Kyoto University,
Graduate School of Economics
Research Project Center Discussion Paper Series

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Discussion Paper No. E-10-015

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March 2011

Export of Deindustrialization and Anti-Balassa-Samuelson Effect: The Consequences of Productivity Growth Differential

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Abstract

This paper focuses on productivity growth differentials between manufacturing and services, deindustrialization, and changes in the real exchange rate. Using a Ricardian trade model with a continuum of goods that introduces nontraded services, the paper investigates these interrelationships. The main results are as follows: (i) if deindustrialization proceeds in both home and foreign countries, then the ratio of home manufacturing employment share to foreign manufacturing employment share and the real exchange rate move in the same direction; (ii) even if the productivity growth differential in the home country is greater than that in the foreign county, the extent of deindustrialization in the home country is not necessarily larger than that in the foreing country. On the contrary, it is possible that the foreign deindustrialization exceeds the home deindustrialization; and (iii) even if the productivity growth differential in the home country is greater than that in the foreign county, the real exchange rate of the home country can depreciate contrary to the expectaion of the Balassa-Samuelson effect.

Keywords: Productivity growth differentials; Deindustrialization; Real exchange rate; Shift in specialization patterns

JEL Classification: F10; F31; O14

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1 Introduction

The purpose of this paper is to theoretically investigate the relationship among productivity growth differentials between manufacturing and services, deindustrialization, and changes in the real exchange rate.[1]

Here, deindustrialization is defined as a decline in the share of employment in industry or manufacturing. Since the employment share of agriculture in developed countries is very small, deindustrialization suggests a rise in the share of employment in services. As many data and empirical studies show, the share of service employment consistently rises while that of industrial employment declines in all developed countries without exception. Deindustrialization in this sense is first analyzed theoretically by Baumol (1967). He proposes a two-sector model that consists of manufacturing and services, and shows that deindustrialization is caused by interaction between productivity growth differentials (between manufacturing and services) and inelastic demand. On the basis of Baumol (1967), many theoretical models have been produced.[2]

However, those are models of closed economy and there are few models that consider deindustrialization in open economy. A noteworthy exception is Spilimbergo (1998). He presents a model that extends the standard Ricardian trade model with a continuum of goods (Dornbusch, Fischer, and Samuelson, 1977; DFS hereafter) to introduce nontraded goods, that is, services. He shows that compared with autarky, free trade spurs deindustrialization and decreases the rate of economic growth. Spilimbergo's model is based on two key assumptions, productivity growth differential[3] and inelastic demand.[4]

In this paper, using Spilimbergo's (1998) model, we attempt to a comparison between two trading countries rather than a comparison between autarky and free trade as in Spilimbergo (1998). In particular, we investigate how a change in a parameter of one country affects the deindustrialization of the other country, which is not investigated in Spilimbergo (1998). As will be shown later, it is possible that even if the productivity growth differential in the home country is greater than that in the foreign country, the extent of the deindustri-

[1] Obstfeld and Rogoff (1996, ch. 4) also consider these three concepts simultaneously.

[2] For the endogenization of productivity growth, see Pugno (2006) and De Vincenti (2007). Fixler and Siegel (1999), Oulton (2001), and Sasaki (2007) discuss the importance of services as intermediate inputs. Bonatti and Felice (2008) considers capital accumulation in both manufacturing and services.

[3] For empirical evidence as to the existence of productivity growth differentials, see Rowthorn and Wells (1987), Rowthorn and Ramaswamy (1997), Kendrick (1985), Bernard and Jones (1996), and Fixler and Siegel (1999).

[4] According to Fuchs (1968), the income elasticity of demand for services is 1.12 in the US 1929–1965. Summers (1985) reports that the income elasticity of demand for services is 0.977 and the price elasticity is very low in a cross-sectional analysis for 34 countries. Following the method of Summers (1985), Falvey and Gemmell (1996) update the data and show that the income elasticity of services is about unity and the price elasticity is 0.32.

alization in the home country is less than that of the foreign country owing to the effect of a shift in trade specialization.

Moreover, using our model, we also analyze the real exchange rate.[5] In the field of international economics, the Balassa-Samuelson (BS) effect is well known as a fundamental factor in determining the movement of the real exchange rate (Balassa, 1964 and Samuelson, 1964). According to the BS effect, a country will experience an appreciation in the real exchange rate if its productivity growth differential between the traded and nontraded goods sectors is greater than the productivity growth differential of its trading partner. However, according to our analysis, it is possible that even if the productivity growth differential in the home country is greater than that in the foreign country, the real exchange rate depreciates, and not appreciates, owing to the effect of a shift in trade specialization.

The remainder of the paper is organized as follows. Section 2 presents our model and then explains the mechanism of deindustrialization. The equilibrium of the model is summarized by two variables: the factorial terms of trade and the index of borderline good. Accordingly, Section 3 conducts a comparative statics analysis with regard to these variables. Section 4 derives the equilibrium real exchange rate and investigates the relationship between deindustrialization and changes in the real exchange rate. Section 5 investigates the effect of productivity growth differential on deindustrialization and the real exchange rate. Section 6 concludes the paper.

2 The Model

Our model is based mainly on the work of Spilimbergo (1998), which is modified for our purpose in some respects.

2.1 Production side

Each sector uses only labor as input and operates with fixed coefficient technology.

$$S = a_s L_s, \quad m(z) = a_m(z) L_m(z), \quad z \in [0, 1], \tag{1}$$

[5] Recent studies on changes in the real exchange rate derive the equilibrium real exchange rate by using an intra-industry trade theory due to product differentiation. Moreover, these studies emphasize other factors besides the traditional BS effect in determining changes in the real exchange rate. See, for instance, Unayama (2003) and Bergin, Glick, and Taylor (2006). The present paper, in contrast, derives the equilibrium real exchange rate by using the traditional inter-industry trade theory. It is true that intra-industry trade arising from product differentiation captures reality to some extent, but inter-industry trade arising from differences in productivity still has significance even among developed countries. For instance, Golub (1994), Golub and Hsieh (2000), and Kasuya and Okada (2007) empirically show the validity of the traditional Ricardian trade theory.

where S and L_s denote the output and the employment of services, respectively, and $m(z)$ and $L_m(z)$ the output and the employment of each manufacturing sector, respectively. a_s and $a_m(z)$ represent the levels of labor productivity of services and the manufacturing sector, respectively. Each manufacturing sector is indexed by a real number z on the closed interval $[0, 1]$.

