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A Multi-Sectoral Balance-of-Payments-Constrained Growth Model with Sectoral Heterogeneity:

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Abstract

This study builds a multi-sectoral balance-of-payments constrained growth model that incorporates some structural heterogeneity between sectors and countries, such as differences in labor productivity, price competition, share of exports and imports, and the quality of commodities. The model in the current paper generates more comprehensive results than Thirlwall (1979), Blecker (1998), and Araujo and Lima (2007), even though it contains their properties and reproduces their implications. Furthermore, as compared with these existing works, the current model sheds more light on the relationship between trade structure, international competition, productivity dynamics, and economic growth. It also shows an example of the fallacy of composition that there are differences between microeconomic and macroeconomic phenomena, by illustrating how changes in nominal wage, the Kaldor–Verdoorn effect, and the degree of market competition in both countries affect the economic growth of the home country.

Keywords: Multi-sectoral Thirlwall's law, International competition, Structural heterogeneity

JEL Classification: F12, O19

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1 Introduction

This study builds a multi-sectoral balance-of-payments-constrained (BOPC) growth model. We comprehensively reveal how the economic growth of the home country is impacted by changes in (i) the wage growth rate, (ii) growth rate of the foreign country, (iii) dynamics of labor productivity, and (iv) increase in international competition. These issues are examined in the presence of heterogeneity in labor productivity, cost-price competition, export and import shares, and the quality of commodities between sectors and countries.

The BOPC growth model is a post-Keynesian, demand-led approach that postulates that the balance-of-payments position of a country imposes a limit on effective demand, to which supply can usually adapt. As is well known and we will show below, in the canonical expression of BOPC growth, the economic growth rate of a country is determined by the so-called Thirlwall's law that originates in Thirlwall (1979).[1] Thirlwall's law implies that a country's growth rate is dependent on the growth rates of other countries; and the ratio of the income elasticities of demand for exports and imports reflects non-price competitiveness. On the basis of this result, an economic policy implication is derived that to be non-price competitive, it is important to increase the attractiveness of the home country's exports, as compared with imported goods. BOPC growth models have revealed the mechanism of economic growth in the open economy context by focusing on a country's quantitative (exports, imports and economic growth) and qualitative (non-price competitiveness) aspects.

Many contributions have been made after the seminal work of Thirlwall (1979). Soukiazis and Cerqueira (2012) comprehensively summarize the recent contributions with regard to the history, theory, and empirical evidence of BOPC growth models. According to Thirlwall (2012), the main research directions are to incorporate capital flows, interest payments on debt, and terms of trade movement; disaggregate the model by commodities (multi-sectoral model) and trading partners; and conduct empirical investigation.

Among other studies, the current study employs the sectorally disaggregated model with some theoretical extensions. Of course, there is much existing literature on multi-sectoral models of BOPC growth. These models are inspired by the structural economic dynamics of Pasinetti (1981, 1993) that emphasizes the structure of demand and production in an economy. Araujo and Teixeira (2003), Araujo and Lima (2007), Araujo et al. (2013), Araujo (2013), Rocha and Lima (2010), Cimoli and Porcile (2010, 2014), Missio and Jayme Jr. (2012), and Bagnai et al. (2012)

[1] Thirlwall (2012) retrospects that the original Thirlwall model is built on the proposition of a limit to the external deficit–GDP ratio, beyond which financial markets get nervous. In other words, its basic idea is that economic growth with an ever-growing deficit is unsustainable. This is why the standard BOPC model starts with the condition of balance of payments equilibrium. It should also be noted that the BOPC model was established with critical implications for the export-led growth model *à la* Kaldor (1970), whose idea was formalized by Dixon and Thirlwall (1975). This is because the export-led growth model ignores the role of import demand and neglects the BOPC in determining the rate of economic growth. See also Blecker (2013) for a comprehensive survey of this topic and Razmi (2013) for a unification of the characteristics of both theories.

are of interest for the current study.

Araujo and Teixeira (2003) constructed a Pasinettian structural change model that incorporates both consumption and capital goods and international trade in them. On this basis, they confirm the validity of Pasinetti's insight on structural change of production and expenditure in the international context. Araujo and Lima (2007) is also a Pasinettian structural dynamic model that recognizes the role of demand-led structural change in economic growth. As we will see below, one of the most important contributions of their study is that it clearly explains the growth mechanism that is based on the structural change from sectors with high income elasticities of demand for imports to that of demand for exports. Araujo et al. (2013) and Araujo (2013) introduce the role of technological progress by using Kaldor–Verdoorn's cumulative causation effect that the current paper also emphasizes. The former reveals that the existence of this effect also enhances BOPC growth, whereas the latter explains that cumulative causation leads to the widening gap in income per capita between rich and poor nations. However, as we indicate below, in these studies, the impacts of changes in the determinants of relative price under international price competition are not clear. That is because these studies suppose that purchasing power parity (PPP) holds over time in each sector. We will complement this remaining issue in this paper. Cimoli and Porcile (2010, 2014) develop the multi-sectoral BOPC model that is based on the Ricardian approach with a continuum of goods *à la* Dornbusch et al. (1977). These studies emphasize the role of technology and structural change that leads to an increase in cost competitiveness for economic growth.

Empirical studies also report that the multi-sectoral Thirlwall's law is a good predictor of the actual growth rate of income. Using the data for Latin American countries and Asian countries, Rocha and Lima (2010) empirically shows that the multi-sectoral Thirlwall's law fits better than the original Thirlwall's law. Bagnai et al. (2012) present a multi-country BOPC model that emphasizes bilateral terms of trade and market shares, but it has a similar implication for the multi-sectoral BOPC model. On applying the model to Sub-Saharan African economies, they found that the multi-country model performs better than the original Thirlwall's law.

These studies have shown important results that are specific to multi-sectoral analysis. Above all, Araujo and Lima (2007) is the closest in spirit to the current study. They showed the mechanism of BOPC growth in a multi-sectoral context, and concluded that the growth rate of per capita income in the home country is also influenced by changes in the share of imports and exports in each sector. Therefore, their model emphasizes the structure of production and non-price competitiveness of each sector as determinants of economic growth.

These models usually assume PPP and especially focus on the role of non-price competition and trade share of each sector. Variations in the terms of trade (the real exchange rate) are normally considered to be irrelevant to BOPC growth. However, this assumption implies that the models do not sufficiently consider some important effects of relative price variations for economic growth. These effects come from changes in nominal wages, sectoral differences in labor productivity, market structure, and price competitiveness of firms. Consequently, the relationship between these intriguing determinants of the relative price (terms of trade) and BOPC growth remains relatively unexplored in the existing literature.[2]

It is against this background that this study further extends a multi-sectoral BOPC model with international competition and productivity dynamics. We set up a model that places heterogeneous industrial structures, such as different growth rates of labor productivity, exports and imports, preferences for commodities, and market competition at the core of the analysis. This is conducted in the following manner. First, this study constructs a multi-sectoral model in which there is international trade between two countries that have multiple sectors. This is conducted in line with Pasinetti (1981, 1993), and Araujo and Lima (2007). Second, it introduces the market competition aspect of BOPC growth based on the Kaleckian model. Extending Blecker (1998)'s idea to multi-sectoral models, we assume international cost–price competition between each sector of both countries. Third, structural aspects of BOPC growth, such as sectoral export and import shares, market structure, price competitiveness, as well as dynamics of productivity differences among sectors are investigated by using the Kaldor–Verdoorn effect.[3]

Thus, this paper reveals how structural heterogeneities, identified as changes in the sectoral composition of exports and imports, sectoral labor productivity dynamics, and intensifying international price competition affect the economic growth rate of the home country. Although some of these attempts have been made in the existing literature, the current paper presents an economic growth model that can be used to comprehensively understand these results. Furthermore, the current paper reveals some important results, hitherto undiscovered, by addressing the four questions mentioned above. We will especially find that: (i) The current model sheds more light on the relationship between the trade structure, international competition, productivity dynamics, and economic growth than Araujo and Lima (2007)'s model. Specifically, (ii) the effect of wage increase on the economic growth rate depends on the sum of cost–price competition elasticity, weighted by the share of exports and imports. Because of this, a rise in the home wage does not necessarily decrease economic growth. This result contrasts with Blecker (1998). (iii) It also

[2]Most studies have assumed that changes in the real exchange rate do not affect economic growth, either because price elasticities of exports and imports are low or the rate does not have a systematic tendency to appreciation or depreciation in the long run. However, it is still important to consider variations in the change in relative prices, especially in terms of industrial sectors, because it still concerns the price competitiveness of each sector and is determined by sectorally different factors such as productivity and the market structure. Bagnai et al. (2012) insist that the assumption of a constant real exchange rate is inappropriate, especially for the analysis of developing countries. Missio and Jayme Jr. (2012) and Araujo et al. (2013) investigate the role of the real exchange rate in innovation and the endogeneity of income elasticities of imports and exports in the multi-sectoral BOPC model. They show that an exchange rate policy that keeps a competitive exchange rate level contributes to faster economic growth by inducing technological progress, under the assumption that PPP holds over time in each sector.

