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Abstract

This study extends Baumol's (1967) two-sector (manufacturing and services) unbalanced growth model to analyze a situation in which, first, services are used for both final consumption and intermediate inputs into manufacturing production, and second, the productivities of the manufacturing and services sectors endogenously evolve. By using this model, we examine the conditions under which the employment share of services increases over time and investigate how the economic growth rate evolves as a result. Our results are summarized as follows. First, if the human capital accumulation function exhibits constant returns to scale with respect to per capita consumption of services, then we obtain a U-shaped relationship between the employment share of services and the economic growth rate. Second, if the human capital accumulation function exhibits decreasing returns to scale with respect to per capita consumption of services, the economic growth rate decreases first, begins to increase after some time, again decreases, and finally, approaches zero.

Keywords: service economy; economic growth; endogenous productivity growth; business services

JEL Classification: J21; J24; O11; O14; O30; O41

1 Introduction

Why does the share of services tend to increase over time? What is the relationship between the tendency toward services and economic growth? This study is an attempt to answer these questions.

Here, we define the tendency toward services as an increase in the employment share of the service sector. This tendency toward services is broadly observed in developed economies.

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Baumol (1967) is a pioneering work that examines the relationship between the tendency toward services and economic growth. He builds a two-sector (manufacturing and services) unbalanced growth model to investigate why the employment share of services increases and the relationship between the tendency toward services and economic growth.[1)] He shows that if the productivity growth of manufacturing is higher than that of services (Baumol's first assumption) and, in addition, if the ratio of demand for services and demand for manufacturing is constant (Baumol's second assumption), then the employment share of services increases over time and the rate of economic growth continues to decline, and finally, approaches the rate of productivity growth of services.

After Baumol's seminal work, many studies on the relationship between the tendency toward services and economic growth have been produced. This study integrates the elements of Sasaki (2007) and Sasaki (2012), both of which develop Baumol's (1967) argument.[2)]

To begin with, Sasaki (2007) introduces intermediate service inputs into the Baumol model and investigates how Baumol's results change. In Baumol (1967), services are used entirely for final consumption. In this respect, Oulton (2001) observes that services are used for intermediate inputs as well as final consumption and models such a situation.[3)] He concludes that as the employment share of services increases, the rate of economic growth also increases. However, in Oulton's model, services are devoted entirely to intermediate inputs into manufacturing, and hence, no services are used for final consumption. Based on this argument, Sasaki (2007) builds a model to capture a situation in which services are used for both intermediate inputs into manufacturing and final consumption. He reaches a conclusion similar to Baumol (1967): if enough time passes, the rate of economic growth declines with the tendency toward services.

Next, Sasaki (2012) introduces endogenous technological progress into the Baumol model and investigates the relationship between the tendency toward service and economic growth. His study is influenced by the work of Pugno (2006). Pugno considers that the consumption of services augments human capital à la Lucas (1988). The consumption of health care and education services will lead to human capital accumulation. Accordingly, the consumption of services increases the productivity of workers, thereby resulting in an increase in the productivity of both manufacturing and services. He incorporates this human capital accumulation effect into Baumol's model and shows that if this effect is relatively strong, the employment shift toward services increases, not decreases, the rate of economic

1) Kapur (2012) builds a model and investigates a situation in which the service sector is divided into two subsectors: a service sector in which productivity stagnates and a service sector in which productivity grows.

2) For a survey of the literature on relationship between the tendency toward services and economic growth, see studies cited in Sasaki (2007, 2012).

3) Services as intermediate inputs are related to the outsourcing of services. For the relationship between the outsourcing of services and economic growth, see Fixler and Siegel (1999).

growth. Sasaki (2012) considers a learning by doing effect in manufacturing as well as Pugno's (2006) human capital accumulation effect. He shows that the employment shift toward services and the rate of economic growth have a U-shaped relationship. That is, if the employment share of services begins to increase from the value of zero, the economic growth rate begins to decline with an increase in the employment share of services, but after some time, it begins to increase with the increase in the employment share of services.

The present study, by integrating the elements of Sasaki (2007) and Sasaki (2012), builds a more general model and investigates the relationship between the employment shift toward services and the economic growth rate. Specifically, services are used for both final consumption and intermediate inputs, human capital is accumulated through the consumption of services, and the productivity of manufacturing increases through learning by doing.

The main results are as follows. First, if the human capital accumulation function exhibits constant returns to scale with respect to per capita consumption of services, then we obtain a U-shaped relationship between the employment share of services and the economic growth rate. Second, if the human capital accumulation function exhibits decreasing returns to scale with respect to per capita consumption of services, the economic growth rate decreases first, begins to increase after some time, again decreases, and finally, approaches zero.

The remainder of the paper is organized as follows. Section 2 builds our model. Section 3 derives the instantaneous equilibrium. Section 4 investigates the equilibrium path by using both an analytical method and numerical simulations. Section 5 concludes the paper.

2 Model

Consider a closed economy that consists of the manufacturing and service sectors. In the manufacturing sector, manufactured goods are produced with labor inputs and intermediate service inputs. In the service sector, services are produced with only labor inputs.[4)] Consumers consume both manufactured goods and services.

