Income Distribution and Economic Growth in a Multi-Sectoral Kaleckian Model

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Discussion Paper No. E-14-011

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October 2014
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Abstract

This study builds an income distribution and growth model within a simple multi-sectoral Kaleckian framework. The model has heterogeneous features in each sector in that the responses of saving and investment to changes in macroeconomic performance differ sectorally, and there are also different sectoral shares of saving and investment. We consider the determinants that establish the economic growth regime (i.e. wage-led and profit-led) and the stable output growth rate adjustment within this framework. By doing so, we reveal the sectoral composition of saving and investment and that elasticity of saving and investment matter for the formation of a growth regime and the stability of the output growth rate at the aggregate level.

Keywords: Multi-sectoral Kaleckian model; Income distribution; Sectoral heterogeneity

JEL Classification: B50, E12, O41

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1 Introduction

This paper presents a simple multi-sectoral economic growth model on the basis of Kaleckian economics. The main focus of the paper is on (i) determinants of output growth rate and its stability, especially at the macroeconomic level, (ii) how sectoral heterogeneity concerns the establishment of a growth regime, and (iii) the differences between sectoral and macroeconomic performance. We discuss these topics in terms of growth regime analysis. In this paper, a growth regime refers to how the rate of change in income distribution affects the aggregate output growth rate, which has been a central topic of the Kaleckian model of growth and distribution.

Since Rowthorn (1981), the Kaleckian model has revealed the relationship between income distribution and economic growth. Briefly, Kaleckian models after Rowthorn (1981) have shown mechanisms of a stagnationist regime, in which a decrease in the share of wages negatively impacts capacity utilization, which is endogenously determined (Dutt (1984); Taylor (1985)). The main contribution of these works is that a rise in wages has a favourable impact on economic growth and employment, in sharp contrast to implications from mainstream economics. Kaleckian models after Bhaduri and Marglin (1990) have shown the diversity of growth regimes (Blecker (2002); Lavoie (2006)). Replacing profit rate with profit share in the investment function, Bhaduri and Marglin (1990) presented that a rise in profit share may also positively affect economic growth and employment; thus, they revealed that there are wage-led and profit-led economic growth regimes. Depending on the relative size of parameters in the IS balance, the growth rate of an economy can either increase or fall with the income distribution. If the profit share stimulates the economic growth rate, a ‘profit-led growth regime’ is established. In contrast, if the wage share stimulates the economic growth rate, a ‘wage-led growth regime’ is established.

Kaleckians have extended the model by Bhaduri and Marglin in several ways. For example, by dynamically endogenizing the income distribution because of conflicting claim models, they investigated the stability conditions of growth and distribution (Cassetti (2003); Sasaki (2014)). In addition, as a response to Sraffian critics, Kaleckians have attempted to present long-run models in which the actual capacity utilization rate adjusts to the normal standard rate (Lavoie (1995, 2003); Duménil and Lévy (1999); Sasaki (2014)). In doing so, they show the conditions for (in-)validity of Kaleckian results, such as the paradox of cost and thrift. Moreover, empirical studies have also been developed. Kaleckians have empirically revealed that a variety of growth regimes such as profit-led and wage-led ones have existed in different countries and periods (Stockham-
mer and Onaran (2004); Hein and Vogel (2008); Storm and Naastepad (2012)).

The model developed in this paper is also Kaleckian in that it incorporates the role of income distribution and an investment function independent of saving. Thus, it has an effective demand-led growth mechanism. Moreover, it presents a novelty whereby aggregate economic growth is explained on the basis of an extension to the multi-sectoral model. In contrast to the standard aggregate Kaleckian model, this multi-sectoral Kaleckian model emphasizes the role of economic structure to determine the economic growth regime and economic growth rate. In this paper, economic structure refers to the sectoral composition of saving and investment; thus, structural change indicates that the sectoral share of these volumes changes. In the model, it is assumed that the economic structure is reflected by the share of saving and investment of each sector, which play an important role in determining the output growth rate. Because consumption is the opposite of saving in a closed economy, the economic structure also embodies a sectoral demand structure (i.e. the sectoral composition of consumption and investment).

Multi-sectoral issues have not been systematically incorporated into Kaleckian models of growth and distribution. The so-called Pasinettian, a representative post-Keynesian stream, has emphasized structural economic dynamics in multi-sectoral closed-economy models (Pasinetti (1981, 1993)) and open-economy models (e.g. the balance-of-payments constraint growth model (Araujo and Lima (2007); Araujo (2012); Araujo et al. (2013))). An important implication of these Pasinettian approaches is that changes in the structure of production lead to changes in the rate of aggregate output growth and employment. Although Pasinettians have revealed that structural change is a driving force for economic growth, the income distribution that Kaleckian models emphasize have little role in these dynamics.

