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Pass-Through, Quality Adjustment, and Imperfect Competition*

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Abstract

How does a change in marginal costs affect the final consumer price in imperfectly competitive markets, where price-setting firms can also adjust product quality? In this paper, we study cost pass-through in such an environment. For both symmetric and heterogeneous firms, we show that pass-through for price and quality can be derived in terms of sufficient statistics that do not depend on any particular demand specification—namely, the first- and second-order elasticities of market demand, the Lerner index of market power, and equilibrium prices and quality choices. In addition, we obtain explicit pass-through formulas under firm symmetry. We then argue that under multinomial and random-coefficient logit demand systems, firms may respond to an increase in operational marginal costs by *both* lowering prices *and* reducing product quality when the number of symmetric firms is sufficiently small. Overall, our numerical analysis suggests that the random-coefficient logit model is more flexible than the multinomial logit model in that it allows price pass-through to exceed one, which is not possible under the multinomial logit. In addition, quality pass-through can be positive under random-coefficient logit demand.

Keywords: Endogenous quality; Pass-through; Sufficient statistics; Oligopoly.

JEL classification: D43; H22; L11; L13.

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1 Introduction

When firms in imperfectly competitive markets can endogenously choose product quality, how does a change in marginal costs shape the eventual prices charged to consumers? To answer this question, this paper provides a framework that considers cost pass-through not only to product prices but also to product quality under imperfect competition.[1] Specifically, we derive pass-through formulas in terms of sufficient statistics that do not depend on demand specification: namely, first- and second-order elasticities of market demand, cost parameters, as well as equilibrium prices and quality choices. We then argue, by considering both multinomial and random-coefficient logit demand models, that firms may respond to an increase in marginal costs by *lowering both* price and quality when the number of symmetric firms is sufficiently small. Overall, our numerical analysis suggests that the random-coefficient logit model is more flexible than the multinomial logit model in that it allows price pass-through to exceed one, which is not possible under the multinomial logit. In addition, quality pass-through can be positive under random-coefficient logit demand.

Little attention has been paid to the endogenous nature of product quality in the existing literature on pass-through under imperfect competition (e.g, Delipalla and Keen 1992; Anderson, de Palma, and Kreider 2001a, b; Weyl and Fabinger 2013; Häkner and Herzing 2016; Adachi and Fabinger 2022). To the best of our knowledge, Cremer and Thisse (1994) show that the cost pass-through to price can be negative in the context of ad valorem taxation when firms are allowed to adjust product quality endogenously. Negative pass-through has also been theoretically investigated and empirically documented in the context of cross-border tariffs—which constitute a form of ad valorem taxation (e.g., Ludema and Yu 2016; Hayakawa, Ito, and Mukunoki 2022). In line with these studies, Doi and Shinkai (2025) examine a more general class of demand function to analyze in what circumstances negative pass-through can arise.

Specifically, Doi and Shinkai (2025) study endogenous quality in a price-setting model of *monopoly*. They show that cost pass-through to prices can be *negative* (at least under linear demand and multinomial logit demand) and that cost pass-through to product quality can be positive (at least under constant-elasticity demand; this never happens under linear or logit demand). While Doi and Shinkai (2025) consider a broader class of demand systems than we do, their analysis is restricted to monopoly. In this paper, we extend their framework to *oligopoly* with multinomial or random-coefficient logit demand. In particular, we derive pass-through formulas in the spirit of Adachi (2023), who emphasizes the role of sufficient statistics such as first- and second-order elasticities.[2] And then, we show that an increase in marginal costs can *lower both* equilibrium prices and equilibrium quality.[3]

We believe that our theory provides a microfoundation for firm behavior behind what is called *shrinkflation* or package-size reduction—a widely observed phenomenon in which

[1] As in this paper, Vives (2008) and Motta and Tarantino (2021), among others, treat with research and development activities as a short-run decision.

[2] In a similar vein, Kroft, Laliberté Leal-Vizcaíno, and Notowidigdo (2023) study the welfare consequences of tax salience under imperfect competition.

[3] See also, e.g., Chu (2010), Fan (2013), Crawford, Shcherbakov, and Shum (2019), Doi and Ohashi (2019), Fan and Yang (2020), Doi (2022), Barahona, Otero, and Otero (2023), Doi, Kono, and Suzaki (2023), and Barwick, Kwon, and Li (2024) for empirical studies that consider endogenous quality.

manufacturers reduce package size with almost no change in unit prices in response to rising marginal costs (Dekimpe and Van Heerde 2023). Although this practice could simply be interpreted as an obscure form of a real price increase, it is more accurately described as a deterioration in product quality. While certain aspects of consumer irrationality likely play an important role in this phenomenon (Chalioti and Serfes 2024), our framework, which endogenizes quality adjustment, enables us to focus on firm behavior without the additional complexity and ambiguity that arise from modeling inattentive consumers.

Our view is that shrinkflation is just one form of cost pass-through—one that can be rationalized once firms are allowed to behave more realistically. That is, firms not only choose prices and quantities, but can also adjust product quality even in the short run, as in the theoretical model developed below. Consistent with Doi and Shinkai's (2025) analysis of monopoly, we find that imperfectly competitive firms may optimally reduce quality in response to higher marginal costs. We interpret this as a rational explanation for shrinkflation, a phenomenon that still awaits thorough empirical investigation (Lee 2024; Janssen and Kasinger 2025).

In a closely related paper, Gaudin (2025) studies firms' quality choices under imperfect competition and elucidates the mechanism through which either under- or over-provision of quality arises. Under-supply refers to situations in which additional quality improvements benefit high–willingness-to-pay consumers more than low–willingness-to-pay consumers. Since the seminal contributions of Spence (1975) and Sheshinski (1976), it has been well understood that these distortions arise because a monopolist's marginal decision fails to account for the average quality experienced by inframarginal consumers. Under oligopoly, strategic interactions further compound this distortion. Gaudin (2025) provides a characterization based on indices similar to diversion ratios, which capture how a firm's price and quality changes spill over to its rivals' demand.

In contrast, our study focuses on objects more directly tied to demand primitives—namely, the first- and second-order price and quality elasticities of demand. Moreover, we adopt a cost structure that differs from Gaudin (2025) in order to flexibly capture industry heterogeneity, in the spirit of Berry and Waldfogel's (2010) empirical analysis of product quality. Most importantly, we examine the properties of both price and quality pass-through. In particular, we focus on when price pass-through is negative (i.e., when an increase in the marginal cost of production *lowers* the product price) and when it is positive—an issue not addressed in Gaudin's (2025) analysis. In this sense, Gaudin's (2025) analysis and ours are complementary.

Finally, this paper is also related to the recent literature that highlights the role of demand curvature in evaluating the effects of policy and other external changes on final prices, and thus consumer and producer surplus. In a series of papers, Miravete, Seim, and Thurk (2023, 2024, 2025) examine how demand curvature affects welfare evaluation in the workhorse model of demand estimation. In line with the popularity in empirical analysis, we study the logit-class of market demand after deriving pass-through formulas based on sufficient statistics. It is important to note that demand curvature plays an important role in pass-through because, curvature still matters, as shown by Kang and Vasserman (2025), even when a policy or external change is not infinitesimal.[4] Even though endogenizing product quality

[4]Kang and Vasserman's (2025) analysis is generalized by von Beringe and Whitmeyer (2025) to allow

complicates the relevant measures of curvature (see Figure 1 below), we emphasize that it is still possible to derive a pass-through characterization in terms of sufficient statistics.

The rest of the paper is organized as follows. Section 2 presents our base model, introducing relevant measures for elasticy and curvature. Section 3 then provides the sufficient-statistics formulas under firm symmetry, followed by Section 4 where a numerical analysis is conducted using the multinomial logit demand. Section 5 then introduce the random-coefficient logit demand and allows firm heterogeneity. Finally, Section 6 concludes the paper.

2 Model

Suppose that J single-product firms produce differentiated goods. Each firm $j = 1, 2, ..., J$ simultaneously chooses its price that consumers face, $p_j \geq 0$, as well as its (one-dimensional) product quality, $x_j \geq 0$.[5,6] Specifically, firm j's profit maximization problem is formulated by:

$$\max_{p_j, x_j} \pi_j = [p_j - (c_j + k_j^V x_j)]q_j(\mathbf{p}, \mathbf{x}) - k_j^F x_j.$$

Here, the demand for firm j, $q_j(\mathbf{p}, \mathbf{x})$ is given as a function of all price and quality variables $\mathbf{p} = (p_1, p_2, ..., p_J)$ and $\mathbf{x} = (x_1, x_2, ..., x_J)$, where it is natural to assume that $\partial q_j(\cdot)/\partial p_j < 0$ and $\partial q_j(\cdot)/\partial x_j > 0$ for each j. Note that the quality variable is measured by the same pecuniary value as price. Each firm j's margincal cost of production is given by a constant, $c_j \geq 0$.

Note that we introduce two coefficients $k_j^V \geq 0$ and $k_j^F \gneq 0$ to express the part that x_j increases the marginal cost per-unit, and the part of the cost that should be deducted irrespective of the amount of production. In the analysis below, we let $c_j = c$, $k_j^V = k^V$, and $k_j^F = k^F$ for any $j = 1, 2, ..., J$ to focus on symmetric equilibrium except for when firm heterogeneiy is considered in numerical analysis in Section 5. In effect, increasing quality by one unit raises the marginal cost by k^V and the fixed cost by k^F. This formulation follows the literature on quality choice (e.g., Berry and Waldfogel, 2010). To simplify the analysis, we impose a linear specification.[7,8]

The difference between Gaudin's (2025) cost structure and ours is that we consider not only the variable part of quality adjustment but also its fixed nature. More specifically, price-setting firm j's profit function in Gaudin's (2025) formulation is written by $\pi_j = [p_j - c(x_j)]q_j(\mathbf{p}, \mathbf{x})$, where $c(x_j)$ is allowed to be non-constant. In contrast, while we assume

for imperfect competition.

[5]We use j for firm index because later in Section 5 we use i for consumer index when we allow consumer heterogeneity.

[6]This timing assumption can be interpreted as the situation where each firm decides on the quality without knowing the quality of other firms' products before deciding on the price.

[7]This linearity assumption can make the derivations transparent. Specifically, with linear costs, only the quantity response and the change in sensitivity ratio arise from a price change. Under nonlinear costs, however, there is an additional effect through the change in the marginal cost of quality.

[8]Naturally, we exclude the case of both $k_j^V = 0$ and $k_j^F = 0$. It is possible to consider the case of $k_j^V < 0$ as in Doi and Shinkai (2025): this enables one to study process innovation as in Gaudin (2025). In this paper, however, we do not consider the case of $k_j^V < 0$ because this paper focuses on quality adjustment.

constant cost for the variable part, we also allow the fixed cost to depend on quality as seen in the profit maximization problem above. In this way, as in Berry and Waldfogel (2010), we are able to use both k^V and k^F to relate to industry characteristics.[9] In addition, while Gaudin (2025) assumes firm symmetry and thus the cost function $c(x_j)$ is common for all firms, we consider firm heterogeneity in terms of production in the numerical analysis below (in Section 5 after establishing the pass-through characterization under firm symmetry).

2.1 Equilibrium under Symmetric Oligopoly

For the following analysis, we further assume that any firm's demand is symmetric. Under symmetric behavior, each firm's demand is denoted by $q(p, x) \equiv q_j(p, ...p, x, ..., x)$.

2.1.1 Price

Under this assumption, the following identity holds:

$$\frac{\partial q}{\partial p}(p, x) = \frac{\partial q_j}{\partial p_j} + \sum_{j' \neq j} \frac{\partial q_j}{\partial p_{j'}}$$

$$\Rightarrow -\frac{\partial q_j}{\partial p_j} = -\frac{\partial q}{\partial p} + (J-1)\frac{\partial q_j}{\partial p_{j'}},$$

which implies the Holmes (1989) identity:

$$\epsilon^p(p, x) = \epsilon^{p,I}(p, x) + (J-1)\epsilon^{p'}(p, x),$$

where

$$\epsilon^p(p, x) \equiv -\frac{\partial q_j}{\partial p_j}(p, ..., p, x, ..., x) \cdot \frac{p}{q(p, x)} > 0$$

is the *own-price elasticity of the firm's demand*, and

$$\begin{cases} \epsilon^{p,I}(p, x) \equiv -\dfrac{\partial q}{\partial p} \cdot \dfrac{p}{q(p, x)} > 0 \\[2ex] \epsilon^{p'}(p, x) \equiv \dfrac{\partial q_j}{\partial p_{j'}} \cdot \dfrac{p}{q(p, x)} > 0 \end{cases}$$

are the *price elasticity of the industry's demand* and the *cross-price elasticity of the firm's demand*, respectively.

Then, the first-order condition with respect to p_j is expressed in terms of sufficient statistics:

$$L(p, x; c, k^V)\epsilon^p(p, x) - 1 = 0 \tag{1}$$

[9] Berry and Waldfogel (2010) use the restaurant industry as an example of an industry with a higher value of k^V and the daily newspaper industry as an example of an industry with a higher value of k^F. In an empirical implementation for the airline industry, Doi (2022) considers the case of $k_j^V = 0$.

because

$$q_j + [p_j - (c + k^V x_j)]\frac{\partial q_j}{\partial p_j} = 0 \tag{2}$$

$$\Leftrightarrow \frac{p_j - (c + k^V x_j)}{p_j} \cdot \left(-\frac{\partial q_j}{\partial p_j}\frac{p_j}{q_j}\right) = 1,$$

where we define the*Lerner index* by

$$L(p, x; c, k^V) \equiv \frac{p - (c + k^V x)}{p},$$

implying that $\partial L/\partial p = (c + k^V x)/p^2$ and $\partial L/\partial x = -k^V/p$.

elasticity. Hence, $\partial \epsilon^p/\partial x < 0$.

2.1.2 Quality

Next, we define the *own-quality elasticity of the firm's demand* by:

$$\epsilon^x(p, x) \equiv \frac{\partial q_j}{\partial x_j}(p, ..., p, x, ..., x) \cdot \frac{x}{q(p, x)} > 0.$$

Similarly, the *quality elasticity of the industry's demand* and the *cross-quality elasticity of the firm's demand* are defined by

$$\begin{cases} \epsilon^{x,I}(p, x) \equiv \dfrac{\partial q}{\partial x} \cdot \dfrac{x}{q(p, x)} > 0 \\ \\ \epsilon^{x'}(p, x) \equiv -\dfrac{\partial q_j}{\partial x_{j'}} \cdot \dfrac{x}{q(p, x)} > 0 \end{cases}$$

respectively. Then, a similar identity

$$\epsilon^x(p, x) = \epsilon^{x,I}(p, x) + (J - 1)\epsilon^{x'}(p, x)$$

holds because

$$\frac{\partial q}{\partial x} = \frac{\partial q_j}{\partial x_j} - (J - 1)\left(-\frac{\partial q_j}{\partial x_{j'}}\right).$$

Then, the first-order condition with respect to x_j is expressed in terms of sufficient statistics:

$$L(p, x; c, k^V)\epsilon^x(p, x)\frac{p}{x} - k^V - \frac{k^F}{q(p, x)} = 0 \tag{3}$$

because

$$[p_j - (c + k^V x_j)]\frac{\partial q_j}{\partial x_j} - k^V q_j - k^F = 0 \tag{4}$$

$$\Leftrightarrow \frac{p_j - (c + k^V x_j)}{p_j} \cdot \left(\frac{\partial q_j}{\partial x_j}\frac{x_j}{q_j}\right)\frac{p_j}{x_j} = k^V + \frac{k^F}{q_j}.$$

Appendix A verifies the second-order conditions and argue how they are simplified under the symmetry assumption.

