is more general than Varian's (1985) algebraic analysis in that he shows that the conjecture holds for any total

[2] See Varian (1989, p.619) for other preceding references.

cost function that depends only on total output, not on its distribution among markets. He shows this result by assuming that the monopolist maximizes her profit at least under uniform pricing.

To understand this point in a clearer manner, I briefly introduce a simplified version of Varian's (1985) argument (see also Varian 1989, pp. 619-621), which proves the conjecture by deriving an upper and lower bound for a change in welfare, $\Delta W \equiv W^* - \overline{W}$, where W^* and $\overline{W}$ are social welfare under price discrimination and under uniform pricing, respectively. Suppose that there are two separate groups/markets, s and w, and there is a representative consumer whose (direct) utility is given by $u(q_s, q_w) + y$, where $q_m \in [0, \infty)$ indicates the amount of consumption in market $m = s, w$, and y is a composite of income spent on other goods.[3] Without loss of generality, I set $y = 0$, and assume concavity and differentiability of $u(q_s, q_w)$. An important step for a further argument is to notice the following relationship that gives inverse demand functions:[4]

$$p_m(q_s, q_w) = \frac{\partial u(q_s, q_w)}{\partial q_m}$$

for $m = s, w$.

Then, let $(\bar{q}_s, \bar{q}_w)$ and (q_s^*, q_w^*) be the pairs of output in both markets under uniform pricing and under price discrimination, respectively, and the associated prices are $\bar{p}$ and (p_s^*, p_w^*). The concavity of $u(\cdot,\cdot)$ is equivalent to:

$$\frac{\partial u(q_s, q_w)}{\partial q_s} \Delta q_s + \frac{\partial u(q_s, q_w)}{\partial q_w} \Delta q_w \geq \Delta u,$$

where $\Delta q_s \equiv q_s^* - \bar{q}_s$, $\Delta q_w \equiv q_w^* - \bar{q}_w$, and $\Delta u \equiv u(q_s^*, q_w^*) - u(\bar{q}_s, \bar{q}_w)$. Additionally, the following relationships hold:

$$\Delta u \leq \bar{p} \cdot (\Delta q_s + \Delta q_w)$$

and

$$\Delta u \geq p_s^* \Delta q_s + p_w^* \Delta q_w.$$

If the marginal cost is constant, the change in the total cost of production, Δc, is equal to $c \cdot (\Delta q_s + \Delta q_w)$. Therefore, since $\Delta W = \Delta u - \Delta c$, the following proposition is obtained.

Proposition 2. (Varian (1985), Fact 3). *When the monopolist is faced with constant marginal cost, welfare change, associated with the regime change from uniform pricing*

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[3] In Varian's (1985) and Schwartz's (1990) original arguments, individual consumer's indirect utility is assumed to exhibit quasi-linear in the vector of prices of the number of groups and in income, which justifies welfare analysis based on the representative consumer's indirect utility.

[4] When the representative consumer's indirect utility is employed, Roy's identity plays this role.

to price discrimination, satisfies the following inequality:

$$(\bar{p} - c)(\Delta q_s + \Delta q_w) \geq \Delta W \geq (p_s^* - c)\Delta q_s + (p_w^* - c)\Delta q_w.$$

The upper bound of this inequality implies that if $\Delta q_s + \Delta q_w = Q^* - \bar{Q} \equiv \Delta Q$, where Q^* and $\bar{Q}$ are aggregate output under price discrimination and under uniform pricing, respectively, is negative, then ΔW is also negative, which supports Claim 1 above. Furthermore, when demand curves are linear, ΔQ is verified to be equal to zero, and hence, the above inequality also formalizes the argument above related to Figure 1.

3. Introducing Consumption Externality

However, Adachi (2002, 2005) shows that in the presence of consumption externality, third-degree price discrimination price discrimination can improve social welfare even if aggregate output does not change. In other words,

Claim 3. *In the presence of consumption externality, an increase in aggregate output is not necessary for social welfare to improve.*

Here, Adachi's (2002, 2005) study begins with a simple observation: one's willingness-to-pay depends on the *composition* of aggregate output. In other words, each consumer may be concerned about how many people within their own group or in the other group consumes the good. For example, students may find it more beneficial use some type of software if more faculty members use it, and faculty members feel the same way. In fact, Pigou (1920) noticed this relationship:

> "The analysis, to be complete, would need to take account of the fact that, in real life, the demand of one purchaser for any r-th unit of a commodity is sometimes, in part, dependent upon the price at which this commodity is being sold *to other purchasers*. When markets are *interdependent* in this way, the issue is complicated, *but the broad results, though rendered less certain, are not, it would appear, substantially altered*."
>
> Pigou (1920, Part 2, Ch. 16 "Discriminating Monopoly," Sect. 8); emphasis is added

To be more specific, let x_m be a positive real number that indexes each consumer in market $m = s, w$. Then, consumer x_m's willingness-to-pay is given by

$$a_m - x_m + \eta \cdot q_n^e,$$

for $m, n = s, w,\ m \neq n,\ \eta \in (-1,1)$ is a constant that exhibits *cross-market network* *effects*, and q_n^e is the expected amount of consumption in market n. Under this linear setting, Adachi (2002) shows that (i) aggregate output does not change, (ii) price discrimination lowers aggregate consumer surplus, but (iii) price discrimination improves social welfare if and only if $1/2 < \eta < 1$.