Given the perfect mobility of domestic labor across sectors, wages are equalized within the country. Competition equalizes the price of each good with its unit labor cost.

$$p_s = \frac{w}{a_s}, \quad p_m(z) = \frac{w}{a_m(z)}, \tag{2}$$

where p_s denotes the price of services, $p_m(z)$ the price of each manufacturing good, and w the money wage in terms of the domestic currency.

2.2 Demand side

Suppose that the social welfare in a country is given by the following CES utility function.

$$U = \left[\alpha M^{\frac{\sigma-1}{\sigma}} + (1-\alpha)S^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 0,\ \sigma \neq 1,\ 0 < \alpha < 1, \tag{3}$$

where M denotes the aggregate consumption of manufacturing goods, S the consumption of services, and σ the elasticity of substitution between M and S. α is the utility weight of the manufacturing consumption. Suppose that M takes the following Cobb-Douglas form.

$$M = \exp\left[\int_0^1 \log m(z)\, dz\right]. \tag{4}$$

The budget constraint is given by $p_m M + p_s S = Y$. Assuming full employment, we can write nominal national income Y as $Y = w(L_m + L_s) = w\bar{L}$, where $L_m \equiv \int_0^1 L_m(z)\, dz$.

The price index for M is given by

$$p_m = \exp\left[\int_0^1 \log p_m(z)\, dz\right], \tag{5}$$

which is the minimum expenditure for buying one unit of M. Moreover, the price index corresponding to the utility function (3) is given by

$$P = \left[\alpha^{\sigma} p_m^{1-\sigma} + (1-\alpha)^{\sigma} p_s^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{6}$$

Maximizing (3) subject to the budget constraint, we obtain the following demand functions.

$$M = \alpha^{\sigma}\left(\frac{p_m}{P}\right)^{-\sigma}\left(\frac{Y}{P}\right), \quad S = (1-\alpha)^{\sigma}\left(\frac{p_s}{P}\right)^{-\sigma}\left(\frac{Y}{P}\right), \tag{7}$$

where Y/P is the real national income. Since the preference is homothetic, the income elasticity of demand is unity. The elasticity of substitution σ also represents the price elasticity of demand.

2.3 Employment share

The manufacturing employment L_m is given by subtracting L_s from $\bar{L}$. To begin with, let us turn to L_s. From the market clearing condition for service, we obtain

$$a_s L_s = (1-\alpha)^{\sigma}\left(\frac{p_s}{P}\right)^{-\sigma}\left(\frac{Y}{P}\right). \tag{8}$$

Substituting $Y = w\bar{L}$ into equation (8) and rearranging it, we have the following employment share of services.

$$\frac{L_s}{\bar{L}} = \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right)^{\sigma}(p_m/p_s)^{1-\sigma}}. \tag{9}$$

Using equation (9), we obtain the employment share of manufacturing.

$$\frac{L_m}{\bar{L}} = \frac{\left(\frac{\alpha}{1-\alpha}\right)^{\sigma}(p_m/p_s)^{1-\sigma}}{1 + \left(\frac{\alpha}{1-\alpha}\right)^{\sigma}(p_m/p_s)^{1-\sigma}}. \tag{10}$$

Each employment share depends on the relative price p_m/p_s. If the elasticity of substitution is smaller than unity, then the manufacturing employment share is increasing in p_m/p_s. Therefore, when the relative price falls, the manufacturing employment share also falls, which is nothing less than deindustrialization. The elasticity of substitution here, as has been stated above, corresponds to the price elasticity of demand. The elasticity of substitution below unity means inelastic demand.

2.4 Free trade

Let us introduce a foreign country to consider trade between the two countries. Suppose that the two countries differ in their labor endowments and production functions in each sector. Suppose also that the manufacturing goods are tradable while services are nontradable, and that there exist no international transportation costs and no trade barriers. We assume that

that the social utility function of the foreign country is given by

$$U^* = \left[\alpha^*(M^*)^{\frac{\sigma-1}{\sigma}} + (1-\alpha^*)(S^*)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \quad 0 < \alpha^* < 1. \tag{11}$$

The parameter α^* in the foreign country is different from that in the home country, but σ is common to the two countries. In what follows, foreign variables are marked with an asterisk "∗".

To consider international trade based on the principle of comparative advantage, we define the relative labor productivity as

$$A(z) \equiv \frac{a_m^*(z)}{a_m(z)}, \quad A'(\cdot) > 0. \tag{12}$$

The sign of the derivative implies that the manufacturing goods are ordered in accordance with diminishing home country comparative advantage.

The home country has a cost advantage over the foreign country in producing z if the following condition holds:

$$\frac{w}{a_m(z)} < \frac{ew^*}{a_m^*(z)}, \tag{13}$$

where w^* is the foreign money wages expressed in terms of its own currency and e is the nominal exchange rate in terms of home currency.

Now let us define $\bar{z}$ such that

$$\frac{ew^*}{w} = A(\bar{z}). \tag{14}$$

Given the relative wage ew^*/w, the home country has a cost advantage in the range $z \in [0, \bar{z})$, while the foreign country has a cost advantage in the range $z \in (\bar{z}, 1]$. Thus, the home (foreign) country exports (imports) the manufacturing goods $z \in [0, \bar{z})$ and imports (exports) the goods $z \in (\bar{z}, 1]$.

