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Abstract

This paper investigates the relationship between trade and economic development using a two-country, non-scale-growth model. Depending on the share of the expenditure for manufactured goods, we obtain two different results with regard to long-run production patterns. If the share of the expenditure is less than or equal to half, the leader country diversifies while the follower country asymptotically specializes in agriculture completely. If, on the other hand, the share of the expenditure is more than half, the leader country completely specializes in manufacturing while the follower country asymptotically specializes in agriculture completely. Whether or not the follower country can catch up with the leader country in the long run depends on two factors: (1) the patterns of production in both countries and (2) the measure of economic welfare that is used, that is, per capita income or per capita consumption.

Keywords: international trade; non-scale growth; uneven development

JEL Classification: F10; F43; O11; O41

1 Introduction

Suppose that the ongoing process of globalization and the progress of information-communication technology promote the transfer of technology in a broader sense. Then, the differences in production technologies across countries will narrow. The tendency of the technologies to proliferate evenly will lead to the convergence of per capita incomes across countries in at least the neoclassical growth model. Even if there are differences in the initial capital stock, per capita incomes in all countries converge to the same level if the parameters of the countries are identical (Solow, 1956).

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However, the reality seems to be different from what the neoclassical growth theory predicts. To better understand the situation, let us see figures 1 and 2. Using the Extended Penn World Tables 3.0 (EPWT), we plot the graphs of world income distribution similar to those in Acemoglu and Ventura (2002).[1] In figures 1 and 2, the horizontal axes measure the log of income per capita of each country relative to the world average in 1980 and the vertical axes measure the log of income per capita relative to the world average in 1990 and 2000. The world average is a simple arithmetic mean. The 45° line is drawn in these figures. Countries on the 45° line are those that grew at the same rate as the world average over the periods 1980–1990 and 1980–2000. Countries above/below the 45° line are those that grew faster/slower than the world average. Comparing the findings for 1980 and those for 1990, we find that most countries are located near the 45° line. However, on comparing the observations for 1980 with those for 2000, we find that countries tend to deviate from the 45° line, which suggests that income distribution diverged with time.

[Figures 1 and 2 around here]

Such disparities in the income level can be said to imply*uneven development*. There are many theoretical studies that investigate uneven development in the context of north-south trade (Molana and Vines, 1989; Conway and Darity, 1991; Dutt, 1996, 2002; and Sarkar, 2001, 2009). These studies emphasize that uneven development is inevitable given north-south asymmetries in economic structures such as patterns of production, income distribution, and consumption.[2]

It is true that economic structures differ between developed and developing countries and north-south trade models are appropriate for analyzing such a situation. However, it is also important to consider that whether or not the level of per capita income in each country would equalize if, as stated above, the ongoing process of globalization promoted the convergence of the economic structures of countries. That is, it is meaningful to investigate the possibility that income disparity across countries would not disappear even if the asymmetries in economic structures that are prerequisite of north-south trade models disappeared.

Acemoglu and Ventura (2002) present a model in which, along the balanced growth path (BGP), the growth rate of income per capita is equalized across countries even though each country's production technology is different. In their model, each country is endowed with a different, constant-returns-to-scale AK production function. In the integrated world

1) EPWT 3.0 is a database that Adalmir Marquetti and Duncan Foley calculate from the Penn World Table 6.2. The period covered is 1963–2003. EPWT 3.0 can be downloaded from http://homepage.newschool.edu/foleyd/epwt/. We choose 97 countries whose data for 1980, 1990, and 2000 are available.

2) Note, however, that models presented in those studies treat uneven development in terms of the rate of capital accumulation, and not the level of income per capita.

equilibrium, countries with low productivity face a higher relative price for their export goods whereas countries with high productivity face a lower relative price for their export goods. This equalizes the return to capital accumulation, and consequently, also growth rate. If each country's growth rate is equal, the initial income gap never narrows.

However, for our purpose, we need to analyze the possibility that the income gap does not narrow even if all countries have the same production technology. To our understanding, Krugman's (1981) model is suitable for such analysis. Using a two-country model with constant-returns-to-scale agricultural and increasing-returns-to-scale manufacturing sectors, he shows that given different initial endowments of capital stock, each country experiences a different path of economic development even if both countries have the same technology. Krugman called this phenomenon uneven development. The manufacturing sector of the capital-rich country will expand whereas that of the capital-poor country will shrink.[3] It should be emphasized that he points out the possibility that uneven development occurs even if each country has the same economic structure.

Nevertheless, Krugman's model has some problems that should be modified. He uses a production function with fixed coefficients and assumes zero population growth. The rate of capital accumulation is equal to the rate of profit. In the long run, the rate of profit is zero, and hence, the capital accumulation stops, which implies that the long-run economic growth rate will be zero even if the economy completely specializes in manufacturing. If the economy gradually specializes in agriculture, the capital stock gradually decreases, and eventually, it vanishes. Moreover, along the transitional dynamics, the rate of profit can be negative. In reality, population growth is not zero, capital stocks in agricultural countries are not zero, economic growth is not zero, and the rate of profit is not negative. If we consider population growth in Krugman's model, even the industrialized country experiences negative growth of income per capita.

In addition, his analysis is not adequate. Uneven development in Krugman's model is the polarization that the capital-rich country can industrialize whereas the capital-poor country cannot, and that is all there is to this. In other words, a detailed analysis has not been conducted with regard to the income gap between the countries and each country's income per capita growth rate.

Based on these observations, we extend Krugman's model to the non-scale-growth case.[4]

3) From the viewpoint of the capital-rich country, the following three long-run equilibria can emerge: (1) the capital rich country diversifies while the capital poor country completely specializes in agriculture; (2) the capital-rich country completely specializes in manufacturing while the capital-poor country completely specializes in agriculture; (3) the capital-rich country completely specializes in manufacturing while the capital-poor country diversifies.

4) For systematic expositions with regard to non-scale growth and scale effects, see Jones (1999) and Christiaans (2004).

We assume that the production function in manufacturing exhibits increasing returns to scale but the extent is not so large: the elasticity of output with respect to capital stock is less than unity. In addition, we assume that the population growth rate is strictly positive. These assumptions ensure that in our model, not only the growth rate of the economy but also the growth rate of real income per capita is strictly positive in the long run. Furthermore, in our model, the economy-wide capital stock continues to increase even though the economy gradually specializes in agriculture. Therefore, the undesirable properties of Krugman's model will be resolved.

Our idea is based on the work of Christiaans (2008). He develops a small-open-economy, non-scale-growth model in which there exist a constant-returns-to-scale agricultural sector and an increasing-returns-to-scale manufacturing sector, and examines the transitional dynamics toward the long-run equilibrium.[5)] He introduces the rest of the world whose structures except for population growth are identical with the home economy, and assumes that the rest of the world is already on the BGP. Our model extends Christiaans' model to the two-country case.

Using the model, we mainly investigate the following two issues: (i) the terms in which uneven development is defined and (ii) the evolution of uneven development with the transition from autarky to free trade. We provide an explanation for these issues.

(i) We can list several indices that measure uneven development. For example, real income per capita and its growth rate are typical measures of uneven development. However, we can also regard real consumption per capita as a measure of economic welfare. In general, utility depends on consumption, and accordingly, we can think of the level of consumption as a measure of economic welfare. As will be shown later, it is possible that uneven development in terms of income and uneven development in terms of consumption are different.[6)] This difference also relates to the issue of catching up. When we discuss whether developing countries can catch up with developed countries, we need to clarify the concept of catching up that we use: catching up in terms of income per capita or in terms of consumption per capita.

(ii) In the usual trade theory, the transition from autarky to free trade is desirable because it improves welfare. In this respect, the criterion used is real income per capita or utility (real consumption per capita). However, as will be shown later, it is possible that at first, the transition from autarky to free trade decreases real income and real consumption. Nevertheless,

5) Christiaans (2008) uses learning-by-doing to express increasing returns to scale.

6) Dutt (1996) states that the definition of uneven development should be based on objective indicators such as the rate of capital and output growth. He also states that subjective indicators such as utility may have little relevance in the context of long-run development processes because preferences change endogenously during the processes. In contrast, we identify consumption with utility. Obviously, consumption is an objective indicator.

it is possible that with the passage of time, the real per capita consumption under free trade exceeds that under autarky. In such cases, we need to examine the*dynamic gains from trade* along with the*static gains from trade*.[7)] Static gains from trade are measured by whether there are gains from trade at the time of switching from autarky to free trade or whether there are gains from trade at each point in time. Dynamic gains from trade are measured by the sum of present discounted values of gains from trade starting from the switching time to some future point in time. Therefore, it is possible that there exist gains from trade in the long run whereas there exist no gains from trade in the short run.

In our model, we obtain two different results concerning long-run production patterns depending on the share of the expenditure for manufactured goods. If the share of the expenditure is less than or equal to half, the leader country diversifies in the long run while the follower country asymptotically specializes in agriculture completely. If, on the other hand, the share of the expenditure is more than half, the leader country specializes in manufacturing completely while the follower country asymptotically specializes in agriculture completely.[8)] Here, the difference between the leader and follower countries lies only in the difference in the initial endowment of capital stock. These differences in the production pattern give a different answer to problems (i) and (ii). For example, if the share of the expenditure is more than half, which leads to the complete specialization of one country in manufacturing, uneven development gets worse with time in either case.

The remainder of the paper is organized as follows. Section 2 presents the basic structure of our model. Section 3 classifies the production patterns. Section 4 analyzes the growth rate of income per capita and that of consumption per capita along the BGP. Section 5 examines the transitional dynamics. Section 6 examines the level of income per capita and that of consumption per capita using numerical simulations, and compares the situation under autarky with that under free trade. Section 7 concludes the paper.