Adachi's (2002) result is further generalized by Hashizume, Ikeda, and Nariu (2021). The inverse demand function in market $m = s, w$ is given by

$$a_m - (1+\zeta)x_m + \zeta q_m^e + \eta \cdot q_n^e,$$

where $\zeta > -1$ is an additional component that exhibits *symmetric within-market* externality, which is derived from the maximization of the representative consumer's net utility,

$$u(x_s, x_w; q_s^e, q_w^e) - p_s x_s - p_w x_w,$$

where

$$u(x_s, x_w; q_s^e, q_w^e) = a_s x_s + a_w x_w - (1+\zeta)\frac{x_s^2 + x_w^2}{2}$$
$$+\zeta \cdot (x_s q_s^e + x_w q_w^e) + \eta \cdot (x_s q_w^e + x_w q_s^e).$$

Under this setting, Hashizume, Ikeda, and Nariu (2021) show that (i) For $\zeta \geq 1/3$, price discrimination never improves social welfare, and (ii) for $\zeta \in (-1, 1/3)$, price discrimination improves social welfare if and only if

$$\frac{1+3\zeta}{2} < \eta < 1.$$

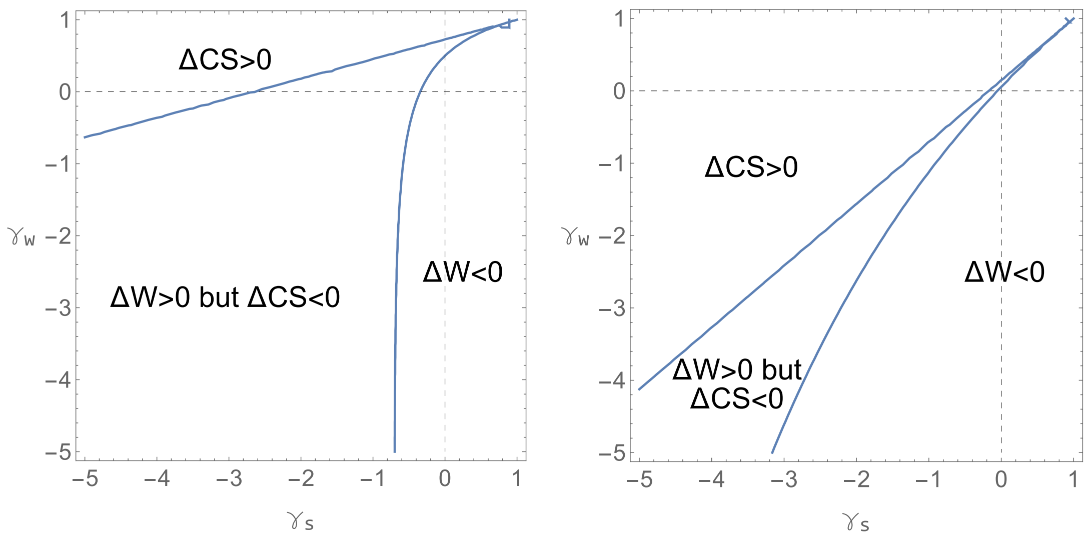
Note that Adachi's (2002) result is nested in this condition as a special case of $\zeta = 0$. In either case, the degree of cross-market network effects must be sufficiently large that willingness-to-pay in both markets can be strengthened. If the degree of within-market externality is strongly negative, then price discrimination is likely to improve social welfare even if η is negative: in this case, an output decrease in the strong market enhances the willingness-to-pay for those consumers, and these beneficial effects are larger than the loss due to an increase in output in the week market where there are also negative within-market externality (recall Figure 1 above).

Subsequently, Adachi (2005) considers *asymmetric* within-market externalities. Namely, for market $m = s, w$, consumer x_m's willingness-to-pay is given by

$$a_m - x_m + \gamma_m \cdot q_m^e,$$

where $\gamma_m < 1$ is within-market externality which may be different across markets. Under this setting, Adachi (2005) shows that (i) aggregate output does not change in this case, either. However, it is shown that (i) price discrimination can improve social welfare, and moreover, (iii) aggregate consumer surplus can increase by price discrimination.

To see Adachi's (2005) results graphically, I assume that (without loss of generality) $a_s = 1$ and $a_w = a \in (0,1)$. Then, Figure 2 shows when price discrimination increases or decreases consumer surplus ($\Delta CS > 0$ or $\Delta CS < 0$) and when it increases or decrease social welfare ($\Delta W > 0$ or $\Delta W < 0$) for the cases of $a = 0.4$ (Panel a) and of $a = 0.9$ (Panel b). First, it is observed that price discrimination necessarily deteriorates social welfare if $\gamma_s > 0$ and $\gamma_w < 0$. This is because the strong market, which exhibits positive externality is where an output decreases, and the oppositive is true for the weak market. More interestingly, (1) if there is negative within-market externality for both markets (i.e., $\gamma_s < 0$ and $\gamma_w < 0$), then the *smaller* a is, the larger the area for $\Delta W > 0$ (see Panel a). However, (2) if there is positive within-market externality in both markets (i.e., $\gamma_s > 0$ and $\gamma_w > 0$) or if the strong market exhibits negative within-market externality whereas the weak market faces positive within-market externality (i.e., $\gamma_s < 0$ and $\gamma_w > 0$), then *larger* a is, the larger the area for $\Delta W > 0$ (see Panel a).



(i) $a = 0.4$ (ii) $a = 0.9$

Figure 2. Welfare effects under asymmetric within-market externality

To Adachi's (2005) results intuitively, note first that for the case of (1), The negative size effect on the absolute value of a change in consumer surplus in the strong market, $|\Delta CS_s|$, is small because $\gamma_s < 0$. Then, a small a is favorable because the effect of $\gamma_w < 0$ on ΔCS_w keeps small. The case of (2) can be similarly understood because ΔCS_w is large enough if the size of the weak market is large for $\gamma_w > 0$. In this way, I have been able to verify that Pigou's (1920) argument needs modification.

4. Market Opening

Pigou (1920, Appendix D, Sect. 10) and Robinson (1933, Book 5, Ch. 15, Sect. 5) also noticed that if price discrimination opens markets which are not served under uniform pricing, total output increases. However, they did not directly relate the increase to changes in welfare.