2.2 Interpretation

Following Bourreau, Jullien, and Lefouili (2026) and Doi and Shinkai (2025), we denote the adjusting quality for a givel level of price p by $\widetilde{x}(p)$:

$$p - (c + k^V \widetilde{x}(p)) \frac{\partial q_j}{\partial x_j} - k^V q(p, \widetilde{x}(p)) - k^F = 0$$

From Equations. (2) and (4), the marginal change in profit with respect to *price* is given by

$$\triangle\ \pi^P(p) \equiv q(p, \widetilde{x}(p)) - r^P(p)[k^V q(p, \widetilde{x}(p)) + k^F]$$

where

$$r^P(p) \equiv \frac{-\frac{\partial q_j}{\partial p_j}(p, \widetilde{x}(p))}{\frac{\partial q_j}{\partial x_j}(p, \widetilde{x}(p))} = \frac{\epsilon^p(p, \widetilde{x}(p))/p}{\epsilon^x(p, \widetilde{x}(p))/\widetilde{x}(p)} > 0.$$

First, a marginal increase in price contributes positively to profit by the amount of q. This effect appears in the first term of $\triangle\ \pi^P(p)$. At the same time, the firms incur an increase in cost of production as well. This effect of reduction is captured by the second term, $r^P(p)(k^V q + k^F)$.

Interestingly, the magnitude of this cost effect is adjusted by $r^P(p)$. In Equation (2), this part corresponds to the profit margin loss adjusted by the output reduction, $[p_j - (c + k^V x_j)](\partial q_j / \partial p_j)$. Equation (4) indicates that the price-cost margin, $p_j - (c + k^V x_j)$, is also equal to $(k^V q + k^F)/(\partial q_j / \partial x_j)$. This is because the firms can also acquire an additional margin by improving the product quality with an additional cost, $k^V q + k^F$. This has an effect through a change in output, $\partial q_j / \partial x_j$. Thus, the effect of the cost increase on the profit margin is discounted/magnified with $\partial q_j / \partial x_j$.

In total, the positive contributuon by the change in p, q must be balanced with the cost of quality change adjusted by $(\partial q_j / \partial p_j)/(\partial q_j / \partial x_j)$. Thus, $r^P(p)$ measures the relative sensitivity of output in response to a change in price in terms of that in quality: a large value of $r^P(p)$ means that a weaker effect of quality improvement on profit, that is, its larger cost.

The quantity effect captures a change in demanded quantity. This is primarily related to the (first-order) elasticities of demand. In contrast, the sensitivity ratio effect results from the (second-order) curvatures of demand.

Similarly, the marginal change in profit with respect to *quality* is written as

$$\triangle\ \pi^X(x) \equiv \left(r^X(x) - k^V\right) q(\widetilde{p}(x), x) - k^F$$

where $\widetilde{p}(x)$ solves Equation (2) for a given x, and

$$r^X(x) \equiv \frac{\frac{\partial q_j}{\partial x_j}(\widetilde{p}(x), x)}{-\frac{\partial q_j}{\partial p_j}(\widetilde{p}(x), x)} = \frac{\epsilon^x(\widetilde{p}(x), x)/x}{\epsilon^p(\widetilde{p}(x), x)/\widetilde{p}(x)} > 0$$

is now the reverse concept of $r^P(p)$: the relative sensitivity of output in response to a change in quality in terms of that in price.

First, a marginal increase in quality contributes positively to profit through the profit margin $p_j - (c + k^V x_j)$ through a change in output that is captured by the term $\partial q_j / \partial x_j$.

The associated marginal cost of quality improvement is the sum of the varying and fixed parts: $k^V q + k^F$. Now, the profit margin is by $q_j/(-\partial q_j/\partial p_j)$ from Equation (2). Thus, the marginal gain is expressed by $[q_j/(-\partial q_j/\partial p_j)] \times (\partial q_j/\partial p_j)$. This means the per-unit marginal gain is $[1/(-\partial q_j/\partial p_j)] \times (\partial q_j/\partial p_j)$ that can be combined with the varying part of cost, k^V. Hence, the marginal change in profit is $\left(r^X(x) - k^V\right) q$, subtracted by the fixed part, k^F.

2.3 Curvatures

To simplify the notations, we define the following variables:

$$D_{pp}(p,x) \equiv \frac{\partial^2 q_j}{\partial p_j^2}(p,...p,x,...,x) < 0,\ D_{pp'}(p,x) \equiv \frac{\partial^2 q_j}{\partial p_j \partial p_{j'}}(p,...p,x,...,x),$$

$$D_{xx}(p,x) \equiv \frac{\partial^2 q_j}{\partial x_j^2}(p,...p,x,...,x) < 0,\ D_{xx'}(p,x) \equiv \frac{\partial^2 q_j}{\partial x_j \partial x_{j'}}(p,...p,x,...,x),$$

for own and cross second-order derivatives within price and quality, respectively, and

$$D_{xp}(p,x) \equiv \frac{\partial^2 q_j}{\partial x_j \partial p_j}(p,...p,x,...,x) = \frac{\partial^2 q_j}{\partial p_j \partial x_j}(p,...p,x,...,x) \equiv D_{px}(p,x)$$

as own second-order derivatives across p and x, and

$$D_{xp'}(p,x) \equiv \frac{\partial^2 q_j}{\partial x_j \partial p_{j'}}(p,...p,x,...,x),\ D_{px'}(p,x) \equiv \frac{\partial^2 q_j}{\partial p_j \partial x_{j'}}(p,...p,x,...,x)$$

as cross second-order derivatives across p and x.

We firstly define the *price curvature of the firm's direct demand* by

$$\begin{aligned} \alpha^{pp}(p,x) &\equiv -\frac{\partial^2 q_j}{\partial p_j^2}(p,...p,x,...,x) \cdot \frac{p}{\frac{\partial q_j}{\partial p_j}(p,...,p,x,...,x)} \\ &= \underbrace{D_{pp}(p,x)}_{<0} \cdot \underbrace{\left(\frac{p}{q(p,x)} \cdot \frac{1}{\epsilon^p(p,x)/p}\right)}_{>0} < 0 \end{aligned}$$

and the *elasticity of the cross-price effect of the firm's direct demand* by

$$\begin{aligned} \alpha^{pp'}(p,x) &\equiv -\frac{\partial^2 q_j}{\partial p_j \partial p_{j'}}(p,...p,x,...,x) \cdot \frac{p}{\frac{\partial q_j}{\partial p_j}(p,...,p,x,...,x)} \\ &= D_{pp'}(p,x) \cdot \underbrace{\left(\frac{p}{q(p,x)} \cdot \frac{1}{\epsilon^p(p,x)/p}\right)}_{>0}. \end{aligned}$$

Similarly, we also define the *quality curvature of the firm's direct demand* by

$$\begin{aligned} \alpha^{xx}(p,x) &\equiv \frac{\partial^2 q_j}{\partial x_j^2}(p,...p,x,...,x) \cdot \frac{x}{\frac{\partial q_j}{\partial x_j}(p,...,p,x,...,x)} \\ &= \underbrace{D_{xx}(p,x)}_{<0} \cdot \underbrace{\left(\frac{x}{q(p,x)} \cdot \frac{1}{\epsilon^x(p,x)/x}\right)}_{>0} < 0 \end{aligned}$$

and the *elasticity of the cross-quality effect of the firm's direct demand* by

$$\begin{aligned}\alpha^{xx'}(p,x) &\equiv \frac{\partial^2 q_j}{\partial x_j \partial x_{j'}}(p,...p,x,...,x)\cdot\frac{x}{\frac{\partial q_j}{\partial x_j}(p,...,p,x,...,x)} \\ &= D_{xx'}(p,x)\cdot\underbrace{\left(\frac{x}{q(p,x)}\cdot\frac{1}{\epsilon^x(p,x)/x}\right)}_{>0}.\end{aligned}$$

Additionally, we need to consider the cross relationship between p and x within the same product and across different products. First, the *price elasticity of the within-product effects between price and quality* and *the price elasticity of the cross-product effect between price and quality* are defined by

$$\begin{aligned}\alpha^{xp}(p,x) &\equiv \frac{\partial^2 q_j}{\partial x_j \partial p_j}(p,...p,x,...,x)\cdot\frac{p}{\frac{\partial q_j}{\partial x_j}(p,...,p,x,...,x)} \\ &= D_{xp}(p,x)\cdot\underbrace{\left(\frac{p}{q(p,x)}\cdot\frac{1}{\epsilon^x(p,x)/x}\right)}_{>0}\end{aligned}$$

and

$$\begin{aligned}\alpha^{xp'}(p,x) &\equiv \frac{\partial^2 q_j}{\partial x_j \partial p_{j'}}(p,...p,x,...,x)\cdot\frac{p}{\frac{\partial q_j}{\partial x_j}(p,...,p,x,...,x)} \\ &= D_{xp'}(p,x)\cdot\underbrace{\left(\frac{p}{q(p,x)}\cdot\frac{1}{\epsilon^x(p,x)/x}\right)}_{>0},\end{aligned}$$

respectively. Second, two quality elasticities are similarly defined:

$$\begin{aligned}\alpha^{px}(p,x) &\equiv \frac{\partial^2 q_j}{\partial p_j \partial x_j}(p,...p,x,...,x)\cdot\frac{x}{\frac{\partial q_j}{\partial p_j}(p,...,p,x,...,x)} \\ &= -D_{px}(p,x)\cdot\underbrace{\left(\frac{x}{q(p,x)}\cdot\frac{1}{\epsilon^p(p,x)/p}\right)}_{>0}\end{aligned}$$

as the *quality elasticity of the within-product effect between price and quality*, and

$$\begin{aligned}\alpha^{px'}(p,x) &\equiv \frac{\partial^2 q_j}{\partial p_j \partial x_{j'}}(p,...p,x,...,x)\cdot\frac{x}{\frac{\partial q_j}{\partial p_j}(p,...,p,x,...,x)} \\ &= -D_{px'}(p,x)\cdot\underbrace{\left(\frac{x}{q(p,x)}\cdot\frac{1}{\epsilon^p(p,x)/p}\right)}_{>0}\end{aligned}$$

as the *quality elasticity of the cross-product effect between price and quality.*

Figure 1 visualizes the relationships between all these curvature measures.

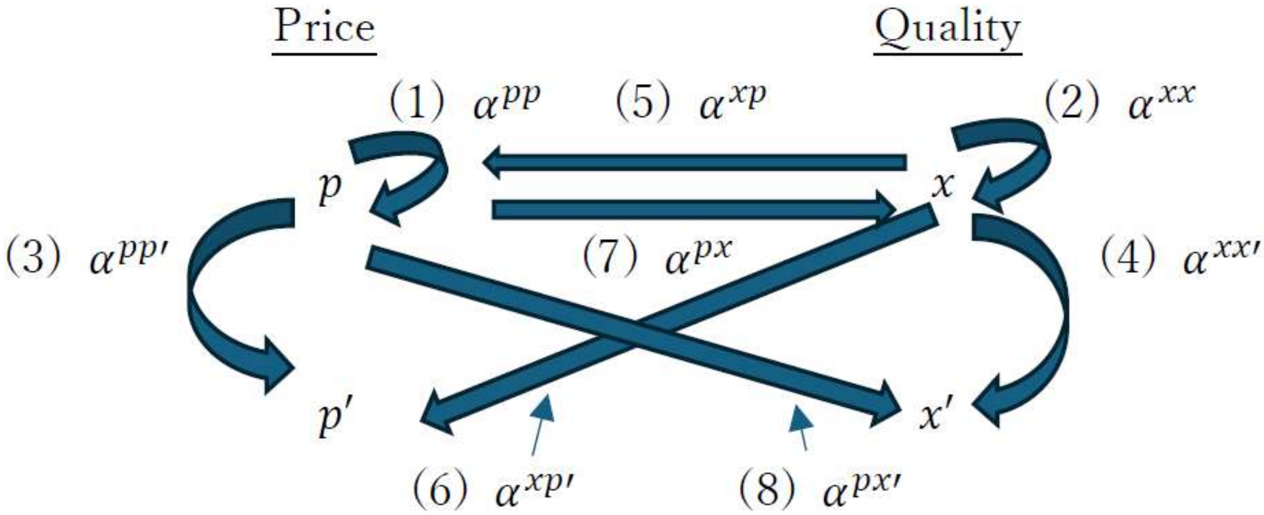

Figure 1: Curvature Relations.

3 Sufficient-Statistics Formulas for Pass-Through with Quality Adjustment

In this section, we provide the sufficient-statistics expressions for pass-through of marginal costs of production, c, with no functional forms for demand being specified. As the following propositions show, these pass-through formulas are expressed in terms of sums (and differences) and multiplications (and ratios) of the elasticities (ϵ's) and curvatures (α's) as well as the endogenous variables (price and quality) and parameters (preference and technology) .

Following Weyl and and Fabinger (2013) and Adachi and Fabinger (2022), among others, we define pass-through of production costs to price and to quantity by $\rho^P \equiv \partial p/\partial c$ and $\rho^X \equiv \partial x/\partial c$, respectively.[10] This measures the change in p or x resulting from a one-unit increase in c. Then, the following proposition is obtained.

Proposition 1. *Define*

$$\det \;= \underbrace{\left(\frac{1}{px}\right)\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)}_{>0} \cdot (H_1 - H_2),$$

where

$$H_1 \equiv \left\{p - \left[(\alpha^{pp} - \epsilon^p) + (J-1)(\alpha^{pp'} + \epsilon^{p'})\right] Lp\right\}$$
$$\times \left\{[(\alpha^{xx} - \epsilon^x) + (J-1)(\alpha^{xx'} + \epsilon^{x'})]Lp - k^V x + \left(\frac{k^F x}{q}\right)\left[1 - (J-1)\left(\frac{\epsilon^{x'}}{\epsilon^x}\right)\right]\right\}$$

[10] Notice that the marginal cost of production could be inclusive of a specific tax $t \geq 0$ so that the effective marginal cost is $c + t$, meaning that cost pass-through here is equivalent to specific tax pass-through. One could also include an ad valerem (value-added) tax as in Adachi and Fabinger (2022). We leave this issue to future research.

and

$$H_2 \equiv \left\{p + \left[(\alpha^{xp} + \epsilon^p) + (J-1)(\alpha^{xp'} - \epsilon^{p'})\right] Lp - \left(\frac{k^F x}{q}\right)\left(\frac{\epsilon^p}{\epsilon^x}\right)\left[1 - (J-1)\left(\frac{\epsilon^{p'}}{\epsilon^p}\right)\right]\right\}$$
$$\times \left\{\left[(\alpha^{px} - \epsilon^x) + (J-1)(\alpha^{px'} + \epsilon^{x'})\right] Lp - k^V x\right\},$$

and it is reasonably assumed to be negative (i.e., $H_1 < H_2$). Then, the cost pass-throughs to price and quality are expressed by

$$\rho^P = \underbrace{\left\{\left(\frac{-1}{\det}\right)\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)\left(\frac{1}{x}\right)\right\}}_{>0}$$
$$\times \underbrace{\left\{\left[(\alpha^{px} - \alpha^{xx}) + (J-1)(\alpha^{px'} - \alpha^{xx'})\right] Lp - \left(\frac{k^F x}{q}\right)\left[1 - (J-1)\left(\frac{\epsilon^{x'}}{\epsilon^x}\right)\right]\right\}}_{\equiv G_1}$$

and

$$\rho^X = \underbrace{\left\{\left(\frac{-1}{\det}\right)\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)\left(\frac{1}{p}\right)\right\}}_{>0}$$
$$\times \underbrace{\left\{\left[(\alpha^{xp} + \alpha^{pp}) + (J-1)(\alpha^{xp'} + \alpha^{pp'})\right] Lp - \left(\frac{k^F x}{q}\right)\left[\left(\frac{\epsilon^p}{\epsilon^x}\right) - (J-1)\left(\frac{\epsilon^{p'}}{\epsilon^x}\right)\right]\right\}}_{\equiv G_2},$$

respectively.