2.1 Firms

We specify the production functions of both sectors as follows:

$$Q_m = A_m \left[\beta^{\frac{1}{\psi}} (hL_m)^{\frac{\psi-1}{\psi}} + (1-\beta)^{\frac{1}{\psi}} S^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}, \quad \beta \in (0,1),\ \psi > 0,\ \psi \neq 1, \tag{1}$$

4) In our model, only labor is the primary factor of production. For an analysis of a service-oriented economy using models that consider capital accumulation, see Kongsamut, Rebelo, and Xie (2001) and Klyuev (2005).

$$Q_s = A_s(hL_s), \tag{2}$$

where Q_m denotes the output of manufacturing; Q_s the output of services; L_m the employment of manufacturing; L_s the employment of services; S the intermediate service inputs into manufacturing; A_m the productivity specific to manufacturing; h the level of human capital; β a positive parameter; and ψ the elasticity of substitution between labor inputs and intermediate service inputs. A_s denotes the productivity specific to services. Human capital is accumulated in workers themselves by consuming services, and accordingly, both L_m and L_s are multiplied by h.

Profits of manufacturing firms π_m and service firms π_s are given as follows:

$$\pi_m = p_m Q_m - (wL_m + p_s S), \tag{3}$$

$$\pi_s = p_s Q_s - wL_s, \tag{4}$$

where p_m denotes the price of manufactured goods and p_s the price of services. The wage rate is denoted by w. Suppose that labor is perfectly free to move between the two sectors. Then, the nominal wages in both sectors are equalized.

2.2 Consumers

We specify the problem of utility maximization of consumers. Suppose that the representative consumer solves the following optimization problem.

$$\max_{c_m, c_s} u = \left[\alpha^{\frac{1}{\sigma}} c_m^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} (c_s + \gamma)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

$$\alpha \in (0,1),\ \sigma > 0,\ \sigma \neq 1,\ \gamma > 0,$$

$$\text{s.t. } p_m c_m + p_s c_s = w, \tag{6}$$

where c_i denotes per capita consumption ($c_i = C_i/L$); σ the elasticity of substitution between the two types of consumption; α a positive parameter governing the weight of expenditure for manufacturing; and γ a positive parameter. Such a non-homothetic preference is also adopted by Iscan (2010). When $\gamma = 0$, the preference is homothetic, and hence, the income elasticities of manufacturing consumption and services consumption are unity. When $\gamma > 0$, the preference is non-homothetic, and hence, the income elasticity of manufacturing demand is less than unity and that of services demand is greater than unity. Introducing a positive γ does not affect the dynamics of the employment share very much. However, it does affect the dynamics of the consumption ratio C_s/C_m. Baumol (1967) assumes that the consumption ratio remains constant over time. The assumption $\gamma > 0$ corresponds to Baumol's Assumption 2.

2.3 Labor and goods markets

Suppose that total labor supply L is constant. Then, the labor market clearing condition is given by

$$L_m + L_s = L. \tag{7}$$

The goods market clearing condition is given by

$$Q_m = C_m, \tag{8}$$

$$Q_s = C_s + S. \tag{9}$$

The manufactured goods are all used for final consumption. On the other hand, services are used for both final consumption and intermediate inputs.

2.4 Productivity growth

Here, we define economic growth used in this paper. In our model, there is no capital accumulation or population growth, and hence, productivity growth is the engine of growth. In what follows, we refer to an increase in total factor productivity (TFP) of the whole economy as economic growth.

The growth rate of TFP of the manufacturing sector $g_{\text{TFP},m}$ is given by

$$g_{\text{TFP},m} = g_{A_m} + \frac{\beta^{\frac{1}{\psi}}(hL_m)^{\frac{\psi-1}{\psi}}}{\beta^{\frac{1}{\psi}}(hL_m)^{\frac{\psi-1}{\psi}} + (1-\beta)^{\frac{1}{\psi}} S^{\frac{\psi-1}{\psi}}} g_h, \tag{10}$$

where $g_x \equiv \dot{x}/x$ denotes the growth rate of a variable x.

On the other hand, the growth rate of TFP of the service sector $g_{\text{TFP},s}$ is given by

$$g_{\text{TFP},s} = g_{A_s} + g_h. \tag{11}$$

Let us specify g_{A_m}, g_{A_s}, and g_h that appear in Equations (10) and (11).

First, the productivity specific to A_s increases exogenously, which is specified as follows:

$$A_s(t) = A_{s,0}(1 + \mu\theta t)^{\frac{1}{\theta}}, \quad \theta \geq 0,\ \mu > 0, \tag{12}$$

where $A_{s,0}$ denotes the initial level of A_s; and μ and θ are positive parameters. When specifying an exogenous and continuous increase in a variable, we usually assume that the variable increases at a constant rate, and hence, we use an exponential function. Instead, based on the study of Groth, Koch, and Steger (2010), we use a specification more flexible than an exponential function. Accordingly, the growth rate of A_s is given by

$$g_{A_s} = \frac{\mu}{1 + \mu\theta t}. \tag{13}$$

From this, we know that if $\theta > 0$, the growth rate of A_s declines over time.

This specification includes some special cases. First, if $\theta = 0$, we obtain

$$A_s = A_{s,0}e^{\mu t}, \tag{14}$$

which is an exponential function. Second, if $\theta \to +\infty$, we obtain

$$A_s = A_{s,0}, \tag{15}$$

which is constant. Third, if $\theta = 1$, we obtain

$$A_s = A_{s,0}(1 + \mu t), \tag{16}$$

which is a linear function of time.

