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A Model of Bank Leverage and Financial Instability**

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The Safer, the Riskier: A Model of Bank Leverage and Financial Instability*

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Abstract

This note provides an example of a case where financial instability can be amplified by stable fundamentals rather than risky fundamentals, using a variation of Diamond and Rajan (2009).

JEL Classification: E3, G01, G21

Keywords: Bank runs, Great moderation, Financial crisis, Maturity mismatch

1 Introduction

The 2008-09 financial crisis directed renewed attention to the anatomy of a financial sector disruptions. Maturity mismatch and over-leverage/credit among other factors are broadly considered to have played critical roles. Lorenzoni (2008) develops a clear-cut, three-period model where simple externality gives rise to over-credit and ensuing fire sales of capital. Jeane and Korinek (2010) demonstrate that overly volatile boom-and-bust cycles can arise from similar over-credit among lenders. In a spirit similar to these models, our model aims to explore what could create the highly leveraged financial sector, resulting in a higher probability of a crisis.¹ Among others, we focus on the widely acknowledged fact that in the run-up to the crisis, the global financial markets as well as the real economy appeared increasingly stable: the era that has been dubbed the Great Moderation.

This note extends the model introduced by Diamond and Rajan (2009, hereafter DR) to better understand the interaction between banks' leverage and the probability of a bank run.² Our model sets forth a framework to determine banks' leverage under a slightly generalized environment. Using the framework, we provide an example of a case where the probability of a financial crisis can increase under a "safer" economy in the absence of bubbles, irrational agents or externality.

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¹These precursors mentioned focus on over-leverage/credit among lenders while we explicitly incorporate the banking sector into the model.

²Following Diamond and Rajan (2001, 2009), we do not distinguish between a bank run and a financial crisis, as we rely on representative banks.

2 The Model

2.1 Agents, Endowment, Technology and Preference

We consider a variation of economy described by DR. Most of the assumptions are maintained in line with the original DR model except for the households' preference, which we replace with that in Allen and Gale (1998) to focus on the aggregate liquidity shock. Entrepreneurs and bankers are risk-neutral, while households have log utility. The economy lasts for three periods. At date 0, households are born with a unit of a good. By assumption, no household consumes at date 0. They deposit all the date-0 endowments into banks, which lend it to entrepreneurs. Each entrepreneur invests a unit of the good to launch a project at date 0. Projects yield random output \tilde{Y}_2 at their completion at date 2, but the outputs are reduced to X_1 if the projects are prematurely liquidated at date 1. Outcomes of projects follow a uniform distribution with a support $[\underline{Y}_2, \bar{Y}_2]$.

We analyze the crisis probability subject to aggregate preference/liquidity shocks. In DR, the random shock arises from the uncertainty over income, the date-2 endowment in particular, and DR consider unobservable two discrete aggregate states where households expect either high income or low income. In contrast, we eliminate uncertainty with respect to households' income while incorporating a more straightforward random shock regarding liquidity preference. Specifically, we take the Allen-Gale utility; $U(C_1, C_2) = \theta \log(C_1) + (1 - \theta) \log(C_2)$, where C_t is consumption at date t and θ is a continuous random variable with a support $\theta \in [0, 1]$. Here, θ represents households' preference for date-1 consumption, while effectively it signals how much liquidity is needed at date 1.³ Using this Allen-Gale utility provides the advantage that we can focus on aggregate uncertainty in a straightforward manner.

2.2 Households

A household chooses its withdrawal w_1 , given deposit face value D and the preference shock θ , to maximize the aforementioned Allen-Gale type utility,

$$\begin{aligned}
 U(C_1, C_2) &= \theta \log C_1 + (1 - \theta) \log C_2, \\
 s.t. \ C_1 &= \begin{cases} e_1 + w_1 & \text{with probability } 1 - \pi \\ e_1 + X_1 & \text{with probability } \pi \end{cases} \\
 C_2 &= \begin{cases} e_2 + r_{12}(D - w_1) & \text{with probability } 1 - \pi \\ e_2 & \text{with probability } \pi, \end{cases}
 \end{aligned} \tag{1}$$

where π denotes the *ex post* probability of a financial crisis (i.e., bank runs). As discussed, θ can be interpreted as a "liquidity shock" because θ determines the needs for liquidity for each period.

