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The change of correlation structure across industries: an analysis in the regime-switching framework*

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Abstract

This paper studies changes of correlation structure across industries in the United States equities market in the regime-switching framework. To capture the irreversible structural change and to separate it from the recurring booming-recession switches, we introduce two Markov chains. We empirically identify the timing of the structural change and confirm that, after the change, the asset return correlations across industries increased. Moreover, the impact of the structural change on correlations is stronger in a recession period than in a booming period.

Key words: Stock market returns; regime-switching models; structural change; correlation risk; the EM algorithm; financial crisis.

JEL Classification: C22, G10

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1 Introduction

The global financial crisis in 2008 shows the importance of understanding of correlations of financial instruments. During the catastrophic shock, prices of financial assets are reported to have moved together; correlations of asset returns, including those traditionally considered weak, rapidly increased. This phenomenon causes significant impacts on derivatives prices and hence on risk management. Accordingly, as part of post-crisis study, there has been increasing number of literature about measuring and managing correlations risk. To name just a few, Driessen et al. [2009] study whether exposure to marketwide correlation shocks affects expected option returns, and Buraschi et al. [2010] propose a new optimal portfolio choice model by allowing correlation across industries to be stochastic. In the former paper, they find that assets sensitive to higher marketwide correlations earn negative excess returns.

In this paper, we attempt to answer the fundamental and crucial question that when the structural change in asset return correlations occurred and to what extent this change has made impacts on means and covariances of asset returns and on investors' portfolio choice problems. To obtain firm results, we restrict our scope to the U.S. domestic stock market and concentrate on the correlation structure across industries. For this purpose, we propose a regime-switching model with two (mutually independent) Markov chains; one reversible and the other irreversible. We introduce the irreversible chain in an attempt to capture certain change that could have occurred in the market. The reversible chain corresponds, as we shall show in Section 3.3.1, to booming and recession periods, a consistent result with Ang and Bekaert [2002] and Okimoto [2008].

We show that (1) this reversible chain alone cannot explain the change in correlation structure well, (2) the irreversible change is estimated to have occurred in May 2003, which roughly coincides with the period when a housing market bubble started partly due to the monetary relaxation policy, and (3) there is a clear evidence that the correlation across industries increased after that period. Moreover, the impact on correlation is stronger in a recession period than in a booming period. This is consistent with the above mentioned phenomenon that the correlations among financial assets increased after the crisis; however, we estimate that the increase in correlation coefficients may have occurred well before the crisis.

We review some of the literature about changes in correlation structure and about Markov switching models. In particular, there exist a large number of studies on the correlation in the international stock markets. For example, Berben and Jensen [2005] investigate the correlations of international stock markets by fitting smooth transition GARCH models to weekly return data. They report correlations among the German, UK and US stock markets have doubled in the period of 1980 $\sim$ 2000. However, the correlations between Japan and other market did not change significantly in this period. Other papers include Karolyi and Stulz [1996], Ramchaned and Susmel [1998], Das and Uppal [2004], and Bekaert et al. [2009].

The Markov regime switching model was introduced by Hamilton [1989] for capturing sudden changes in economic time series data. The advantage of using the Markov regime switching model includes that it may capture some moment properties, frequently observed in the real markets, such as auto-correlation, volatility clustering, asymmetric correlation, non-normal skewness and kurtosis (e.g., Timmermann [2000]). In the international markets, Ang and Bekaert [2002] estimate the parameters of the reversible Markov switching model assuming the jointly normal returns in the period of 1972 $\sim$ 1997. They claim that the multivariate Markov regime switching model succeeds in replicating the asymmetric correlation pattern reported by Longin and Solnik [2001]. However, they mention to fail to reject the null hypothesis that the correlations among regimes are equal. Pettenuzzo and Timmermann [2011] apply an irreversible Markov switching to return prediction models and study the U.S. financial market. They identify random changes or "breaks" to the parameters of the prediction model that uses dividend yield or short term interest rate.

This paper is one of the few studies that incorporate both reversible and irreversible Markov chains and identify economic implications that each chain contains. In particular, the enriched model allows us to identify the timing when the correlations across the industries increased. The paper is organized as follows: Section 2 introduces our Markov switching model. Section 3 presents estimation methodology and results. The latter part consists of the following: First we test if the reversible Markov chain captures a stationary component of asset returns, then estimate whether and when the irreversible structural change occurred. Given the estimated timing of the structural change, we compare means and covariances before and after the structural change. In section 4, we examine the regime switching effects through the Monte Carlo simulations. Section 5 concludes, and some technicalities about the estimation procedure, model specification and auxiliary tables are provided in the Appendix.

2 The Model

We consider a discrete-time, finite horizon regime switching model, where t varies from 0 to T with $T > 0$ fixed. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space which hosts a coupled Markov chain $Z = (Z_t)_{t=0}^T$ that explains regime switch and some stochastic process that drives the log-return process $Y := (Y_t)_{t=0}^T$. The variable Z_t indicates a regime at time t and we assume that an evolution of regime Z is described by a couple of two independent Markov chains $(S_t)_{t=0}^T$ and $(D_t)_{t=0}^T$, so that we denote $Z = (S, D)$.

The first component $(S_t)_{t=0}^T$ captures a reversible transition in the regimes of asset returns. We assume that S is a stationary Markov chain with two regimes $\{0, 1\}$ and the time-invariant transition probability matrix

$$\mathcal{P} \quad := \quad \begin{pmatrix} \mathbb{P}(S_{t+1} = 0|S_t = 0) & \mathbb{P}(S_{t+1} = 1|S_t = 0) \\ \mathbb{P}(S_{t+1} = 0|S_t = 1) & \mathbb{P}(S_{t+1} = 1|S_t = 1) \end{pmatrix} = \begin{pmatrix} p_{00} & 1 - p_{00} \\ 1 - p_{11} & p_{11} \end{pmatrix}$$

where p_{00} and p_{11} are constant parameters to be estimated.

The second component $(D_t)_{t=0}^T$ captures an irreversible structural change in the asset returns. In particular, we assume that the structural change can happen only once. This can be modeled via a Markov chain consisting of two regimes $D_t = 0$ or $D_t = 1$ for all $t \geq 0$ where $D_t = 0$ represents the regime before a structural change happens and $D_t = 1$ is after the change and is absorbing. Accordingly, the transition probability matrix of D is time invariant and defined by

$$\mathcal{Q} := \begin{pmatrix} \mathbb{P}(D_{t+1} = 0|D_t = 0) & \mathbb{P}(D_{t+1} = 1|D_t = 0) \\ \mathbb{P}(D_{t+1} = 0|D_t = 1) & \mathbb{P}(D_{t+1} = 1|D_t = 1) \end{pmatrix} = \begin{pmatrix} q_{00} & 1 - q_{00} \\ 0 & 1 \end{pmatrix}$$

where q_{00} is a constant parameter to be estimated. By construction, $D_0 = 0$ and D_t cannot return to regime 0 once D_t moves, at some time t, to regime 1. Furthermore, for a comparison purpose, we also consider the model without any structural change, which is done by setting $q_{00} = 1$. Note that in this case the model reduces to a conventional regime-switching model with Markov chain S.

The process Y is a vector of log returns of N-industry stock indices defined by

$$Y_t = \mu_{Z_t} + \Sigma_{Z_t}^{1/2} e_t,$$

where, for each $t \geq 0$, μ_{Z_t} is the N-dimensional vector, $\Sigma_{Z_t}^{1/2}$ is the $N \times N$ matrix which satisfies $\Sigma_{Z_t}^{1/2}(\Sigma_{Z_t}^{1/2})^\top = \Sigma_{Z_t}$, and $(e_t)_{t=0}^T$ is an N-dimensional i.i.d standard normal vector.

The mean and covariance of process $(Y_t)_{t=1}^T$ are affected by the regime described by $(Z_t)_{t=1}^T$ in the following way:

$$\mu_{Z_t} = \sum_{s=0}^{1}\sum_{d=0}^{1} \mathbf{1}\{S_t = s, D_t = d\}\mu_{s,d},\ \Sigma_{Z_t} = \sum_{s=0}^{1}\sum_{d=0}^{1} \mathbf{1}\{S_t = s, D_t = d\}\Sigma_{s,d}$$

where $\mu_{s,d}$ $(s = 0, 1, d = 0, 1)$ is a constant N-dimensional vector and $\Sigma_{s,d}$ $(s = 0, 1, d = 0, 1)$ is an $N \times N$ constant positive definite symmetric matrix.

By this formulation, the conditional mean and covariance of Y_t given Z_t are

$$E(Y_t|Z_t) = \mu_{Z_t}, \quad \mathrm{Var}(Y_t|Z_t) = \Sigma_{Z_t}.$$

Accordingly, the distribution of Y_t given Z_t is

$$Y_t|Z_t \sim N(\mu_{Z_t}, \Sigma_{Z_t})$$

where $N(A, B)$ is the normal distribution with mean A and covariance B.

The standard assumptions about dependence structure of the model are as follows:

Assumption 2.1 *Let $\mathcal{F}_t := \sigma\{Y_s : 0 \le s \le t\}$ be the σ-algebra generated by the log-return process Y.*

(i) S_u and D_v are independent for any pair of u and v; $0 \le u, v \le T$.

(ii) For any $t \ge 0$, given S_t, S_{t+1} is independent of $\mathcal{F}_t$, and given D_t, D_{t+1} is independent of $\mathcal{F}_t$.

We shall employ these assumptions in the sequel.

3 Estimation

3.1 Data

We use the monthly log returns (price returns) of Standard & Poor's 500 sector indices classified into ten industries, following the Global Industry Classification Standard (GICS). The log-returns covered the period from February 1995 to December 2011, a total of 203 data points. All the data are obtained from the "Thomson Reuters Datastream". Table.1 shows the acronyms of the industries.

3.2 Method

By using the log return process Y of S&P 500 sector indices, we estimate the two unknown components of the model. The first component is an evolution of the regimes, where we can estimate only the probability of being at particular regimes of $Z = (S, D)$. The second component is the mean vector μ_{Z_t} and covariance matrix Σ_{Z_t} that are distinct for each regime. We jointly estimate these components based on an iteration method. In particular, we use the Expectation-Maximization (EM) algorithm as introduced by Dempster, Lairf and Rubin [1977] after suitably making it fit to our framework. By construction, the initial state is $Z_0 = (0, 0)$ or $Z_0 = (1, 0)$, and we denote by ρ_s the initial marginal probability of being at regime s, that is $\mathbb{P}(S_0 = s)$.