[3]Introducing this effect into the multi-sectoral model with international competition may well be reasonable, because productivity dynamics is one of the determinants of cost and price competitiveness. Furthermore, it is this circulative effect for the economic growth rate that Kaldorian export-led growth has emphasized under cumulative causation (Dixon and Thirlwall (1975); Setterfield and Cornwall (2002)). However, as is indicated in the preceding footnote, the canonical export-led model ignores the role of imports, and thus, BOPC condition.

shows an example of a fallacy of composition that the Keynesian model has emphasized so far. At the industrial level, a rise in wage rate in the home country necessarily deteriorates each sector's trade balance, whereas its impact on exports and imports at the macroeconomic level is not necessarily the same. Thus, there are differences between microeconomic and macroeconomic phenomena.

This paper is organized as follows. Section 2 builds an extended version of the multi-sectoral BOPC model with heterogeneous industrial structures. Section 3 first derives the economic growth rate under the multi-sectoral BOPC model with international competition, and then, this section explains the generality of the model. Furthermore, by way of comparative static analysis, this section presents several theoretical and political implications that are specific to the multi-sectoral version of the BOPC growth model with structural heterogeneity. Section 4 presents our conclusion.

2 Setup of the Model

2.1 The BOPC Condition, Export and Import Demand Functions

The following is a list of the main notations for the home country used in this paper. The subscript i indicates a variable for the ith sector. Y_D: total output (total income), Y_i: output (income), E_i: employment, q_i: labor productivity, p_i: price of the commodity, x_i: export demand, w: nominal wage rate, e: nominal exchange rate between two countries, m_i: import demand, c_i: unit labor cost, z_i: gross markup ratio. The same variables in the foreign country are expressed by adding the subscript F to the variable (for example, the nominal wage rate in the foreign country is w_F).

We consider the international trade of commodities between two countries that we call the home country and foreign country. Our focus is on the determinants of economic growth in the home country. Both countries have n sectors; each sector produces the same commodity in both countries, but with different productivities. Each sector is presented by the index $i = 1, 2, ...n$ and the ith sector produces commodity i. Each sector of both countries has heterogeneous characteristics related to export and import shares, productivity growth, pricing, and the market competition structure. We assume that the commodity i produced in each country is an imperfect substitute, and the ith sector of the home country is in competition with the same sector in the foreign country across the cost–price level of production.

There is an important difference between Araujo and Lima (2007) and the current model with regard to trade goods and specialization. Whereas the former supposes international trade under complete specialization, the latter does not make this assumption. In Araujo and Lima (2007), it is only when the price of commodity i in the home country is lower than that of the partner country that the home country has comparative advantage in producing commodity i and can, therefore, export it. Similarly, it is only when the price of commodity i in the home country is higher than in the partner country that the home country imports the commodity. In contrast, we suppose intra-industry trade in which an economy both exports and imports commodity i, for which the ith sector is in competition with the trade partner country. When we consider the circumstances of international competition, such a supposition is reasonable.

In this paper, the balance-of-payments-constrained (BOPC) condition is measured in nominal terms at a macroeconomic level. Although the trade balance may not be in equilibrium in each sector, the trade has to be balanced at the aggregate level. In a multi-sector context, the total value of exports (imports) comprises the total value of exports (imports) from each sector of the economy. Therefore, the trade balance at the aggregate level is given by

$$\sum_{i=1}^{n} p_i x_i = \sum_{i=1}^{n} p_{Fi} e m_i, \tag{1}$$

where the left-hand side (LHS) represents the total value of exports in the home country, and the right-hand side (RHS) represents the total value of imports in the home country in one period. In order for this trade balance to be maintained over time, it is necessary that the time rate of change in total exports and imports should be equal. Therefore,

$$\sum_{i=1}^{n} \frac{p_i x_i}{\sum_{i=1}^{n} p_i x_i}(\hat{p}_i + \hat{x}_i) = \sum_{i=1}^{n} \frac{p_{Fi} e m_i}{\sum_{i=1}^{n} p_{Fi} e m_i}(\hat{p}_{Fi} + \hat{e} + \hat{m}_i)$$
$$\sum_{i=1}^{n} \nu_i(\hat{p}_i + \hat{x}_i) = \sum_{i=1}^{n} \mu_i(\hat{p}_{Fi} + \hat{e} + \hat{m}_i). \tag{2}$$

Eq.(2) is the BOPC condition in the growth term, where the hat symbol represents the growth rate of each variable. $\nu_i \equiv \frac{p_i x_i}{\sum_{i=1}^{n} p_i x_i} \in [0, 1]$ denotes the market share of the ith industry in a country's total exports (in volume), and $\mu_i \equiv \frac{p_{Fi} e m_i}{\sum_{i=1}^{n} p_{Fi} e m_i} \in [0, 1]$ denotes the market share of the ith industry in the country's total imports (in volume). We assume that these terms are exogenous and constant; they are historically given or determined by the specialization patterns of the country.[4] If the ith sector of the home country produces a commodity that is only for domestic use, its share of exports is zero (that is, $\nu_i = 0$). Similarly, if the home country does not import a commodity i from the foreign country, its share of imports is zero (that is, $\mu_i = 0$). It should also be noted that $\sum_{i=1}^{n} \nu_i = 1$ and $\sum_{i=1}^{n} \mu_i = 1$ by definition.

Following Thirlwall (1979), we assume that the export and import demand functions for each commodity are given by the Cobb–Douglas functional form. First, the export demand (the foreign demand) function for commodity i is given by

$$x_i = \bar{x}_i \left(\frac{p_i}{e p_{Fi}}\right)^{-\varepsilon_{Fi}} Y_F^{\eta_{Fi}}, \tag{3}$$

[4] Strictly speaking, both ν_i and μ_i are endogenous variables and change over time, because the exports and imports of the ith sector are defined in eqs. (3) and (5), respectively. However, the model becomes analytically untraceable if we treat them as endogenous variables. Therefore, these values are assumed to be constant and determined by the historical context of the economy.

where $\bar{x_i}$ is a constant term, $\varepsilon_{Fi} \in (0,1)$ is the relative price elasticity, and $\eta_{Fi} \geq 0$ is the income elasticity of demand for exports of commodity i. This formalization means that if the real exchange rate, measured by the price of commodity i, appreciates (that is, a fall in ep_{Fi}/p_i), exports of commodity i decrease. Eq. (3) also means that booms in the foreign country (that is, a rise in Y_F) induce higher exports of commodity i.

By taking logarithms of eq. (3) and differentiating with respect to time, the growth rate of exports of commodity i is obtained as follows:

$$\hat{x}_i = -\varepsilon_{Fi}(\hat{p}_i - \hat{e} - \hat{p}_{Fi}) + \eta_{Fi}\hat{Y}_F. \tag{4}$$

This is the dynamic form of exports of commodity i.

Second, the import demand function for commodity i is given by

$$m_i = \bar{m}_i \left(\frac{ep_{Fi}}{p_i}\right)^{-\varepsilon_{Di}} Y_D^{\eta_{Di}}, \tag{5}$$

where $\bar{m_i}$ is a constant term, $\varepsilon_{Di} \in (0,1)$ is the relative price elasticity, and $\eta_{Di} \geq 0$ is the income elasticity of demand for imports of commodity i. When the real exchange rate, measured by the price of commodity i, depreciates (that is, a rise in ep_{Fi}/p_i), imports of commodity i decrease. Eq. (5) also means that an increase in the home country's income (that is, a rise in Y_D) induces higher imports of commodity i.

The dynamic form of the import demand function is derived by following the same procedure as above. That yields

$$\hat{m}_i = \varepsilon_{Di}(\hat{p}_i - \hat{e} - \hat{p}_{Fi}) + \eta_{Di}\hat{Y}_D. \tag{6}$$

Jointly with $\varepsilon_{Fi} \in (0,1)$ and $\varepsilon_{Di} \in (0,1)$, we assume that the Marshall–Lerner condition with respect to trade in the ith industry holds. That is, $\varepsilon_{Fi} + \varepsilon_{Di} > 1$.

2.2 Price, Production Cost, and International Competition

Our model of pricing, production cost, and international competition is developed by disaggregating Blecker (1998)'s model. Blecker (1998) examined the relationship between BOPC growth and changes in wage cost (or living standards) by employing a partial exchange-rate-pass-through model. According to this model, when home production becomes more costly relative to foreign production as measured by relative unit labor costs, the markup rate is reduced so as to keep the commodity more competitive and preserve market share. The relative unit labor cost is also affected by the nominal exchange rate that may fluctuate depending on conditions in the international financial markets. By introducing this idea into the current multi-sectoral model, we formalize the relationship between international competition, production cost, and commodity pricing in each sector.