Some Kaleckians have also presented multi-sectoral models, but such attempts are rare. For example, Dutt (1990, 1997) and Park (1998-99) showed multi-sectoral Kaleckian models and argued the over-determination problem particular to such models. The central topic among their discussion is how to remove this problem through the introduction of classical competition and moving-average rates of accumulation into the models. Consequently, their work did not focus on sectoral heterogeneity issues pertaining to income generation. More recently, Araujo and Teixeira (2011, 2012) presented a multi-sectoral version of a post-Keynesian growth model. They presented two major implications by connecting a Pasinettian model of structural change and a Kaleckian model of economic growth. The first implication is that by developing post-Keynesian
multi-sectoral growth models, they determine the condition that realizes the natural rate of profit in a Pasinettian sense. As the natural rate of profit determines the constant mark-up rate over time in their models, they also show that Pasinettian structural dynamics depends on income distribution and on evolution patterns of demand and technological progress. The second implication is that their model presents the possibility that different sectors may have different growth regimes. Even if a sector is operating in a wage-led growth regime, other sectors may be operating in a profit-led one. These contributions cannot be obtained using the aggregate Kaleckian model of growth and distribution.

Issues remain for the multi-sectoral Kaleckian model that should be investigated in more detail. For example, it is not clear how aggregate (macroeconomic) growth rate is concerned with sectoral features in these models. Dutt (1990, 1997) and Park (1998-99) concentrated on solving over-determination problems, and they focused on sectoral accumulation rates. However, aggregate output growth is not the main emphasis. This is also an issue in Araujo and Teixeira (2011, 2012), who showed that each sector has a different growth regime; however, its connection with aggregate output growth is not examined. Moreover, while these studies setup disaggregated models, they do not examine how sectoral heterogeneity is concerned with aggregate output growth rate and stability. These studies consider sectorally different saving and investment functions and thus the sectoral accumulation rate. Consequently, the different economic structures concerning the shares of saving and investment in each sector play no role in determining aggregate growth rates. Then, in the existing literature, it is not sufficiently clear how structural changes in an economy lead to changes in the rate of aggregate output growth and employment.

In order to solve these problems, we tackle research questions (i), (ii), and (iii) mentioned in the introduction. By revealing these questions, this study further extends the Kaleckian model, and we provide new insights to these questions. A more detailed discussion is provided later, but we briefly summarize the main results to each question as follows: (i) The current multi-sectoral model reveals that in addition to the elasticity parameters of saving and investment of each sector, the structure of the economy also plays an important role as a determinant of economic growth rate and its stability, especially at the macroeconomic level. Thus, the multi-sectoral model clarifies that the sectoral reallocation of saving and investment volume is another source.

1The natural rate of profit in the Pasinettian sense in this study is the rate that endows each sector with the units of productive capacity required to fulfil demand.
of stable economic growth. (ii) This also reveals that the structure of the economy plays an important role in determining the type of growth regime. A change in the sectoral saving and investment composition may transform a growth regime into another type of regime, which is an important difference between the standard aggregate Kaleckian model and the multi-sectoral Kaleckian model concerning growth-regime formation. (iii) The aggregate output growth rate that equilibrates aggregate investment and saving balance over time may differ from the one that equilibrates each sectoral balance over time. Consequently, there is a different mechanism for growth-regime formation at the sectoral and macroeconomic levels.

The remainder of this paper is organized as follows. Section 2 presents the basic structure of our multi-sectoral Kaleckian model, which is defined using sectoral saving and investment. Section 3 examines the relationship between the rate of change in income distribution and the aggregate output growth while focusing on the differences between sectoral properties and aggregate outcomes. In this section, we also consider the conditions for stable economic growth and the determination of the growth regime. Section 4 concludes.

2 Model

We start the analysis with the presumption of model. There are $n$ sectors of production in a closed economy without a government sector. Each sector has heterogeneous characteristics in that it has different saving and investment behaviour and different shares of saving and investment in the economy. There are two classes, workers and capitalists, in each sector. The workers provide labour and save a portion of wage income that they earn. The capitalists also save the profit that they receive. Following a standard Kaleckian model, it is assumed that capitalists’ propensity to save is higher than workers’ propensity to save. Moreover, the capitalists are also the owners of the firms in each sector, and the firms in each sector implement investment according to the levels of the macroeconomic variables. It is assumed that the aggregate output is produced as a result of all firms’ production in the economy, which is conducted using a Leontief-type production function with constant output–capital and output–labour coefficients.

The following lists the main notations for the national economic variables used in this paper. $X$: total output (total income), $L$: total employment, $S$: saving, $I$: investment, $w$: nominal wage rate, $\sigma$: wage share, $1 - \sigma$: profit share. These variables are also a function of time. Using
subscript $i$, these variables indicate variables for sector $i$.

