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Fertility, Education, Inequality, and Economic Growth

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Fertility, Education, Inequality, and Economic Growth*

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Abstract

This study extends the Barro–Becker–Bewley model of endogenous fertility and intergenerational transfers by incorporating human capital investment in children and life-cycle savings. Calibrating the model to the U.S. economy, this study quantitatively analyzes the potential effects of child-related policies such as child allowances and education subsidies. Child allowances raise fertility in the short run but reduce investment in human and physical capital, thereby suppressing economic growth. Education subsidies reduce fertility in the short run but enhance welfare across all generations. The effects of child-related policies on income and wealth inequality and intergenerational income mobility are generally limited.

JEL Classification Numbers: C61, D15, H31, I24, J13.

Key Words: dynamic general equilibrium; heterogeneous agents; overlapping generations.

1 Introduction

Over the past several decades, various child-related policies have been introduced in many countries. These policies aim to improve the welfare of children and young households, reduce poverty rates and inequality, and promote economic growth. In some countries, they also aim to prevent further declines in birth rates. However, the outcomes of these diverse child-related policies are difficult to predict empirically, except for short-term direct effects. Therefore, the purpose of this study is to provide a comprehensive theoretical and quantitative analysis of child-related policies to complement existing empirical research.

The present study extends the Barro–Becker model (Becker and Barro, 1988; Barro and Becker, 1989) on endogenous fertility and intergenerational transfers by incorporating education expenditure or human capital investment in children and life-cycle saving decisions (de la

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Croix and Doepke, 2003). In addition, this study transforms this extended model into a Bewley-type incomplete market model by incorporating idiosyncratic labor income shocks, human capital shocks, and wealth shocks.[1] Calibrating the model to the U.S. economy, this study quantitatively analyzes the potential effects of child-related policies—such as child allowances, education subsidies, and childcare and paid leave subsidies—on heterogeneous household decision-making, economic growth, and the distribution of income and wealth.

The model economy in this study consists of heterogeneous households living through two adult periods, a representative firm, and the government. Households are heterogeneous regarding initial assets, human capital (labor productivity), and idiosyncratic labor income shocks. In each period, young households choose their consumption, savings, number of children (fertility), inter vivos transfers to children, and education expenditures (human capital investment) for children to maximize their lifetime utility and dynastic value. Child-related policies first affect each household's number of children, inter vivos transfers, and education expenditure. Second, the decisions of young (parent) households influence the initial assets and human capital of the next generation (children), amplifying or dampening the policy effects in the second period and beyond.

The contributions of this study are as follows. First, it constructs a Barrow–Becker–Bewley model incorporating human capital investment to serve as a foundation for comprehensively analyzing various child-related policies concerning fertility rates, intergenerational transfers, and education expenditures. Second, it empirically verifies that both the income elasticity and working ability elasticity of the number of children are positive, though not particularly high. Third, by combining this model with empirical findings, it quantitatively demonstrates the potential short-run and long-run effects of child-related policies on household behavior and the aggregate economy.

The policy experiments in the present study suggest that behavioral responses to child-related policies in the model economy vary across households depending on their human capital and wealth. However, the average impact of these policies on income and wealth inequality is limited. This is due to the positive but low income (working ability) elasticity of fertility and the fact that altruistic households' optimal decisions involve intergenerational risk sharing, which is likely to mitigate income and wealth inequality. Policy experiments also suggest that any child-related policy aimed at promoting economic growth and improving the welfare of future generations will ultimately raise fertility rates, not only in the long run but even in the short run. Furthermore, because the calibrated economy is dynamically efficient, as in the current U.S. economy, child-related policies that promote capital accumulation—physical capital, human capital, or both—are likely to raise fertility rates in the second period and beyond.

The previous literature that the present study is based on are as follows. Loury (1981) and Laitner (1988, 1991) construct Bewley-type OLG-dynasty models with intergenerational transfers. Becker and Barro (1988) and Barro and Becker (1989) construct deterministic dy-

[1]Children do not inherit labor income shocks but partially inherit human capital shocks of their parents.

nasty models of optimal fertility, consumption, and intergenerational transfers in a small-open economy and a closed economy, respectively. Alvarez (1999) combines the above features and constructs a Bewley-type dynasty model, in which households choose their optimal fertility, consumption, and intergenerational transfers.

Nishiyama (2002) constructs a altruistically linked four-period OLG-dynasty model , in which the parents and their (adult) children are altruistically linked, and these households strategically choose the optimal sizes of inter vivos transfers (in both directions) and bequests. de la Croix and Doepke (2003) construct a two-period deterministic OLG model, in which the young households (parents) receive a warm-glow utility from their children and choose the optimal number of children and education spending for children.

More recent studies that construct an OLG model or a dynasty model with fertility decision, intergenerational transfers, and/or education spending include the following. Stantcheva (2015) constructs a dynasty model, in which the households are heterogeneous with respect to their initial wealth and human capital, and they choose the education spending and bequests; and this study analyzes the effects of income, education, and bequest taxes. Córdoba et al. (2016) constructs a dynasty model, in which the households are heterogeneous with respect to their initial wealth and labor productivity, and they choose the optimal number of children and bequests per child; and this study evaluates the effect of endogenous fertility on social mobility and long-run inequality.

Daruich and Kozlowski (2020) construct an altruistically linked OLG model, in which the adult households in each period are heterogeneous with respect to their wealth, human capital, education attainment, human capital shocks, and they choose the discrete number of children, children's consumption, and inter vivos transfers that affect the children's discrete choices on schooling; and this study evaluates the effect of endogenous fertility and transfers on intergenerational mobility. Zhou (2025) also constructs an altruistically linked OLG model with single and married households, in which the households are heterogeneous with respect to their wealth and human capital, and married households choose the number of children, time input (at home childcare), and monetary input (education spending) for their children to maximize the lifetime and dynastic utility; and this study evaluates the effect of endogenous fertility, childcare, and education spending on macroeconomic variables.

The rest of this paper is presented as follows: Section 2 provides the empirical background of the quantitative model, Section 3 describes the Barro–Becker–Bewley model with human capital investment, Section 4 describes the calibration strategy to the U.S. economy and shows the baseline economy, Section 5 explains the possible effects of six child-related policies in equilibrium transition paths, and Section 6 concludes the paper. Appendix describes the computational algorithms used to find the optimal decisions of heterogeneous households and to solve the model for equilibrium transition paths.

2 Empirical Background

This section first discusses the empirical relationship between the number of children in each married household and the household's total income, labor income, or hourly wages. Next, this section discusses the effects of hourly wages and the number of children on the working hours of husbands and wives.

2.1 Income and Hourly Wages on the Number of Children

We first estimate the effects of total income, labor income, and hourly wages on the number of children in each married household, using the combined cross-section data from the Survey of Consumer Finances (SCF) of years 2016 and 2019. We only use the data of married households with wives' ages between 35 and 54 and with strictly positive income and labor income. The models we estimate are

$$(1) \qquad kids = \beta_0 + \beta_1 \ln income + \boldsymbol{\gamma}\mathbf{X} + u,$$

$$(2) \qquad kids = \beta_0 + \beta_1 \ln wageinc + \boldsymbol{\gamma}\mathbf{X} + u,$$

$$(3) \qquad kids = \beta_0 + \beta_1 \ln hwage + \boldsymbol{\gamma}\mathbf{X} + u,$$

$$(4) \qquad kids = \beta_0 + \beta_1 \ln hwage_m + \beta_2 \ln hwage_f + \boldsymbol{\gamma}\mathbf{X} + u,$$

where $kids$ is the number of children, $income$ is the household's total income, $wageinc$ is the household's total labor income, $hwage$ is the household's hourly wage, and $hwage_m$ and $hwage_f$ are the hourly wages of husband and wife, respectively; and $\mathbf{X}$ are the controls,

$$(5) \qquad \boldsymbol{\gamma}\,\mathbf{X} = \gamma_1\, age_f + \gamma_2\, age_f^2 + \gamma_3\, educ\_hs_m + \gamma_4\, educ\_hs_f + \gamma_5\, race\_wh + \gamma_6\, year_{2016},$$

where age_f is the wife's age, $educ\_hs_m$ and $educ\_hs_f$ are dummy variables for the husband and wife having a high school diploma (or higher), respectively, $race\_wh$ takes the value 1 if the husband is white and non-Hispanic, and $year_{2016}$ takes the value 1 if the household is from the 2016 SCF.

The top panel of Table 1 shows the summary statistics of the SCF subset of married households with wives' ages between 35 and 54, and the first four columns of Table 2 show the results of the weighted least squares (WLS) regressions on the number of children. The income semi-elasticity of (the effect of the rate of change in income on) the number of children is 0.1613, the wage income semi-elasticity is 0.1279, the hourly wage semi-elasticity is 0.1947, and the semi-elasticity with respect to the husband's hourly wage is 0.1628. These estimates imply that children are normal goods, and the household's income effects are in general positive. The coefficient of the wife's hourly wage is negative but insignificant. This is partly because a larger share of wives, relative to husbands, are not working in the market and their hourly wages

Table 1: Summary Statistics of the Subsets of SCF 2016 and 2019

	Mean	SD	Min	Max	N
Married Households with Wives' Ages between 35 *and* 54					
$kids$	1.5541	1.2449	0.	7.	2606
$\ln income$	11.7285	0.8479	7.7666	18.8776	2606
$\ln wageinc$	11.5352	0.9126	2.4683	16.1175	2606
$\ln hwage$	3.2668	0.6848	2.3026	8.1069	2606
$\ln hwage_m$	3.2376	0.7630	2.3026	8.4780	2606
$\ln hwage_f$	2.9109	0.6540	2.3026	7.5104	2606
age_f	44.1600	5.7351	35.	54.	2606
$educ\_hs_m$	0.8834	0.3210	0.	1.	2606
$educ\_hs_f$	0.9144	0.2798	0.	1.	2606
$race_wh$	0.6440	0.4789	0.	1.	2606
Married Households with Wives' Ages between 25 *and* 54					
$hours_m$	2151.95	882.62	0.	6760.	3563
$hours_f$	1473.30	994.98	0.	5200.	3563
$kids$	1.5010	1.2598	0.	7.	3563
$\ln hwage_m$	3.1769	0.7125	2.3026	9.0628	3563
$\ln hwage_f$	2.8697	0.6178	2.3026	7.5104	3563
$health_m$	1.9564	0.7190	1.	4.	3563
$health_f$	1.8770	0.7230	1.	4.	3563
age_m	41.9362	9.8801	21.	86.	3563
age_f	39.3668	8.3633	25.	54.	3563
$educ\_hs_m$	0.8910	0.3116	0.	1.	3563
$educ\_hs_f$	0.9236	0.2658	0.	1.	3563
$race_wh$	0.6391	0.4803	0.	1.	3563

Note: Numbers of mean and standard deviation are calculated with the SCF weights. Numbers of income and wage income are bottom coded at $1, and numbers of hourly wages are bottom coded at $10. Hourly wages of those with zero working hours are set to $10. The health states are set to 2 (Good) if they are 0 (inappropriate).

are imputed to be $10. Yet, this is also possibly because wives tend to spend more time for childcare, and the wives' working ability is positively related to the cost of raising children.

When we divide the above semi-elasticities by the average number of children, 1.5541, in Table 1, the income elasticity of the number of children is 0.1038, the wage income elasticity is 0.0823, the hourly wage elasticity is 0.1253, and the husband's wage elasticity is 0.1048. This study uses one of these elasticities of the average number of children to construct the initial steady-state (baseline) economy in Section 4.

2.2 Hourly Wages and the Number of Children on Working Hours

Next, we estimate the effects of the couple's hourly wages and the number of children on each spouse's working hours, using the cross-section data from the Survey of Consumer Finances of years 2016 and 2019. We only use the data of married households with wives' ages between 25 and 54 and with strictly positive income and labor income. The models we

Table 2: The Effects of Income, Labor Income, and Hourly Wages on the Number of Children and Working Hours

	kids				$hours_m$	$hours_f$
	(1)	(2)	(3)	(4)	(6)	(7)
$\ln income$	0.1613					
	(0.0238)					
$\ln wageinc$		0.1279				
		(0.0220)				
$\ln hwage$			0.1947			
			(0.0287)			
$\ln hwage_m$				0.1628	356.94	-275.17
				(0.0260)	(19.48)	(19.73)
$\ln hwage_f$				-0.0145	-125.87	758.63
				(0.0335)	(19.86)	(26.95)
$kids$					34.27	-113.77
					(8.85)	(10.91)
$health_m$					-155.86	35.68
					(18.05)	(19.81)
$health_f$					56.36	-57.40
					(18.94)	(19.21)
age_m					56.41	
					(8.24)	
age_m^2					-0.75	
					(0.10)	
age_f	0.2344	0.2355	0.2472	0.2426		62.99
	(0.0762)	(0.0759)	(0.0763)	(0.0767)		(14.77)
age_f^2	-0.0034	-0.0034	-0.0035	-0.0035		-0.81
	(0.0009)	(0.0009)	(0.0009)	(0.0009)		(0.18)
$educ\_hs_m$	-0.2941	-0.2758	-0.2782	-0.2834	172.14	92.06
	(0.0682)	(0.0669)	(0.0643)	(0.0648)	(42.59)	(44.61)
$educ\_hs_f$	-0.0910	-0.0703	-0.0553	-0.0157	-78.51	409.50
	(0.0769)	(0.0766)	(0.0756)	(0.0757)	(54.12)	(50.76)
$race\_wh$	-0.1518	-0.1361	-0.1383	-0.1372	109.70	-26.03
	(0.0462)	(0.0457)	(0.0453)	(0.0454)	(26.11)	(24.29)
$year_{2016}$	0.0135	0.0116	0.0300	0.0233	19.47	15.99
	(0.0426)	(0.0425)	(0.0430)	(0.0432)	(20.89)	(22.26)
$intercept$	-3.5009	-3.1816	-2.6097	-2.4116	385.62	-1244.96
	(1.6765)	(1.6558)	(1.6696)	(1.6754)	(187.71)	(282.36)
N	2606	2606	2606	2606	3559	3559
R^2	0.1145	0.1122	0.1149	0.1136	0.1518	0.2953
SD	0.0004	0.0003	0.0003	0.0002	0.0011	0.0015

Note: All regression results are estimated by weighted least squares (WLS) using the merged data set of SCF 2016 and 2019.

estimate are

$$(6) \quad hours_m = \beta_0 + \beta_1 \ln hwage_m + \beta_2 \ln hwage_f + \beta_3\, kids + \boldsymbol{\delta}\mathbf{X}_m + u,$$

(7) $$hours_f = \beta_0 + \beta_1 \ln hwage_m + \beta_2 \ln hwage_f + \beta_3\, kids + \boldsymbol{\delta}\mathbf{X}_f + u,$$

where $hours_m$ and $hours_f$ are annual working hours of husbands and wives, respectively, and $\mathbf{X}_m$ and $\mathbf{X}_f$ are the controls,

(8) $$\boldsymbol{\delta}\mathbf{X}_m = \delta_1\, health_m + \delta_2\, health_f + \delta_3\, age_m + \delta_4\, age_m^2 + \delta_5\, educ\_hs_m + \delta_6\, educ\_hs_f + \delta_7\, race\_wh + \delta_8\, year_{2016},$$

(9) $$\boldsymbol{\delta}\mathbf{X}_f = \delta_1\, health_m + \delta_2\, health_f + \delta_3\, age_f + \delta_4\, age_f^2 + \delta_5\, educ\_hs_m + \delta_6\, educ\_hs_f + \delta_7\, race\_wh + \delta_8\, year_{2016},$$

where age_m is the husband's age, and $health_m$ and $health_f$ are the health conditions of husband and wife, respectively, each of which takes 1 when the condition is excellent, 2 when it is good, 3 when it is fair, and 4 when it is poor.

