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Formalizing Debt-led and Debt-burdened Growth Regimes with Endogenous Macrodynamics of the Minskian Financial Structure:

A Long-Run Analysis

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Abstract

In this paper, we formally derive a version of the Minskian taxonomy of the firms' financial structure (hedge, speculative, and Ponzi types), under the economic growth context in the long run. As for the economic growth, we formalize the mechanism of debt-led (debt-burdened) growth where the economy expands as the debt variables increase (decrease). By explicitly introducing the relationship between the finance growth regime and Minskian taxonomy in a dynamic model, the model in this paper enables us to evaluate whether or not the economic growth regime is sounded in terms of the firms' financial positions.

1 Introduction

The purpose of this paper is to examine the dynamic characteristics of growth and finance in the long run. It also aims at generalizing the finance growth regimes and considering their relationships with the Minskian financial structure. In this paper, the finance growth regime indicates debt-led or debt-burdened growth, whereas the Minskian financial structure refers to hedge, speculative, and Ponzi finance of the Minskian taxonomy.

Many post-Keynesian studies have focused on the relationship between income distribution and aggregate demand or growth regimes, which demonstrate either wage-led or profit-led growth regimes (Bhaduri [2007]; Lavoie [2010]). The profit-led growth regime indicates a phase wherein the output expands as the profit variables increase. On the contrary, the wage-led growth regime indicates a phase wherein the output expands as the wage variables increase. These studies are inspired from the discussions on growth and distribution by Robinson, Kaldor, and Kalecki.

Minsky (Minsky [1975]; Minsky [1982]; Minsky [1986]) is another intellectual source of post-Keynesian economics. Minsky explained the fluctuation in investment in a capitalist economy by focusing on not only the income distribution but also the link between investment and finance. According to him, capital accumulation is affected by the long-run expectation of an entrepreneur with regard to cash flow. When economic booms continue in an economy and the entrepreneurs' expectations become optimistic, we have active capital accumulation. However, with capital accumulation, the firms are more dependent on external finance. As for the result, the firms' leverage ratio gradually increases, and their financial position thus becomes fragile.

The Minskian and Kaleckian models with debt accumulation try to link economic growth with the Minskian financial structure. Foley [2003] is a seminal paper that examined the Minskian taxonomy in the context of post-Keynesian growth theory. He extended the Taylor and O'connell [1985] type of short-run growth model, and explained that the economy tends to be closer to the Ponzi regime because of a low growth rate and profit rate. On the basis of the model in Foley [2003], Lima and Meirelles [2007] also investigate the stability of debt and the interest rates dynamics under the Minskian taxonomy. They showed given an interest rate that is lower than the growth rate, the long-run equilibrium solution under the hedge finance would be stable through the procyclical banking markup policy. In contrast, they showed that the Ponzi regime would be unstable regardless of the types of interest rate policy. Moreover, Charles [2008b] presents a dynamic model with a Minskian financial structure that tends to be unstable. He explained the unstable mechanism by intro-

ducing a parameter that describes the state of the financial structure à la Minsky.

Other noteworthy works in this field are on the formalization of economic growth in terms of debt-led or debt-burdened regimes. If a rise in the debt ratio (in a number of cases, the debt-capital ratio) stimulates capital accumulation, then this capital accumulation pattern is referred to as a debt-led pattern. On the contrary, if the rise in debt ratio restrains capital accumulation, the capital accumulation pattern is referred to as a debt-burdened pattern.

Although the above studies on the Minskian taxonomy are examined in terms of economic growth, they do not consider the type of finance growth regimes in a sufficient manner. For example, Lima and Meirelles [2007] assume that the saving rates of productive and financial capitalists are equal in analyzing the macrodynamics of the debt regime. As for the results, their model cannot capture the mechanism of debt-led and debt-burdened growth. On the other hand, the model of Foley [2003] covers only debt-burdened growth, due to the depressing effects of an increase in the interest rate and capital inflow. Moreover, the models of Charles [2008a] and Charles [2008b] are viable only under debt-burdened growth, which is due to the large negative impact of the interest payments on investment. These studies can explain only a phase of the finance growth regimes, and thus, are shallow.

In contrast, Hein [2006] and Hein [2007] allow both debt-led and debt-burdened growth regimes. While the former is always stable, the latter is unstable in the long run when the debt-capital ratio also evolves. However, the results obtained by Hein [2007] are in question, since Hein and Shoder [2009] empirically shows that debtburdened growth actually exists in the U.S. and Germany. Thus, from an empirical point of view, it is hard to consider that the debt ratio does not shrink or expand infinitely. Sasaki and Fujita [2010] constructed a model that allows both stable debtled and stable debt-burdened growth regimes. They relaxed the assumption in the models of Hein [2006] and Hein [2007] that the retention ration of the firms is unity, and proved that the long-run equilibrium value of the debt-capital ratio is positive with plausible interest rates irrespective of whether the long-run equilibrium is debtled growth or debt-burdened growth. While this is a general model that allows both stable debt-led and stable debt-burdened growth regimes, it still does not consider the Minskian financial structure. The same is true for the recently popular stock-flow consistent models (Lavoie and Godley [2001]; Dos Santos and Zezza [2008]; Treeck [2007]).

To summarize, the existing literature cannot generally consider the relationship among the debt-led and debt-burdened growth regimes and the Minskian financial structure. Some works have revealed the mechanism of finance growth regimes (Hein [2006]; Hein [2007]; Sasaki and Fujita [2010]), without considering the Minskian taxonomy. Other works have examined the stability of the Minskian taxonomy (Foley [2003]; Lima and Meirelles [2007]; Charles [2008b]), without considering the possibility of debt-led and debt-burdened growth regimes. Therefore, the relationship among the debt-led and debt-burdened growth regimes and the Minskian financial structure is yet to be clarified.

The novelty of this paper is that we explicitly explain the relationship between the finance growth regime and the Minskian taxonomy in a dynamic model, whereas related literature discusses these mechanisms separately. Thus, we formalize not only the mechanisms of the finance growth regime more generally (i.e., our model formalizes both the debt-led and debt-burdened growth regimes) but also considers how their association with the Minskian taxonomy (i.e., hedge, speculative, and Ponzi regimes). Indeed, if these relationships are captured by a macrodynamic model, we can understand whether or not the economic growth regime is sounded in terms of the firms' financial positions.