Recall that $(\mathcal{F}_t)_{t=0}^T$ is the information of observable process $(Y_t)_{t=0}^T$ up to time t and let $\Theta^{(k)}$ be the candidate of parameters $\mathcal{P}, \mathcal{Q}, \mu_s, (s = 0, 1)$, $\Sigma_s, (s = 0, 1)$, and $\rho_s, (s = 0, 1)$ in

Table. 1: The Industry Codes

	Sector Name
ENE	Energy
MAT	Materials
IND	Industrials
CD	Consumer Discretionary
CS	Consumer Staples
HC	Health Care
FIN	Financials
IT	Information and Technology
TEL	Telecom services
U	Utilities

the kth iteration of the EM algorithm. Following Hamilton [1990], the updating formulae for our model parameters in the $(k+1)$ th iteration are

$$\begin{aligned}
\mu_{s,d}^{(k+1)} &= \frac{\sum_{t=0}^{T} Y_t \mathbb{P}(S_t = s, D_t = d | \mathcal{F}_T; \Theta^{(k)})}{\sum_{t=0}^{T} \mathbb{P}(S_t = s, D_t = d | \mathcal{F}_T; \Theta^{(k)})} \\
\Sigma_{s,d}^{(k+1)} &= \frac{\sum_{t=0}^{T} (Y_t - \mu_{s,d}^{(k+1)})(Y_t - \mu_{s,d}^{(k+1)})^\top \mathbb{P}(S_t = s, D_t = d | \mathcal{F}_T; \Theta^{(k)})}{\sum_{t=0}^{T} \mathbb{P}(S_t = s, D_t = d | \mathcal{F}_T; \Theta^{(k)})} \\
p_{ss}^{(k+1)} &= \frac{\sum_{t=1}^{T} \mathbb{P}(S_t = s, S_{t-1} = s | \mathcal{F}_T; \Theta^{(k)})}{\sum_{t=1}^{T} \mathbb{P}(S_{t-1} = s | \mathcal{F}_T; \Theta^{(k)})} \\
q_{00}^{(k+1)} &= \frac{\sum_{t=1}^{T} \mathbb{P}(D_t = 0, D_{t-1} = 0 | \mathcal{F}_T; \Theta^{(k)})}{\sum_{t=1}^{T} \mathbb{P}(D_{t-1} = 0 | \mathcal{F}_T; \Theta^{(k)})} \\
\rho_s^{(k+1)} &= \mathbb{P}(S_0 = s) = \mathbb{P}(S_0 = s | \mathcal{F}_T; \Theta^{(k)}) \quad s = 0, 1, \ d = 0, 1,
\end{aligned}$$

where $\mathbb{P}(\cdot \mid \cdot , \Theta^{(k)})$ is the probability calculated under the parameter set $\Theta^{(k)}$. Hamilton [1990] shows if we repeat updating the parameters using these formulae, then the sequence of the parameters obtained by this algorithm converges, as $k \to \infty$, to the maximum likelihood estimators. The EM algorithm is based on the following probabilities of being at a particular regime:

$$\begin{aligned}
&\mathbb{P}(S_t = s, D_t = d | \mathcal{F}_T; \Theta^{(k)}) \\
&\mathbb{P}(S_t = s, S_{t-1} = \hat{s} | \mathcal{F}_T; \Theta^{(k)}) \\
&\mathbb{P}(D_t = d, D_{t-1} = \hat{d} | \mathcal{F}_T; \Theta^{(k)}) \\
&\mathbb{P}(S_0 = s | \mathcal{F}_T; \Theta^{(k)}),
\end{aligned}$$

which we estimate by the method Kim [1994] proposed. For the detail of the derivation, see Appendix.

3.3 Results

The objective of this subsection is to investigate whether and when an irreversible structural change occurred and how it affected a stationary component of asset returns. In particular, we want to see how the irreversible change, if exists, altered the correlation structure of asset returns.

3.3.1 Identifying the Markov chains (S, D)

We first want to test whether a two-state reversible Markov switching model reasonably captures a stationary component of asset returns. See Figures 1 and 2 where we show the estimated probability; more precisely, *smoothed probability:*[1] $\mathbb{P}(S_t = s, S_{t-1} = \hat{s}|\mathcal{F}_T; \Theta^{(*)})$, where $\Theta^{(*)}$ is the set of estimated parameters by the EM algorithm.

Figure. 1: The estimated probability (smoothed probability) of being at regime $S_t = 0$ and NBER recession dates. We assume $q_{00} = 1$.

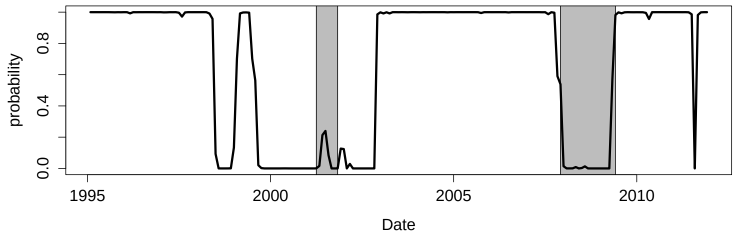


Figure. 2: The estimated probability (smoothed probability) of being at regime $S_t = 0$ and NBER recession dates. We assume $q_{00} \neq 1$.

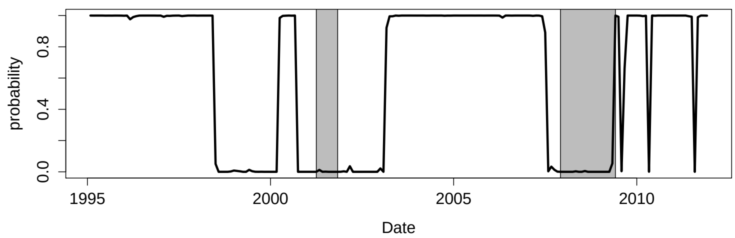


Figure 1 shows both NBER recession dates (shaded regions) and the probability of being at regime $S_t = 0$ estimated by the model without a structural change ($q_{00} = 1$), whereas Figure 2 replaces the latter probability with the one estimated by the model with a structural change ($q_{00} \neq 1$). Because both Figures indicate a nearly identical probability, it supports our assumption that the irreversible structural change in asset returns happens, independently of the stationary and reversible transition in asset returns. Furthermore, we conduct the CHP test used widely for detecting an existence of Markov switching.[2] The result is reported in Table 2, where the test is performed sector by sector. The test statistic of each sector exceeds 10% critical value, so that the data indicate the existence of Markov switching structure. Given the above results, our two-state Markov switching model seems to capture a stationary component of asset returns.

[1]See also (6.1a) and (6.1b) in the Appendix for the details.

[2]The CHP test is developed by Carrasco, Hu and Ploberger[2004]. The null hypothesis of this test is that a time series is not Markov switching. The test statistic of this test is computed by the bootstrap method.

Figure 1 and 2 also identify a relation between regime S_t and an economic environment in the U.S. Although regime $S_t = 0$ does not necessarily represent a period of the economic boom, the estimated probability being at regime $S_t = 0$ for $t \geq 0$ and a booming period in the U.S. economy almost coincide after 2005. Thus, asset returns depend on an economic environment more strongly in the recent period. Therefore, for simplicity, we call $S_t = 0$ a booming regime and $S_t = 1$ a recession regime. Next, we estimate whether and when

Table. 2: The results of the CHP test. We try 500 times parametric bootstraps.

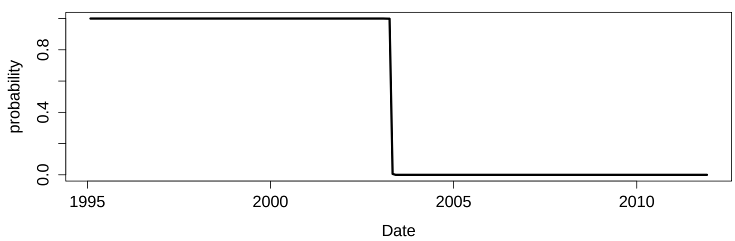
Sector	statistics	10% critical values	5% critical values
ENE	3.24877	2.52963	3.26007
MAT	6.32881	2.75980	3.55003
IND	11.71194	2.55863	3.19111
CD	12.08934	2.48899	3.06581
CS	16.98629	2.35370	2.81645
HC	6.52737	2.62898	3.12237
FIN	14.35005	2.55416	3.37105
IT	8.68406	2.63233	3.29138
TEL	8.83678	2.71052	3.57918
U	9.18174	2.73153	3.41308

the irreversible structural change happened. Table 3 shows the AIC statistics for the model with and without the irreversible structural change. These statistics imply that the model with the irreversible structural change fits the data better than the model without the irreversible structural change. Thus, the data suggests the existence of the irreversible structural change. As for the timing of the irreversible structural change, Figure 3 reports the estimated probability of being at regime $D_t = 0$. Note that again we show the smoothed probability $\mathbb{P}(D_t = d, D_{t-1} = \hat{d} | \mathcal{F}_T; \Theta^{(*)})$.

Table. 3: The AICs of our model

AICs	
$q_{00} = 1$	$0 < q_{00} < 1$
11176.14881	10947.55287

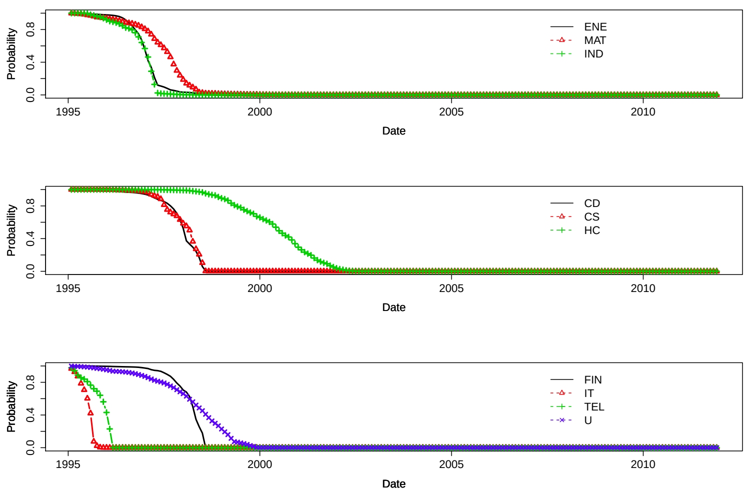
Figure. 3: The estimated probability (smoothed probability) of being at $D_t = 0$ computed by jointly using multi-sector returns.



The result implies that the irreversible structure change happened in May 2003. This roughly corresponds to a period when a housing bubble started in the U.S., which is well in advance to the Lehman shock.