Proof. See Appendix B. □

It is observed that

$$\rho^P \gtrless 0 \Leftrightarrow L \cdot (pq)[(\alpha^{px} - \alpha^{xx}) + (J-1)(\alpha^{px'} - \alpha^{xx'})] \gtrless (k^F x)\left[1 - (J-1)\left(\frac{\epsilon^{x'}}{\epsilon^x}\right)\right]$$

and

$$\rho^X \gtrless 0 \Leftrightarrow L \cdot (pq)[(\alpha^{xp} + \alpha^{pp}) + (J-1)(\alpha^{xp'} + \alpha^{pp'})] \gtrless (k^F x)\left[\left(\frac{\epsilon^p}{\epsilon^x}\right) - (J-1)\left(\frac{\epsilon^{p'}}{\epsilon^x}\right)\right].$$

In the case of monopoly ($J = 1$), it is simple to see that $\rho^P \gtrless 0 \Leftrightarrow L \cdot (pq)(\alpha^{px} - \alpha^{xx}) \gtrless k^F x$ and $\rho^X \gtrless 0 \Leftrightarrow L \cdot (pq)(\alpha^{xp} + \alpha^{pp}) \gtrless (\epsilon^p/\epsilon^x)(k^F x)$.

4 Multinomial Logit Demand

In this section, we apply the pass-through formulas obtained above to one of the representative classes of parametric demand: mutlinomial logit demand. In structural demand analysis, the underlying structure is provided by discrete-choice modeling: each consumer's (or household's) probability of purchasing a product—typically represented by a logit function—is aggregated across all consumers in the market to obtain the product's market share. Hence, the logit-type market demand has played an important role in the structural analysis of market interactions with product differentiation since Berry (1994) and Berry, Levinsohn, and Pakes (1995).

In light of its popularity, this section considers multinomial logit demand under the assumption of symmetric firms. This allows us to apply the pass-through formulas derived in the previous section to the multinomial logit case. Note that when we consider firm heterogeneity in the next section, we instead solve for ρ_j^P and ρ_j^X for each $j = 1, 2, .., J$ directly from the first-order conditions for profit maximization.

4.1 Formulation

Below, we analyze the effects of a change in the number of firms, J, on ρ^P and ρ^X.[11] We mainly focus on changes in the number of firms, J, because they are related to entry and consolidation.

Suppose that each firm j faces the following share/demand function:

$$q_j(p_j, \mathbf{p}_{-j}, x_j, \mathbf{x}_{-j}; \omega, \beta, \gamma, \delta) = \frac{\exp(\omega - \beta p_j + \gamma x_j^\delta)}{1 + \sum_{k=1}^{J} \exp(\omega - \beta p_k + \gamma x_k^\delta)} \in (0, 1),$$

where $\omega > 0$ is now the product-specific utility, $\beta > 0$ is the *price responsiveness* of the representative consumer, $\gamma > 0$ is the (relative) *weight for quality*, and $\delta \in (0, 1)$ is, again, the intensity of preference for quality.[12] Then, under symmetric pricing, each firm's market share is given by

$$q(p, x; J) = \frac{\exp(\omega - \beta p + \gamma x^\delta)}{1 + J \cdot \exp(\omega - \beta p + \gamma x^\delta)}.$$

The symmetric equilibrium (p, x) is, then, numerically obtained by solving the system of

[11] Ritz (2024) provides a related analysis to study the effects of competition on cost pass-through. In particular, Ritz (2024) shows that pass-through may be lower for a higher degree of quantity/price competition if production costs are sufficiently convex because higher marginal costs due to a larger amount of production "put the brakes on" pass-through.

[12] Anderson, de Palma, and Thisse (1987) argue that the (gross) indirect utility of the representative consumer is given by

$$V(\boldsymbol{p}, \boldsymbol{x}) = \frac{\ln \left[\sum_{k=1}^{J} \exp\left(\omega - \beta p_k + \gamma x_k^\delta\right) \right]}{\beta}.$$

This demand form can also be microfounded by the random utility model (see, e.g., Anderson, de Palma, and Thisse 1992, Ch. 2).

equations:

$$\begin{cases} \beta \cdot \left(1 - \dfrac{\exp(\omega - \beta p + \gamma x^{\delta})}{1 + J \cdot \exp(\omega - \beta p + \gamma x^{\delta})}\right)[p - (c + k^V x)] - 1 = 0 \\ (\gamma\delta)\left(1 - \dfrac{\exp(\omega - \beta p + \gamma x^{\delta})}{1 + J \cdot \exp(\omega - \beta p + \gamma x^{\delta})}\right)\left(\dfrac{\exp(\omega - \beta p + \gamma x^{\delta})}{1 + J \cdot \exp(\omega - \beta p + \gamma x^{\delta})}\right) \\ \qquad \times [p - (c + k^V x)]x^{\delta - 1} - \left(k^V \cdot \dfrac{\exp(\omega - \beta p + \gamma x^{\delta})}{1 + J \cdot \exp(\omega - \beta p + \gamma x^{\delta})} + k^F\right) = 0 \end{cases} \tag{5}$$

and Table 1 summarizes the elasticities and curvatures that are used to compute the pass-throughs.[13,14]

The formulas can be simplified a little using the property of elasticities and curvatures of the multinomial logit demand:

$$\begin{aligned} \rho^P &= \left\{\left(\frac{-1}{\det}\right)\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)\left(\frac{1}{x}\right)\right\} \\ &\quad \times \{[\underbrace{(\alpha^{px} - \alpha^{xx})}_{=1-\delta} + (J-1)\underbrace{(\alpha^{px'} - \alpha^{xx'})}_{=0}]Lp - \left(\frac{k^F x}{q}\right)[1 - (J-1)\underbrace{\left(\frac{\epsilon^{x'}}{\epsilon^x}\right)}_{=\frac{q}{1-q}}]\} \\ &= \left\{\left(\frac{-1}{\det}\right)\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)\left(\frac{1}{x}\right)\right\}[\underbrace{(1-\delta)Lp - \frac{k^F x}{q}}_{=(1-\delta-\epsilon^x)Lp + k^V x} + (J-1)\left(\frac{k^F x}{1-q}\right)] \\ &= \left\{\left(\frac{-1}{\det}\right)\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)\left(\frac{1}{x}\right)\right\}[\underbrace{(1-\delta-\epsilon^x)Lp}_{\gtreqless 0} + \underbrace{k^V x + (J-1)\left(\frac{k^F x}{1-q}\right)}_{>0}] \end{aligned}$$

and

$$\begin{aligned} \rho^X &= \left\{\left(\frac{-1}{\det}\right)\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)\left(\frac{1}{p}\right)\right\} \\ &\quad \times \{[\underbrace{(\alpha^{xp} + \alpha^{pp})}_{=0} + (J-1)\underbrace{(\alpha^{xp'} + \alpha^{pp'})}_{=0}]Lp - \left(\frac{k^F x}{q}\right)\underbrace{\left[\left(\frac{\epsilon^p}{\epsilon^x}\right) - (J-1)\left(\frac{\epsilon^{p'}}{\epsilon^x}\right)\right]}_{=\left(\frac{\beta p}{(\gamma\delta)x^{\delta-1}}\right)\left(\frac{1-Jq}{1-q}\right)}\} \\ &= -\left\{\left(\frac{-1}{\det}\right)\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)\left(\frac{1}{p}\right)\right\}\left(\frac{k^F x}{q}\right)\left(\frac{\beta p}{\gamma\delta x^{\delta-1}}\right)\left(\frac{1-Jq}{1-q}\right) < 0. \end{aligned}$$

[13]The elasticities of the industry demand are given by $\epsilon^{p,I}(p,x) = \beta p \cdot [1 - Jq(p,x)]$ and $\epsilon^{x,I}(p,x) = (\gamma\delta)x^{\delta-1} \cdot [1 - Jq(p,x)]$.

[14]See Appendix A for the expression of the second-order condition. Table 3 in Appendix C summurizes the first- and second-order derivatives that are used in obtaining the expressions in Table 1.

Table 1: Elasticities and Curvatures of Multinomial Logit Demand

(i) Elasticities

$\epsilon^p(p,x)$	$\beta p \cdot [1 - q(p,x)]$
$\epsilon^{p'}(p,x)$	$\beta p \cdot q(p,x)$
$\epsilon^x(p,x)$	$(\gamma\delta)x^\delta \cdot [1 - q(p,x)]$
$\epsilon^{x'}(p,x)$	$(\gamma\delta)x^\delta \cdot q(p,x)$

(ii) Curvatures

$\alpha^{pp}(p,x)$	$\beta p \cdot [1 - 2q(p,x)]$
$\alpha^{pp'}(p,x)$	$-\beta p \cdot [1 - 2q(p,x)] \left(\frac{q(p,x)}{1 - q(p,x)} \right)$
$\alpha^{xx}(p,x)$	$-(1-\delta) + (\gamma\delta)x^\delta \cdot [1 - 2q(p,x)]$
$\alpha^{xx'}(p,x)$	$-(\gamma\delta)x^\delta \cdot [1 - 2q(p,x)] \left(\frac{q(p,x)}{1 - q(p,x)} \right)$
$\alpha^{xp}(p,x)$	$-\beta p \cdot [1 - 2q(p,x)]$ $(= -\alpha^{pp})$
$\alpha^{xp'}(p,x)$	$\beta p \cdot [1 - 2q(p,x)] \left(\frac{q(p,x)}{1 - q(p,x)} \right)$ $(= -\alpha^{pp'} = -\alpha^{xp} \cdot [q/(1-q)])$
$\alpha^{px}(p,x)$	$(\gamma\delta)x^\delta \cdot [1 - 2q(p,x)]$ $(= 1 - \delta + \alpha^{xx})$
$\alpha^{px'}(p,x)$	$-(\gamma\delta)x^\delta \cdot [1 - 2q(p,x)] \left(\frac{q(p,x)}{1 - q(p,x)} \right)$ $(= \alpha^{xx'} = -\alpha^{px} \cdot [q/(1-q)])$

4.2 Numerical Analysis

Setting In the following numerical analysis, the supply-side parameters are given by $c = 1$, and $k^V = 1$, and $k^F = 0.5$. The number of firms (J) is set to be $1, 2, 3, 5, 10$ or 20.

For demand parameters, we fix the constant term and the exponent of the quality ($\omega = 0$ and $\delta = 0.5$) and try a variety of combinations of the price and quality coefficients (β, γ).[15] The values of β and γ are set as follows. First, we specify sufficiently wide ranges for β and γ, as explained in detail below. Next, each range is evenly divided into 100 values. Finally, all combinations of these values are examined, resulting in 10,000 (100×100) parameter combinations. For a given set of parameter values, we calculate the equilibrium ρ^P and ρ^X.

The ranges of β and γ are determined as follows. First, we set the initial ranges as $\beta \in [0.8, 2.6]$ and $\gamma \in [10, 14.5]$. Second, each range is evenly divided into four values, and equilibria are computed for all 16 (4×4) parameter combinations. Third, for each combination, we examine whether there exists an equilibrium that satisfies the second-order conditions, whether the resulting profit has a positive value, and whether the total market share is not extreme, that is, lies within the range of 0.01 to 0.99. If all these three conditions are satisfied, the parameter combination is regarded as "adequate." Fourth, we check whether the lower bound of the initial range yields at least one adequate combination. If so, the lower bound is extended by 20% of the range. The same procedure is applied to the upper bound. Lastly, the second step through the fourth step are repeated until neither the lower nor the upper bounds of both parameters are further extended (i.e., none of the combinations related to the bounds are adequate).

Results Figures 2 graphically shows how ρ^P and ρ^X are related. Dots are for the parameter values under which we can find the equilibrium with the satisfied second-order conditions, positive profits, and the total market within a range of $[0.01, 0.99]$.[16]

First, we notice that the cost pass-through to price, ρ^P never exceeds one. As shown in the next section, however, the random-coefficient logit demand is more flexible in this dimention because $\rho^P > 1$ is possible. It is also interesing that both $\rho^P < 0$ and $\rho^X < 0$ are posssible: firms responds to an increase of marginal costs by lowering the product price as well as the product quality. However, we only observe this happens if the number of firms is less than or equal to three.

When $J = 1$, many dots are located around $(\rho^X, \rho^P) = (0, 1)$ and the region with $\rho^X < 0$ and $\rho^P \approx 0$. The latter case can especially be interpreted as "shrinkfltion." However, if J is equal to or greater than two, the bunching around $\rho^X < 0$ and $\rho^P \approx 0$ disappears. Finally, as J increases, the set of combinations converges to $(\rho^X, \rho^P) = (0, 1)$, as expected from the consequences of perfect competition.

[15] We can confirm that the main results are robust if alternative values for ω and δ are considered.

[16] The reason for relating ρ^P to the vertical axis is that the price is called higher or lower rather than bigger or smaller.