We will derive a condition for balanced trade. The home country spends a part of its national income Y, $E_m \equiv p_m M$, on the manufacturing goods, and the fraction $1 - \bar{z}$ of E_m is spent on the imported goods. In a similar way, the foreign country spends a part of its national income Y^*, $E_m^* \equiv p_m^* M^*$, on the manufacturing goods, and the fraction $\bar{z}$ of E_m^* is spent on the imported goods. From these, the value of imports of the home country is $(1 - \bar{z})E_m$ in terms of the home currency while that of the foreign country is $\bar{z}E_m^*$ in terms of the foreign currency. Thus, the balanced trade condition is given by

$$(1-\bar{z})E_m = e\bar{z}E_m^*. \tag{15}$$

Note that $\bar{z}E_m^*$ is multiplied by e.

Let us return to E_m and E_m^*. There is just one factor of production and there are no profits in the model, so that the expenditure spent on the manufacturing goods has to be equal to the value of total production in the manufacturing, which is consequently equal to the income earned by the employees in the manufacturing sector. Thus, we have

$$E_m = w\int_0^{\bar{z}} L_m(z)\,dz, \quad E_m^* = w^*\int_{\bar{z}}^1 L_m^*(z)\,dz. \tag{16}$$

We define $L_m = \int_0^{\bar{z}} L_m(z)\,dz$ and $L_m^* = \int_{\bar{z}}^1 L_m^*(z)\,dz$. Note that $L_m = \int_0^1 L_m(z)\,dz$ under autarky. Substituting equation (16) into equation (15), we can rewrite the balanced trade condition as

$$\omega = \frac{L_m}{L_m^*}\cdot\frac{1-\bar{z}}{\bar{z}}, \quad \text{where } \omega \equiv \frac{ew^*}{w}. \tag{17}$$

The relative wage ω also means the inverse of the factorial terms of trade. What is important in equation (16) is L_m/L_m^*. In the usual DFS model L_m/L_m^* is replaced by $\bar{L}/\bar{L}^*$, which is constant and exogenously given. In our model, on the other hand, there appears the manufacturing employment ratio between the two countries, which is a function of p_m/p_s, not constant (see equation (10)).

The prices of the manufacturing goods after trade in terms of the home currency are given by

$$p_m(z) = \begin{cases} \dfrac{w}{a_m(z)} & z \in [0,\bar{z}], \\ \dfrac{ew^*}{a_m^*(z)} & z \in [\bar{z},1]. \end{cases} \tag{18}$$

Using these prices, we obtain the price index for the manufacturing goods in each country:

$$p_m = \exp\left[\int_0^{\bar{z}} \log\frac{w}{a_m(z)}\,dz + \int_{\bar{z}}^1 \log\frac{ew^*}{a_m^*(z)}\,dz\right], \tag{19}$$

$$p_m^* = \exp\left[\int_0^{\bar{z}} \log\frac{w}{ea_m(z)}\,dz + \int_{\bar{z}}^1 \log\frac{w^*}{a_m^*(z)}\,dz\right]. \tag{20}$$

Note that $p_m = ep_m^*$ because the law of one price holds for the traded goods and both countries are identical in the index for the aggregate consumption of manufacturing goods.

Therefore, purchasing power parity (PPP hereafter) for traded goods holds in our model.

The relative prices between manufacturing and services are given by

$$\frac{p_m}{p_s} = a_s \omega^{1-\bar{z}} \exp\left[-\int_0^{\bar{z}} \log a_m(z)\, dz - \int_{\bar{z}}^1 \log a_m^*(z)\, dz\right], \tag{21}$$

$$\frac{p_m^*}{p_s^*} = a_s^* \omega^{-\bar{z}} \exp\left[-\int_0^{\bar{z}} \log a_m(z)\, dz - \int_{\bar{z}}^1 \log a_m^*(z)\, dz\right]. \tag{22}$$

For analytical tractability, we specify the production functions as follows:

$$a_m(z) = \exp(T_m + 1 - z), \quad a_s = \exp(T_s), \tag{23}$$

$$a_m^*(z) = \exp(T_m^* + z), \quad a_s^* = \exp(T_s^*), \tag{24}$$

where T_m (T_m^*) and T_s (T_s^*) capture the overall level of technology in the manufacturing and services, respectively, which are given exogenously.[6] Under these specifications, the relative labor productivity in manufacturing is rewritten as follows:

$$A(z) = \exp[T_m^* - T_m + 2z - 1], \tag{25}$$

which implies that $\partial A(\cdot)/\partial z = 2\exp[T_m^* - T_m + 2z - 1] > 0$

2.5 Equilibrium of the model

The equilibrium values of $\bar{z}$ and ω are given by the intersection of the curve given by equation (14) and the curve given by equation (17), which are restated as follows:

$$\text{AA} : \omega = A(\bar{z}), \tag{26}$$

$$\text{BB} : \omega = \frac{L_m(\omega, \bar{z})}{L_m^*(\omega, \bar{z})} \frac{1-\bar{z}}{\bar{z}}. \tag{27}$$

The AA curve determines $\bar{z}$ with ω given, while the BB curve determines ω with $\bar{z}$ given. Since L_m and L_m^* depend on the relative prices, which in turn depend on ω and $\bar{z}$, we can write L_m and L_m^* as $L_m(\omega, \bar{z})$ and $L_m^*(\omega, \bar{z})$, respectively. The AA curve is upward-sloping in $(\bar{z}, \omega)$ space because $A'(\cdot) > 0$. The shape of the BB curve is not evident at this stage because $L_m(\cdot)/L_m^*(\cdot)$ is not constant. However, if the BB curve were upward-sloping, the equilibrium could be unstable, so that comparative static analysis below might be meaningless. Following Spilimbergo (1998), we assume that the BB curve is downward-sloping in

[6] Such functional forms follow Obstfeld and Rogoff (1996, ch. 4).

our analysis below.[7] Figure 1 shows the AA and BB curves, the intersection of which gives the equilibrium values of $\bar{z}$ and ω.

[Figure 1 to be inserted here]

3 Comparative statics analysis

This section conducts a comparative statics analysis with regard to ω, $\bar{z}$, and the commodity terms of trade.