To identify the above mentioned irreversible structural change, it it crucial to estimate the model by jointly using multi-sector returns. To see this further, we report, in Figure 4, the probability of being at regime $D_t = 0$ obtained by estimating the model consisting of single-sector returns.

Figure. 4: The estimated probability (smoothed probability) of being at $D_t = 0$ computed by using individual sector returns.



Almost all the sector returns imply that the probability of being at regime $D_t = 0$ started to decline soon after the beginning of the data. Moreover, probability declined gradually. This means that we cannot infer when the irreversible structural change had happened. On the contrary, in our multi-sector estimation (see Figure 3 again), the probability of being at regime $D_t = 0$ declined dramatically from 1 to 0 at a single period. This confirms that the multi-sector estimation is a key to infer the timing of the irreversible structural change.

3.3.2 Impacts of Structural Change on Means and Covariances

Given the estimated period of the structural change, we then compute mean vectors and covariance matrices for each regime S_t before and after the structural change.

Table 4 summarizes means and volatilities of each sector returns for regime S_t before and after the irreversible structural change. We confirm that at the booming regime, means are positive, and at the recession regime, means are negative. Moreover, volatilities are higher at the recession regime than the ones at the booming regime. For each regime D_t, we test the null hypothesis that means and volatilities at the booming regime are equal to those at the recession regime. As shown in Table 5, we reject the null hypothesis at the 10% significant level except for means of a few industries. This confirms that means and volatilities at the booming regime are statistically different from those at the recession regime. Although not reported, we can obtain a similar result from the model without an irreversible structural change ($q_{00} = 1$).

On the other hand, for each regime S_t, we test the null hypothesis that means and volatilities before the irreversible structural change are equal to those after the irreversible structural

Table. 4: The estimated means and volatilities of each sector returns before and after the structural change. Numbers in parenthesis are standard errors.

Structure	$D_t = 0$			
Regime	$S_t = 0$		$S_t = 1$	
Parameters	$\mu_{0,0}$	$\sigma_{0,0}$	$\mu_{1,0}$	$\sigma_{1,0}$
ENE	1.57075	3.88612	-0.36353	5.72157
	(0.55620)	(0.39228)	(0.76000)	(0.57147)
MAT	0.47291	5.03320	-0.17774	6.62812
	(0.71882)	(0.50926)	(0.87490)	(0.66249)
IND	2.01714	3.03514	-0.70598	7.21063
	(0.43479)	(0.30021)	(0.91320)	(0.71933)
CD	1.73846	3.31970	-0.37356	7.87306
	(0.47651)	(0.33369)	(0.99014)	(0.78614)
CS	2.04822	3.07534	-0.65082	5.72344
	(0.44095)	(0.30856)	(0.78783)	(0.57240)
HC	2.69476	3.86397	-0.39476	6.06147
	(0.55452)	(0.38431)	(0.81199)	(0.60639)
FIN	2.97084	3.84344	-0.72781	7.62850
	(0.55157)	(0.37761)	(0.98163)	(0.76172)
IT	2.10595	6.38989	-0.45462	11.27995
	(0.91731)	(0.64797)	(1.47962)	(1.12858)
TEL	0.91528	4.92119	-1.09830	6.64434
	(0.70564)	(0.49827)	(0.91291)	(0.66511)
U	1.63341	3.30147	-1.69237	6.72387
	(0.47450)	(0.33654)	(0.93622)	(0.67318)

Structure	$D_t = 1$			
Regime	$S_t = 0$		$S_t = 1$	
Parameters	$\mu_{0,1}$	$\sigma_{0,1}$	$\mu_{1,1}$	$\sigma_{1,1}$
ENE	2.17236	4.99054	-2.81418	11.09907
	(0.56563)	(0.40050)	(2.23657)	(1.55978)
MAT	1.65968	4.35947	-3.00875	14.14337
	(0.49681)	(0.35291)	(2.84785)	(1.98930)
IND	1.64592	3.45150	-3.62732	11.12342
	(0.39647)	(0.28563)	(2.25876)	(1.56574)
CD	1.50946	3.59369	-2.98208	12.53005
	(0.40829)	(0.28816)	(2.51968)	(1.76575)
CS	1.00987	2.20861	-1.10848	5.23442
	(0.24998)	(0.17645)	(1.05449)	(0.73651)
HC	0.95226	3.14991	-2.06832	7.08042
	(0.35582)	(0.25149)	(1.42060)	(0.99774)
FIN	0.96026	3.50571	-5.32789	16.98377
	(0.39856)	(0.28174)	(3.40925)	(2.39363)
IT	1.51039	4.60216	-2.57060	10.19298
	(0.52165)	(0.36759)	(2.05421)	(1.43360)
TEL	1.23062	4.10870	-2.56394	7.74930
	(0.46498)	(0.32789)	(1.54959)	(1.09356)
U	1.46032	3.82859	-2.18498	6.21907
	(0.43257)	(0.30567)	(1.24755)	(0.87440)

p_{00}	p_{11}	q_{00}	log likelihood
0.95084	0.91748	0.98990	-5209.77644
(0.12707)	(0.05956)	(0.09879)	

change. Table 6 shows that we cannot reject the null hypothesis at the 10% significant level for most of means and volatilities. Thus, the irreversible structural change does not have a statistically significant impact on means and volatilities.

Table. 5: The result of the hypothesis testing : the null hypotheses are the equality of means and that of standard deviations across the two regimes $S_t = 0$ and $S_t = 1$.

before structural change ($D_t = 0$)				
Null hypotheses	equal means across regimes		equal volatilities across regimes	
Sectors	Wald statistics	P-values	Wald statistics	P-values
ENE	4.19217	0.04061	6.97835	0.00825
MAT	0.32726	0.56728	3.63524	0.05657
IND	7.27516	0.00699	28.62131	0.00000
CD	3.70178	0.05435	28.32125	0.00000
CS	8.95713	0.00276	16.57704	0.00005
HC	9.89443	0.00166	9.34964	0.00223
FIN	10.85320	0.00099	19.73349	0.00001
IT	2.15478	0.14213	14.06822	0.00018
TEL	3.05441	0.08052	4.29605	0.03820
U	10.04249	0.00153	20.68266	0.00001

after structural change ($D_t = 1$)				
Null hypotheses	equal means across regimes		equal volatilities across regimes	
Sectors	Wald statistics	P-values	Wald statistics	P-values
ENE	4.63153	0.03139	14.36410	0.00015
MAT	2.58760	0.10770	23.46006	0.00000
IND	5.21964	0.02233	23.13004	0.00000
CD	3.07845	0.07934	24.95700	0.00000
CS	3.79579	0.05138	15.94733	0.00007
HC	4.23335	0.03964	14.56399	0.00014
FIN	3.34359	0.06747	31.30537	0.00000
IT	3.67592	0.05520	14.26197	0.00016
TEL	5.47631	0.01928	10.14686	0.00145
U	7.57783	0.00591	6.64519	0.00994

By contrast, the irreversible structural change has a stronger effect on the correlation structure across sector returns, which is summarized in Table 7 and Figure 5. (The full correlation coefficients are reported in Tables 12 and 13 in Appendix.) Before the irreversible structural change, the correlation structure of the recession regime is slightly tilted toward the positive direction than that of the booming regime. This resembles the result obtained by the model estimated without the irreversible structural change. On the other hand, after the irreversible structural change, the difference in the correlation structure between the recession regime and the booming regime has widened because correlations in the recession regime has increased dramatically. Moreover, the correlation structure of the booming regime has also shifted toward the positive direction after the irreversible structural change. These results imply that the irreversible structural change has strengthened the comovement of asset returns both in the booming and recession regimes. Thus, our model seems to capture not only the hike in correlations in the Lehman shock period but also a gradual elevation of correlation since early 2000's.

To confirm the above observations, for each of the booming and recession regime, we test whether the irreversible structural change has increased or decreased correlations. For the booming regime, Table 7 shows that, at the 10% critical level, 38% of correlations has increased, whereas 2% of correlations has decreased. As for the recession regime, the same table shows that 76% of correlations has increased, whereas none of correlations has decreased. Furthermore, before and after the irreversible structural change, we test whether correlations at the booming regime are equal to those at the recession regime. Before the irreversible

Table. 6: The results of the hypothesis testing : the null hypotheses are the equality of means and that of standard deviations across the two regimes $D_t = 0$ and $D_t = 1$.

booming regime ($S_t = 0$)				
Null hypotheses	equal means across structures		equal volatilities across structures	
Sectors	Wald statistics	P-values	Wald statistics	P-values
ENE	0.57607	0.44786	3.87924	0.04889
MAT	1.84715	0.17412	1.18125	0.27710
IND	0.39822	0.52801	1.01064	0.31475
CD	0.13305	0.71529	0.38606	0.53438
CS	4.19592	0.04052	5.94326	0.01477
HC	6.99059	0.00819	2.42000	0.11980
FIN	8.72971	0.00313	0.51448	0.47321
IT	0.31857	0.57247	5.75882	0.01641
TEL	0.13896	0.70931	1.84631	0.17421
U	0.07259	0.78761	1.34512	0.24613

recession regime ($S_t = 1$)				
Null hypotheses	equal means across structures		equal volatilities across structures	
Sectors	Wald statistics	P-values	Wald statistics	P-values
ENE	1.09316	0.29577	10.47844	0.00121
MAT	0.91760	0.33811	12.84881	0.00034
IND	1.47889	0.22395	5.16055	0.02311
CD	0.95380	0.32875	5.80874	0.01595
CS	0.12275	0.72607	0.27488	0.60008
HC	1.06907	0.30116	0.76193	0.38273
FIN	1.71018	0.19096	13.87607	0.00020
IT	0.72039	0.39602	0.35516	0.55121
TEL	0.67488	0.41135	0.74544	0.38792
U	0.10090	0.75075	0.20932	0.64730

Figure. 5: The horizontal axis is correlation coefficients and the vertical axis shows the number of pairs of industries. The three histograms are the cases: $q_{00} = 1$, $q_{00} \neq 1$ before the structural change, and $q_{00} \neq 1$ after the structural change.