Next, we specify the productivity specific to A_m. We assume that A_m is an increasing function of the knowledge stock K_m.

$$A_m = K_m^{\phi}, \quad \phi > 0, \tag{17}$$

where ϕ denotes the elasticity of A_m with respect to K_m. We assume that the knowledge stock depends on the production experience that is accumulated until now; we specify the knowledge stock as follows:

$$K_m = \exp\left[\int_{-\infty}^{t} \frac{L_m(\tau)}{L(\tau)}\, d\tau\right]. \tag{18}$$

Note that the production experience K_m is measured by L_m/L. We use the manufacturing employment share, and not the level of manufacturing employment, to ascertain that the dynamics of the model hold even when the labor force grows. In addition, we use manufacturing employment, and not manufacturing output, for the following reason. In our model, labor is the sole factor of production; then, an increase in output has a one-to-one relationship with an increase in employment. Therefore, for simplicity, we measure production experience by employment of manufacturing, not by output of manufacturing.

Substituting Equation (19) into Equation (18) and differentiating the resultant expression with respect to time, we obtain

$$\dot{A}_m = \left(\phi \frac{L_m}{L}\right) A_m. \tag{19}$$

That is, A_m becomes an increasing function of the employment share of manufacturing.[5)]

Then, we specify the accumulation of human capital h. According to Pugno (2006), human capital is accumulated through consumption services.[6)]

$$\dot{h} = \delta c_s^{\lambda}, \quad \delta > 0,\ 0 < \lambda < 1, \tag{20}$$

where δ denotes a positive parameter and captures the efficiency of human capital accumulation. Note that Equation (20) is different from the specifications of Pugno (2006) and Sasaki (2012). They assume that $\lambda = 1$ whereas we assume that $0 < \lambda \leq 1$. The reason is that we investigate a broader situation, which includes a knife-edge case $\lambda = 1$. As will be shown later, if we assume that $0 < \lambda < 1$, economic growth is not sustainable in the very long run. Instead, we introduce productivity specific to services, that is, A_s and consider the case in which A_s increases over time.

We derive the growth rate of TFP of the whole of economy. Note that in our model, services are used for intermediate inputs into manufacturing. In this case, as Oulton (2001) and Sasaki (2007) adopt, it is appropriate to use Domar aggregation presented by Domar (1961).[7)] Using Domar aggregation, we obtain the growth rate of TFP g_{TFP} as follows:

$$g_{\mathrm{TFP}} = \frac{p_m Q_m}{\mathrm{GDP}} g_{\mathrm{TFP},m} + \frac{p_s Q_s}{\mathrm{GDP}} g_{\mathrm{TFP},s}. \tag{21}$$

Gross domestic product (GDP) is given by $p_m C_m + p_s C_s = wL$. Equation (21) considers that services are used for intermediate inputs, and hence, the sum of the weights exceeds unity, that is, $p_m Q_m + p_s Q_s > \mathrm{GDP}$.

5) De Vincenti (2007) adopts a specification in which the growth rate of manufacturing productivity is a decreasing function of the employment share of manufacturing.

6) In addition, the relationship between health services and economic growth is analyzed in Van Zon and Muysken (2001).

7) For Domar aggregation, see also ten Raa and Schettkat (2001).

3 Instantaneous equilibrium

At some point in time, A_m, A_s, and h are given. We obtain the instantaneous equilibrium as follows:

1. Solving the utility maximization problem of consumers, we obtain demand functions for manufacturing and services, which depend on p_m, p_s, and w.
2. Solving the profit maximization problem of firms, we obtain the optimal ratio of L_m and S, which depends on p_s and w.
3. Solving the zero profit conditions of firms, we obtain p_m and p_s, which depend on A_m, A_s, h, and w.
4. Solving the goods market clearing condition, we obtain L_m and L_s.

From the utility maximization problem, we obtain demand functions for manufacturing and services.

$$C_m = \frac{\alpha}{1-\alpha}\left(\frac{p_s}{p_m}\right)^{\sigma}\left[\frac{w - \frac{\alpha}{1-\alpha}p_m\left(\frac{p_s}{p_m}\right)^{\sigma}\gamma}{p_s + \frac{\alpha}{1-\alpha}p_m\left(\frac{p_s}{p_m}\right)^{\sigma}} + \gamma\right]L, \tag{22}$$

$$C_s = \frac{w - \frac{\alpha}{1-\alpha}p_m\left(\frac{p_s}{p_m}\right)^{\sigma}\gamma}{p_s + \frac{\alpha}{1-\alpha}p_m\left(\frac{p_s}{p_m}\right)^{\sigma}} \cdot L. \tag{23}$$

From the profit maximization problem of manufacturing and service firms, that is, the equalization between the marginal rate of substitution and the relative price, we obtain

$$S = \frac{1-\beta}{\beta}A_s^{\psi}(hL_m). \tag{24}$$

That is, intermediate service inputs are linear with respect to effective labor hL_m.

From the zero profit conditions, we obtain

$$p_m = \frac{w}{\beta^{\frac{1}{\psi-1}}A_m h\left(1 + \frac{1-\beta}{\beta}A_s^{\psi-1}\right)^{\frac{1}{\psi-1}}}, \tag{25}$$

$$p_s = \frac{w}{A_s h}. \tag{26}$$

Hence, the relative price is given by

$$\frac{p_s}{p_m} = \beta^{\frac{1}{\psi-1}} \frac{A_m}{A_s} \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)^{\frac{1}{\psi-1}} . \tag{27}$$

With Equation (22), we can express Q_m as a function of L_m.

$$Q_m = \beta^{\frac{1}{\psi-1}} A_m h L_m \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)^{\frac{\psi}{\psi-1}} . \tag{28}$$

That is, the output of manufacturing is linear with respect to the employment of manufacturing.