The bottom panel of Table 1 shows the summary statistics of the SCF subset of married households with wives' ages between 25 and 54, and last two columns of Table 2 show the results of the WLS regressions on the working hours of husband and wife. The hourly wage semi-elasticity of husband's working hours 356.94 hours, and the hourly wage semi-elasticity of wife's working hours 758.63 hours. The cross elasticity of the working hours with respect to their spouses' hourly wages are both negative, which supports the assumption that the married households determine their labor supply jointly and altruistically. The effect of an additional child on the husband's working hours is $+34.27$ hours and that on the wife's working hours is -133.77.

If there were no time costs of raising children at all but only financial costs of that, this study presumes, both the husband and wife would increase their labor supply equally by 34.27 hours. However, the wife would reduce her working hours by 113.77, according to the estimate, and the wife would spend the difference, $34.27 + 113.77 = 148.07$ hours, for the at-home childcare of the additional child. It might be the case that the husband would also spend some additional hours for the childcare in addition to working longer hours. It is also likely that they would purchase some additional childcare services from the market. Therefore, this study assumes that the lower bound of the time cost of childcare per child to be 148 hours or 4.1% of the average working hours of married households, $2152 + 1473 = 3625$ hours, in Table 1.

3 Model Economy

The economy consists of a large number of heterogeneous and overlapping households that are altruistically linked to their descendants, a large number of representative firms with constant-returns-to-scale production technology, and a government that can commit to its fiscal policy schedule. Time is discrete, and all variables are growth adjusted with the long-run growth rate ρ in this model economy. Households live through two periods, and there are two types of households, young (working-age) and old (partially-retired), in each period in the economy. Households are heterogeneous with respect to their initial wealth, a_t, human capital, h_t, and a working (earning) ability shock, ε_t. These households can work for $1+\zeta$ periods, where $\zeta \in [0,1]$. Representative firms, in each period, choose the capital input and the labor input to maximize their profits. The government follows the fiscal policy rule from period t and onward.

Let Ω_t be a time series of the government policy variables and the factor prices,

$$(10) \quad \Omega_t = \{\tau_{k,s}, \tau_{h,s}, \tau_{c,s}, \tau_{n,s}, \tau_{b,s}, \tau_{e,s}, \tau_{p,s}, tr_{p,s}, \vartheta_s, \psi_s, g_s, r_s, w_s\}_{s=t}^{\infty},$$

where $\tau_{k,t}$ is a capital income tax rate, $\tau_{h,t}$ is a labor income tax rate, $\tau_{c,t}$ is a consumption tax rate, $\tau_{n,t}$ is a child tax (allowance if negative) rate, $\tau_{b,t}$ is an estate (inter vivos transfer) tax rate, $\tau_{e,t}$ is an education tax (subsidy if negative) rate, $\tau_{p,t}$ is a payroll tax rate for public pension, $tr_{p,s}$ is a public pension benefit, ϑ_t is public education spending per child, ψ_t is childcare and paid leave subsidies (in hours) per child, g_t is the other government spending per household, r_t is the real interest rate, and w_t is the real wage rate per efficiency unit of labor.

3.1 Heterogeneous Households' Problem

Let's consider a young household born in period t. Let $a_t \in A = [0, \infty)$ be the initial wealth of this household, let $h_t \in H = (0, \infty)$ be the human capital, let $\varepsilon_t > 0$ be the i.i.d. working ability shock, where the expected value of $\exp(\varepsilon_t)$ is normalized to unity. Let c_t be the consumption per unit of family members in the first period, let s_t be the wealth at the end of the first (young) period, let d_{t+1} be growth-adjusted consumption in the second (old) period, let n_t be the number of children of this household, let b_t be the inter vivos transfer per unit of children, and let e_t be the education spending per unit of children.[2] Then, the household's optimization problem is

$$(11) \quad v(a_t, h_t, \varepsilon_t; \Omega_t) = \max_{c_t, s_t, n_t, b_t, e_t} \left\{ u(c_t) + \tilde{\beta} u(d_{t+1}) + \tilde{\gamma}\, \Phi(n_t) \mathbb{E}_t \big[v(a_{t+1}, h_{t+1}, \varepsilon_{t+1}; \Omega_{t+1}) \big] \right\}$$

subject to the budget constraint and the laws of motion of the state variables,

$$(12) \quad (1+\tau_{c,t})(1+n_t)^{\xi} c_t + s_t + \tau_{n,t} n_t + (1+\tau_{b,t}) b_t n_t + (1+\tau_{e,t}) e_t n_t$$
$$= (1 + (1-\tau_{k,t}) r_t) a_t + (1-\tau_{h,t}-\tau_{p,t}) w_t h_t \exp(\varepsilon_t)(1-(\phi-\psi_t) n_t),$$

$$(13) \quad (1+\tau_{c,t+1}) d_{t+1}$$
$$= (1 + (1-\tau_{k,t+1}) r_{t+1}) \frac{s_t}{1+\rho} + (1-\tau_{h,t+1}-\tau_{p,t+1}) w_{t+1} \frac{h_t \exp(\varepsilon_t)\, \zeta}{1+\rho} + tr_{p,t+1},$$

[2] The unit of children, n_t, in the model economy is 2.0 in the real economy.

$$(14)\quad a_{t+1} = \frac{1}{1+\rho} b_t \exp(\epsilon_{a,t+1}) \geq 0, \qquad \epsilon_{a,t+1} \sim N(-\sigma^2_{\epsilon_a}/2, \sigma^2_{\epsilon_a}),$$

$$(15)\quad h_{t+1} = \frac{1}{1+\rho}\Lambda(e_t, h_t, \vartheta_t; \bar{h}_t) \exp(\epsilon_{h,t+1}) > 0, \qquad \epsilon_{h,t+1} \sim N(-\sigma^2_{\epsilon_h}/2, \sigma^2_{\epsilon_h}),$$

$$(16)\quad \varepsilon_{t+1} \sim N(-\sigma^2_{\varepsilon}/2, \sigma^2_{\varepsilon}),$$

$$(17)\quad c_t > 0, \qquad s_t \geq 0, \qquad n_t \geq 0, \qquad b_t \geq 0, \qquad e_t \geq 0,$$

where $v(a_t, h_t, \varepsilon_t)$ is a value function, $u(c_t)$ is a period utility function, $\tilde{\beta} \in (0, 1)$ is a growth-adjusted discount factor, $\tilde{\gamma} \in (0, \tilde{\beta}\,]$ is the growth-adjusted and discounted degree of altruism, $\Phi(n_t)$ is a scale function of n_t children, $(1 + n_t)^\xi$ is the adult-equivalent number of family members, ϕ is the time cost of childcare per dependent child, $\Lambda(e_t, h_t, \vartheta_t; \bar{h}_t)$ is a human capital accumulation function; and $\epsilon_{a,t}$ and $\epsilon_{h,t}$ are the i.i.d. shocks to inter vivos transfers to their children and the human capital of their children, respectively.

The period utility function is one of constant relative risk aversion,

$$(18)\quad u(c_t) = \frac{c_t^{1-\sigma} - c_{\min}^{1-\sigma}}{1-\sigma}, \qquad u'(c_t) > 0, \qquad u''(c_t) < 0,$$

where σ is the coefficient of relative risk aversion, and $c_{\min}$ is a shifting parameter of the utility function such that $u(c_t) \geq 0$ for all $c_t \geq c_{\min}$.[3] When the discount factors per period are β and γ, the growth-adjusted discount factors are

$$(19)\quad \tilde{\beta} = (1+\rho)^{1-\sigma}\beta, \qquad \tilde{\gamma} = (1+\rho)^{1-\sigma}\gamma.$$

The scaling function of n_t children is a standardized version of exponential child discounting as in Córdoba et al. (2016),

$$(20)\quad \Phi(n_t) = \frac{1-\exp(-\mu n_t)}{1-\exp(-\mu)}, \qquad \Phi(0) = 0, \qquad \Phi(1) = 1, \qquad \lim_{n_t \to \infty} \Phi(n_t) = \frac{1}{1-\exp(-\mu)},$$

where μ is inversely related to the elasticity of $\Phi(n_t)$ with respect to n_t, and for all $n_t > 0$,

$$\lim_{\mu \to 0} \Phi(n_t) = n_t, \qquad \lim_{\mu \to \infty} \Phi(n_t) = 1.$$

The inter-generational law of motion of human capital is one in de la Croix and Doepke (2003),

$$(21)\quad \Lambda(e_t, h_t, \vartheta_t; \bar{h}_t) = B(\vartheta_t + e_t)^\eta\, h_t^\tau\, \bar{h}_t^\kappa,$$

where B is the scale parameter of human capital, and $\bar{h}_t$ is the average level of human capital

[3]The elasticity of inter-temporal substitution, $1/\sigma$, must be less than one; otherwise, the household's total consumption would be negatively correlated to the number of family members. Yet, the utility value, $u(c_t)$, must be mostly positive; otherwise the optimal number of children would be zero.

of young households in period t.

Appendix to this paper describes the Kuhn–Tucker conditions and the complementarity problem to solve the household's optimization problem, (11)–(17). Solving this problem for the optimal decisions yields the decision rules, $c_t(a_t, h_t, \varepsilon_t; \Omega_t)$, $s_t(a_t, h_t, \varepsilon_t; \Omega_t)$, $n_t(a_t, h_t, \varepsilon_t; \Omega_t)$, $b_t(a_t, h_t, \varepsilon_t; \Omega_t)$, $e_t(a_t, h_t, \varepsilon_t; \Omega_t)$, and

$$(22)\quad d_{t+1}(a_t, h_t, \varepsilon_t; \Omega_t) = \frac{1}{1+\tau_{c,t+1}} \times \left[(1 + (1-\tau_{k,t+1})r_{t+1}) \frac{s_t(a_t, h_t, \varepsilon_t; \Omega_t)}{1+\rho} + (1-\tau_{h,t+1}-\tau_{p,t+1})w_{t+1} \frac{h_t \exp(\varepsilon_t)\zeta}{1+\rho} + tr_{p,t+1} \right].$$

The value function of the household born in period t is obtained as

$$(23)\quad v(a_t, h_t, \varepsilon_t; \Omega_t) = u(c_t(a_t, h_t, \varepsilon_t; \Omega_t)) + \tilde{\beta} u(d_{t+1}(a_t, h_t, \varepsilon_t; \Omega_t)) + \tilde{\gamma}\Phi(n_t(a_t, h_t, \varepsilon_t; \Omega_t)) \mathbb{E}_t \big[v(a_{t+1}(a_t, h_t, \varepsilon_t, \epsilon_{a,t+1}; \Omega_t), h_{t+1}(a_t, h_t, \varepsilon_t, \epsilon_{h,t+1}; \Omega_t), \varepsilon_{t+1}; \Omega_{t+1}) \big],$$

where

$$(24)\quad a_{t+1}(a_t, h_t, \varepsilon_t, \epsilon_{a,t+1}; \Omega_t) = \frac{1}{1+\rho} b_t(a_t, h_t, \varepsilon_t; \Omega_t) \exp(\epsilon_{a,t+1}),$$

$$(25)\quad h_{t+1}(a_t, h_t, \varepsilon_t, \epsilon_{h,t+1}; \Omega_t) = \frac{1}{1+\rho} \Lambda(e_t(a_t, h_t, \varepsilon_t; \Omega_t), h_t, \vartheta_t; \bar{h}_t) \exp(\epsilon_{h,t+1}).$$

Note that consumption of the old household, born in period $t-1$, after a policy change in period t is

$$(26)\quad d_t(a_{t-1}, h_{t-1}, \varepsilon_{t-1}; \Omega_{t-1}, \Omega_t) = \frac{1}{1+\tau_{c,t}} \times \left[(1 + (1-\tau_{k,t})r_t) \frac{s_{t-1}(a_{t-1}, h_{t-1}, \varepsilon_{t-1}; \Omega_{t-1})}{1+\rho} + (1-\tau_{h,t}-\tau_{p,t})w_t \frac{h_{t-1} \exp(\varepsilon_{t-1})\zeta}{1+\rho} + tr_{p,t} \right],$$

and that the value (lifetime utility) function evaluated with a policy change in period t is

$$(27)\quad v(a_{t-1}, h_{t-1}, \varepsilon_{t-1}; \Omega_{t-1}, \Omega_t) = u(c_{t-1}(a_{t-1}, h_{t-1}, \varepsilon_{t-1}; \Omega_{t-1})) + \tilde{\beta} u(d_t(a_{t-1}, h_{t-1}, \varepsilon_{t-1}; \Omega_{t-1}, \Omega_t)) + \tilde{\gamma}\Phi(n_{t-1}(a_{t-1}, h_{t-1}, \varepsilon_{t-1}; \Omega_{t-1})) \times \mathbb{E}_{t-1} \big[v(a_t(a_{t-1}, h_{t-1}, \varepsilon_{t-1}, \epsilon_{a,t}; \Omega_{t-1}), h_t(a_{t-1}, h_{t-1}, \varepsilon_{t-1}, \epsilon_{h,t}; \Omega_{t-1}), \varepsilon_t; \Omega_t) \big].$$

3.2 The Joint Distribution of Households

The population of young households is growth adjusted and normalized to unity in each period. Let $X_t(a_t, h_t, \varepsilon_t)$ be a cumulative distribution function of the heterogeneous households born in period t, and let $x_t(a_t, h_t, \varepsilon_t)$ be the corresponding density function, where

$$(28)\quad \int_{A\times H\times \mathbf{R}} dX_t(a_t, h_t, \varepsilon_t) = 1.$$

Let $\bar{n}_t$ be the total (and average) number of children of young households in period t,

$$(29)\quad \bar{n}_t = \int_{A\times H\times \mathbf{R}} n_t(a_t, h_t, \varepsilon_t; \Omega_t)\, dX_t(a_t, h_t, \varepsilon_t),$$

which is equal to the gross population growth rate from period t to period $t+1$. Then, the law of motion of $x_t(a_t, h_t, \varepsilon_t)$ is

$$(30)\quad x_{t+1}(a_{t+1}, h_{t+1}, \varepsilon_{t+1}) = \frac{1}{\bar{n}_t} \int_{A\times H\times \mathbf{R}^3} \mathbb{1}_{\{a_{t+1}=a_{t+1}(a_t,h_t,\varepsilon_t,\epsilon_{a,t+1};\Omega_t)\}} \mathbb{1}_{\{h_{t+1}=h_{t+1}(a_t,h_t,\varepsilon_t,\epsilon_{h,t+1};\Omega_t)\}} \\ \times n_t(a_t, h_t, \varepsilon_t; \Omega_t)\, dF_a(\epsilon_{a,t+1})\, dF_h(\epsilon_{h,t+1})\, dX_t(a_t, h_t, \varepsilon_t),$$

where $F_a(\epsilon_{a,t})$ and $F_h(\epsilon_{h,t})$ are the cumulative distribution functions of inter vivos transfer shock, $\epsilon_{a,t}$, and human capital shock, $\epsilon_{h,t}$, respectively.