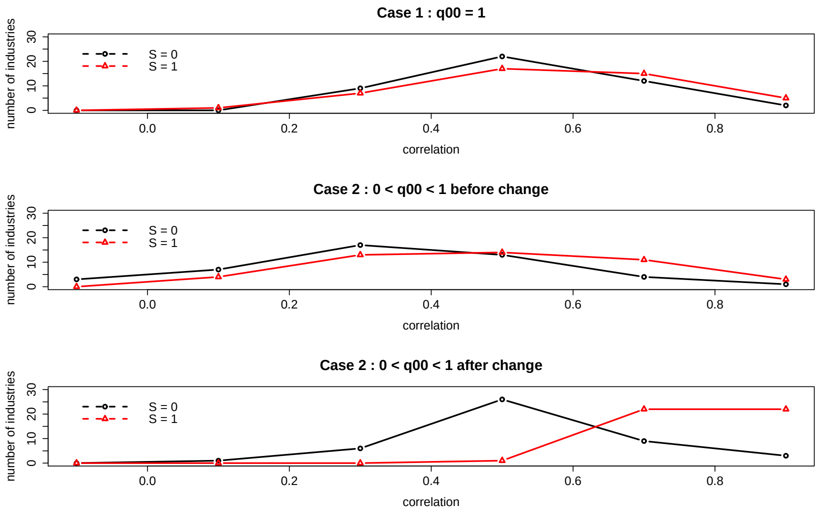


Table. 7: The summary table for changes of correlations at 10% significant level. Numbers in parentheses are percentages of all pairs.

Case 1 : $q_{00} = 1$

	correlation in $S = 0 < S = 1$	correlation in $S = 0 > S = 1$
Number of changes	26 (58%)	19 (42%)
Number of significant changes	10 (22%)	2 (4%)

Case 2 : $0 < q_{00} < 1$

Fix S.

$S = 0$	correlation in $D = 0 < D = 1$	correlation in $D = 0 > D = 1$
Number of changes	36 (80%)	9 (20%)
Number of significant changes	17 (38%)	1 (2%)

$S = 1$	correlation in $D = 0 < D = 1$	correlation in $D = 0 > D = 1$
Number of changes	43 (96%)	2 (4%)
Number of significant changes	34 (76%)	0 (0%)

Fix D.

$D = 0$	correlation in $S = 0 < S = 1$	correlation in $S = 0 > S = 1$
Number of changes	36 (80%)	9 (20%)
Number of significant changes	11 (24%)	0 (0%)

$D = 1$	correlation in $S = 0 < S = 1$	correlation in $S = 0 > S = 1$
Number of changes	45 (100%)	0 (0%)
Number of significant changes	36 (80%)	0 (0%)

structural change, Table 7 shows that, at the 10% critical level, 24% of correlations are higher in the recession regimen, whereas 2% of correlations are higher in the booming regime. After the irreversible structural change, the same table also shows that 80% of correlations are higher in the recession regime, whereas none of correlations are higher in the booming regime. Thus, the test statistics confirm the trend observed in Figure 5. See Tables 14 and 15 in the Appendix for the hypothesis testing results on correlation coefficients before and after the structural change, based on which we created the summary table (that is, Table 7).

The result after the irreversible structural change also strongly contrasts with that of hypothesis testing of correlations without the irreversible structural change, where the only 22% of correlations are higher in the recession regime at 10% significant level. See Table 16 in Appendix for the full result.

4 Asset Allocation Problem

To confirm the importance of the information about the structural change, we look at the asset allocation problems in the regime-switching market. We consider the global minimum-variance portfolios and the tangency portfolios. We simulate the performances of these portfolios by the Monte Carlo simulations.

We assume that several investors exist and consider the performances of their portfolios. These investors are identified by the levels of knowledge about the regimes and structure. First type investor has the full information of the regimes and structure. She knows the real regime and the real structure of the market at every time and rebalances her portfolio in response to regime switches and structural change optimally. Next, we assume that second type investor has only the information of the regimes. She thinks that the initial structure will not change during her investment horizon, so she rebalances a portfolio in response to only regime switches. Finally, third type investor has only the information of the structure. She thinks that the initial regime will not change. Therefore, she chooses his portfolio using only the information of the structure.

Let W_0^F and $\phi^F = (\phi_{F,1}, \cdots, \phi_{F,10})^\top$ be the initial wealth and portfolio of the full information investor. Then, the wealth after one month, W_1^F, is,

$$W_1^F = W_0^F \left(\sum_{i=1}^{10} \phi_{F,i} \exp\left\{ Y_1^{(i)} \right\} \right),$$

where $Y_1^{(i)}$ is the i th component of the log-return Y_1. The prices of assets have the log-normal distributions. Therefore, the expectation value of wealth W_1^F of the full information investor given $S_0 = s, D_0 = d$ is,

$$E(W_1^F | S_0 = s, D_0 = d) = W_0^F \phi_F^\top g_{s,d}(\mu, \Sigma),$$

where $g_{s,d}$ is the $\mathbb{R}^{10}$ - valued function as follows,

$$\begin{aligned} g_{s,d}(\mu, \Sigma) &:= \sum_{s',d'} P_{s',d'}^{s,d} \begin{pmatrix} \exp\{\mu_{s',d'}^1 + \sigma_{s',d'}^1/2\} \\ \vdots \\ \exp\{\mu_{s',d'}^{10} + \sigma_{s',d'}^{10}/2\} \end{pmatrix}, \\ P_{s',d'}^{s,d} &:= \mathbb{P}(S_1 = s', D_1 = d' \mid S_0 = s, D_0 = d). \end{aligned}$$

$\mu_{s',d'}^i$ is the i th component of the vector $\mu_{s',d'}$ and $\sigma_{s',d'}^i$ is the $i \times i$ th element of the matrix $\Sigma_{s',d'}$.

The variance of her wealth W_1^F given $S_0 = s, D_0 = d$ is,

$$\mathrm{Var}(W_1^F | S_0 = s, D_0 = d) = (W_0^F)^2 \phi_F^\top H_{s,d}(\mu, \Sigma)\phi_F,$$

where $H_{s,d}$ is the $\mathbb{R}^{10\times 10}$ - valued function as follows,

$$\begin{aligned}[H_{s,d}(\mu,\Sigma)]_{i,j} &= \sum_{s',d'} P^{s,d}_{s',d'} \exp\left\{\mu^i_{s',d'} + \mu^j_{s',d'} + \frac{1}{2}(\sigma^i_{s',d'} + \sigma^j_{s',d'} + \sigma^{i,j}_{s',d'})\right\} \\ &\quad - \left(\sum_{s',d'} P^{s,d}_{s',d'} \exp\left\{\mu^i_{s',d'} + \frac{1}{2}\sigma^i_{s',d'}\right\}\right)\left(\sum_{s',d'} P^{s,d}_{s',d'} \exp\left\{\mu^j_{s',d'} + \frac{1}{2}\sigma^j_{s',d'}\right\}\right), \\ &\quad i,j = 1,\ldots,10.\end{aligned}$$

$[H_{s,d}(\mu,\Sigma)]_{i,j}$ is the $i \times j$ th element of the matrix $H_{s,d}(\mu,\Sigma)$ and $\sigma^{i,j}_{s',d'}$ is the $i \times j$ th element of the matrix $\Sigma_{s',d'}$.

The partial information investors consider the smilier moments conditioned their available information. Only regime information (ignore structure) investor's portfolio is ϕ_S and only structure information (ignore regime) investor's portfolio is ϕ_D. The investors compute global minimum-variance portfolios and tangency portfolios under their available information.

We assumed that initial wealths of investors, $W_0^A, A = F, S, D$ equal 1 and that the investors cannot short-selling. No short-selling means that $\phi_{A,i} \geq 0,\ i = 1, \cdots, 10, A = F, S, D$. The portfolio is normalized $\sum_{i=1}^{10} \phi_{A,i} = 1,\ A = F, S, D$.

In practice, we use the estimated parameters of three models – the full model ($0 < q_{00} < 1$, full information investor's assumption), only recursive regime switching model ($q_{00} = 1$, only regime information investor's assumption) and only structure change model (only structure information investor's assumption, see Appendix 6.2, Model $D1$) – to compute the moments of the industry indices considered by the investors.

Next, we estimate the performances of these portfolios under the full model's market environment. We generate the log-return processes by the Monte Carlo method using the estimated parameters of full model. The investors react to their available information and rebalance the minimum-variance portfolios and the tangency portfolios. The rebalancing interval of their portfolios is one month. For example, if the regime is extreme ($S_t = 0$) and the structure is 1 ($D_t = 1$) at time t, then the first type investor chooses the optimal portfolio for one month with the information of $S_t = 0$ and $D_t = 1$, but the second type investor chooses the optimal portfolio with the information of $S_t = 0$ and $D_t = 0$ since he has only the information of the market regime and assumes that $D_t = 0$. Their investment horizons are 1, 3, 6, 12, 24, 36, 60, 96 and 120 months. We examine the simulations in the initial state $S_0 = 0$ or 1 and the initial structure $D_0 = 1$. Therefore, we consider the performance after the structural change.

Figure 6 and 7 show the pie charts of the global minimum-variance portfolios and tangency portfolios when the investment horizon is one month.

The only structure information investor do not change her portfolio regardless of the value of initial state variable, S_0. The investors use these portfolios when rebalancing their portfolios. Table .4 shows the simulation results. Figure .4 shows the Sharpe ratios of the simulations. If the investors use the strategy of the global minimum variance portfolios, when compare the Sharpe ratio, the negative effect of ignoring the structure D (only using the information of the recursive regime S) is larger than ignoring the recursive regime S (only using the information of structure D). On the other hand, the negative effect of ignoring the recursive regime is larger than ignoring the structure when the strategy of the tangency portfolios are adopted. Thus, we conclude that both the recursive state and structure are important information of the investment.

Figure. 6: The pie charts of global minimum variance portfolios.