Following Kaleckian standard markup pricing, the price level of commodity i is determined by

$$p_i = z_i w q_i^{-1}, \tag{7}$$

where it is assumed that the level and growth rate of nominal wages across industries are unique in the country. That is, we assume $w_i = w$ and $\hat{w}_i = \hat{w}$.

The gross markup ratio of the ith industry z_i is endogenized in order to consider the relationship between international competition, production cost, and commodity price. In accordance with the formalization in Blecker (1998), the gross markup ratio is given by the following constant-elasticity function:

$$z_i = \bar{z}_i \left(\frac{w q_i^{-1}}{e w_F q_{Fi}^{-1}} \right)^{-\theta_i}, \tag{8}$$

where $\bar{z}_i$ is a positive constant, $wq_i^{-1} \equiv c_i$ indicates the ith industry's unit labor cost in the home country, and $w_F q_{Fi}^{-1} \equiv c_{Fi}$ indicates the ith industry's unit labor cost in the foreign country. $\theta_i \in [0, 1)$ is the elasticity of the gross markup that reflects the degree of international cost–pricing competition.

Eq. (8) implies that a rise in the unit labor cost of the ith industry in the home country relative to the other country leads to a price reduction by the firms in this sector. This reduction is brought about by cutting the markup ratio in order to keep the firms' products more competitive. For example, a large θ_i means firms in this sector i consider cost–pricing competition is severe. When there is a relative rise in the unit labor cost of the ith industry in the home country, firms in this sector significantly decrease their gross markup ratio and commodity price, so as to preserve their price competitiveness in the international trade. On the contrary, a small θ_i means firms in this sector consider cost–pricing competition to be not as severe. If θ_i is zero, markup pricing is independent of international competition across production costs, and firms in this sector behave almost monopolistically in international trade.

Dynamics of the price and gross markup ratio for commodity i are given by

$$\hat{p}_i = \hat{z}_i + \hat{w} - \hat{q}_i = \hat{z}_i + \hat{c}_i, \tag{9}$$

$$\hat{z}_i = -\theta_i(\hat{c}_i - \hat{e} - \hat{c}_{Fi}). \tag{10}$$

By using these two equations, we get the rate of change in the price of commodity i as follows:

$$\hat{p}_i = (1 - \theta_i)\hat{c}_i + \theta_i \hat{e} + \theta_i \hat{c}_{Fi}. \tag{11}$$

Eq. (11) means the rate of change in the price of commodity i is determined by the unit labor cost in the home country as well as the foreign country, and the nominal exchange rate, weighted by the degree of competition. As cost–pricing competition becomes severe, firms in the ith sector have to take the change in the unit labor cost in the foreign country and exchange rate fluctuations into consideration.

The existing literature on the BOPC model has assumed that productivity dynamics are given exogenously (Thirlwall (1979); Blecker (1998); Araujo and Lima (2007)). However, such formalization has serious problems, especially in examining sectoral differences in productivity that significantly impact the export and import performance. As an extension of the existing literature, we endogenize the dynamics of unit labor cost by using a Kaldorian perspective.

The unit labor cost in the ith sector is given by $c_i \equiv w/q_i$. This can be rewritten as follows:

$$c_i \equiv w\left(\frac{E_i}{Y_i}\right) = w\left(\frac{Y_D}{Y_i} \cdot \frac{E_i}{Y_D}\right). \tag{12}$$

We assume that income in the ith industry is proportional to the total income in the country ($Y_i = a_i Y_D$). Under this assumption, the dynamics of unit labor cost is then $\hat{c}_i = \hat{w} - \hat{q}_i$, where $\hat{q}_i \equiv \hat{Y}_D - \hat{E}_i$ is the growth rate of labor productivity. We suppose that it is endogenously determined by the Kaldor–Verdoorn mechanism. That is,

$$\hat{q}_i \equiv \hat{Y}_D - \hat{E}_i = \gamma_i \hat{Y}_D, \tag{13}$$

where $\gamma_i \in [0, 1]$ represents the Kaldor–Verdoorn coefficient that is specific to each industrial sector and is affected by the presence of dynamic increasing returns to scale, other externalities, and the size of the market. Using eq. (13), the rate of change in unit labor costs in the ith sector of the home country is obtained as follows:

$$\hat{c}_i = \hat{w} - \gamma_i \hat{Y}_D. \tag{14}$$

Following Blecker (1998), an analogous set of pricing equations is assumed to hold for the foreign country. The price level of commodity i produced by the foreign country is given by

$$p_{Fi} = z_{Fi} w_F q_{Fi}^{-1}. \tag{15}$$

The gross markup ratio of the set by the ith sector in the foreign country is assumed to be determined in a manner similar to that of the home country. It is given by

$$z_{Fi} = \overline{z}_{Fi}\left(\frac{w q_i^{-1}}{e w_F q_{Fi}^{-1}}\right)^{\theta_{Fi}}, \tag{16}$$

where $\overline{z}_{Fi}$ is a positive constant and $\theta_{Fi} \in [0, 1)$ is the elasticity of the gross markup of the ith sector in the foreign country. It represents the degree of international cost–pricing competition. The implication is the same as in eq. (8). That is, we regard a large value of θ_{Fi} as a case of intensive competition in this sector of the foreign country, and a small value implies monopolistic competition. In addition, we assume a joint restriction on the degree of competition that $\theta_i \in [0, 1)$, $\theta_{Fi} \in [0, 1)$ and $\theta_i + \theta_{Fi} \in [0, 1)$. This condition is necessary for ruling out the case that a rise in relative unit labor costs in the home country would cause extreme profit-squeeze behavior.

Finally, the unit labor cost in the ith sector of the foreign country is given by $c_{Fi} \equiv w_F/q_{Fi}$, which can be rewritten as follows:

$$c_{Fi} \equiv w_F\left(\frac{E_{Fi}}{Y_{Fi}}\right) = w\left(\frac{Y_F}{Y_{Fi}} \cdot \frac{E_{Fi}}{Y_F}\right), \tag{17}$$

where it is assumed that the income share of the ith industry in the total income is constant in the foreign country ($Y_{Fi} = a_{Fi}Y_F$). We assume that labor productivity is also affected by the Kaldor–Verdoorn effect in the ith foreign sector. In a similar manner to the above manipulation, the dynamics of labor productivity are given by $\hat{q}_{Fi} = \gamma_{Fi}\hat{Y}_F$, where $\gamma_{Fi} \in [0, 1]$ represents the Kaldor–Verdoorn effect that is specific to each industrial sector in the foreign country. Using eq. (17), the dynamics of unit labor costs in the ith sector of the home country are obtained as follows:

$$\hat{c}_{Fi} = \hat{w}_F - \gamma_{Fi}\hat{Y}_F. \tag{18}$$

By the same token, dynamic pricing in the ith industry in the foreign country is obtained as follows:

$$\hat{p}_{Fi} = (1 - \theta_{Fi})\hat{c}_{Fi} - \theta_{Fi}\hat{e} + \theta_{Fi}\hat{c}_i. \tag{19}$$

2.3 Terms of Trade and Growth Rates of Exports and Imports

Let the terms of trade (real exchange rate) in each sector be $r_i = ep_{Fi}/p_i$. The rate of change in this term is determined by the following equation:

$$\hat{r}_i = \hat{p}_{Fi} + \hat{e} - \hat{p}_i. \tag{20}$$

By substituting eqs. (11) and (19) into (20), we obtain the determinants of the dynamics of terms of trade in each sector as follows:

$$\hat{r}_i = -\phi_i(\hat{c}_i - \hat{e} - \hat{c}_{Fi}), \tag{21}$$

where $\phi_i \equiv (1 - \theta_i - \theta_{Fi}) \in (0, 1]$ summarizes the degree of international competition. If the ith industry in both countries is not in competition ($\theta_i = \theta_{Fi} = 0$), the value of ϕ_i is equal to unity. A change in the relative unit labor cost directly affects the evolution of terms of trade in the ith industry. On the contrary, if the ith industry in only one of these countries is subjected to severe competitive pressure (that is, $\theta_i \simeq 1$ or $\theta_{Fi} \simeq 1$), or both are subjected to equally strong competitive pressure (fifty–fifty), the value of ϕ_i is close to zero. In this case, a change in the relative unit labor cost does not affect the evolution of terms of trade for the ith industry.[5]

From the discussion above, the nominal value of exports in the ith sector is given by p_ix_i. Let us remark that the growth rate of x_i, given by eq.(4), depends on the terms of trade in the ith