2.1 Saving and Investment

It is assumed that saving and investment behaviour in sector $i$ are affected mainly by changes in output and income distribution at the macroeconomic level. Then, the responses of saving and investment to changes in these variables vary by sector using the different elasticities of saving and investment. Thus, we formalize this by saying that each sector has heterogeneous saving and investment reactions to the same impact on output and income distribution at the aggregate level.

For analytical purposes, we define that the saving and investment demand functions for each sector are given using a Cobb–Douglas functional form. First, the saving function of sector $i$ is given by

$$S_i = s_i(wL)\alpha_i(X - wL)\beta_i, \quad (1)$$

where $s_i$ is a constant term that represents saving behaviour particular to sector $i$, $\alpha_i \in (0, 1)$ is the saving elasticity caused by a change in wage income, and $\beta_i \in (0, 1)$ is the saving elasticity caused by a change in profit income. In defining the income distribution, it is assumed that price level $p$ is constant and equal to unity.\(^2\) Following the standard Kaleckian model, it is assumed that the sectoral saving elasticity caused by changes in profit income is higher than that caused by changes in wage income, that is, $\alpha_i < \beta_i$.\(^3\) As $wL = \sigma X$ and $X - wL = (1 - \sigma)X$, Equation (1) is rewritten as follows:

$$S_i = s_i(\sigma X)^\alpha ((1 - \sigma)X)^\beta_i$$

$$= s_i \sigma^\alpha (1 - \sigma)^\beta_i X^{\alpha + \beta_i}. \quad (2)$$

By taking the logarithms of Equation (2) and differentiating with respect to time, the growth rate of saving in sector $i$ is obtained as follows:

$$\hat{S}_i = (\alpha_i - \beta_i)\hat{\sigma} + (\alpha_i + \beta_i)\hat{X}, \quad (3)$$

\(^2\)This is for the simplicity and does not affect the main results as long as we suppose that the rate of change in the distribution share is an exogenous variable.

\(^3\)Most Kaleckian and Kaldorian models are constructed using propensity to save from wages and profits, whereas the current model is constructed in terms of elasticity of saving using these variables.
where \( \delta = \sigma/(1 - \sigma) \) is the ratio of wage and profit. The hat symbol represents the rate of change in each variable (e.g. \( \hat{S}_i = \frac{dS_i}{dt} \)). According to empirical studies, although the wage share has gradually been decreasing in the era of neoliberalism, it is still larger than profit share (Stockhammer (2013)). Therefore, it is assumed that \( \delta > 1 \). Because it is also assumed that \( \alpha_i < \beta_i, \alpha_i - \beta_i \delta < 0 \) holds. Consequently, we obtain \( \partial \hat{S}_i / \partial \hat{\sigma} < 0 \) from Equation (3). A rise in the rate of change in the wage share causes a fall in the rate of change in saving in sector \( i \). In addition, \( \partial \hat{S}_i / \partial \hat{X} > 0 \) holds, indicating that an increase in the rate of change in aggregate output leads to an increase in the rate of change in saving in sector \( i \).

Second, the investment function of sector \( i \) is given by

\[
I_i = A_i X^\theta (X - wL)^{\gamma_i},
\]

where \( A_i \) is an autonomous investment term in sector \( i \) that grows at a constant rate. The investment function is also defined as a Cobb–Douglas type, in which \( \theta_i \in (0, 1) \) is the investment elasticity to a change in output, and \( \gamma_i \in (0, 1) \) is the investment elasticity to a change in profit income. This formalization is similar to Bhaduri and Marglin (1990) in that it introduces both an accelerator effect and a profit effect into the investment function.\(^4\) As \( X - wL = (1 - \sigma)X \), Equation (4) is rewritten as follows:

\[
I_i = A_i (1 - \sigma)^{\gamma_i} X^{\gamma_i + \theta_i}.
\]

By taking the logarithms of Equation (5) and differentiating with respect to time, the growth rate of investment in sector \( i \) is obtained as follows:

\[
\hat{I}_i = \hat{A}_i + (\gamma_i + \theta_i) \hat{X} - \gamma_i \delta \hat{\sigma},
\]

where \( \hat{A}_i \) is a constant growth term of investment in sector \( i \). As \( \partial \hat{I}_i / \partial \hat{X} > 0 \) holds, a rise in the rate of change in aggregate output leads to a rise in the rate of change in investment in sector \( i \). In contrast, because \( \partial \hat{I}_i / \partial \hat{\sigma} < 0 \) holds, a rise in the rate of change in wages leads to a fall in the rate of change in investment of that sector. The former represents an accelerator effect, and the latter represents a profit-squeeze effect on the dynamics of sectoral investment behaviour.