2 Model

Consider a world that consists of Home (leader country) and Foreign (follower country). Both countries produce homogeneous manufactured and agricultural goods. The manufactured good is used for consumption and investment while the agricultural good is used only for consumption.

7) For static and dynamic gains from trade, see also Baldwin (1992), Mazumdar (1996), and Redding (1999).

8) Skott and Sethi (2000) also obtain such a relationship between expenditure patterns and trade patterns. They build a two-country model with agriculture and manufacturing. In their model, as in our model, agricultural and manufacturing sectors are constant returns to scale and increasing returns to scale, respectively; however, in contrast to our model, labor is the sole factor of production.

2.1 Production

Firms produce manufactured goods X_i^M with labor input L_i^M and capital stock K_i and produce agricultural goods X_i^A with only labor input L_i^A. Here, $i = 1$ and $i = 2$ denote Home and Foreign, respectively. Both countries have the same production functions, which are specified as follows:

$$
\begin{aligned}
X_i^M &= A_i K_i^{\alpha} (L_i^M)^{1-\alpha}, \quad \text{where } A_i = K_i^{\beta} && (1)\\
&= K_i^{\alpha+\beta} (L_i^M)^{1-\alpha}, \quad 0 < \alpha < 1,\ 0 < \beta < 1,\ \alpha + \beta < 1, && (2)\\
X_i^A &= L_i^A. && (3)
\end{aligned}
$$

Here, A_i in equation (1) expresses an externality associated with capital accumulation, which captures the learning-by-doing effect à la Arrow (1962). Substituting A_i into equation (1), we obtain equation (2), which shows that manufacturing production is increasing returns to scale and β corresponds to the extent of the increasing returns. Equation (3) shows that agricultural production is constant returns to scale.

Suppose that labor supply is equal to population and that population is fully employed. Moreover, suppose that population grows at a constant rate n_i and initial population is unity in each country:[9)]

$$
L_i(t) = L_i^M(t) + L_i^A(t) = e^{n_i t}, \quad n_i > 0. \tag{4}
$$

Let p denote the price of manufactured goods relative to agricultural goods. Then, the profits of manufacturing and agricultural firms are respectively given by

$$
\begin{aligned}
\pi_i^M &= pX_i^M - w_i L_i^M - p r_i K_i, && (5)\\
\pi_i^A &= X_i^A - w_i L_i^A, && (6)
\end{aligned}
$$

where w_i denotes wage in terms of agricultural goods and r_i denotes the profit rate.

From the profit-maximizing conditions, we obtain the following relations:

$$
\begin{aligned}
p \frac{\partial X_i^M}{\partial L_i^M} &= w_i = 1, && (7)\\
\frac{\partial X_i^M}{\partial K_i} &= r_i \text{ with } A_i \text{ given.} && (8)
\end{aligned}
$$

9) This assumption is not essential to our results. Even if the initial population levels are different, we can obtain similar results.

From equation (7), we find that wage is unity as long as agricultural production is positive. Note that we assume Marshallian externality in deriving equation (8): profit-maximizing firms regard A_i as exogenously given. Accordingly, firms do not internalize the effect of A_i.

2.2 Consumption

For ease of simplification, we make the classical assumption that wage income and profit income are entirely devoted to consumption and saving, respectively. Let us define real consumption per capita c_i as $c_i = C_i/L = (C_i^M)^\gamma (C_i^A)^{1-\gamma}/L$, where C_i denotes the economy-wide real consumption. In this case, a fraction γ of wage income is spent on C_i^M and the rest $1-\gamma$ is spent on C_i^A. The same assumption is also adopted by Krugman (1981) and Christiaans (2008):

$$pC_i^M = \gamma w_i L_i, \tag{9}$$

$$C_i^A = (1-\gamma) w_i L_i. \tag{10}$$

Moreover, the following relationship between investment and saving holds:

$$pI_i = pr_i K_i. \tag{11}$$

From this equation, the rate of capital accumulation leads to

$$\frac{\dot{K}_i}{K_i} = r_i. \tag{12}$$

That is, the rate of capital accumulation is equal to the rate of profit. A dot over a variable denotes the time derivative of the variable (e.g., $\dot{K}_i \equiv dK_i/dt$).

3 Classification of production patterns

In our model, two countries with two industries engage in international trade, and consequently, seven possible production patterns exist.[10] From the view point of Home, the following four patterns are sufficient for our purpose.[11]

Case 1 Both countries produce both goods, that is, both countries diversify.

10) Under autarky, both goods are produced. For the analysis under autarky, see Appendix A.

11) The remaining three production patterns are as follows: (1) Home completely specializes in agriculture while Foreign completely specializes in manufacturing, (2) Home completely specializes in agriculture while Foreign diversifies, and (3) Home diversifies while Foreign completely specializes in manufacturing.

Case 2 Home completely specializes in manufacturing while Foreign completely specializes in agriculture.

Case 3 Home diversifies while Foreign completely specializes in agriculture.

Case 4 Home completely specializes in manufacturing while Foreign diversifies.

3.1 Case 1: Both countries diversify

In Case 1, the market-clearing conditions for both goods are as follows:

$$X_1^M + X_2^M = C_1^M + C_2^M + I_1 + I_2, \tag{13}$$

$$X_1^A + X_2^A = C_1^A + C_2^A. \tag{14}$$

Rewriting equation (13) with equation (8), we have

$$p(X_1^M + X_2^M) = \gamma(w_1 L_1 + w_2 L_2) + p\alpha(X_1^M + X_2^M). \tag{15}$$

From equation (7), we obtain $L_i^M = [p(1-\alpha)K_i^{\alpha+\beta}]^{\frac{1}{\alpha}}$, which is substituted into equation (2) to get $X_i^M = K_i^{\alpha+\beta}[p(1-\alpha)K_i^{\alpha+\beta}]^{\frac{1-\alpha}{\alpha}}$. Substituting this expression into equation (15), we obtain the price of manufactured goods:

$$p^{\frac{1}{\alpha}} = \frac{\gamma(L_1 + L_2)}{(1-\alpha)^{\frac{1}{\alpha}}\left(K_1^{\frac{\alpha+\beta}{\alpha}} + K_2^{\frac{\alpha+\beta}{\alpha}}\right)}. \tag{16}$$

Substituting equation (16) into $L_i^M = [p(1-\alpha)K_i^{\alpha+\beta}]^{\frac{1}{\alpha}}$, we obtain the share of manufacturing employment in both countries:

$$\theta_1^M \equiv \frac{L_1^M}{L_1} = \frac{\gamma[1 + (L_2/L_1)]}{1 + (K_2/K_1)^{\frac{\alpha+\beta}{\alpha}}}, \tag{17}$$

$$\theta_2^M \equiv \frac{L_2^M}{L_2} = \frac{\gamma[1 + (L_1/L_2)]}{1 + (K_1/K_2)^{\frac{\alpha+\beta}{\alpha}}}. \tag{18}$$

Along the BGP in which both countries diversify, the share of manufacturing employment in both countries has to be constant and the rate of capital accumulation has to be the same in both countries. For this to occur, it is necessary that the growth rate of population in both countries be the same ($n_1 = n_2 = n$). As we know from equation (4), when $n_1 = n_2 = n$, we have $L_1 = L_2$, that is, population is identical in both countries. In the following analysis, we assume that $n_1 = n_2 = n$ in every case to ensure the continuity of the analysis. As stated above, the rate of capital accumulation is equal to the rate of profit. In this case, the rate of profit is given by $r_i = \alpha K_i^{\alpha+\beta-1}(\theta_i^M L)^{1-\alpha}$. Because r_i is constant along the BGP, the rate of capital accumulation along the BGP leads to

$$g_{K_1}^* = g_{K_2}^* = \frac{1-\alpha}{1-\alpha-\beta} n = \phi n > 0, \quad \text{where } \phi \equiv \frac{1-\alpha}{1-\alpha-\beta} > 0, \tag{19}$$

which shows that the rate of capital accumulation is proportionate to the growth rate of population. In addition, from equation (16), the growth rate of the terms of trade along the BGP leads to

$$g_p^* = -\frac{\beta}{1-\alpha-\beta} n = -\varepsilon n < 0, \quad \text{where } \varepsilon \equiv \frac{\beta}{1-\alpha-\beta} > 0. \tag{20}$$

That is, along the BGP, the relative price of manufactured goods is decreasing at a constant rate.

Let us derive the dynamics of capital stock. Considering the BGP growth rate of capital stock, we introduce new variables (scale-adjusted capital stock): $k_1 \equiv K_1/L^\phi$ and $k_2 \equiv K_2/L^\phi$. Using these new variables, we can rewrite the share of manufacturing employment in both countries, (17) and (18), as follows:

$$\theta_1^M = \frac{2\gamma}{1 + (k_2/k_1)^{\frac{\alpha+\beta}{\alpha}}}, \tag{21}$$

$$\theta_2^M = \frac{2\gamma}{1 + (k_1/k_2)^{\frac{\alpha+\beta}{\alpha}}}. \tag{22}$$

With these equations, we can derive the equations of motion for the scale-adjusted capital stock as follows:

$$\dot{k}_1 = \alpha k_1^{\alpha+\beta} \left[\frac{2\gamma}{1 + (k_2/k_1)^{\frac{\alpha+\beta}{\alpha}}} \right]^{1-\alpha} - \phi n k_1, \tag{23}$$

$$\dot{k}_2 = \alpha k_2^{\alpha+\beta} \left[\frac{2\gamma}{1 + (k_1/k_2)^{\frac{\alpha+\beta}{\alpha}}} \right]^{1-\alpha} - \phi n k_2. \tag{24}$$

These differential equations describe the dynamics of capital stock when both countries diversify.