Then, total private consumption, other than education spending, in period t, C_t, is

$$(31)\quad C_t = \int_{A\times H\times \mathbf{R}} (1 + n_t(a_t, h_t, \varepsilon_t; \Omega_t))^{\xi} c_t(a_t, h_t, \varepsilon_t; \Omega_t)\, dX_t(a_t, h_t, \varepsilon_t) \\ + \frac{1}{\bar{n}_{t-1}} \int_{A\times H\times \mathbf{R}} d_t(a_{t-1}, h_{t-1}, \varepsilon_{t-1}; \Omega_{t-1}, \Omega_t)\, dX_{t-1}(a_{t-1}, h_{t-1}, \varepsilon_{t-1}),$$

where $1/\bar{n}_{t-1}$ is the population of old households in this period. Total private wealth at the beginning of period t, W_t, is

$$(32)\quad W_t = \int_{A\times H\times \mathbf{R}} a_t\, dX_t(a_t, h_t, \varepsilon_t) \\ + \frac{1}{\bar{n}_{t-1}} \int_{A\times H\times \mathbf{R}} \frac{s_{t-1}(a_{t-1}, h_{t-1}, \varepsilon_{t-1}; \Omega_{t-1})}{1+\rho}\, dX_{t-1}(a_{t-1}, h_{t-1}, \varepsilon_{t-1}),$$

and total labor supply in efficiency units in period t, L_t^s, is

$$(33)\quad L_t^s = \int_{A\times H\times \mathbf{R}} h_t \exp(\varepsilon_t)(1 - (\phi - \psi_t) n_t(a_t, h_t, \varepsilon_t; \Omega_t))\, dX_t(a_t, h_t, \varepsilon_t) \\ + \frac{1}{\bar{n}_{t-1}} \int_{A\times H\times \mathbf{R}} \frac{h_{t-1} \exp(\varepsilon_{t-1})\zeta}{1+\rho}\, dX_{t-1}(a_{t-1}, h_{t-1}, \varepsilon_{t-1}).$$

Total inter vivos transfers at the end of period t, B_t, is

$$(34)\quad B_t = \int_{A\times H\times \mathbf{R}} b_t(a_t, h_t, \varepsilon_t; \Omega_t) n_t(a_t, h_t, \varepsilon_t; \Omega_t)\, dX_t(a_t, h_t, \varepsilon_t),$$

and total private education spending in period t, E_t, is

$$(35)\quad E_t = \int_{A\times H\times \mathbf{R}} e_t(a_t, h_t, \varepsilon_t; \Omega_t) n_t(a_t, h_t, \varepsilon_t; \Omega_t)\, dX_t(a_t, h_t, \varepsilon_t).$$

3.3 The Representative Firm's Problem

The production function is assumed to be one of Cobb–Douglas,

$$(36)\quad Y_t = F(K_t, L_t) = A\, K_t^{\alpha} L_t^{1-\alpha},$$

where Y_t is total output, K_t is the capital input, and L_t is the labor input, all of which are productivity growth adjusted and per young household, and A is total factor productivity.

The representative firm's profit maximization problem is

$$(37)\quad \max_{K_t, L_t} A\, K_t^{\alpha} L_t^{1-\alpha} - (r_t + \delta) K_t - w_t L_t,$$

where δ is the depreciation rate of the capital stock, and the first order conditions are

$$(38)\quad r_t = \alpha A \left(\frac{K_t}{L_t}\right)^{\alpha-1} - \delta, \qquad w_t = (1-\alpha) A \left(\frac{K_t}{L_t}\right)^{\alpha}.$$

The market clearing conditions of the capital and labor markets are

$$(39)\quad K_t = W_t, \qquad L_t = L_t^s.$$

3.4 The Government's Budget

Total government consumption, other than public education spending in period t, G_t, is

$$(40)\quad G_t = \left(1 + \frac{1}{\bar{n}_{t-1}}\right) g_t,$$

total public education spending in period t, Θ_t, is

$$(41)\quad \Theta_t = \int_{A\times H\times \mathbf{R}} \vartheta_t n_t(a_t, h_t, \varepsilon_t; \Omega_t)\, dX_t(a_t, h_t, \varepsilon_t) = \vartheta_t \bar{n}_t,$$

total cost of childcare and paid leave subsidies in period t, Ψ_t, is

$$(42)\quad \Psi_t = \int_{A\times H\times \mathbf{R}} w_t \bar{h}_t \psi_t n_t(a_t, h_t, \varepsilon_t; \Omega_t)\, dX_t(a_t, h_t, \varepsilon_t) = w_t \bar{h}_t \psi_t \bar{n}_t,$$

where $\bar{h}_t$ is the average human capital of the young households,

$$(43)\quad \bar{h}_t = \int_{A\times H\times\mathbf{R}} h_t \, dX_t(a_t, h_t, \varepsilon_t),$$

and total public pension benefits to old households in period t, P_t, is

$$(44)\quad P_t = \frac{1}{\bar{n}_{t-1}} \int_{A\times H\times\mathbf{R}} tr_{p,t} \, dX_{t-1}(a_{t-1}, h_{t-1}, \varepsilon_{t-1}) = \frac{tr_{p,t}}{\bar{n}_{t-1}}.$$

Let's assume, for simplicity, that the government' net wealth or debt is zero. Then, the government's budget constraint of the general account is

$$(45)\quad \tau_{c,t}C_t + \tau_{k,t}r_tK_t + \tau_{h,t}w_tL_t + \tau_{b,t}B_t + \tau_{e,t}E_t + \tau_{n,t}\bar{n}_t = G_t + \Theta_t + \Psi_t,$$

and the budget constraint of the public pension account is

$$(46)\quad \tau_{p,t}w_tL_t = P_t.$$

When the government budget constraints hold, gross investment in period t, I_t, is

$$(47)\quad I_t = Y_t - C_t - E_t - G_t - \Theta_t - \Psi_t,$$

and the low of motion of growth-adjusted capital stock is

$$(48)\quad K_{t+1} = \frac{1}{1+\rho}\frac{1}{\bar{n}_t}\left[\, I_t + (1-\delta)K_t \,\right].$$

3.5 Recursive Competitive Equilibrium

The recursive competitive equilibrium of this model economy is defined as follows.

DEFINITION Recursive Competitive Equilibrium: Let $(a_t, h_t, \varepsilon_t)$ be the individual state of households, and let Ω_t be a time series of the government policy variables and the factor prices,

$$\Omega_t = \{\tau_{k,s}, \tau_{h,s}, \tau_{c,s}, \tau_{n,s}, \tau_{b,s}, \tau_{e,s}, \tau_{p,s}, tr_{p,s}, \vartheta_s, \psi_s, g_s, r_s, w_s\}_{s=t}^{\infty}.$$

The value functions of households, $\{v(a_s, h_s, \varepsilon_s; \Omega_s)\}_{s=t}^{\infty}$, the decision rules of households,

$$\{c(a_s, h_s, \varepsilon_s; \Omega_s), s(a_s, h_s, \varepsilon_s; \Omega_s), n(a_s, h_s, \varepsilon_s; \Omega_s), b(a_s, h_s, \varepsilon_s; \Omega_s), e(a_s, h_s, \varepsilon_s; \Omega_s)\}_{s=t}^{\infty},$$

and the distribution of households, $\{X_s(a, h, \varepsilon)\}_{s=t}^{\infty}$, are in a recursive competitive equilibrium if, for all $s = t, \dots, \infty$,

- each household solves the optimization problem, (11)–(17), taking Ω_s as given;

- the firm solves its profit maximization problem, (37)–(38);
- the government policy schedule satisfies conditions (40)–(46); and
- the factor markets are cleared as shown in equation (39).

The economy is in a steady-state equilibrium and thus on the balanced growth path if, in addition,

- $\Omega_{s+1} = \Omega_s$ and $X_{s+1}(a, h, \varepsilon) = X_s(a, h, \varepsilon)$ for all $s = t, \ldots, \infty$.

3.6 Social Welfare Measures

This study employs the compensating variations measure to evaluate changes in social welfare. The compensating variation of a household of state (a, h, ε) is a one-time negative wealth transfer that restores the baseline value (lifetime utility) in the economy after a policy change. Suppose that the economy is in an initial steady-state equilibrium in period $t = 0$ and that the government introduces a policy change at the beginning of period 1. Then, the compensating variations of young households at the beginning of period $t = 1, \ldots, \infty$ are calculated as $cv_t(a, h, \varepsilon; \Omega_t)$ such that

$$v(a - cv_t(a, h, \varepsilon; \Omega_t), h, \varepsilon; \Omega_t) = v(a, h, \varepsilon; \Omega_0), \tag{49}$$

and the compensating variations of initial old households, born in period 0, at the beginning of period 1 are calculated as $cv_0(a, h, \varepsilon; \Omega_0, \Omega_1)$ such that

$$v(a - cv_0(a, h, \varepsilon; \Omega_0, \Omega_1), h, \varepsilon; \Omega_0, \Omega_1) = v(a, h, \varepsilon; \Omega_0). \tag{50}$$

The average compensating variations by generation cohort are calculated as

$$\bar{cv}_t = \int_{A \times H \times \mathbf{R}} cv_t(a, h, \varepsilon; \Omega_t)\, dX_t(a, h, \varepsilon), \tag{51}$$

$$\bar{cv}_0 = \int_{A \times H \times \mathbf{R}} cv_0(a, h, \varepsilon; \Omega_0, \Omega_1)\, dX_0(a, h, \varepsilon). \tag{52}$$

Note that $X_1(a, h, \varepsilon) = X_0(a, h, \varepsilon)$ if the economy in period $t = 0$ is in a steady-state equilibrium.

4 Calibration and the Baseline Economy

We assume that the baseline economy is in a steady-state equilibrium, so that the economy is on a balanced-growth path with stylized tax and pension systems.

4.1 Main Parameter Values and Government Policy Assumptions

Table 3 shows the main parameter values and baseline government policy values of the model economy. The capital income share, α, of the production function is set to 0.36. The annual capital depreciation rate is assumed to be 5%, and the capital depreciation rate per period, δ, is set to $1 - 0.95^{30} = 0.7854$. The annual productivity growth rate is assumed to be 1%, and the productivity growth rate per period, ρ, is set to $1.01^{30} - 1 = 0.3478$. The households in this economy are assumed to work for 40 years, and the old households work for 10 years out of 30 years. Thus, the share of working years in the old period, ζ, is set to $10/30 = 0.3333$.

The annual capital–output ratio is assumed to be 3.0, and the capital–output ratio, K_t/Y_t, is targeted to be $3.0/30 = 0.10$ in the baseline (initial steady-state) economy. The subjective discount factor of time, β, is set to 0.5277 so that $K_t/Y_t = 0.10$. The coefficient of relative risk aversion, σ, of the utility function is assumed to be 1.5. Then, the growth-adjusted discount factor, $\tilde{\beta}$, is set to $(1+\rho)^{1-\sigma}\,\beta = 0.4545$. The annual discount factor is $0.4545^{1/30} = 0.9741$. The degree of parental altruism toward their children is assumed to be 0.8, that is, the parents care about their children 20% less than they care about themselves. Then, the growth-adjusted discount and altruism factor, $\tilde{\gamma}$, is set to $0.8\,\tilde{\beta} = 0.3636$.

The education elasticity of child's human capital, η, is set to 0.5. The parents' human capital elasticity of children's human capital, τ, is set to 0.2, following the estimates in de la Croix and Doepke (2003). The average human capital elasticity of child's human capital, κ, is set to 0, that is, the model economy does not account for the externalities of human capital investment. The hazard rate, μ, of the exponential discount factor, $\Phi(n_t)$, of children's value is set to 1.7397 so that the hourly wage (working ability) elasticity of the number of children is 0.125, which is equal to the estimated income semi-elasticity, 0.1613, divided by the average number of children, 1.5541, in Section 2.[4] When μ is 1.7397, $\lim_{n\to\infty}\Phi(n_t) = 1.2130$. The parameter, ξ, of the adult-equivalent number of family members for consumption is set to 0.5. The time cost of childcare per unit of children, ϕ, is set to 0.10.[5]

The wage rate per effective labor, w_t, the average level of human capital, $\bar{h}_t$, and the average number of children, $\bar{n}_t$, are all normalized to unity in the baseline economy. The scaling parameter, A, of the production function is set so that $w_t = 1.0$ when $K_t/Y_t = 0.10$ in the baseline economy. The scaling parameter, B, of the human capital function is set so that $\bar{h}_t = 1.0$ in the baseline economy, and the shifting parameter, $c_{\min}$, of the utility function is set to 0.1014

[4]If we calibrate the model so that the income elasticity of the number of children is 0.104 instead, the hazard rate, μ, is increased to 1.9560. Yet, the effects of this parameter change on policy responses are modest. For example, introducing the child allowance of 0.01 would increase the number of children by 2.4% and 2.1% in the first two periods rather than 2.6% and 2.2% as in Table 5 in Section 5.

[5]The lower bound of the time cost of childcare per child is estimated to be about 140 hours per child or 4.1% of the average working hours of married households, as discussed in Section 2, and it is 8.2% per unit of children in the model economy. Considering a possible depreciation of human capital during the childcare, ϕ is set to 10.0% or 0.10 in this study.

Table 3: Main Parameter Values and Baseline Government Policy Values

Main Parameters			
Share parameter of capital income	α	0.3600	
Discount factor	β	0.5277	$K/Y = 3.0/30$ (baseline)
Growth-adjusted discount factor	$\tilde{\beta}$	0.4545	$(1+\rho)^{1-\sigma}\beta$
Growth-adjusted discount and altruism factor	$\tilde{\gamma}$	0.3636	$0.8\,\tilde{\beta}$
Capital depreciation rate	δ	0.7854	$1-0.95^{30}$ (5% annual rate)
Share of working years in the old period	ζ	0.3333	$10/30$ (retire at age 65)
Education elasticity of human capital	η	0.5000	
Parents' human capital elasticity of human capital	τ	0.2000	de la Croix and Doepke (2003)
Average human capital elasticity of human capital	κ	0.0000	
Hazard rate of exponential child discount factor	μ	1.7397	Wage elasticity of $n_t = 0.125$
Adult equivalent scale for consumption	ξ	0.5000	
Long run productivity growth rate	ρ	0.3478	$1.01^{30}-1$ (1% annual rate)
Coefficient of relative risk aversion	σ	1.5000	
Time cost of childcare per unit of children	ϕ	0.1000	
Scaling and Other Parameters			
Scaling parameter (TFP) of production function	A	3.0482	$w_t = 1.0$ (baseline)
Scaling parameter of human capital function Λ	B	3.2810	$\bar{h}_t = 1.0$ (baseline)
Shifting parameter of utility function u	$c_{\min}$	0.1014	$\bar{n}_t = 1.0$ (baseline)
SD of log labor income shocks ε_t	σ_ε	0.2000	
SD of log wealth shocks $\epsilon_{a,t}$	σ_a	0.1000	
SD of log human capital shocks $\epsilon_{h,t}$	σ_h	0.1000	
Baseline Government Policies			
Capital income tax rate	$\tau_{k,t}$	0.1000	
Labor income tax rate	$\tau_{h,t}$	0.1000	
Consumption tax rate	$\tau_{c,t}$	0.1000	
Child tax rate	$\tau_{n,t}$	0.0000	
Estate (gift inter vivos) tax rate	$\tau_{b,t}$	0.0000	
Education tax rate	$\tau_{e,t}$	0.0000	
PAYG public pension (payroll) tax rate	$\tau_{p,t}$	0.1000	
PAYG public pension benefit	$tr_{p,t}$	0.1146	
Public education spending per child	ϑ_t	0.0716	$\vartheta_t \bar{n}_t = 0.04\,Y_t$
Childcare and paid leave subsidies per child	ψ_t	0.0000	
Other government consumption	g_t	0.1069	

Note: A unit of the number of children, n_t, in this study is 2 children in the real economy. Thus, $\bar{n}_t = 1$ in the baseline economy is corresponding to the total fertility rate of 2.0. The child tax and the education tax are considered as allowance and subsidy, respectively, when the tax rates are negative.

so that $\bar{n}_t = 1.0$ in the baseline economy. The average consumption of young households per unit of adult family members, $\bar{c}_t$, is 0.4244 as in Table 4. So, in this economy, parents do not want to raise children if they expect their children's future consumption to be less than 24% of the average consumption.