About 60 years later, Hausman and MacKie-Mason (1988) gave a formal discussion in relation to the existence of scale economies, which is remedied by such authors as Layson (1994) and Nahata, Ostaszewski and Sahoo (1990):

Claim 4. *If introduction of price discrimination opens new markets, i.e., markets not being served under uniform pricing, it can increase Marshallian welfare if marginal cost is nonincreasing.*

To express this statement more formally, I again use the two-market version of Varian's (1985) treatment that is described above. Let market w not be served under uniform pricing, i.e., $\bar{q}_w = 0$. Then, the uniform price, $\bar{p}$, and the output, $\bar{q}_s$, are just equal to the monopoly price and the monopoly output, respectively.

If the monopolist is faced with independent (inverse) demands, that is, $p_m(q_s, q_w) \equiv p_m(q_m)$ for $m = s, w$, price discrimination *must* lead to a Pareto welfare gain: since $\Delta q_s = 0$ with $p_s^* = \bar{p} > c$ and $\Delta q_w > 0$, price discrimination generates consumer surplus in market w, although it is unchanged in market s. Note that this argument depends on the assumption that marginal cost is constant. When it is decreasing, the result of Pareto welfare gain is also obtained: by price discrimination, total output becomes greater, and hence, the corresponding marginal cost is lower. Therefore, the price in market s lowers and the output increases, i.e., consumer surplus in market s increases. When marginal cost is increasing, price discrimination necessarily harms consumers in market s by raising the price, and even Marshallian welfare may decrease. Therefore, the following claim is partly verified in the sense that independency between demands in assumed.

Claim 5. *When there are only two markets, only the one of which is served under uniform pricing, price discrimination can lead to a Pareto welfare improvement if marginal cost is nonincreasing.*

Moreover, the above argument does not depend on the number of markets: Claim 4 above is also verified in the case of independent demands.

Then, what happens if dependent demands such as complementary and substitutable demands are assumed? Note that even if price discrimination opens a new market, it does *not necessarily* mean an increase in welfare, let alone a Pareto improvement. Due to interdependency between demands, it may negatively affect output in markets that are originally served under uniform pricing, that is, an increase in output of market w ($\Delta q_w > 0$) may work to decrease output in market s, that is, $\Delta q_s < 0$. Hence, it may result in a decrease in welfare. This is the case in the presence of sustitutability between demands.

In the presence of complementarity between demands, however, the monopolist may now exploit gains accrued to consumers in market s, i.e., may decrease consumer surplus in market s though willingness-to-pay of them goes up due to opening of market w. The following assumption is made to exclude those negative effects of market opening.

Assumption 6. (Hausman and MacKie-Mason's (1988) Definition 1, modified). *Two demand functions are such that changing the price in one market does not reduce the consumer surplus in the other market, at an unchanged price in the latter.*

When the monopolist price-discriminates, she is weakly better off since she always has an option to set a uniform price. The following is nonetheless a formal expression of Claim 5.

Proposition 7. (Hausman and MacKie-Mason (1988), Proposition 1). *Suppose there are two different demand functions satisfying Assumption 1, and one market is not served under uniform pricing. If marginal cost is constant or decreasing, then price discrimination always (weakly) yields a Pareto improvement.*

When there are more than two markets, except that only one market is served under uniform pricing, a Pareto improvement is not usually expected. This is because since uniform price depends on some weighted average of demand elasticity in each (opening) market, it is expected that price discrimination usually makes discriminatory prices rise in some markets and fall in other markets.[5] As in the proposition above, the

[5] Hausman and MacKie-Mason (1988, pp. 256-7) conclude that in this case a Pareto improvement is not possible. However, this is not true. In fact, Nahata, Ostaszewski and Sahoo (1990) show that third-degree price discrimination may either lower or rise price in all markets

following formal assertion of Claim 4 is obtained.

Proposition 8. (Hausman and MacKie-Mason's (1988) Proposition 3, modified) *Suppose that markets which satisfy Assumption 1.6 and are not served under uniform pricing can now be served by price discrimination. If marginal cost is constant or decreasing, then price discrimination yields a welfare gain.*

Hausman and MacKie-Mason (1988) show general conditions under which Marshallian welfare is improved by price discrimination. If marginal cost is constant or decreasing, then price discrimination yields a welfare gain.

However, as in Section 3 above, Okada and Adachi (2013) point out a counter case by explicitly considering cross-market externality. Specifically, they argue that it is welfare improving to close the weak market under price discrimination---even though the monopolist is willing to open it in this situation---if negative externality is sufficiently large and the weak market is relatively small. This is because consumer surplus in the strong market decreases sharply so that it outweighs the welfare gain in the weak market. In this way, private incentives and social gains do not necessarily align once consumption externality is considered in this context of market opening.

5. Oligopolistic Third-Degree Price Discrimination

I now turn to third-degree price discrimination *under imperfect competition*. In particular, I aim to update the following statement---made about ten years ago---in an empirical paper on intertemporal price discrimination:

> "The welfare implications of third-degree price discrimination by a monopolist were studied by Robinson (1933), and later formalized by Schmalensee (1981), Varian (1985), and Aguirre, Cowan, and Vickers (2010) among others. The impact of discrimination on welfare is ambiguous. In oligopoly situations there are *virtually no predictions* as to how discrimination impacts welfare."
>
> Hendel and Nevo (2013, p. 2723); emphasis added

Third-degree price discrimination---in which identical units of final products and services are sold at different prices across different consumers segmented by identifiable characteristics such as location, age, time of use, and so on---is only possible under

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by assuming that marginal cost is constant and some sub-market pro t functions are multiple-peaked.