We define the commodity terms of trade as follows:

$$p_T \equiv \frac{p_{m,\mathrm{EX}}}{e p^*_{m,\mathrm{EX}}}, \tag{28}$$

where $p_{m,\mathrm{EX}}$ denotes the price of home export goods and $p^*_{m,\mathrm{EX}}$ the price of foreign export goods. These prices are respectively specified as follows:

$$p_{m,\mathrm{EX}} = \exp\left[\frac{1}{\bar{z}}\int_0^{\bar{z}} \log p_m(z)\,dz\right], \quad p^*_{m,\mathrm{EX}} = \exp\left[\frac{1}{1-\bar{z}}\int_{\bar{z}}^1 \log p^*_m(z)\,dz\right]. \tag{29}$$

Substituting (29) into (28), we obtain

$$p_T = \frac{1}{\omega}\exp\left[T^*_m - T_m + \bar{z} - \tfrac{1}{2}\right]. \tag{30}$$

Since we have $\omega = A(\bar{z})$ at the equilibrium, we can rewrite equation (30) as

$$p_T = \exp\left[-\bar{z} + \tfrac{1}{2}\right]. \tag{31}$$

Note that $\bar{z}$ in equation (31) is the equilibrium value of $\bar{z}$. From (31), we can see that a rise in $\bar{z}$, that is, the expansion of the range of manufacturing goods produced at home leads to a deterioration in the home commodity terms of trade.

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[7]Under our specifications, the relative prices of the domestic and foreign countries are given by

$$\frac{p_m}{p_s} = \omega^{1-\bar{z}}\exp\left[T_s - \bar{z}T_m - (1-\bar{z})T^*_m + \bar{z}^2 - \bar{z} - \tfrac{1}{2}\right],$$
$$\frac{p^*_m}{p^*_s} = \omega^{-\bar{z}}\exp\left[T^*_s - \bar{z}T_m - (1-\bar{z})T^*_m + \bar{z}^2 - \bar{z} - \tfrac{1}{2}\right].$$

Substituting these prices in equation (27) yields an implicit function for the BB curve, in which $\bar{z}$ and ω are the endogenous variables while $\bar{L}$, $\bar{L}^*$, T_m, T_s, T^*_m, T^*_s, α, α^*, and σ are the parameters. Appendix presents conditions for the downward-sloping BB curve.

In what follows, we consider cases where σ is smaller than unity. When the elasticity of substitution is not unity, L_m/L_m^* in the BB curve depends on the relative prices between manufacturing and services, so that the BB curve depends on the labor productivity both in manufacturing and in services.[8]

■ **Changes in relative labor endowments**

Suppose that $\bar{L}/\bar{L}^*$ increases: the labor endowment in the home country increases relative to that in the foreign country. While the AA curve does not shift, the BB curve rotates clockwise around $(1, 0)$. As a result, both ω and $\bar{z}$ rise. That is, the equilibrium factorial terms of trade deteriorates and the range of manufacturing goods produced at home expands. Moreover, the home commodity terms of trade deteriorate.

[Figure 2 to be inserted here]

■ **Changes in the home manufacturing productivity**

Suppose that the manufacturing labor productivity in the home country increases. To begin with, the AA curve shifts downward. In addition, the BB curve rotates on a certain point as shown in Figure 3. Figure 3 describes a case where both countries are symmetric concerning all the parameters and labor endowments before T_m rises. In this case, the BB curve rotates around the initial equilibrium point as in Figure 3. Note that the BB curve does not generally rotate around the equilibrium point.[9] In any case, however, the AA curve shifts more largely than the BB curve, so that ω declines and $\bar{z}$ increases, which leads to a deterioration in the home commodity terms of trade.

[Figure 3 to be inserted here]

■ **Changes in the home services productivity**

Suppose that the productivity of services sector in the home country T_s increases. While the AA curve does not shift, the BB curve rotates clockwise around $(1, 0)$. This results in the rise of ω and the expansion of the range of manufacturing goods produced at home, which deteriorates the home commodity terms of trade.

[Figure 4 to be inserted here]

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[8]When $\sigma = 1$, L_m/L_m^* remains constant. Thus, the AA and BB curves shift separately, not simultaneously. Moreover, labor productivity in services does not appear in the BB curve.

[9]For this, see Appendix.

■ **Changes in the utility weight of manufacturing consumption**

A rise in α means a demand shift toward manufacturing goods in the home country.[10] The parameter α appears in the BB curve. Then the rise of α rotates the BB curve clockwise around $(1, 0)$, which is similar to the case of a rise in $\bar{L}/\bar{L}^*$. A fall in α, in contrast, means a demand shift toward services in the home country. In this case, both ω and $\bar{z}$ fall, thereby improving the home commodity terms of trade.

Table 1 summarizes the results above. From the table we see that a rise in productivity both in manufacturing and in services deteriorates the home commodity terms of trade.

[Table 1 to be inserted here]

4 Real exchange rate

The real exchange rate, ε, can be defined as $\varepsilon \equiv eP^*/P$. A fall in ε means that the home country experiences appreciation in the real exchange rate relative to the foreign country. Substituting P and P^* into the above definition yields

$$\varepsilon = \left[\left(\frac{\alpha^*}{\alpha} \right)^{\sigma} \cdot \frac{1 + \left(\frac{1-\alpha^*}{\alpha^*}\right)^{\sigma} (p_m^*/p_s^*)^{\sigma-1}}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{\sigma} (p_m/p_s)^{\sigma-1}} \right]^{\frac{1}{1-\sigma}}, \tag{32}$$

where the relative prices are given by

$$\frac{p_m}{p_s} = \exp\left[-(T_m - T_s) - (\bar{z} - 1)^2 - \tfrac{1}{2} \right], \tag{33}$$

$$\frac{p_m^*}{p_s^*} = \exp\left[-(T_m^* - T_s^*) - \bar{z}^2 - \tfrac{1}{2} \right]. \tag{34}$$

Note that $\bar{z}$ is evaluated at the equilibrium.