In (1), the one-period interest rate from period one to two is denoted as r_{12} . When the households can smooth out their consumption, the intertemporal first-order condition for consumption is

$$\frac{\theta}{1 - \theta} \left(\frac{C_1}{C_2} \right)^{-1} = r_{12}. \tag{2}$$

³While we describe the utility function as an Allen-Gale type here, in principle the main results remain unchanged if we use a Diamond and Dyvbig (1983) utility. In essence, in light of the purpose of this note, the two types of utility functions work identically.

Meanwhile, the budget constraint holds with equality $C_1 + C_2/r_{12} = e_1 + D + e_2/r_{12}$, and the withdrawal can be written as

$$w_1 = \theta \left(\frac{e_2}{r_{12}} + D \right) - (1 - \theta) e_1. \quad (3)$$

2.3 Entrepreneurs and Bankers

Entrepreneurs and bankers in our model replicate those in DR. As discussed in Diamond and Rajan (2001), each banker is a relationship lender that has obtained special knowledge of the entrepreneurs' business, and this knowledge assures her collection skill to acquire a fraction γY_2 of the output from the entrepreneurs. The collection skill is assumed to be not transferable to other lenders. We also follow the Diamond and Rajan's (2001) argument for the reason why bankers issue demand deposits D : as a commitment device to compensate for the lack of transferability of their collection skill and to promote liquidity creation. In line with this argument, the bankers need to determine the face value of deposits before observing θ in a perfectly competitive banking sector. As discussed in Allen and Gale (1998), the perfect competition converts banks' optimization problem into an optimal risk-sharing problem. As a result, banks maximize the households' expected utility:

$$U(C_1, C_2) = \int_0^{\theta^*} \left\{ \theta \log \left[\theta \left(\frac{e_2}{r_{12}} + D + e_1 \right) \right] + (1 - \theta) \log \left[(1 - \theta) r_{12} \left(\frac{e_2}{r_{12}} + D + e_1 \right) \right] \right\} dF(\theta) \\ + \int_{\theta^*}^1 [\theta \log(e_1 + X_1) + (1 - \theta) \log(e_2)] dF(\theta), \quad (4)$$

where θ^* is the threshold level of θ that precipitates bank runs for $\theta > \theta^*$ and $F(\theta)$ is the cumulative distribution function of θ . More specifically, θ^* is the level of liquidity shock at which the total withdrawal equals the liquidation value of the banker's total assets. Because a larger liquidity shock increases households' withdrawal, by definition, a larger θ^* points to a higher crisis probability. Using θ^* , we can define the probability of a crisis as $\pi = 1 - F(\theta^*)$ for any $D \in [X_1, \infty)$.

In this model, bank runs are precipitated if the realized θ reveals that full payment on deposit is impossible at date 1. In the case of a run, the bankers liquidate all of the entrepreneurs' projects, repay X_1 to households, and loses all their assets. Let the banker's assets be $A(r_{12})$. As assumed in DR, bankers receive perfect signals at date 1 on the date-2 value for \tilde{Y}_2 of each entrepreneur.⁴ Then, the banker's assets $A(r_{12})$ are given by,

$$A(r_{12}) = \frac{1}{\bar{Y}_2 - \underline{Y}_2} \int_{\underline{Y}_2}^{Y_2(r_{12})} X_1 dY_2 + \frac{1}{\bar{Y}_2 - \underline{Y}_2} \int_{Y_2(r_{12})}^{\bar{Y}_2} \frac{\gamma Y_2}{r_{12}} dY_2, \quad (5)$$

where the first and second terms of the equation indicate the project values of liquidation and continuation, respectively. In (5), $Y_2(r_{12})$ denotes the threshold return of projects satisfying $Y_2(r_{12}) = r_{12}X_1/\gamma$. Bankers liquidate a project, whose return cannot exceed the opportunity cost, to provide liquidity for the households' withdrawal demand.

⁴Allen and Gale (1998) make a similar assumption for this interim signal.

2.4 Equilibrium

If banks do not experience a run, the liquidity market clearing condition indicates

$$\theta \left(\frac{e_2}{r_{12}} + D \right) - (1 - \theta) e_1 = \frac{1}{\bar{Y}_2 - \underline{Y}_2} \left[\frac{r_{12} X_1}{\gamma} - \underline{Y}_2 \right] X_1, \quad (6)$$

which uniquely determines the equilibrium interest rate r_{12} . The left-hand side of the equation points to liquidity demand (3), while the the right-hand side indicates supply from the project liquidation shown in (5).