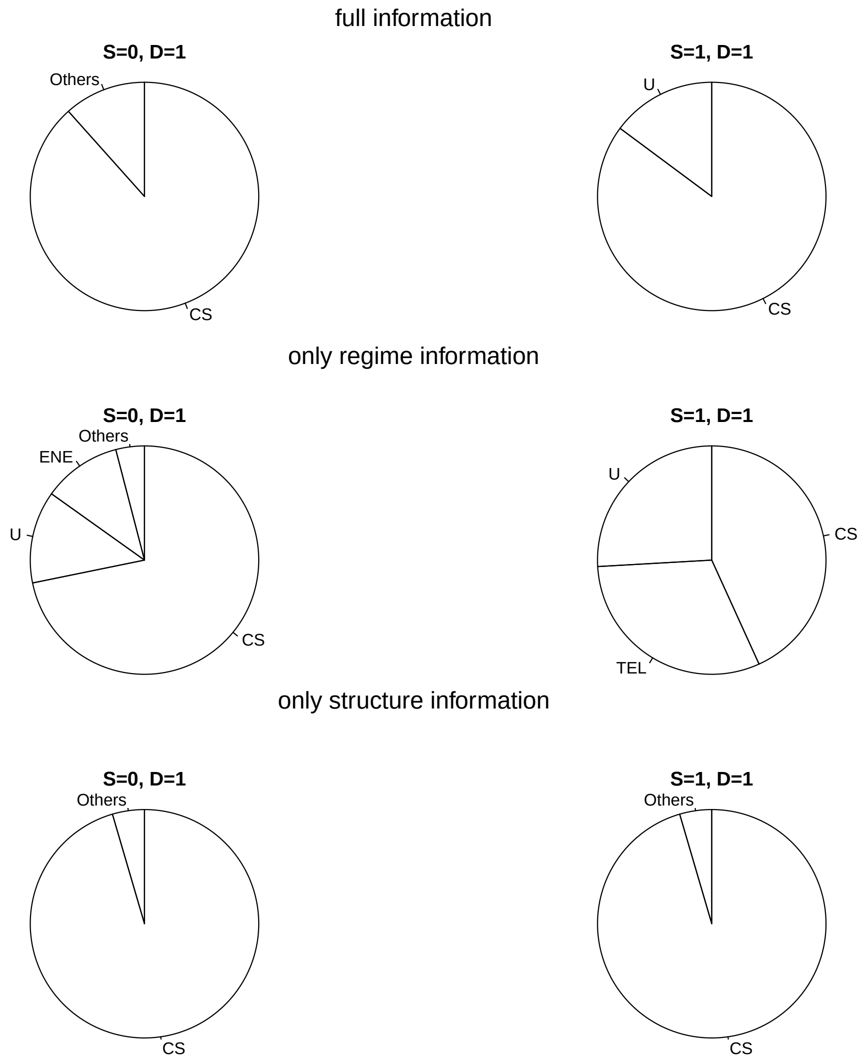


Figure. 7: The pie charts of tangency portfolios.

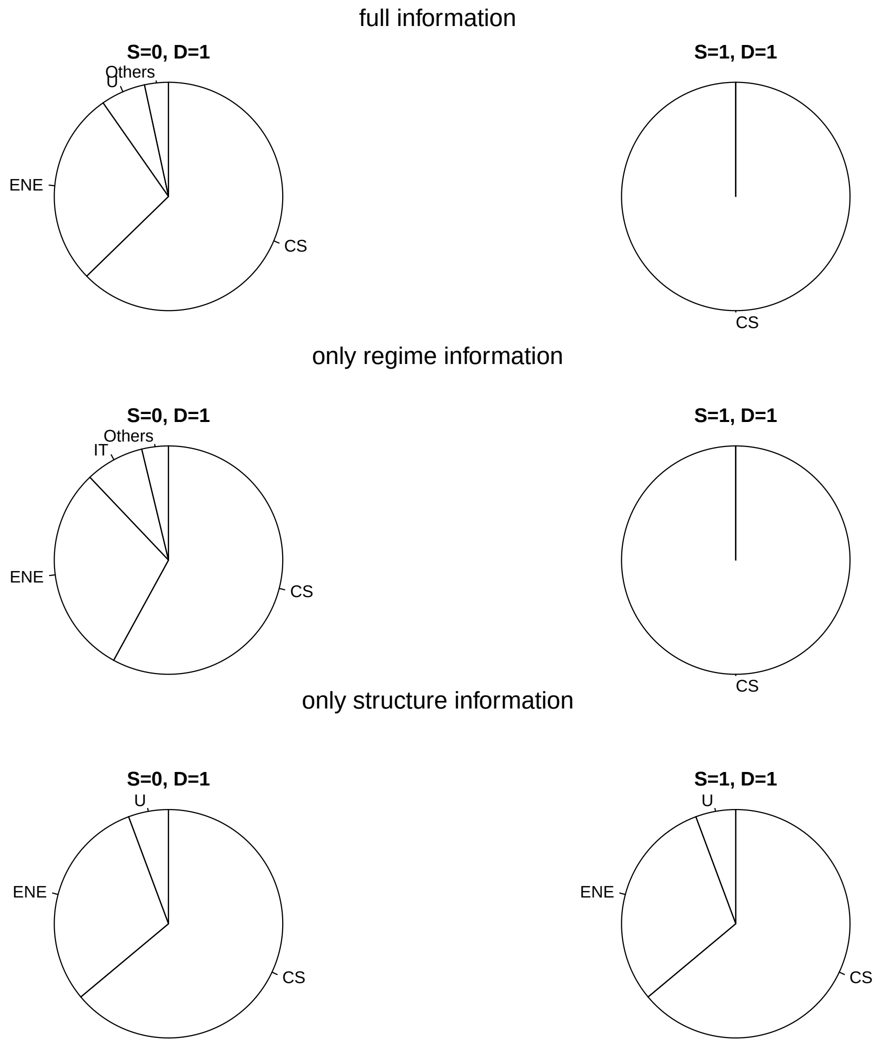


Table. 8: The simulation results of the performances of the wealth with three types of investment strategies. Type 1: all information available, type 2: only regime information (ignore structure), type 3: only structure information (ignore regime). We show the means of the simulations. Numbers in parentheses are the standard deviations of them. Number of trials is 30000. The initial structure is $D_0 = 1$. The risk-free rate is 0. The initial wealth is 1.

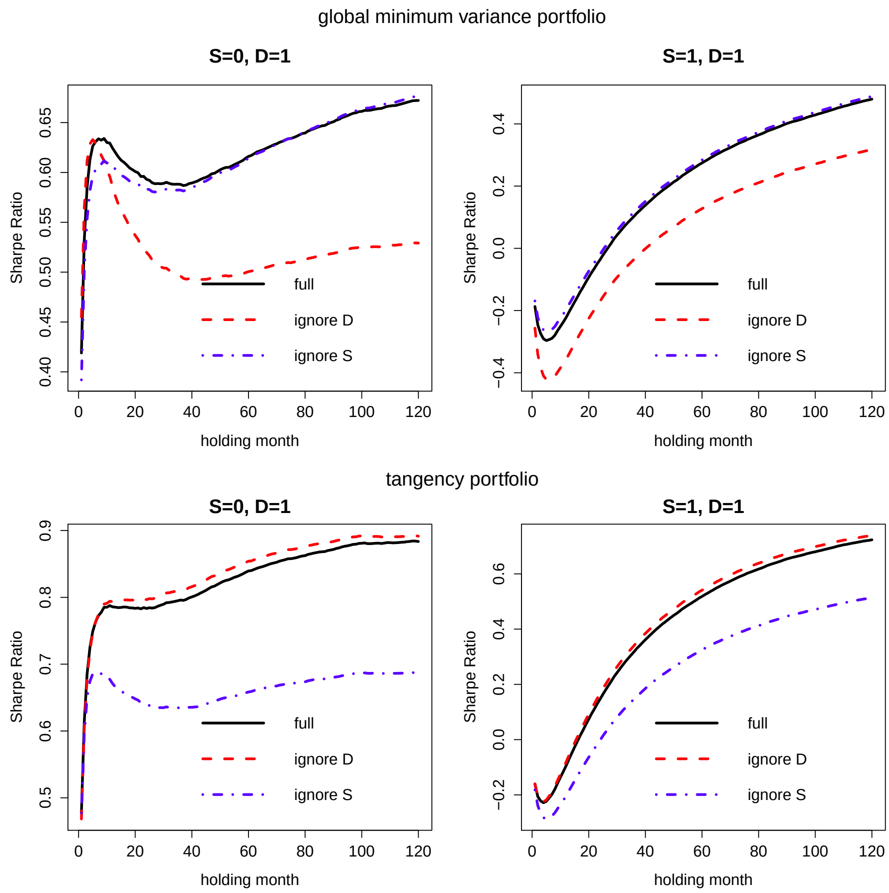
global minimum variance portfolios

	initial state $S_0 = 0$			initial state $S_0 = 1$		
month	type 1	type 2	type 3	type 1	type 2	type 3
1	1.01022	1.01152	1.00964	0.99065	0.98609	0.99151
	(0.02438)	(0.02532)	(0.02459)	(0.04998)	(0.05419)	(0.05021)
3	1.02810	1.03127	1.02658	0.97685	0.96489	0.97905
	(0.04802)	(0.05113)	(0.04806)	(0.08452)	(0.09169)	(0.08474)
6	1.04992	1.05456	1.04732	0.96528	0.94553	0.96880
	(0.07915)	(0.08666)	(0.07852)	(0.11833)	(0.12887)	(0.11809)
12	1.08443	1.08939	1.08030	0.96217	0.93281	0.96683
	(0.13523)	(0.15259)	(0.13240)	(0.16992)	(0.18815)	(0.16795)
24	1.13960	1.14052	1.13323	0.99084	0.95234	0.99530
	(0.23537)	(0.27030)	(0.22834)	(0.25058)	(0.28009)	(0.24563)
36	1.18944	1.18487	1.18109	1.03292	0.98737	1.03632
	(0.32216)	(0.37264)	(0.31096)	(0.31951)	(0.35730)	(0.31187)
60	1.29717	1.27909	1.28442	1.12495	1.06466	1.12554
	(0.48236)	(0.55753)	(0.46310)	(0.45530)	(0.50748)	(0.44190)
96	1.48171	1.44112	1.46084	1.27754	1.19168	1.27299
	(0.73179)	(0.84234)	(0.69798)	(0.66493)	(0.73676)	(0.64107)
120	1.61433	1.55405	1.58729	1.39403	1.28791	1.38523
	(0.91387)	(1.04702)	(0.86838)	(0.82180)	(0.90739)	(0.78879)

tangency portfolios

	initial state $S_0 = 0$			initial state $S_0 = 1$		
month	type 1	type 2	type 3	type 1	type 2	type 3
1	1.01316	1.01373	1.01340	0.99193	0.99193	0.98841
	(0.02754)	(0.02932)	(0.02808)	(0.05032)	(0.05032)	(0.06364)
3	1.03676	1.03835	1.03692	0.98101	0.98115	0.97144
	(0.05366)	(0.05657)	(0.05681)	(0.08561)	(0.08584)	(0.10720)
6	1.06711	1.07020	1.06619	0.97438	0.97499	0.95791
	(0.08794)	(0.09198)	(0.09616)	(0.12134)	(0.12226)	(0.14916)
12	1.11864	1.12471	1.11356	0.98309	0.98523	0.95645
	(0.15099)	(0.15717)	(0.16923)	(0.17863)	(0.18140)	(0.21350)
24	1.20990	1.22211	1.19280	1.04026	1.04691	0.99937
	(0.26796)	(0.27883)	(0.30167)	(0.27498)	(0.28216)	(0.31671)
36	1.29937	1.31858	1.26792	1.11547	1.12771	1.06011
	(0.37613)	(0.39273)	(0.42218)	(0.36303)	(0.37520)	(0.41013)
60	1.50013	1.53627	1.43415	1.28576	1.31163	1.19717
	(0.59575)	(0.62789)	(0.65941)	(0.55131)	(0.57623)	(0.60662)
96	1.86679	1.93794	1.73208	1.59053	1.64468	1.43474
	(0.98705)	(1.05435)	(1.06877)	(0.88180)	(0.93592)	(0.94213)
120	2.15274	2.25534	1.95561	1.83797	1.91796	1.62394
	(1.30461)	(1.40806)	(1.39264)	(1.15845)	(1.24246)	(1.21490)

Figure. 8: The Sharpe ratios. In computing the Sharpe ratios, we assume that the risk-free rate equal 0.