As has been shown above, the equilibrium value of $\bar{z}$ depends on each parameter, so that we can express it as $\bar{z} = \bar{z}(T_m, T_m^*, T_s, T_s^*, \alpha, \alpha^*, \bar{L}, \bar{L}^*)$. From equations (33) and (34) together with (32), we can state that changes in the real exchange rate depend both on the direct effect of productivity growth and on the indirect effect of a shift in specialization patterns via the productivity growth. Besides the usual BS effect, the effect of a shift in specialization patterns affects the real exchange rate.

In our model, both deindustrialization and changes in the real exchange rate are consequences, so that there is no causal relationship between them. However, we can interrelate

[10] With p_m/p_s being constant, a rise in α increases M/S irrespective of the size of σ.

them because both the manufacturing employment share and the real exchange rate depend on the relative price. This is what we are turning to in what follows.

Let us consider the ratio of manufacturing employment share in the home and foreign countries. Which country experiences more rapid deindustrialization depends on the decreasing rates of the relative prices p_m/p_s and p_m^*/p_s^*. In a similar way, whether the real exchange rate appreciates or depreciates depends on the rates of changes in the relative prices. Accordingly, we can investigate the relationship between the real exchange and the ratio of manufacturing employment share.

Let $l_m \equiv L_m/\bar{L}$ and $l_m^* \equiv L_m^*/\bar{L}^*$ be the manufacturing employment share in the home and foreign countries, respectively. Then, the ratio of the manufacturing employment share, $\phi \equiv l_m/l_m^*$, is given by

$$\phi = \frac{1 + \left(\frac{1-\alpha^*}{\alpha^*}\right)^{\sigma} (p_m^*/p_s^*)^{\sigma-1}}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{\sigma} (p_m/p_s)^{\sigma-1}}. \tag{35}$$

Suppose that both l_m and l_m^* decline for any reason. If α and α^* are constant, then a fall in ϕ means that home deindustrialization is more rapid than foreign deindustrialization. From equations (32) and (35), we obtain the following relationship between ϕ and ε:

$$\phi = \left(\frac{\alpha}{\alpha^*}\right)^{\sigma} \varepsilon^{1-\sigma}. \tag{36}$$

When $\sigma < 1$, both ϕ and ε move in the same direction given that α and α^* are constant.

Result 1. *Suppose that the elasticity of substitution between the aggregate consumption of manufacturing goods and the consumption of services is smaller unity and that the utility weight of manufacturing consumption in both countries is constant. Then, an economy in which deindustrialization is more rapid than its trading partner experiences an appreciation of the real exchange rate. Moreover, changes in ϕ is smaller than changes in ε, that is, the ratio of manufacturing share in the home and foreign countries changes less than the real exchange rate.*

5 Numerical analysis

Section 3 reveals how equilibrium values of ω and $\bar{z}$ change with a change in each parameter. This subsection analyzes changes in variables such as ε and ϕ by using numerical simulations.[11]

Let us derive indirect utility functions because we are interested in changes in welfare.

$$V = a_s \left[\alpha^\sigma \left(\frac{p_m}{p_s} \right)^{1-\sigma} + (1-\alpha)^\sigma \right]^{\frac{1}{\sigma-1}}, \quad V^* = a_s^* \left[(\alpha^*)^\sigma \left(\frac{p_m^*}{p_s^*} \right)^{1-\sigma} + (1-\alpha^*)^\sigma \right]^{\frac{1}{\sigma-1}}, \tag{37}$$

where V and V^* are per capita utility, which are not directly dependent on population.

To start with, we set the initial parameters so that both countries will be symmetric.

$$T_m = 0,\ T_m^* = 0,\ T_s = 0,\ T_s^* = 0,\ \alpha = 1/2,\ \alpha^* = 1/2,\ \bar{L} = 1,\ \bar{L}^* = 1,\ \sigma = 2/3. \tag{38}$$

In this benchmark case, each endogenous variable is determined as listed in Table 2 (see the column "Benchmark").

[Table 2 to be inserted here]

Next, we change each parameter to analyze how the endogenous variables change. Since the home and foreign countries are symmetric in their structures, we focus on the parameters of the home country only. Compared with the benchmark case, each parameter is set as follows:

$$\bar{L} = 2,\ T_m = 1,\ T_s = 1,\ \alpha = 1/3. \tag{39}$$

Table 2 summarizes the results of these parameters.

Some results are worth commenting. When T_m rises, the real exchange rate appreciates and the ratio of manufacturing employment share declines. In addition, both l_m and l_m^* decline. This result suggests that a rise in the home manufacturing productivity can cause deindustrialization in the foreign country. We can consider this case to be a case where the productivity growth differential between manufacturing and services in the home country is greater than that in the foreign country because T_m rises whereas T_s, T_m^*, and T_s^* remain constant. Therefore, we are able to compare this case with the usual BS case. In the usual BS

[11] This numerical analysis is conducted as follows. To begin with, eliminating ω from equations (26) and (27), we derive an equation for $\bar{z}$:

$$\exp[T_m^* - T_m + 2\bar{z} - 1] = \frac{\bar{L}}{\bar{L}^*} \cdot \frac{1 + \left(\frac{1-\alpha^*}{\alpha^*}\right)^\sigma \exp\left[(1-\sigma)\left\{(T_m^* - T_s^*) + \bar{z}^2 + \frac{1}{2}\right\}\right]}{1 + \left(\frac{1-\alpha}{\alpha}\right)^\sigma \exp\left[(1-\sigma)\left\{(T_m - T_s) + \bar{z}^2 - 2\bar{z} + \frac{3}{2}\right\}\right]} \cdot \frac{1-\bar{z}}{\bar{z}}.$$

Next, substituting numerical values into the parameters of the equation above, we can solve it for $\bar{z}$. Using the numerical value of $\bar{z}$, we can obtain other endogenous variables. To solve the equation of $\bar{z}$, we use the "FindRoot" command, a built-in function of *Mathematica*.

case, the real exchange rate appreciates, and in this case, the real exchange rate appreciates as well.