Similar to Theorem 1 in DR, we have the following results: (i) The equilibrium is unique, if it exists; (ii) If there is a set of $\{r_{12}, w_1\}$ solving (3) and (6) with $r_{12} > 0$ and $w_1 \in (0, D)$, the price and the allocation are the equilibrium. Otherwise, $\{r_{\max}, D\}$ is the equilibrium price and allocation, where $r_{\max} = \gamma \bar{Y}_2 / X_1$.⁵

2.5 Banker's Leverage

Now we can state our first main result:

Proposition 1 *The optimal level of D is unique.*

To expedite the illustration, we define the households' lifetime income $m \equiv e_2/r_{12} + D + e_1$ under full consumption smoothing and assume that m is monotonically increasing in D around the neighborhood of X_1 .⁶ This assumption requires a condition,

$$\lim_{D \rightarrow X} \int_0^1 \frac{\partial r_{12}}{\partial D} \frac{e_2}{r_{12}} dF(\theta) < 1,$$

which implies that the discounting effect of a higher r_{12} on m does not exceed the outright effect of an increase in D . Under this assumption, households do not accept any offer of D if D is strictly smaller than X_1 . Then, we examine the two extreme cases: $D = X_1$ and D approaching infinity.

Suppose $D = X_1$. With this D , banks always remain solvent. In this case,

$$U(C_1, C_2) = \int_0^1 \left\{ \log m + (1 - \theta) \log r_{12} + \log \left[\theta^\theta (1 - \theta)^{1-\theta} \right] \right\} dF(\theta).$$

If this is the optimum, the inequality,

$$\lim_{D \rightarrow X} \frac{\partial U}{\partial D} = \int_0^1 \left[\frac{1}{m} \frac{\partial m}{\partial D} + \frac{(1 - \theta)}{r_{12}} \frac{\partial r_{12}}{\partial D} \right] dF(\theta) \leq 0 \quad (7)$$

needs to hold. The first term inside the brackets is by assumption strictly positive. The second term is non-negative because each element, $1 - \theta$, r_{12} , and its derivative with respect to D are all non-negative for any $\theta \in [0, 1]$. Hence, this contradicts the inequality (7).

⁵The maximum level of interest rate r_{\max} is obtained by solving $X_1 = \{X_1^2 / [\gamma (\bar{Y}_2 - \underline{Y}_2)]\} r_{\max} - \underline{Y}_2 X_1 / (\bar{Y}_2 - \underline{Y}_2)$.

⁶This assumption assures the necessity of the financial transaction. Otherwise, there would be no point in discussing banks.

On the other hand, suppose that D takes an infinitely large value. With an infinitesimally small $\theta_\varepsilon > 0$, bank runs take place, because the liquidity demand under θ_ε exceeds the maximum liquidity supply:

$$\theta_\varepsilon \left(\frac{e_2}{r_{\max}} + D + e_1 \right) - e_1 > \frac{1}{\bar{Y}_2 - \underline{Y}_2} \left[\frac{X_1}{\gamma} r_{\max} - \underline{Y}_2 \right] X_1.$$

Therefore, for any $\theta \in (0, 1]$, the expected utility is

$$\begin{aligned} U &= \int_0^1 [\theta \log(e_1 + X_1) + (1 - \theta) \log(e_2)] dF(\theta) \\ &= \int_0^1 U_{run} dF(\theta) < \int_0^1 U_{no-run} dF(\theta), \end{aligned}$$

where we define the utilities under a run and in the absence of a run; $U_{run} = \theta \log(e_1 + X) + (1 - \theta) \log(e_2)$ and $U_{no-run} = \theta \log(\theta m) + (1 - \theta) \log[(1 - \theta) r_{12} m]$, respectively. With any strictly positive probability of a crisis, it is evident to show

$$\int_0^1 U_{run} dF(\theta) < \int_0^{\theta^*} U_{no-run} dF(\theta) + \int_{\theta^*}^1 U_{run} dF(\theta).$$

Hence, the infinitely large D cannot be the optimum.