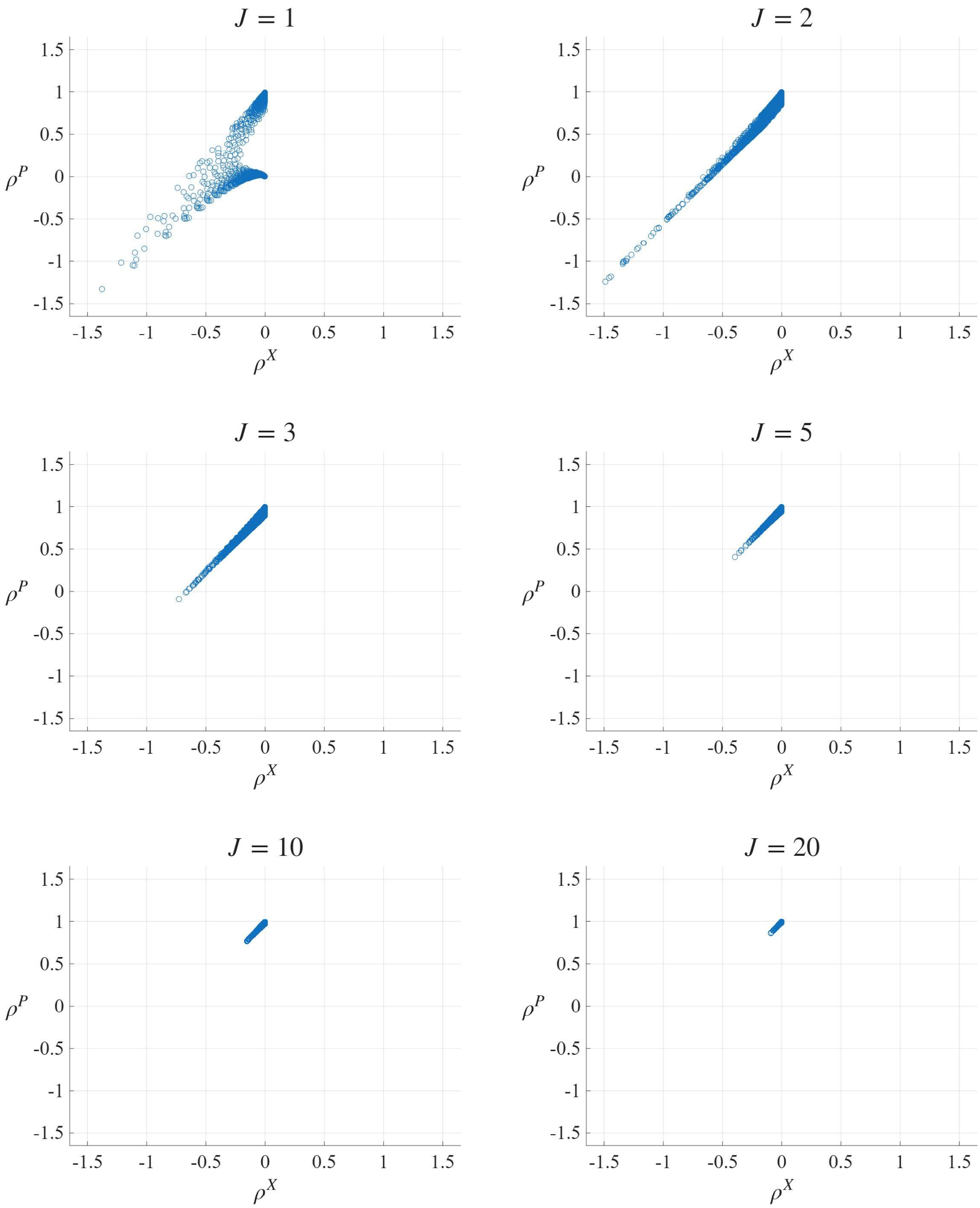

Figure 2: Relationships between ρ^P and ρ^X for each J for multinomial logit demand

5 Random-Coefficient Logit Demand

As is well known, multinomial logit demand has an important limitation: the cross-price elasticity with respect to a change in any product's price is the same for all other products because it is determined solely by that product's own price, market share, and the price coefficient. The random-coefficient logit model overcomes this limitation. In addition, Miravete, Seim, and Thurk (2023) emphasize that the random-coefficient logit model allows for greater flexibility in the estimated degree of cost pass-through through a wide range of possible demand curvatures.

5.1 Formulation

In light of these attractive features, this section introduces the random-coefficient logit demand as a generalization of the multinomial logit demand under the assumption of symmetric firms. Specifically, we assume that the demand is expressed by market share, and is given by the random-coefficient logit demand that can also include consumer heterogeneity in a tractable manner: for each heterogeneous firm $j = 1, 2, , ..., J$,

$$q_j(p_j, \mathbf{p}_{-j}, x_j, \mathbf{x}_{-j}; \boldsymbol{\omega}, \gamma, \delta, \overline{\beta}, \sigma_\beta^2) = \int_{i \in \mathcal{I}} \underbrace{\frac{\exp(\omega_j - \beta_i p_j + \gamma_i x_j^\delta)}{1 + \sum_{j'=1,2,...,J} \exp(\omega_{j'} - \beta_i p_{j'} + \gamma_i x_{j'}^\delta)}}_{\equiv \Pr_{ij}(\mathbf{p},\mathbf{x})} dG(i),$$

where the price and quality coefficients for each heterogenous consumer $i \in \mathcal{I}$ in the market is drawn according to $(\beta_i, \gamma_i)^{\mathrm{T}} \sim iid\ N((\overline{\beta}, \overline{\gamma})^{\mathrm{T}}, \Sigma)$ and the covariance matrix is given by:

$$\Sigma = \begin{pmatrix} \sigma_\beta^2 & \rho\sigma_\beta\sigma_\gamma \\ \rho\sigma_\beta\sigma_\gamma & \sigma_\gamma^2 \end{pmatrix}$$

with the correlation coefficient $\rho \in (-1, 1)$.

Under firm symmetry, the equilibrium pair (p, x) satisfies:

$$\begin{cases} \left(\int_{i\in\mathcal{I}} \beta_i \Pr_{ij}(1 - \Pr_{ij}) dG(i)\right) [p - (c + k^V x)] - q(p, x) = 0 \\ \delta x^{\delta-1} \left(\int_{i\in\mathcal{I}} \gamma_i \Pr_{ij}(1 - \Pr_{ij}) dG(i)\right) [p - (c + k^V x)] - k^V q(p, x) - k^F = 0 \end{cases}$$

where $\Pr_{ij}$ is identical for all j's given the number of symmetric firms J:

$$\Pr{}_{ij}(J) = \frac{\exp(\omega - \beta_i p + \gamma_i x^\delta)}{1 + J \cdot \exp(\omega - \beta_i p + \gamma_i x^\delta)}$$

and aggregate market demand is given by

$$q(p, x; J) = \int_{i\in\mathcal{I}} \Pr{}_{ij}(J) dG(i).$$

We are able to keep using the sufficient-statistics formulas for pass-through derived in Section 3 above.

If firm heterogeneity is allowed, we first need to relax the symmetry in the production side: production costs and quality costs are denoted by $\mathbf{c} = (c_1, c_2, ..., c_J)$, $\mathbf{k}^V = (k_1^V, k_2^V, ..., k_J^V)$, and $\mathbf{k}^F = (k_1^F, k_2^F, ..., k_J^F)$. From Equations (2) and (4), each firm j's first-order conditions are:

$$\begin{cases} F_j(\mathbf{p}, \mathbf{x}; c_j, k_j^V) \equiv L_j(p_j, x_j; c_j, k_j^V)\epsilon_j^p(\mathbf{p}, \mathbf{x}) - 1 = 0 \\ G_j(\mathbf{p}, \mathbf{x}; c_j, k_j^V, k_j^F) \equiv L_j(p_j, x_j; c_j, k_j^V)\epsilon_j^x(\mathbf{p}, \mathbf{x})\dfrac{p_j}{x_j} - k_j^V - \dfrac{k_j^F}{q_j(\mathbf{p}, \mathbf{x})} = 0 \end{cases}$$

where

$$L_j = L(p_j, x_j; c_j, k_j^V) \equiv \frac{p_j - (c_j + k_j^V x_j)}{p_j}$$

is firm j's markup ratio,

$$\epsilon_j^p(\mathbf{p}, \mathbf{x}) \equiv -\frac{\partial q_j}{\partial p_j}(\mathbf{p}, \mathbf{x}) \cdot \frac{p_j}{q_j(\mathbf{p}, \mathbf{x})} = \frac{p_j \int_{i\in\mathcal{I}} \beta_i \Pr_{ij}(1 - \Pr_{ij})dG(i)}{\int_{i\in\mathcal{I}} \Pr_{ij} dG(i)} > 0$$

is firm j's own price elasitity, and

$$\epsilon_j^x(\mathbf{p}, \mathbf{x}) \equiv \frac{\partial q_j}{\partial x_j}(\mathbf{p}, \mathbf{x}) \cdot \frac{x_j}{q_j(\mathbf{p}, \mathbf{x})} = \frac{\delta x_j^\delta \int_{i\in\mathcal{I}} \gamma_i \Pr_{ij}(1 - \Pr_{ij})dG(i)}{\int_{i\in\mathcal{I}} \Pr_{ij} dG(i)} > 0$$

is firm j's own quality elasticity. The equilibrium $\{(p_1, x_1), (p_2, x_2), ..., (p_J, x_J)\}$ is obtained by solving these $2 \times J$ equations.

Analogously, firm j's cross price elasitity and cross quality elasticity are defined by

$$\epsilon_{j'}^{p'}(\mathbf{p}, \mathbf{x}) \equiv \frac{\partial q_j}{\partial p_{j'}}(\mathbf{p}, \mathbf{x}) \cdot \frac{p_{j'}}{q_j(\mathbf{p}, \mathbf{x})} = \frac{p_{j'} \int_{i\in\mathcal{I}} \beta_i \Pr_{ij} \Pr_{ij'} dG(i)}{\int_{i\in\mathcal{I}} \Pr_{ij} dG(i)}$$

and

$$\epsilon_{j'}^{x'}(\mathbf{p}, \mathbf{x}) \equiv -\frac{\partial q_j}{\partial x_{j'}}(\mathbf{p}, \mathbf{x}) \cdot \frac{x_{j'}}{q_j(\mathbf{p}, \mathbf{x})} = \frac{\delta x_{j'}^\delta \int_{i\in\mathcal{I}} \gamma_i \Pr_{ij} \Pr_{ij'} dG(i)}{\int_{i\in\mathcal{I}} \Pr_{ij} dG(i)},$$

respectively.[17] Table 2 shows the expressions for the elasticities and curvatures.

Now, the cost pass-through for price and quality is given by:

$$\begin{pmatrix} \rho_1^P \\ \vdots \\ \rho_J^P \\ \rho_1^X \\ \vdots \\ \rho_J^X \end{pmatrix} = -M^{-1} \begin{pmatrix} -\dfrac{\epsilon_1^p}{p_1} \\ \vdots \\ -\dfrac{\epsilon_J^p}{p_J} \\ -\dfrac{\epsilon_1^x}{x_1} \\ \vdots \\ -\dfrac{\epsilon_J^x}{x_J} \end{pmatrix},$$

[17] See Appendix A for the expression of the second-order condition.

Table 2: Elasticities and Curvatures of Random-Coefficient Logit Demand

(i) Elasticities

ϵ_j^p	$p_j \cdot \dfrac{\int_{i\in\mathcal{I}} \beta_i \Pr_{ij}(1-\Pr_{ij}) dG(i)}{\int_{i\in\mathcal{I}} \Pr_{ij}\, dG(i)}$
$\epsilon_{j'}^{p'}$	$p_{j'} \cdot \dfrac{\int_{i\in\mathcal{I}} \beta_i \Pr_{ij} \Pr_{ij'} dG(i)}{\int_{i\in\mathcal{I}} \Pr_{ij} dG(i)}$
ϵ_j^x	$\delta x_j^\delta \cdot \dfrac{\int_{i\in\mathcal{I}} \gamma_i \Pr_{ij}(1-\Pr_{ij}) dG(i)}{\int_{i\in\mathcal{I}} \Pr_{ij}\, dG(i)}$
$\epsilon_{j'}^{x'}$	$\delta x_{j'}^\delta \cdot \dfrac{\int_{i\in\mathcal{I}} \gamma_i \Pr_{ij} \Pr_{ij'} dG(i)}{\int_{i\in\mathcal{I}} \Pr_{ij} dG(i)}$

(ii) Curvatures

α_{jj}^{pp}	$p_j \cdot \dfrac{\int_{i\in\mathcal{I}} \beta_i^2 \Pr_{ij}(1-\Pr_{ij})(1-2\Pr_{ij}) dG(i)}{\int_{i\in\mathcal{I}} \beta_i \Pr_{ij}(1-\Pr_{ij}) dG(i)}$
$\alpha_{jj'}^{pp'}$	$-p_{j'} \cdot \dfrac{\int_{i\in\mathcal{I}} \beta_i^2 \Pr_{ij}\Pr_{ij'}(1-2\Pr_{ij}) dG(i)}{\int_{i\in\mathcal{I}} \beta_i \Pr_{ij}(1-\Pr_{ij}) dG(i)}$
α_{jj}^{xx}	$x_j \cdot \dfrac{\int_{i\in\mathcal{I}} \gamma_i \Pr_{ij}(1-\Pr_{ij}) \left[(\delta-1)x_j^{-1} + \left(\delta x_j^{\delta-1}\right)\gamma_i(1-2\Pr_{ij})\right] dG(i)}{\int_{i\in\mathcal{I}} \gamma_i \Pr_{ij}(1-\Pr_{ij}) dG(i)}$
$\alpha_{jj'}^{xx'}$	$-\delta x_{j'}^\delta \cdot \dfrac{\int_{i\in\mathcal{I}} (\gamma_i)^2 \Pr_{ij}\Pr_{ij'}(1-2\Pr_{ij}) dG(i)}{\int_{i\in\mathcal{I}} \gamma_i \Pr_{ij}(1-\Pr_{ij}) dG(i)}$
α_{jj}^{xp}	$-p_j \cdot \dfrac{\int_{i\in\mathcal{I}} (\beta_i\gamma_i) \Pr_{ij}(1-\Pr_{ij})(1-2\Pr_{ij}) dG(i)}{\int_{i\in\mathcal{I}} \gamma_i \Pr_{ij}(1-\Pr_{ij}) dG(i)}$
$\alpha_{jj'}^{xp'}$	$p_{j'} \cdot \dfrac{\int (\beta_i\gamma_i) \Pr_{ij}\Pr_{ij'}(1-2\Pr_{ij}) dG(i)}{\int_{i\in\mathcal{I}} \gamma_i \Pr_{ij}(1-\Pr_{ij}) dG(i)}$
α_{jj}^{px}	$\delta x_j^\delta \cdot \dfrac{\int_{i\in\mathcal{I}} (\beta_i\gamma_i) \Pr_{ij}(1-\Pr_{ij})(1-2\Pr_{ij}) dG(i)}{\int_{i\in\mathcal{I}} \beta_i \Pr_{ij}(1-\Pr_{ij}) dG(i)}$
$\alpha_{jj'}^{px'}$	$-\delta x_{j'}^\delta \cdot \dfrac{\int_{i\in\mathcal{I}} (\beta_i\gamma_i) \Pr_{ij}\Pr_{ij'}(1-2\Pr_{ij}) dG(i)}{\int_{i\in\mathcal{I}} \beta_i \Pr_{ij}(1-\Pr_{ij}) dG(i)}$

where

$$M \equiv \begin{pmatrix} \frac{\partial F_1}{\partial p_1} & \cdots & \cdots & \frac{\partial F_1}{\partial p_J} & \frac{\partial F_1}{\partial x_1} & \cdots & \cdots & \frac{\partial F_1}{\partial x_J} \\ \vdots & & & \vdots & \vdots & & & \vdots \\ \frac{\partial F_J}{\partial p_1} & \ddots & & \frac{\partial F_J}{\partial p_J} & \frac{\partial F_J}{\partial x_1} & \ddots & & \frac{\partial F_J}{\partial x_J} \\ \frac{\partial G_1}{\partial p_1} & & \ddots & \frac{\partial G_1}{\partial p_J} & \frac{\partial G_1}{\partial x_1} & & \ddots & \frac{\partial G_1}{\partial x_J} \\ \vdots & & & \vdots & \vdots & & & \vdots \\ \frac{\partial G_J}{\partial p_1} & \cdots & \cdots & \frac{\partial G_J}{\partial p_J} & \frac{\partial G_J}{\partial x_1} & \cdots & \cdots & \frac{\partial G_J}{\partial x_J} \end{pmatrix}.$$

Interestingly, as Appendix D shows, each element of M is expressed in terms of sufficient statistics again: elasticities, curvatures, endogenous variables for price and quality, and the parameters with no specification of market demand.