When T_s rises, the manufacturing employment share of the home country increases while that of the foreign country decreases. The relative price of the home country rises, but the level of utility increases. The home utility is decreasing in the relative price and increasing in the level of productivity in services. When T_s rises, the latter positive effect dominates the former negative effect, and consequently the utility of the home country rises. Moreover, the rise of T_s in the home country leads to a decline in the relative price of the foreign country, thereby increasing the foreign utility.

[Table 3 to be inserted here]

In contrast to the case above, we will change some of the parameters simultaneously. In this case, we will obtain interesting results. Let us set the productivity parameters to $T_m = 1$, $T_s = 1/2$, $T_m^* = 1/2$, and $T_s^* = 0$. This means that the productivity growth differential between manufacturing and services in the home country is greater than that in the foreign country.[12]

From Table 3, we obtain the following result with regard to deindustrialization.

Result 2. *Suppose that the growth rate of productivity in manufacturing is greater than that in services. Suppose also that this sectoral difference in productivity growth is larger in the home country than that in the foreign country. Then, depending on the size of the effect of a shift in trade specialization, it is possible that the extent of the deindustrialization in the foreign country is greater than that of the home country.*

According to Baumol's (1967) closed economy model, other things being equal, a country whose productivity growth differential is large shows the large extent of deindustrialization. However, with condideration for international trade, it is possible that the extent of deindustrialization of a country is smaller than that of the trading partner even though the productivity growth differential is large. Obstfeld and Rogoff (1996, p. 225) point out that the extent of deindustrialization in Japan is small though the productivity growth differential is large. Then, they conclude that it is difficult to explain the decline in manufacturing share

[12]We must note the following. Let us write benchmark parameters and parameters after the change as $T_{m,0}$, $T_{s,0}$ and $T_{m,1}$, $T_{s,1}$, respectively. Then, $(T_{m,1} - T_{m,0}) - (T_{s,1} - T_{s,0})$ does *not* represent the productivity growth differential. Recall that the levels of the productivity are $a_m(z) = \exp(T_m + 1 - z)$ and $a_s = \exp(T_s)$. Thus, the productivity growth differential is given by $\exp(T_{m,1} - T_{m,0}) - \exp(T_{s,1} - T_{s,0})$. When $T_m = 1$, $T_s = 1/2$, $T_m^* = 1/2$, and $T_s^* = 0$, the home productivity growth differential, compared with the benchmark case, will be $\exp(1) - \exp(1/2) \approx 1.07$ and the foreign productivity growth differential will be $\exp(1/2) - \exp(0) \approx 0.65$, so that the former is certainly larger than the latter. That is, the productivity growth differential between manufacturing and services in the home country is greater than that in the foreign country.

by the productivity growth differential. However, as our analysis shows, when the productivity growth differential in the home country is large, the extent of deindustrialization in the trading partner can be greater than that in the home: we might call this phenomenon the *export of deindustrialization*.

From Table 3, we obtain the following result with regard to the real exchange rate.

Result 3. *Suppose that the growth rate of productivity in manufacturing is greater than that in services. Suppose also that this sectoral difference in productivity growth is larger in the home country than that in the foreign country. Then, depending on the size of the effect of a shift in trade specialization, it is possible that the real exchange rate depreciates in contrast to what the usual BS effect expects.*

According to the usual BS effect, the real exchange rate in this case should appreciate because the productivity growth differential between manufacturing and services in the home country is greater than that in the foreign country. However, we observe an depreciation in the real exchange rate. With the effect of a shift in specialization patterns, it is possible that the real exchange rate depreciates, which is opposite to what the BS effect expects (the anti-BS effect).

Recent empirical studies challenge the goodness of fit of the BS effect as an explanation for the long-term movement of the real exchange rate (Lee and Tang, 2007 and literature cited therein). These studies emphasize that PPP does not hold even for traded goods (e.g., Engel, 1999). In the intra-industry trade theory mentioned in footnote 5, PPP for traded goods does not hold. However, we show that it is possible that the real exchange rate moves in the direction opposite to the expectation of the BS effect even though PPP for traded goods holds.

Why does it happen? When the elasticity of substitution is smaller than unity, the real exchange rate is increasing in p_m/p_s whereas it is decreasing in p^*_m/p^*_s. Each relative price will fall when the growth rate of productivity in manufacturing exceeds that in services in each country. A fall in p_m/p_s has an effect of decreasing ε (i.e., appreciation), and on the other hand a fall in p^*_m/p^*_s has an effect of increasing ε (i.e., depreciation). This means that the real exchange rate appreciates (depreciates) if the decreasing rate of the home relative price is larger (smaller) than the decreasing rate of the foreign relative price. These relative prices are given by equations (33) and (34). As has been stated above, changes in the relative prices depend both on the direct effect of productivity growth differentials and on the indirect effect of a shift in $\bar{z}$. In benchmark case, we have $\bar{z} = 1/2$. When $\bar{z}$ becomes larger than 1/2, the indirect effect in (33), that is, $(-(\bar{z}-1)^2 - \frac{1}{2})$ is increasing in $\bar{z}$ while the indirect effect in (34), that is, $(-\bar{z}^2 - \frac{1}{2})$ is decreasing in $\bar{z}$. This suggests that whereas a shift in specialization has an effect of diminishing the decreasing rate of p_m/p_s in the home country, the shift in specialization has an effect of enhancing the decreasing rate of p_m^*/p_s^* in the foreign country. From this it follows that the real exchange rate can depreciate even though the productivity growth differential between manufacturing and services in the home country is greater than that in the foreign country.

Note that the same mechanism works behind Results 2 and 3 because from Result 1, ϕ and ε move in the same direction as long as $0 < \sigma < 1$

6 Concluding remarks

We have analyzed deindustrialization and changes in the real exchange rate by using a Ricardian trade model with a continuum of goods that introduces nontraded services. The main results are summarized as follows: (i) if deindustrialization proceeds in both countries, then the ratio of manufacturing employment share and the real exchange rate move in the same direction; (ii) even if the productivity growth differential in the home country is greater than that in the foreign county, the extent of deindustrialization in the home country is not necessarily larger than that in the foreing country. On the contrary, it is possible that the foreign deindustrialization exceeds the home deindustrialization; and (iii) even if the productivity growth differential in the home country is greater than that in the foreign county, the real exchange rate of the home country can depreciate contrary to the expectaion of the Balassa-Samuelson effect.