For both countries' diversification to last, additional conditions are required. First, we assume that both countries' capital stocks are strictly positive, that is, $k_1 > 0$ and $k_2 > 0$. Second, let us express Home's agricultural output as a function of k_1 and k_2.

$$X_1^A = \left[1 - \frac{2\gamma}{1 + (k_2/k_1)^{\frac{\alpha+\beta}{\alpha}}} \right] L. \tag{25}$$

The condition $X_1^A > 0$ is given by

$$\left(\frac{k_2}{k_1}\right)^{\frac{\alpha+\beta}{\alpha}} > 2\gamma - 1. \tag{26}$$

When both $k_1 > 0$ and $k_2 > 0$, equation (26) necessarily holds if $\gamma \leq 1/2$. That is, if the expenditure share for manufactured goods is less than or equal to half, then agricultural output is strictly positive, which means that both countries always diversify. If, on the other hand, $\gamma > 1/2$, we can rewrite equation (26) as follows:

$$k_2 > (2\gamma - 1)^{\frac{\alpha}{\alpha+\beta}} k_1. \tag{27}$$

This means that if k_1 and k_2 satisfy equation (27), Home's agricultural output is strictly positive. In the same way, we can derive the condition that Foreign's agricultural output is strictly positive when $\gamma > 1/2$ as follows:

$$k_2 < \frac{k_1}{(2\gamma - 1)^{\frac{\alpha}{\alpha+\beta}}}. \tag{28}$$

In the phase diagram that will be introduced later, the area composed of equalities (27) and (28) forms a*diversification cone*. As long as a combination of k_1 and k_2 is inside the cone, both countries diversify. However, if a combination of k_1 and k_2 is outside the cone, one country's agricultural output becomes zero and the country completely specializes in manufacturing while the other country diversifies.

3.2 Case 2: Home produces only manufactured goods while Foreign produces only agricultural goods

In Case 2, only Home accumulates capital stock and the market clearing conditions for both goods are as follows:

$$X_1^M = C_1^M + C_2^M + I_1, \tag{29}$$
$$X_2^A = C_1^A + C_2^A. \tag{30}$$

Using these equations, we obtain the terms of trade:

$$p = \frac{\gamma L}{(1-\alpha)(1-\gamma)X_1^M}. \tag{31}$$

The rate of capital accumulation is given by

$$\frac{\dot{K}_1}{K_1} = r_1 = \alpha K_1^{\alpha+\beta-1} L^{1-\alpha}. \tag{32}$$

From this, the rate of capital accumulation and the growth rate of the terms of trade lead to

$$g_{K_1}^{*} = \phi n, \tag{33}$$

$$g_p^{*} = -\varepsilon n. \tag{34}$$

The equation of motion for the scale-adjusted capital stock is given by

$$\dot{k}_1 = \alpha k_1^{\alpha+\beta} - \phi n k_1. \tag{35}$$

The dynamics in this case are stable: if both countries start with this production pattern, then the situation is sustainable.

3.3 Case 3: Home diversifies while Foreign produces only agricultural goods

In Case 3, only Home accumulates capital stock and the market clearing conditions for both goods are as follows:

$$X_1^M = C_1^M + C_2^M + I_1, \tag{36}$$

$$X_1^A + X_2^A = C_1^A + C_2^A. \tag{37}$$

Using these equations, we obtain the terms of trade:

$$p = \frac{2\gamma L}{(1-\alpha) X_1^M}. \tag{38}$$

With p now determined, we can obtain the share of manufacturing employment in Home:

$$\theta_1^M = 2\gamma. \tag{39}$$

For $0 < \theta_1^M < 1$ to hold, we need

$$\gamma < \frac{1}{2}. \tag{40}$$

The rate of capital accumulation is given by

$$\frac{\dot{K}_1}{K_1} = r_1 = \alpha K_1^{\alpha+\beta-1}(L_1^M)^{1-\alpha} = \alpha K_1^{\alpha+\beta-1}(2\gamma L)^{1-\alpha}. \tag{41}$$

From this, we obtain the BGP growth rate as follows:

$$g_{K_1}^* = \phi n, \tag{42}$$

$$g_p^* = -\varepsilon n. \tag{43}$$

Considering the BGP growth rate, we obtain the equation of motion for the scale-adjusted capital stock:

$$\dot{k}_1 = \alpha(2\gamma)^{1-\alpha} k_1^{\alpha+\beta} - \phi n k_1. \tag{44}$$

These dynamics are stable: if both countries start with this production pattern, such a situation is sustainable given $\gamma < 1/2$.

3.4 Case 4: Home produces only manufactured goods while Foreign diversifies

In Case 4, both countries accumulate capital stock. The market clearing conditions for both goods are as follows:

$$X_1^M + X_2^M = C_1^M + C_2^M + I_1 + I_2, \tag{45}$$

$$X_2^A = C_1^A + C_2^A. \tag{46}$$

From this, the terms of trade are given by

$$p = \frac{\gamma L}{(1-\alpha)[(1-\gamma)X_1^M + X_2^M]}. \tag{47}$$

Rewriting equation (47) leads to

$$(1-\alpha)^{\frac{1}{\alpha}} K_2^{\frac{\alpha+\beta}{\alpha}} p^{\frac{1}{\alpha}} = \gamma L - (1-\alpha)(1-\gamma) K_1^{\alpha+\beta} L^{1-\alpha} p. \tag{48}$$

Equation (48) is an equation that determines p. Although we cannot find p explicitly, we can see that p is uniquely determined:[12)]

$$p = p(K_1, K_2, L). \tag{49}$$

In this case, the rate of capital accumulation is given by

$$\frac{\dot{K}_1}{K_1} = r_1 = \alpha K_1^{\alpha+\beta-1} L^{1-\alpha}, \tag{50}$$

$$\frac{\dot{K}_2}{K_2} = r_2 = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} p^{\frac{1-\alpha}{\alpha}} K_2^{\frac{\beta}{\alpha}}. \tag{51}$$

Note that p in equation (51) should be replaced by p that is determined through equation (49). From equation (50), we find that the BGP growth rate of capital accumulation in Home is given by $g^*_{K_1} = \phi n$.

The BGP growth rate of capital accumulation in Foreign and the terms of trade can be calculated as follows. Along the BGP, K_2 should grow at a constant rate and Foreign's share of manufacturing employment should be constant, which yields the following system of equations:

$$\frac{1}{\alpha} g_p + \frac{\alpha+\beta}{\alpha} g_{K_2} - n = 0, \tag{52}$$

$$\frac{1-\alpha}{\alpha} g_p + \frac{\beta}{\alpha} g_{K_2} = 0. \tag{53}$$

From this, we obtain the BGP growth rate:

$$g^*_{K_2} = \phi n, \tag{54}$$

$$g^*_p = -\varepsilon n. \tag{55}$$

Let us derive the dynamic equations for the scale-adjusted capital stock. The BGP growth rate of capital stock in both countries is the same as in Cases 1–3. Because the terms of trade p continue to decline at a constant rate along the BGP, we introduce a new variable $\pi \equiv pL_2^{\varepsilon}$ (scale-adjusted terms of trade). With this variable, we can derive the

12) Let us consider the left-hand and right-hand sides of equation (48) to be the functions of p. Then, the left-hand side is an increasing function of p that passes through the origin and the right-hand side is a straight line with a negative slope and a positive intercept. From this, p is determined by the intersection of the two functions.

following dynamic equations of the scale-adjusted capital stock:

$$\dot{k}_1 = \alpha k_1^{\alpha+\beta} - \phi n k_1, \tag{56}$$

$$\dot{k}_2 = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \pi^{\frac{1-\alpha}{\alpha}} k_2^{\frac{\alpha+\beta}{\alpha}} - \phi n k_2. \tag{57}$$

Further, π that appears in the above equations satisfies the following equation:

$$(1-\alpha)^{\frac{1}{\alpha}} k_2^{\frac{\alpha+\beta}{\alpha}} \pi^{\frac{1}{\alpha}} = \gamma - (1-\alpha)(1-\gamma) k_1^{\alpha+\beta} \left(\frac{L_1}{L_2}\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \pi. \tag{58}$$

Equation (58) is a rewritten form of equation (48). From equation (58), π becomes a function of k_1 and k_2:[13)]

$$\pi = \pi(k_1, k_2), \quad \pi_{k_1} < 0,\ \pi_{k_2} < 0, \tag{59}$$

where the signs of the partial derivatives are obtained by completely differentiating equation (58).

In the steady state in this case, $\dot{k}_1 = 0$, $\dot{k}_2 = 0$, and equation (58) have to be satisfied simultaneously. However, as a later analysis with phase diagram will show, such a situation does not exist. The loci of $\dot{k}_1 = 0$ and $\dot{k}_2 = 0$ do not intersect.

Nevertheless, as the later analysis with phase diagram will also show, we can know the long-run situation. In the long run, k_1 converges to a value determined by $\dot{k}_1 = 0$, k_2 asymptotically converges to zero, and π will be constant:

$$\pi^* = \frac{\gamma}{(1-\alpha)(1-\gamma)\left(\frac{\alpha}{\phi n}\right)^{\frac{\alpha+\beta}{1-\alpha-\beta}}}. \tag{60}$$

4 Long-run growth rates

We use the real income per capita and real consumption per capita as measures of economic welfare. To obtain these measures, we need to define the price index. In the following analysis, we use the consumer price index. Let p_c denote the consumer price index that is consistent with the expenditure minimizing problem. Then, the price index is given by $p_c = p^\gamma$.[14)]

Let $y_{i,A}$ and $y_{i,M}$ denote the real income per capita when the economy completely spe-

13) We can determine the scale-adjusted terms of trade π with the same logic as that used in p.