[5]In the case of fifty–fifty competition, the terms of trade becomes independent of a change in the relative unit labor cost, because commodity prices in both countries change almost proportionally. For instance, let us consider $\theta_i \simeq 0.5$ and $\theta_{Fi} \simeq 0.5$ by using eqs. (11) and (19). A decrease in unit labor cost in the foreign country reduces $\hat{p}_{Fi}$ by nearly 0.5 points, whereas firms in the home country reduce their commodity price $\hat{p}_i$ by nearly 0.5 points owing to cost–price competition. Consequently, the relative price, p_i/p_{Fi} remains almost constant. However, such a change in the unit labor cost has a different impact on the markup ratio of the two countries. It raises the markup ratio of the foreign country and reduces that of the home country. In other words, in this international competition model, intensive competition affects the profitability of each industry.

industry and the expansion of foreign income. Using eqs. (4), (11), and (21), the growth rate of nominal exports of this sector is

$$\begin{aligned}\hat{p}_i + \hat{x}_i &= \hat{p}_i - \varepsilon_{Fi}(\hat{p}_i - \hat{e} - \hat{p}_{Fi}) + \eta_{Fi}\hat{Y}_F \\ &= \hat{c}_i \underbrace{(1 - \theta_i - \varepsilon_{Fi}\phi_i)}_{1-A_i} + (\hat{e} + \hat{c}_{Fi}) \underbrace{(\theta_i + \varepsilon_{Fi}\phi_i)}_{A_i} + \eta_{Fi}\hat{Y}_F. \end{aligned} \tag{22}$$

The changes in the unit labor costs in each economy's sector follow eqs. (14) and (18). By substituting these equations into eq. (22), we get the growth rate of nominal exports in the ith sector as follows:

$$\begin{aligned}\hat{p}_i + \hat{x}_i &= (1 - A_i)(\hat{w} - \gamma_i\hat{Y}_D) + A_i(\hat{e} + \hat{w}_F - \gamma_{Fi}\hat{Y}_F) + \eta_{Fi}\hat{Y}_F \\ &= [\hat{w} - (\hat{w} - \hat{w}_F - \hat{e})A_i] - (1 - A_i)\gamma_i\hat{Y}_D + (\eta_{Fi} - A_i\gamma_{Fi})\hat{Y}_F. \end{aligned} \tag{23}$$

The nominal value of imports adjusted by the exchange rate in the ith sector is defined by $p_{Fi}em_i$. In a manner similar to the above formalization, we can derive the growth rate of this value as follows:

$$\begin{aligned}\hat{p}_{Fi} + \hat{e} + \hat{m}_i &= \hat{p}_{Fi} + \hat{e} + \varepsilon_{Di}(\hat{p}_i - \hat{e} - \hat{p}_{Fi}) + \eta_{Di}\hat{Y}_D \\ &= \hat{c}_i \underbrace{(\theta_{Fi} + \varepsilon_{Di}\phi_i)}_{B_i} + (\hat{e} + \hat{c}_{Fi}) \underbrace{(1 - \theta_{Fi} - \varepsilon_{Di}\phi_i)}_{1-B_i} + \eta_{Di}\hat{Y}_D. \end{aligned} \tag{24}$$

Furthermore, using eqs. (14) and (18), we get

$$\begin{aligned}\hat{p}_{Fi} + \hat{e} + \hat{m}_i &= B_i(\hat{w} - \gamma_i\hat{Y}_D) + (1 - B_i)(\hat{e} + \hat{w}_F - \gamma_{Fi}\hat{Y}_F) + \eta_{Di}\hat{Y}_D \\ &= [\hat{w}_F + \hat{e} + (\hat{w} - \hat{w}_F - \hat{e})B_i] - (1 - B_i)\gamma_{Fi}\hat{Y}_F + (\eta_{Di} - B_i\gamma_i)\hat{Y}_D. \end{aligned} \tag{25}$$

The economic implications of the parameters A_i and B_i should be mentioned. $A_i \equiv \theta_i + \varepsilon_{Fi}\phi_i = \varepsilon_{Fi}(1 - \theta_{Fi}) + (1 - \varepsilon_{Fi})\theta_i \in (0, 1)$ is the degree of competition, weighted by the price elasticity of the demand for exports. Similarly, $B_i \equiv \theta_{Fi} + \varepsilon_{Di}\phi_i = \varepsilon_{Di}(1 - \theta_i) + (1 - \varepsilon_{Di})\theta_{Fi} \in (0, 1)$ is also the degree of competition, but weighted by the price elasticity of the demand for imports. In other words, A_i and B_i represent the complex effect of the cost–price competition structure on imports and exports in the ith sector. If the ith industries in both countries are not competing with each other, then $\theta_i = 0$ and $\theta_{Fi} = 0$ will hold. Consequently, $\phi_i = 1$, $A_i = \varepsilon_{Fi}$, and $B_i = \varepsilon_{Di}$. In addition, because we assumed that $\varepsilon_{Fi} + \varepsilon_{Di} > 1$, it is true that $1 - A_i - B_i = \phi_i(1 - \varepsilon_{Fi} - \varepsilon_{Di}) \leq 0$ holds. This is the Marshall–Lerner condition for each sector that takes international competition into consideration.

Finally, in order to compare the industrial performance with the macroeconomic consequences, it is useful to illustrate the impact of an increase in the home wage rate on the change in trade balance at the sectoral level. As we will show below, industrial performance and macroeconomic performance are not parallel.

Let tb_i be the difference between the growth rate of nominal exports and imports. This is given by

$$tb_i = \left\{[\hat{w} - (\hat{w} - \hat{w}_F - \hat{e})A_i] - (1 - A_i)\gamma_i \hat{Y}_D + (\eta_{Fi} - A_i\gamma_{Fi})\hat{Y}_F\right\}$$
$$- \left\{[\hat{w}_F + \hat{e} + (\hat{w} - \hat{w}_F - \hat{e})B_i] - (1 - B_i)\gamma_{Fi}\hat{Y}_F + (\eta_{Di} - B_i\gamma_i)\hat{Y}_D\right\}. \tag{26}$$

When tb_i is zero, the balance of payments in the ith sector is in equilibrium. Differentiating tb_i with respect to $\hat{w}$ gives the impacts of a rise in home wage on the balance of payments in the ith sector as follows:

$$\frac{\partial tb_i}{\partial \hat{w}} = 1 - A_i - B_i = \phi_i(1 - \varepsilon_{Fi} - \varepsilon_{Di}) \leq 0. \tag{27}$$

Thus, a rise in the home wage rate necessarily deteriorates the trade balance of each sector, because under the Marshall–Lerner condition, it has a larger impact on import growth than on export growth. In this case, *ceteris paribus*, a fall in the GDP growth rate in the home country is required to restrain the import growth, in order to recover the balance of payments of this sector. Thus, a high wage rate and high economic growth rate must always be in tradeoff under the BOPC condition at the sectoral level. On the contrary, this is not necessarily the case at the macroeconomic level.

3 Multi-Sector BOPC Growth with Sectoral Heterogeneity

3.1 Derivation of the Growth Rate and its Generality

This section demonstrates that our model has generality that includes the important aspects of Thirlwall (1979), Blecker (1998), and Araujo and Lima (2007) on the BOPC growth rate. A multi-sectoral BOPC growth condition is given by eq. (2), which means that the time rate of change of both total exports and total imports should be equal. By substituting eqs. (23) and (25) into eq. (2), this condition is rewritten as

$$\sum_{i=1}^{n} \nu_i \left\{\hat{w} - (\hat{w} - \hat{w}_F - \hat{e})A_i - (1 - A_i)\gamma_i \hat{Y}_D + (\eta_{Fi} - A_i\gamma_{Fi})\hat{Y}_F\right\} =$$
$$\sum_{i=1}^{n} \mu_i \left\{\hat{w}_F + \hat{e} + (\hat{w} - \hat{w}_F - \hat{e})B_i - (1 - B_i)\gamma_{Fi}\hat{Y}_F + (\eta_{Di} - B_i\gamma_i)\hat{Y}_D\right\}. \tag{28}$$