\(^4\)More precisely, there are differences between the formalization in the model by Bhaduri and Marglin (1990) and the current model. The investment function used in Bhaduri and Marglin (1990) consists of a capacity utilization rate and the share of profits, where the former represents the accelerator effect and the latter represents the profit effect. The investment function in Equation (4) includes output and profit income, where the former plays a role in the accelerator effect and the latter plays a role in the profit effect.
2.2 Balanced Economic Growth

In this paper, the goods market equilibrium is defined in real terms at the macroeconomic level. Although there may be disequilibria in some sectors, the aggregate demand and supply must be balanced at the aggregate level. Balanced economic growth is defined by such a situation whereby the equilibrium between aggregate demand and supply is sustained over time. In an economy without government and foreign sectors, this situation is defined by the equilibrium between total saving and investment.

In a multi-sector context, the total volume of saving (investment) is the sum of saving (investment) from each sector of the economy. Therefore, the equilibrium in the goods market at the aggregate level is given by

\[ \sum_{i=1}^{n} S_i = \sum_{i=1}^{n} I_i, \]  

where the left-hand side (LHS) represents the total volume of saving, and the right-hand side (RHS) represents total volume of investment at an initial period. In order for this equilibrium to be maintained over time, it is necessary for the time rate of change in total saving and investment to be equal. Therefore,

\[ \sum_{i=1}^{n} \frac{S_i}{S_i} \dot{S}_i = \sum_{i=1}^{n} \frac{I_i}{I_i} \dot{I}_i = \sum_{i=1}^{n} \nu_i \dot{S}_i = \sum_{i=1}^{n} \mu_i \dot{I}_i. \]

Equation (8) is the equilibrium condition for aggregate demand and supply in the growth term. In this equation, \( \nu_i \equiv \frac{S_i}{\sum S_i} \in [0, 1] \) denotes the share of sector \( i \)'s saving in a country's total saving, and \( \mu_i \equiv \frac{I_i}{\sum I_i} \in [0, 1] \) denotes the share of sector \( i \)'s investment in the country's total investment.

We assume that these terms are exogenous and constant and that they are historically given or determined by such factors as demand structure and its evolution patterns in the economy.\(^5\) In addition, \( \sum_{i=1}^{n} \nu_i = 1 \) and \( \sum_{i=1}^{n} \mu_i = 1 \) by definition.

\(^5\)More precisely, \( \nu_i, \mu_i, \) and \( \delta \) are endogenous variables and change over time, because the saving and investment of sector \( i \) are defined in Equations (1) and (4) and \( \delta \) is defined using \( \sigma/(1 - \sigma) \). As these variables change over time, the values of \( \nu_i, \mu_i, \) and \( \delta \) differ over periods. However, the model becomes analytically untraceable if we treat
The multi-sectoral growth condition is given by Equation (8), which means that the time rate of change of both total saving and total investment should be equal over time. By substituting Equations (3) and (6) into Equation (8), this condition is rewritten as follows:

\[
\sum_{i=1}^{n} v_i \left[ (\alpha_i - \beta_i \delta) \hat{\sigma} + (\alpha_i + \beta_i) \hat{X} \right] = \sum_{i=1}^{n} \mu_i \left[ \hat{A}_i + (\gamma_i + \theta_i) \hat{X} - \gamma_i \hat{\sigma} \right].
\] (9)

The LHS represents the growth rate of total saving, and the RHS represents that of total investment. The difference of these two terms approximates the growth rate of excess supply or demand. This must be zero when the aggregate demand and supply are balanced over time. After some algebraic manipulation to solve \( \hat{X} \), the aggregate output growth rate in the multi-sectoral version of the Kaleckian model is derived as follows:

\[
\hat{X} = \frac{\sum_{i=1}^{n} \mu_i \hat{A}_i - \left( \sum_{i=1}^{n} v_i (\alpha_i - \beta_i \delta) + \sum_{i=1}^{n} \mu_i \gamma_i \delta \right) \hat{\sigma}}{\sum_{i=1}^{n} v_i (\alpha_i + \beta_i) - \sum_{i=1}^{n} \mu_i (\gamma_i + \theta_i)}.
\] (10)

For this equation, we assume later that

\[
\Theta \equiv \sum_{i=1}^{n} v_i (\alpha_i + \beta_i) - \sum_{i=1}^{n} \mu_i (\gamma_i + \theta_i) > 0.
\]

This condition assures a stable output growth rate adjustment, which is similar to the so-called ‘Keynesian stability condition’. We assume that the growth rate of the wage share is an exogenous variable and takes a constant value. Then, aggregate output growth is determined by the change in income distribution over time and the structure of the economy concerning the sectoral composition of saving and investment and autonomous investment demand. Because it is assumed that aggregate output is produced using a Leontief-type production function with constant output–labour coefficients and that the price level is constant over time, there is no productivity growth and inflation at the macroeconomic level. Accordingly, \( \hat{L} = \hat{X} \) holds, and the growth rate of labour demand over time is determined by the aggregate output growth rate. Therefore, the current model is also a Keynesian model of employment dynamics.\textsuperscript{6}