14) Strictly speaking, we have $p_c = \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)} p^\gamma$. However, we use $p_c = p^\gamma$ because the constant terms have no effect on our results.

cializes in agriculture and the real income per capita when the economy completely specializes in manufacturing, respectively. Then, we have $y_{i,A} = X_i^A/(p_c L) = 1/p^\gamma$ and $y_{i,M} = (pX_i^M)/(p_c L) = (p^{1-\gamma}X_i^M)/L$. Remember that the BGP growth rate of K and that of p are $g_K^* = \phi n$ and $g_p^* = -\varepsilon n$, respectively. Using these growth rates, we can obtain the following growth rates of real income per capita: $g_{y_{i,A}}^* = g_{y_{i,M}}^* = \gamma\varepsilon n > 0$.

Let $y_{i,D}$ denote the real income per capita when the economy diversifies. Then, we have $y_{i,D} = (pX_i^M + X_i^A)/(p_c L)$. Given that along the BGP, each sector's employment share is constant, we find that the growth rates of pX_i^M and X_i^A under diversification are equal to the growth rates of pX_i^M and X_i^A under complete specialization, respectively. Therefore, the growth rate of $y_{i,D}$ is equal to the growth rates of $y_{i,A}$ and $y_{i,M}$.

$$g_{y_{i,D}}^* = g_{y_{i,A}}^* = g_{y_{i,M}}^* = \gamma\varepsilon n > 0. \tag{61}$$

From the above analysis, we find that the BGP growth rate of real income per capita is equal in every case.[15)]

We now focus on real consumption per capita. In our model, consumption consists of only wage income, and hence, c_i is equal to real wage in terms of the consumer price index, that is, $c_i = w_i/p_c$. As long as both goods are produced, we have $w = 1$, from which we obtain $c_i = 1/p_c$ under autarky and under diversification in free trade. Under complete specialization in manufacturing, we obtain $c_i = w_i/p_c = (1-\alpha)\pi k_i^{\alpha+\beta}/p_c$. Because both π and k_i are constant in the long-run equilibrium, the BGP growth rate of real consumption per capita is equal to the absolute value of the rate of change in the consumer price index, and therefore, is equal to the growth rate of real income per capita:

$$g_{c_i}^* = g_{y_{i,A}}^* = g_{y_{i,M}}^* = g_{y_{i,D}}^* = \gamma\varepsilon n > 0. \tag{62}$$

The result that a country specializing in a low-growth sector (agriculture in our model) can attain the same growth rate as a country specializing in a high-growth sector (manufacturing in our model) is also shown in Felbermayr (2007). In his model, the price of high-growth investment goods relative to low-growth consumption goods continues to decline along the BGP. This continuous change in terms of trade equalizes the growth rates of both countries. In our model, the price of manufactured goods relative to agricultural goods continues to decline in the long run.[16)] It follows from this that a country specializing

15) We obtain real income by deflating nominal income with p_c. In contrast, like Christiaans (2008), we can also use p to deflate nominal income. Even in this case, three growth rates are identical and given by $g_{y_{i,A}} = g_{y_{i,M}} = g_{y_{i,D}} = \varepsilon n > 0$.

16) This property derives from the basic framework of Christiaans' (2008) model. For a model in which the relative price continues to decrease along the BGP, see also Wong and Yip (1999).

completely in agriculture whose long-run output growth is zero can attain a positive growth in real income per capita.

5 Transitional dynamics

5.1 Transition to an industrial or a non-industrial country

The above analysis concerns a situation where both countries' production patterns are fixed to the corresponding pattern at the time when the two countries start. In what follows, we analyze the transitional dynamics of both countries with initial endowments of capital stock being given historically. Here, we assume that both countries already engage in free trade at the initial time. Note that even when k_i approaches zero, K_i continues to increase because the growth rate of K_i is equal to the rate of profit and the rate of profit is always positive.

As stated above, the dynamical systems differ depending on whether the share of the expenditure for manufactured goods is less than or equal to 1/2, or more than 1/2.

[Figure 3 around here]

Figure 3 is a phase diagram for $\gamma \leq 1/2$ and corresponds to Cases 1 and 3.[17] Both countries' economic structures are identical except for the initial endowments of capital stock, and accordingly, the phase diagram is symmetric with respect to the 45° line. E_1 is a steady state in which both countries diversify, corresponding to Case 1. E_2 is a steady state in which Home diversifies while Foreign asymptotically specializes in agriculture completely,[18] corresponding to Case 3. E_3 is a steady state in which Foreign diversifies while Home asymptotically specializes in agriculture completely. As figure 3 shows, E_1 is a saddle point, and is thus, unstable. Accordingly, unless the initial conditions are exactly located either at E_1 or on the 45° line, both countries move toward either E_2 or E_3. This means that in the long run, one country diversifies while the other country asymptotically specializes in agriculture completely.

[Figure 4 around here]

Figure 4 is a phase diagram for $\gamma > 1/2$ and corresponds to Cases 1, 2, and 4.[19] The

17) The coordinates of the points in figure 3 are given in Appendix C.
18) The word "asymptotically" implies that the agricultural output converges to zero but it never vanishes because we assume that Foreign's capital stock is strictly positive. For this, see also Christiaans (2008)
19) The coordinates of the points in figure 4 are given in Appendix C.

phase diagram in this case is more complicated than that in $\gamma \leq 1/2$.[20] We now explain the loci of $\dot{k}_1 = 0$ and $\dot{k}_2 = 0$.

First, we explain the locus of $\dot{k}_1 = 0$ extending from P_1 to P_2 and the locus of $\dot{k}_2 = 0$ extending from P_3 to P_4. These loci correspond to the dynamics in which both countries diversify, that is, Case 1. The intersection of these loci E'_1 is a steady state in which both countries diversify. In this case, it is necessary that the dynamics be located in the diversification cone.

Second, we explain the locus of $\dot{k}_2 = 0$ extending from O to P_3 and the locus of $\dot{k}_1 = 0$ extending from P_2 to E'_2. These loci correspond to the dynamics in which Home completely specializes in manufacturing while Foreign diversifies, that is, Case 4.

Finally, we explain the locus of $\dot{k}_1 = 0$ extending from O to P_1 and the locus of $\dot{k}_2 = 0$ extending from P_4 to E'_3. These loci correspond to the dynamics in which Foreign completely specializes in manufacturing while Home diversifies.

As with the case of $\gamma \leq 1/2$, E'_1 is a saddle point, and is thus, unstable. As the arrows in figure 4 show, both countries converge to either E'_2 or E'_3. E'_2 is a long-run equilibrium in which Home completely specializes in manufacturing while Foreign asymptotically specializes in agriculture completely, corresponding to Case 2. E'_3 is a long-run equilibrium in which Foreign completely specializes in manufacturing while Home asymptotically specializes in agriculture completely. That is, in the long run, one country completely specializes in manufacturing while the other country asymptotically specializes in agriculture completely.

We now further explain the transitional dynamics toward a long-run equilibrium. We have $k_1 = k_2$ on the 45° line. Because $k_1 \equiv K_1/L_1^{\phi}$, $k_2 \equiv K_2/L_2^{\phi}$; further, $L_1 = L_2$ and $k_1 = k_2$ together imply $K_1 = K_2$. Whether a combination of both countries' initial capital stock lies above or below the 45° line determines the long-run equilibrium to which both countries converge. When the initial capital stock of Home is larger than that of Foreign, both countries converge to E_2 if $\gamma \leq 1/2$ and to E'_2 if $\gamma > 1/2$. That is, a country whose initial capital stock is larger than that of its trade partner diversifies or completely specializes in manufacturing in the long run. In contrast, a country whose initial capital stock is smaller than that of its trade partner asymptotically specializes in agriculture completely in the long run irrespective of the size of γ.

From the above analysis, we obtain the following proposition:

Proposition 1. *Suppose that both countries differ in their initial endowments of capital stock. If the manufacturing employment share is less than or equal to* $1/2$*, then in the long-*

20) In our model, a regime switch occurs from diversification to complete specialization. That is, the systems of differential equations can change across boundaries, which yields a discontinuity in the right-hand sides of the differential equations. For the behavior of the system on the boundary, see Honkapohja and Ito (1983) and Marrewijk and Verbeek (1993).

run equilibrium, one country diversifies while the other country asymptotically specializes in agriculture completely. If, in contrast, the share of manufacturing employment is more than $1/2$*, then in the long-run equilibrium, one country completely specializes in manufacturing while the other country asymptotically specializes in agriculture completely.*

Why do we obtain such results?[21] To begin with, we consider the case of $\gamma \leq 1/2$. Suppose that the initial capital stock in Home is larger than that in Foreign without loss of generality. When both countries diversify at the initial point in time, the wage and the relative price in Home are the same as those in Foreign, respectively. Because there exist increasing returns to scale in manufacturing, the rate of profit in Home is higher than that in Foreign.[22] The rate of capital accumulation is equal to the rate of profit, which implies that Home's capital stock grows faster than Foreign's capital stock. Then, at the next point in time, the capital stock in Home is still larger than that in Foreign, from which the rate of profit in Home is still higher than that in Foreign, which implies that Home's capital stock grows faster than Foreign's capital stock. This process continues. That is, the difference in the initial endowments of capital stock expands cumulatively. When $\gamma > 1/2$, the same argument holds.

5.2 Different paths converging to equilibrium

This subsection analyzes the transitional dynamics of the model using numerical simulations.[23] For this purpose, we need to specify the parameters. We use $\alpha = 0.3$, $\beta = 0.2$, and $n = 0.02$.[24] In addition, we use $\gamma = 0.4$ for the case where the share of the expenditure for manufactured goods is less than or equal to 1/2 and $\gamma = 0.6$ for the case where the expenditure share is more than 1/2.