Given that the two extreme cases cannot be the optimum, the strict concavity of log utility assures the uniqueness of the optimal $D \in (X_1, \infty)$.

3 Crisis Probability and the Banks' Leverage

3.1 Numerical Examples

The numerical example that we give here broadly follows the parameter set chosen by DR. Let $e_1 = 1.2$, $X_1 = 0.95$, $\bar{Y}_2 = 2.5$, $\underline{Y}_2 = 0.5$, and $\gamma = 0.9$. Instead of the two discrete states for $e_2^s \in \{e_2^H, e_2^L\}$ in DR, we take a single constant value $e_2 = 2.1$ (i.e., the simple average of e_2^H and e_2^L chosen by DR) and assume that a liquidity shock θ is generated from the beta distribution with two parameters that give us the mean and the standard deviation of θ . The marked difference in our model from DR lies in the fully endogenous probability of a crisis, $\pi = 1 - F(\theta^*)$, which effectively replaces the exogenously given probability for an “exuberant” state to materialize in DR.

Under the parameter set with a mean of 0.25 and a standard deviation of 0.07 for θ , the banks sets the level of the deposit face value at 1.42, striking a proper balance between the return from high leverage and the risk of a run. The resulting *ex post* probability of a run is 19 percent. Figure 1 plots the households' utility over a variety of deposit face values. The figure also articulates the sub-components of the utility. The smooth bell shape of the utility can be understood as the weighted average of the two sub-components, (i) the expected utility in the absence of a run $E(U|no\ run)$ and (ii) the expected utility under a run $E(U|run)$. In the figure, the probability of a crisis is represented by the ratio of the distance along the vertical axis between the solid and the upper dashed lines to that between the upper and lower dashed lines.

3.2 Discussion: The Safer, the More Secure?

It came as surprise to many people at a time when the 2008-09 financial crisis followed the Great Moderation. The model discussed in this note suggests a possible explanation of why the financial crisis unfolded when people recognized that they are now living in a safer world – the era of Great Moderation.⁷

As discussed so far, banks in our model have a strong incentive to raise D when they find a lower probability of an extreme liquidity shock (i.e., a large θ) – put differently, when they recognize, correctly, that the fundamentals are safe. If banks keep D unchanged in the face of a transition of the economy from a riskier to a safer one, the probability of a financial crisis decreases. This, however, may not always be true if we take the banks’ response into account. Banks raise D in response to stabilized fundamentals, and the banks’ leveraging effect can result in a higher crisis probability, despite the probability’s initial reduction reflecting the stabilization of the economy. In other words, it is not always true that the “safer” the economy is, the more secure you are.

To substantiate “the safer, the riskier” case, we perform an experiment. In our model, the underlying random shock is θ . Now suppose that, for some reason, the distribution of θ , the fundamentals of the economy, undergoes a change from Case 1 to Case 2 as indicated in Figure 2. If bankers do not react to this change in the distribution by keeping their leverage level unchanged, this change in the distribution is favorable to households, as an extremely large θ is now less likely to materialize. In this experiment, the mean of θ decreases from 0.25 to 0.15 with the standard deviation unchanged at 0.07. Recall that the *ex post* probability of a crisis was 19 percent in the initial “risky” distribution with the mean of 0.25 (Case 1 in Table 1). With the lower mean of 0.15 in the “safer” distribution, the probability of a crisis is lower at only 2.5 percent in the absence of changes in banks’ leverage. In contrast, with the elevated leverage reflecting the banks’ response to changes in the distribution, the resulting *ex post* probability of a crisis actually increases from 19 to 22 percent (Case 2 in Table 1). The results of the experiment are summarized in Table 1.

The key to understanding this result lies in θ^* . If banks do not react to the change in the distribution of θ , θ^* remains unchanged and the resulting crisis probability (region A in Figure 2) declines. In contrast, if banks react rationally, θ^* decreases from θ_R^* to θ_S^* , giving rise to a higher crisis probability (region B in Figure 2). The economic interpretation of the decrease in θ^* is that the “safer” distribution incentivizes banks to raise leverage, taking on more risks. The higher leverage makes banks more vulnerable to liquidity shocks and the elevated vulnerability can endanger the economy. As a result, the improved fundamentals can leave the economy exposed to higher crisis risks.