Our attempt also contributes to Minsky's explanations in his financial instability hypothesis. His argument with regard to this issue emphasizes that the debt ratio increases with the boom in capital accumulation. In terms of the financial growth regime, this implies that the economy is a debt-led growth regime. However, as explained above, the growth regime can be both debt-led and debt-burdened. Therefore, by considering them together, we can have a more detailed understanding of the possibility of the financial instability hypothesis in the case where capital accumulation may expand as the debt ratio decreases.

The remainder of this paper is organized as follows. Section 2 presents the basic structure of our model, where we classify the finance growth regimes and the Minskian financial structure. Section 3 examines the dynamic characteristics of the debt-led and debt-burdened growth regimes, while focusing on their relationships with the Minskian financial structure. Section 4 concludes the paper.

2 Model

The following is a list of the main notations used in this paper. X: output (total income), X^{*}: potential output, K: capital stock, E: effective employment level, $1 - \pi$: wage share, π : profit share, $X^*/K = v$: potential output-capital ratio (constant and set to unity for simplicity), u = X/K: output-capital ratio (effective demand), $r = \pi u$: profit rate, S: total savings, I: investment demand, g: actual rate of capital accumulation in the short-run, g_L : actual rate of capital accumulation in the long-

run, W: nominal wage, R: profit (net operating revenue), *i*: nominal interest rate, λ : debt-capital ratio, *t*: time.

The economy is closed and has no government fiscal expenditure. A single good that is used for both investment and consumption is produced with labour and capital, which are combined through a fixed coefficient technology. For simplicity, we assume that there is no technological change in production. Three classes—firms, workers, and capitalists (who own the firms)—are supposed to exist in the economy. The workers provide labour and receive wage income, with the wage bill being WE. The firms receive net operating revenue R, which is the surplus over wages. Hence, the functional distribution of income is given by

$$PX = WE + R, (1)$$

where PX is the total income.

From equation (1), the relationship among profit rate, capacity utilization, and profit share can be expressed as

$$r = \pi u, \tag{2}$$

where we assume the income distribution share to be constant. The profit rate r is then defined as the net operating revenue R divided by the value of capital stock.

Following Lima and Meirelles [2007] and many post-Keynesian analyses, we assume that the firms can make use of loans, which are financed by capitalists under a given interest rate, *i*. The firms must therefore pay debt service *iD* on their debt stock, along with a dividend to the capitalists in each period. We assume the dividend rate to be $(1-s_F)$, and the retention rate to be s_F . Hence, the firms' net profit is R-iD, and the capitalists' total income is $(1 - s_F)(R - iD) + iD$.

We assume that the three classes different saving behaviours. The firms retain a constant fraction s_F of their net profits, capitalists save a constant fraction s_C of their income, while workers spend all their wage income. At the macroeconomic level, the total savings *S* is composed of the profits retained by the firms and the savings from the capitalists' income. Therefore, the aggregate savings as a proposition of the capital stock is given by

$$\frac{S}{K} = s_C[(1 - s_F)(r - i\lambda) + i\lambda] + s_F(r - i\lambda).$$
(3)

The firms make an investment plan I, which is given by the following desired investment function:

$$g = \frac{I}{K} = \alpha + \beta r - \gamma i \lambda, \tag{4}$$

where α, β and γ are positive parameters, and g denotes investment in the short run as a ratio of the existing physical capital stock. α stands for the motivation to accumulate that might be affected by the long-run economic trends. The profit rate r is defined as the net operating revenue R divided by the value of capital stock. We also assume that the investment is negatively affected by $i\lambda$ due to the interest payments. In this setting, we basically follow Lima and Meirelles [2007]. The only difference lies in the forms of investment function. In our model, the interest payments negatively affect the capital accumulation rate, while in the model of Lima and Meirelles [2007], the interest rate restrains the capital accumulation rate.

2.1 Behaviour of the Model in the Short run

The short run is defined as a time span along which the capital stock K and the debt stock D are taken as given. The disequilibrium between investment and savings is adjusted through the changes in capacity utilization.

The short-run equilibrium value of capacity utilization rate is obtained from eqs. (3) and (4) as follows:

$$u = \frac{\alpha + \{s_F(1 - s_C) - \gamma\}i\lambda}{\pi(\Delta - \beta)},\tag{5}$$

where $\Delta = s_F(1 - s_C) + s_C$. The short-run equilibrium is stable if the denominator of this eq. (5) is positive. Since $\pi \in (0, 1)$, the necessary and sufficient condition for stability is $\Delta - \beta > 0$. This is well known as the Keynesian stability condition, which we assume to be the case. We also assume that the numerator of eq. (5) is positive, which will ensure a positive equilibrium value of *u*.

Using the short-run equilibrium value of the capacity utilization rate, we can obtain the short-run profit rate and accumulation rate as follows:

$$r = \frac{\alpha + \{s_F(1 - s_C) - \gamma\}i\lambda}{(\Delta - \beta)},\tag{6}$$

$$g = A + Bi\lambda,\tag{7}$$

where $A = \frac{\alpha \Delta}{\Delta - \beta}$ and $B = \frac{s_F (1 - s_C)(\beta - \gamma) - \gamma s_C}{(\Delta - \beta)}$.

Some characteristics should be mentioned regarding the short-run capacity utilization, profit rate, and capital accumulation rate¹. In our model, a rise in the profit share will lead to a decrease in the capacity utilization. This is because the redistribution of income from the firms who save to the workers who do not, raises consumption demand, which increases capacity utilization. This case is referred to as

¹The results below are explained by almost the same mechanism as in Lima and Meirelles [2007].

the stagnationist regime in the post-Keynesian literature. However, a change in the income distribution share does not have any impact on the profit rate and the rate of capital accumulation. This is because an increase in the profit share will lower the capacity utilization to the same extent.