In figure 5, we superimpose dynamic paths starting at different initial conditions upon the phase diagram of the case where the expenditure share is less than or equal to 1/2. We choose $S_1(20, 15)$ and $S_2(60, 45)$ as the initial values. Both initial values converge to the long-run equilibrium $E_2(84, 0)$. As stated above, at E_2, Home diversifies while Foreign asymptotically specializes in agriculture completely. Starting from S_1, Foreign industrializes at first, but it becomes an agricultural country in the end.

21) Here, we mention the differences in specialization patterns between our model and Krugman (1981). In Krugman's model, complete specialization in manufacturing occurs when a country reaches the long-run equilibrium: the country is diversified before it reaches the long-run equilibrium. In our model, in contrast, complete specialization in manufacturing occurs before a country reaches the long-run equilibrium.

22) If production is constant returns to scale, the rates of profit in both countries are equalized despite the difference in capital stock.

23) For numerical simulations, we use *Mathematica* 7.

24) Under perfect competition, the parameter α corresponds to the profit share, and therefore, $\alpha = 0.3$ is reasonable. We owe the value of the extent of externality β to the simulation of Graham and Temple (2006).

[Figure 5 around here]

In figure 6, we superimpose dynamic paths starting at different initial conditions upon the phase diagram of the case where the manufacturing expenditure share is more than 1/2.[25] We choose $S'_1(20, 5)$, $S'_2(40, 30)$, $S'_3(100, 60)$, and $S'_4(150, 100)$ as the initial values. All initial values converge to the long-run equilibrium $E'_2(115, 0)$. As stated above, at E'_2, Home completely specializes in manufacturing while Foreign asymptotically specializes in agriculture completely. Starting from S'_1, Home always completely specializes in manufacturing while Foreign industrializes at first but gradually becomes an agricultural country. Starting from S'_2, S'_3, and S'_4, both countries remain diversified at first but they eventually cross the boundary of the diversification cone, and consequently, Home completely specializes in manufacturing while Foreign gradually becomes an agricultural country.

[Figure 6 around here]

6 Comparison between free trade and autarky

6.1 Uneven development in terms of income and consumption

Thus far, we conducted our analysis on the assumption that both countries are under free trade at the initial point in time. Now, we compare real income per capita and real consumption per capita under free trade and under autarky.[26]

To obtain income and consumption, we need to express the output and terms of trade in each case as functions of k_1, k_2, and π. Because we know the time paths of k_1, k_2, and π from the numerical simulations, we can obtain the time paths of income and consumption using the numerical values of k_1, k_2, and π.[27]

First, we investigate the case where $\gamma \leq 1/2$. Figure 7 shows the time paths of real income per capita in Home and Foreign. The parameters used are the same as before. We choose $S_1(20, 15)$ as an initial condition both under free trade and autarky. This implies that Home is the leader while Foreign is the follower at the initial point in time.

In figure 7, solid and broken lines correspond to free trade and autarky, respectively. Home's income under free trade continues to exceed that under autarky. In contrast, Foreign's income under free trade continues to fall short of that under autarky. In usual trade

25) In this case, we need to derive the time path of π. For the derivation, see Appendix D.

26) For the transition from autarky to free trade, see Appendixes A and B.

27) The equations of manufacturing output, agricultural output, and terms of trade in each case are given in Appendix E. Case 3 is an extreme situation of Case 1: if we let $k_2 \to 0$ in Case 1, then we obtain Case 3. In a similar way, Case 2 is an extreme situation of Case 4: if we let $k_2 \to 0$, then we obtain Case 2.

models, real income necessarily increases when the economy switches from autarky to free trade, but this is not the case in our model. When switching from autarky to free trade, Home experiences an increase in the price of manufactured goods, which leads to a decrease in consumption and an increase in investment. In this case, the extent of the increase in investment exceeds the extent of the decrease in consumption, thereby leading to the increase in the real income. In Foreign, the opposite holds. When switching from autarky to free trade, Foreign experiences an increase in consumption and a decrease in investment. In this case, the extent of the decrease in consumption exceeds the increase in consumption, thereby leading to the fall in real income.

Figure 8 shows the time paths of real consumption per capita in Home and Foreign. This is an interesting case. For a while, Home experiences lower consumption under free trade than that under autarky, but this situation reverses at some point in time. In Foreign, consumption under free trade is always larger than that under autarky. At the initial point in time, Home has a comparative advantage in manufacturing while Foreign enjoys the same in agriculture. The relative price of manufactured goods under free trade necessarily lies in between Home's and Foreign's prices under autarky, and accordingly, under free trade, the consumer price index of Home increases and that of Foreign decreases. Because real consumption per capita is given by the inverse of the consumer price index, Home's real consumption per capita under free trade is smaller than that under autarky, and Foreign's real consumption per capita under free trade is larger than that under autarky. The consumer price index continues to fall with the passage of time. However, before the countries reach the BGP, the rate of decline in the consumer price index under free trade is larger than that under autarky. Therefore, eventually, Home's real consumption per capita under free trade exceeds that under autarky.

Figure 9 shows the time paths of relative income y_1/y_2 and relative consumption c_1/c_2. Under autarky, both y_1/y_2 and c_1/c_2 approach unity with the passage of time, and hence, Foreign can catch up with Home. Under free trade, y_1/y_2 increases up to a certain value, that is, the income gap widens over time. In contrast, c_1/c_2 is always unity, that is, Foreign can catch up with Home in terms of consumption level.

[Figures 7, 8, and 9 around here]

Second, we investigate the case where $\gamma > 1/2$. The parameters used are the same as before. We choose $S'_2(40, 30)$ as an initial condition both under free trade and autarky.

Figure 10 shows the time paths of real income per capita in Home and Foreign. The time paths under free trade bend around $t = 60$ because after that, Home completely specializes in manufacturing, and hence, the switching of the system of the differential equations occurs.

From figure 10, we find that under free trade, real income per capita in Home increases whereas that in Foreign decreases as compared to under autarky, which is similar to the case where $\gamma \leq 1/2$.

Figure 11 shows the time paths of real consumption per capita in Home and Foreign. As with the case where $\gamma \leq 1/2$, this is an interesting case. For a while, Home's real consumption under free trade is less than that under autarky, but the former becomes greater than the latter with the passage of time. In Foreign, a reverse relation is observed. Foreign's real consumption under free trade is larger than that under autarky, but the former becomes less than the latter with the passage of time.

Figure 12 shows the time paths of relative income and relative consumption. Under autarky, as in the case of $\gamma \leq 1/2$, both relative income and relative consumption approach unity over time: Foreign can catch up with Home. Under free trade, the relative income increases up to a certain value, and hence, the income gap widens. The relative consumption under free trade exhibits an interesting behavior. Up to around $t = 60$ where both countries diversify, the relative consumption under free trade is unity, that is, Foreign continues to catch up with Home until $t = 60$. After that, Home completely specializes in manufacturing, and the relative consumption increases up to a certain value, that is, the gap in welfare between Home and Foreign widens.

[Figures 10, 11, and 12 around here]

Summarizing the above analysis, we obtain the following results.

Result 1. *Suppose that uneven development is measured in terms of real income per capita. Then, the follower country cannot catch up with the leader country irrespective of the share of the expenditure for manufactured goods.*

Result 2. *Suppose that uneven development is measured in terms of real consumption per capita. If the share of the expenditure for manufactured goods is smaller than or equal to* $1/2$*, then the follower country catches up with the leader country. If, on the other hand, the share of the expenditure is more than* $1/2$*, then the follower country cannot catch up with the leader country.*[28]

At the time when a country switches from autarky to free trade, real income increases and real consumption decreases or real income decreases and real consumption increases. These results originate in the differences in the propensity to save from wage income and from profit income. If we assume that a constant fraction of national income (wages plus

28) Similar results are obtained in Skott and Sethi (2000). They define uneven development in terms of real wage. Real wage in their model is identical to real consumption per capita and the level of utility.

profits) is saved like Solow (1956), real income and real consumption move proportionately, and hence, we can identify real income with real consumption. However, if the propensity to save differs according to income categories as in our model, we cannot identify real income with real consumption.

6.2 Static and dynamic gains from trade

The preceding subsection shows that when $\gamma > 1/2$ under free trade, uneven development in terms of real consumption per capita occurs (figure 12). However, even in this case, there exists a situation where Foreign is better off under free trade than under autarky. In the case where relation between consumption under free trade and that under autarky is reversed with the passage of time, we can compare the integral of the present discounted values of real consumption per capita under free trade with that under autarky to judge which is better, free trade or autarky. If both countries switch from autarky to free trade at time zero, we can define the definite integral of $c(t)$ from $t = 0$ to $t = T$ as follows:

$$W_i^j = \int_0^T c_i^j(t)\exp(-\rho t)\,dt, \quad \rho > 0,\ i = 1, 2,\ j = \mathrm{AT}, \mathrm{FT}, \tag{63}$$

where ρ denotes the rate of time preference. The superscript j denotes whether the economy is under autarky ($j = \mathrm{AT}$) or under free trade ($j = \mathrm{FT}$). For example, W_1^{FT} denotes the integral of the present discounted values of real consumption per capita in Home under free trade.

Using equation (63), we compute the integral of consumption.[29] Table 1 shows the results when $\gamma = 0.4$ and $T = 100$. When $\rho = 0.02$, the welfare of Home declines when switching from autarky to free trade. In contrast, when $\rho = 0.01$, the welfare increases when switching.

Table 2 shows the results when $\gamma = 0.6$ and $T = 200$. When $\rho = 0.02$, both countries are rendered better off by switching to free trade. However, when $\rho = 0.01$, the welfare of Foreign declines when switching from autarky to free trade.

[Tables 1 and 2 around here]

We can summarize the above results as follows:

Result 3. *Suppose that economic welfare is measured in terms of real consumption per capita. Even if autarky is better for the economy than free trade in the short run, there exists a situation where the economy is rendered better off under free trade in the long run because*

29) If we let $T \to \infty$, we need the condition $\rho > \gamma\varepsilon n$ so that the integral should not diverge.

of the dynamic gains from trade. In contrast, even if free trade is better for the economy than autarky in the short run, there exists a situation where the economy is rendered worse off under free trade in the long run.