imperfect competition. This is because under perfect competition, the law of one price should prevail: all consumers must face an identical price of the same good. However, in reality, firms usually recognize their market power at least to some extent: they do not necessarily lose all of their customers even if their price is higher than the rivals' prices. This is arguably due to product differentiation across firms based on branding strategy or simply to the daily situation where a small number of firms coordinately compete each other by taking advantage of the fact that it is costly for consumers to move and search (physically or virtually). They create the sources of imperfect competition, and, as Robinson (1933, p.180) points out, "there can be some degree of price discrimination."[6]

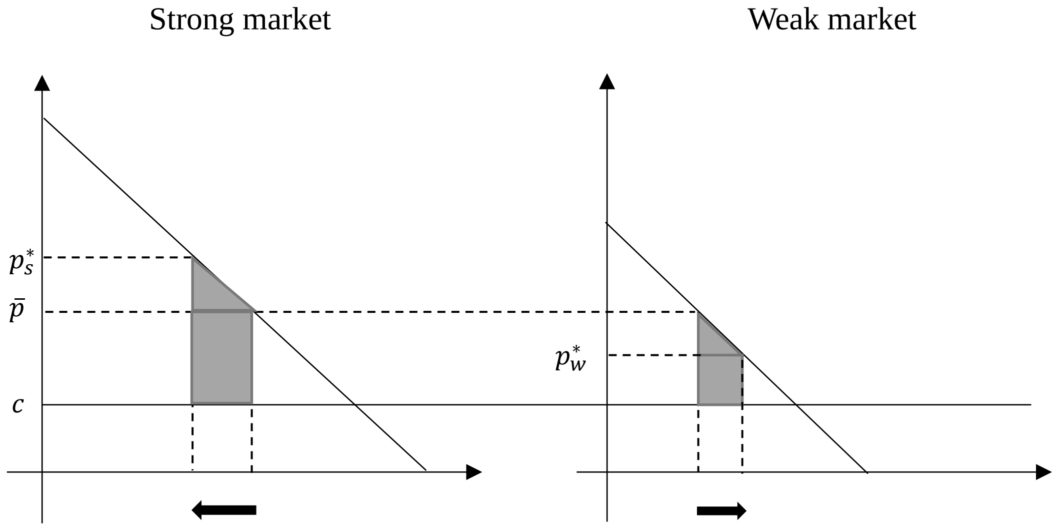

Figure 3. Welfare changes by price discrimination

Figure 3 depicts how social welfare is affected by price discrimination in a canonical situation where there are two separate markets with common constant marginal costs, $c \geq 0$. When price discrimination is banned, the uniform price $\bar{p}$ prevails. If it is allowed, two opposite effects arise: in the market where the discriminator price, p_S^*, becomes higher, the associated change in output is negative, and hence, welfare loss (the dark trapezoid in the left panel) is observed. On the other hand, in the other market where the discriminatory price, p_W^*, is lower than the uniform price, output expands, and welfare is improved (the dark trapezoid in the right panel). Here, as above, the former market is called the "strong" market, and the latter the "weak" market. As shown in this figure, it is

[6] Additionally, no arbitrage must take price; otherwise, consumers who face a lower discriminatory price may resell the good to other consumers with a slightly lower price than they face, possibly stealing a large of demand that sellers could have satisfy.

not obvious which trapezoid is larger. However, Aguirre, Cowan, and Vickers (2010) provide a sufficient condition for welfare improvement under monopoly: the demand curvature in the weak market is sufficiently large as compared to that in the strong market. This is because an increase in output in the weak market is relatively larger.

Based on this result, one might be tempted to extend it to the case of oligopoly, conjecturing that price discrimination improves social welfare if the intensity of competition in the weak market is stronger. However, in an oligopoly model of linear demand with horizontal product differentiation, Adachi and Matsushima (2014) show the opposite is true: the degree of substitution in the *strong* market must be sufficiently higher. This result is due to the equilibrium effect of oligopoly that does not arise from Aguirre, Cowan, and Vickers' (2010) case of monopoly: if, instead, the degree of substation in the weak market is sufficiently larger, then the equilibrium uniform price is already low due to price competition so that there is little room for price discrimination to expand output and hence increase social welfare in the weak market. In contrast, if the degree of substitution in the strong market is sufficiently larger, the output decrease due to the price increase can be kept relatively small. Then, the welfare gain in the weak market can outweigh the loss in the weak market, improving social welfare. It becomes clear that the intrinsic nature of market demand related to curvature and the effect of competition must be distinguished in understanding welfare effects.

Adachi (2022) furthermore considers a fairly general model of oligopolistic third-degree price discrimination with general demands, (a reasonable amount of) cost differences across markets,[7] and firm heterogeneity to show that it is possible to generalize Adachi and Matsushima's (2014) finding. To study the welfare effects of oligopolistic third-degree price discrimination, Adachi (2022) proposes a *sufficient statistics approach* that utilizes a relevant set of economically interpretable measures such as elasticity to a minimum extent to conduct policy evaluation (Chetty 2009, Kleven 2021, Adachi and Fabinger 2022). I here emphasize the usefulness of this approach to the context of third-degree price discrimination. Interested readers should consult Adachi and Matsushima (2014) and Adachi (2022) for technical details.

[7] Differential pricing---driven by cost differences across markets---is studied by Chen and Schwartz (2015) who assume monopoly, and later by Chen, Li, and Schwartz (2021) for the case of oligopoly. The present survey is in line with the tradition of the existing literature on third-degree price discrimination in which price differences are mainly driven by demand differences across markets.