From the goods market clearing condition $C_m = Q_m$, we obtain

$$\frac{\alpha}{1-\alpha} \left(\frac{p_s}{p_m}\right)^{\sigma} \left[\frac{w - \frac{\alpha}{1-\alpha} p_m \left(\frac{p_s}{p_m}\right)^{\sigma} \gamma}{p_s + \frac{\alpha}{1-\alpha} p_m \left(\frac{p_s}{p_m}\right)^{\sigma}} + \gamma \right] L = \beta^{\frac{1}{\psi-1}} A_m h L_m \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)^{\frac{\psi}{\psi-1}} . \tag{29}$$

Solving equation (29) for L_m/L, we obtain

$$\frac{L_m}{L} = \frac{\frac{\alpha}{1-\alpha} \left(\frac{p_s}{p_m}\right)^{\sigma} \left[\frac{w - \frac{\alpha}{1-\alpha} p_m \left(\frac{p_s}{p_m}\right)^{\sigma} \gamma}{p_s + \frac{\alpha}{1-\alpha} p_m \left(\frac{p_s}{p_m}\right)^{\sigma}} + \gamma \right]}{\beta^{\frac{1}{\psi-1}} A_m h \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)^{\frac{\psi}{\psi-1}}}. \tag{30}$$

Consumption of services is given by $C_s = Q_s - S$, which is rewritten in per capita terms as

$$c_s = A_s h \frac{L_s}{L} - \frac{1-\beta}{\beta} A_s^{\psi} h \left(1 - \frac{L_s}{L}\right). \tag{31}$$

From this, the rate of human capital accumulation is given by

$$\frac{\dot{h}}{h} = \frac{\delta}{h} \left[A_s h \frac{L_s}{L} - \frac{1-\beta}{\beta} A_s^{\psi} h \left(1 - \frac{L_s}{L}\right) \right]^{\lambda} . \tag{32}$$

For the rate of human capital accumulation to be positive, we need

$$\frac{L_s}{L} > \frac{1}{1 + \frac{1-\beta}{\beta} A_s^{1-\psi}}. \tag{33}$$

The ratio of manufacturing output to GDP and the ratio of services output to GDP are given by the following equations, respectively,

$$\frac{p_m Q_m}{\mathrm{GDP}} = \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right) \frac{L_m}{L}, \tag{34}$$

$$\frac{p_s Q_s}{\mathrm{GDP}} = \frac{L_s}{L} = 1 - \frac{L_m}{L}. \tag{35}$$

The growth rate of TFP of manufacturing is given by

$$g_{\mathrm{TFP},m} = g_{A_m} + \frac{1}{1 + \frac{1-\beta}{\beta} A_s^{\psi-1}} g_h. \tag{36}$$

Therefore, the growth rate of TFP for the whole economy is given by

$$g_{\mathrm{TFP}} = \frac{L_m}{L}\left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right) g_{\mathrm{TFP},m} + \left(1 - \frac{L_m}{L}\right) g_{\mathrm{TFP},s}. \tag{37}$$

Suppose that the employment share of services increases over time, and finally, reaches $L_s/L = 1$. Then, the growth rate of TFP for the whole economy is given by

$$g_{\mathrm{TFP}} = g_h = \delta A_s^{\lambda} h^{\lambda-1}. \tag{38}$$

That is, the growth rate of TFP for the whole economy is equal to the growth rate of human capital. If $0 < \lambda < 1$ and if h takes a large value in the long run, we have $g_{\mathrm{TFP}} \to 0$. In contrast, if $\lambda = 1$, we have $g_{\mathrm{TFP}} = \delta A_s$. Hence, if A_s is constant, we obtain sustainable growth, that is, $g_{\mathrm{TFP}} > 0$.

4 Equilibrium path and numerical simulations

Each sector's employment share, each sector's productivity growth, and the productivity growth of the whole economy depend on A_m and h. Moreover, $\dot{A}_m$ and $\dot{h}$ also depend on A_m and h. As a result, if we give the initial values of A_m and h, and if we examine the system of differential equations of A_m and h, we can obtain the time paths of A_m and h. In addition, by using this result, we can know the time paths of all variables. However, the differential equations of out model are non-linear, and hence, analytical solutions are hard to obtain. Accordingly, we use numerical simulations, if needed.

4.1 Case in which $\lambda = 1$

Let $\lambda = 1$ in Equation (20). This case is investigated in Pugno (2006) and Sasaki (2012), although they do not consider intermediate services inputs. Then, if $\theta \to +\infty$ and $A_{s,0} = 1$, we obtain $A_s(t) = 1$.[8)]

In this case, the growth rate of TFP for the whole economy leads to

$$g_{\mathrm{TFP}} = \beta^{-1} \frac{L_m}{L}(g_{A_m} + \beta g_h) + \left(1 - \frac{L_m}{L}\right) g_h. \tag{39}$$

From this, we see that the elasticity of substitution between labor inputs and intermediate services inputs in manufacturing, that is, ψ, never affects the result.

If the employment share of services increases and L_s/L approaches unity, then the limit of each sector's productivity growth is given by

$$\lim_{L_s/L \to 1} g_{A_m} = 0, \tag{40}$$

$$\lim_{L_s/L \to 1} g_h = \delta, \tag{41}$$

$$\lim_{L_s/L \to 1} g_{\mathrm{TFP},m} = \beta\delta, \tag{42}$$

$$\lim_{L_s/L \to 1} g_{\mathrm{TFP}} = g_h = \delta. \tag{43}$$

The growth rate of productivity specific to manufacturing approaches zero, the rate of human capital accumulation approaches δ, the growth rate of manufacturing TFP approaches $\beta\delta$, and the growth rate of TFP for the whole economy approaches δ, which is the same as the rate of human capital accumulation.