In turn, the impact of a change in the debt-capital ratio on the short-run equilibrium values of capacity utilization and capital accumulation rate is ambiguous:

$$\frac{\partial u}{\partial \lambda} = \left[\frac{s_F(1-s_C) - \gamma}{\pi(\Delta - \beta)}\right]i,\tag{8}$$

$$\frac{\partial g}{\partial \lambda} = Bi = \left[\frac{s_F(1 - s_C)(\beta - \gamma) - \gamma s_C}{\Delta - \beta}\right]i.$$
(9)

A change in the debt-capital ratio also affects the level of effective demand. If the reaction coefficient of investment to interest payments γ is larger than the product of the firms' retention ratio and the capitalists' propensity to consume $s_F(1 - s_C)$, an increase in the debt-capital ratio decreases the capacity utilization rate. In the opposite case, an increase in the debt-capital ratio raises the capacity utilization rate. The short-run finance growth regimes depend on the profit effect on investment, in addition to the interest payment and the savings parameters. If $s_F(1 - s_C)(\beta - \gamma) - \gamma s_C > 0$, then the short-run finance growth regime is debt-led growth. In this case, an increase in the debt-capital ratio raises the capital accumulation rate. In the opposite case ($(s_F(1 - s_C)(\beta - \gamma) - \gamma s_C < 0)$), the short-run finance growth regime is debt-led growth regime is debt-burdened growth. Here, an increase in the debt-capital ratio decreases the capital ratio rate.

2.2 Behavior of the Model in the Long Run

In the short run, the capacity utilization adjusts the imbalance in *IS*. As a result, the short-run capital accumulation rate is determined as $g = A + Bi\lambda$. However, we suppose that in the long run, with capacity utilization and profit rate as determined in the short run, the capital stock *K* and the debt stock *D* also evolve². As both the debt stock and the capital stock change, the long-run capital accumulation rate and debt ratio, too, change.

We consider the evolution of the long-run rate of capital accumulation g_L with the dynamics of the debt ratio λ . Following Foley [2003], the long-run rate of capital accumulation is assumed to be determined according to the dynamic form of the

²The post-Keynesian macrodynamic models with such periodic analysis include the household debt models in Dutt [2005] and Dutt [2006b] and the model in Charles [2008b].

short-run accumulation rate³. That is, in the long run, the dynamics of the capital accumulation rate follow the change in the short-run accumulation rate due to the financial activity over time. This is obtained by differentiating eq. (7) with respect to time:

$$\dot{g_L} = Bi\dot{\lambda}.\tag{10}$$

In addition, we suppose that the firms' new loans in each period $\dot{D} = dD/dt$ are equal to the difference between the long-run capital accumulation and the firms' net profit, as we will derive later. That is, $\dot{D} = g_L K - (rK - iD)$. Since $\lambda = D/K$, we have

$$\dot{\lambda} = g_L - r + i\lambda - g_L \lambda. \tag{11}$$

Eqs. (10) and (11) define the long-run economic dynamics. These two equations comprise the so-called zero root system wherein one of the eigen values of the Jacobian matrix is zero. In a two-dimensional model, the other eigen value is the trace of the Jacobian matrix⁴. In the long-run steady state wherein $g_L = \dot{\lambda} = 0$, we have

$$g_L = \frac{r - i\lambda}{1 - \lambda}.$$
 (12)

Since the profit rate r is determined in the short-run eq. (6), in the long-run steady state, we get

$$g_L = \frac{1}{(1-\lambda)(\Delta-\beta)} \left[\alpha - (s_C + \gamma - \beta)i\lambda \right], \tag{13}$$

where we assume $s_C + \gamma - \beta > 0$, so that we can obtain economically meaningful solutions. Using this equation, we will depict below the steady state locus of this economy in the (λ, g_L) plane. By differentiating eq. (13) with respect to λ , we get the slope of the steady state locus:

$$\frac{\partial g_L}{\partial \lambda} = \frac{1}{(1-\lambda)^2 (\Delta - \beta)} \left[\alpha - (s_C + \gamma - \beta)i \right],\tag{14}$$

All points on this locus constitute the long-run steady state. However, we have to consider another condition that determines the transitional dynamics. From eq. (10),

³In Foley [2003], the short-run capital accumulation is determined by a given interest rate and the state of confidence. The long-run accumulation rate is determined by the exogenous target equilibrium rate of accumulation, while the dynamics are derived from the short-run capital accumulation rate.

⁴Giavazzi and Wyplosz [1985] explain the solution of the zero root system. Bhaduri [2007], Dutt [2006a], and Lavoie [2010] are the dynamic analyses using the zero root system in the post-Keynesian economics. However, these studies examine growth and distribution, and not financial issues.

we have $g_L = Bi\lambda$. Therefore, the equation for the transitional dynamics is obtained by integrating this equation with respect to time:

$$g_L(t) = Bi\lambda(t) + g_L(0) - Bi\lambda(0), \tag{15}$$

where the sign of *B* determines whether the growth regime is debt-led or debtburdened. In this system, it is easy to understand that the dynamics for the long-run steady state depend on the initial values of g_L and λ in eq. (15). Hence, the long-run steady state of the growth regime has path dependency. If the initial condition varies, the long-run position of the steady state also varies accordingly.

The local stability of the dynamical system eqs. (10) and (11) can be examined through a routine calculation of the Jacobian matrix. However, we consider a larger neighbourhood domain of the steady state, focusing on global stability. $\lambda \in (0, 1)$ is assumed as an upper and lower limit for the debt ratio. We state the following scalar function in order to examine global stability:

$$V(t) = \frac{1}{2}(g_L - g_L^*)^2.$$
 (16)

This scalar function satisfies the condition for using the second method of Liapunov (Gandolfo [1997])⁵. Therefore, stability within the domain is guaranteed by this method.