As in the previous section, we impose firm symmetry below and conduct a numerical analysis of how the number of firms, J, affects the cost pass-throughs.

5.2 Numerical Analysis

Setting We proceed with the same as in Section 4. For the parameters regarding the distribution of random-coefficients, we set $\sigma_\beta^2 = 0.5$, $\sigma_\gamma^2 = 0.5$, and $\rho = 0$ as a baseline.[18]

Results Figure 3 illustrates the relationship between ρ^P and ρ^X for different numbers of firms ($J = 1, 2, 3, 5, 10$, and 20). In addition, Figure 4 presents this relationship for $J = 1$ under different values of the random coefficient distribution parameters $(\sigma_\beta, \sigma_\gamma, \rho)$. The baseline values of these parameters are $(0.5, 0.5, 0)$, and the corresponding graph is included in Figure 3.

Notably, it is now possible that $\rho^P > 1$ in contrast to the case of multinomial logit demand. A positive value of $\sigma_\beta^2 > 0$ is necessary for ρ^P to be more than one, as shown in Figure 4. Note also that ρ^P can also be negative. It is also important that in contrast to multinomial logit demand, $\rho^X > 0$ is now possible: the necessary conditions is $\sigma_\beta^2 > 0$ or $\sigma_\gamma^2 > 0$ as shown in Figure 4. Also, Figure 4 shows that a change in the covariance parameter (ρ) has little effect on the qualitative results.

When $J = 1$, many dots apprear around $(\rho^X, \rho^P) = (0, 1)$ and the area with $\rho^X \approx 0$ and $\rho^P > 0$: in contrast to multinomial logit demand, the dots representing "shrinkfltion" rarely appear when $\sigma_\beta > 0$. When $J \geq 2$, the dots around $\rho^X \approx 0$ and $\rho^P > 0$ disappear and $\rho^X < 0$. As J increases, ρ^X converges to 0. Additionally, as J increases, while the lower bound of ρ^P converges to 1, the upper bound remains more than one (about 1.08): the pass-through rates greater than one (i.e."over-shifting") are observed in many empirical settings (see Footnote 3 of Miravete, Seim, and Thurk 2023).

[18] We generate 1,000 individuals for the numerical integration. To guarantee the positive β_i and γ_i, σ_β^2 and σ_β^2 are adjusted when they are large compared to $|\beta|$ and $|\gamma|$. Specifically, they are adjusted to be $|\beta|/4$ if $|\beta|/4$ is less than 0.5.

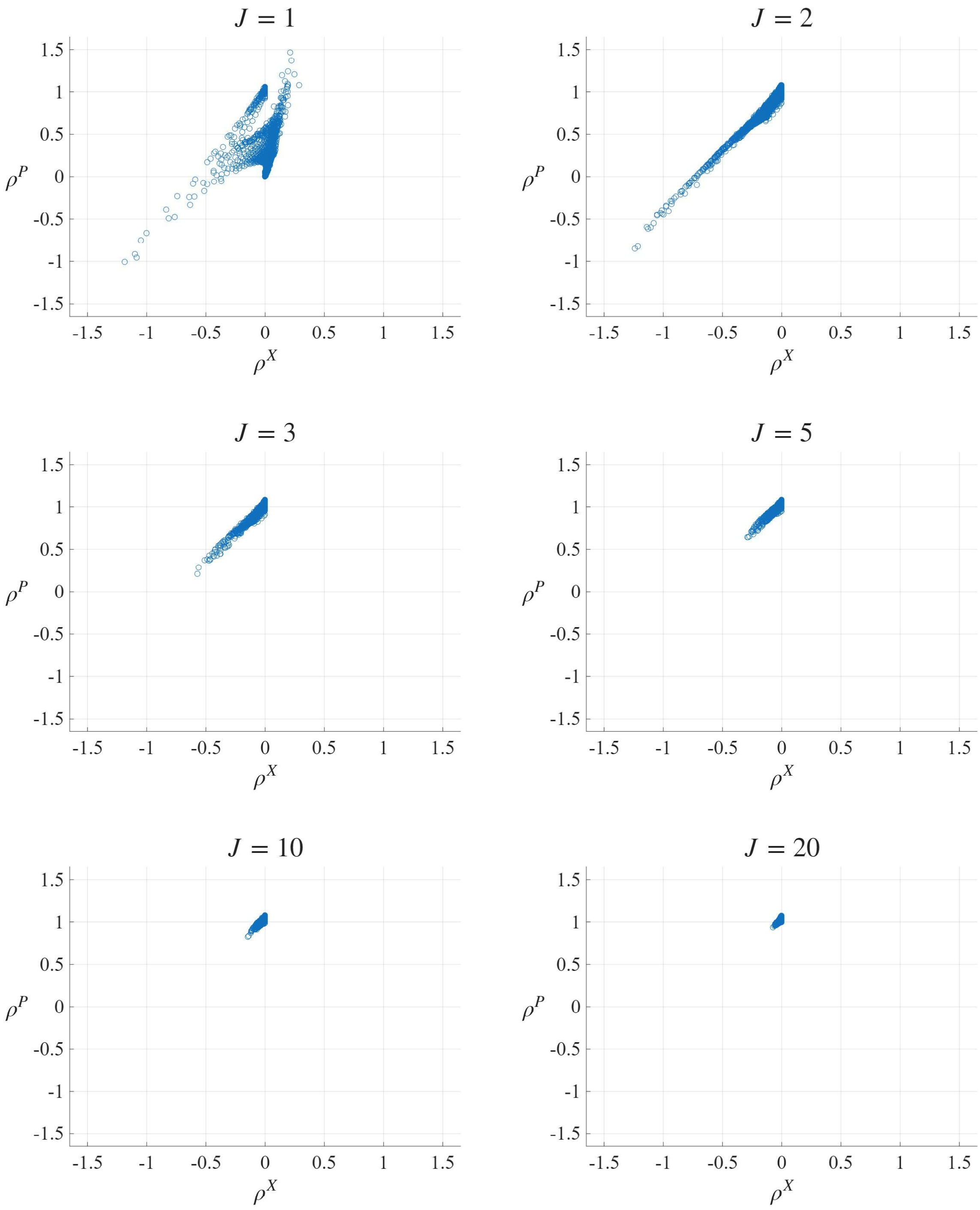

Figure 3: Relationships between ρ^P and ρ^X for each J for random-coefficient logit demand

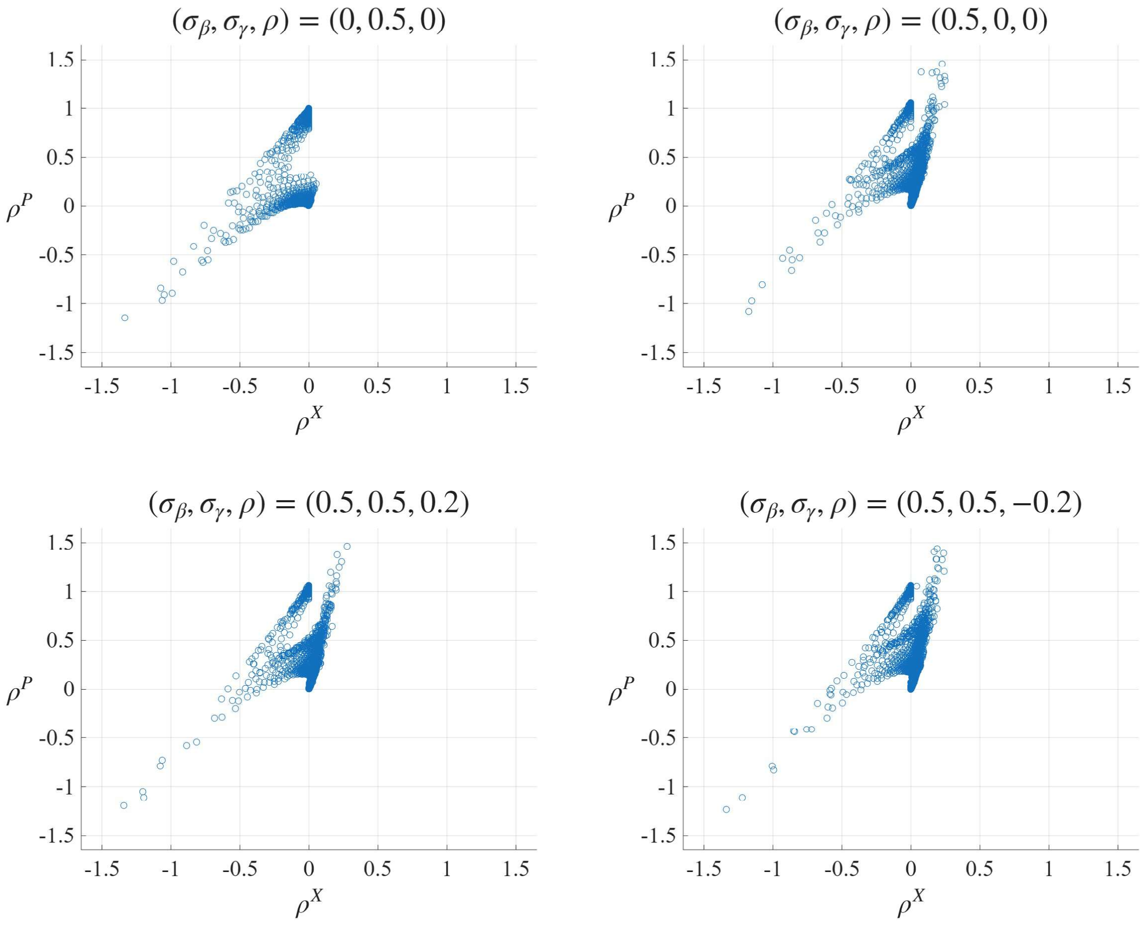

Figure 4: Relationship between ρ^P and ρ^X with $J = 1$: Role of the Random Coefficient Distribution Parameters

6 Concluding Remarks

This paper has studied cost pass-through in consideration of endogenous choice of product quality under imperfect competition to provide the formulas in terms of sufficient statistics. In this environment, an increase in marginal costs can lead to declines in both the equilibrium price and product quality when the number of symmetric firms is sufficiently small. More broadly, our numerical results highlight the greater flexibility of the random-coefficient logit model relative to the multinomial logit, in that it allows price pass-through to exceed unity—a feature ruled out by the multinomial logit. Furthermore, quality pass-through can be positive under random-coefficient logit demand.

In this paper, we employ a short-run analysis in the sense that the number of firms, J, is fixed. It is important and interesting to endogenize J by introducing a certain degree of firm heterogeneity with respect to, e.g., cost (see, e.g., Schröer and Sørensen 2021; Kroft, Laliberté Leal-Vizcaíno, and Notowidigdo 2024).

Appendices

A Second-Order Conditions for the Profit Function

First, note that the second-order conditions are:

$$\begin{cases} 2\dfrac{\partial q_j}{\partial p_j} + [p_j - (c_j + k_j^V x_j)]\dfrac{\partial^2 q_j}{\partial p_j^2} < 0 \\ \left(2\dfrac{\partial q_j}{\partial p_j} + [p_j - (c_j + k_j^V x_j)]\dfrac{\partial^2 q_j}{\partial p_j^2}\right)\left(-2k_j^V\dfrac{\partial q_j}{\partial x_j} + [p_j - (c_j + k_j^V x_j)]\dfrac{\partial^2 q_j}{\partial x_j^2}\right) \\ \qquad - \left(\dfrac{\partial q_j}{\partial x_j} - k_j^V\dfrac{\partial q_j}{\partial p_j} + [p_j - (c_j + k_j^V x_j)]\dfrac{\partial^2 q_j}{\partial x_j \partial p_j}\right)^2 > 0 \end{cases}$$

for each $j = 1, ..., J$.

Then, if the symmetry assumption is imposed, they are expressed in terms of elasticities and curvatures

$$\begin{aligned} \frac{\partial^2 \pi_j}{\partial p_j^2} &= 2\frac{\partial q_j}{\partial p_j} + [p - (c + k^V x)]\frac{\partial^2 q_j}{\partial p_j^2} \\ &= \frac{q \cdot \epsilon^p}{p}\left[-2 + L \cdot \alpha^{pp}\right] < 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 \pi_j}{\partial x_j^2} &= -2k^V\frac{\partial q_j}{\partial x_j} + [p - (c + k^V x)]\frac{\partial^2 q_j}{\partial x_j^2} \\ &= \frac{q \cdot \epsilon^x}{x}\left(-2k^V + L \cdot \alpha^{xx} \cdot \frac{p}{x}\right) < 0, \end{aligned}$$

and

$$\frac{\partial^2 \pi_j}{\partial p_j^2} \cdot \frac{\partial^2 \pi_j}{\partial x_j^2} > \left(\frac{\partial^2 \pi_j}{\partial x_j \partial p_j}\right)^2,$$

where

$$\begin{aligned}
\frac{\partial^2 \pi_j}{\partial x_j \partial p_j} &= \frac{\partial q_j}{\partial x_j} - k^V \frac{\partial q_j}{\partial p_j} + [p - (c + k^V x)] \frac{\partial^2 q_j}{\partial x_j \partial p_j} \\
&= \frac{q \cdot \epsilon^x}{x} (L \cdot \alpha^{xp}) + k^V \left(\frac{q}{p} \right) \epsilon^p.
\end{aligned}$$

Note here that either p or x should appear non-linearly in demand. Otherwise, all the eight α's are zero, and it is verified that

$$\frac{\partial^2 \pi_j}{\partial p_j^2} \cdot \frac{\partial^2 \pi_j}{\partial x_j^2} = 4 \left\{ \underbrace{\left(\frac{q}{x} \right) \epsilon^x}_{\equiv A > 0} \cdot \underbrace{k^V \left(\frac{q}{p} \right) \epsilon^p}_{\equiv B > 0} \right\},$$

whereas

$$\frac{\partial^2 \pi_j}{\partial x_j \partial p_j} = \underbrace{\left(\frac{q}{x} \right) \epsilon^x}_{= A} + \underbrace{k^V \left(\frac{q}{p} \right) \epsilon^p}_{= B} = A + B,$$

implying that

$$\begin{aligned}
&\frac{\partial^2 \pi_j}{\partial p_j^2} \cdot \frac{\partial^2 \pi_j}{\partial x_j^2} > \left(\frac{\partial^2 \pi_j}{\partial x_j \partial p_j} \right)^2 \\
&\Leftrightarrow 4AB > (A + B)^2 \\
&\Leftrightarrow \frac{A + B}{2} < \sqrt{AB},
\end{aligned}$$

which never holds for $A > 0$ and $B > 0$.