5.1 Modeling Oligopolistic Competition

In this subsection, I introduce a standardized model of third-degree price discrimination under price discrimination. Oligopolistic firms are assumed to be symmetric, and for simplicity, I assume the number of firms is two, indexed by 1 and 2. The entire market can be segmented into submarkets---simply called markets as above---based on unambiguous traits so that firms can set different prices on these markets. However, government regulation or social customs may not permit such *price discrimination*: this case is called *uniform pricing*. I focus on the simplest case where there are only two markets. Then, under the assumption of firm symmetry, firms agree on the raking of two market prices under price discrimination. As already mentioned above, I follow Robinson (1933) to call the market with the higher price the “strong” market and the other market with the lower price the “weak” market. They are indexed by s and w, respectively, below.

Specifically, our market demands for firm $i = 1,2$ in market $m = s, w$ are denoted by $x_{im} = x_{im}(p_{im}, p_{-i,m})$, where p_{im} and p_{-im} are firm i’s and $-i$’s ($-i = 1, 2$, and $-i \neq i$) prices in market m.[8] In is natural to assume that $\partial x_{im}/\partial p_{im} < 0$ and $\partial x_{im}/\partial p_{-i,m} > 0$: given the other firm’s price, firm i’s demand decreases if it raises its own price, and given its own price, it increases if the rival price becomes higher. Furthermore, $x_{im}(\cdot)$ is assumed to be twice continuously differentiable. Given this demand structure, firm i’s best response under price discrimination is defined by

$$BR_{im}(p_{-i,m}) \equiv \underset{p_{im}}{\text{argmax}}(p_{im} - c_m) x_{im}\left(p_{im}, p_{-i,m}\right),$$

where $c_m \geq 0$ is constant marginal cost in market m that is common for both firms. Following Corts (1998), I provide a formal definition for strong and weak markets for firm i: market s is strong (that is, market w is weak) if and only if

$$BR_{is}(p_{-i}) > BR_{iw}(p_{-i})$$

for all $p_{-i} \geq 0$. Best-response symmetry arises if both firms agree on this ranking; otherwise, the situation is called best-response asymmetry. Below, I solely focus on the case of firm symmetry: Adachi (2022) extends this type of model to include heterogenous firms, keeping best-response symmetry, and Corts (1998) analyzes the welfare effects of

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[8] As Adachi (2022) discusses, these demands in market m are micro-founded by the maximization problem of the representative consumer’s (net) quasi-linear utility

$$U_m(x_{1m}, x_{2m}) - p_{1m}x_{1m} - p_{2m}x_{2m},$$

with respect to $x_{1m} > 0$ and $x_{2m} > 0$. Typically, it is assumed that $\partial U_m/\partial x_{im} > 0$ and $\partial^2 U_m/\partial x_{im}^2 < 0$ for $i = 1,2$, and $\partial^2 U_m/(\partial x_{1m}\partial x_{2m}) < 0$.

oligopolistic third-degree price discrimination by allowing best-response asymmetry.[9]

Under symmetric pricing, I am able to define symmetric firm demand in market m, $q_m(p) \equiv x_{im}(p,p)$ with symmetric price p. As in Stole (2007, p.2235), I introduce three measures to capture market competitiveness. First, the *price elasticity of the industry's demand,*

$$\epsilon_m^I(p) \equiv -\frac{pq_m''(p)}{q_m(p)} > 0,$$

is the concept that shows to what extent the industry as a whole loses consumers by a price change. Under monopoly, these consumers face no other firms to switch to. However, in oligopoly, if one firm raises its price, given the other firm's price, escaping consumers are categorized into (i) those who give up purchasing any goods, and (ii) those who switch to the other firm. Holmes' (1989, p.246) identity,

$$\epsilon_m^{own}(p) = \epsilon_m^I(p) + \epsilon_m^{cross}(p),$$

expresses this relationship in a formal manner, indicating that the *own price elasticity of the firm's demand,*

$$\epsilon_m^{own}(p) \equiv -\frac{p}{q_m(p)}\frac{\partial x_{1m}}{\partial p_1}(p,p) > 0,$$

has two components: (i) $\epsilon_m^I(p)$ and (ii) the *cross price elasticity of the firm's demand,*

$$\epsilon_m^{cross}(p) \equiv \frac{p}{q_m(p)}\frac{\partial x_{2m}}{\partial p_1}(p,p) > 0.$$

Now, I introduce other useful concepts related to oligopolistic competition that Stole (2008) do not mention. First, I define the *conduct parameter* in market m.[10] by

$$\theta_m(p) \equiv 1 - \frac{\epsilon_m^{cross}(p)}{\epsilon_m^{own}(p)} \in [0,1]$$

This is a normalized measure that captures competitiveness: if the industry is monopoly so that $\epsilon_m^{cross} = 0$, $\theta_m = 1$. On the other hand, if the firms have no market power as in Betrand competition with homogenous goods or perfect competition in the literal sense so that $\epsilon_m^{cross} = \epsilon_m^{own}$, then $\theta_m = 0$. Under any type of imperfect competition, θ_m lies between 0 and 1: the intensity of competition is stronger as θ_m is closer to one.

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[9] More specifically, Corts (1998) shows that welfare predictions of allowing price discrimination can be unambiguously obtained because discriminatory prices either jump or drop in all markets under best-response asymmetry.

[10] Obviously, this index is not a parameter but rather an endogenous variable. The origin of this name comes from the empirical industrial organization literature that targets the estimation of this "parameter" (see Bresnahan 1989, Genesove and Mullin 1998, and Corts 1999). Adachi (2022, Table 1) shows that $\theta_m(p)$ is actually a constant if either linear or CES (constant elasticity of substitution) demand is imposed.