Substituting eq. (16) in eqs. (15) and (13), and differentiating eq. (16) with respect to time, we obtain

$$\dot{V(t)} = \left(\frac{1}{1-\lambda}\right)(g_L - r + i\lambda - g_L\lambda)^2 \left[Bi - \frac{\alpha - (s_C + \gamma - \beta)i}{(1-\lambda)^2(\Delta - \beta)}\right].$$
 (17)

Therefore, for $\lambda \in (0, 1)$, global stability requires $Bi - \frac{\alpha - (s_C + \gamma - \beta)i}{(1 - \lambda)^2(\Delta - \beta)} < 0$. While *B* determines whether the growth regime is debt-led or debt-burdened, the second term in the large parentheses in equation (14) represents the slope of long-run steady state locus. In other words, the combination of the type of growth regime and the slope of the long-run steady state locus plays an important role in global stability. As we have seen, the sign of *B* can be either positive or negative depending on the finance growth regime. As for $\alpha - (s_C + \gamma - \beta)i$, it can be either positive or negative depending on the discussion above, we get the following proposition.

⁵In fact, it satisfies **Theorem 23.1** in Gandolfo [1997]. For $\lambda \in (0, 1)$, all partial derivatives are assumed to exist with respect to $g_L - g_L^*$, and are deemed to be continuous. In addition, V(t) is positive definite.

Proposition 1. The global stability of a dynamic system is assured, if the absolute value of Bi is smaller than that of $\frac{\alpha - (s_C + \gamma - \beta)i}{(1 - \lambda)^2(\Delta - \beta)}$.

In this section, we showed that the long-run steady state of the growth regime has path dependency. We also presented a condition for the global stability of the dynamics of this economy. In the next section, we will show that the long-run steady state of the locus is also affected by the Minskian financial structure, which is related to global stability.

2.3 Minskian Financial Structure

Foley [2003] and Lima and Meirelles [2007] formalized what Minsky descriptively explained as the financial instability hypothesis. They classified the conditions for hedge, speculative, and Ponzi regimes, and examined the dynamic stability of each regime.

Following their studies⁶, we set the cash flow identity such that it equates the firm's source of funds from net operating revenue *R* and new borrowing that is the change in debt \dot{D} , to its usage of funds for long-run investment $g_L K$ and interest payments *iD*. Therefore, we have

$$\dot{D} = g_L K - R + iD. \tag{18}$$

Using this relationship, we define the Minskian financial structure as follows:

Hedge financial structure The hedge financing firms can fulfil their contractual payment obligations through their net operating revenue obtained as the result of economic activity in the short run. By considering current borrowing and debt service as contractual payment obligations, we formalize the hedge finance by the following

⁶Here, we employ the accounting categories of Lima and Meirelles [2007]. By abstracting from the payment of dividends for simplicity, the cash flow identity equates the firms' source of funds from net operating revenue *R* and new borrowing \dot{D} , to its usage of funds for long-run capital accumulation $g_L K$ and debt service iD. We then have $R + \dot{D} \equiv g_L K + iD$. Hence, the change in debt is derived by $\dot{D} = g_L K - R + iD$. By normalizing through the positive capital stock *K*, we can express the change in debt as in eq. (18).

equation⁷:

$$R \ge \dot{D} + iD. \tag{19}$$

We will examine this condition on per capita basis. By dividing both sides of eq. (19) with positive *K*, we get:

$$R/K \ge \dot{D}/K + iD/K. \tag{20}$$

Using eqs. (6) and (18), we investigate the following equation:

$$R/K - (D/K + iD/K) = r - (g_L - r + i\lambda + i\lambda)$$
$$= \frac{(\alpha - (s_C + \gamma - \beta)i\lambda)}{(\Delta - \beta)(1 - \lambda)}(1 - 2\lambda).$$
(21)

Therefore, with $\alpha - (s_C + \gamma - \beta)i\lambda > 0$, the financial structure is hedge finance if $0 < \lambda \le 1/2$, and is not hedge finance, if $\lambda > 1/2$. This result is suggestive, since it implies that the firms must ensure that their debt ratio is less than half, in order to retain hedge finance.

Speculative financial structure According to Minsky's characterization, the speculative firms can meet their debt service even as they are unable to repay the principle from their net operating revenue. Our model formalizes speculative finance as a case where a firm's net operating revenue can cover its debt service, while the debt ratio is more than half (i.e., $R/K < \dot{D}/K + iD/K$). The mathematical formalization in the speculative financial structure is this as follows:

$$R \ge iD. \tag{22}$$

As in the above exercise, by dividing both sides of eq. (22) with positive *K*, we consider the condition for speculative finance:

$$R/K \ge iD/K. \tag{23}$$

⁷Minskian studies varies in their mathematical formalization of the Minskian taxonomy: ((Minsky [1986]), Foley [2003], Lima and Meirelles [2007] and Charles [2008b]). In particular, our definition of hedge finance is different from that employed by Foley [2003] and Lima and Meirelles [2007]. While firms do not borrow in the hedge financial structure in their models (i.e., $\dot{D} < 0$), the negative interest payment effect remains. In contrast, our model allows the firms to borrow even in the hedge financial structure. This modification will be persuasive when we employ an investment function with the negative interest payment effect.

Using eq. (6), we can obtain the boundary for speculative finance:

$$R/K - iD/K = r - i\lambda$$
$$= \frac{1}{(\Delta - \beta)(1 - \lambda)} (\alpha - (s_C + \gamma - \beta)i\lambda).$$
(24)

Therefore, given $\lambda > 1/2$, if condition $\alpha - (s_C + \gamma - \beta)i\lambda \ge 0$ is satisfied, the financial structure is of the speculative type.

Ponzi financial structure In Minsky's original argument, the Ponzi firms are unable to pay not only the principle but also the debt service from their net operating revenue. In other words, the Ponzi firms are borrowing to pay part of their debt service. Such a condition for Ponzi finance can be derived directly from the above results. This means that a firm's net operating revenue cannot cover its debt service. The mathematical formalization in the Ponzi financial structure is this as follows:

$$R < iD. \tag{25}$$

By referring to eq. (24), we can obtain the boundary for Ponzi finance:

$$R/K - iD/K = r - i\lambda$$
$$= \frac{1}{(\Delta - \beta)(1 - \lambda)} (\alpha - (s_C + \gamma - \beta)i\lambda).$$
(26)

Therefore, given $\lambda > 1/2$, if condition $\alpha - (s_C + \gamma - \beta)i\lambda < 0$ is satisfied, the financial structure is of the Ponzi type.