The standard deviation of log labor income shocks, ε_t, is assumed to be 0.20. The standard deviations of log wealth (inter vivos transfer) shocks, $\epsilon_{a,t}$, and log human capital shocks, $\epsilon_{h,t}$, are both assumed to be 0.10. Without the latter two shocks, the household's wealth, a_t, and human capital, h_t, are perfect substitutes, that is, either inter vivos transfers, b_t, or education spending, e_t, is binding at the lower bound 0. The sizes of the standard deviations of these shocks determine the elasticity of substitution between b_t and e_t.

The capital income tax rate, $\tau_{k,t}$, the labor income tax rate, $\tau_{h,t}$, and the consumption tax rate, $\tau_{c,t}$, are all set to 10% in the baseline economy. The child tax rate, $\tau_{n,t}$, the estate (inter vivos transfer) tax rate, $\tau_{b,t}$, and the education tax rate, $\tau_{e,t}$, are all normalized to 0% in the baseline economy. The payroll tax rate, $\tau_{p,t}$, for the pay-as-you-go public pension is assumed to be 10%, and the public pension benefit per old household, $tr_{p,t}$, is set to 0.1146 so that the government pension budget is balanced in the baseline economy. Public education spending per child, ϑ_t, is set to 0.0716 so that total public education spending, $\vartheta_t \bar{n}_t$, is 4.0% of GDP.[6] Childcare and paid leave subsidies (in hours) per child, ψ_t, are normalized to 0. The other government consumption per household is set to 0.1069 so that the rest of the government budget is also balanced in the baseline economy.

4.2 Baseline Economy

Table 4 shows the averages of individual variables, aggregate variables, and inequality statistics in the baseline economy. The average initial (beginning-of-period) wealth of young households, $\bar{a}_t$, is 0.0801, which is 8.0% of the average human capital, $w_t \bar{h}_t$, in the baseline economy. The average consumption of young households per unit of adult family members, $\bar{c}_t$, is 0.4244, and the average saving is 0.1335. The average number of children, $\bar{n}_t$, is normalized to unity, and one unit of children in the model economy is corresponding to two children in the real economy. The average inter vivos transfer to children, $\bar{b}_t$, is 0.1040 per unit of children, and the average private education spending, $\bar{e}_t$, is 0.0964 per unit of children.

The average consumption of old households, $\bar{d}_t$, is 0.6022, and it is larger than $\bar{c}_t$, because the equilibrium interest rate is higher than the subjective discount rate of time. The average initial wealth of children, $\bar{a}_{t+1}$, is 0.0771 per unit of children, and the average human capital of children, $\bar{h}_{t+1}$, is 0.9921. Both $\bar{a}_{t+1}$ and $\bar{h}_{t+1}$ are smaller than $\bar{a}_t$ and $\bar{h}_t$ because the former two averages are calculated without considering the number of children of each household. The value or the lifetime dynastic utility of young households, $\bar{v}_t$, is 7.4946. Total consumption (excluding education spending), C_t, is 1.2043, total capital stock, K_t, is 0.1791, total labor supply in efficiency units, L_t, is 1.1463, and total output, Y_t, is 1.7912. The interest rate, r_t, is 2.8146 per period or 4.56% annual rate, and the wage rate, w_t, is normalized to unity in the baseline economy.

Regarding inequalities, the Gini coefficient of consumption (including private education spending) is 0.1628, the Gini coefficient of labor income (both young and old households) is 0.3053, and the Gini coefficient of beginning-of-period wealth of young and old households is

[6] Total expenditures for public elementary and secondary schools in the United States were $870 billion in 2019-20 and $927 billion in 2020-21, according to Institute of Education Sciences (2023, 2024), which are about 4.0% of GDP in these years.

Table 4: Main Variables and Inequality Statistics in the Baseline Economy

Individual Variables (population weighted average)			
Initial wealth (young)	$\bar{a}_t$	0.0801	
Human capital	$\bar{h}_t$	1.0000	normalized to unity
Consumption (young)	$\bar{c}_t$	0.4244	
Saving	$\bar{s}_t$	0.1335	
Number of children	$\bar{n}_t$	1.0000	normalized to unity
Inter vivos transfer per unit of children	$\bar{b}_t$	0.1040	
Education expenditure per unit of children	$\bar{e}_t$	0.0964	
Consumption (old)	$\bar{d}_{t+1}$	0.6022	
Initial wealth of children	$\bar{a}_{t+1}$	0.0771	
Human capital of children	$\bar{h}_{t+1}$	0.9921	
Value (lifetime utility)	$\bar{v}_t$	7.4946	
Aggregate Variables (per young household)			
Consumption	C_t	1.2043	$\bar{c}_t + \bar{d}_t/\bar{n}_t$
Capital stock	K_t	0.1791	$\bar{a}_t + \bar{s}_t/(\bar{n}_t(1+\rho))$
Labor supply	L_t	1.1463	
Output	Y_t	1.7912	
Gross interest rate	$1 + r_t$	3.8146	4.56% annual rate
Wage rate	w_t	1.0000	normalized to unity
Gini Coefficients			
Consumption	(c_t, d_t)	0.1763	
Consumption and education	$(c_t+e_t n_t, d_t)$	0.1628	
Labor income (young)	$(\tilde{h}_t l_t)$	0.1511	
Labor income (young and old)	$(\tilde{h}_t l_t, \tilde{h}_{t-1}\zeta)$	0.3053	
Wealth (young and old)	(a_t, s_{t-1})	0.3318	
Wealth before inter vivos transfers	$(0, b_{t-1}+s_{t-1})$	0.6698	
Intergenerational Correlations			
Human capital	(h_t, h_{t+1})	0.7707	
Working ability	$(\tilde{h}_t, \tilde{h}_{t+1})$	0.4573	
Labor income	$(\tilde{h}_t l_t, \tilde{h}_{t+1} l_{t+1})$	0.4324	
OLS Coefficients (Semi-elasticities)			
Income elasticity of n_t	$(n_t, \ln(\tilde{h}_t l_t + r_t a_t))$	0.1489	
Labor income elasticity of n_t	$(n_t, \ln \tilde{h}_t l_t)$	0.1235	
Working ability elasticity of n_t	$(n_t, \ln \tilde{h}_t)$	0.1250	

Note: The aggregate consumption, C_t, includes the consumption of dependent children. The average total consumption of young households, $\tilde{c}_t$, is the population weighted average of $(1 + n_t)^{\xi} c_t$. The working ability is defined as $\tilde{h}_t = h_t e^{\varepsilon_t}$, and the labor income of a young household is $\tilde{h}_t l_t$, where $l_t = 1 - (\phi - \psi_t) n_t$. The variables in $t + 1$ and $t - 1$ are productivity growth adjusted.

0.3318. The wealth Gini coefficient is much smaller than that observed in the data because the present study assumes that all intergenerational transfers are inter vivos transfers rather than bequests. If we calculate the wealth Gini coefficient just before inter vivos transfers are made, the Gini coefficient is calculated as 0.6698 in the baseline economy. Regarding intergenerational mobility, the intergenerational correlation of human capital is 0.7707, and the correlation

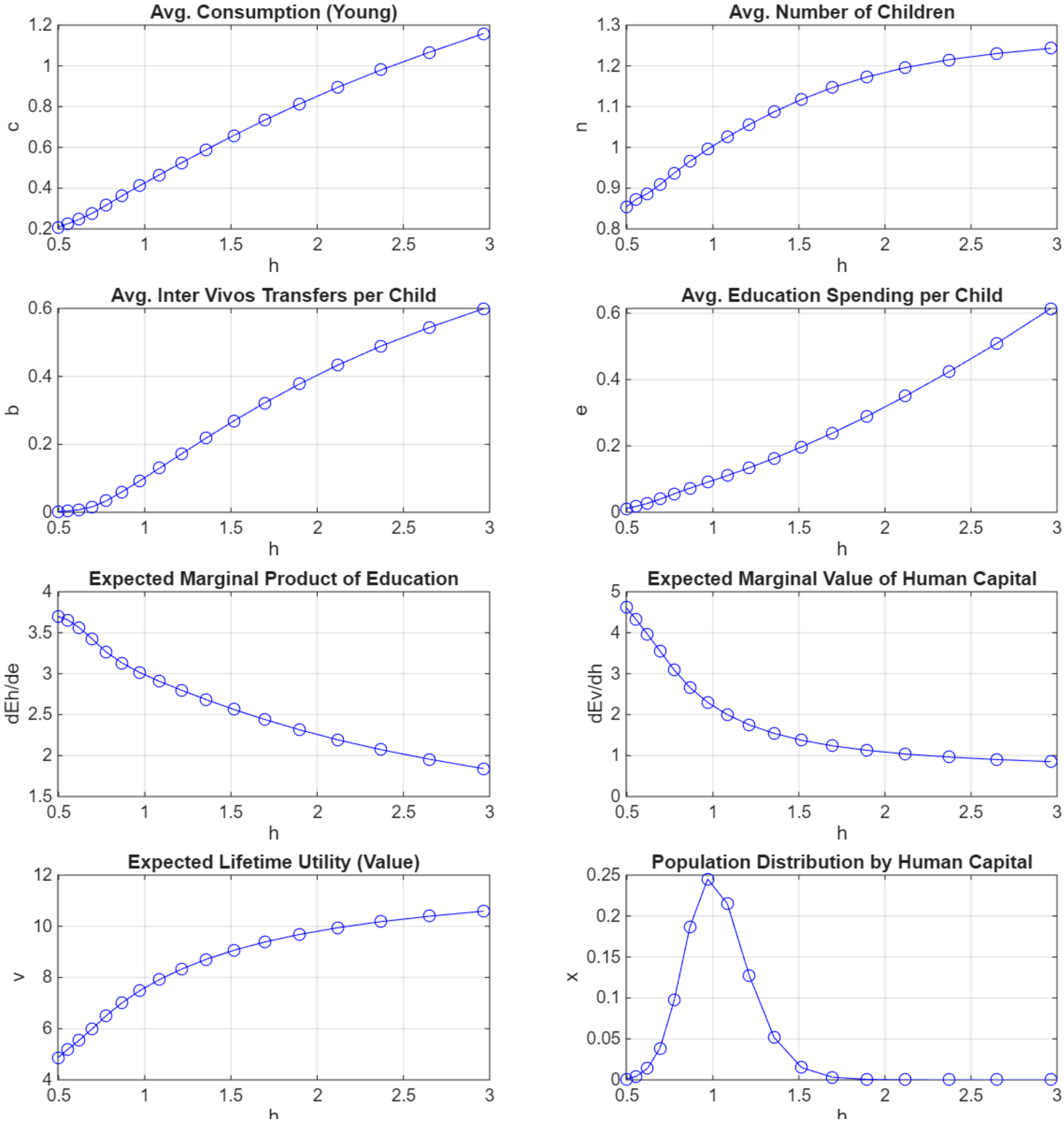

Figure 1: The Household Behaviors by Human Capital in the Baseline Economy

of labor income of young households is calculated as 0.4324. The income elasticity of the number of children is 0.1489, the wage income elasticity is 0.1235, and the working ability (hourly wage) elasticity is targeted to 0.1250 in the baseline economy.

Figure 1 shows the averages of individual variables of the young households conditional on the human capital, h_t, as well as the marginal distribution of human capital in the baseline economy. The average consumptions of young and old households are both increasing and slightly concave in h_t. The average number of children is also increasing in h_t, but it is more strongly concave by the assumption of $\Phi(n_t)$ and μ. The upper bound of the average number of children by h_t is around 1.25 in the baseline economy or 2.5 children in the real economy.[7]

[7]In the model economy, the household's wealth, a_t, and working ability shocks, ε_t, affects the number of

As Table 2 in Section 2 shows, the number of children is increasing in the husband's working ability but decreasing in the wife's working ability. Yet, it is increasing in combined human capital, as the permanent income measure of a household. Thus, children are normal goods in this model economy.

The inter vivos transfer, b_t, and education spending, e_t, are highly substitutable. The cost of the inter vivos transfer is independent of the parent's h_t, but the return to education spending is increasing in h_t (or the cost of children's human capital is decreasing in h_t). Therefore, inter vivos transfers per unit of children, b_t, are on average increasing and slightly concave in h_t. The average education spending per unit of children, e_t, is increasing and slightly convex in h_t. The parents' education spending generates a skewed distribution of human capital in the long run.

5 Policy Experiments

The altruistic (child-related) part of the household's optimization problem is, abstracting from all shocks, productivity growth, and human capital externality,

$$(53)\quad \max_{n_t,b_t,e_t} \tilde{\gamma}\Phi(n_t)v(a_{t+1},h_{t+1})$$

subject to the budget constraint,

$$\begin{aligned}(54)\quad & (1+\tau_{c,t})\big[\,(1+n_t)^{\xi}-1\,\big]c_t \\ & \quad + \big[\,\tau_{n,t}+(1+\tau_{b,t})b_t+(1+\tau_{e,t})e_t+(1-\tau_{h,t}-\tau_{p,t})w_t h_t(\phi-\psi_t)\,\big]n_t \\ & \quad = (1-\varsigma)\,\big[\,(1+(1-\tau_{k,t})r_t)a_t+(1-\tau_{h,t}-\tau_{p,t})w_t h_t\,\big] \equiv \mathrm{inc}_t,\end{aligned}$$

and the laws of motion of the state variables,

$$(55)\quad a_{t+1}=b_t\geq 0, \qquad h_{t+1}=B(\vartheta_t+e_t)^{\eta}\,h_t^{\tau}>0,$$

where ς is the share of disposable income and wealth used for the parents' own consumption and saving.

The first order conditions for the interior solution are

$$\begin{aligned}(56)\quad n_t:\quad & \tilde{\gamma}\Phi'(n_t)v(a_{t+1},h_{t+1})=\lambda_t\big[\,(1+\tau_{c,t})\xi(1+n_t)^{\xi-1}c_t \\ & \quad +\tau_{n,t}+(1+\tau_{b,t})b_t+(1+\tau_{e,t})e_t+(1-\tau_{h,t}-\tau_{p,t})w_t h_t(\phi-\psi_t)\,\big]\equiv\lambda_t\mathrm{mc}_{n,t},\end{aligned}$$

$$(57)\quad b_t:\quad \tilde{\gamma}\Phi(n_t)v_a(a_{t+1},h_{t+1})=\lambda_t(1+\tau_{b,t})n_t\equiv\lambda_t\mathrm{mc}_{b,t},$$

children. In the real economy, in addition, the preference heterogeneity would affect the number of children as well.

$$(58)\quad e_t: \quad \tilde{\gamma}\Phi(n_t)v_h(a_{t+1}, h_{t+1})B\eta(\vartheta_t + e_t)^{\eta-1}\, h_t^{\tau} = \lambda_t(1+\tau_{e,t})n_t \equiv \lambda_t \mathrm{mc}_{e,t},$$

where $\lambda_t > 0$ is the Lagrange multiplier and

$$\lambda_t = \frac{u'(c_t)}{(1+\tau_{c,t})(1+n_t)^{\xi}}$$

in the full model. Let's call the left-hand sides the marginal values of n_t, b_t, and e_t, and call the right-hand sides the marginal costs of of n_t, b_t, and e_t. Note that the marginal cost of n_t is increasing in b_t, e_t and h_t, and that the marginal costs of b_t and e_t are both increasing in n_t. This property generates the trade off between the quantity and quality of children (Becker and Lewis, 1973).