4 Concluding Remarks

The main results of this note need to be taken with two caveats at a minimum. First, we do not claim that a variety of factors not considered here do not play critical roles. Rather, we believe that overoptimism and myopic behavior both by banks and households can prompt financial crises by means of a number of channels. Apart from them, this note provides an example of a case where, even if we assume that agents are fully rational, the probability of a financial crisis can rise in a “safer” economy. Second, this model includes only one source of random shock, that is, the

⁷See Bernanke (2004) for example.

preference/liquidity shock, while the fundamentals would depend on a number of random shocks, such as technology shocks.

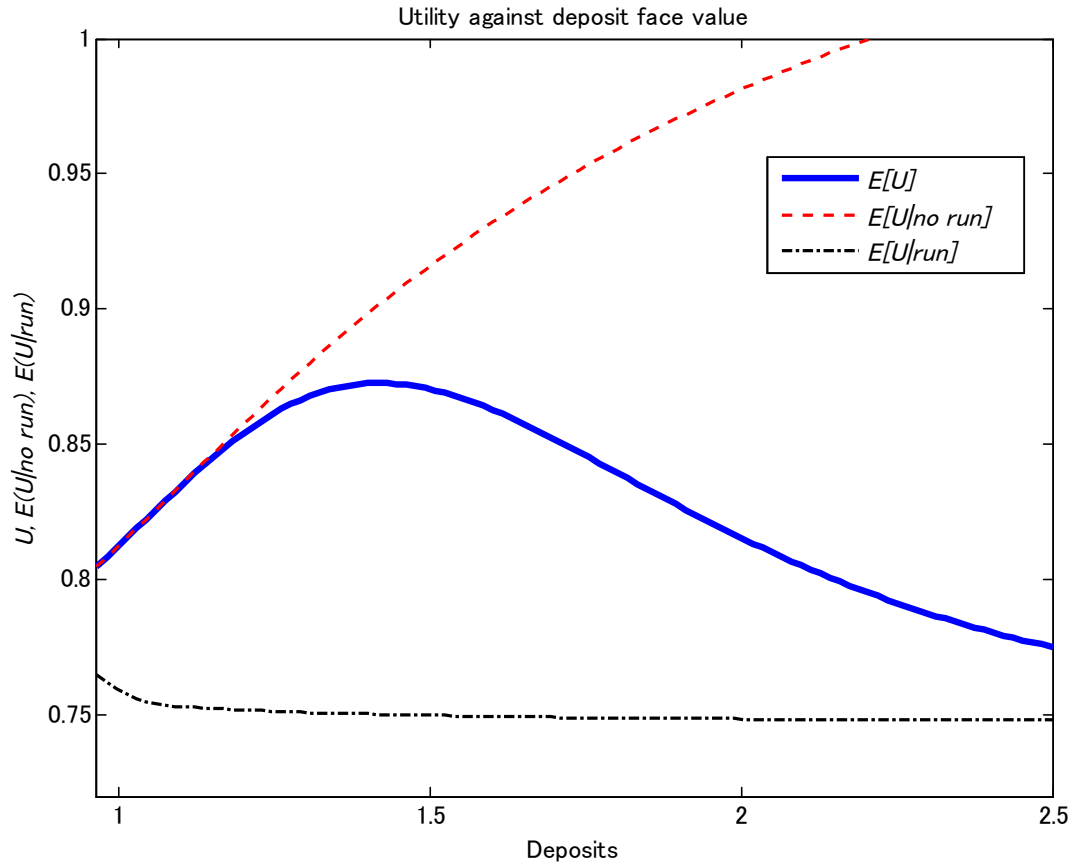
We believe that developing an infinite horizon dynamic model can promote better understanding of financial crises, exploiting our model's compatibility with the standard dynamic stochastic general equilibrium models. To examine whether the main results of this note hold in this class of models would be worth doing and such standardized models would be a promising avenue for future research in understanding the roles and outcomes of government/central bank's interventions in an attempt to promote the financial and economic stability.⁸