The value of $g_{\mathrm{TFP}}(0)$, that is, the growth rate of TFP for the whole economy when the employment share of services is zero, must be positive.

$$g_{\mathrm{TFP}}(0) = \beta^{-1}[\phi - \delta(1 - \beta)]. \tag{44}$$

From this, we need the following condition:

$$\phi - \delta(1 - \beta) > 0. \tag{45}$$

Hereafter, we assume that this condition holds.

Comparing $g_{\mathrm{TFP}}(0)$ with $g_{\mathrm{TFP}}(1)$, which is obtained when $L_s/L = 1$, we can see whether an increase in the employment share of services increases or decreases the rate of economic

8) For the main equations in this case, see the Appendix.

growth in the long run. If $g_{\text{TFP}}(0) < g_{\text{TFP}}(1)$, the rate of economic growth increases in the long run. In contrast, if $g_{\text{TFP}}(0) > g_{\text{TFP}}(1)$, the rate of economic growth decreases in the long run. From this, we obtain

$$g_{\text{TFP}}(0) \lesseqgtr g_{\text{TFP}}(1) \iff \phi \lesseqgtr \delta. \tag{46}$$

With the calculation of g_{TFP}, we find that it is a quadratic function of L_s/L, which is given by

$$\begin{aligned} g_{\text{TFP}} &= \frac{\phi}{\beta}\left(\frac{L_s}{L}\right)^2 + \frac{\delta - 2\phi}{\beta}\frac{L_s}{L} + \frac{\phi - \delta(1-\beta)}{\beta} \\ &= \frac{\phi}{\beta}\left(\frac{L_s}{L} - \frac{2\phi - \delta}{2\phi}\right)^2 + \delta\left(\frac{4\beta\phi - \delta}{4\beta\phi}\right). \end{aligned} \tag{47}$$

This result is almost the same as that of Sasaki (2012). With $\beta = 1$, Equation (47) is the same as that of Sasaki (2012). Hence, Proposition 3 of Sasaki (2012) holds as is.

Proposition 1. *If $2\phi < \delta$, the growth rate of* TFP *increases with the employment shift toward services and converges to δ. If $2\phi > \delta$, the growth rate of* TFP *decreases until the employment share of services reaches $(L_s/L)^* = (2\phi - \delta)/2\phi$, and from then onward, it increases with the employment shift toward services, finally converging to δ.*

This proposition is shown in Figures 1–3.

[Figures 1–3 around here]

When deriving Proposition 1, we assume that the employment share of services increases over time. In our model, productivity growth and employment share are dependent on each other. Therefore, we must analyze the dynamics of employment share and productivity growth simultaneously.

Because the abovementioned system is non-linear and somewhat complicated, we use numerical simulations to analyze the dynamics. Here, we investigate the following three cases.

$$\text{Case 1} : \phi < \delta,\ 2\phi < \delta, \tag{48}$$

$$\text{Case 2} : \phi < \delta,\ 2\phi > \delta, \tag{49}$$

$$\text{Case 3} : \phi > \delta. \tag{50}$$

In Cases 1 and 2, we have $g(0) < g(1)$, and in Case 3, we have $g(0) > g(1)$.

Table 1: List of parameters ($\lambda = 1$)

	α	β	γ	δ	ϕ
Case 1	0.5	0.5	1	0.02	0.015
Case 2	0.5	0.9	1	0.03	0.02
Case 3	0.5	0.5	1	0.02	0.04

We set the parameters as in Table 1 and the initial values as $A_m(0) = h(0) = 1$. Then, we change the elasticity of substitution σ from 0.1 to 0.9 in intervals of 0.1.

Figures 4–9 show the results of the numerical simulation. Figures 4, 6, and 8 show the time paths of the employment share of services. Figures 5, 7, and 9 show the time paths of the growth rate of TFP for the whole economy. In all cases, the employment share of services increases over time. In Cases 1 and 2, the growth rate of TFP first declines, then increases, and finally, converges to δ. In Case 3, the growth rate of TFP increases constantly over time, which is similar to the result of Oulton (2001).

[Figures 4–9 around here]

These results are obtained given that $\theta \to +\infty$. However, if θ is sufficiently larger than unity, similar results are also obtained.

4.2 Case in which $0 < \lambda < 1$

Let $0 < \lambda < 1$ in Equation (20). In this case, analytical solutions are hard to obtain, and thus, we resort to numerical simulations.

We set the parameters as in Table 2 and the initial values as $A_m(0) = h(0) = 1$. Then, we change the elasticity of substitution σ from 0.1 to 0.9 in intervals of 0.1.

Table 2: List of parameters ($0 < \lambda < 1$)

	α	β	γ	δ	ϕ	ψ	λ	μ	θ
Case 4	0.5	0.5	1	0.01	0.02	1.2	0.9	0.03	0.5
Case 5	0.5	0.5	1	0.01	0.02	0.2	0.9	0.03	0.5

As stated in Equation (36), as long as enough time passes, the growth rate of TFP for the whole economy approaches zero. However, along with transitional dynamics, the employment share of services and the growth rate of TFP display interesting behavior. In Case 4, the employment share of services monotonically increases whereas the growth rate of TFP first declines, then increases, and finally, declines after some time. In Case 5, the employment share of services first increases, then declines, and finally, increases after some time. In this case, the growth rate of TFP shows similar behavior to that in Case 4.