5 Concluding Remarks

We identify the change in correlation structure across industries. This knowledge should be important in the fields of asset allocation, derivatives pricing, and risk management. However, there is still an issue why this structural change happened. This issue has been studied recently. For example, Branch and Evans [2010] construct a model that contains multiple regimes and use the concept of the bounded rationality: the econometric misspecification approach to asset pricing questions.

Since we have estimated when this change occurred and this change has not been reversed yet, a reasonable approach from this point of view is to investigate if there exist some fundamental events occurred in the domestic or international capital markets around May 2003 and in what mechanism the correlation structure has been distorted.

6 Appendix

6.1 The EM Algorithm

Based on Hamilton [1989], [1990] and Kim [1994], we explain the EM algorithm we use in this paper. For notational simplicity, we treat the case of $q_{00} = 1$ since the extension to the general case is straightforward. First we introduce the *full-information* likelihood function, which means that we hypothesize that we can observe unobserved variables. For the random vectors, $Y = (Y_0, \cdots, Y_T)$ and $S = (S_0, \cdots, S_T)$, the full-information likelihood function is

$$\begin{aligned} f(Y, S; \Theta) &= \prod_{t=0}^{T} \left(\sum_{s=0}^{1} \mathbf{1}\{S_t = s\} f(Y_t | \mathcal{F}_{t-1}, S_t = s; \Theta) \right) \\ &\times \prod_{t=1}^{T} \left(\sum_{s=0}^{1} \sum_{\widehat{s}=0}^{1} \mathbf{1}\{S_t = s, S_{t-1} = \widehat{s}\} \mathbb{P}(S_t = s | S_{t-1} = \widehat{s}) \right) \\ &\times \sum_{s=0}^{1} \mathbf{1}\{S_1 = s\} \mathbb{P}(S_1 = s) \end{aligned}$$

where $\mathcal{F}_t$ is the σ-algebra generated by Y up to time t and Θ is the distribution parameters and the conditional density $f(Y_t | \mathcal{F}_{t-1}, S_t = s; \Theta)$ is

$$f(Y_t | \mathcal{F}_{t-1}, S_t = s; \Theta) = \frac{1}{\sqrt{(2\pi)^N \det \Sigma_s}} \exp\left\{ -\frac{1}{2} (Y_t - \mu_s)^\top \Sigma_s^{-1} (Y_t - \mu_s) \right\}.$$

For the "expectation" step, we start with the initial parameter set $\Theta^{(0)}$ to compute

$$\begin{aligned} Q(\Theta; \Theta^{(0)}) &= E^{\Theta^{(0)}}[\log f(Y, S; \Theta) | \mathcal{F}_T] \\ &= \sum_{t=0}^{T} \sum_{s=0}^{1} \mathbb{P}(S_t = s | \mathcal{F}_T; \Theta^{(0)}) \log f(Y_t | \mathcal{F}_{t-1}, S_t = s; \Theta) \\ &+ \sum_{t=1}^{T} \sum_{s=0}^{1} \sum_{\widehat{s}=0}^{1} \mathbb{P}(S_t = s, S_{t-1} = \widehat{s} | \mathcal{F}_T; \Theta^{(0)}) \log \mathbb{P}(S_t = s | S_{t-1} = \widehat{s}) \\ &+ \sum_{s=0}^{1} \mathbb{P}(S_1 = s | \mathcal{F}_T; \Theta^{(0)}) \log \mathbb{P}(S_1 = s). \end{aligned}$$

Next, we search the parameters maximizing $Q(\Theta; \Theta^{(0)})$ function,

$$\Theta^{(1)} = \arg\max_{\Theta \in \overline{\Theta}} Q(\Theta; \Theta^{(0)})$$

where $\overline{\Theta}$ is the parameter space of this model. This step is the "maximization" step. We then continue these steps from $k = 1, 2, \ldots$.

Hamilton [1990] shows if we repeat these steps to infinity, then the sequence of the parameters obtained by this algorithm converges to the maximum likelihood estimators. According to Hamilton [1990], the updating formulae of the parameters in the $(k+1)$th iteration are

$$\begin{aligned}
\mu_s^{(k+1)} &= \frac{\sum_{t=0}^T Y_t \mathbb{P}(S_t = s|\mathcal{F}_T; \Theta^{(k)})}{\sum_{t=0}^T \mathbb{P}(S_t = s|\mathcal{F}_T; \Theta^{(k)})} \quad s = 0, 1 \\
\Sigma_s^{(k+1)} &= \frac{\sum_{t=0}^T (Y_t - \mu_s^{(k+1)})(Y_t - \mu_s^{(k+1)})^\top \mathbb{P}(S_t = s|\mathcal{F}_T; \Theta^{(k)})}{\sum_{t=0}^T \mathbb{P}(S_t = s|\mathcal{F}_T; \Theta^{(k)})} \quad s = 0, 1 \\
p_{ss}^{(k+1)} &:= \mathbb{P}(S_{t+1} = s|S_t = s; \Theta^{(k+1)}) = \frac{\sum_{t=1}^T \mathbb{P}(S_t = s, S_{t-1} = s|\mathcal{F}_T; \Theta^{(k)})}{\sum_{t=1}^T \mathbb{P}(S_{t-1} = s|\mathcal{F}_T; \Theta^{(k)})} \quad s = 0, 1 \\
\rho_s^{(k+1)} &:= \mathbb{P}(S_0 = s; \Theta^{(k+1)}) = \mathbb{P}(S_1 = s|\mathcal{F}_T; \Theta^{(k)}) \quad s = 0, 1.
\end{aligned}$$

These updating formulae are obtained by the first order conditions of the maximization of $Q(\Theta; \Theta^{(0)})$ with respect to Θ.

By looking at the above formulae, we see that we need to compute

$$\mathbb{P}(S_t = s|\mathcal{F}_T; \Theta^{(k)}), t = 0, \cdots, T \tag{6.1a}$$

$$\mathbb{P}(S_t = s, S_{t-1} = s|\mathcal{F}_T; \Theta^{(k)}), t = 1, \cdots, T \tag{6.1b}$$

which are called *smoothed probabilities*.

Let us suppose that we have estimated up to kth iteration and explain how to update to the $(k+1)$th estimates. Now we do the following:

<u>Forward calculation:</u> Assume further that we have obtained $\mathbb{P}(S_{t-1} = s|\mathcal{F}_{t-1}; \Theta^{(k)})$, then we have, for the next time step t,

$$\mathbb{P}(S_t = s|\mathcal{F}_{t-1}; \Theta^{(k)}) = \sum_{\widehat{s}=0}^{1} \mathbb{P}(S_t = s|S_{t-1} = \widehat{s}) \mathbb{P}(S_{t-1} = \widehat{s}|\mathcal{F}_{t-1}; \Theta^{(k)}), \quad s = 0, 1. \tag{6.2}$$

By using this, we compute $\mathbb{P}(S_t = s|\mathcal{F}_t; \Theta^{(k)}), s = 0, 1$ in the following way: By Bayes' rule, we obtain

$$\begin{aligned}
\mathbb{P}(S_t = s|\mathcal{F}_t; \Theta^{(k)}) &= \mathbb{P}(S_t = s|Y_t, \mathcal{F}_{t-1}; \Theta^{(k)}) \\
&= \frac{\mathbb{P}(Y_t \in \mathrm{d}y|S_t = s, \mathcal{F}_{t-1}; \Theta^{(k)})}{\mathbb{P}(Y_t \in \mathrm{d}y|\mathcal{F}_{t-1}; \Theta^{(k)})} \mathbb{P}(S_t = s|\mathcal{F}_{t-1}; \Theta^{(k)}) \\
&= \frac{f(Y_t|S_t = s, \mathcal{F}_{t-1}; \Theta^{(k)}) \mathbb{P}(S_t = s|\mathcal{F}_{t-1}; \Theta^{(k)})}{\sum_{\widehat{s}=0}^1 f(Y_t|S_t = \widehat{s}, \mathcal{F}_{t-1}; \Theta^{(k)}) \mathbb{P}(S_t = \widehat{s}|\mathcal{F}_{t-1}; \Theta^{(k)})}, \quad s = 0, 1
\end{aligned} \tag{6.3}$$

where $\mathbb{P}(S_t = s|\mathcal{F}_t; \Theta^{(k)})$ is called the *filtered probability*. As a proxy for the filtered probability at $t = 0$, we use $\mathbb{P}(S_0 = s; \Theta^{(k)})$. We then repeat this procedure *forwards* up to time T. In other words, we shall obtain the whole set of probabilities (6.2) and (6.3) for $t = 0, \ldots, T$. Recall k is still fixed.