The LHS represents the growth rate of total exports and the RHS represents that of total imports. Hence, the difference of these two terms approximates the growth rate of net exports that is zero when the trade is balanced over time. After some algebraic manipulation for solving $\hat{Y}_D$, we get the economic growth rate of the multi-sectoral BOPC model under international competition,

$$\begin{aligned}\hat{Y}_D &= \frac{1}{\Theta}\Bigg\{\hat{Y}_F\left[\sum_{i=1}^{n}\nu_i\eta_{Fi} + \sum_{i=1}^{n}\gamma_{Fi}[\mu_i(1-B_i)-\nu_iA_i]\right] \\ &\quad + \hat{w}\sum_{i=1}^{n}[\nu_i(1-A_i)-\mu_iB_i] + (\hat{w}_F+\hat{e})\sum_{i=1}^{n}[\nu_iA_i-(1-B_i)\mu_i]\Bigg\} \\ &= \frac{1}{\Theta}\Bigg\{\sum_{i=1}^{n}\nu_i\eta_{Fi}\hat{Y}_F + \hat{w}\sum_{i=1}^{n}[\nu_i(1-A_i)-\mu_iB_i] + \sum_{i=1}^{n}(\hat{w}_F-\gamma_{Fi}\hat{Y}_F+\hat{e})[\nu_iA_i-(1-B_i)\mu_i]\Bigg\},\end{aligned} \tag{29}$$

where we define $\Theta_i \equiv \mu_i\eta_{Di} + \gamma_i[\nu_i(1-A_i)-\mu_iB_i]$ and

$$\Theta \equiv \sum_{i=1}^{n}\mu_i\eta_{Di} + \sum_{i=1}^{n}\gamma_i[\nu_i(1-A_i)-\mu_iB_i]. \tag{30}$$

We assume the value of Θ is positive. Araujo and Lima (2007) call their formalization of the BOPC growth rate as "the multi-sectoral Thirlwall's law" that is constructed in a pure labor economy on the basis of Pasinetti (1981, 1993)'s structural economic dynamics model. According to their multi-sectoral Thirlwall's law, the growth rate of per capita income in the home country is a result of changes in the composition of demand or the structure of production that come from changes in the share of each sector. Inspired by not only Araujo and Lima (2007) but also Blecker (1998), our multi-sectoral model further incorporates the role of international competition and the Kaldor–Verdoorn effect. It may be possible to say that the current model is more comprehensive than these existing representative BOPC models, because it can show the implications of Thirlwall (1979), Blecker (1998), and Araujo and Lima (2007) as special cases.[6] Let us deduce propositions from the current model, while paying attention to the relationship with these models.

Proposition 1. *If we aggregate the multi-sectoral model into one sector, our model results in Blecker (1998)'s model with endogenous productivity growth.*

Proof. In the case where there is only one sector, $i = n = 1$. Under this, we denote $\nu_i = \mu_i = 1$, $\gamma_i = \gamma$, $\gamma_{Fi} = \gamma_F$, $\eta_{Di} = \eta_D$, $\eta_{Fi} = \eta_F$, $\theta_i = \theta$, $\theta_{Fi} = \theta_F$, $\varepsilon_{Di} = \varepsilon_D$, and $\varepsilon_{Fi} = \varepsilon_F$. Then, eq. (29) becomes

$$\hat{Y}_D = \frac{(\hat{w}-\hat{w}_F-\hat{e})(1-A-B) + \hat{Y}_F[\eta_F - \gamma_F(1-A-B)]}{\eta_D + (1-A-B)\gamma}. \tag{31}$$

[6] Nell (2003) and Bagnai et al. (2012) also call their model "the generalized version of Thirlwall's Law." According to Nell (2003), Thirlwall's BOP constrained growth model is a specific case involving a bilateral trade relationship between one country and the "rest of the world." In their paper, the specific case is generalized into multilateral trade relations between an individual country and blocks of countries. Thus, their model is general in the sense that Thirlwall's law is extended to a multi-country setting. It shows that the BOPC growth is determined by not only income and relative price elasticity of bilateral imports and exports, but also by the bilateral import and export market shares. Although our model supposes trade between two countries, it incorporates trade between multiple sectors (commodities), the effect of international competition, and the role of the Kaldor–Verdoorn effect in each sector.

As in Blecker (1998), eq. (31) can be expressed in terms of relative wage dynamics.

$$\hat{w} - \hat{w}_F - \hat{e} = \frac{\eta_D \hat{Y}_D - \eta_F \hat{Y}_F}{(1 - \theta - \theta_F)(1 - \varepsilon_D - \varepsilon_F)} + \gamma \hat{Y}_D - \gamma_F \hat{Y}_F, \tag{32}$$

where $\gamma \hat{Y}_D - \gamma_F \hat{Y}_F$ is the endogenously determined productivity difference between two countries that is given exogenously in Blecker (1998). Eq.(32) is essentially the same as the formalization driven in Blecker (1998). □

Blecker (1998) explained that it is not generally possible for a country to achieve full-employment growth with balanced trade simultaneously, while maintaining a relatively high wage growth. Let us illustrate this explanation in a simple case, where the productivity difference is zero, by putting $\gamma = \gamma_F = 0$. In this case, the BOPC condition is given by

$$\hat{w} - \hat{w}_F - \hat{e} = \frac{\eta_D \hat{Y}_D - \eta_F \hat{Y}_F}{(1 - \theta - \theta_F)(1 - \varepsilon_D - \varepsilon_F)}. \tag{33}$$

Because we assumed $\varepsilon_D + \varepsilon_F > 1$, the sign of the denominator of eq. (33) is negative. Thus, there is a tradeoff between wage rate and economic growth, and the so-called wage-led growth is not possible in the BOPC context.[7] If the wage rate in the home country is relatively high as compared to that in the foreign country at a given growth rate $\hat{Y}_D$, it causes a trade deficit in the home country. In order to reduce the trade deficit, the home country has to either cut the nominal wage growth so as to increase price competitiveness, or reduce the economic growth rate so as to decrease imports. According to Blecker (1998), the former corresponds to the neoclassical strategy and the latter corresponds to an uncompetitive case in the post-Keynesian sense. The current model also presents Blecker's implications for international competitiveness.

Proposition 2. *The aggregate model leads to the original Thirlwall's law, if we assume that purchasing power parity (PPP) holds.*

Proof. If PPP holds, $\hat{r} = -\phi(\hat{c} - \hat{e} - \hat{c}_F) = 0$. By using eqs. (14) and (18), the changes in terms of trade remain constant as long as the following condition holds.

$$\hat{c} - \hat{c}_F - \hat{e} = (\hat{w} - \hat{w}_F - \hat{e}) - (\gamma \hat{Y}_D - \gamma_F \hat{Y}_F) = 0. \tag{34}$$

Therefore, for PPP to hold, it is necessary that $\hat{w} = \hat{w}_F + \hat{e}$ and $\gamma \hat{Y}_D = \gamma_F \hat{Y}_F = 0$. By substituting these conditions into eq. (32), we get

$$\hat{Y}_D = \frac{\eta_F}{\eta_D} \hat{Y}_F. \tag{35}$$

This is nothing but the original Thirlwall's law. □

[7] Wage-led growth normally refers to an increase in wage share that raises the economic growth rate by stimulating consumption and investment demand (Rowthorn (1981); Bhaduri and Marglin (1990)). In these Kaleckian growth models, the wage works as the source of demand as well as a cost for production. BOPC literature, however, focuses more on the role of wage as a production cost.

Eq. (35) means that long-run growth depends on the economic growth rate of the foreign country multiplied by the ratio of exports to imports income elasticity. If the home country aims to grow, it must be able to improve income elasticity that represents non-price competitiveness; for example, it must focus on providing quality commodities that satisfy consumers' preferences rather than on cost–price competition. Thirlwall (1979) also applied this equation to developed countries over the periods 1951–1973 (–1976) and found a correspondence between the actual growth rate and the growth rate predicted by Thirlwall's law.

Proposition 3. *The current multi-sectoral model generates a result that is close to Araujo and Lima (2007), if we assume no inflation in all sectors of both countries and PPP.*

Proof. When PPP holds at zero inflation rate in both countries, it is satisfied that $\hat{p}_i = \hat{p}_{Fi} = \hat{e} = 0$. Then, the rate of change in exports and imports in the ith sector is $\hat{p}_i + \hat{x}_i = \eta_{Fi}\hat{Y}_F$ and $\hat{p}_{Fi} + \hat{e}_i + \hat{m}_i = \eta_{Di}\hat{Y}_D$, respectively. By using these conditions and eq. (2), we get the following result:

$$\hat{Y}_D = \frac{\sum_{i=1}^{n} \nu_i \eta_{Fi}}{\sum_{i=1}^{n} \mu_i \eta_{Di}} \hat{Y}_F. \tag{36}$$

Like Araujo and Lima (2007), by summing over eq. (4) under zero inflation and using some algebraic manipulation, we get

$$\hat{Y}_F = \frac{\sum_{i=1}^{n} \hat{x}_i}{\sum_{i=1}^{n} \eta_{Fi}}. \tag{37}$$

Substituting eq. (37) into eq. (36), we obtain

$$\hat{Y}_D = \frac{\sum_{i=1}^{n} \nu_i \eta_{Fi}}{\sum_{i=1}^{n} \mu_i \eta_{Di} \sum_{i=1}^{n} \eta_{Fi}} \sum_{i=1}^{n} \hat{x}_i. \tag{38}$$

This result is close to what Araujo and Lima (2007) call "the multi-sectoral Thirlwall's law." □

Whereas Araujo and Lima (2007) derived "the multi-sectoral Thirlwall's law" that shows the growth rate of per capita income in labor coefficient terms in a pure labor economy, we derived eq. (38) in nominal and national income terms. In both cases, the structure of the economy—reflected by the ratio of the sum of income elasticities for exports and imports, weighted by the share of each industry—is important for economic growth. According to Araujo and Lima (2007), a major implication of the multi-sectoral Thirlwall's law is that changes in the composition of demand or structure of production, arising from changes in the share of each sector in aggregate exports or imports, are also important for economic growth per capita. Thirlwall (1979) explained that given the income elasticities of imports and exports, the growth rate of the foreign country is the only determinant of the home country's growth rate. In contrast, the novelty of Araujo and Lima (2007) is that the home country can still raise its growth rate if it can manage to change the sectoral composition of exports and imports. That is, the overall growth rate is also determined by the structural change that changes the composition of exports and imports such that the weighted income elasticity of exports grows faster than that of imports.