\textsuperscript{6}Under this assumption, the rate of change in income distribution corresponds to the sectorally weighted average of the nominal wage growth rate, which is also assumed to be exogenous and constant. When the labour productivity growth is zero at the aggregate level, \( \hat{\sigma} = \sum_{i=1}^{n} \frac{u_i}{\sum_{i=1}^{n} u_i} \hat{b}_i \). Moreover, if the nominal wage growth rate is the same rate \( \hat{w}^c \) across all sectors, \( \hat{\sigma} = \hat{w}^c \) results.
3 Distribution and Growth in a Multi-Sectoral Kaleckian Model

In this section, we examine the driving force of economic growth in the multi-sectoral Kaleckian model through comparative statistics. In doing so, we also consider different properties concerning growth regime formation and stability conditions between the conventional aggregate model and the current multi-sectoral model. With regard to stability, the sign of $\Theta$ defined previously is critical, and with regard to the growth regime, the sign of $\Omega$ defined later plays an important role. Both variables are concerned with structural aspects of the economy.

3.1 Comparative Statics Analysis

An important contribution of the Kaleckian model is that it reveals the impact of change in income distribution on the economic growth rate. Therefore, we distinguish the growth regime based on these rates.

By differentiating Equation (10) with respect to the rate of change in the wage share, we obtain

$$\frac{\partial \hat{X}}{\partial \hat{\sigma}} = -\frac{\Omega}{\Theta},$$

(11)

where $\Omega$ is defined by

$$\Omega = \sum_{i=1}^{n} \nu_i (\alpha_i - \beta_i \delta) + \sum_{i=1}^{n} \mu_i \gamma_i \delta.$$  

(12)

The growth regime is determined according to the sign of $\Omega$. If the sign of $\Omega$ is positive, the economy has a profit-led growth regime, whereas if it is negative, the economy has a wage-led growth regime. In the profit-led growth regime (wage-led growth regime), a rise in the rate of change in the profit share (wage share) leads to a higher economic growth rate.

$\Omega$ is defined in Equation (12), where the value of $\alpha_i - \beta_i \delta$ is negative for all sectors. Then, given the sectoral composition of saving and investment and the wage–profit ratio, when the negative difference of elasticities of saving between wage income and profit income are small (i.e. relatively large $\alpha_i$ and small $\beta_i$) and the elasticities of investment to profit income are high (i.e. large $\gamma_i$), a profit-led growth regime tends to be established. Inversely, when the negative difference of elasticities of saving between wage income and profit income are large (i.e. relatively small $\alpha_i$ and large $\beta_i$) and the elasticities of investment to profit income are small (i.e. small $\gamma_i$), a wage-led growth regime tends to be established.
Such an implication concerning the establishment of the growth regime is similar to the standard aggregate Kaleckian model à la Bhaduri and Marglin. As Blecker (2002) shows, when the propensity to save from wage share is large and that from profit share is small with a large profit effect on capital accumulation, a rise in the wage share tends to decrease the output level and economic growth. These conditions thus generate the profit-led growth regime. The current model shows a similar mechanism not in terms of propensity coefficient but in terms of the elasticities of each variable.

Moreover, the current multi-sectoral Kaleckian model creates novelty for the establishment of the growth regime. It is important to recognize that the determinants of growth regimes are concerned with not only elasticity parameters for saving and investment but also with their sectoral composition. The structure of the economy, reflected by the share of saving and investment of each sector, also plays an important role in determining the type of growth regime. The importance of structural dynamics for economic growth has been emphasized by Pasinetti (1981, 1993), Araujo and Lima (2007), and Araujo (2013). By applying a structural dynamics point of view to the growth regime-formation issue, we can show that the current multi-sectoral Kaleckian model can clearly present the importance of sectoral characteristics in this issue.

First, a change in sectoral saving composition may transform one growth regime to another growth regime. Suppose that the difference of saving elasticities to profit and wage in sector \( j \) is much larger than that in sector \( i \). Then, \(|\alpha_i - \beta_i| < |\alpha_j - \beta_j|\) is assumed. As \( \sum_{i=1}^{n} \gamma_i = 1 \), a change in the share of saving in a sector indicates a sectoral shift of saving composition to another sector. A shift of saving composition from sector \( i \) to sector \( j \) implies that \(-\Delta \nu_i = \Delta \nu_j\). Given the elasticity parameters, such a change in sectoral saving composition has a negative impact on the value of \( \Omega \) that determines the type of growth regime. That is,

\[
\Delta \Omega = -[(\alpha_i - \beta_i) - (\alpha_j - \beta_j)] \Delta \nu_j < 0. \tag{13}
\]

Thus, a sectoral shift of saving to a sector with a larger difference of saving elasticities leads to a lower value of \( \Omega \). A structural change with regard to saving composition from sector \( i \) to sector \( j \) then enhances the wage-led growth regime property in the economy.