It is shown that the financial structure can be distinguished according to the debt position and the interest rate. We now summarize the basic condition. If $0 < \lambda \le 1/2$, the financial structure is hedge finance. If $1/2 < \lambda \le \alpha/(s_C + \gamma - \beta)i$, the financial structure is speculative finance. If $1/2 < \alpha/(s_C + \gamma - \beta)i < \lambda$, the financial structure is Ponzi finance.

2.4 Growth Trajectory and Financial Structure

Our model involves the endogenous transformation of the Minskian financial structure. In particular, it shows two cases with regard to the regime change. As $\frac{\alpha}{(s_C + \gamma - \beta)i}$ describes the boundary between the speculative and Ponzi financial structures, we denote it by λ_{SP} . In addition, the relationship between λ_{SP} and 1 plays an important role in the endogenous transformation of the Minskian financial structure.

Given a constant positive parameter α , we can depict the relationship among the slope of the steady state locus, the Minskian financial structure, and the economic

growth rate as in Figure 1. In this figure, we define a function $R_j(\lambda) = \alpha - (s_C + \gamma - \beta)i\lambda$, which concerns the steady state path eq. (13), and the financial structure of (21), (24), and (25). Using this function, we can integrally consider the relationships between the growth trajectory and the change in financial structure. The slope of the steady state locus, $g_L = \lambda = 0$, can be distinguished by evaluating the sign of $R_j(\lambda)$ at $\lambda = 1$. If $R_j(1) > 0$, the slope is upwards, but if $R_j(1) < 0$, the slope is downwards.

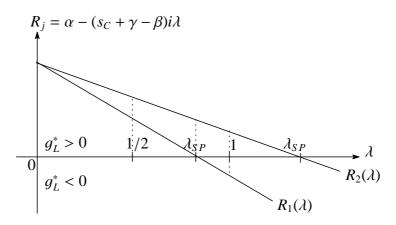


Figure 1: Discriminant for the Slope of the Steady State Locus, and the Economic Growth Rate

Note: $R_j(\lambda)$: i = 1, 2 is defined on the basis of $\alpha - (s_C + \beta - \gamma)i\lambda$; λ_{SP} is given by $\alpha/(s_C + \gamma - \beta)i$, which distinguishes between the speculative and Ponzi financial structures. For $0 < \lambda \le 1/2$, the financial structure is of the hedge type; for $1/2 < \lambda \le \lambda_{SP}$, it is of the speculative type; for $\lambda_{SP} < \lambda$, it is of the Ponzi type. As this figure indicates, if $\lambda_{SP} > 1$, the Ponzi regime does not arise and the slope of the steady state locus $g_L = \lambda = 0$ is always upwards. On the other hand, $\lambda_{SP} < 1$ results in an endogenous change in the Minskian financial structure and the slope of the steady state locus $g_L = \lambda = 0$ is always downwards.

Using eq. $R_j(\lambda)$, we distinguish some cases. If $\alpha/(s_C+\gamma-\beta)i > 1$, $\alpha-(s_C+\gamma-\beta)i > 0$ and the slope of the steady state locus in eq. (14) is always positive. As $R_2(\lambda)$ in Figure 1 indicates, this parametrical configuration does not result in the Ponzi financial structure given the possible debt ratio $\lambda \in (0, 1)$. The financial structure is of the hedge type for $0 < \lambda \le 1/2$, and of the speculative type for $1/2 < \lambda < 1$. We collectively refer to these cases as the non-Ponzi regime.

On the other hand, if $\alpha/(s_C + \gamma - \beta)i < 1$, then $\alpha - (s_C + \gamma - \beta)i < 0$ and the slope of the steady state locus in eq. (14) is always negative regardless of the financial structure. As $R_1(\lambda)$ in Figure 1 indicates, this parametrical configuration allows three types of Minskian financial structure. For $0 < \lambda \le 1/2$, the financial structure is of the hedge type; for $1/2 < \lambda \le \lambda_{SP}$, the financial structure is of the speculative type. For these two financial structures, the growth rate at the steady state (13) is still positive. However, for $\lambda_{SP} < \lambda$, the financial structure is of the Ponzi type, and the growth rate at the steady state is negative. We refer to this case as the Ponzi regime.

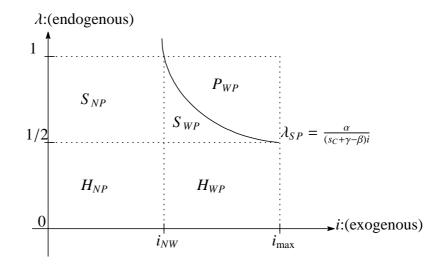


Figure 2: Condition for the Minskian Financial Structure

Note: $i_{NW} = \frac{\alpha}{(s_C + \gamma - \beta)}$ and $i_{max} = \frac{2\alpha}{(s_C + \gamma - \beta)}$; *H* refers to the hedge financial structure; *S*, the speculative financial structure; *P*, the Ponzi financial structure. These are the conditions at the long-run steady state. The subscript *NP* implies that the regime is of the non-Ponzi type, the subscript *WP* implies that the financial structure is of the Ponzi type. The curve derived by function λ_{SP} distinguishes the area between the speculative finance and Ponzi financial structure.

Interestingly enough, the above finance conditions affect the growth rate in the long-run steady state. Under a non-Ponzi regime, the condition for the hedge and speculative financial structure always ensures that $\alpha - (s_C + \gamma - \beta)i\lambda > 0$, which indicates that the long-run equilibrium capital accumulation rate in eq. (13) is always positive. On the contrary, under the Ponzi regime, $\alpha - (s_C + \gamma - \beta)i\lambda < 0$, which indicates that the long-run equilibrium capital accumulation rate in eq. (13) may be negative. In this sense, the long-run growth rate is not independent of the financial structure.