3.2 Comparative Statics: Importance of Sectoral Structure

The above propositions are already well known. However, one of the features of the current model is that it enables comprehensive understanding of the important aspects of the existing models. Furthermore, the current model has another novelty that is clearly explained by disaggregating the model. One of its most important implications is that even if an economic phenomenon holds true at the industry level, it may not do so at the macroeconomic level. The structure of the economy, reflected by the share of imports and exports of each sector, also plays an important role in understanding this implication. Below, we will explain this by way of comparative statics on the effects of changes in the nominal wage, the Kaldor–Verdoorn effect, and the condition of market competition in both countries.

Let us begin with the comparative statics on the changes in the nominal wage rates in both countries. In Blecker's model, an increase in the relative nominal wages necessarily leads to stagnation in the growth rate of the home country when the Marshall–Lerner condition is assured. In contrast, the results of the current model are not necessarily so. In this regard, we get the following proposition.

Proposition 4. *A change in the home and foreign wage rates has a contrasting effect on economic growth. When a rise in the home wage increases economic growth, a rise in the foreign wage decreases economic growth. When a rise in the home wage decreases economic growth, a rise in the foreign wage increases economic growth.*

Proof. By differentiating $\hat{Y}_D$ with regard to $\hat{w}$ and $\hat{w}_F$ in eq. (29), we get

$$\frac{\partial \hat{Y}_D}{\partial \hat{w}} = \frac{1}{\Theta}\left[1 - \sum_{i=1}^{n} \nu_i A_i - \sum_{i=1}^{n} \mu_i B_i\right], \tag{39}$$

$$\frac{\partial \hat{Y}_D}{\partial \hat{w}_F} = -\frac{1}{\Theta}\left[1 - \sum_{i=1}^{n} \nu_i A_i - \sum_{i=1}^{n} \mu_i B_i\right], \tag{40}$$

where $1 - \sum_{i=1}^{n} \nu_i A_i - \sum_{i=1}^{n} \mu_i B_i \lesseqgtr 0$. Therefore, when $1 > \sum_{i=1}^{n} \nu_i A_i + \sum_{i=1}^{n} \mu_i B_i$, a rise in the home wage increases economic growth, whereas a rise in the foreign wage decreases economic growth.

In contrast, when $1 < \sum_{i=1}^{n} \nu_i A_i + \sum_{i=1}^{n} \mu_i B_i$, a rise in the home wage decreases economic growth, whereas a rise in the foreign wage increases economic growth. □

Because changes in relative prices are mostly ignored in both the original and multi-sectoral Thirlwall's laws, this proposition is not obtained. In contrast, Blecker (1998)'s model focuses on the role of international competitiveness. However, as his model supposes an aggregated case, $1 - \sum_{i=1}^{n} \nu_i A_i - \sum_{i=1}^{n} \mu_i B_i$ results in $(1 - \theta - \theta_F)(1 - \varepsilon_D - \varepsilon_F)$ that is necessarily negative by the Marshall–Lerner condition. Hence, a change in the nominal wage in the home country always has a negative relationship with the economic growth rate in his model.

In contrast to the existing literature, a rise in the home wage rate does not necessarily decrease economic growth in the current model. Furthermore, it should be emphasized that there are differences between industrial and macroeconomic dynamics in international trade. To illustrate this, let us consider the impact of changes in the nominal wage rate on the nominal exports and imports of the ith sector. From eqs. (23) and (25), with a rise in the home wage rate, the growth rate of nominal exports increases by $(1 - A_i)$ and that of nominal imports by B_i. Consequently, the difference between the export and import growth rates is $(1 - A_i - B_i) = \phi_i(1 - \varepsilon_{Fi} - \varepsilon_{Di})$. This is necessarily non-positive under the Marshall–Lerner condition. Thus, a wage increase in the home country necessarily deteriorates the trade balance of each industrial sector.

However, eq. (28), states that at the macroeconomic level, the overall growth rate of nominal exports increases by $\sum_{i=1}^{n} \nu_i(1 - A_i)$ and that of nominal imports by $\sum_{i=1}^{n} \mu_i B_i$ as a consequence of a rise in wage rate. Thus, the difference between the export and import growth rates at the macroeconomic level is given by $1 - \sum_{i=1}^{n} \nu_i A_i - \sum_{i=1}^{n} \mu_i B_i$. This is not necessarily non-positive, even if we assume the Marshall–Lerner condition.

This is because the macroeconomic performance of exports and imports depends on the sum of price-competition elasticities, weighted by the market share of each sector. More intuitively, even if a rise in the wage rate decreases the net exports of an industry because the Marshall–Lerner condition works strongly there, when its import and export shares (that is, ν_i and μ_i) are small, the macroeconomic impact of the rise in wage rate is not so large as to decrease the overall growth rate of net exports. Similarly, even when the shares of exports and imports are large in another sector, but its Marshall–Lerner condition is not as strong ($\varepsilon_{Di} + \varepsilon_{Fi} \simeq 1$), the macroeconomic impact of the rise in wage rate is not so large as to decrease the overall growth rate of net exports. Thus, the overall performance of exports and imports is determined as the sum of these weighted average impacts. If it leads to a higher growth of exports than imports, the economic growth rate of the home country ($\hat{Y}_D$) must rise, so that the BOPC condition can be satisfied.

Because we introduced the Kaldor–Verdoorn effect into the multi-sectoral BOPC model as an important extension, we will examine its impact on macroeconomic growth. In this case too, changes in this effect have differing impacts on exports and imports at the industrial and macroeconomic levels.

Proposition 5. *A rise in the effect of dynamic increasing returns to scale (that is, a rise in the Verdoorn coefficient γ_i) in the home country decreases its economic growth rate, if a cut in the domestic unit labor cost decreases the degree of contribution of exports more than that of imports in that sector.*

Proof. By totally differentiating $\hat{Y}_D$ and γ_i with respect to eq. (29), and rearranging the result, we get

$$\frac{d\hat{Y}_D}{d\gamma_i} = \frac{\hat{Y}_D}{\Theta}(-\nu_i(1 - A_i) + \mu_i B_i). \tag{41}$$

If $-\nu_i(1 - A_i) + \mu_i B_i$ is negative, an increase in the Kaldor–Verdoorn effect has a negative impact on economic growth. In contrast, if $-\nu_i(1 - A_i) + \mu_i B_i$ is positive, an increase in this effect has a positive impact on economic growth. □

It is important to understand why such a mechanism works. By referring to eqs. (22) and (24), we can understand that when there is a cut in the unit labor cost in the *i*th sector of the home country, $-(1 - A_i)$ measures the change in the growth rate of nominal exports of the *i*th sector, whereas $-B_i$ measures that of nominal imports. In an aggregate model, $-\nu_i(1 - A_i) - (-\mu_i B_i) = -(1-A-B)$ is always positive. Thus, a decrease in the unit labor cost has less impact on the growth rate of nominal exports than that of nominal imports of the *i*th sector. As a result, this sector contributes to a rise in the economic growth rate. Therefore, in the aggregated model, the Kaldor–Verdoorn effects necessarily have a positive impact on the growth rate of the home country. However, these effects are weighted by each sector's share of exports and imports in the current multi-sectoral model. Even if a cut in the unit labor cost decreases imports more than exports in the *i*th sector of the home country, when the share of imports of the *i*th sector (μ_i) is small and its exports share (ν_i) is large, its impact on the exports and imports at the macroeconomic level may be reversed. In the disaggregated model, an increase in the Kaldor–Verdoorn effect may have a negative impact on the economic growth of the home country.