Next, it is also important to introduce two measures for second-order elasticity. First, I define the *curvature of the firm's (direct) demand* in market m by

$$\alpha_m^{own}(p) \equiv -\frac{p}{\partial x_{1m}(p,p)/\partial p_1}\frac{\partial^2 x_{1m}}{\partial p_1^2}(p,p)$$

as a measure that captures the convexity/concavity of the firm's direct demand: given the rival's price p under symmetric pricing, α_m^{own} is positive if $\partial x_{Am}(\cdot,p)/\partial p_A$, is convex, and it becomes negative if $\partial x_{Am}(\cdot,p)/\partial p_A$, is concave. Note that this curvature can be defined under monopoly. However, the next concept—the *elasticity of the cross-price effect of the firm's direct demand* in market m—appears only in oligopoly:

$$\alpha_m^{cross}(p) \equiv -\frac{p}{\partial x_{1m}(p,p)/\partial p_A}\frac{\partial^2 x_{1m}}{\partial p_2 \partial p_1}(p,p).$$

This measures how the degree of switching behavior when the firm raises its price (i.e., giving up the firm's good but purchasing the rival's good) is affected by the rival's price: α_m^{cross} is larger if the associated change in $|\partial x_{im}/\partial p_i|$ is larger by a change in p_{-i}.

As a digression, I note that the industry-level curvature that may correspond to ϵ_m^I is not necessary in the analysis below. A corresponding measure would be defined by

$$\alpha_m^I(p) \equiv -\frac{q_m(p)q_m''(p)}{[q_m'(p)]^2},$$

and one could establish a curvature version of Holmes' (1989):

$$\alpha_m^I = \left(\frac{1}{\epsilon_m^I}\right)^2 (\alpha_m^{own} + 2\alpha_m^{cross})\epsilon_m^{own} + \frac{q_m}{(q_m')^2}\frac{\partial^2 x_{2m}}{\partial p_1^2}$$

by using the identity,

$$q_m'' = \frac{\partial^2 x_{1m}}{\partial p_1^2} + 2\frac{\partial^2 x_{1m}}{\partial p_2 \partial p_1} + \frac{\partial^2 x_{2m}}{\partial p_1^2}$$

and the definitions of α_m^{own}, α_m^{cross}, ϵ_m^I, and ϵ_m^{own}. Under monopoly, this is reduced to

$$\alpha_m^I = \left(\frac{1}{\epsilon_m^I}\right)^2 \alpha_m^{own} \cdot \epsilon_m^{own}$$
$$= \frac{\alpha_m^{own}}{\epsilon_m^{own}}$$

because $\epsilon_m^I = \epsilon_m^{own}$. This monopoly version is already shown by Aguire, Cowan, and Vickers (2010, p.1603).[11]

Turning back to our main argument, I conclude this subsection by defining the *profit margin* and the *mark-up rate* (i.e., the Lerner index) by

$$\mu_m(p) \equiv p - c_m$$

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[11] In Aguire, Cowan, and Vickers' (2010, p.1603) notation, the corresponding expression is $\sigma_m = \alpha_m/\eta_m$.

and

$$L_m(p) \equiv \frac{\mu_m(p)}{p},$$

respectively.

5.2 Output Effects

In this subsection, I explain Adachi's (2022) result that provides an alternative expression for Holmes' (1989) necessary and sufficient condition for an increase of aggregate output by price discrimination under symmetry oligopoly.

Under price discrimination, output in the weak market expands owning to a lower price, whereas the strong market shrinks. Does aggregate output increase or decrease? By assuming no cost differentials ($c \equiv c_s = c_w$) as most of the papers on third-degree price discrimination do, Holmes (1989, p.247) claims that aggregate output increases by a regime change from uniform pricing to price discrimination in oligopoly if and only if

$$\underbrace{\left[\frac{\mu_w}{-q'_w(p_w)} \cdot \frac{d}{dp_w}\left(\frac{\partial x_{A,w}}{\partial p_A}\right) - \frac{\mu_s}{-q'_s(p_s)} \cdot \frac{d}{dp_s}\left(\frac{\partial x_{A,s}}{\partial p_A}\right)\right]}_{adjusted\ concavity} + \underbrace{\left[\frac{\epsilon_s^{cross}}{\epsilon_s^I} - \frac{\epsilon_w^{cross}}{\epsilon_w^I}\right]}_{elasticity\ ratio} > 0.$$

The first term is related to what is coined (somewhat inappropriately) by Robinson (1933, p.193) in the case of monopoly: if and only if the demand curvature in the weak market is sufficiently large as compared to that in the strong market, price discrimination increases aggregate output. The second term is an additional twist due to oligopoly. To understand this in a clear manner, note that for $m = s, w$,

$$\frac{\epsilon_m^{cross}}{\epsilon_m^I} = \frac{1}{\theta_m} - 1$$

from the definition of θ_m. Therefore, $\epsilon_m^{cross}/\epsilon_m^I$ captures the intensity of competition in market m.

Adachi (2022) shows that Holmes's (1989) inequality above is equivalently given by an economically interpretable expression:

$$\underbrace{\left[\frac{L_w}{\theta_w} \cdot (\alpha_w^{own} + \alpha_w^{cross}) - \frac{L_s}{\theta_s} \cdot (\alpha_s^{own} + \alpha_s^{cross})\right]}_{adjusted\ concavity} + \underbrace{\left[\frac{1}{\theta_s} - \frac{1}{\theta_w}\right]}_{reverse\ conduct\ ratio} > 0.$$

The proof is given in Adachi (2022). Note that in that proof, the common marginal cost assumption (i.e., $c_s = c_w$) is not necessary.

This expression makers clear how the pure effects of competition (the reverse conduct ratio) is separated from the effects that are mainly related to the demand curvature. Obviously, from a more fundamental level, θ_m is also affected by the demand curvature. The opposite approach would be to firstly assume a fully parametrized functional form for market demand and then derive a similar condition based on "deep" parameters of market demand. The downside of this standard approach is that a parametric form of market demand itself might not be a fundamental object that exist in market but rather a researcher's supposition that is (arbitrary or empirically) chosen under the assumption that firms behave as if this demand form selected by the researcher is their market demand. Instead, our sufficient characteristics characterization does not depend on the choice of market demand, and is able to provide an economically much clear interpretation because our sufficient statistics have economic meanings. In contrast, not all parameters of demand functions permit an economic interpretation. In the next subsection, I will keep this stance for studying the welfare effects of oligopolistic third-degree price discrimination.