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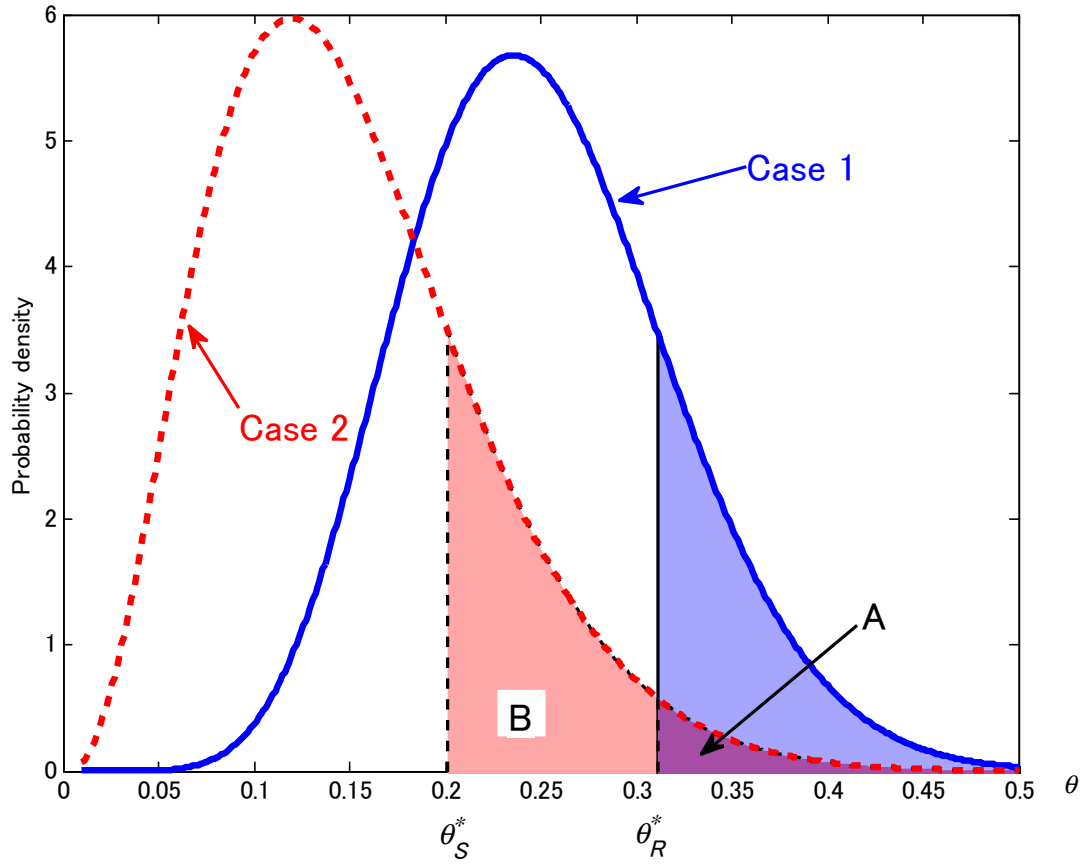
⁸ Angeloni and Faia (2010) and Gertler and Kiyotaki (2011), who introduce banks in this class of models, are marked examples of this direction, despite the absence of maturity mismatch.

Figure 1: Banks' leverage and utility



Note: The solid line represents the utility level against the face value of deposits. The upward-sloping dashed line is the expected utility conditional on no bank run, and the downward-sloping dashed line is the expected utility conditional on a bank run. The calibration is based on the assumption that a liquidity shock follows a beta distribution with a mean of 0.25 and a standard deviation of 0.07.

Figure 2: Comparison of distribution for θ



Note: The solid line represents the probability density function based on a beta distribution with a mean of 0.25 and a standard deviation of 0.07 (Case 1). The dashed line is the probability density function of a beta distribution with a smaller mean of 0.15 but with the same standard deviation (Case 2). Here θ_R^* is the threshold value of a liquidity shock that precipitates a bank run under Case 1, while θ_S^* is the threshold value corresponding to Case 2.

Table 1: Numerical examples of bank run probability π

	Case 1	Case 2	Case 3
μ_θ	0.250	0.150	0.150
σ_θ	0.070	0.070	0.065
π	0.191	0.217	0.204
θ^*	0.311	0.201	0.201
D	1.418	2.056	2.056

Note: Each column calibrates the mean (μ_θ) and standard deviation (σ_θ) of θ under the assumption that θ follows the beta distribution. The probability of bank runs π , the threshold level of θ^* , and the level of deposits D are computed from the calibrated moments.