Corollary 1. *A rise in the effect of dynamic increasing returns to scale (that is, a rise in the Verdoorn coefficient γ_{Fi}) in the ith sector of the foreign country increases the economic growth rate of the home country, if a cut in the foreign unit labor cost decreases the degree of contribution of imports more than that of exports in that sector.*

Proof. By totally differentiating $\hat{Y}_D$ and γ_{Fi} with respect to eq. (29), and rearranging the result, we get

$$\frac{d\hat{Y}_D}{d\gamma_{Fi}} = \frac{\hat{Y}_F}{\Theta}(-\nu_i A_i + \mu_i(1 - B_i)). \tag{42}$$

If $-\nu_i A_i + \mu_i(1 - B_i)$ is negative, an increase in the Kaldor–Verdoorn effect in the foreign country has a negative impact on the home country's economic growth. In contrast, if $-\nu_i A_i + \mu_i(1 - B_i)$ is positive, an increase in this effect has a positive impact on the home country's economic growth. □

$-A_i$ measures the change in growth rate of nominal exports of the ith sector, whereas $-(1-B_i)$ measures that of nominal imports, when there is a cut in the unit labor cost in the ith sector of the foreign country. In the aggregate model, a rise in the effect of dynamic increasing returns to scale in the foreign country necessarily decreases the economic growth rate of the home country, because export growth decreases more than import growth by the Marshall–Lerner condition. In contrast, it is not necessarily so in this multi-sectoral version, because this condition is weighted by the sectoral composition. Even if there is a cut in the unit labor cost in the ith sector of the foreign country by the Kaldor–Verdoorn effect, when the share of imports of the ith sector (μ_i) is large and its exports share (ν_i) is small, nominal import growth decreases more than export growth at the macroeconomic level. This leads to a trade surplus at the macroeconomic level. In order to recover the BOP condition, the economic growth rate of the home country must rise. Thus, this exercise also shows the importance of distinguishing the industrial and macroeconomic impacts of changes in productivity growth.

According to the generalized Thirlwall's law, foreign income is one of the important sources of effective demand that contributes to a rise in economic growth. However, in the current extended model, the effect of a rise in foreign income is more complicated.

Proposition 6. *The effect of foreign economic growth is both positive and negative, depending on the volume and cost effects on the nominal export and import growth.*

Proof. By differentiating $\hat{Y}_D$ with respect to $\hat{Y}_F$ in eq. (29), we get

$$\frac{\partial \hat{Y}_D}{\partial \hat{Y}_F} = \frac{1}{\Theta}\left[\sum_{i=1}^{n} \nu_i \eta_{Fi} + \sum_{i=1}^{n} \gamma_{Fi}[-\nu_i A_i + \mu_i(1 - B_i)]\right]. \tag{43}$$

As we showed in eq. (42), the sign of $-\nu_i A_i + \mu_i(1-B_i)$ can be both negative and positive. If it is positive for all i, an expansion of the foreign economy necessarily contributes to the economic growth of the home country. However, if the sum of $\gamma_{Fi}[-\nu_i A_i + \mu_i(1 - B_i)]$ is negative and it offsets the first term in the RHS of eq.(43), an expansion of the foreign economy leads to a low rate of economic growth in the home country. □

The first term on the RHS in eq. (43) represents that a rise in $\hat{Y}_F$ leads to an increase in foreign demand. Therefore, we call it the volume effect. As the multi-sectoral Thirlwall's law has indicated (Araujo and Lima (2007)), this effect works strongly when the sectoral income elasticity of demand for imports is lower and that of demand for exports is higher. In addition to the volume effect, a rise in $\hat{Y}_F$ also has a positive impact on productivity growth and reduces the unit labor cost in the foreign country. This causes the relative price to change. This impact on both nominal export and import growth at the macroeconomic level is represented by the second term in eq. (43); therefore, we call it the cost effect. As we showed in Corollary 1, the cost effect on the growth rate of the home country works negatively, when it leads to a higher decrease in nominal import growth than export growth at the macroeconomic level. If this impact is strong enough to offset the volume effect, there is stagnation in the home country's economic growth.[8]

Inspired by Blecker (1998), another novel feature of the current paper is introducing competition between two countries in the multi-sectoral BOPC model. The aggregated BOPC model shows that increased pressure on price competition decreases the growth rate of nominal imports more than that of nominal exports, and consequently, a trade surplus is generated. Under the BOPC condition, this surplus is adjusted by a rise in the growth rate of the home country, *ceteris paribus*, at a given relative rate of home wages. However, as eq. (33) also implies, there is a tradeoff between economic growth and the relative rate of home wage increase. This tradeoff is more rigid if the competitive pressure for cost–pricing behavior (that is, a rise in θ) is more severe. In this case, even if there is a cut in the relative rate of home wages, its impact on the economic growth rate becomes limited.

In contrast, as we showed above, a rise in the home wage rate does not necessarily decrease economic growth in our multi-sectoral model, because its impacts on economic growth depend on the weighted-average elasticities of market structure–price–net exports (A_i and B_i) and the composition of export and import shares (μ_i and ν_i, respectively). Therefore, our investigation gives a new implication about the effect of changes in the market competition structure on growth and income distribution. In order to compare with the essence of Blecker (1998), let us examine this impact in a simple case where productivity growth is zero ($\gamma_i = 0$ and $\gamma_{Fi} = 0$). Then, we get the following proposition.

Proposition 7. *The impact of an increase in international competition in the ith sector of the home country on economic growth depends on its share of exports and imports.*

Proof. Suppose the Verdoorn coefficient is zero as a simple case. Let TB be the difference between the growth rate of nominal exports and imports, given by

$$TB = \sum_{i=1}^{n} \nu_i \left\{\hat{w} - (\hat{w} - \hat{w}_F - \hat{e})A_i + \eta_{Fi}\hat{Y}_F\right\} - \sum_{i=1}^{n} \mu_i \left\{\hat{w}_F + \hat{e} + (\hat{w} - \hat{w}_F - \hat{e})B_i + \eta_{Di}\hat{Y}_D\right\}. \quad (44)$$

By differentiating TB with respect to θ_i, we get

$$\frac{\partial TB}{\partial \theta_i} = -(\hat{w} - \hat{w}_F - \hat{e})[\nu_i(1 - \varepsilon_{Fi}) - \mu_i\varepsilon_{Di}]. \quad (45)$$

When the sign of $\hat{w} - \hat{w}_F - \hat{e}$ is positive, the sign of $\partial TB/\partial\theta_i$ depends on that of $\nu_i(1 - \varepsilon_{Fi}) - \mu_i\varepsilon_{Di}$. Although the Marshall–Lerner condition stipulates $\varepsilon_{Fi} + \varepsilon_{Di} > 1$, eq. (45) also includes the

[8]In a special case in which the Kaldor–Verdoorn effect is perfect in each sector of the foreign country ($\gamma_{Fi} = 1$), substituting this into eq. (43) shows that the effect of a rise in the foreign economic growth rate is expressed by the sum of the impacts of the multi-sector Thirlwall's law and the increase in the foreign wage.

share of exports and imports. Therefore, the sign of $\nu_i(1-\varepsilon_{Fi})-\mu_i\varepsilon_{Di}$ may be either positive or negative. For example, consider that the export share of the ith sector is relatively large and its import share is small. In this case, the sign of $\nu_i(1-\varepsilon_{Fi})-\mu_i\varepsilon_{Di}$ can be positive and increasing price competition in this sector would have negative impacts on the balance of payments of the home country. Consequently, the growth rate of the home country must fall in order to recover the BOPC condition. □

Proposition 7 presents an implication that is in sharp contrast to Blecker (1998). In Blecker (1998), both ν_i and μ_i are unity and the Marshall–Lerner condition results in nominal imports falling more than nominal exports because of severe competition (that is, $TB>0$). This disequilibrium in trade balance is reduced by a rise in the economic growth rate of the home country. In contrast, Proposition 7 reveals that if price competition becomes more severe in sector whose export share is relatively large, the economy will stagnate, because such competition has negative impacts on the macroeconomic as well as the sectoral trade balance. In the current model, even if the Marshall–Lerner condition is satisfied at the sectoral level, the share of exports and imports may reverse the impact of a change in the degree of competition on the growth rates of exports and imports.

By the same token, the impact of an increase in international competition in the ith sector of the foreign country on the home country's economic growth can be summarized in the following corollary.