Result 4. *Suppose that economic welfare is measured in terms of real consumption per capita. If the share of the expenditure for manufactured goods is more than* $1/2$*, then the follower country cannot catch up with the leader country. Nevertheless, there exists a situation where free trade is better for the follower country than autarky.*

Whether or not dynamic gains from trade exist depends on the two factors: the extent to which people discount future economic welfare and the time period for which people calculate economic welfare.

7 Conclusions

In this paper, we have investigated the relationship between trade and economic development using a two-country, non-scale growth model. In the model, we have assumed that two countries coexist with manufacturing and agricultural sectors and that both countries have identical economic structures except for the initial endowment of capital stock. Moreover, we have assumed that agricultural production is constant returns to scale while manufacturing production is increasing returns to scale. Depending on the share of the expenditure for manufactured goods, we obtain the following two situations: (1) if the share of the expenditure is less than or equal to 1/2, then one country diversifies (produces both goods) while the other country asymptotically specializes in agriculture completely and (2) if the expenditure share is more than 1/2, then one country completely specializes in manufacturing while the other country asymptotically specializes in agriculture completely.

In our model, regardless of the situation the economy belongs to (autarky or free trade) and regardless of the sector the economy specializes in (manufacturing or agriculture), the long-run growth rates of real income per capita and real consumption per capita in both countries are equalized.

Under autarky, the follower country can catch up with the leader country. Under free trade, on the other hand, whether or not the follower country can catch up with the leader country depends on two factors: the production patterns of the two countries and the measure of economic welfare used. We must note that even though the follower country cannot catch up with the leader country, it is possible that free trade is better for the follower country than autarky.

The results of our analysis are based on a specific model and specific assumptions. In particular, that the population growth rates in both countries are the same is quite a limiting assumption. If the population growth in the leader country is faster, then in the long run, the leader's per capita income relative to the follower's per capita income continues to increase, and hence, income disparity continues to expand.

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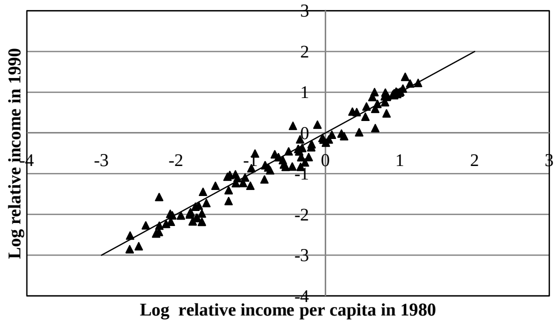

Figure 1: Income per capita in 1980 and 1990 relative to the world average. Source: EPWT 3.0

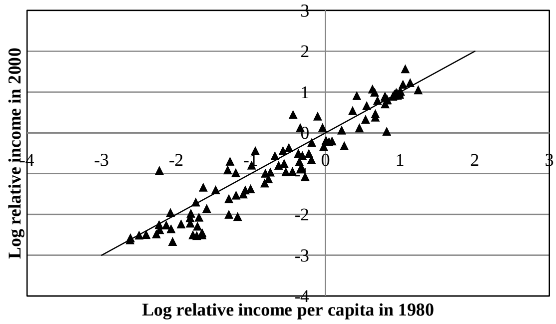

Figure 2: Income per capita in 1980 and 2000 relative to the world average. Source: EPWT 3.0

Table 1: Relationship between the rate of time preference and dynamic gains from trade ($\gamma = 0.4$, $T = 100$)

	Autarky→Free trade ($\rho = 0.02$)	Autarky→Free trade ($\rho = 0.01$)
Home	88.4345→88.1086	133.243→133.419
Foreign	85.5502→88.1086	129.362→133.419

Table 2: Relationship between the rate of time preference and dynamic gains from trade ($\gamma = 0.6$, $T = 200$)

	Autarky→Free trade ($\rho = 0.02$)	Autarky→Free trade ($\rho = 0.01$)
Home	175.954→183.8977	354.051→389.572
Foreign	168.64→170.8159	343.226→338.820

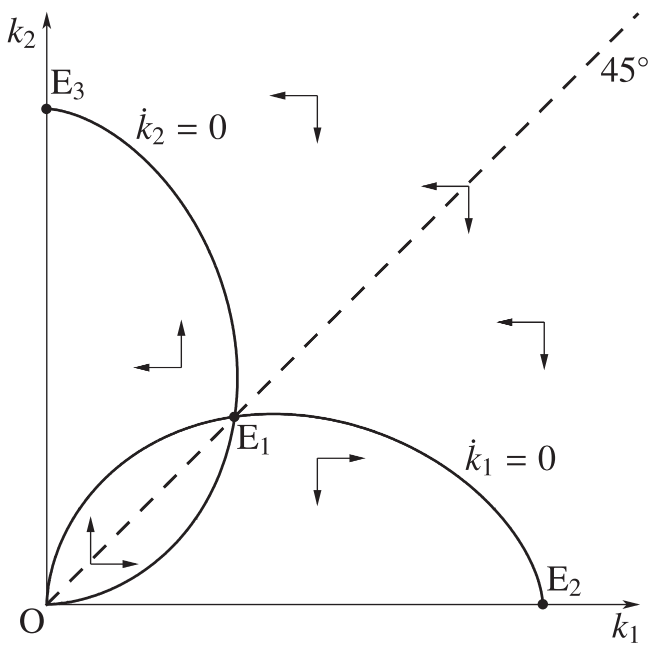

Figure 3: Phase diagram when the share of the expenditure for manufactured goods is less than or equal to 1/2

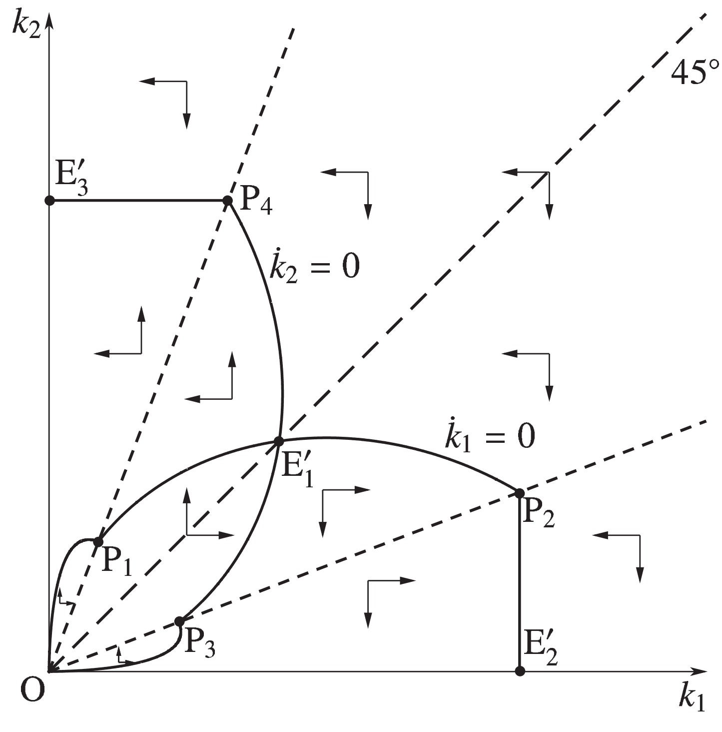

Figure 4: Phase diagram when the share of the expenditure for manufactured goods is more than 1/2

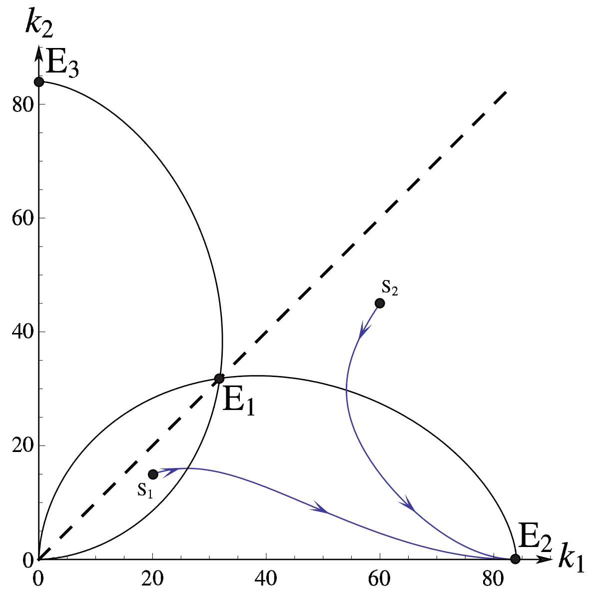

Figure 5: Convergence to an equilibrium from different initial points (α = 0.3, β = 0.2, γ = 0.4, n = 0.02)

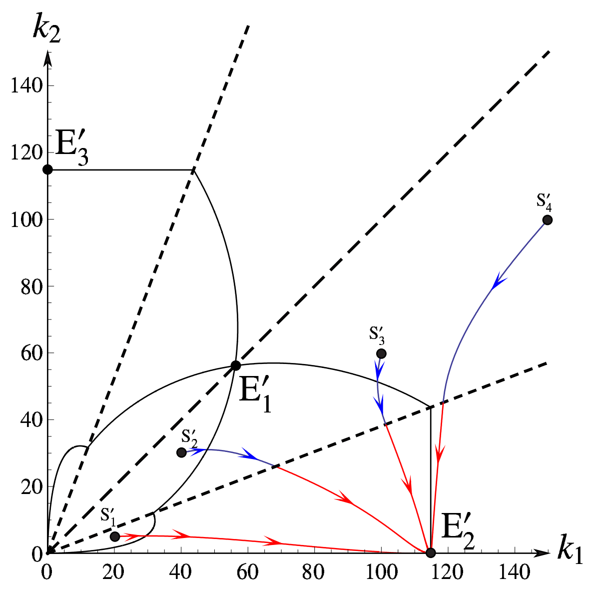

Figure 6: Convergence to an equilibrium from different initial points (α = 0.3, β = 0.2, γ = 0.6, n = 0.02)