[Figures 10–13 around here]

The time paths of the growth rate of TFP in Cases 4 and 5 can be explained as follows. First, as the employment share of services increases, the learning by doing effect of manufacturing weakens, which lowers the economic growth rate. However, as the employment share of services increases further, human capital accumulation proceeds further, and productivity specific to services increases with the passage of time. These contribute to an increase in the growth rate of TFP. Nevertheless, as enough time passes, there is a diminishing positive effect of an increasing services employment share on the growth rate of TFP. In the long run, the growth rate of TFP approaches zero. The phase of decreasing → increasing→ is similar to the time path obtained in Sasaki (2012) while the phase of increasing → decreasing→ is similar to the time path obtained in Sasaki (2007).

5 Conclusion

This study extended Baumol's unbalanced growth model to investigate the relationship between the tendency toward services and economic growth. In our model, the productivity growth of manufacturing and that of services were endogenously determined and services were used for intermediate inputs into manufacturing as well as for final consumption.

We showed that the specification of human capital accumulation affects the results. If the human capital accumulation function has constant returns to scale with respect to per capita consumption of services, the result is similar to that of Sasaki (2012), who does not consider intermediate service inputs. We obtain a U-shaped relationship between the employment share of services and economic growth. In contrast, if the human capital accumulation function has decreasing returns to scale with respect to per capita consumption of services, we obtain the combination of a U-shaped and an inverted U-shaped relationship between the employment share of services and economic growth.

Our model considers both the case of services playing the role of intermediate inputs and the case of consumption of educational and health services leading to human capital accumulation. Hence, it is more realistic than the existing literature. The main result is that the relationship between the tendency toward services and economic growth is not monotonous. This result has implications for empirical analysis regarding the relationship between the tendency toward services and economic growth.

Hartwig (2012) empirically tested the three hypotheses about the relationship between the tendency toward services and economic growth: Baumol's (1967) hypothesis—a shift in employment share toward the services sector decreases the rate of economic growth;

Kaldor's (1957) hypothesis—the growth rate of per capita real GDP is almost constant despite the presence of structural changes; and Pugno's (2006) hypothesis—a shift in the employment share toward services increases the economic growth rate. He concluded that no evidence is found in support of Pugno's hypothesis and it less evident whether structural change is compatible with balanced growth or whether it leads to long-term stagnation.

Our model can explain why such ambiguous empirical results are obtained. The relationship between the employment shift toward services and economic growth is not monotonous. Consequently, if we perform a cross-country analysis for countries whose stages of development are diversified, we are likely to obtain ambiguous results. Moreover, if we perform a time series analysis for a specific country, we are likely to obtain different results depending on which period of data is used.

Appendix

In case of $\lambda = 1$, $\theta \to +\infty$, and $A_{s,0} = 1$, the main equations are obtained as follows.

From the profit maximization conditions, we obtain

$$S = \frac{1-\beta}{\beta}(hL_m). \tag{51}$$

From the zero profit conditions, we obtain

$$p_m = \frac{w}{hA_m}, \tag{52}$$

$$p_s = \frac{w}{h}. \tag{53}$$

Accordingly, the relative price is given by

$$\frac{p_s}{p_m} = A_m. \tag{54}$$

Hence, in this case, the relative price depends only on the productivity specific to manufacturing.

Using Equation (24), we can express the output of manufacturing by L_m.

$$Q_m = \beta^{-1} A_m h L_m. \tag{55}$$

From the goods marker clearing condition $C_m = Q_m$, we obtain

$$\frac{\alpha}{1-\alpha}\left(\frac{p_s}{p_m}\right)^{\sigma}\left[\frac{w - \frac{\alpha}{1-\alpha} p_m \left(\frac{p_s}{p_m}\right)^{\sigma}\gamma}{p_s + \frac{\alpha}{1-\alpha} p_m \left(\frac{p_s}{p_m}\right)^{\sigma}} + \gamma\right] L = \beta^{-1} A_m h L_m. \tag{56}$$

Solving this equation for L_m/L, we obtain

$$\frac{L_m}{L} = \frac{\frac{\alpha}{1-\alpha}\left(\frac{p_s}{p_m}\right)^{\sigma}\left[\frac{w - \frac{\alpha}{1-\alpha} p_m \left(\frac{p_s}{p_m}\right)^{\sigma}\gamma}{p_s + \frac{\alpha}{1-\alpha} p_m \left(\frac{p_s}{p_m}\right)^{\sigma}} + \gamma\right]}{\beta^{-1} A_m h}. \tag{57}$$

Per capita consumption of services $C_s = Q_s - S$ is given by

$$c_s = \frac{C_s}{L} = h\frac{L_s}{L} - \frac{1-\beta}{\beta} h\left(1 - \frac{L_s}{L}\right). \tag{58}$$

Hence, the accumulation of human capital leads to

$$\dot{h} = \frac{\delta}{\beta} h\left[\frac{L_s}{L} - (1-\beta)\right]. \tag{59}$$