This section analyzes various policy reform plans aimed to reduce the marginal costs (or increase the marginal values) of raising children, making inter vivos transfers, and providing education spending on the individual decisions and the aggregate economy. More specifically, this section conducts the following three sets of policy experiments:

1. increasing child allowances (reducing $\tau_{n,t}$ by 0.01 or 1% of baseline $w_t\bar{h}_t$) versus increasing education subsidies (reducing $\tau_{e,t}$ by 10.10 percentage points);

2. increasing inter vivos transfer subsidies (reducing $\tau_{b,t}$ by 9.10 percentage points) versus decreasing labor income taxes (reducing $\tau_{h,t}$ by 1.10 percentage points);

3. increasing public education expenditure (increasing ϑ_t by 0.01 from 0.0716) versus increasing childcare and paid leave subsidies (increasing ψ_t by 0.0127 or 1.27% of time endowment).

The costs of each policy reform are assumed to be financed by raising the consumption tax rate, $\tau_{c,t}$, so that the government's combined budget, the sum of equations (45) and (46), is balanced each period, that is,

$$(59)\quad \tau_{c,t} = [\,(G_t + \Theta_t(\vartheta_t) + \Psi_t(\psi_t) + P_t(tr_{p,t})) \\ -(\tau_{k,t}r_tK_t + \tau_{h,t}w_tL_t + \tau_{b,t}B_t + \tau_{e,t}E_t + \tau_{n,t}\bar{n}_t)\,]\, C_t^{-1}.$$

The sizes of the policy changes are set so that the policy costs during the first period are the same, a 1.09 percentage point increase in $\tau_{c,t}$.

5.1 Increasing Child Allowances vs. Education Subsidies

This subsection compares the possible effects of increasing child allowances (reducing $\tau_{n,t}$ by 0.01, or 1.0% of the average human capital of young households in the baseline) and increasing education subsidies (reducing $\tau_{e,t}$ by 10.10 percentage points), both financed by increasing consumption taxes (adjusting $\tau_{c,t}$ to balance the government's budget in each period). Figure 2 shows the behavioral effects of these two policy changes in the first period by human capital, and Table 5 shows the individual, aggregate, and distributional effects of these policy changes over the transition paths.

When the child tax rate, $\tau_{n,t}$, was reduced permanently by 0.01, the marginal cost of children, $\text{mc}_{n,t}$, fell by 0.01 (on average by 1.9%), and the average number of children, $\bar{n}_t$, would increase by 2.6% in the first period. The increase in the number of children, n_t, however, is heterogeneous and decreasing in the household's human capital, h_t. The increase in n_t is on average around 5.9% when $h_t = 0.6$, and it is 0.8% when $h_t = 2.0$. The increase in n_t would increase the marginal costs of inter vivos transfers per child, $\text{mc}_{b,t}$, and education spending per child, $\text{mc}_{e,t}$, proportionately, and it would decrease both $\bar{b}_t$ and $\bar{e}_t$ by on average 1.7% and 2.5%, respectively, in the first period. The decreases in b_t and e_t are also heterogeneous and positively correlated to the increases in n_t for each h_t.

These changes in b_t and e_t in the first period would decrease the initial wealth of children, a_{t+1}, and the human capital of children, h_{t+1}. The averages of these, $\bar{a}_t$, and $\bar{h}_t$, at the beginning of the second period would decrease by 2.5% and 0.9%, respectively. Because of the negative income effect, inc_t, of decreasing $\bar{a}_t$ and $\bar{h}_t$ in the second period, the population growth rate or the increase in the average number of children, $\bar{n}_t$, would fall to 2.2% (from the baseline economy) in the second period. Total capital stock (per young household) at the beginning of the second and third periods would decrease by 1.6% and 2.6%, respectively. Total effective labor (per young household) in the second and third periods would decrease by 1.4% and 2.2%, respectively, and total output (per young household) would decrease by 1.5% and 2.3%, respectively, in these periods. The consumption tax rate, $\tau_{c,t}$, would increase by 1.09 percentage points in the first period, and it would increase by 1.06 and 1.41 percentage points (from the baseline economy) in the second and third periods.

The policy reform would improve the welfare of young households with $h_t < 1.2$ and decline the welfare of those with $h_t > 1.2$ slightly in the first period. The welfare of young households would improve by 0.2% in period 1 and decline by 0.1% as a percentage of total wealth (capital stock) per young household under the compensating variation measure. The welfare of initial old households would decline on average by 0.9% in the same welfare measure because of the increase in the consumption tax rate. The effect of this policy reform on labor income inequality is small, and the Gini coefficient of labor income of young households would increase only from 0.151 to 0.152 in the first and second periods. The intergenerational correlation of human capital would increase from 0.457 to 0.466 in the first period and to 0.468 in the second period. The Gini coefficient of household wealth would increase from 0.670 to 0.674 in the second period and 0.676 in the third period. This is because the inter vivos transfers of households with $h_t < 1.5$ would decrease but those with $h_t > 1.5$ would slightly increase.

When the education tax rate, $\tau_{e,t}$, was reduced permanently by 10.1 percentage points, the marginal cost of education, $\text{mc}_{e,t}$, fell by 10.1%, and the marginal cost of children, $\text{mc}_{n,t}$,

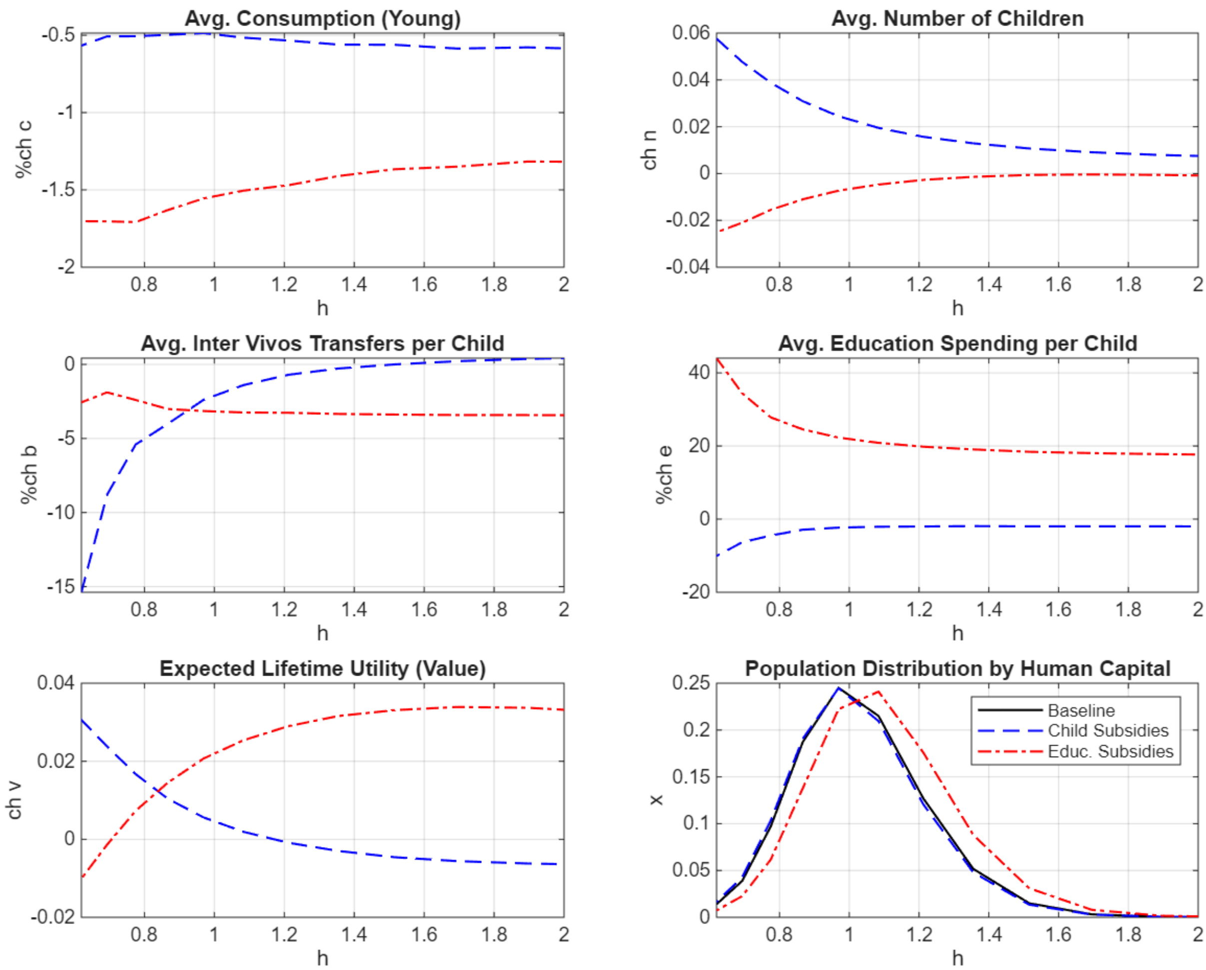

Figure 2: Policy Experiments: Child Allowances vs. Education Subsidies (policy effects by human capital in the first period)

would fall on average by 1.8%. The average education spending per child, $\bar{e}_t$, would increase by 22.0%, the average inter vivos transfers per child, $\bar{b}_t$, would decrease by 3.2%, and the average number of children, $\bar{n}_t$, would decrease by 0.8% in the first period. The percent increase in education spending, e_t, is heterogeneous and decreasing in the household's human capital, h_t. The increase in e_t is on average 44% when $h_t = 0.6$, and it is 17% when $h_t = 2.0$. The number of children, n_t, would on average decrease slightly from the baseline economy.

These changes in b_t and e_t in the first period would decrease the initial wealth of children, a_{t+1}, and increase the human capital of children, h_{t+1}. The average wealth of children, $\bar{a}_t$, would decrease by 2.7%, and the average human capital of children, $\bar{h}_t$, would increase by 6.2%, at the beginning of the second period. Because of the positive income effect, inc_t, of increasing $\bar{a}_t$ and $\bar{h}_t$ combined in the second period, the average number of children, $\bar{n}_t$, would increase by 0.2% (from the baseline economy) in the second period. Total capital stock at the beginning of the second period would decrease by 0.6% and that at the beginning of third period would increase by 1.5%. Total effective labor in the second and third periods would increase by 5.0% and 8.2%, respectively, and total output would increase by 2.9% and 5.7%, respectively,

Table 5: Policy Experiments: Child Allowances vs. Education Subsidies

	Baseline	Child Allowances				Education Subsidies			
Period t	0	1	2	3	∞	1	2	3	∞
Individual Variables (population-weighted average, % ch. from the baseline)									
$\bar{a}_t$	0.080	-0.0	-2.5	-3.6	-6.9	-0.0	-2.7	1.5	15.6
$\bar{h}_t$	1.000	0.0	-0.9	-1.8	-4.3	0.0	6.2	9.0	19.9
$\bar{c}_t$	0.424	-0.5	-1.5	-2.5	-5.3	-1.5	1.4	4.4	15.7
$\bar{s}_t$	0.133	1.7	0.3	-0.5	-3.1	0.2	1.6	4.4	14.5
$\bar{n}_t$	1.000	2.6	2.2	1.8	0.9	-0.8	0.2	1.3	4.7
$\bar{b}_t$	0.104	-1.7	-2.9	-3.8	-6.4	-3.2	1.2	4.2	15.7
$\bar{e}_t$	0.096	-2.5	-4.8	-6.4	-11.2	22.0	27.6	33.8	58.9
$\bar{d}_{t+1}$	0.602	-0.0	-1.3	-2.2	-4.9	1.0	4.7	7.7	19.1
$\bar{a}_{t+1}$	0.077	-1.7	-2.9	-3.8	-6.4	-3.2	1.2	4.2	15.7
$\bar{h}_{t+1}$	0.992	-0.7	-1.6	-2.2	-4.1	6.1	8.9	11.1	19.7
$\bar{v}_t$	7.495	0.1	-0.5	-1.0	-2.4	0.3	1.8	3.2	7.7
$\bar{cv}_t$	0.000	0.2	-0.1	-0.8	-2.6	1.6	2.4	4.0	11.0
Initial Old Households (population-weighted average, % ch. from the baseline)									
$\bar{d}_t$	0.602	-1.0				-1.0			
$\bar{v}_{t-1}$	7.495	-0.0				0.0			
$\bar{cv}_{t-1}$	0.000	-0.9				0.5			
Aggregate Variables (per young household, % ch. from the baseline)									
C_t	1.204	-0.5	-1.8	-2.7	-5.5	-1.3	1.7	4.6	15.4
K_t	0.179	0.0	-1.6	-2.6	-5.3	0.0	-0.6	1.5	12.2
L_t	1.146	-0.2	-1.4	-2.2	-4.5	0.1	5.0	8.2	18.2
Y_t	1.791	-0.1	-1.5	-2.3	-4.8	0.0	2.9	5.7	16.0
$1+r_t$	3.815	-0.1	0.1	0.3	0.5	0.0	3.4	4.0	3.2
w_t	1.000	0.1	-0.1	-0.2	-0.3	-0.0	-2.0	-2.3	-1.9
Government Policy Variables (%, ch. in p.p. from the baseline)									
$\tau_{n,t}$	0.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00
$\tau_{e,t}$	0.00	0.00	0.00	0.00	0.00	-10.10	-10.10	-10.10	-10.10
$\tau_{c,t}$	0.00	**1.09**	1.06	1.41	2.52	**1.09**	0.32	-0.66	-3.88
Gini Coefficients									
$gini(c_t, d_t)$	0.176	0.177	0.179	0.180	0.181	0.179	0.176	0.177	0.183
$gini(\tilde{h}_t l_t)$	0.151	0.152	0.152	0.153	0.153	0.151	0.151	0.152	0.157
$gini(a_t, s_{t-1})$	0.332	0.332	0.338	0.342	0.353	0.332	0.330	0.324	0.315
$gini(0, b_{t-1}+s_{t-1})$	0.670	0.670	0.674	0.676	0.678	0.670	0.668	0.667	0.672
Intergenerational Correlations									
$corr(\tilde{h}_t, \tilde{h}_{t+1})$	0.457	0.466	0.468	0.469	0.469	0.461	0.467	0.475	0.531
$corr(\tilde{h}_t l_t, \tilde{h}_{t+1} l_{t+1})$	0.432	0.443	0.445	0.446	0.445	0.435	0.442	0.449	0.508
OLS Coefficients									
$\hat{\beta}(n_t, \ln(\tilde{h}_t l_t + r_t a_t))$	0.149	0.111	0.112	0.112	0.113	0.170	0.168	0.167	0.163
$\hat{\beta}(n_t, \ln \tilde{h}_t l_t)$	0.124	0.083	0.084	0.085	0.087	0.146	0.145	0.143	0.145
$\hat{\beta}(n_t, \ln \tilde{h}_t)$	0.125	0.085	0.086	0.087	0.089	0.147	0.146	0.145	0.146

Note: The compensating variation in wealth, $\bar{cv}_t$, in each period is the negative of population-weighted average as a percentage of baseline wealth per young household, K_0. The compensating variations of initial old households are measured at the beginning of period 1.

in these periods. The consumption tax rate, $\tau_{c,t}$, would increase by 1.09 percentage points in the first period, but it would increase only by 0.32 percentage points (from the baseline economy) in the second period and decrease by 0.66 percentage points in the third period.