Multinomial and Random-Coefficient Logit Demands

The second-order conditions

$$\begin{cases}
2 \frac{\partial q_j}{\partial p_j} + Lp \frac{\partial^2 q_j}{\partial p_j^2} < 0 \\
\left(2 \frac{\partial q_j}{\partial p_j} + Lp \frac{\partial^2 q_j}{\partial p_j^2} \right) \left(-2k^V \frac{\partial q_j}{\partial x_j} + Lp \frac{\partial^2 q_j}{\partial x_j^2} \right) \\
\qquad - \left(\frac{\partial q_j}{\partial x_j} - k^V \frac{\partial q_j}{\partial p_j} + Lp \frac{\partial^2 q_j}{\partial x_j \partial p_j} \right)^2 > 0
\end{cases}$$

for each $j = 1, 2, ..., J$ are simplified to

$$\begin{cases}
-2 + \beta Lp(1 - 2q) < 0 \\
\beta(\gamma\delta) x^{\delta-1} [-2 + \beta Lp(1 - 2q)] \\
\times \left\{ -2k^V + Lpx^{-1} [(\delta - 1) + (\gamma\delta) x^\delta (1 - 2q)] \right\} \\
- \left[(\gamma\delta) x^{\delta-1} + \beta k^V - \beta Lp(\gamma\delta) x^{\delta-1} (1 - 2q) \right]^2 > 0.
\end{cases}$$

for the case of multinomial logit demand under firm symmetry.

For the case of random-coefficient logit demand with firm heterogeneity, each term is given by:

$$
\begin{aligned}
&2\frac{\partial q_j}{\partial p_j} + Lp\frac{\partial^2 q_j}{\partial p_j^2} \\
&= -2\left(\int_{i\in\mathcal{I}}\left[\beta_i \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})\right]dG(i)\right) + Lp\left(\int_{i\in\mathcal{I}}\left[\beta_i^2 \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})(1-2\mathrm{Pr}_{ij})\right]dG(i)\right),
\end{aligned}
$$

$$
\begin{aligned}
&-2k^V\frac{\partial q_j}{\partial x_j} + Lp\frac{\partial^2 q_j}{\partial x_j^2} \\
&= \left[-2k^V\delta x^{\delta-1} + Lp\delta(\delta-1)x^{\delta-2}\right]\left(\int_{i\in\mathcal{I}}\left[\gamma_i \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})\right]dG(i)\right) \\
&\quad + Lp(\delta x^{\delta-1})^2\left(\int_{i\in\mathcal{I}}\left[\gamma_i^2 \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})(1-2\mathrm{Pr}_{ij})\right]dG(i)\right)
\end{aligned}
$$

and

$$
\begin{aligned}
&\frac{\partial q_j}{\partial x_j} - k^V\frac{\partial q_j}{\partial p_j} + Lp\frac{\partial^2 q_j}{\partial x_j \partial p_j} \\
&= \delta x_j^{\delta-1}\left(\int_{i\in\mathcal{I}}\left[\gamma_i \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})\right]dG(i)\right) + k^V\left(\int_{i\in\mathcal{I}}\left[\beta_i \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})\right]dG(i)\right) \\
&\quad - Lp\delta x^{\delta-1}\left(\int_{i\in\mathcal{I}}\left[\beta_i\gamma_i \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})(1-2\mathrm{Pr}_{ij})\right]dG(i)\right).
\end{aligned}
$$

In the numerical analysis, we check whether each pair of candidate parameters satisfies the second-order condition.

B Proof of Proposition 1

First, define the following function from Equations (1) and (3) by:

$$
\begin{bmatrix} F(p,x;c,k^V) \\ G(p,x;c,k^V,k^F) \end{bmatrix} = \begin{bmatrix} L(p,x;c,k^V)\epsilon^p(p,x) - 1 \\ L(p,x;c,k^V)\epsilon^x(p,x)\dfrac{p}{x} - k^V - \dfrac{k^F}{q(p,x)} \end{bmatrix}.
$$

By letting $\det \equiv (\partial F/\partial p)(\partial G/\partial x) - (\partial F/\partial x)(\partial G/\partial p)$, the implicit function theorem implies that:

$$\begin{bmatrix} \rho_{mc}^{P} \\ \\ \rho_{mc}^{X} \end{bmatrix} = -\begin{bmatrix} \dfrac{\partial F}{\partial p} & \dfrac{\partial F}{\partial x} \\ \\ \dfrac{\partial G}{\partial p} & \dfrac{\partial G}{\partial x} \end{bmatrix}^{-1} \begin{bmatrix} \dfrac{\partial F}{\partial c} \\ \\ \dfrac{\partial G}{\partial c} \end{bmatrix}$$

$$= \left(\frac{-1}{\det}\right) \begin{bmatrix} \dfrac{\partial G}{\partial x} & -\dfrac{\partial F}{\partial x} \\ \\ -\dfrac{\partial G}{\partial p} & \dfrac{\partial F}{\partial p} \end{bmatrix} \begin{bmatrix} \dfrac{\partial F}{\partial c} \\ \\ \dfrac{\partial G}{\partial c} \end{bmatrix}$$

$$= \left(\frac{-1}{\det}\right) \begin{bmatrix} \dfrac{\partial G}{\partial x} \cdot \dfrac{\partial F}{\partial c} - \dfrac{\partial F}{\partial x} \cdot \dfrac{\partial G}{\partial c} \\ \\ -\dfrac{\partial G}{\partial p} \cdot \dfrac{\partial F}{\partial c} + \dfrac{\partial F}{\partial p} \cdot \dfrac{\partial G}{\partial c} \end{bmatrix},$$

which is further simplified as

$$\begin{bmatrix} \rho_{mc}^{P} \\ \\ \rho_{mc}^{X} \end{bmatrix} = \left(\frac{-1}{\det}\right) \begin{bmatrix} -\dfrac{\partial G}{\partial x} \cdot \dfrac{\epsilon^{p}}{p} + \dfrac{\partial F}{\partial x} \cdot \dfrac{\epsilon^{x}}{x} \\ \\ -\dfrac{\partial F}{\partial p} \cdot \dfrac{\epsilon^{x}}{x} + \dfrac{\partial G}{\partial p} \cdot \dfrac{\epsilon^{p}}{p} \end{bmatrix}$$

because

$$\begin{bmatrix} \dfrac{\partial F}{\partial c} \\ \\ \dfrac{\partial G}{\partial c} \end{bmatrix} = -\begin{bmatrix} \dfrac{\epsilon^{p}}{p} \\ \\ \dfrac{\epsilon^{x}}{x} \end{bmatrix} .$$

First, notice that

$$\begin{aligned} \frac{\partial F}{\partial p} &= \frac{\partial L}{\partial p} \cdot \epsilon^{p} + L \cdot \frac{\partial \epsilon^{p}}{\partial p} \\ &= (1-L) \cdot \left(\frac{\epsilon^{p}}{p}\right) + L \cdot \frac{\partial \epsilon^{p}}{\partial p} \qquad (6) \\ &= (1-L) \cdot \left(\frac{\epsilon^{p}}{p}\right) \\ &\quad -L \cdot \left\{\alpha^{pp} + (J-1)\alpha^{pp'} - \left[1 + \epsilon^{p} - (J-1)\epsilon^{p'}\right]\right\} \cdot \left(\frac{\epsilon^{p}}{p}\right) \\ &= \left(\frac{\epsilon^{p}}{p}\right) \cdot \left\{1 - \left[(\alpha^{pp} - \epsilon^{p}) + (J-1)(\alpha^{pp'} + \epsilon^{p'})\right] L\right\} \end{aligned}$$

because it is verified that

$$
\begin{aligned}
\frac{\partial \epsilon^p}{\partial p} &= \left[\left(-\frac{\partial^2 q_j}{\partial p_j^2}\right) + (J-1)\left(-\frac{\partial^2 q_j}{\partial p_j \partial p_{j'}}\right)\right]\left(\frac{p}{q}\right) - \left(\frac{1}{q}\right)\left(\frac{\partial q_j}{\partial p_j}\right)\left(1 - \frac{p}{q}\frac{\partial q}{\partial p}\right) \\
&= \left(\frac{-1}{p}\right)\left(-\frac{p}{q}\frac{\partial q_j}{\partial p_j}\right)\left[\left(-\frac{p \cdot \partial^2 q_j / \partial p_j^2}{\partial q_j / \partial p_j}\right) + (J-1)\left(-\frac{p \cdot \partial^2 q_j / (\partial p_j \partial p_{j'})}{\partial q_j / \partial p_j}\right)\right] \\
&\quad - \left(\frac{-1}{p}\right)\left(-\frac{p}{q}\frac{\partial q_j}{\partial p_j}\right)\left(1 + \epsilon^{p,I}\right) \\
&= -\left[\alpha^{pp} + (J-1)\alpha^{pp'} - \left(1 + \epsilon^{p,I}\right)\right]\left(\frac{\epsilon^p}{p}\right).
\end{aligned}
$$

Second, notice that

$$
\begin{aligned}
\frac{\partial F}{\partial x} &= \frac{\partial L}{\partial x} \cdot \epsilon^p + L \cdot \frac{\partial \epsilon^p}{\partial x} \\
&= -k^V \cdot \left(\frac{\epsilon^p}{p}\right) + L \cdot \frac{\partial \epsilon^p}{\partial x} \\
&= -k^V \cdot \left(\frac{\epsilon^p}{p}\right) \\
&\quad + L\left[\alpha^{px} + (J-1)\alpha^{px'} - \epsilon^x + (J-1)\epsilon^{x'}\right]\left(\frac{p}{x}\right)\left(\frac{\epsilon^p}{p}\right) \\
&= \left(\frac{\epsilon^p}{p}\right) \cdot \left\{\left[(\alpha^{px} - \epsilon^x) + (J-1)(\alpha^{px'} + \epsilon^{x'})\right]\left(\frac{Lp}{x}\right) - k^V\right\}
\end{aligned}
\tag{7}
$$

because

$$
\begin{aligned}
\frac{\partial \epsilon^p}{\partial x} &= \left[\left(-\frac{\partial^2 q_j}{\partial p_j \partial x_j}\right) + (J-1)\left(-\frac{\partial^2 q_j}{\partial p_j \partial x_{j'}}\right)\right]\left(\frac{p}{q}\right) - \left(\frac{1}{x}\right)\left(-\frac{p}{q}\frac{\partial q_j}{\partial p_j}\right) \cdot \left(\frac{x}{q}\frac{\partial q}{\partial x}\right) \\
&= \frac{1}{x}\left(-\frac{p}{q}\frac{\partial q_j}{\partial p_j}\right)\left[\left(\frac{x \cdot \partial^2 q_j / (\partial p_j \partial x_j)}{\partial q_j / \partial p_j}\right) + (J-1)\left(\frac{x \cdot \partial^2 q_j / (\partial p_j \partial x_{j'})}{\partial q_j / \partial p_j}\right)\right] \\
&\quad - \frac{1}{x}\left(-\frac{p}{q}\frac{\partial q_j}{\partial p_j}\right) \cdot \epsilon^{x,I} \\
&= \left[\alpha^{px} + (J-1)\alpha^{px'} - \epsilon^{x,I}\right]\left(\frac{p}{x}\right)\left(\frac{\epsilon^p}{p}\right).
\end{aligned}
$$

Third, notice that

$$
\begin{aligned}
\frac{\partial G}{\partial p} &= \frac{\partial (Lp)}{\partial p}\cdot\left(\frac{\epsilon^x}{x}\right)+\left(\frac{Lp}{x}\right)\cdot\frac{\partial \epsilon^x}{\partial p}-\frac{k^F}{q^2}\left(-\frac{\partial q}{\partial p}\frac{p}{q}\right)\frac{q}{p} \\
&= \left(\frac{\epsilon^x}{x}\right)+\left(\frac{Lp}{x}\right)\cdot\frac{\partial \epsilon^x}{\partial p}-k^F\cdot\frac{\epsilon^{p,I}}{pq} \qquad (8)\\
&= \left(\frac{\epsilon^x}{x}\right) \\
&\quad +\left(\frac{Lp}{x}\right)[\alpha^{xp}+(J-1)\alpha^{xp'}+\epsilon^p-(J-1)\epsilon^{p'}]\left(\frac{x}{p}\right)\left(\frac{\epsilon^x}{x}\right) \\
&\quad -\frac{k^F}{pq}\left[\epsilon^p-(J-1)\epsilon^{p'}\right]\left(\frac{x}{\epsilon^x}\right)\left(\frac{\epsilon^x}{x}\right) \\
&= \left(\frac{\epsilon^x}{x}\right)\left\{1+\left[(\alpha^{xp}+\epsilon^p)+(J-1)(\alpha^{xp'}-\epsilon^{p'})\right]L-\left(\frac{k^F x}{pq}\right)\left(\frac{\epsilon^p}{\epsilon^x}\right)\left[1-(J-1)\left(\frac{\epsilon^{p'}}{\epsilon^p}\right)\right]\right\}
\end{aligned}
$$

because

$$
\begin{aligned}
\frac{\partial \epsilon^x}{\partial p} &= \left[\frac{\partial^2 q_j}{\partial x_j \partial p_j}+(J-1)\frac{\partial^2 q_j}{\partial x_j \partial p_{j'}}\right]\frac{x}{q}+\left(\frac{1}{p}\right)\left(\frac{x}{q}\frac{\partial q_j}{\partial x_j}\right)\left(-\frac{p}{q}\frac{\partial q}{\partial p}\right) \\
&= \frac{1}{p}\left(\frac{x}{q}\frac{\partial q_j}{\partial x_j}\right)\left[\left(\frac{p\cdot \partial^2 q_j/(\partial x_j \partial p_j)}{\partial q_j/\partial x_j}\right)+(J-1)\left(\frac{p\cdot \partial^2 q_j/(\partial x_j \partial p_{j'})}{\partial q_j/\partial x_j}\right)\right] \\
&\quad +\frac{1}{p}\left(\frac{x}{q}\frac{\partial q_j}{\partial x_j}\right)\epsilon^{p,I} \\
&= [\alpha^{xp}+(J-1)\alpha^{xp'}+\epsilon^{p,I}]\left(\frac{x}{p}\right)\left(\frac{\epsilon^x}{x}\right).
\end{aligned}
$$