Second, a similar implication can be obtained with regard to investment composition. Suppose that the investment elasticity to profit in sector \( j \) is much larger than that in sector \( i \). That is, \( \gamma_i < \gamma_j \) is assumed. As \( \sum_{i=1}^{n} \mu_i = 1 \), a change in the share of investment in a sector implies a
sectoral shift of investment composition to another sector. The shift of investment composition from sector \( i \) to sector \( j \) that significantly shifts investment elasticity to profit is \( -\Delta \mu_i = \Delta \mu_j \). Given the elasticity parameters, such a change in sectoral investment composition has a positive impact on the value of \( \Omega \). That is,

\[
\Delta \Omega = -(\gamma_i - \gamma_j)\Delta \mu_j > 0, \tag{14}
\]

and a sectoral shift of investment toward a sector with higher elasticity of investment to profit leads to a larger value of \( \Omega \). A structural change with regard to investment composition from sector \( i \) to sector \( j \) enhances the profit-led growth regime property in the economy.

In sum, the current study reveals that the establishment of a growth regime is also concerned with the structural change of saving and investment. In a closed economy without a government, because the opposite of saving is consumption by definition, the sectoral shares of saving and investment also embody the sectoral structure of demand composition. Hence, the current multi-sectoral Kaleckian model also reveals the importance of the demand structure of the economy to the establishment of the growth regime. The share of saving and investment is not uniform across sectors in an economy. In other words, sectoral heterogeneity of demand and its structural change are determinants of the growth regime.

Finally, an autonomous growth of investment demand in each sector also matters for high economic growth.

\[
\frac{\partial \dot{X}}{\partial A_i} = \frac{\mu_i}{\Theta} > 0. \tag{15}
\]

Here, a rise in autonomous investment demand in a sector leads to a higher growth rate. Therefore, the current model also involves a demand-led growth feature.

### 3.2 Sectoral Properties and Aggregate Outcomes

This section shows the differences of sectoral properties and aggregate dynamics on the basis of the current multi-sectoral Kaleckian model in more detail. In doing so, we first consider the Keynesian stability condition, and then we argue the difference concerning the growth regime formation between multi-sectoral and aggregate Kaleckian models.
3.2.1 Sectoral and Aggregate Stability

In examining the growth mechanism in the previous section, we have assumed the Keynesian stability condition. This condition is summarized as $\Theta > 0$. This is based on the adjustment of the output growth rate according to the investment–saving gap in the growth term at the macroeconomic level. When the growth rate of total investment is higher than that of the saving rate (i.e. where there is excess demand in the growth term), the total output growth rate increases to solve the disequilibrium in the goods market over time. In mathematical terms, the adjustment process at the macroeconomic level is as follows:

$$\frac{d\hat{X}}{dt} = \Lambda \left[ d \frac{dt}{dt} \log \sum_{i=1}^{n} I_i - d \frac{dt}{dt} \log \sum_{i=1}^{n} S_i \right],$$

where $\Lambda$ is a positive adjustment parameter. The stability condition is $\frac{d\hat{X}}{dt} < 0$, and it is reduced to the following inequality from Equations (3) and (6):

$$\Theta \equiv \sum_{i=1}^{n} \nu_i(\alpha_i + \beta_i) - \sum_{i=1}^{n} \mu_i(\gamma_i + \theta_i) > 0.$$

A stable adjustment requires that the sectorally weighted average saving elasticities to changes in output growth must be larger than the sectorally weighted average investment elasticities to these changes.

With regard to the stability condition, three implications can be derived from the current multi-sectoral Kaleckian model. First, the shares of saving and investment of each sector also play an important role in assuring stability. Because $\Theta$ includes the sectoral composition of saving and investment, changes in this composition affect the stability condition. For instance, when the elasticities of saving to both profit and wage income are higher in sector $i$ than in sector $j$, that is, $\alpha_i + \beta_i > \alpha_j + \beta_j$, a sectoral shift of saving from sector $i$ to sector $j$ reduces the stability. When there is such a shift of saving composition, we have $-\Delta \nu_i = \Delta \nu_j$ with $\alpha_i + \beta_i > \alpha_j + \beta_j$. Consequently, the value of $\Theta$ changes by