The policy reform would decline the welfare of young households with $h_t < 0.7$ and improve the welfare of those with $h_t > 0.7$ in the first period. The welfare of young households in period 1 would improve on average by 1.6% as a percentage of total wealth per young household under the compensating variation measure. The welfare of initial old households would also improve on average by 0.5% in the same welfare measure because of the welfare gains of the children. The effect of this policy reform on labor income inequality is small, and the Gini coefficient of labor income of young households would increase from 0.151 to 0.152 in the third period. The intergenerational correlation of human capital would increase more significantly from 0.457 to 0.461 in the first period and to 0.467 in the second period. The Gini coefficient of household wealth would decrease only slightly from 0.670 to 0.668 in the second period and to 0.667 in the third period.

The Winner: Education Subsidies

5.2 Cutting Estate Taxes vs. Labor Income Taxes

This subsection compares the possible effects of cutting estate (inter vivos transfer) taxes (reducing $\tau_{b,t}$ by 9.10 percentage points) and cutting labor income taxes (reducing $\tau_{h,t}$ by 1.10 percentage points), both financed by increasing consumption taxes. Figure 3 shows the behavioral effects of these two policy changes in the first period by human capital, and Table 6 shows the individual, aggregate, and distributional effects of these policy changes over the transition paths.

When the estate tax rate, $\tau_{b,t}$, was reduced permanently by 9.10 percentage points, the marginal cost of inter vivos transfers, $\text{mc}_{b,t}$, fell by 9.10%, and the marginal cost of children, $\text{mc}_{n,t}$, would fall on average by 1.8%. The average inter vivos transfers per child, $\bar{b}_t$, would increase by 17.6%, the average education spending per child, $\bar{e}_t$, would decrease by 5.5%, and the average number of children, $\bar{n}_t$, would increase by 0.4% in the first period. The percent increase in inter vivos transfers, b_t, is heterogeneous and decreasing in the household's human capital, h_t. The increase in b_t is on average 41% when $h_t = 0.6$, and it is 12% when $h_t = 2.0$.

Then, these changes in b_t and e_t in the first period would increase the initial wealth of children, a_{t+1}, and decrease the human capital of children, h_{t+1}. The average wealth of children, $\bar{a}_t$, would increase by 17.3%, and the average human capital of children, $\bar{h}_t$, would decrease by 1.6%, at the beginning of the second period. Because of the positive income effect, inc_t, of increasing $\bar{a}_t$ and $\bar{h}_t$ combined in the second period, the average number of children, $\bar{n}_t$, would increase by 1.8% (from the baseline economy) in the second period. Total capital stock at the beginning of the second and third periods would increase by 6.0% and 9.3%, respectively. Total effective labor in the second and third periods would decrease by 1.5% and 0.7%, respectively,

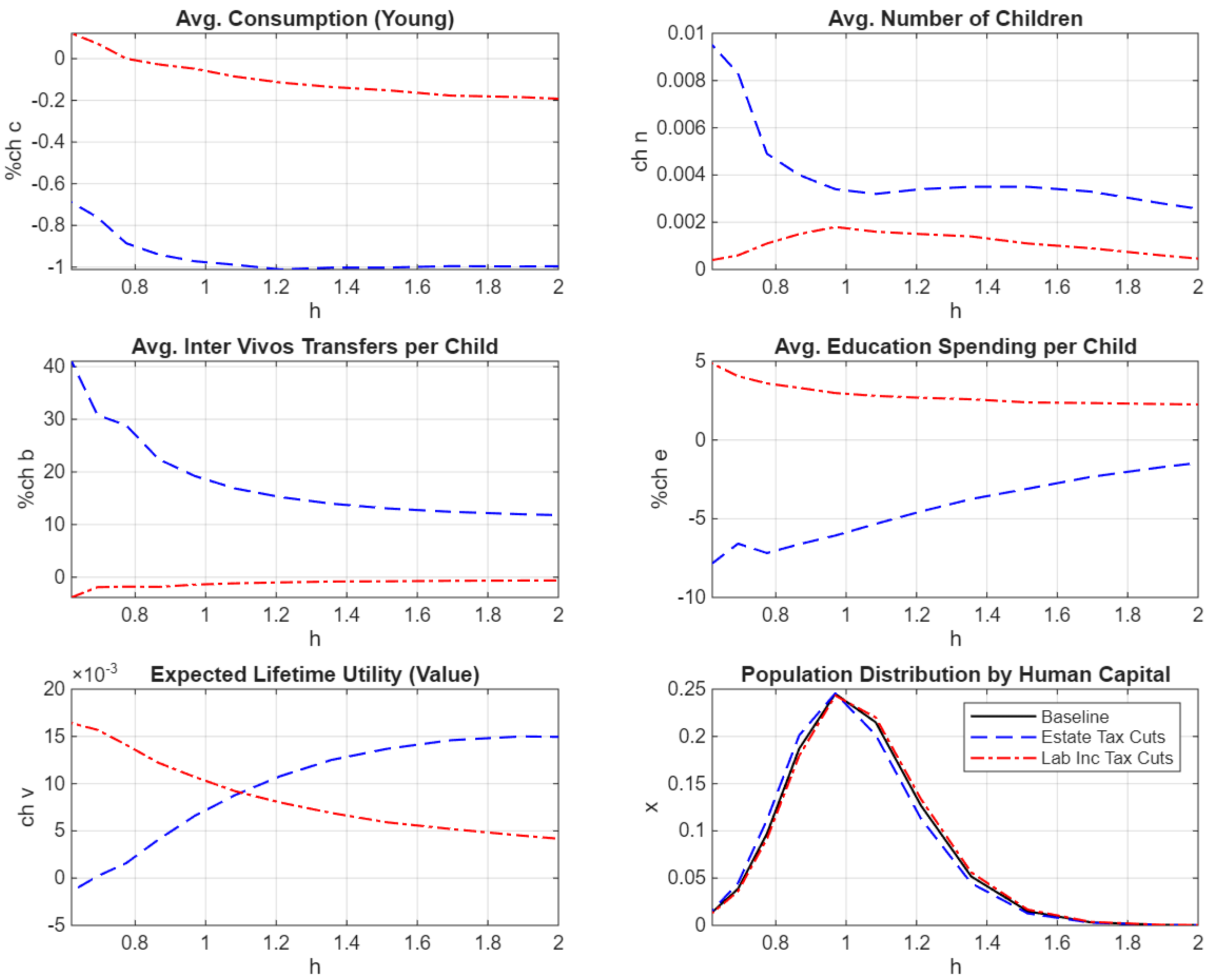

Figure 3: Policy Experiments: Estate Tax Cuts vs. Labor Income Tax Cuts (policy effects by human capital in the first period)

and total output would increase by 1.2% and 2.8%, respectively, in the second and third periods. The consumption tax rate, $\tau_{c,t}$, would increase by 1.09 percentage points in the first period, and it would increase by 0.89 and 0.16 percentage points (from the baseline economy) in the second and third periods.

The policy reform would decline on average the welfare of young households with $h_t < 0.7$ and improve the welfare of those with $h_t > 0.7$ in the first period. The welfare of young households would improve on average by 0.6% and 2.2%, respectively, in the first and second periods, as a percentage of total wealth per young household under the compensating variation measure. The welfare of initial old households would decline on average by 0.6% in the same welfare measure because of the increase in the consumption tax rate. The effect of this policy reform on labor income inequality is very small, and the Gini coefficient of labor income of young households would increase slightly from 0.151 in the baseline economy to 0.152 in the third period. The intergenerational correlation of human capital would increase steadily from 0.457 to 0.460 and 0.465 in the first and second periods, respectively. The Gini coefficient of household wealth before making inter vivos transfers would increase from 0.670 to 0.673 and

Table 6: Policy Experiments: Estate Tax Cuts vs. Labor Income Tax Cuts

	Baseline	Estate Tax Cuts				Labor Income Tax Cuts			
Period t	0	1	2	3	∞	1	2	3	∞
Individual Variables (population-weighted average, % ch. from the baseline)									
$\bar{a}_t$	0.080	-0.0	17.3	19.3	30.7	-0.0	-1.2	-0.7	0.6
$\bar{h}_t$	1.000	0.0	-1.6	0.2	7.3	0.0	0.8	1.1	2.0
$\bar{c}_t$	0.424	-1.0	2.1	4.1	12.1	-0.1	0.2	0.5	1.6
$\bar{s}_t$	0.133	-2.7	3.0	4.8	12.0	1.1	1.1	1.4	2.3
$\bar{n}_t$	1.000	0.4	1.8	2.4	4.8	0.1	0.2	0.3	0.7
$\bar{b}_t$	0.104	17.6	19.6	22.0	31.2	-1.2	-0.7	-0.4	0.5
$\bar{e}_t$	0.096	-5.5	2.1	5.9	21.0	2.9	3.2	3.8	5.7
$\bar{d}_{t+1}$	0.602	-3.6	-1.0	0.9	8.5	0.3	0.6	0.9	1.9
$\bar{a}_{t+1}$	0.077	17.6	19.6	22.0	31.2	-1.2	-0.7	-0.4	0.5
$\bar{h}_{t+1}$	0.992	-1.6	0.3	1.7	7.2	0.8	1.1	1.3	2.0
$\bar{v}_t$	7.495	0.1	1.4	2.3	5.6	0.1	0.3	0.4	0.9
$\bar{cv}_t$	0.000	0.6	2.2	4.0	9.4	0.7	0.9	1.0	1.6
Initial Old Households (population-weighted average, % ch. from the baseline)									
$\bar{d}_t$	0.602	-1.0				-0.6			
$\bar{v}_{t-1}$	7.495	-0.0				0.0			
$\bar{cv}_{t-1}$	0.000	-0.6				0.0			
Aggregate Variables (per young household, % ch. from the baseline)									
C_t	1.204	-0.9	-0.7	1.0	8.5	-0.3	0.2	0.5	1.5
K_t	0.179	0.0	6.0	9.3	17.5	0.0	0.0	0.2	1.1
L_t	1.146	-0.0	-1.5	-0.7	5.8	-0.0	0.6	1.0	1.8
Y_t	1.791	-0.0	1.2	2.8	9.8	-0.0	0.4	0.7	1.6
$1+r_t$	3.815	-0.0	-4.3	-5.6	-6.1	-0.0	0.4	0.5	0.4
w_t	1.000	0.0	2.7	3.5	3.9	0.0	-0.2	-0.3	-0.2
Government Policy Variables (%, ch. in p.p. from the baseline)									
$\tau_{b,t}$	0.00	-9.10	-9.10	-9.10	-9.10	0.00	0.00	0.00	0.00
$\tau_{h,t}$	10.00	0.00	0.00	0.00	0.00	-1.10	-1.10	-1.10	-1.10
$\tau_{c,t}$	0.00	**1.09**	0.89	0.16	-2.29	**1.09**	0.92	0.82	0.46
Gini Coefficients									
$gini(c_t, d_t)$	0.176	0.178	0.168	0.169	0.174	0.176	0.176	0.176	0.177
$gini(\tilde{h}_t l_t)$	0.151	0.151	0.151	0.152	0.156	0.151	0.151	0.151	0.152
$gini(a_t, s_{t-1})$	0.332	0.332	0.328	0.327	0.322	0.332	0.333	0.332	0.333
$gini(0, b_{t-1}+s_{t-1})$	0.670	0.670	0.673	0.674	0.678	0.670	0.670	0.670	0.671
Intergenerational Correlations									
$corr(\tilde{h}_t, \tilde{h}_{t+1})$	0.457	0.460	0.465	0.474	0.519	0.459	0.460	0.462	0.467
$corr(\tilde{h}_t l_t, \tilde{h}_{t+1} l_{t+1})$	0.432	0.435	0.441	0.449	0.496	0.434	0.436	0.437	0.442
OLS Coefficients									
$\hat{\beta}(n_t, \ln(\tilde{h}_t l_t + r_t a_t))$	0.149	0.146	0.147	0.147	0.146	0.150	0.150	0.150	0.150
$\hat{\beta}(n_t, \ln \tilde{h}_t l_t)$	0.124	0.121	0.119	0.121	0.125	0.125	0.125	0.125	0.126
$\hat{\beta}(n_t, \ln \tilde{h}_t)$	0.125	0.122	0.121	0.122	0.126	0.126	0.126	0.127	0.128

Note: The compensating variation in wealth, $\bar{cv}_t$, in each period is the negative of population-weighted average as a percentage of baseline wealth per young household, K_0. The compensating variations of initial old households are measured at the beginning of period 1.

0.674 in the second and third period, and the Gini coefficient of household wealth after making transfers would decrease temporarily from 0.332 to 0.328 in the second period.

When the labor income tax rate, $\tau_{h,t}$, was reduced permanently by 1.10 percentage points, the disposable labor income for child-related expense, $(1 - \tau_{h,t} - \tau_{p,t})w_t h_t(1 - \varsigma)$, would increase by 1.4%, and the marginal cost of children, $\mathrm{mc}_{n,t}$, would *increase* on average by 0.6%. In addition, the marginal values of education and children would both rise. The average education spending per child, $\bar{e}_t$, and the average number of children, $\bar{n}_t$, would increase by 2.9% and 0.1%, respectively, and the inter vivos transfers per child, $\bar{b}_t$, would decrease by 1.2% in the first period. The percent increase in education spending, e_t, is decreasing modestly in the household's human capital, h_t. The change in the number of children, n_t, and the percent change in inter vivos transfers, b_t, do not depend on the household's human capital level very much.

These changes in b_t and e_t in the first period would decrease the initial wealth of children, a_{t+1}, and increase the human capital of children, h_{t+1}. The average wealth of children, $\bar{a}_t$, would decrease by 1.2%, and the average human capital of children, $\bar{h}_t$, would increase by 0.8%, at the beginning of the second period. The average number of children, $\bar{n}_t$, would increase by 0.2% (from the baseline economy) in the second period. Total capital stock would not change in the second period and would increase by 0.2% in the third period. Total effective labor would increase by 0.6% and 1.0%, respectively, in the second and third periods, and total output would increase by 0.4% and 0.7%, respectively, in these periods. The consumption tax rate, $\tau_{c,t}$, would increase by 1.09 percentage points in the first period, and it would increase by 0.92 and 0.82 percentage points in the second and third periods.

The welfare of young households in the first period would improve on average by 0.7% as a percentage of total wealth per young household under the compensating variation measure. The welfare of initial old households would not change. The effect of this policy reform on labor income inequality is very small, and the Gini coefficient of labor income of young households would stay at 0.151 through the third period. The intergenerational correlation of human capital would increase a little from 0.457 to 0.459 and 0.460 in the first and second periods. The Gini coefficient of household wealth would stay at 0.670 through the third period.