Finally, notice that

$$
\begin{aligned}
\frac{\partial G}{\partial x} &= \frac{\partial (Lp)}{\partial x}\cdot\left(\frac{\epsilon^x}{x}\right)+Lp\cdot\frac{\partial(\epsilon^x/x)}{\partial x}-\frac{k^F}{q^2}\cdot\left(-\frac{\partial q}{\partial x}\frac{x}{q}\right)\left(\frac{q}{x}\right) \\
&= -\left(k^V+\frac{Lp}{x}\right)\cdot\left(\frac{\epsilon^x}{x}\right)+\left(\frac{Lp}{x}\right)\cdot\frac{\partial \epsilon^x}{\partial x}+k^F\cdot\frac{\epsilon^{x,I}}{xq} \qquad (9)\\
&= -\left(k^V+\frac{Lp}{x}\right)\cdot\left(\frac{\epsilon^x}{x}\right) \\
&\quad +\left(\frac{Lp}{x}\right)\left[\alpha^{xx}+(J-1)\alpha^{xx'}+(1-\epsilon^{x,I})\right]\left(\frac{\epsilon^x}{x}\right) \\
&\quad +\left(\frac{k^F}{q}\right)\left[\frac{\epsilon^x}{x}-(J-1)\left(\frac{\epsilon^{x'}}{x}\right)\right] \\
&= \left(\frac{\epsilon^x}{x}\right)\left\{\left[(\alpha^{xx}-\epsilon^x)+(J-1)(\alpha^{xx'}+\epsilon^{x'})\right]\left(\frac{Lp}{x}\right)+\left(\frac{k^F}{q}\right)\left[1-(J-1)\left(\frac{\epsilon^{x'}}{\epsilon^x}\right)\right]-k^V\right\}
\end{aligned}
$$

because

$$\begin{aligned}
\frac{\partial \epsilon^x}{\partial x} &= \left[\frac{\partial^2 q_j}{\partial x_j^2} + (J-1)\frac{\partial^2 q_j}{\partial x_j \partial x_{j'}}\right]\frac{x}{q} + \frac{1}{q}\frac{\partial q_j}{\partial x_j}\left(1 - \frac{x}{q}\frac{\partial q}{\partial x}\right) \\
&= \frac{1}{x}\cdot\left(\frac{x}{q}\frac{\partial q_j}{\partial x_j}\right)\left[\left(\frac{x\cdot\partial^2 q_j/\partial x_j^2}{\partial q_j/\partial x_j}\right) + (J-1)\left(\frac{x\cdot\partial^2 q_j/(\partial x_j \partial x_{j'})}{\partial q_j/\partial x_j}\right)\right] + \frac{1}{x}\left(\frac{x}{q}\frac{\partial q_j}{\partial x_j}\right)(1-\epsilon^{x,I}) \\
&= \left[\alpha^{xx} + (J-1)\alpha^{xx'} + (1-\epsilon^{x,I})\right]\left(\frac{\epsilon^x}{x}\right).
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\frac{\det}{\left(\frac{1}{px}\right)\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)} \\
&\quad= \left\{p - \left[(\alpha^{pp}-\epsilon^p) + (J-1)(\alpha^{pp'}+\epsilon^{p'})\right]Lp\right\} \\
&\quad\times\left\{\left[(\alpha^{xx}-\epsilon^x) + (J-1)(\alpha^{xx'}+\epsilon^{x'})\right]Lp + \left(\frac{k^F x}{q}\right)\left[1-(J-1)\left(\frac{\epsilon^{x'}}{\epsilon^x}\right)\right] - k^V x\right\} \\
&\qquad - \left\{p + \left[(\alpha^{xp}+\epsilon^p) + (J-1)(\alpha^{xp'}-\epsilon^{p'})\right]Lp - \left(\frac{k^F x}{q}\right)\left(\frac{\epsilon^p}{\epsilon^x}\right)\left[1-(J-1)\left(\frac{\epsilon^{p'}}{\epsilon^p}\right)\right]\right\} \\
&\quad\times\left\{\left[(\alpha^{px}-\epsilon^x) + (J-1)(\alpha^{px'}+\epsilon^{x'})\right]Lp - k^V x\right\} \\
&\quad= \left\{p - \left[(\alpha^{pp}-\epsilon^p) + (J-1)(\alpha^{pp'}+\epsilon^{p'})\right]Lp\right\}\left\{\left[(\alpha^{xx}-\epsilon^x) + (J-1)(\alpha^{xx'}+\epsilon^{x'})\right]Lp - k^V x\right\} \\
&\qquad + \left\{p - \left[(\alpha^{pp}-\epsilon^p) + (J-1)(\alpha^{pp'}+\epsilon^{p'})\right]Lp\right\}\left(\frac{k^F x}{q}\right)\left[1-(J-1)\left(\frac{\epsilon^{x'}}{\epsilon^x}\right)\right] \\
&\qquad - \left\{p + \left[(\alpha^{xp}+\epsilon^p) + (J-1)(\alpha^{xp'}-\epsilon^{p'})\right]Lp\right\}\left\{\left[(\alpha^{px}-\epsilon^x) + (J-1)(\alpha^{px'}+\epsilon^{x'})\right]Lp - k^V x\right\} \\
&\quad + \left(\frac{k^F x}{q}\right)\left(\frac{\epsilon^p}{\epsilon^x}\right)\left[1-(J-1)\left(\frac{\epsilon^{p'}}{\epsilon^p}\right)\right]\left\{\left[(\alpha^{px}-\epsilon^x) + (J-1)(\alpha^{px'}+\epsilon^{x'})\right]Lp - k^V x\right\} \\
&= \left[(\alpha^{xx}-\alpha^{px}) + (J-1)(\alpha^{xx'}-\alpha^{px'})\right](Lp)(1-v)p \\
&\quad + \left[(\alpha^{pp}+\alpha^{xp}) + (J-1)(\alpha^{pp'}+\alpha^{xp'})\right](Lp)k^V x \\
&\quad - \left\{\left[(\alpha^{pp}-\epsilon^p) + (J-1)(\alpha^{pp'}+\epsilon^{p'})\right]\left[(\alpha^{xx}-\epsilon^x) + (J-1)(\alpha^{xx'}+\epsilon^{x'})\right]\right. \\
&\qquad \left. + \left[(\alpha^{xp}+\epsilon^p) + (J-1)(\alpha^{xp'}-\epsilon^{p'})\right]\left[(\alpha^{px}-\epsilon^x) + (J-1)(\alpha^{px'}+\epsilon^{x'})\right]\right\}(Lp)^2 \\
&\qquad + \left(\frac{k^F x}{q}\right)\left[1-(J-1)\left(\frac{\epsilon^{x'}}{\epsilon^x}\right)\right]\left\{p - \left[(\alpha^{pp}-\epsilon^p) + (J-1)(\alpha^{pp'}+\epsilon^{p'})\right]Lp\right\} \\
&\quad - \left(\frac{k^F x}{q}\right)\left(\frac{\epsilon^p}{\epsilon^x}\right)\left[1-(J-1)\left(\frac{\epsilon^{p'}}{\epsilon^p}\right)\right]\left\{k^V x - \left[(\alpha^{px}-\epsilon^x) + (J-1)(\alpha^{px'}+\epsilon^{x'})\right]Lp\right\}.
\end{aligned}$$

It is natural to assume that this expression takes a negative value. To see this, if the demand is linear in price (i.e., all the curvatures are zero except for α^{xx}),

$$\frac{\det}{\left(\frac{1}{px}\right)\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)}$$
$$= \alpha^{xx}(Lp)p$$
$$-\underbrace{\left\{\left[\epsilon^p-(J-1)\epsilon^{p'}\right]\left[\epsilon^x-(J-1)\epsilon^{x'}-\alpha^{xx}\right]-\left[\epsilon^p-(J-1)\epsilon^{p'}\right]\left[\epsilon^x-(J-1)\epsilon^{x'}\right]\right\}}_{=-\alpha^{xx}\cdot\epsilon^{p,I}}(Lp)^2$$
$$+\left(\frac{k^F x}{q}\right)\left\{\underbrace{\left[1-(J-1)\left(\frac{\epsilon^{x'}}{\epsilon^x}\right)\right]}_{=\epsilon^{x,I}/\epsilon^x}\left(p+\epsilon^{p,I}Lp\right)-\underbrace{\left(\frac{\epsilon^p}{\epsilon^x}\right)\left[1-(J-1)\left(\frac{\epsilon^{p'}}{\epsilon^p}\right)\right]}_{=\epsilon^{p,I}/\epsilon^x}\left(kx+\epsilon^{x,I}Lp\right)\right\},$$

implying that

$$\det \gtrless 0 \Leftrightarrow \alpha^{xx}(Lp)\underbrace{\left\{p+\epsilon^{p,I}(Lp)\right\}}_{=(\epsilon^p+\epsilon^{p,I})\cdot(Lp)}+\left(\frac{k^F x}{q\epsilon^x}\right)\left[\epsilon^{x,I}\cdot p-\epsilon^{p,I}\cdot(kx)\right]\gtrless 0$$
$$\Leftrightarrow \underbrace{\alpha^{xx}(Lp)^2(\epsilon^p+\epsilon^{p,I})\epsilon^x}_{<0}+\left(\frac{k^F x}{q}\right)\left[\epsilon^{x,I}\cdot p-\epsilon^{p,I}\cdot(kx)\right]\gtrless 0.$$

It is also natural to assume that $\epsilon^{x,I}$ is relatively small in comparison to $\epsilon^{p,I}$. Hence, we are able to assume that $\det < 0$.

Finally, it is verified that (from Equations 9 and 7)

$$\begin{aligned}
-\frac{\partial G}{\partial x}\cdot\frac{\epsilon^p}{p}+\frac{\partial F}{\partial x}\cdot\frac{\epsilon^x}{x} &= \left(\frac{Lp}{x}\right)\left(\frac{\epsilon^x}{x}-\frac{\partial\epsilon^x}{\partial x}\right)\left(\frac{\epsilon^p}{p}\right)+L\left(\frac{\partial\epsilon^p}{\partial x}\right)\left(\frac{\epsilon^x}{x}\right)-k^F\left(\frac{\epsilon^{x,I}}{xq}\right)\left(\frac{\epsilon^p}{p}\right)\\
&= -\left(\frac{Lp}{x}\right)\left[\alpha^{xx}+(J-1)\alpha^{xx'}-\epsilon^{x,I}\right]\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)\\
&\quad+\left(\frac{Lp}{x}\right)\left[\alpha^{px}+(J-1)\alpha^{px'}-\epsilon^{x,I}\right]\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)\\
&\quad-k^F\left(\frac{\epsilon^{x,I}}{xq}\right)\left(\frac{\epsilon^p}{p}\right)\\
&= L\left(\frac{p}{x}\right)\left[(\alpha^{px}-\alpha^{xx})+(J-1)(\alpha^{px'}-\alpha^{xx'})\right]\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)-k^F\left(\frac{\epsilon^{x,I}}{xq}\right)\left(\frac{\epsilon^p}{p}\right)\\
&= \left\{L\left(\frac{p}{x}\right)\left[(\alpha^{px}-\alpha^{xx})+(J-1)(\alpha^{px'}-\alpha^{xx'})\right]-\frac{k^F}{q}\right\}\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)\\
&\quad+\frac{k^F}{q}(J-1)\left(\frac{\epsilon^{x'}}{\epsilon^x}\right)\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right),
\end{aligned}$$

and that (from Equations 6 and 8)

$$
\begin{aligned}
-\frac{\partial F}{\partial p}\cdot\frac{\epsilon^x}{x}+\frac{\partial G}{\partial p}\cdot\frac{\epsilon^p}{p} &= \left(\frac{Lp}{x}\right)\left(\frac{\partial\epsilon^x}{\partial p}\right)\left(\frac{\epsilon^p}{p}\right)+L\left(\frac{\epsilon^p}{p}-\frac{\partial\epsilon^p}{\partial p}\right)\left(\frac{\epsilon^x}{x}\right)-k^F\left(\frac{\epsilon^{p,I}}{pq}\right)\left(\frac{\epsilon^p}{p}\right)\\
&= L[\alpha^{xp}+(J-1)\alpha^{xp'}+\epsilon^{p,I}]\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)\\
&\quad +L\left[\alpha^{pp}+(J-1)\alpha^{pp'}-\epsilon^{p,I}\right]\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)\\
&\quad -k^F\left(\frac{\epsilon^{p,I}}{pq}\right)\left(\frac{\epsilon^p}{p}\right)\\
&= L\left[(\alpha^{xp}+\alpha^{pp})+(J-1)(\alpha^{xp'}+\alpha^{pp'})\right]\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)-k^F\left(\frac{\epsilon^{p,I}}{pq}\right)\left(\frac{\epsilon^p}{p}\right)\\
&= L\left[(\alpha^{xp}+\alpha^{pp})+(J-1)(\alpha^{xp'}+\alpha^{pp'})\right]\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)\\
&\quad -k^F\left(\frac{\epsilon^p-(J-1)\epsilon^{p'}}{pq}\right)\left(\frac{\epsilon^p}{p}\right),\\
&= L\left[(\alpha^{xp}+\alpha^{pp})+(J-1)(\alpha^{xp'}+\alpha^{pp'})\right]\left(\frac{\epsilon^p}{p}\right)\left(\frac{\epsilon^x}{x}\right)-\frac{k^F}{q}\left(\frac{\epsilon^p}{p}\right)^2\\
&\quad +\frac{k^F}{q}(J-1)\left(\frac{\epsilon^{p'}}{p}\right)\left(\frac{\epsilon^p}{p}\right),
\end{aligned}
$$

both of which yield the desired result.

C First- and Second-Order Derivatives for Multinomial and Random-Coefficient Logit Demands

Tables 3 and 4 summarize the first- and second-order derivatives for multinomial logit demand under firm heterogeneity and random-coefficient logit demand with the possibility of firm heterogeneity, respectively.

D Derivation of Element Expressions in M

Below, we show how each of the eight elements in M is simplified in terms of the sufficient statistics.