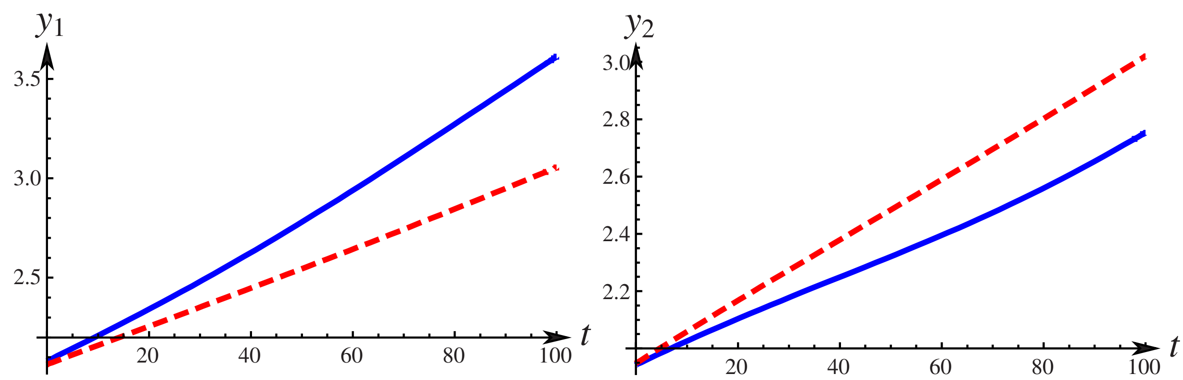

Figure 7: Comparison between per capita real income under free trade and that under autarky ($\gamma = 0.4$)

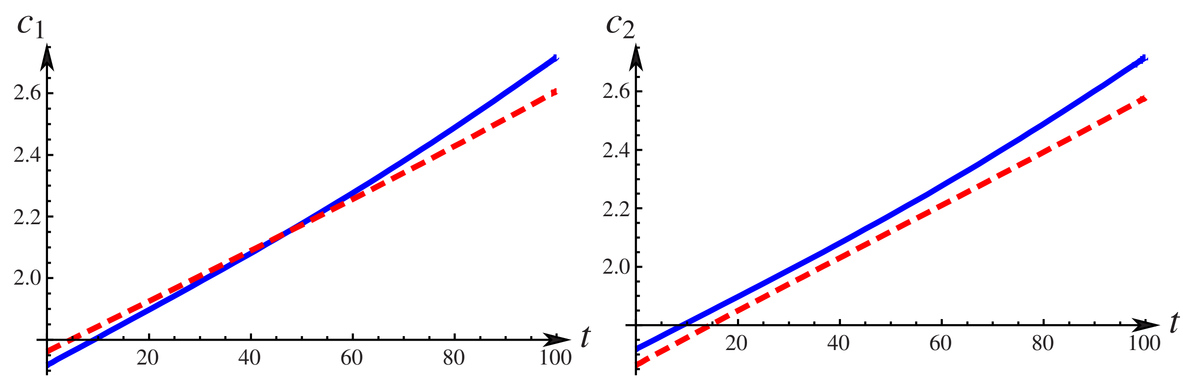

Figure 8: Comparison between per capita real consumption under free trade and autarky ($\gamma = 0.4$)

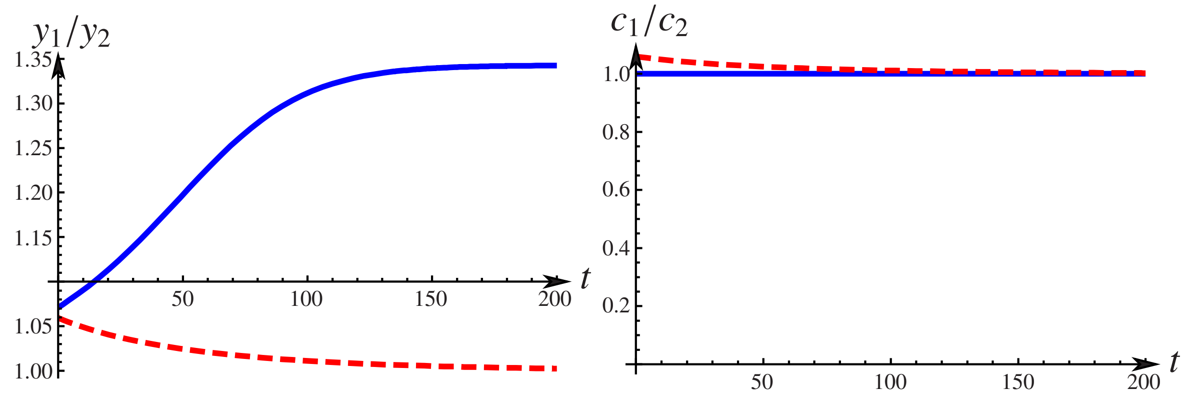

Figure 9: Relative income and relative consumption under free trade and autarky ($\gamma = 0.4$)

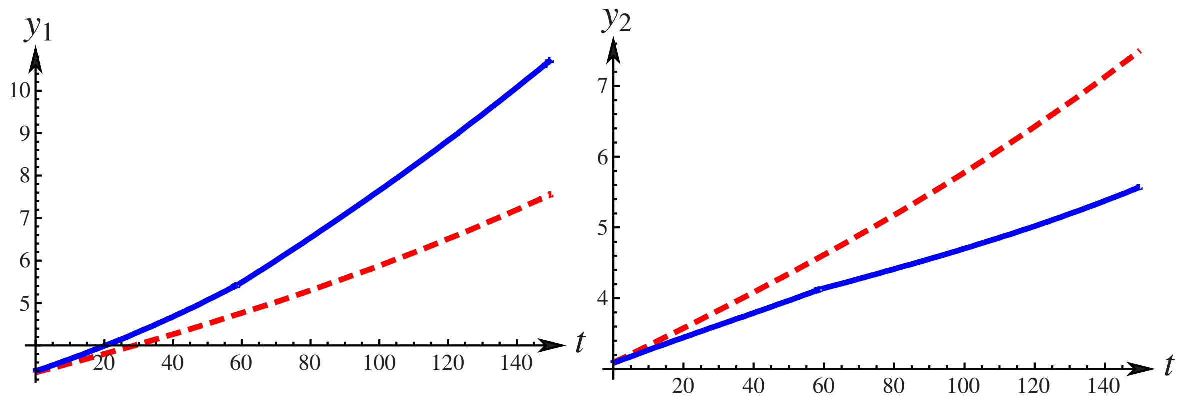

Figure 10: Comparison between per capita real income under free trade and autarky (γ = 0.6)

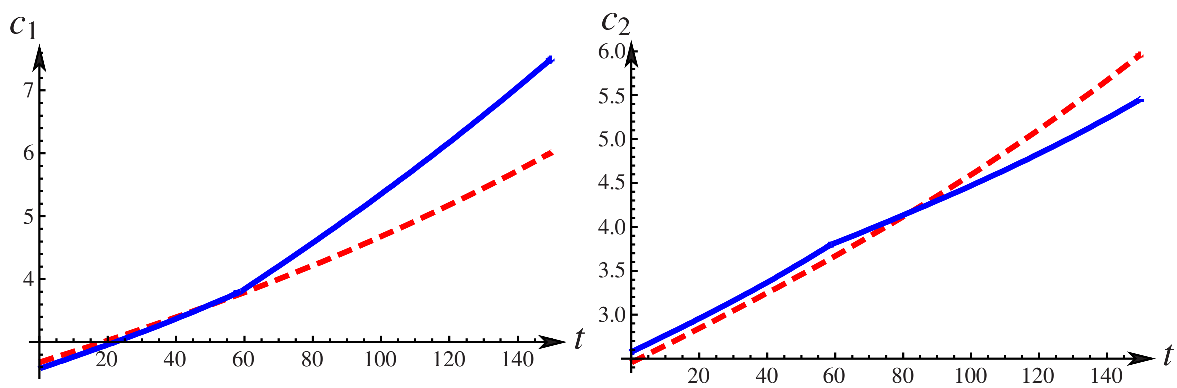

Figure 11: Comparison between per capita real consumption in free trade and autarky (γ = 0.6)

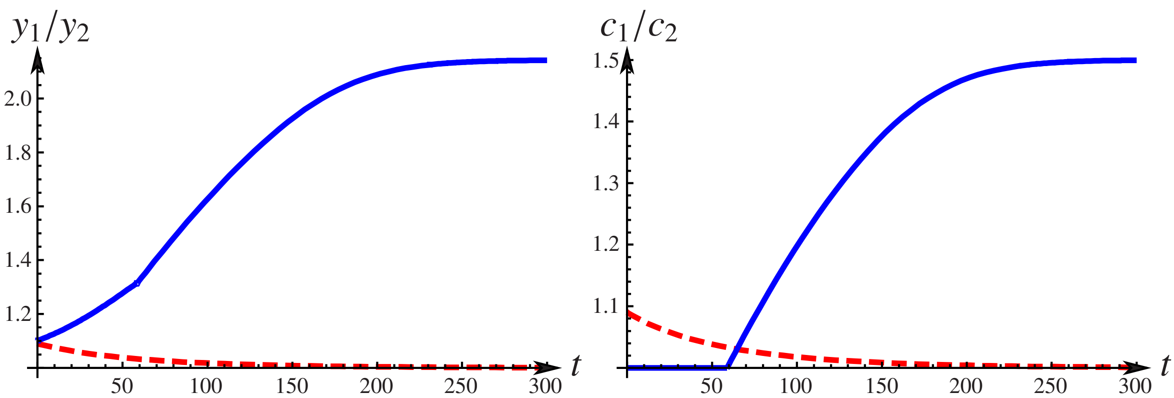

Figure 12: Relative income and relative consumption in free trade and autarky (γ = 0.6)

Appendix

A Equilibrium under autarky

Under autarky, both goods have to be produced. The market clearing conditions are as follows:

$$X_i^M = C_i^M + I_i, \tag{A-1}$$

$$X_i^A = C_i^A. \tag{A-2}$$

Note that we have $w_i = 1$ under autarky. From the market clearing condition for manufactured goods, we obtain p, which is used to derive each sector's employment share:

$$L_i^M / L_i = \gamma, \tag{A-3}$$

$$L_i^A / L_i = 1 - \gamma. \tag{A-4}$$

Therefore, under autarky, each sector's employment share is constant.