Backward calculations: For computing (6.1a) and (6.1b), first we use Bayes' rule to write

$$
\begin{aligned}
&\mathbb{P}(S_t = s, S_{t+1} = \widehat{s}|\mathcal{F}_T; \Theta^{(k)}) \\
&\qquad = \mathbb{P}(S_t = s|S_{t+1} = \widehat{s}, \mathcal{F}_T; \Theta^{(k)})\mathbb{P}(S_{t+1} = \widehat{s}|\mathcal{F}_T; \Theta^{(k)}) \\
&\qquad \approx \mathbb{P}(S_t = s|S_{t+1} = \widehat{s}, \mathcal{F}_t; \Theta^{(k)})\mathbb{P}(S_{t+1} = \widehat{s}|\mathcal{F}_T; \Theta^{(k)}) \\
&\qquad = \frac{\mathbb{P}(S_{t+1} = \widehat{s}|S_t = s, \mathcal{F}_t; \Theta^{(k)})\mathbb{P}(S_t = s|\mathcal{F}_t; \Theta^{(k)})}{\mathbb{P}(S_{t+1} = \widehat{s}|\mathcal{F}_t; \Theta^{(k)})}\mathbb{P}(S_{t+1} = \widehat{s}|\mathcal{F}_T; \Theta^{(k)}) \\
&\qquad = \frac{\mathbb{P}(S_{t+1} = \widehat{s}|S_t = s; \Theta^{(k)})\mathbb{P}(S_t = s|\mathcal{F}_t; \Theta^{(k)})}{\mathbb{P}(S_{t+1} = \widehat{s}|\mathcal{F}_t; \Theta^{(k)})}\mathbb{P}(S_{t+1} = \widehat{s}|\mathcal{F}_T; \Theta^{(k)}) \qquad (6.4)
\end{aligned}
$$

where in the last line we use Assumption 2.1-(ii). We compute *backwards* starting with $t = T - 1$ down to $t = 0$. All the probabilities in the last line are known[3] and hence we obtain

$$
\mathbb{P}(S_t = s|\mathcal{F}_T; \Theta^{(k)}) = \sum_{\widehat{s}=0}^{1} \mathbb{P}(S_t = s, S_{t+1} = \widehat{s}|\mathcal{F}_T; \Theta^{(k)}). \qquad (6.5)
$$

for all $t = T, \cdots, 1$. The resulting probabilities in (6.5) and (6.4) for $t = 0, \cdots, T$ are the smoothed probabilities (6.1a) and (6.1b), respectively. Plugging (6.1a) and (6.1b) into the recursive formulae for parameter estimation above, we have updated for the $(k+1)$th iteration. Note that the smoothed probabilities, which are used in the updating formulae for parameter estimation, have rich information since they estimate the probabilities of being in certain regime at time t by using the full observations.

6.2 Model Specification

In addition to the above two models, we apply other Markov regime switching models to the U.S. stock market. The models are as follows,

1. once structural change without recursive regimes model (Model $D1$),
2. twice structural changes without recursive regimes model (Model $D2$),
3. twice structural changes with recursive regimes model (Model $D2S2$).

The recursive regime is captured by the variable $S_t = 0$ or 1. The table 9 shows the AICs of these three models.

Table. 9: The AICs of above models

AICs		
Model $D1$	Model $D2$	Model $D2S2$
11323.000	11246.466	11082.006

The AIC of the full model ($0 < q_{00} < 1$) in the previous section is 10947.553. Therefore, the full model is the most appropriate model in the view of the AIC. In the point of view of the hypothesis testing, the full model is the most parsimonious. The table 10 shows the summary of the hypothesis testing that the correlations are identical across states.

[3]Except for the last one, we already have the whole sets of probabilities ($t = 0 \cdots T$) in (6.4): Namely, $\mathbb{P}(S_{t+1} = \widehat{s}|S_t = s; \Theta^{(k)})$ is $p_{ss}^{(k)}$ obtained in the kth iteration, and $\mathbb{P}(S_t = s|\mathcal{F}_t; \Theta^{(k)})$ and $\mathbb{P}(S_{t+1} = \widehat{s}|\mathcal{F}_t; \Theta^{(k)})$ are from (6.2) and (6.3), respectively. Finally, for the last one, if we set $t = T - 1$, then $\mathbb{P}(S_T = \widehat{s}|\mathcal{F}_T; \Theta^{(k)})$ is known again by (6.2). With all these, we obtain (6.5) at $t = T - 1$, that is, $\mathbb{P}(S_{T-1} = s|\mathcal{F}_T; \Theta^{(k)})$. This one is then plugged for the next time step $t = T - 2$ into (6.4).

Table. 10: The summary table for changes of correlations at 10% significant level. Numbers in parentheses are percentages of all pairs.

Model $D1$

	correlation in $D = 0 < D = 1$	correlation in $D = 0 > D = 1$
Number of changes	43 (96%)	2 (4%)
Number of significant changes	36 (80%)	0 (0%)

Model $D2$

	correlation in $D = 0 < D = 1$	correlation in $D = 0 > D = 1$
Number of changes	29 (64%)	16 (36%)
Number of significant changes	5 (11%)	3 (7%)

	correlation in $D = 1 < D = 2$	correlation in $D = 1 > D = 2$
Number of changes	44 (98%)	1 (2%)
Number of significant changes	36 (80%)	0 (0%)

	correlation in $D = 2 < D = 0$	correlation in $D = 2 > D = 0$
Number of changes	1 (2%)	44 (98%)
Number of significant changes	0 (0%)	40 (91%)

In the model $D1$, the correlations structural change is obviously. But, we can not reject the hypotheses that means of log-returns are identical across structure for all sector. And, we reject only five hypotheses that the variance are the same. Hence, we conclude that the model $D1$ does not capture the market boomings and recessions.

The model $D2$ also appears that the correlations structural change. However, the differences of means and variances are not identified clearly, like the full model.

Table 11 shows the summary of the correlations equal test in the model $D2S2$. In the model $D2S2$, the correlation structural change is appeared. However, the means and variances changes is not appeared clearly.

In conclusion, our proposed model – once structural change with recursive regime switching model is appropriate to distinguish the recursive market states (the dynamics of the individual means and variances) and the irreversible structural change (the dynamics of the correlations).

Table. 11: The summary table for changes of correlations at 10% significant level. Numbers in parentheses are percentages of all pairs.

Model $D2S2$

Fix D.

$D=0$	correlation in $S=0<S=1$	correlation in $S=0>S=1$
Number of changes	4 (9%)	41 (91%)
Number of significant changes	0 (0%)	13 (29%)

$D=1$	correlation in $S=0<S=1$	correlation in $S=0>S=1$
Number of changes	12 (27%)	33 (73%)
Number of significant changes	0 (0%)	4 (9%)

$D=2$	correlation in $S=0<S=1$	correlation in $S=0>S=1$
Number of changes	3 (7%)	42 (93%)
Number of significant changes	1 (2%)	14 (31%)

Fix $S=0$.

	correlation in $D=0<D=1$	correlation in $D=0>D=1$
Number of changes	24 (53%)	21 (47%)
Number of significant changes	2 (4%)	6 (13%)

	correlation in $D=1<D=2$	correlation in $D=1>D=2$
Number of changes	44 (98%)	1 (2%)
Number of significant changes	0 (0%)	31 (69%)

	correlation in $D=2<D=0$	correlation in $D=2>D=0$
Number of changes	2 (4%)	43 (96%)
Number of significant changes	0 (0%)	36 (80%)

Fix $S=1$.

	correlation in $D=0<D=1$	correlation in $D=0>D=1$
Number of changes	36 (80%)	9 (20%)
Number of significant changes	4 (9%)	0 (0%)

	correlation in $D=1<D=2$	correlation in $D=1>D=2$
Number of changes	36 (80%)	9 (20%)
Number of significant changes	12 (27%)	1 (2%)

	correlation in $D=2<D=0$	correlation in $D=2>D=0$
Number of changes	6 (13%)	39 (87%)
Number of significant changes	1 (2%)	20 (44%)

6.3 Tables

Table. 12: The estimated correlation coefficients when $D_t = 0$. Numbers in parenthesis are standard errors. The summary is shown in Table 7.

Before structural change ($D_t = 0$)

booming regime ($S_t = 0$)

Sector	ENE	MAT	IND	CD	CS	HC	FIN	IT	TEL	U
MAT	0.638	1.000								
	(0.084)									
IND	0.574	0.686	1.000							
	(0.095)	(0.075)								
CD	0.195	0.520	0.617	1.000						
	(0.133)	(0.102)	(0.087)							
CS	0.232	0.215	0.487	0.235	1.000					
	(0.127)	(0.128)	(0.105)	(0.131)						
HC	0.311	0.286	0.553	0.317	0.826	1.000				
	(0.121)	(0.123)	(0.095)	(0.125)	(0.045)					
FIN	0.379	0.400	0.697	0.412	0.487	0.569	1.000			
	(0.116)	(0.113)	(0.070)	(0.116)	(0.107)	(0.095)				
IT	0.279	0.486	0.419	0.383	0.164	0.247	0.276	1.000		
	(0.132)	(0.110)	(0.118)	(0.122)	(0.137)	(0.132)	(0.131)			
TEL	0.013	0.016	0.142	0.307	0.421	0.428	0.368	0.124	1.000	
	(0.078)	(0.101)	(0.116)	(0.121)	(0.115)	(0.112)	(0.115)	(0.132)		
U	0.268	-0.045	0.328	-0.052	0.328	0.337	0.503	-0.175	0.160	1.000
	(0.128)	(0.129)	(0.120)	(0.132)	(0.126)	(0.125)	(0.105)	(0.135)	(0.134)	

recession regime ($S_t = 1$)

Sector	ENE	MAT	IND	CD	CS	HC	FIN	IT	TEL	U
MAT	0.572	1.000								
	(0.095)									
IND	0.613	0.797	1.000							
	(0.088)	(0.052)								
CD	0.440	0.730	0.874	1.000						
	(0.114)	(0.066)	(0.033)							
CS	0.404	0.446	0.486	0.420	1.000					
	(0.118)	(0.113)	(0.108)	(0.116)						
HC	0.524	0.360	0.636	0.540	0.683	1.000				
	(0.102)	(0.123)	(0.084)	(0.100)	(0.075)					
FIN	0.608	0.659	0.843	0.827	0.545	0.717	1.000			
	(0.089)	(0.080)	(0.041)	(0.045)	(0.099)	(0.069)				
IT	0.234	0.358	0.681	0.715	0.117	0.353	0.614	1.000		
	(0.133)	(0.123)	(0.076)	(0.069)	(0.138)	(0.123)	(0.088)			
TEL	0.186	0.263	0.401	0.419	0.212	0.316	0.477	0.551	1.000	
	(0.136)	(0.131)	(0.118)	(0.117)	(0.135)	(0.127)	(0.109)	(0.099)		
U	0.370	0.321	0.347	0.234	0.408	0.386	0.298	0.170	0.183	1.000
	(0.122)	(0.127)	(0.124)	(0.133)	(0.118)	(0.120)	(0.129)	(0.137)	(0.136)	

Table. 13: The estimated correlation coefficients when $D_t = 1$. Numbers in parenthesis are standard errors. The summary is shown in Table 7.