Table 3: First- and Second-Order Derivatives for Multinomial Logit Demand

1st-Order	
$\frac{\partial q}{\partial p}$	$-\beta q(1-Jq)$
$\frac{\partial q}{\partial x}$	$\gamma\delta x^{\delta-1}q(1-Jq)$
$\frac{\partial q_j}{\partial p_j}$	$-\beta q_j(1-q_j)$
$\frac{\partial q_j}{\partial p_{j'}}$	$\beta q_j q_{j'}$
$\frac{\partial q_j}{\partial x_j}$	$(\gamma\delta)x_j^{\delta-1}q_j(1-q_j)$
$\frac{\partial q_j}{\partial x_{j'}}$	$-(\gamma\delta)x_{j'}^{\delta-1}q_j q_{j'}$
2nd-Order	
$D_{pp}\left(\equiv \frac{\partial^2 q_j}{\partial p_j^2}\right)$	$\beta^2 q_j(1-q_j)(1-2q_j)$
$D_{pp'}\left(\equiv \frac{\partial^2 q_j}{\partial p_j \partial p_{j'}}\right)$	$-\beta^2 q_j q_{j'}(1-2q_j)$
$D_{xx}\left(\equiv \frac{\partial^2 q_j}{\partial x_j^2}\right)$	$(\gamma\delta)x_j^{\delta-1}q_j(1-q_j)\left[(\delta-1)x_j^{-1}+(\gamma\delta)x_j^{\delta-1}(1-2q_j)\right]$
$D_{xx'}\left(\equiv \frac{\partial^2 q_j}{\partial x_j \partial x_{j'}}\right)$	$-(\gamma\delta)^2(x_j x_{j'})^{\delta-1}q_j q_{j'}(1-2q_j)$
$D_{xp}\left(\equiv \frac{\partial^2 q_j}{\partial x_j \partial p_j}\right)$	$-\beta(\gamma\delta)x_j^{\delta-1}q_j(1-q_j)(1-2q_j)$
$D_{xp'}\left(\equiv \frac{\partial^2 q_j}{\partial x_j \partial p_{j'}}\right)$	$\beta(\gamma\delta)x_j^{\delta-1}q_j q_{j'}(1-2q_j)$
$D_{px'}\left(\equiv \frac{\partial^2 q_j}{\partial p_j \partial x_{j'}}\right)$	$\beta(\gamma\delta)x_{j'}^{\delta-1}q_j q_{j'}(1-2q_j)$

Table 4: Random-Coefficient Logit Demand (with Firm Heterogeneity)

1st-Order	
$\frac{\partial q_j}{\partial p_j}$	$-\int_{i\in\mathcal{I}}\beta_i \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})dG(i)$
$\frac{\partial q_j}{\partial p_{j'}}$	$\int_{i\in\mathcal{I}}\beta_i \mathrm{Pr}_{ij}\mathrm{Pr}_{ij'}dG(i)$
$\frac{\partial q_j}{\partial x_j}$	$\delta x_j^{\delta-1}\int_{i\in\mathcal{I}}\gamma_i \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})dG(i)$
$\frac{\partial q_j}{\partial x_{j'}}$	$-\delta x_{j'}^{\delta-1}\int_{i\in\mathcal{I}}\gamma_i \mathrm{Pr}_{ij}\mathrm{Pr}_{ij'}dG(i)$
2nd-Order	
$\frac{\partial^2 q_j}{\partial p_j^2}$	$\int_{i\in\mathcal{I}}\beta_i^2 \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})(1-2\mathrm{Pr}_{ij})dG(i)$
$\frac{\partial^2 q_j}{\partial p_j \partial p_{j'}}$	$-\int_{i\in\mathcal{I}}\beta_i^2 \mathrm{Pr}_{ij}\mathrm{Pr}_{ij'}(1-2\mathrm{Pr}_{ij})dG(i)$
$\frac{\partial^2 q_j}{\partial x_j^2}$	$\delta x_j^{\delta-1}\int_{i\in\mathcal{I}}\gamma_i \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})\left[(\delta-1)x_j^{-1}+\left(\delta x_j^{\delta-1}\right)\gamma_i(1-2\mathrm{Pr}_{ij})\right]dG(i)$
$\frac{\partial^2 q_j}{\partial x_j \partial x_{j'}}$	$-\delta^2(x_j x_{j'})^{\delta-1}\int_{i\in\mathcal{I}}(\gamma_i)^2 \mathrm{Pr}_{ij}\mathrm{Pr}_{ij'}(1-2\mathrm{Pr}_{ij})dG(i)$
$\frac{\partial^2 q_j}{\partial p_j \partial x_j}$	$-\delta x_j^{\delta-1}\int_{i\in\mathcal{I}}(\beta_i\gamma_i)\mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})(1-2\mathrm{Pr}_{ij})dG(i)$
$\frac{\partial^2 q_j}{\partial x_j \partial p_{j'}}$	$\delta x_j^{\delta-1}\int_{i\in\mathcal{I}}(\beta_i\gamma_i)\mathrm{Pr}_{ij}\mathrm{Pr}_{ij'}(1-2\mathrm{Pr}_{ij})dG(i)$
$\frac{\partial^2 q_j}{\partial p_j \partial x_{j'}}$	$\delta x_{j'}^{\delta-1}\int_{i\in\mathcal{I}}(\beta_i\gamma_i)\mathrm{Pr}_{ij}\mathrm{Pr}_{ij'}(1-2\mathrm{Pr}_{ij})dG(i)$

D.1 $\partial F_j / \partial p_j$

$$\begin{aligned}\frac{\partial F_j}{\partial p_j} &= (1-L_j)\left(\frac{\epsilon_j^p}{p_j}\right) + L_j \frac{\partial \epsilon_j^p}{\partial p_j} \\ &= (1-L_j)\left(\frac{\epsilon_j^p}{p_j}\right) - L_j \cdot \left(\alpha_{jj}^{pp} - 1 - \epsilon_j^p\right)\left(\frac{\epsilon_j^p}{p_j}\right) \\ &= \left[1 - L_j \cdot \left(\alpha_{jj}^{pp} - \epsilon_j^p\right)\right]\left(\frac{\epsilon_j^p}{p_j}\right),\end{aligned}$$

where

$$\alpha_{jj}^{pp} \equiv -\frac{\partial^2 q_j}{\partial p_j^2}\frac{p_j}{\partial q_j/\partial p_j} = p_j \cdot \frac{\int_{i\in\mathcal{I}} \beta_i^2 \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})(1-2\mathrm{Pr}_{ij})dG(i)}{\int_{i\in\mathcal{I}} \beta_i \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})dG(i)},$$

because

$$\frac{\partial \epsilon_j^p}{\partial p_j} = -\left(\alpha_{jj}^{pp} - 1 - \epsilon_j^p\right)\left(\frac{\epsilon_j^p}{p_j}\right).$$

D.2 $\partial F_j / \partial p_{j'}$

$$\begin{aligned}\frac{\partial F_j}{\partial p_{j'}} &= L_j \frac{\partial \epsilon_j^p}{\partial p_{j'}} \\ &= L_j \cdot \left(\alpha_{jj'}^{pp'} - \epsilon_{j'}^{p'}\right)\left(\frac{\epsilon_j^p}{p_{j'}}\right),\end{aligned}$$

where

$$\alpha_{jj'}^{pp'} \equiv -\frac{\partial^2 q_j}{\partial p_j \partial p_{j'}}\frac{p_{j'}}{\partial q_j/\partial p_j} = -p_{j'} \cdot \frac{\int_{i\in\mathcal{I}} \beta_i^2 \mathrm{Pr}_{ij}\mathrm{Pr}_{ij'}(1-2\mathrm{Pr}_{ij})dG(i)}{\int_{i\in\mathcal{I}} \beta_i \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})dG(i)},$$

because

$$\frac{\partial \epsilon_j^p}{\partial p_{j'}} = \left(\alpha_{jj'}^{pp'} - \epsilon_{j'}^{p'}\right)\left(\frac{\epsilon_j^p}{p_{j'}}\right).$$

D.3 $\partial F_j / \partial x_j$

$$\begin{aligned}\frac{\partial F_j}{\partial x_j} &= -\frac{k_j^V}{p_j}\epsilon_j^p + L_j \frac{\partial \epsilon_j^p}{\partial x_j} \\ &= \left[-\frac{k_j^V x_j}{p_j} + L_j \cdot \left(\alpha_{jj}^{px} - \epsilon_j^p\right)\right]\left(\frac{\epsilon_j^p}{x_j}\right),\end{aligned}$$

where

$$\alpha_{jj}^{px} \equiv \frac{\partial^2 q_j}{\partial p_j \partial x_j}\frac{x_j}{\partial q_j/\partial p_j} = \delta x_j^\delta \cdot \frac{\int_{i\in\mathcal{I}} (\beta_i\gamma_i) \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})(1-2\mathrm{Pr}_{ij})dG(i)}{\int_{i\in\mathcal{I}} \beta_i \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})dG(i)},$$

because

$$\frac{\partial \epsilon_j^p}{\partial x_j} = \left(\alpha_{jj}^{px} - \epsilon_j^p\right)\left(\frac{\epsilon_j^p}{x_j}\right).$$

D.4 $\partial F_j / \partial x_{j'}$

$$\begin{aligned} \frac{\partial F_j}{\partial x_{j'}} &= L_j \frac{\partial \epsilon_j^p}{\partial x_{j'}} \\ &= L_j \cdot \left(\alpha_{jj'}^{px'} + \epsilon_{j'}^{x'} \right) \left(\frac{\epsilon_j^p}{x_{j'}} \right), \end{aligned}$$

where

$$\alpha_{jj'}^{px'} \equiv \frac{\partial^2 q_j}{\partial p_j \partial x_{j'}} \frac{x_{j'}}{\partial q_j / \partial p_j} = -\delta x_{j'}^{\delta} \cdot \frac{\int_{i \in \mathcal{I}} (\beta_i \gamma_i) \mathrm{Pr}_{ij} \mathrm{Pr}_{ij'} (1 - 2\mathrm{Pr}_{ij}) dG(i)}{\int_{i \in \mathcal{I}} \beta_i \mathrm{Pr}_{ij} (1 - \mathrm{Pr}_{ij}) dG(i)},$$

because

$$\frac{\partial \epsilon_j^p}{\partial x_{j'}} = \left(\alpha_{jj'}^{px'} + \epsilon_{j'}^{x'} \right) \left(\frac{\epsilon_j^p}{x_{j'}} \right).$$

D.5 $\partial G_j / \partial p_j$

$$\begin{aligned} \frac{\partial G_j}{\partial p_j} &= \left(\frac{\epsilon_j^x}{x_j} \right) + \left(\frac{L_j p_j}{x_j} \right) \frac{\partial \epsilon_j^x}{\partial p_j} - k_j^F \frac{\epsilon_j^p}{p_j q_j} \\ &= \left(\frac{\epsilon_j^x}{x_j} \right) + \left(\frac{L_j p_j}{x_j} \right) \left(\alpha_{jj}^{xp} + \epsilon_j^p \right) \left(\frac{\epsilon_j^x}{p_j} \right) - k_j^F \frac{\epsilon_j^p}{p_j q_j} \\ &= \left[1 + L_j \cdot \left(\alpha_{jj}^{xp} + \epsilon_j^p \right) \right] \left(\frac{\epsilon_j^x}{x_j} \right) - k_j^F \frac{\epsilon_j^p}{p_j q_j}, \end{aligned}$$

where

$$\alpha_{jj}^{xp} \equiv \frac{\partial^2 q_j}{\partial x_j \partial p_j} \frac{p_j}{\partial q_j / \partial x_j} = -p_j \cdot \frac{\int_{i \in \mathcal{I}} (\beta_i \gamma_i) \mathrm{Pr}_{ij} (1 - \mathrm{Pr}_{ij}) (1 - 2\mathrm{Pr}_{ij}) dG(i)}{\int_{i \in \mathcal{I}} \gamma_i \mathrm{Pr}_{ij} (1 - \mathrm{Pr}_{ij}) dG(i)},$$

because

$$\frac{\partial \epsilon_j^x}{\partial p_j} = \left(\alpha_{jj}^{xp} + \epsilon_j^p \right) \left(\frac{\epsilon_j^x}{p_j} \right).$$

D.6 $\partial G_j / \partial p_{j'}$

$$\begin{aligned} \frac{\partial G_j}{\partial p_{j'}} &= \left(\frac{L_j p_j}{x_j} \right) \frac{\partial \epsilon_j^x}{\partial p_{j'}} + \frac{\epsilon_{j'}^{p'}}{p_{j'} q_j} \\ &= \left(\alpha_{jj'}^{xp'} - \epsilon_{j'}^{p'} \right) \left(\frac{L_j p_j}{x_j} \right) \left(\frac{\epsilon_j^x}{p_{j'}} \right) + \frac{\epsilon_{j'}^{p'}}{p_{j'} q_j}, \end{aligned}$$

where

$$\alpha_{jj'}^{xp'} \equiv \frac{\partial^2 q_j}{\partial x_j \partial p_{j'}} \frac{p_{j'}}{\partial q_j / \partial x_j} = p_{j'} \cdot \frac{\int (\beta_i \gamma_i) \mathrm{Pr}_{ij} \mathrm{Pr}_{ij'} (1 - 2\mathrm{Pr}_{ij}) dG(i)}{\int_{i \in \mathcal{I}} \gamma_i \mathrm{Pr}_{ij} (1 - \mathrm{Pr}_{ij}) dG(i)},$$

because

$$\frac{\partial \epsilon_j^x}{\partial p_{j'}} = \left(\alpha_{jj'}^{xp'} - \epsilon_{j'}^{p'} \right) \left(\frac{\epsilon_j^x}{p_{j'}} \right).$$

D.7 $\partial G_j/\partial x_j$

$$\begin{aligned}\frac{\partial G_j}{\partial x_j} &= \left(-k_j^V + \frac{k_j^F}{q_j}\right)\left(\frac{\epsilon_j^x}{x_j}\right) + \left(\frac{L_j p_j}{x_j}\right)\left(\frac{\partial \epsilon_j^x}{\partial x_j} - \frac{\epsilon_j^x}{x_j}\right)\\ &= \left[\left(-k_j^V + \frac{k_j^F}{q_j}\right) + \left(\alpha_{jj}^{xx} - \epsilon_j^x\right)\left(\frac{L_j p_j}{x_j}\right)\right]\left(\frac{\epsilon_j^x}{x_j}\right),\end{aligned}$$

where

$$\alpha_{jj}^{xx} \equiv \frac{\partial^2 q_j}{\partial x_j^2}\frac{x_j}{\partial q_j/\partial x_j} = x_j \cdot \frac{\int_{i\in\mathcal{I}} \gamma_i \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})\left[(\delta-1)x_j^{-1} + \left(\delta x_j^{\delta-1}\right)\gamma_i(1-2\mathrm{Pr}_{ij})\right]dG(i)}{\int_{i\in\mathcal{I}} \gamma_i \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})dG(i)},$$

because

$$\frac{\partial \epsilon_j^x}{\partial x_j} = \left(\alpha_{jj}^{xx} + 1 - \epsilon_j^x\right)\left(\frac{\epsilon_j^x}{x_j}\right).$$

D.8 $\partial G_j/\partial x_{j'}$

$$\begin{aligned}\frac{\partial G_j}{\partial x_{j'}} &= \left(\frac{L_j p_j}{x_j}\right)\frac{\partial \epsilon_j^x}{\partial x_{j'}} - \frac{k_j^F}{q_j}\left(\frac{\epsilon_{j'}^{x'}}{x_{j'}}\right)\\ &= \left(\alpha_{jj'}^{xx'} + \epsilon_{j'}^{x'}\right)\left(\frac{L_j p_j}{x_j}\right)\left(\frac{\epsilon_j^x}{x_{j'}}\right) - \frac{k_j^F}{q_j}\left(\frac{\epsilon_{j'}^{x'}}{x_{j'}}\right),\end{aligned}$$

where

$$\alpha_{jj'}^{xx'} \equiv \frac{\partial^2 q_j}{\partial x_j \partial x_{j'}}\frac{x_{j'}}{\partial q_j/\partial x_j} = -\delta x_{j'}^{\delta} \cdot \frac{\int_{i\in\mathcal{I}} (\gamma_i)^2 \mathrm{Pr}_{ij}\mathrm{Pr}_{ij'}(1-2\mathrm{Pr}_{ij})dG(i)}{\int_{i\in\mathcal{I}} \gamma_i \mathrm{Pr}_{ij}(1-\mathrm{Pr}_{ij})dG(i)},$$

because

$$\frac{\partial \epsilon_j^x}{\partial x_{j'}} = \left(\alpha_{jj'}^{xx'} + \epsilon_{j'}^{x'}\right)\left(\frac{\epsilon_j^x}{x_{j'}}\right).$$

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