A shift in specialization patterns in the model corresponds to structural change in manufacturing sectors. Thus, we can conclude that when investigating the manufacturing employment shift and the movement of the long-term real exchange rate, we should consider structural change as well as productivity growth differentials.

Comparative statics analysis in this paper assumes that trade is balanced before and after changes in the parameters because our model is a static one. If, however, we introduce dynamic optimization into the model, then the effect of a change in a parameter in the dynamic model might differ with that in the static model because trade need not be balanced at each point in time. Moreover, the correlation between the ratio of the manufacturing employment share and the real exchange rate should be empirically analyzed. These issues will be left to future research.

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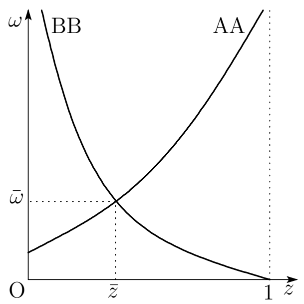

Figure 1: Equilibrium

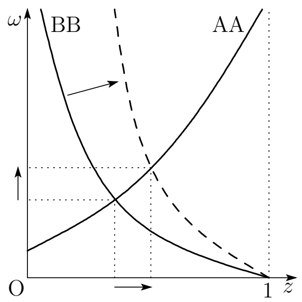

Figure 2: A rise in $\bar{L}/\bar{L}^*$

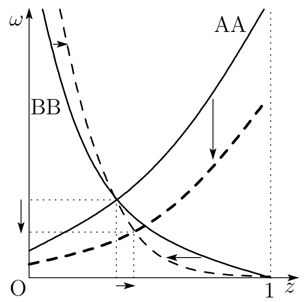

Figure 3: A rise in T_m

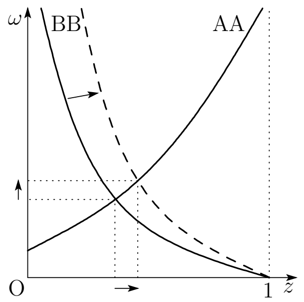

Figure 4: A rise in T_s

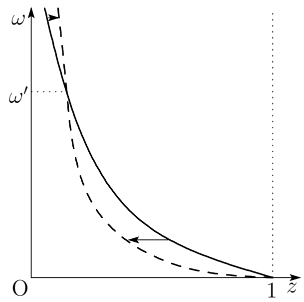

Figure 5: A large ω'

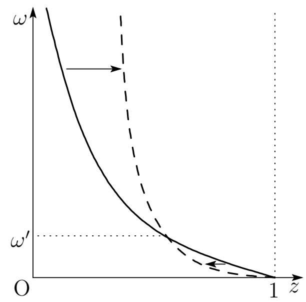

Figure 6: A small ω'

Table 1: Results of comparative statics analysis of $\bar{z}$, ω, and p_T

	$\bar{z}$	ω	p_T
$\bar{L}/\bar{L}^* \uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
$T_m \uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
$T_s \uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
$\alpha \downarrow$	$\downarrow$	$\downarrow$	$\uparrow$

Table 2: Results of numerical analysis

	Benchmark	$\bar{L} \uparrow$	$T_m \uparrow$	$T_s \uparrow$	$\alpha \downarrow$
$\bar{z}$	0.5	0.621	0.640	0.531	0.449
$\bar{\omega}$	1	1.275	0.487	1.063	0.903
p_T	1	0.886	0.869	0.970	1.052
ε	1	1.147	0.647	1.738	0.897
ϕ	1	1.047	0.865	1.202	0.736
l_m	0.438	0.447	0.367	0.523	0.325
l_m^*	0.438	0.427	0.425	0.435	0.442
p_m/p_s	0.472	0.526	0.196	1.323	0.448
p_m^*/p_s^*	0.472	0.412	0.403	0.458	0.496
V	0.711	0.678	1.012	1.178	0.691
V^*	0.711	0.754	0.761	0.721	0.696

Table 3: Simultaneous change in parameters

	Benchmark	After the change
$\bar{z}$	0.5	0.589
$\bar{\omega}$	1	0.724
p_T	1	0.915
ε	1	1.113
ϕ	1	1.036
l_m	0.438	0.404
l_m^*	0.438	0.390
p_m/p_s	0.472	0.311
p_m^*/p_s^*	0.472	0.260
V	0.711	1.398
V^*	0.711	0.910

A Appendix

■ The slope of the BB curve

The BB curve is given as follows:

$$\omega = \frac{L_m(\omega,\bar{z})}{L_m^*(\omega,\bar{z})}\frac{1-\bar{z}}{\bar{z}}. \tag{A-1}$$

Note that $\bar{z}$ in the equation above is not a value obtained by the intersection with the AA curve but a value on the BB curve.

Applying the implicit function theorem to (A-1), we can totally differentiate it as follows:

$$\begin{aligned} d\omega = {} & \frac{1-\bar{z}}{\bar{z}}\left[\frac{\partial L_m}{\partial \omega}\frac{1}{L_m^*} - \frac{\partial L_m^*}{\partial \omega}\frac{L_m}{(L_m^*)^2}\right]d\omega \\ & + \left[\frac{1-\bar{z}}{\bar{z}}\frac{\partial L_m}{\partial \bar{z}}\frac{1}{L_m^*} - \frac{1-\bar{z}}{\bar{z}}\frac{\partial L_m^*}{\partial \bar{z}}\frac{L_m}{(L_m^*)^2} - \frac{1}{\bar{z}}\frac{L_m}{L_m^*} - \frac{1-\bar{z}}{\bar{z}^2}\frac{L_m}{L_m^*}\right]d\bar{z}. \end{aligned} \tag{A-2}$$