For the rate of human capital accumulation to be positive, we need

$$\frac{L_s}{L} > 1 - \beta. \tag{60}$$

The ratio of manufacturing output to GDP and the ratio of services output to GDP are given by the following equations, respectively,

$$\frac{p_m Q_m}{\text{GDP}} = \beta^{-1}\frac{L_m}{L}, \tag{61}$$

$$\frac{p_s Q_s}{\text{GDP}} = \frac{L_s}{L} = 1 - \frac{L_m}{L}. \tag{62}$$

The growth rate of manufacturing TFP is given by

$$g_{\text{TFP},m} = g_{A_m} + \beta g_h. \tag{63}$$

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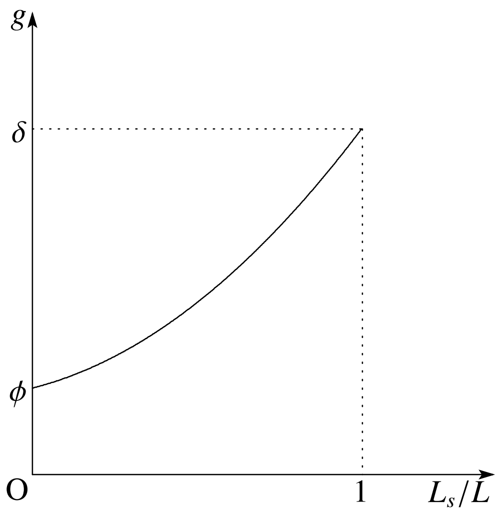

Figure 1: The relationship between L_s/L and g_{TFP} ($\phi < \delta$ and $2\phi < \delta$)

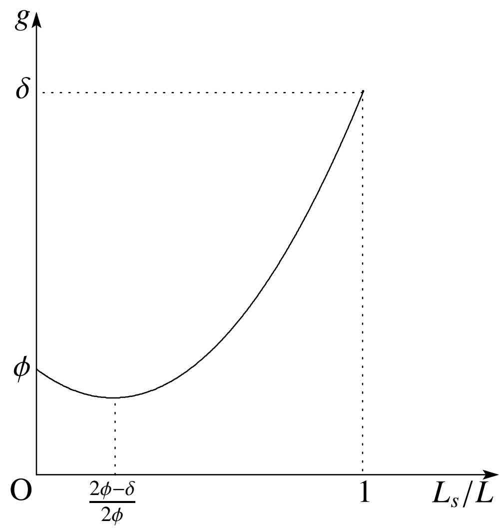

Figure 2: The relationship between L_s/L and g_{TFP} ($\phi < \delta$ and $2\phi > \delta$)

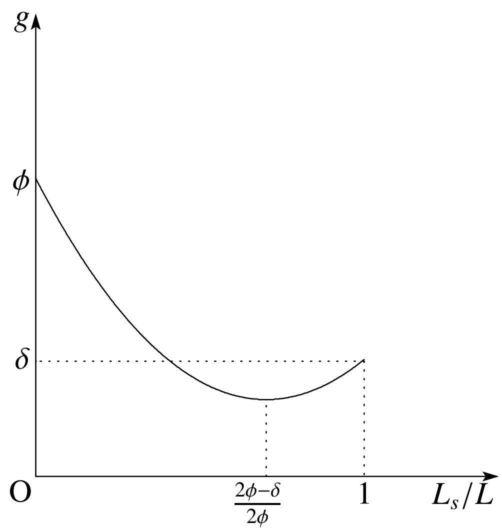

Figure 3: The relationship between L_s/L and g_{TFP} ($\phi > \delta$)