Corollary 2. *The impact of an increase in international competition in the ith sector of the foreign country on the home country's economic growth also depends on the share of exports and imports.*

Proof. Suppose the Verdoorn coefficient is zero as a simple case. By differentiating TB in eq. (44) with respect to θ_{Fi}, we get

$$\frac{\partial TB}{\partial \theta_{Fi}}=-(\hat{w}-\hat{w}_F-\hat{e})[-\nu_i\varepsilon_{Fi}+\mu_i(1-\varepsilon_{Di})]. \tag{46}$$

When the sign of $\hat{w}-\hat{w}_F-\hat{e}$ is positive, the sign of $\partial TB/\partial\theta_{Fi}$ depends on that of $-\nu_i\varepsilon_{Fi}+\mu_i(1-\varepsilon_{Di})$. Eq. (46) also includes the share of exports and imports. Therefore, the sign of $-\nu_i\varepsilon_{Fi}+\mu_i(1-\varepsilon_{Di})$ may be either positive or negative. □

Given a relative rate of home wage increase, more intensive international competition in the ith sector of the foreign country necessarily has a positive impact on the economic growth of the home country in Blecker (1998), by the Marshall–Lerner condition. However, if we disaggregate the model and focus on the sectoral composition of imports and exports, another possibility is discovered. If the import share of the ith sector is large, whereas the export share is small, the sign of $-\nu_i\varepsilon_{Fi}+\mu_i(1-\varepsilon_{Di})$ can be positive. Then, increasing price competition in this sector in the foreign country has negative impacts on the balance of payments of the home country. Consequently, the growth rate of the home country must fall in order to recover the BOPC condition.

In sum, first, when the home country has a large share of exports and a small share of imports for commodity i, if firms in the ith sector of the home country become more sensitive to the relative unit labor cost in their pricing, owing to more intensive international price competition in this sector (that is, a rise in θ_i), the balance of payments of the home country is negatively impacted; therefore, the growth rate of the home country decreases. Under the same structure, the same impact on the ith sector of the foreign country (that is, a rise in θ_{Fi}) leads to a current account surplus in the home country and its growth rate increases. Second, when the home country has a large share of imports and a small share of exports, if firms in the ith sector of the home country become more sensitive to the relative unit labor cost in their pricing, owing to more intensive international price competition in this sector, the balance of payments of the home country are positively impacted; therefore, the growth rate of the home country rises. Under the same structure, however, the same impact on the ith sector of the foreign country leads to a current account deficit, and thus, the growth rate of the home country falls.

3.3 Summary of the Results

We summarize the results obtained in the preceding sections. Table 1 summarizes the relationship between the representative existing BOPC models and the current model.

Table 1: The structure of model

The current model eq. (29) is close to	If we add assumptions that
Original Thirlwall's law	All sectors are aggregated into one sector and PPP holds
Blecker model	All sectors are aggregated into one sector and labor productivity is exogenous
Araujo and Lima model	PPP holds at each sector under no inflation in both countries

The structure of the current model comprehensively includes the main characteristics of Thirlwall (1979), Blecker (1998), and Araujo and Lima (2007) and it can easily be reduced to them by adding assumptions. In other words, the current model is more generalized and can consider their analytical scope as special cases.

Introducing sectoral heterogeneity in production costs and the degree of price–competition into a multi-sectoral model is a novelty of this study. By doing so, this study attempts to contribute to identifying the exact mechanism that determines BOPC growth, beyond what has been shown by Thirlwall (1979), Blecker (1998), and Araujo and Lima (2007). Table 2 summarizes the main results obtained by the comparative statics that have not been conducted in sufficient detail in the above literature. When the export share of the ith industry is large and its import share

Table 2: The impacts on the growth rates of home counrty

	If the the sum of price-competition elasticities weighted by market share is	
A rise in:	less than unity	more than unity
$\hat{w}$	positive	nagative
$\hat{w}_F$	negative	positive
	When the value of	
A rise in:	ν_i is large and μ_i is small	ν_i is small and μ_i is large
γ_i	negative	positive
γ_{Fi}	negative	positive
θ_i	negative	positive
θ_{Fi}	positive	negative

is small, a rise in the Verdoorn coefficient in both countries has a negative effect on the growth rate of the home country. This is because it deteriorates the trade balance of the home country at the macroeconomic level, and as a result, the home country's growth rate must be lowered in order to satisfy the BOPC condition. However, their impacts are reversed, when the export share of the *i*th industry is small but its import share is large.

In addition, a rise in θ_i—an increase in price competition pressure on the *i*th sector in the home country—also has a negative impact on the balance of payments, and as a result, the growth rate of the home country decreases. This occurs because the export share of the *i*th industry is large and its import share is small. In the same structure of production, a rise in θ_{Fi} has a positive impact on the balance of payments of the home country, and the home country's growth rate increases.

The impact of a rise in the foreign economic growth rate does not fit into this table. This is because it depends on both the volume and cost effects, as shown in the first and second terms, respectively, on the RHS of eq.(43). In the standard multi-sectoral BOPC model, its effect is positive because only the volume effect works. However, if we introduce the Kaldor–Verdoorn effect, the story becomes complicated. A boom in the foreign economy also has a positive impact on productivity growth and reduces the unit labor cost in the foreign country. When the nominal import growth decreases more than export growth at the macroeconomic level because of this cost effect, and this impact is strong enough to offset the volume effect, economic growth stagnates.

Last but not the least, one of the most important results obtained in this paper is that a wage increase in the home country has a different impact on the trade balance at the industrial and macroeconomic levels. In the preceding literature, a rise in the home wage rate increases the production cost and deteriorates the terms of trade. As a result, the economy stagnates. Thus, it is in the interests of each industry and the national economy to improve the trade balance. However, the current model has a different implication, especially when the sum of price-competition elasticity for exports and imports weighted by their market share is less than unity. In this case, although a rise in the home wage still deteriorates the trade balance in each sector, as is shown in eq. (27), it has a favorable effect on the macroeconomic growth of the home country. This is because the national economy is composed of heterogeneous sectors that have different shares of exports and imports. Under such an economic structure, even though a wage increase could have a potential positive effect on macroeconomic growth, it may not be realized in the economy. This is because each industry has reason to oppose the increase in the home wage rate that may shrink its export share and deteriorate the sectoral trade balance. The current model, thus, implies the structural difficulty of increasing wages in an economy composed of heterogeneous sectors.

4 Conclusion

The importance of the multi-sectoral BOPC model lies in the finding that despite the absence of international growth, an economy can still grow at a higher rate by bringing about structural changes. This study has built a multi-sectoral BOPC model that incorporates some structural heterogeneity, such as differences in labor productivity, price competition, export and import shares, and the quality of commodities between sectors and countries. On the basis of the model, this study investigated the effects of different shocks on the economic growth rate of the home country.

It is shown that the multi-sectoral Thirlwall's law developed in the current paper generates more comprehensive results than Thirlwall (1979), Blecker (1998), and Araujo and Lima (2007). The current model contains their properties and reproduces their implications. First, similar to the original Thirlwall (1979)'s law, one country's growth rate is directly related to other countries' growth rates and the income elasticity of demand for exports, whereas it is inversely related to the income elasticity of demand for imports. Second, similar to the original Blecker (1998) model, there is a tradeoff between high growth rate and wage rate. This tradeoff becomes severer when cost–price competition gets intensive; this is reflected in a rise in the degree of exchange rate pass-through. Third, similar to the original Araujo and Lima (2007) model, changes in the composition of demand or structure of production that are manifested as changes in the export and import shares of each sector are also important for economic growth.

In addition to these results, this extension of the model also generates some novel implications for industrial structure, cost-price competition, and the determinants of economic growth rates in the BOPC context. First, in contrast to Blecker (1998)'s model, a rise in the home wage does not necessarily decrease economic growth. It depends on the sum of price-competition elasticities, weighted by export and import shares. Second, the model sheds more light on the relationship between industrial structure, international competition, and economic growth than Araujo and Lima (2007)'s model. This is shown by means of comparative statics on the effects of changes in the nominal wage, the Kaldor–Verdoorn effect, and the degree of market competition in both countries. For example, suppose that unit labor cost in the home country is higher than in other countries. A more competitive pressure on the ith sector of the home country decreases the economic growth rate, when its export share is large and import share is small. Third, our theoretical investigation also shows that there are differences between microeconomic and macroeconomic phenomena. The results newly obtained in this paper show the importance of the fallacy of composition that the Keynesian theory has emphasized so far. As we explained in Proposition 4, for example, an increase in the wage rate has different impacts on the exports and imports at the industrial and macroeconomic levels. At the industrial level, a rise in the wage rate of the home country necessarily deteriorates each sector's trade balance, owing to the Marshall–Lerner condition. In contrast, its impact on exports and imports at the macroeconomic level does not necessarily generate the same result. This is because the impact of the home wage increase on macroeconomic exports and imports also depends on the industrial structure that is reflected in the share of exports and imports. When the economy is composed of an industrial structure that has a small share of exports and imports, an increase in the home wage does not always lead to a decrease in the rate of economic growth. Furthermore, the industrial structure of the economy also matters for how changes in the growth of the foreign economy, the Kaldor–Verdoorn effect, and the market structure in each sector impact macroeconomic growth in the home country.

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