$$\Delta \Theta = [\alpha_j + \beta_j - (\alpha_i + \beta_i)] \Delta \nu_j,$$

which is a negative value. Therefore, a sectoral shift of saving to a sector with lower elasticities of saving reduces the value of $\Theta$. In other words, the stability condition is harder to satisfy than before such a structural change.
The same mechanism is true for investment composition. When the elasticities of investment to both profit income and output are lower in sector \( i \) than in sector \( j \), that is, \( \gamma_i + \theta_i < \gamma_j + \theta_j \), a sectoral shift of investment from sector \( i \) to sector \( j \) also reduces the stability. When there is such a shift of investment composition, we have \( -\Delta \mu_i = \Delta \mu_j \) with \( \gamma_i + \theta_i < \gamma_j + \theta_j \). Consequently, the value of \( \Theta \) changes by

\[
\Delta \Theta = [\gamma_i + \theta_i - (\gamma_j + \theta_j)]\Delta \mu_j,
\]

which is a negative value. Therefore, the adjustment of aggregate output growth rate may become unstable because of structural change. The value of \( \Theta \) is reduced by such structural change that the share of investment rises in a sector where the elasticities of investment to both profit income and output are high, while the share of investment falls in a sector where these elasticities are low.

Second, there are different conditions concerning stability at the sectoral and aggregate level. As we have shown, the stability condition at the aggregate level is given by Equation (17). Suppose that when there is disequilibrium at the sectoral level, it is also solved by the change in the aggregate output growth rate but at a different speed particular to each sector, \( \Lambda_i \). Thus, we express that sectoral disequilibrium has a different impact on aggregate output growth. When the growth rate of investment is higher than that of saving in sector \( i \), the aggregate output growth rate increases to solve the disequilibrium in the goods market for sector \( i \) over time. In mathematical terms, the adjustment process for the sectoral equilibrium is as follows:

\[
\frac{dX}{dt} = \Lambda_i \left[ \frac{d}{dt} \log I_i - \frac{d}{dt} \log S_i \right],
\]

where \( \Lambda_i \) is a positive adjustment parameter that indicates the effect of the output growth adjustment caused by disequilibrium in sector \( i \). The stability condition for the sectoral equilibrium is \( \frac{dX}{dt} < 0 \), which we denote as \( \Theta_i > 0 \). By substituting Equations (3) and (6) into (20), the stability condition is precisely denoted by the following inequality:

\[
\Theta_i \equiv \alpha_i + \beta_i - (\gamma_i + \theta_i) > 0.
\]

Stable adjustment using aggregate output growth for the sectoral IS equilibrium simply requires that saving elasticities to changes in output growth are larger than investment elasticities to such changes. The sectoral shares of saving and investment do not concern the stability condition for
the sectoral equilibrium. Thus, the stability condition for the sectoral level does not correspond to the stability condition for the aggregate level.

The third implication, which is somewhat related to the second one, is that aggregate stability may still be assured even if unstable conditions remain in some sectors. This is because the sectoral stability condition does not have to be satisfied in some sectors for aggregate stability to be assured. Consequently, even if there is instability in one sector, depending on the sectoral composition of saving and investment and their elasticities in other sectors, aggregate economic growth may be stable. Let us illustrate a simple case of a two-sector economy, in which sector 2 is unstable and sector 1 is stable. It is assumed that $\alpha_1 + \beta_1 - (\gamma_1 + \theta_1) > 0$ and $\alpha_2 + \beta_2 - (\gamma_2 + \theta_2) < 0$. As there are only two sectors, when the saving share of sector 1 is $v_1$, that of sector 2 is $1 - v_1$. Similarly, when the investment share of sector 1 is $\mu_1$, that of sector 2 is $1 - \mu_2$. Based on Equation (17), the stability condition at the aggregate level is given by

$$\Theta \equiv v_1(\alpha_1 + \beta_1 - \alpha_2 - \beta_2) - \mu_1(\gamma_1 + \theta_1 - \gamma_2 - \theta_2) + (\alpha_2 + \beta_2 - \gamma_2 - \theta_2).$$  (22)

The third term on the RHS of this equation is negative by assumption, whereas the signs of the first and second terms are not determined \textit{a priori}. The sectoral composition of saving and investment as well as the elasticities of saving and investment matter for the sign of $\Theta$. When $\alpha_1 + \beta_1 > \alpha_2 + \beta_2$, a rise (fall) in $v_1$ contributes to (de-)stabilization, and when $\gamma_1 + \theta_1 < \gamma_2 + \theta_2$, a rise (fall) in $\mu_1$ also contributes to (de-)stabilization.\footnote{It should be noted that such an illustration as $\alpha_1 + \beta_1 > \alpha_2 + \beta_2$ and $\gamma_1 + \theta_1 < \gamma_2 + \theta_2$ is compatible with the assumption of $\alpha_1 + \beta_1 - (\gamma_1 + \theta_1) > 0$ and $\alpha_2 + \beta_2 - (\gamma_2 + \theta_2) < 0$. For example, inequality $\alpha_1 + \beta_1 > \gamma_2 + \theta_2 > \alpha_2 + \beta_2 > \gamma_1 + \theta_1$ satisfies all conditions without inconsistency.} Thus, if the sum of these two terms is much larger than the last term, aggregate stability is still assured regardless of instability in sector 2. In this case, the potential instability to the output growth rate caused by sector 2 is offset by the stability mechanisms in sector 1 and the structure of the economy. Consequently, aggregate stability is established.