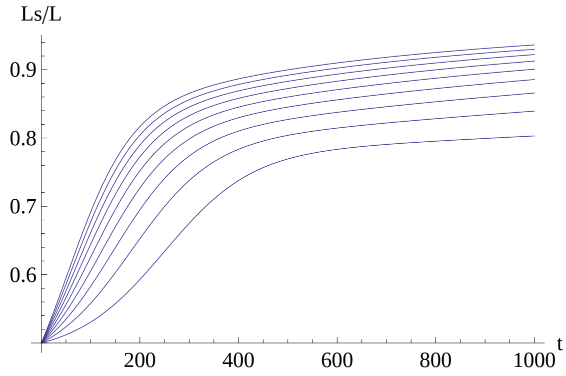

Figure 4: Employment share of services in Case 1

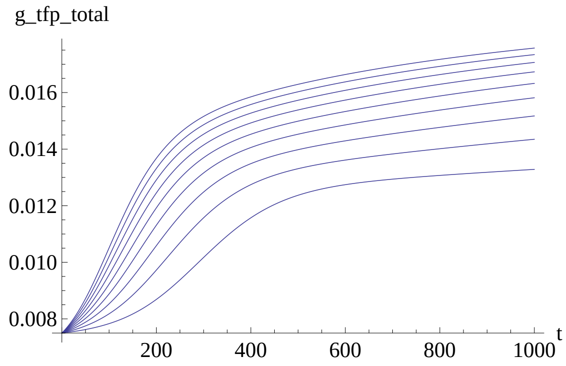

Figure 5: TFP growth rate in Case 1

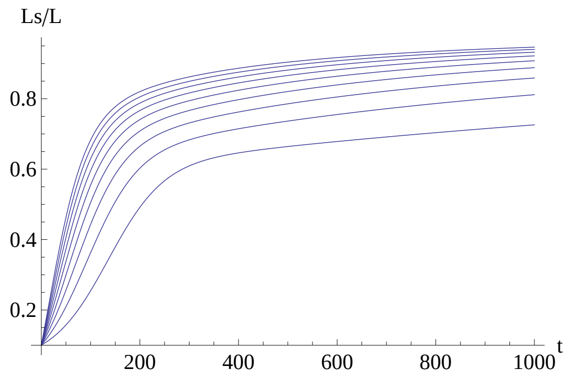

Figure 6: Employment share of services in Case 2

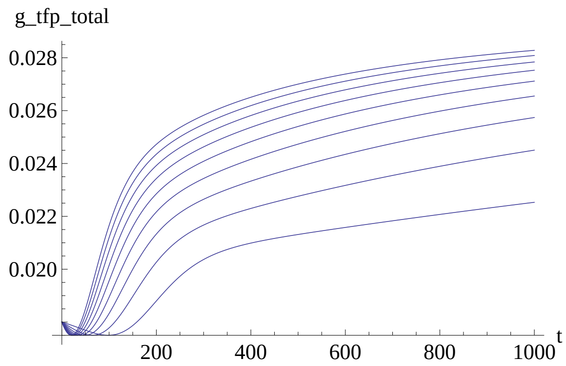

Figure 7: TFP growth rate in Case 2

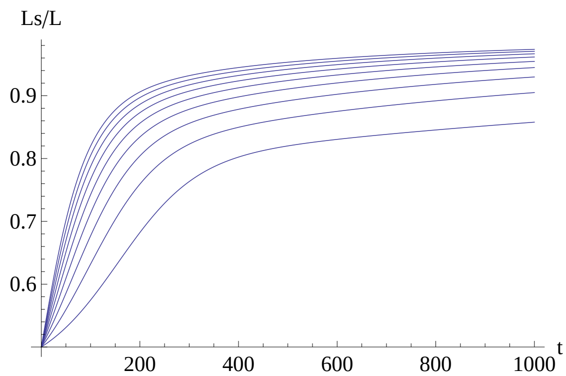

Figure 8: Employment share of services in Case 3

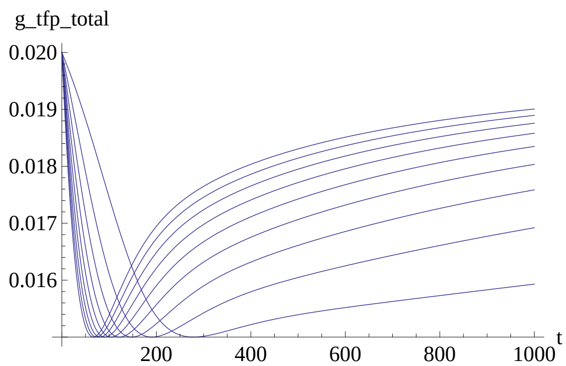

Figure 9: TFP growth rate in Case 3

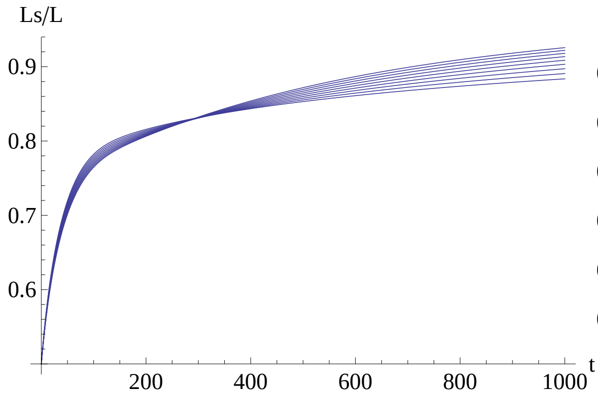

Figure 10: Employment share of services in Case 4

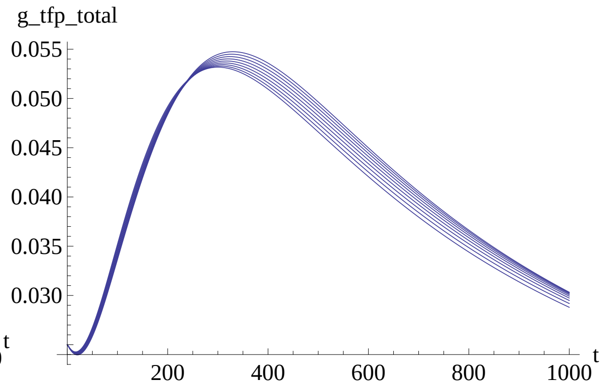

Figure 11: TFP growth rate in Case 4

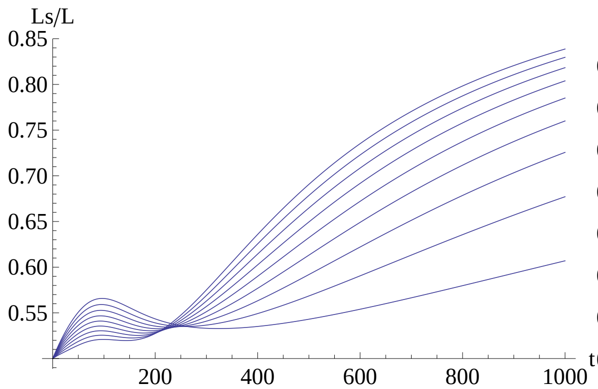

Figure 12: Employment share of services in Case 5

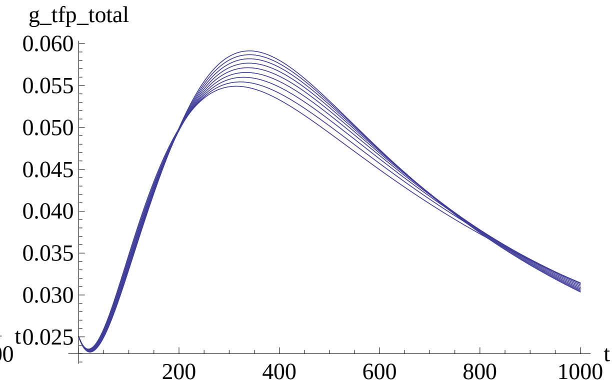

Figure 13: TFP growth rate in Case 5