The profit rate is given by $r_i = \alpha K_i^{\alpha+\beta-1} (\gamma L_i)^{1-\alpha}$, and is constant along the BGP. From this, the BGP growth rates of K_i and p are given by $g_{K_i}^* = \phi n$ and $g_p^* = -\varepsilon n$, respectively. Hence, the dynamics of the scale-adjusted capital stock are given by

$$\dot{k}_i = \alpha \gamma^{1-\alpha} k_i^{\alpha+\beta} - \phi n_i k_i. \tag{A-5}$$

The steady state is a situation where $\dot{k}_i = 0$, from which we obtain

$$k_i^* = \left(\frac{\alpha \gamma^{1-\alpha}}{\phi n_i} \right)^{\frac{1}{1-\alpha-\beta}}. \tag{A-6}$$

The steady state is stable. The equilibrium value of k_i^* under autarky is equal to that under both countries' diversification. Moreover, the growth rates of real per capita income and real per capita consumption under autarky are equal to those under free trade.

B Comparative advantage

Under autarky, the relative price of manufactured goods is given by

$$p_i = \frac{(\gamma L)^\alpha}{(1-\alpha) K_i^{\alpha+\beta}}. \tag{B-7}$$

If $K_1 > K_2$, that is, $k_1 > k_2$, then we have $p_1 < p_2$. This means that if $K_1 > K_2$ ($k_1 > k_2$), then Home has a comparative advantage in manufacturing while Foreign has a comparative advantage in agriculture.

If (k_1, k_2) is located below the 45° line in the phase diagram at the time when both countries switch from autarky to free trade, Home has a comparative advantage in manufacturing while Foreign has a comparative advantage in agriculture, and vice versa. If (k_1, k_2) is located on the 45° line at the time when both countries switch from autarky to free trade, no trade occurs.

C Coordinates of the points in figures 3 and 4

The coordinates of the important points under figures 3 and 4 can be obtained analytically.

First, with regard to figure 3, we have

$$\mathrm{E}_1 : k_1 = \left(\frac{\alpha\gamma^{1-\alpha}}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \qquad k_2 = \left(\frac{\alpha\gamma^{1-\alpha}}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \tag{C-8}$$

$$\mathrm{E}_2 : k_1 = \left(\frac{2^{1-\alpha}\alpha\gamma^{1-\alpha}}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \qquad k_2 = 0, \tag{C-9}$$

$$\mathrm{E}_3 : k_1 = 0, \qquad k_2 = \left(\frac{2^{1-\alpha}\alpha\gamma^{1-\alpha}}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}. \tag{C-10}$$

Second, with regard to figure 4, we have

$$\mathrm{E}_1' : k_1 = \left(\frac{\alpha\gamma^{1-\alpha}}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \qquad k_2 = \left(\frac{\alpha\gamma^{1-\alpha}}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \tag{C-11}$$

$$\mathrm{E}_2' : k_1 = \left(\frac{\alpha}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \qquad k_2 = 0, \tag{C-12}$$

$$\mathrm{E}_3' : k_1 = 0, \qquad k_2 = \left(\frac{\alpha}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \tag{C-13}$$

$$\mathrm{P}_1 : k_1 = (2\gamma-1)^{\frac{\beta}{(\alpha+\beta)(1-\alpha-\beta)}}\left(\frac{\alpha}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \qquad k_2 = (2\gamma-1)^{\frac{1-\alpha}{1-\alpha-\beta}}\left(\frac{\alpha}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \tag{C-14}$$

$$\mathrm{P}_2 : k_1 = \left(\frac{\alpha}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \qquad k_2 = (2\gamma-1)^{\frac{\alpha}{\alpha+\beta}}\left(\frac{\alpha}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \tag{C-15}$$

$$\mathrm{P}_3 : k_1 = (2\gamma-1)^{\frac{1-\alpha}{1-\alpha-\beta}}\left(\frac{\alpha}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \qquad k_2 = (2\gamma-1)^{\frac{\beta}{(\alpha+\beta)(1-\alpha-\beta)}}\left(\frac{\alpha}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \tag{C-16}$$

$$P_2 : k_1 = (2\gamma - 1)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{\alpha}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \qquad k_2 = \left(\frac{\alpha}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}. \tag{C-17}$$

D Calculation of transitional dynamics in Case 4

In Case 4, we cannot analytically find the value of p that is determined by the market clearing conditions, and hence, the analysis in this case is not as easy as that in the other three cases. However, we can numerically analyze the transitional dynamics.

To begin with, we set arbitrary values of the initial capital stock $k_1(0)$ and $k_2(0)$. Substituting these values into the trade balance condition, we can numerically compute the initial terms of trade $\pi(0)$.

Next, differentiating both sides of the trade balance condition with respect to time, we find an equation that corresponds to $\dot{\pi}$:

$$\dot{\pi} = -\frac{(1-\alpha)(1-\gamma)k_1^{\alpha+\beta-1}\pi\dot{k}_1 + (1-\alpha)^{\frac{1}{\alpha}}\frac{\alpha+\beta}{\alpha}k_2^{\frac{\beta}{\alpha}}\pi^{\frac{1}{\alpha}}\dot{k}_2}{\frac{(1-\alpha)^{\frac{1}{\alpha}}}{\alpha}k_2^{\frac{\alpha+\beta}{\alpha}}\pi^{\frac{1-\alpha}{\alpha}} + (1-\alpha)(1-\gamma)k_1^{\alpha+\beta}}. \tag{D-18}$$

Substituting the equations of $\dot{k}_1$ and $\dot{k}_2$ into the right-hand side of the above equation, we can express $\dot{\pi}$ as a function of k_1, k_2, and π, which is a differential equation for π.

From the above procedure, we can obtain a system of the three differential equations $\dot{k}_1$, $\dot{k}_2$, and $\dot{\pi}$. Because we already know the initial values $k_1(0)$, $k_2(0)$, and $\pi(0)$, we can obtain the time paths of $k_1(t)$, $k_2(t)$, and $\pi(t)$.

E Output and relative prices in Cases 1–4

[Case 1]

$$X_1^M = \frac{(2\gamma)^{1-\alpha}k_1^{\alpha+\beta}}{\left[1 + (k_2/k_1)^{\frac{\alpha+\beta}{\alpha}}\right]^{1-\alpha}}L^{\phi}, \qquad X_1^A = \left[1 - \frac{2\gamma}{1 + (k_2/k_1)^{\frac{\alpha+\beta}{\alpha}}}\right]L, \tag{E-19}$$

$$X_2^M = \frac{(2\gamma)^{1-\alpha}k_2^{\alpha+\beta}}{\left[1 + (k_1/k_2)^{\frac{\alpha+\beta}{\alpha}}\right]^{1-\alpha}}L^{\phi}, \qquad X_2^A = \left[1 - \frac{2\gamma}{1 + (k_1/k_2)^{\frac{\alpha+\beta}{\alpha}}}\right]L, \tag{E-20}$$

$$p = \frac{(2\gamma)^{\alpha}}{(1-\alpha)\left(k_1^{\frac{\alpha+\beta}{\alpha}} + k_2^{\frac{\alpha+\beta}{\alpha}}\right)^{\alpha}}L^{-\varepsilon}. \tag{E-21}$$

[Case 2]

$$X_1^M = k_1^{\alpha+\beta} L^{\phi}, \qquad X_1^A = 0, \tag{E-22}$$

$$X_2^M = 0, \qquad X_2^A = L, \tag{E-23}$$

$$p = \frac{\gamma}{(1-\alpha)(1-\gamma)k_1^{\alpha+\beta}} L^{-\varepsilon}. \tag{E-24}$$

[Case 3]

$$X_1^M = (2\gamma)^{1-\alpha} k_1^{\alpha+\beta} L^{\phi}, \qquad X_1^A = (1-2\gamma)L, \tag{E-25}$$

$$X_2^M = 0, \qquad X_2^A = L, \tag{E-26}$$

$$p = \frac{(2\gamma)^{\alpha}}{(1-\alpha)k_1^{\alpha+\beta}} L^{-\varepsilon}. \tag{E-27}$$

[Case 4]

$$X_1^M = k_1^{\alpha+\beta} L^{\phi}, \qquad X_1^A = 0, \tag{E-28}$$

$$X_2^M = (1-\alpha)^{\frac{1-\alpha}{\alpha}} \pi^{\frac{1-\alpha}{\alpha}} k_2^{\frac{\alpha+\beta}{\alpha}} L^{\phi}, \qquad X_2^A = [1-(1-\alpha)^{\frac{1}{\alpha}} \pi^{\frac{1}{\alpha}} k_2^{\frac{\alpha+\beta}{\alpha}}]L, \tag{E-29}$$

$$p = \pi L^{-\varepsilon}. \tag{E-30}$$