The change in the financial structure can also be explained in terms of the interest rate and debt ratio, as in Figure 2. If the interest rate is smaller than i_{NW} , the economy does not include the Ponzi financial structure, regardless of the value of debt ratio. In this case, for $\lambda \in (0, 1)$, λ is always smaller than λ_{SP} that distinguishes the area for the Ponzi regime. In this case, given an interest rate, the financial structure can transform only between the hedge and speculative financial structure according to the debt level. On the other hand, when the interest rate lies between i_{NW} and i_{max} , the economy involves three types of financial structure including the Ponzi finance. If $i \in (i_{NW}, i_{max})$, the financial structure changes according to the value of λ . While for $\lambda \leq 1/2$, the financial structure is always of the hedge type, for $\lambda > 1/2$, it changes depending on the relationship with λ_{SP} . If $1/2 < \lambda \leq \lambda_{SP}$, the financial structure is still speculative, but in the case of $1/2 < \lambda_{SP} < \lambda$, it results in the Ponzi finance. Thus, if the interest rate is relatively high ($i \in (i_{NW}, i_{max})$), the financial structure can transform among the hedge, speculative, and Ponzi finance types according to the debt ratio. Thus, a rise in the debt ratio leads the economy only to the speculative finance when the interest rate is relatively low. However, if the interest rate is high, an increase in the debt ratio easily leads to the endogenous transformation of the Minskian financial structure from the hedge to Ponzi type.

In the next section, we will investigate the dynamic stability of the debt-led and debt-burdened growth regimes under each Minskian financial structure.

3 Growth Regimes and Endogenous Macrodynamics of the Minskian Financial Structure

3.1 Dynamics of the Finance Growth Regimes: Non-Ponzi Case

The conditions for the non-Ponzi case are $\frac{\alpha}{(s_C + \beta - \gamma)i} > 1$ and $\frac{\alpha}{(s_C + \beta - \gamma)} > i$ (in terms of the interest rate). In this case, the slope of $g_L = \lambda = 0$ is always positive, from eq. (14). Under these conditions, the finance growth regime can either be debt-led if B > 0 (for example, due to a large s_F), or be debt-burdened if B < 0 (for example, due to a small s_F). Let us consider the global stability of each case. Figure 3 illustrates the case of debt-led growth under a non-Ponzi financial structure.

Proposition 2. Debt-led growth under a non-Ponzi regime is conditionally stable.

Proof. The sign of *Bi* is positive under debt-led growth, whereas the slope of the $\dot{g}_L = \dot{\lambda} = 0$ locus is also positive. Therefore, if and only if the absolute value of *Bi* is smaller than that of $\dot{g}_L = \dot{\lambda} = 0$, global stability is assured.

Suppose first that global stability is assured. As the arrows in this figure indicate, if one starts from initial position A, the transitional dynamics trace the solid line depicted by eq. (15). When the global stability condition is assured, a possible case is that it leads to point E_1 where the locus $\dot{g}_L = \dot{\lambda} = 0$ intersects the transitional dynamics line. On the contrary, if one starts from initial position B, the transitional dynamics trace the solid line from B, which too is depicted by eq. (15). When the global stability condition is assured, it leads to point E_2 , where the locus $\dot{g}_L = \dot{\lambda} = 0$ intersects the transitional dynamics line. The steady state in the long run differs according to the initial position, and thus, the dynamics have hysteresis.

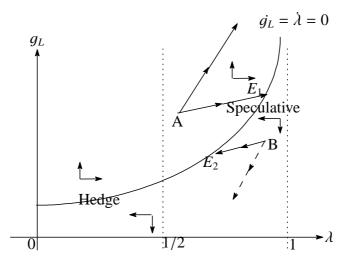


Figure 3: Debt-led Growth under a Non-Ponzi Regime

Note: The hedge finance corresponds to the steady state locus defined in $\lambda \in (0, 1/2)$, whereas the speculative finance corresponds to the steady state locus defined in $\lambda \in (1/2, 1)$. The solid lines depict the stable path to the steady state, and the broken lines depict the unstable path.

As proposition 2 stipulates, however, convergence may not occur. If the slope of the transitional dynamics is much larger than that of the locus $\dot{g}_L = \dot{\lambda} = 0$, we have a state where both the economic growth and debt ratio are low. For example, when the economy starts from initial position B with a high *Bi*, it may ride on the unstable path depicted by the broken line. The path from B with a high value of *Bi* does not intersect the locus $\dot{g}_L = \dot{\lambda} = 0$. Thus, the cumulative contraction of these two variables continues without some policy stimulation for economic growth.

On the other hand, even if the economy starts from initial position A with a high *Bi*, it will lead to a steady state. This is because the slope of $g_L = \dot{\lambda} = 0$ also increases, according to the rise in the debt ratio. As a result, the transitional path intersects the locus $g_L = \dot{\lambda} = 0$ when the debt ratio and the capital accumulation rate are high ($\lambda \doteq 1$). This is mathematically explained as follows. The slope of the locus $g_L = \dot{\lambda} = 0$ is given by eq. (14), and we get $\lim_{\lambda \to 1} \partial g_L / \partial \lambda = \infty$. On the contrary, the value of *Bi* is always finite in the neighbourhood of $\lambda = 1$. Therefore, we get V(t) < 0 in eq. (17), and the stability condition is assured. Hence, when high debt-led economic growth depends on the debt, the hedge financial structure can be never recovered.

We now investigate global stability in the case of debt-burdened growth under a non-Ponzi financial structure.

Proposition 3. Debt-burdened growth under a non-Ponzi regime is globally stable.

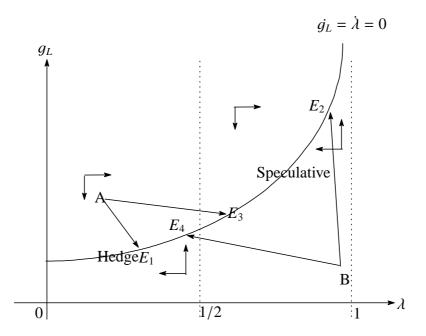


Figure 4: Debt-burdened Growth under a Non-Ponzi Regime Note: The hedge finance corresponds to the steady state locus defined in $\lambda \in (0, 1/2)$; the speculative finance corresponds to the steady state locus defined in $\lambda \in (1/2, 1)$. The solid lines depict the stable path to the steady state.