5.3 Welfare Analysis

Suppose that oligopolistic firms are engaged in price discrimination: the symmetric equilibrium prices are denoted by p_s^* in the strong market and p_w^* in the weak market. When is the current regime justified in terms of social welfare? In other words, under what conditions is a ban on price discrimination welfare deteriorating?

Adachi (2022) provides a sufficient condition in terms of sufficient statistic for when such a ban lowers social welfare. To show that result, I first define *cost pass-through* in market $m = s, w$, by $\rho_m \equiv \partial p_m / \partial c_m$. In particular, I focus on pass-through at the discriminatory price, $\rho_m^* \equiv \partial p_m^* / \partial c_m$. Adachi (2022) verifies that this pass-through is expressed in terms of the demand elasticities and curvatures defined above:

$$\rho_m^* = \frac{1}{2 - \frac{(\epsilon_m^{cross})^* + (\alpha_m^{own})^* + (\alpha_m^{cross})^*}{(\epsilon_m^{own})^*}}.$$

The conduct parameter and the profit margin under price discrimination are similarly denoted by $\theta_m^* \equiv \theta_m(p_m^*)$ and $\mu_m^* \equiv \mu_m(p_m^*)$, respectively. Then, Adachi (2022) shows the following proposition.

Proposition 9. *A ban on price discrimination is welfare deteriorating if*

$$\mu_s^* \theta_s^* \rho_s^* < \mu_w^* \theta_w^* \rho_w^*$$

holds.

Note that this sufficient condition does not specify a form of market demand.[12] Furthermore, as in the previous subsection, the common cost assumption is not necessary. Adachi (2022) also argues that a vector-matrix version of this inequality can be obtained when the assumption of firm symmetry is relaxed, but best-response symmetry is still kept.

Figure 4 provides a graphical illustration of this result. The marginal change of welfare in either market is depicted as a colored rectangle. Obviously, the height is nothing but the profit margin μ_m^*. Then, the marginal change in quality corresponds to quantity pass-through, and Adachi (2022) verifies that (the absolute value of) this measure is expressed by $\theta_m^* \rho_m^*$. Therefore, each rectangle's area is $\mu_m^* \times \theta_m^* \rho_m^*$, and under the monotonicity condition, the inequality holds, and thus a regime shift from price discrimination to uniform pricing lowers social welfare.

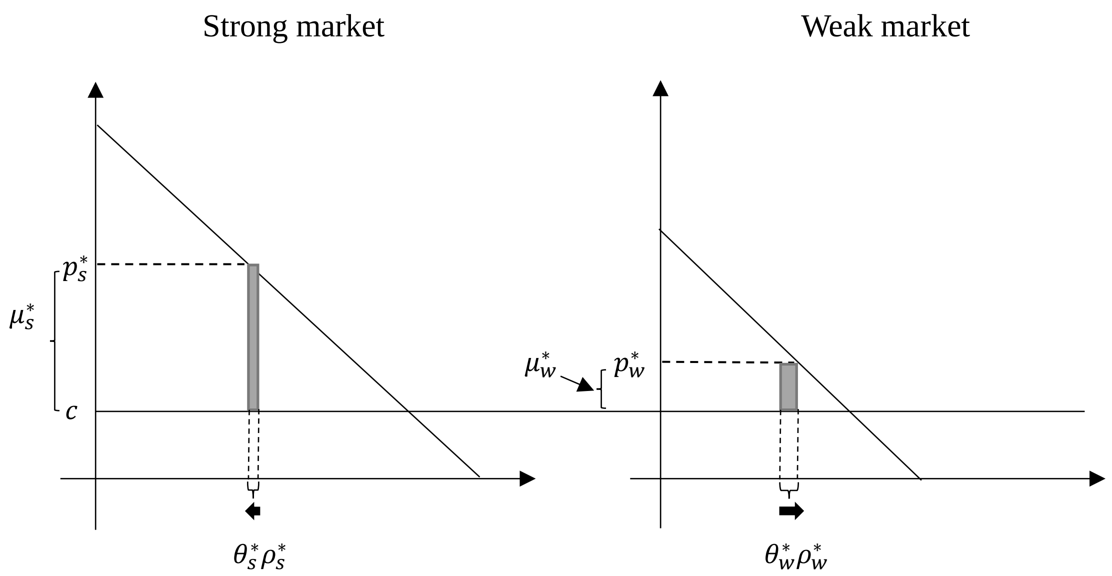

Figure 4. Welfare predictions in terms of sufficient statistics

Lastly, I discuss the relationship between the above sufficient condition provided by Adachi (2022) and the necessary and sufficient condition shown by Adachi and Matsushima (2014) under the assumption of linear demand. To do so, I employ the

[12] Behind this result, the monotonicity condition called the increasing ratio condition (IRC)---an oligopoly version of Aguirre, Cowan, and Vickers' (2010) condition with the same name---must hold. Adachi (2022) verifies that this condition is satisfied for the three classes of market demand in consideration: linear, CES (constant elasticity of substitution), and multinomial logit.

following linear demand functions in each market $m = s, w$:

$$\begin{cases} x_{1m}(p_{1m}, p_{2m}) = \dfrac{1}{(1-\delta_m^2)\beta_m}[\omega_m(1-\delta_m) - p_{1m} + \delta_m p_{2m}] \\ x_{2m}(p_{1m}, p_{2m}) = \dfrac{1}{(1-\delta_m^2)\beta_m}[\omega_m(1-\delta_m) - p_{2m} + \delta_m p_{1m}], \end{cases}$$

which are derived from the representative consumer's (gross) utility (see Footnote 2):

$$U_m(x_{1m}, x_{2m}) = \omega_m \cdot (x_{1m} + x_{2m}) - \frac{1}{2}(\beta_m x_{1m}^2 + 2\gamma_m x_{1m}x_{2m} + \beta_m x_{2m}^2),$$

where $\omega_m > 0$ captures the *market size*, and $\delta_m \equiv \gamma_m/\beta_m \in [0,1)$ is the degree of *brand substitution* between firms. In this case, the conduct parameter is a constant, $\theta_m = 1 - \delta_m$: a fiercer level of competition is associated with a greater value of δ_m. Below, I employ an innocuous normalization, $\beta_s = \beta_w = 1$.