The Winner: Cutting Estate Taxes

5.3 Increasing Public Education vs. Childcare Paid Leave Subsidies

This subsection compares the possible effects of increasing public education (increasing ϑ_t by 0.01, or 1.0% of the average human capital of young households in the baseline) and increasing childcare and paid leave subsidies (increasing ψ_t by 0.0127, or 1.27% of the time endowment of young households), both financed by increasing consumption taxes. Figure 4 shows the behavioral effects of these two policy changes in the first period by human capital, and Table 7 shows the individual, aggregate, and distributional effects of these policy changes

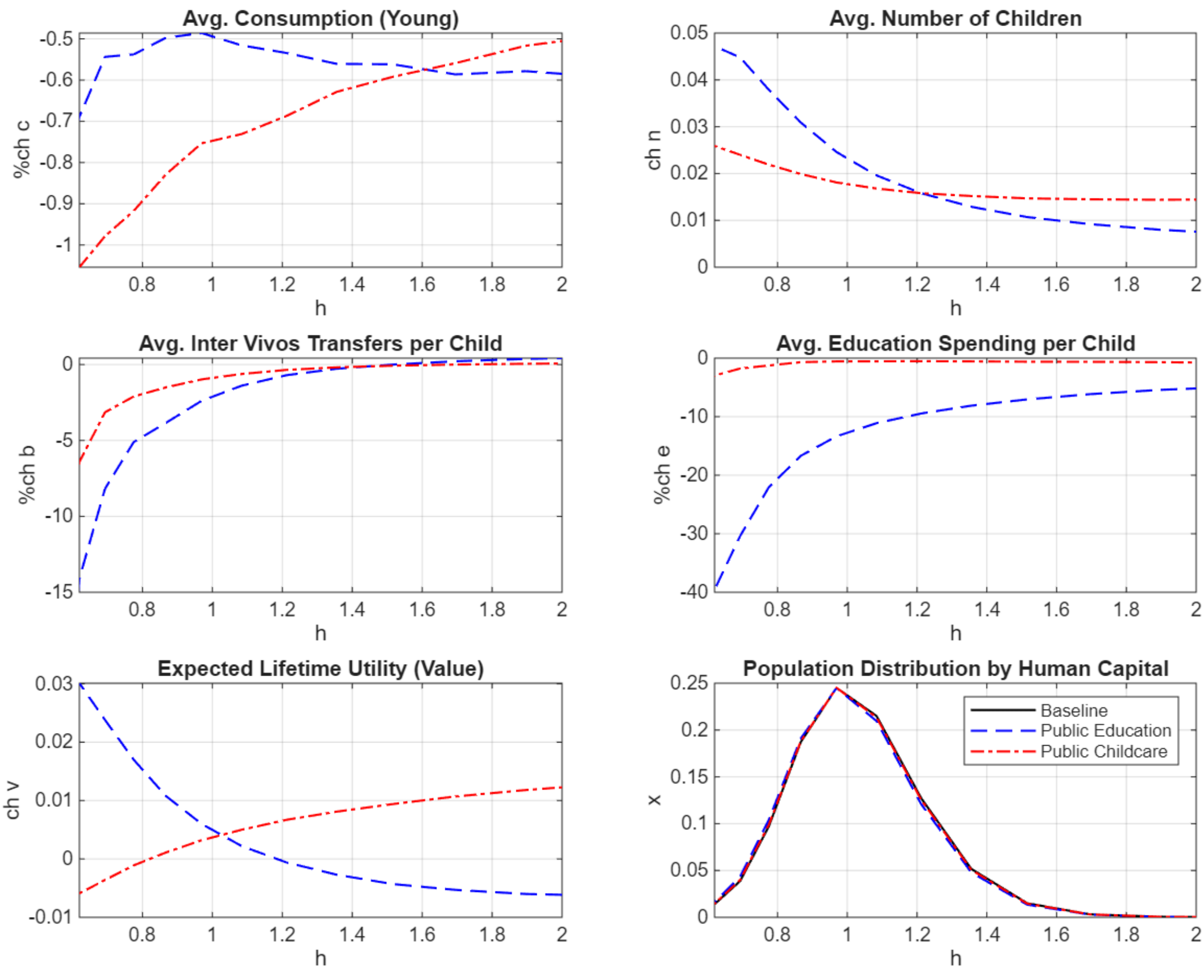

Figure 4: Policy Experiments: Public Education vs. Childcare Paid Leave Subsidies (policy effects by human capital in the first period)

over the transition paths.

When the public education per child, ϑ_t, was increased permanently by 0.01, it would generate almost the same effect of decreasing the child tax rate per child, $\tau_{n,t}$, by the same amount. This is because the public education, ϑ_t, and private education, e_t, are perfect substitutes in the model economy, and the private education spending is larger than 0.01 for all households except for those with very small wealth, a_t, and very low human capital, h_t, in the baseline economy. So, the increase in public education would mostly be canceled out by the decrease in private education, and the reduction in the cost of private education would have the same effect of child subsidies.[8]

When childcare and paid leave subsidies per child, ψ_t, were increased permanently by 0.0127, the marginal cost of children, $\mathrm{mc}_{n,t}$, would fall by on average 1.9%. So, the average number of children, $\bar{n}_t$, would increase by 1.8% in the first period. The increase in $\bar{n}_t$ would increase the marginal costs of inter vivos transfers, $\mathrm{mc}_{b,t}$, and the education spending, $\mathrm{mc}_{e,t}$,

[8]Public education as well as many other paternalistic policies would be more important if the degree of parental altruism is heterogeneous and positively correlated to the parents' income or human capital.

Table 7: Policy Experiments: Public Education vs. Childcare Paid Leave Subsidies

	Baseline	Public Education				Childcare Paid Leave Subsidies			
Period t	0	1	2	3	∞	1	2	3	∞
Individual Variables (population-weighted average, % ch. from the baseline)									
$\bar{a}_t$	0.080	-0.0	-2.5	-3.5	-6.6	-0.0	-1.0	-1.3	-2.3
$\bar{h}_t$	1.000	0.0	-0.9	-1.7	-4.0	0.0	-0.3	-0.5	-1.3
$\bar{c}_t$	0.424	-0.5	-1.5	-2.4	-5.1	-0.8	-1.0	-1.3	-2.2
$\bar{s}_t$	0.133	1.6	0.4	-0.5	-2.9	1.3	0.9	0.6	-0.2
$\bar{n}_t$	1.000	2.5	2.1	1.8	0.9	1.8	1.7	1.6	1.3
$\bar{b}_t$	0.104	-1.7	-2.8	-3.7	-6.2	-0.7	-1.1	-1.4	-2.2
$\bar{e}_t$	0.096	-12.8	-15.0	-16.6	-21.1	-0.7	-1.4	-1.9	-3.4
$\bar{d}_{t+1}$	0.602	-0.0	-1.2	-2.1	-4.7	0.1	-0.3	-0.6	-1.5
$\bar{a}_{t+1}$	0.077	-1.7	-2.8	-3.7	-6.2	-0.7	-1.1	-1.4	-2.2
$\bar{h}_{t+1}$	0.992	-0.7	-1.5	-2.1	-3.9	-0.2	-0.5	-0.7	-1.2
$\bar{v}_t$	7.495	0.1	-0.5	-0.9	-2.3	0.0	-0.1	-0.3	-0.7
$\bar{cv}_t$	0.000	0.2	-0.1	-0.7	-2.4	0.3	0.3	0.1	-0.4
Initial Old Households (population-weighted average, % ch. from the baseline)									
$\bar{d}_t$	0.602	-1.0				-0.8			
$\bar{v}_{t-1}$	7.495	-0.0				-0.0			
$\bar{cv}_{t-1}$	0.000	-0.8				-0.6			
Aggregate Variables (per young household, % ch. from the baseline)									
C_t	1.204	-0.5	-1.8	-2.7	-5.2	-0.6	-1.2	-1.5	-2.3
K_t	0.179	0.0	-1.6	-2.5	-5.1	0.0	-0.7	-1.0	-1.9
L_t	1.146	-0.2	-1.4	-2.1	-4.3	1.0	0.4	0.2	-0.5
Y_t	1.791	-0.1	-1.4	-2.2	-4.6	0.6	-0.0	-0.3	-1.0
$1+r_t$	3.815	-0.1	0.1	0.3	0.5	0.6	0.7	0.7	0.8
w_t	1.000	0.1	-0.1	-0.2	-0.3	-0.4	-0.4	-0.4	-0.5
Government Policy Variables (%, ch. in p.p. from the baseline)									
ϑ_t	7.16	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
ψ_t	0.00	0.00	0.00	0.00	0.00	1.27	1.27	1.27	1.27
$\tau_{c,t}$	0.00	**1.09**	1.05	1.39	2.44	**1.09**	0.96	1.06	1.38
Gini Coefficients									
$gini(c_t, d_t)$	0.176	0.177	0.179	0.180	0.180	0.178	0.179	0.179	0.179
$gini(\tilde{h}_t l_t)$	0.151	0.152	0.152	0.152	0.152	0.151	0.152	0.152	0.152
$gini(a_t, s_{t-1})$	0.332	0.332	0.337	0.342	0.351	0.332	0.334	0.336	0.338
$gini(0, b_{t-1}+s_{t-1})$	0.670	0.670	0.674	0.675	0.678	0.670	0.673	0.673	0.674
Intergenerational Correlations									
$corr(\tilde{h}_t, \tilde{h}_{t+1})$	0.457	0.464	0.464	0.464	0.461	0.460	0.461	0.461	0.460
$corr(\tilde{h}_t l_t, \tilde{h}_{t+1} l_{t+1})$	0.432	0.441	0.441	0.441	0.437	0.439	0.439	0.439	0.439
OLS Coefficients									
$\hat{\beta}(n_t, \ln(\tilde{h}_t l_t + r_t a_t))$	0.149	0.115	0.116	0.116	0.117	0.140	0.140	0.140	0.140
$\hat{\beta}(n_t, \ln \tilde{h}_t l_t)$	0.124	0.087	0.088	0.090	0.091	0.117	0.118	0.118	0.119
$\hat{\beta}(n_t, \ln \tilde{h}_t)$	0.125	0.089	0.090	0.091	0.093	0.118	0.119	0.119	0.120

Note: The compensating variation in wealth, $\bar{cv}_t$, in each period is the negative of population-weighted average as a percentage of baseline wealth per young household, K_0. The compensating variations of initial old households are measured at the beginning of period 1.

and $\bar{b}_t$ and $\bar{e}_t$ would both decrease by 0.7% in the first period. Then, these decreases would lower the average wealth of children, $\bar{a}_t$, by 1.0% and the average human capital of children, $\bar{h}_t$, by 0.3% in the second period. Because the negative income effect, inc_t, of decreasing $\bar{a}_t$ and $\bar{h}_t$ in the second period, the population growth rate, $\bar{n}_t$, would fall slightly to 1.7% in the second period.

Total capital stock at the beginning of the second and third periods would decrease by 0.7% and 1.0%, respectively. Total effective labor would increase by 1.0% and 0.4% (from the baseline economy) in the first and second periods, respectively. Total output would increase by 0.6% in the first period and decrease very slightly in the second period. The consumption tax rate, $\tau_{c,t}$, would increase by 1.09 percentage points in the first period, and it would increase by 0.96 and 1.06 percentage points in the second and third periods.

The policy reform would on average decline the welfare of young households with $h_t < 0.8$ and improve the welfare of those with $h_t > 0.8$ in the first period. The welfare of young households would improve on average by 0.3% in the first period as a percentage of total wealth per young household under the compensating variation measure. The welfare would improve by 0.3% and 0.1% in the second and third periods. The welfare of initial old households would decline on average by 0.6% in the same welfare measure because of the increase in the consumption tax rate. The effect of this policy reform on labor income inequality is very small, and the Gini coefficient of labor income would increase from 0.151 to 0.152 in the second and third periods. The intergenerational correlation of human capital would increase from 0.457 to 0.460 in the first period and to 0.461 in the second period. The Gini coefficient of household wealth would increase a little from 0.670 to 0.673 in the second and third periods. This is because childcare and paid leave subsidies would benefit high income households more than low income households.

The Winner: Childcare and Paid Leave Subsidies (with a small margin)

6 Concluding Remarks

The present study constructs a Barro–Becker–Bewley model with parental human capital investment of de la Croix and Doepke (2003). In the model economy, heterogeneous households choose the number of children, education spending for children, and inter vivos transfers to children, in addition to they choose their own consumption and saving. Calibrating the model to the U.S. economy, this study then analyzes the possible effects of family-related policies.

- Increasing child allowances reduces the marginal cost of raising children. This policy increases the number of children but decreases per-child education expenditures and inter vivos transfers. Therefore, this policy declines the average welfare of both existing initial old households and future young households.

- Increasing education subsidies reduces both the marginal cost of child-rearing and the marginal cost of education expenditure. This policy increases education spending significantly but reduces the number of children and inter vivos transfers in the short run. Overall, this policy improves the average welfare of both current households (including early elderly households) and future households.

- Increasing inter vivos transfer subsidies reduces both the marginal cost of child rearing and the marginal cost of inter vivos transfers. This policy increases inter vivos transfers and the number of children while reducing education expenditures in the short run. This policy improves the average welfare of current and future young households but declines the average welfare of initial old households.

Because the number of children, education spending for children, and inter vivos transfer to children are highly substitutable to each other, known as the quantity–quality tradeoff, the optimal policy implied by the quantitative analysis would be a combination of the above three policy changes. For example, if child allowance, education subsidy, and inter vivos transfer subsidy rates are set to $(\tau_{n,t}, \tau_{e,t}, \tau_{b,t}) = (-0.003, -0.047, -0.030)$, the marginal cost of children, education, and inter vivos transfers would decrease more proportionately by 2.0%, 4.7%, and 3.0%, respectively. Then, this policy would increase the number of children, education spending, and inter vivos transfers both in the short run and in the long run, and it would also improve the welfare, on average, of all current and future households.

The model developed in the present study is highly stylized and, to keep its transparency, it intentionally abstracts from several aspects that are possibly important to analyze the behaviors of married households.

- The model does not consider the timings of childbirths, education spending, and intergenerational transfers. One period in the model economy is one generation, and a married couple determine the number of children, education spending, and inter vivos transfers.

- The model does not incorporate the human capital heterogeneity (wage gap) between married couples. Accordingly, the model does not consider the allocation of working and childcare hours between them either.

- The model does not incorporate single households and decisions on marriages and divorces. Most single households without children would be worse off from any child-related policies unless they were appropriately compensated.

These aspects of the household's decisions will be addressed in the future. Regarding the lack of single households in the model economy, the average number of children is normalized to unity in the baseline economy, that is, the average total fertility rate is normalized to 2.0. Suppose, more realistically, that the 50% of all households are married and the other 50% are single (male or female) households. Then, the two thirds of women in the economy are married, and the total fertility rate will be 1.33. Even though this study abstracts from all the above aspects, the policy implications provided in the present study would not change the policy implications significantly.

A Solving the Bewley-type Dynastic OLG Model

This study solves the Bewley-type dynastic OLG model for a steady state equilibrium and an equilibrium transition paths with Gauss-Jacobi iterations to obtain the fixed point of Ω_t. The study solves the altruistically dynastic households' optimization problem by using the marginal value function iteration. The individual household's optimal decisions at each state in each period are obtained by solving the Kuhn-Tucker conditions (or the corresponding complementarity problem) with a nonlinear equation solver, hybrd1, in MINPACK of Fortran90. For more details of the computational algorithm and methods, see Nishiyama and Smetters (2014).