Proof. The sign of *Bi* is negative under debt-burdened growth, whereas the slope of the locus $\dot{g}_L = \dot{\lambda} = 0$ is positive. Therefore, the global stability condition $\dot{V(t)} < 0$ being assured in equation (17) is trivial.

Given proposition 3, convergence occurs in this regime. Some possible cases are depicted in the phase diagram (Figure 4). The direction depends on the initial value of the long-run growth rate and debt ratio, and the impact of debt on the economic growth. Let us consider four cases. First, if the initial position is near the area corresponding to the hedge financial structure, then the economic growth is strong, i.e., the absolute value of *Bi* is large. This case is depicted by the path from A to E_1 . Second, even if the initial position is near the area corresponding to the hedge financial structure will arise because the negative impact of debt burden on the economic growth are and the initial position is near the area corresponding to the hedge financial structure, the area corresponding to the hedge financial structure, the area corresponding to the hedge financial structure, the speculative finance will arise because the negative impact of debt burden on the economic growth is weak, i.e., the absolute value of *Bi* is small. This is depicted by the path from A to E_3 , wherein the dynamics involve an endogenous change in financial structure between the hedge and speculative types.

Third, if initially, the debt ratio is high and the growth rate is low, the speculative financial structure will arise due to the large negative impact of debt burden on the economic growth, i.e., the large absolute value of *Bi*. This case is depicted by the

path from B to E_2 . Lastly, even from the same initial position as in the third case, the hedge finance will arise because the negative impact of debt burden on the economic growth is weak, i.e., the absolute value of *Bi* is small. This case is depicted by the path from B to E_4 . This is also a case wherein the dynamics involve an endogenous change in financial structure between the hedge and speculative types.

3.2 Dynamics of the Finance Growth Regimes: Ponzi Case

As we have formalized above, if the interest rate is set at a high level, it may result in an endogenous change in the firms' financial position from hedge to Ponzi. The conditions for the Ponzi case are $\frac{\alpha}{(s_C + \beta - \gamma)i} < 1$ and $\frac{\alpha}{(s_C + \beta - \gamma)} < i$ (in term of the interest rate). In this case, the slope of $g_L = \dot{\lambda} = 0$ is always negative, from eq. (14). Here, both debt-led (B > 0) and debt-burdened (B < 0) growth regimes are possible. As in the previous section, let us consider the global stability of each regime. Figure 5 illustrates a phase diagram of debt-led growth under the Ponzi financial structure. The stability of the debt-led growth regime under the Ponzi financial structure can be obtained as follows.

Proposition 4. Debt-led growth under the Ponzi regime is globally unstable.

Proof. The sign of *Bi* is positive under debt-led growth. On the other hand, the slope of the locus $\dot{g}_L = \dot{\lambda} = 0$ is always negative. Therefore, *Bi* is always larger than $\dot{g}_L = \dot{\lambda} = 0$. As a result, this economy is globally unstable.

Using Figure 5, we consider some possibilities concerning the dynamic properties of this economy. In the macrodynamics of an economy with the Ponzi regime, we can explain that the economy has hysteresis and that the steady state in the long run differs according to the initial position. However, as proposition 4 stipulates, convergence never occurs in this case. If one starts from initial position A, the transitional dynamics trace the broken line depicted by eq. (15). Since this economy is globally unstable, the transitional dynamic path from A never intersects the steady state locus. As a result, it necessarily leads to point E_1 , where the value of λ is close to unity. In this example, we initially start from the hedge financial structure, which will go through a high growth phase. However, if the economy originally is close to the Ponzi financial structure (e.g., at point B), it will experience negative economic growth, as indicated by the broken line to point E_2 . In both cases, the firms' financial positions are never sounded, since the firms' debt ratio is high and their equity ratio is almost squeezed in these cases.

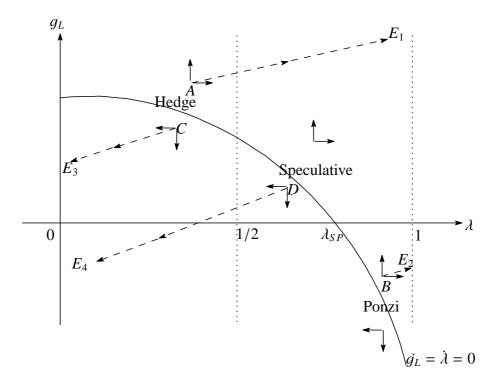


Figure 5: Debt-led Growth under the Ponzi Regime

Note: The hedge financial structure corresponds to the steady state locus defined in $\lambda \in (0, 1/2)$; the speculative financial structure corresponds to the steady state locus defined in $\lambda \in (1/2, \lambda_{SP})$; the Ponzi financial structure is on the steady state locus defined in $\lambda \in (\lambda_{SP}, 1)$. The solid lines depict the stable path to the steady state, and the broken lines depict the unstable path.

On the contrary, if one starts from initial position C, the transitional dynamics trace the broken line that leads to point E_3 . In addition, the transitional dynamic path near the area corresponding to the speculative financial structure (area near D) leads to point E_4 . As we have shown above, the direction of each path depends on the value of *Bi*. Therefore, if the impact of *Bi* is relatively strong—that is, the slope of the transitional dynamics is steep—we may easily have negative growth.

Finally, we consider the dynamic property of the debt-burdened growth regime under the Ponzi regime.

Proposition 5. *The global stability of the debt-burdened growth regime under the Ponzi regime is conditionally stable.*

Proof. The sign of *Bi* is negative under debt-burdened growth, whereas the slope of the locus $\dot{g}_L = \dot{\lambda} = 0$ is also negative under the Ponzi regime. Therefore, if and only if the absolute value of *Bi* is larger than that of $\dot{g}_L = \dot{\lambda} = 0$, global stability is assured.