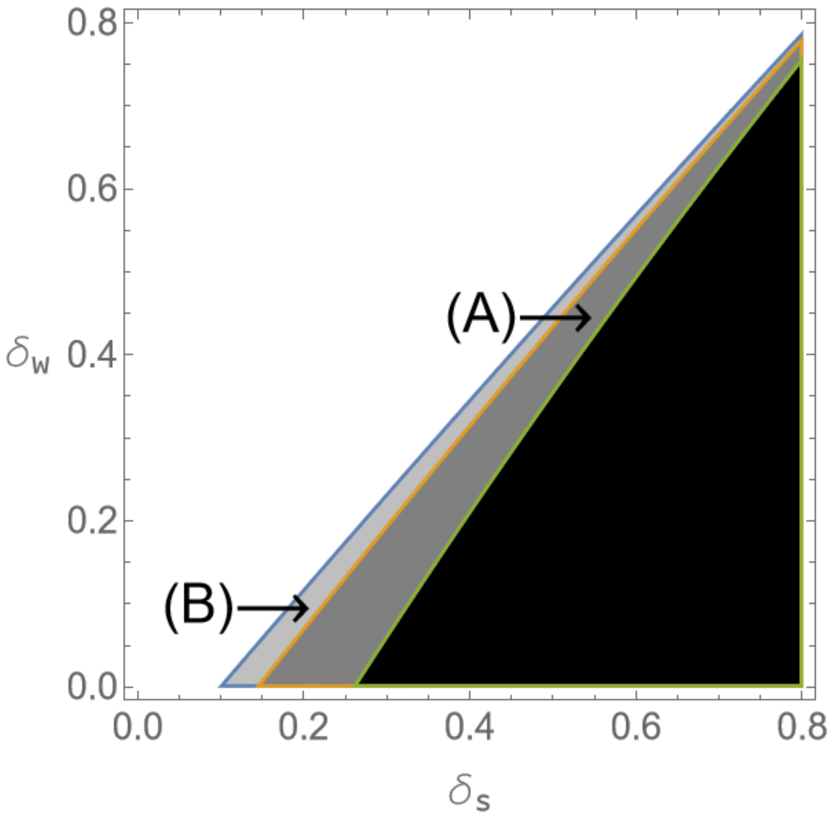

Figure 5. The region for ΔW (A+B) and the region for $\mu_s^*\theta_s^*\rho_s^* < \mu_w^*\theta_w^*\rho_w^*$ (A)

Under the assumption of common marginal costs ($c_s = c_w$), Adachi and Matsushima's (2014) Lemma 1 shows that the welfare change from uniform pricing to price discrimination, ΔW, is given by

$$\Delta W = -\frac{p_s^* - \bar{p}}{1+\delta_s}(p_s^* + \bar{p}) + \frac{\bar{p} - p_w^*}{1+\delta_w}(\bar{p} + p_w^*).$$

Assuming that $\omega_s = 1$, $\omega_w = 0.85$, and $c_s = c_w = 0$ as in Adachi and Matsushima's (2014) Figure 4, Figure 5 below depicts the region of (δ_s, δ_w) where $\Delta W > 0$. First, the black area is excluded to ensure that market s is indeed strong (i.e., $p_s^* \geq p_w^*$). Then, area (A) corresponds to $\mu_s^* \theta_s^* \rho_s^* < \mu_w^* \theta_w^* \rho_w^*$, and the combined areas (A) and (B) is where $\Delta W > 0$ holds. As observed, the area given by our sufficient condition almost covers the area for $\Delta W > 0$. This figure clearly shows that substation in the *strong* market must be sufficiently higher. This is one important prediction from my analysis, and I have now been able to update Hendel and Nevo's (2013) assessment of oligopolistic third-degree price discrimination.

6. Conclusion

This survey has briefly explained the state of art in the theoretical literature of oligopolistic third-degree price discrimination. First, I focus on two well-known results in the literature: (i) an increase in aggregate output is necessary for price discrimination to increase social welfare, and (ii) price discrimination leads to a Pareto welfare improvement if one of the two markets is not served under uniform pricing. Interestingly, cross- and inter-market consumption externality is key to reverse the existing results. This finding may suggest that our understanding of imperfect competitive behavior would be enriched by considering externality as evidenced in the economics of platforms (Belleflamme and Peitz 2021) that has only a vicennial tradition.

Second, I have stressed that there is no great divide between monopoly and oligopoly because both situations can be treated systematically in terms of the conduct parameter. As long as firm symmetry is imposed, intuition based on graphical expositions remains valid for both cases. This argument readily carries over to the case of heterogenous firms, although actual expressions are more involved (see Adachi 2022). This should be particularly helpful for implementing an empirical study using a theoretical model (see, e.g., Adams and Williams 2019). In the next ten years to come, I also expect to see more advances in theoretical research on third-degree price discrimination in imperfectly competitive intermediate markets: see Gaudin and Lestage (2022) for a recent attempt.

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