A.1 The Complementarity Problem of Each Household

Let $\mathbf{s}_t = (a_t, h_t, \varepsilon_t)$ be the household's state vector, and let $\mathbf{d}_t = (c_t, s_t, n_t, b_t, e_t)^\top$ be its decision vector. Let $f(\mathbf{d}_t; \mathbf{s}_t, \Omega_t)$ be the objective function on the right-hand-side of equation (11), that is,

$$(60)\quad f(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) = u(c_t) + \tilde{\beta} u(d_{t+1}) + \tilde{\gamma}\, \Phi(n_t) \mathbb{E}_t\big[\, v(a_{t+1}, h_{t+1}, \varepsilon_{t+1}; \Omega_{t+1})\,\big],$$

and let $g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) = 0$ be the budget constraint in equation (12), that is,

$$(61)\quad \begin{aligned} g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) = {} & (1 + \tau_{c,t})(1 + n_t)^{\xi} c_t + s_t + \tau_{n,t} n_t + (1 + \tau_{b,t}) b_t n_t + (1 + \tau_{e,t}) e_t n_t \\ & - (1 + (1 - \tau_{k,t}) r_t) a_t - (1 - \tau_{h,t} - \tau_{p,t}) w_t h_t (1 - (\phi - \psi_t) n_t). \end{aligned}$$

Let λ_t be the Lagrange multiplier for the budget constraint, let $\mu_t = (\mu_{c,t}, \mu_{s,t}, \mu_{n,t}, \mu_{b,t}, \mu_{e,t})^\top$ be the Lagrange multipliers for the inequality constraints in equation (17), and let $\mathbf{d}_{\min} = (\epsilon, \epsilon, 0, 0, 0)^\top$ be the lower bound of $\mathbf{d}_t$, where ϵ is a small number, such as 10^{-3}. Then, the Lagrangian of this one period problem is

$$(62)\quad \mathcal{L}(\mathbf{d}_t, \lambda_t, \mu_t; \mathbf{s}_t, \Omega_t) = f(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) - \lambda_t g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) - \mu_t^\top (\mathbf{d}_{\min} - \mathbf{d}_t)$$

subject to equations (13)–(15), and the Kuhn-Tucker conditions are the first order (stationary) condition,

$$(63)\quad \nabla f(\mathbf{d}_t; \mathbf{s}_t, \Omega_t)^\top - \nabla g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t)^\top \lambda_t + \mu_t = 0,$$

and the complementarity slackness conditions,

$$(64)\quad g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) \le 0, \qquad \lambda_t \ge 0, \qquad \lambda_t g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) = 0,$$

$$(65)\quad \mathbf{d}_t \ge \mathbf{d}_{\min}, \qquad \mu_t \ge 0, \qquad \mu_{i,t}(\mathbf{d}_{i,\min} - \mathbf{d}_{i,t}) = 0 \qquad \forall i.$$

The above complementarity problem can be expressed compactly as the nonlinear system of equations,

$$(66)\quad \min\left\{\max\left[\begin{pmatrix}\nabla f(\mathbf{d}_t;\mathbf{s}_t,\Omega_t)^\top - \nabla g(\mathbf{d}_t;\mathbf{s}_t,\Omega_t)^\top\lambda_t\\ g(\mathbf{d}_t;\mathbf{s}_t,\Omega_t)\end{pmatrix},\begin{pmatrix}\mathbf{d}_{\min}-\mathbf{d}_t\\ 0-\lambda_t\end{pmatrix}\right],\begin{pmatrix}\mathbf{d}_{\max}-\mathbf{d}_t\\ \lambda_{\max}-\lambda_t\end{pmatrix}\right\} = \mathbf{0},$$

where $\mathbf{d}_{\max}$ and $\lambda_{\max}$ are the upper bounds of $\mathbf{d}_t$ and λ_t. Adding non-binding constraints, $\mathbf{d}_t \leq \mathbf{d}_{\max}$ and $\lambda_t \leq \lambda_{\max}$, usually improves the computational stability when we solve the problem of wide-range of heterogeneous households with a Newton-type nonlinear equation solver. We also replace the $\min(a,b)$ and $\max(a,b)$ operators with the Fischer-Burmeister functions,

$$(67)\quad \phi^-(a,b) \equiv a+b-\sqrt{a^2+b^2}, \qquad \phi^+(a,b) \equiv a+b+\sqrt{a^2+b^2},$$

respectively, to make the above system of equations differentiable without affecting the solutions.

A.2 The first Order Conditions

The first order conditions for an interior solution, $\nabla f(\mathbf{d}_t;\mathbf{s}_t,\Omega_t)^\top - \nabla g(\mathbf{d}_t;\mathbf{s}_t,\Omega_t)^\top\lambda_t = 0$, of the household's problem in each period are

$$(68)\quad c_t: \quad u'(c_t) - \lambda_t(1+\tau_{c,t})(1+n_t)^\xi = 0,$$

$$(69)\quad s_t: \quad \tilde\beta u'(d_{t+1})\frac{\partial d_{t+1}}{\partial s_t} - \lambda_t = 0,$$

$$(70)\quad n_t: \quad \tilde\gamma\Phi'(n_t)\,\mathbb{E}_t\big[\,v(a_{t+1},h_{t+1},\varepsilon_{t+1};\Omega_{t+1})\,\big] - \lambda_t\big[\,(1+\tau_{c,t})\xi(1+n_t)^{\xi-1}c_t$$
$$+\tau_{n,t} + (1+\tau_{b,t})b_t + (1+\tau_{e,t})e_t + (1-\tau_{h,t}-\tau_{p,t})w_t h_t\exp(\varepsilon_t)(\phi-\psi_t)\,\big] = 0,$$

$$(71)\quad b_t: \quad \tilde\gamma\Phi(n_t)\,\mathbb{E}_t\left[\,v_a(a_{t+1},h_{t+1},\varepsilon_{t+1};\Omega_{t+1})\frac{\partial a_{t+1}}{\partial b_t}\,\right] - \lambda_t(1+\tau_{b,t})n_t = 0,$$

$$(72)\quad e_t: \quad \tilde\gamma\Phi(n_t)\,\mathbb{E}_t\left[\,v_h(a_{t+1},h_{t+1},\varepsilon_{t+1};\Omega_{t+1})\frac{\partial h_{t+1}}{\partial e_t}\,\right] - \lambda_t(1+\tau_{e,t})n_t = 0,$$

where the value function is (23), the marginal value functions are (76) and (77), and

$$(73)\quad \frac{\partial d_{t+1}}{\partial s_t} = \frac{1}{1+\rho}\frac{1+(1-\tau_{k,t+1})r_{t+1}}{1+\tau_{c,t+1}},$$

$$(74)\quad \frac{\partial a_{t+1}}{\partial b_t} = \frac{1}{1+\rho}\exp(\epsilon_{a,t+1}),$$

$$(75)\quad \frac{\partial h_{t+1}}{\partial e_t} = \frac{1}{1+\rho}B\eta(\vartheta_t+e_t)^{\eta-1}h_t^\tau\bar h_t^\kappa\exp(\epsilon_{h,t+1}).$$

Note that the marginal cost of n_t includes the terms with b_t and e_t, and the marginal costs of b_t and e_t are both proportional to n_t. These create the trade-off between the quantity and quality of children.

The marginal value functions of the household born in period t are

$$(76)\quad v_a(a_t, h_t, \varepsilon_t; \Omega_t) = (1 + (1 - \tau_{k,t})r_t)\lambda_t(a_t, h_t, \varepsilon_t; \Omega_t),$$

$$\begin{aligned}(77)\quad v_h(a_t, h_t, \varepsilon_t; \Omega_t) &= (1 - \tau_{h,t} - \tau_{p,t})w_t \exp(\varepsilon_t)(1 - (\phi - \psi_t)n_t(a_t, h_t, \varepsilon_t; \Omega_t))\lambda_t(a_t, h_t, \varepsilon_t; \Omega_t)\\ &+ \tilde{\beta}(1 - \tau_{h,t+1} - \tau_{p,t+1})w_{t+1}\frac{\exp(\varepsilon_t)\zeta}{1+\rho}\frac{u'(d_{t+1}(a_t, h_t, \varepsilon_t; \Omega_t))}{1+\tau_{c,t+1}}\\ &+ \tilde{\gamma}\Phi(n_t(a_t, h_t, \varepsilon_t; \Omega_t))\frac{1}{1+\rho}\Lambda_h(e_t(a_t, h_t, \varepsilon_t; \Omega_t), h_t, \vartheta_t; \bar{h}_t)\\ &\times \mathbb{E}_t\big[\exp(\epsilon_{h,t})v_h(a_{t+1}(a_t, h_t, \varepsilon_t, \epsilon_{a,t+1}; \Omega_t), h_{t+1}(a_t, h_t, \varepsilon_t, \epsilon_{h,t+1}; \Omega_t), \varepsilon_{t+1}; \Omega_{t+1})\big].\end{aligned}$$

where

$$(78)\quad \lambda_t(a_t, h_t, \varepsilon_t; \Omega_t) = \frac{u'(c_t(a_t, h_t, \varepsilon_t; \Omega_t))}{(1+\tau_{c,t})(1 + n_t(a_t, h_t, \varepsilon_t; \Omega_t))^{\xi}}.$$

A.3 Solving the Bewley Model for an Equilibrium

Steady-State Equilibrium. The computational algorithm for solving the Bewley dynastic model for a steady-state equilibrium with a time-invariant government policy schedule, Ω, is as follows.

1. Set the initial guesses of government's policy variables and factor prices,

$$\Omega^0 = (\tau_k^0, \tau_h^0, \tau_c^0, \tau_n^0, \tau_b^0, \tau_e^0, \tau_p^0, tr_p^0, \vartheta^0, \psi^0, g^0, r^0, w^0).$$

2. Given Ω^0, find the decision rules and value function of households and the distribution of households.

   (a) Set the initial guesses of the value and marginal value functions,

$$\mathbf{v}^0(a, h, \varepsilon; \Omega^0) = (v^0(a, h, \varepsilon; \Omega^0), v_a^0(a, h, \varepsilon; \Omega^0), v_h^0(a, h, \varepsilon; \Omega^0)).$$

   (b) Given $\mathbf{v}^0(a, h, \varepsilon; \Omega^0)$, find the decision rules of households and the state transition functions,

$$\begin{aligned}\mathbf{d}(a, h, \varepsilon; \Omega^0) = (&c(a, h, \varepsilon; \Omega^0), s(a, h, \varepsilon; \Omega^0), n(a, h, \varepsilon; \Omega^0), b(a, h, \varepsilon; \Omega^0),\\ &e(a, h, \varepsilon; \Omega^0), d'(a, h, \varepsilon; \Omega^0), a'(a, h, \varepsilon; \Omega^0), h'(a, h, \varepsilon; \Omega^0)).\end{aligned}$$

   (c) Find the value and marginal value functions, $\mathbf{v}^1(a, h, \varepsilon; \Omega^0)$, that are corresponding to the decision rules, $\mathbf{d}(a, h, \varepsilon; \Omega^0)$, as fixed points by policy function iteration.[9]

   (d) If the relative difference, $\|\mathbf{v}^1(a, h, \varepsilon; \Omega^0) - \mathbf{v}^0(a, h, \varepsilon; \Omega^0)\|_\infty / \|1 + \mathbf{v}^0(a, h, \varepsilon; \Omega^0)\|_\infty$, is small enough, then move to Step 3. Otherwise, update $\mathbf{v}^0(a, h, \varepsilon; \Omega^0)$ by using $\mathbf{v}^1(a, h, \varepsilon; \Omega^0)$ and return to 2 (b).

[9] We need to update the value function with policy function iteration, in each iteration step, because the Kuhn–Tucker conditions include not only the marginal value functions but also the value function itself.

3. Find the steady-state distribution of households $X(a,h,\varepsilon)$ by fixed-point iteration.

4. Compute aggregate variables, $(C, W, L^s, B, E, K, L, Y, G, \Theta, \Psi, P)$, and the government's policy variables and factor prices,

$$\Omega^1 = (\tau_k^1, \tau_h^1, \tau_c^1, \tau_n^1, \tau_b^1, \tau_e^1, \tau_p^1, tr_p^1, \vartheta^1, \psi^1, g^1, r^1, w^1).$$

5. If the relative difference, $\|\Omega^1 - \Omega^0\|_\infty/(1+\|\Omega^0\|_\infty)$, is small enough, then stop. Otherwise, update Ω^0 by using Ω^1 and return to Step 2.

It will suffice to iterate the economy with $(K/L)^0$ instead of (r^0, w^0) in Step 5.

Equilibrium Transition Path. Assume that the economy is in the initial steady-state equilibrium in period 0 and that a new policy schedule, Ω_1, is introduced at the beginning of period 1. Choose a sufficiently large number T such that the economy is said to reach the new steady-state equilibrium within T period. Let Ω_t be the time series of government's policy variables and factor prices as of period t,

$$\Omega_t = \{\tau_{k,s}, \tau_{h,s}, \tau_{c,s}, \tau_{n,s}, \tau_{b,s}, \tau_{e,s}, \tau_{p,s}, tr_{p,s}, \vartheta_s, \psi_s, g_s, r_s, w_s\}_{s=t}^T.$$

Then, the computational algorithm for solving the model for an equilibrium transition path, from the initial steady state to the final steady state, is as follows.

1. Set the initial guesses of government's policy variables and factor prices,

$$\Omega_1^0 = \{\tau_{k,t}^0, \tau_{h,t}^0, \tau_{c,t}^0, \tau_{n,t}^0, \tau_{b,t}^0, \tau_{e,t}^0, \tau_{p,t}^0, tr_{p,t}^0, \vartheta_t^0, \psi_t^0, g_t^0, r_t^0, w_t^0\}_{t=1}^T.$$

2. Given Ω_T^0, compute the final steady-state equilibrium in period T.

3. For period $t = T-1, T-2, \dots, 1$, compute *backward* the decision rules of households and the state transition functions,

$$\begin{aligned}\mathbf{d}_1(a,h,\varepsilon;\Omega_t^0) = \{&c_t(a,h,\varepsilon;\Omega_t^0), s_t(a,h,\varepsilon;\Omega_t^0), n_t(a,h,\varepsilon;\Omega_t^0), b_t(a,h,\varepsilon;\Omega_t^0),\\ &e_t(a,h,\varepsilon;\Omega_t^0)), d_{t+1}(a,h,\varepsilon;\Omega^0), a_{t+1}(a,h,\varepsilon;\Omega^0), h_{t+1}(a,h,\varepsilon;\Omega^0)\}_{t=1}^{T-1},\end{aligned}$$

and update the value and marginal value functions,

$$\mathbf{v}_1(a,h,\varepsilon;\Omega_t^0) = \{v_t(a,h,\varepsilon;\Omega_t^0), v_{a,t}(a,h,\varepsilon;\Omega_t^0), v_{h,t}(a,h,\varepsilon;\Omega_t^0)\}_{t=1}^{T-1}.$$

4. Set the initial distribution of households $X_1(a,h,\varepsilon) = X_0(a,h,\varepsilon)$. For period $t = 1, 2, \dots, T-1$, compute *forward* aggregate variables, $\{C_t, W_t, L_t^s, B_t, E_t, K_t, L_t, Y_t, G_t, \Theta_t, \Psi_t, P_t\}_{t=1}^{T-1}$, and the government's policy variables and factor prices,

$$\Omega_1^1 = \{\tau_{k,t}^1, \tau_{h,t}^1, \tau_{c,t}^1, \tau_{n,t}^1, \tau_{b,t}^1, \tau_{e,t}^1, \tau_{p,t}^1, tr_{p,t}^1, \vartheta_t^1, \psi_t^1, g_t^1, r_t^1, w_t^1\}_{t=1}^T,$$

by computing the distribution function, $\{X_{t+1}(a,h,\varepsilon)\}_{t=1}^{T-2}$, recursively.

5. If the relative difference, $\|\Omega_1^1 - \Omega_1^0\|_\infty/(1 + \|\Omega_1^0\|_\infty)$, is small enough, then stop. Otherwise, update Ω_1^0 by using Ω_1^1 and return to Step 2.

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