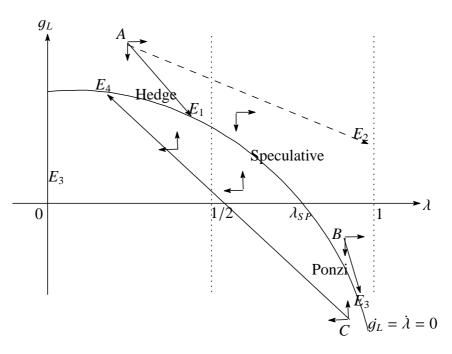


Figure 6: Debt-burdened Growth under the Ponzi Regime

Note: The hedge financial structure corresponds to the steady state locus defined in $\lambda \in (0, 1/2)$; the speculative financial structure corresponds to the steady state locus defined in $\lambda \in (1/2, \lambda_{SP})$; the Ponzi financial structure is on the steady state locus defined in $\lambda \in (\lambda_{SP}, 1)$. The solid lines depict the stable path to the steady state, and the broken lines depict the unstable path.

As in the discussion above, dynamic stability can be distinguished according to the absolute value of Bi and the slope of $g_L = \lambda = 0$. The transitional dynamics from initial position A results in E_1 (for example, in the stable case due to the large absolute value of Bi). In contrast, the transitional dynamics from A never intersect the locus $g_L = \lambda = 0$, and leads to E_2 (for example, in the unstable case due to the small absolute value of Bi).

Interestingly, if the economy is near the area corresponding to the Ponzi financial structure, its financial structure may remain of the Ponzi type. For example, when the economy starts from initial position B, it rides on the stable path depicted by the solid line. Since the debt ratio is close to unity in this situation, the absolute value of the slope of the locus $\dot{g}_L = \dot{\lambda} = 0$ as calculated by eq. (14) is close to infinity. However, the absolute value of Bi remains finite even in the neighbourhood of $\lambda = 1$. Therefore, we get V(t) < 0 in eq. (17), and the stability condition is assured. Thus, the transitional path intersects the locus $\dot{g}_L = \dot{\lambda} = 0$ when the debt ratio is high and growth is negative (position E_3). Hence, if the economy depends (by nature) on a high debt ratio, it is possible its financial structure remains of the Ponzi type. The steady state point E_3 is not desirable for the firms' financial position

and macroeconomic condition.

However, since the economy has hysteresis, other cases may also be possible. Even if the economy initially depends on a high debt ratio, it may trace a financially sounded stable dynamics path. An example is depicted by the solid line from position C in Figure 6. In this case, even if the debt ratio is high at the beginning, the economy may expand while reducing the debt ratio. As a result, the economy will lead to the steady state E_4 , where the financial structure is of the hedge type.

4 Conclusion

Thus far, the Minskian and Kaleckian models with debt accumulation have examined the mechanism of debt-led and debt-burdened growth regimes, and the Minskian financial structure separately. However, the interrelationships between the finance growth regimes and the Minskian financial structure have been ambiguous. In contrast, this paper explicitly examines the relationship between the finance growth regime and the Minskian taxonomy in a dynamic model. Our macrodynamic model sheds light on whether or not the economic growth regime is sounded while focusing on the firms' financial positions.

Table 1 summarizes our results on the interrelationships between the finance growth regimes and the Minskian financial structure. The main results are summarized as follows. (i) Given the parameters in the *IS* balance, the interest rate determines the type of Minskian financial structure under both Ponzi and non-Ponzi financial structures. In this sense, the monetary policy via the interest rate plays an important role in preventing the deterioration of the firms' financial structure. If the interest rate is set as relatively low, only the hedge and speculative financial structures will arise. If it is set as relatively high, not only the hedge and speculative financial structures, but also the Ponzi type will appear. (ii) The debt ratio and the growth rate of the economy change in the long-run dynamics, in which the finance growth regimes concern the direction of these variables. According to the debt ratio determined at the steady state, the financial structure of the economy is determined finally. The higher the debt ratio, the more unlikely is the hedge financial structure to arise. (iii) Our results show that whereas debt-led growth is conditionally stable at the most, debt-burdened growth is conditionally stable at the least. Therefore, our model can explain that a Minskian phase, where the debt ratio increases with capital accumulation, may involve unstable dynamics. In addition, under the Ponzi regime, nonconditional stability is not assured in both finance growth regimes.

		Non Ponzi Regime	Ponzi Regime
Interest rate		$0 < i < i_{NW}$	$i_{NW} < i < i_{\max}$
	Hedge	$0 < \lambda \le 1/2$	$0 < \lambda \le 1/2$
Debt ratio	Speculative	$1/2 < \lambda < 1$	$1/2 < \lambda \leq \lambda_{SP}$
	Ponzi	-	$\lambda_{SP} < \lambda < 1$
Stability	Debt-led growth	conditionally stable	unstable
	Debt-burdened growth	stable	conditionally stable

Table 1: Minskian Taxonomy and Stablity of Economic Growth

Note: $\lambda_{SP} = \frac{\alpha}{(s_C + \gamma - \beta)i}$, $i_{NW} = \frac{\alpha}{(s_C + \gamma - \beta)}$ and $i_{\max} = \frac{2\alpha}{(s_C + \gamma - \beta)}$.

Introducing the Minskian taxonomy in a dynamic model plays an important role for the post-Keynesian economic growth theory, because it enables us to evaluate whether or not the firms' finance position is robust in the process of economic growth. Even if an economy attains high growth, it is not financially desirable as long as the financial structure has the momentum to be fragile.

We hope the results in this paper will provide useful foundations for further research, as there exist only a few works on the post-Keynesian economic growth and the financial structure. For this purpose, some extensions are required. For instance, our model can evaluate the Minskian financial structure only in the steady state. Therefore, an extended model that can examine the Minskian financial structure in the transitional dynamics should be presented. In addition, in our model, the Minskian financial structure changes from the hedge to speculative type at $\lambda = 1/2$. This boundary may be deterministic, even though it is economically understandable. This result may come from the definition of the hedge financial structure in eq. (21). In fact, the firms' financial regime may be of the non-hedge type depending on the profitability conditions, even if their debt ratio is smaller than half. An extended model that captures the changing financial structure at a nondeterministic debt ratio will be interesting. Lastly, our analysis focused on external finance via borrowing. However, other methods such as equity finance are possible in general. We did not included these methods into our model for simplicity. If, however, we introduce these methods into the model, then the growth trajectory will be also affected by them, because the firms will not necessarily depend on debt finance. These issues will be left for future research.

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