After structural change ($D_t = 1$)

booming regime ($S_t = 0$)

Sector	ENE	MAT	IND	CD	CS	HC	FIN	IT	TEL	U
MAT	0.668	1.000								
	(0.063)									
IND	0.497	0.807	1.000							
	(0.086)	(0.040)								
CD	0.423	0.702	0.829	1.000						
	(0.093)	(0.058)	(0.035)							
CS	0.164	0.433	0.525	0.634	1.000					
	(0.110)	(0.092)	(0.082)	(0.068)						
HC	0.211	0.372	0.521	0.595	0.678	1.000				
	(0.108)	(0.097)	(0.083)	(0.073)	(0.061)					
FIN	0.383	0.570	0.648	0.735	0.526	0.461	1.000			
	(0.097)	(0.077)	(0.066)	(0.052)	(0.082)	(0.089)				
IT	0.389	0.609	0.785	0.833	0.489	0.577	0.644	1.000		
	(0.096)	(0.071)	(0.044)	(0.035)	(0.086)	(0.075)	(0.067)			
TEL	0.238	0.473	0.483	0.531	0.380	0.560	0.410	0.487	1.000	
	(0.107)	(0.088)	(0.087)	(0.081)	(0.097)	(0.078)	(0.095)	(0.086)		
U	0.491	0.402	0.416	0.439	0.440	0.525	0.404	0.452	0.588	1.000
	(0.086)	(0.095)	(0.094)	(0.091)	(0.091)	(0.082)	(0.095)	(0.090)	(0.074)	

recession regime ($S_t = 1$)

Sector	ENE	MAT	IND	CD	CS	HC	FIN	IT	TEL	U
MAT	0.927	1.000								
	(0.028)									
IND	0.834	0.925	1.000							
	(0.061)	(0.029)								
CD	0.799	0.926	0.947	1.000						
	(0.072)	(0.028)	(0.021)							
CS	0.794	0.854	0.912	0.883	1.000					
	(0.074)	(0.054)	(0.033)	(0.044)						
HC	0.709	0.774	0.805	0.761	0.829	1.000				
	(0.099)	(0.080)	(0.070)	(0.084)	(0.062)					
FIN	0.601	0.820	0.865	0.897	0.750	0.638	1.000			
	(0.127)	(0.065)	(0.050)	(0.039)	(0.087)	(0.118)				
IT	0.837	0.914	0.903	0.890	0.819	0.786	0.807	1.000		
	(0.060)	(0.033)	(0.037)	(0.041)	(0.065)	(0.076)	(0.069)			
TEL	0.710	0.735	0.735	0.804	0.697	0.603	0.706	0.820	1.000	
	(0.099)	(0.092)	(0.091)	(0.070)	(0.102)	(0.127)	(0.100)	(0.065)		
U	0.863	0.797	0.703	0.670	0.767	0.686	0.446	0.722	0.675	1.000
	(0.051)	(0.072)	(0.100)	(0.110)	(0.082)	(0.105)	(0.159)	(0.095)	(0.108)	

Table. 14: The results of the hypothesis testing: the null hypothesis is the equality of correlations between two regimes $S_t = 0$ and $S_t = 1$. Marks $*$, $**$, and $***$ indicate rejecting the null at significant level 10%, 5%, and 1%, respectively. Numbers in parentheses are P-values. The summary is shown in Table 7.

before structural break ($D_t = 0$)

	ENE	MAT	IND	CD	CS	HC	FIN	IT	TEL	U
MAT	0.266									
	(0.606)									
IND	0.087	1.487								
	(0.768)	(0.223)								
CD	1.966	2.955*	7.654***							
	(0.161)	(0.086)	(0.006)							
CS	0.983	1.822	0.000	1.109						
	(0.321)	(0.177)	(0.993)	(0.292)						
HC	1.801	0.178	0.422	1.910	2.629					
	(0.180)	(0.673)	(0.516)	(0.167)	(0.105)					
FIN	2.427	3.447*	3.188*	11.141***	0.158	1.588				
	(0.119)	(0.063)	(0.074)	(0.001)	(0.691)	(0.208)				
IT	0.057	0.597	3.453*	5.567**	0.056	0.341	4.553**			
	(0.811)	(0.440)	(0.063)	(0.018)	(0.812)	(0.559)	(0.033)			
TEL	1.221	2.207	2.448	0.444	1.383	0.434	0.473	6.683**		
	(0.269)	(0.137)	(0.118)	(0.505)	(0.240)	(0.510)	(0.491)	(0.010)		
U	0.334	4.071	0.012	2.320	0.215	0.079	1.526	3.196*	0.014	
	(0.563)	(0.044)	(0.914)	(0.128)	(0.643)	(0.778)	(0.217)	(0.074)	(0.905)	

after structural break ($D_t = 1$)

	ENE	MAT	IND	CD	CS	HC	FIN	IT	TEL	U
MAT	13.998***									
	(0.000)									
IND	10.257***	5.872**								
	(0.001)	(0.015)								
CD	10.180***	12.180***	8.252***							
	(0.001)	(0.000)	(0.004)							
CS	22.570***	15.590***	19.128***	9.559***						
	(0.000)	(0.000)	(0.000)	(0.002)						
HC	11.580***	10.216***	6.896***	2.232	3.028*					
	(0.001)	(0.001)	(0.009)	(0.135)	(0.082)					
FIN	1.854	6.091**	6.782***	6.080**	3.500*	1.442				
	(0.173)	(0.014)	(0.009)	(0.014)	(0.061)	(0.230)				
IT	15.616***	15.211***	4.340**	1.078	9.308***	3.814*	2.877*			
	(0.000)	(0.000)	(0.037)	(0.299)	(0.002)	(0.051)	(0.090)			
TEL	10.474***	4.167**	3.966**	6.489**	5.009**	0.083	4.560**	9.523***		
	(0.001)	(0.041)	(0.046)	(0.011)	(0.025)	(0.773)	(0.033)	(0.002)		
U	13.861***	10.890***	4.360**	2.615	7.135***	1.451	0.051	4.257**	0.437	
	(0.000)	(0.001)	(0.037)	(0.106)	(0.008)	(0.228)	(0.822)	(0.039)	(0.508)	

Table. 15: The results of the hypothesis testing: the null hypothesis is the equality of correlations between two regimes $D_t = 0$ and $D_t = 1$. Marks $*$, $**$, and $***$ indicate rejecting the null at significant level 10%, 5%, and 1%, respectively. Numbers in parentheses are P-values. The summary is shown in Table 7.

booming regime ($S_t = 0$)

	ENE	MAT	IND	CD	CS	HC	FIN	IT	TEL	U
MAT	0.081									
	(0.776)									
IND	0.369	2.026								
	(0.544)	(0.155)								
CD	1.975	2.385	5.127**							
	(0.160)	(0.123)	(0.024)							
CS	0.168	1.919	0.081	7.305***						
	(0.682)	(0.166)	(0.776)	(0.007)						
HC	0.388	0.300	0.065	3.694*	3.792*					
	(0.533)	(0.584)	(0.798)	(0.055)	(0.052)					
FIN	0.001	1.536	0.257	6.490**	0.084	0.696				
	(0.981)	(0.215)	(0.612)	(0.011)	(0.772)	(0.404)				
IT	0.459	0.873	8.456***	12.615***	4.080**	4.683**	6.266**			
	(0.498)	(0.350)	(0.004)	(0.000)	(0.043)	(0.030)	(0.012)			
TEL	2.921*	11.596***	5.543**	2.371	0.072	0.929	0.082	5.281**		
	(0.087)	(0.001)	(0.019)	(0.124)	(0.788)	(0.335)	(0.775)	(0.022)		
U	2.095	7.820***	0.336	9.374***	0.514	1.585	0.492	14.976***	7.830***	
	(0.148)	(0.005)	(0.562)	(0.002)	(0.473)	(0.208)	(0.483)	(0.000)	(0.005)	

recession regime ($S_t = 1$)

	ENE	MAT	IND	CD	CS	HC	FIN	IT	TEL	U
MAT	12.821***									
	(0.000)									
IND	4.284**	4.726**								
	(0.038)	(0.030)								
CD	7.103***	7.456***	3.445*							
	(0.008)	(0.006)	(0.063)							
CS	7.852***	10.621***	14.271***	13.934***						
	(0.005)	(0.001)	(0.000)	(0.000)						
HC	1.684	8.005***	2.405	2.879*	2.246					
	(0.194)	(0.005)	(0.121)	(0.090)	(0.134)					
FIN	0.002	2.409	0.115	1.385	2.407	0.338				
	(0.963)	(0.121)	(0.734)	(0.239)	(0.121)	(0.561)				
IT	17.196***	19.155***	7.024***	4.712**	21.044***	8.910***	2.979*			
	(0.000)	(0.000)	(0.008)	(0.030)	(0.000)	(0.003)	(0.084)			
TEL	9.772***	8.685***	4.985**	8.009***	8.250***	2.556	2.397	5.184**		
	(0.002)	(0.003)	(0.026)	(0.005)	(0.004)	(0.110)	(0.122)	(0.023)		
U	13.940***	10.649***	4.981**	6.379**	6.274**	3.534*	0.520	10.945***	7.975***	
	(0.000)	(0.001)	(0.026)	(0.012)	(0.012)	(0.060)	(0.471)	(0.001)	(0.005)	

Table. 16: The results of the hypothesis testing in the reduced model of $q_{00} = 1$: the null hypothesis is the equality of correlations between two regimes $S_t = 0$ and $S_t = 1$. Marks $*$, $**$, and $***$ indicate rejecting the null at significant level 10%, 5%, and 1%, respectively. Numbers in parentheses are P-values.

	ENE	MAT	IND	CD	CS	HC	FIN	IT	TEL	U
MAT	0.949									
	(0.330)									
IND	1.376	0.003								
	(0.241)	(0.958)								
CD	1.806	4.326**	5.965**							
	(0.179)	(0.038)	(0.015)							
CS	6.146**	4.842**	0.954	0.042						
	(0.013)	(0.028)	(0.329)	(0.838)						
HC	8.081***	3.174*	2.098	0.011	0.297					
	(0.004)	(0.075)	(0.147)	(0.917)	(0.586)					
FIN	2.058	3.996**	3.696*	3.262*	0.028	0.024				
	(0.151)	(0.046)	(0.055)	(0.071)	(0.867)	(0.878)				
IT	0.081	0.630	0.027	0.172	1.473	0.346	0.221			
	(0.776)	(0.428)	(0.869)	(0.678)	(0.225)	(0.557)	(0.638)			
TEL	0.507	0.030	0.082	0.327	3.523*	1.784	0.036	7.341***		
	(0.476)	(0.862)	(0.774)	(0.568)	(0.061)	(0.182)	(0.850)	(0.007)		
U	0.672	0.078	0.001	0.031	0.176	0.204	0.420	0.182	10.922***	
	(0.412)	(0.780)	(0.981)	(0.861)	(0.675)	(0.651)	(0.517)	(0.670)	(0.001)	

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