Here, we introduce the following elasticities:

$$\eta = \frac{\partial L_m}{\partial \bar{z}}\frac{\bar{z}}{L_m}, \quad \eta^* = \frac{\partial L_m^*}{\partial \bar{z}}\frac{\bar{z}}{L_m^*}, \quad \theta = \frac{\partial L_m}{\partial \omega}\frac{\omega}{L_m}, \quad \theta^* = \frac{\partial L_m^*}{\partial \omega}\frac{\omega}{L_m^*}.$$

Substituting these into equation (A-2) and rearranging it, we have

$$\frac{d\omega}{d\bar{z}} = \frac{\omega}{\bar{z}}\frac{\eta - \eta^* - 1}{1-\theta+\theta^*}. \tag{A-3}$$

From this, the BB curve is downward-sloping if the following condition holds.

$$\frac{\eta - \eta^* - 1}{1-\theta+\theta^*} < 0. \tag{A-4}$$

Now we derive the elasticities. Considering the fact that L_m depends on p_m/p_s, which in turn depends on $\bar{z}$ and ω, we obtain the following relations:

$$\eta = \bar{z}D(1-\sigma)\frac{L_s}{\bar{L}}, \tag{A-5}$$

$$\eta^* = \bar{z}D(1-\sigma)\frac{L_s^*}{\bar{L}^*}, \tag{A-6}$$

$$\theta = (1-\bar{z})(1-\sigma)\frac{L_s}{\bar{L}}, \tag{A-7}$$

$$\theta^* = -\bar{z}(1-\sigma)\frac{L_s^*}{\bar{L}^*}, \tag{A-8}$$

where $D \equiv -\log\omega + (T^*_m - T_m + 2\bar{z} - 1)$. First, from these relations we obtain

$$1 - \theta + \theta^* = 1 - (1-\sigma)\left[(1-\bar{z})\frac{L_s}{\bar{L}} + \bar{z}\,\frac{L^*_s}{\bar{L}^*}\right]. \tag{A-9}$$

Since $0 < \sigma < 1$ by supposition and the employment share in services is smaller than or equal to unity, $1-\theta+\theta^* > 0$ holds. Therefore, the denominator of the left-hand side of (A-4) is always positive. Second, we obtain

$$\eta - \eta^* - 1 = \bar{z}D(1-\sigma)\left(\frac{L_s}{\bar{L}} - \frac{L^*_s}{\bar{L}^*}\right) - 1. \tag{A-10}$$

Since $1 - \theta + \theta^* > 0$ holds, (A-10) needs to be negative so that the BB curve will be downward-sloping. Analysis in the text assumes $\eta - \eta^* - 1 < 0$.

■ A shift in the BB curve with a rise in T_m

How the BB curve shifts with a rise in T_m depends on the sign of the following derivative:

$$\left.\frac{d\omega}{dT_m}\right|_{\bar{z}=\text{const.}}. \tag{A-11}$$

Applying the implicit function theorem to the BB curve, we can totally differentiating it as follows:

$$\left.\frac{d\omega}{dT_m}\right|_{\bar{z}=\text{const.}} = -\frac{\omega\bar{z}(1-\sigma)}{1-\theta+\theta^*}\left(\frac{L_s}{\bar{L}} - \frac{L^*_s}{\bar{L}^*}\right). \tag{A-12}$$

Since $1 - \theta + \theta^* > 0$ from the analysis above, the sign of (A-12) depends on which is larger, $L_s/\bar{L}_s$ or $L^*_s/\bar{L}^*_s$. If $L_s/\bar{L} > L^*_s/\bar{L}^*$, then the sign of (A-12) is negative. Consequently, the BB curve shifts downward with a rise in T_m.

Let us analyze the size of the employment share in services. The inequality $L_s/\bar{L} > L^*_s/\bar{L}^*$ is identical with

$$\frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right)^{\sigma}(p_m/p_s)^{1-\sigma}} > \frac{1}{1 + \left(\frac{\alpha^*}{1-\alpha^*}\right)^{\sigma}(p^*_m/p^*_s)^{1-\sigma}}. \tag{A-13}$$

Substituting the relative prices into (A-13) and computing it further, we finally obtain the following relation:

$$\omega < \frac{a^*_s}{a_s}\left[\left(\frac{\alpha^*}{1-\alpha^*}\right)\Big/\left(\frac{\alpha}{1-\alpha}\right)\right]^{\frac{\sigma}{1-\sigma}} \equiv \omega'. \tag{A-14}$$

From this it follows that if $\omega < \omega'$, then $L_s/\bar{L} > L^*_s/\bar{L}^*$. This means that the part of the BB curve such that $\omega < \omega'$ shifts downward with a rise in T_m. On the other hand, the part of the BB curve such that $\omega > \omega'$ shifts upward with a rise in T_m. The point of the BB curve such that $\omega = \omega'$ does not shift with a rise in T_m. Summarizing these results, we can conclude that the BB curve rotates on the point of $\omega = \omega'$ as shown in Figures 3, 5, and 6.

In the benchmark case, $\omega' = 1$ and the intersection of the AA and BB curves is $\omega = 1$, so that the BB curve happens to rotate on the initial equilibrium point.

■ A shift in the BB curve with a rise in T_s

How the BB curve shifts with a rise in T_s depends on the sign of the following derivative:

$$\left.\frac{d\bar{z}}{dT_s}\right|_{\omega=\text{const.}} = \frac{\omega(1-\sigma)}{1-\theta+\theta^*} \cdot \frac{L_s}{\bar{L}}. \tag{A-15}$$

When $\sigma < 1$, the sign of (A-15) is positive. Therefore, the BB curve shifts upward with a rise in T_s.

■ A shift in the BB curve with a rise in α

How the BB curve shifts with a rise in α depends on the sign of the following derivative:

$$\left.\frac{d\bar{z}}{d\alpha}\right|_{\omega=\text{const.}} = \frac{\omega\sigma}{\alpha(1-\alpha)(1-\theta+\theta^*)} \cdot \frac{L_s}{\bar{L}}. \tag{A-16}$$

The sign of (A-16) is always positive. Therefore, the BB curve shifts upward with a rise in α.