These three implications can never be obtained in the standard Kaleckian model, as it is normally constructed at the aggregate level; thus, sectoral decomposition properties are not clear in such a model. With regard to the stability of the output growth rate adjustment, the important implication is that the sectorally different shares of saving and investment also play an important role for the realization of stable output growth. Their sectoral shares in an economy are not uniform and vary; therefore, such sectoral heterogeneity generates both stabilizing and destabilizing
factors for the aggregate output growth rate adjustment.

3.2.2 Sectoral and Aggregate Growth Regimes

In addition to the stability condition, there is another important result concerning the establishment of the growth regime. That is, the growth regime in each sector differs from that at the aggregate level. Even if the wage-led growth condition is dominant for the aggregate output growth rate that realizes equilibrium in a sector, it is possible that the profit-led growth condition is dominant for this rate in another sector. This is because the growth regime at the aggregate level depends on not only saving and investment elasticities but also on the sectoral composition of these variables.

We illustrate this phenomenon using a simple economy of two sectors, 1 and 2. Suppose that both sectors are stable. An equilibrium of saving and investment is realized over time in sector 1, written as \( \hat{S}_1 = \hat{I}_1 \). In this case, the output growth rate that realizes the IS balance in this sector is obtained from Equations (3), (6), and (20):

\[
\hat{X}_1 = \frac{\hat{A}_1 - [\alpha_1 - \beta_1 \delta + \gamma_1 \delta] \hat{\delta}}{\alpha_1 + \beta_1 - (\gamma_1 + \theta_1)},
\]  

(23)

where \( \hat{X}_1 \) is an aggregate output growth rate that equilibrates the sectoral investment and saving balance over time. When the Keynesian stability condition is assured, the sign of \( \alpha_1 - \beta_1 \delta + \gamma_1 \delta \) determines the sectoral growth regime. The output growth \( \hat{X}_1 \) that equilibrates sector 1’s IS balance is positively affected by an increase in the rate of change in the profit share when \( \alpha_1 - \beta_1 \delta + \gamma_1 \delta \) is positive. In contrast, output growth \( \hat{X}_1 \) is positively affected by a rise in the rate of change in the wage share when \( \alpha_1 - \beta_1 \delta + \gamma_1 \delta \) is negative. The former case can be called a sectoral profit-led growth regime and the latter case can be called a sectoral wage-led growth regime.

However, the aggregate output growth rate \( \hat{X} \), which equilibrates the aggregate IS balance over time, may differ from \( \hat{X}_i \), which equilibrates each sectoral IS balance over time. Consequently, there are different mechanisms of growth regime formation at the sectoral and macroeconomic levels. In the two-sector economy shown, suppose that sector 2 involves conditions for a sectoral wage-led growth regime. Then, it is assumed that \( \alpha_2 - \beta_2 \delta + \gamma_2 \delta < 0 \). As there are only two sectors, when the saving share of sector 1 is \( \nu_1 \), that of sector 2 is \( 1 - \nu_1 \). Similarly, when the investment share of sector 1 is \( \mu_1 \), that of sector 2 is \( 1 - \mu_2 \). On the basis of Equation (12), the
discriminant for the growth regime at the aggregate level is then given by

$$\Omega \equiv (\alpha_2 - \beta_2\delta + \gamma_2\delta) + [v_1(\alpha_1 - \beta_1\delta) + \mu_1\gamma_1\delta] - [v_1(\alpha_2 - \beta_2\delta) + \mu_1\gamma_2\delta].$$

(24)

The first term on the RHS of this equation has a negative sign in case of a sectoral wage-led growth regime by assumption. However, the signs of the second and third terms are not determined \textit{a priori}, because they are weighted by the sectoral share of saving and investment. Consequently, the sectoral composition of saving and investment and the elasticities of saving and investment matter for the sign of $\Omega$. If the sum of these two terms is positive and much larger than the absolute value of the first term, the aggregate growth regime is profit-led even if sector 2 has a condition for a sectoral wage-led growth regime. As mentioned previously, each sector has different shares of saving and investment. Hence, such heterogeneous properties among sectors are determinants of the type of growth regime.

Finally, it may be possible to say that the current multi-sectoral model is more comprehensive than the standard aggregate Kaleckian model, because it can reproduce its basic implications. There is only one sector in the aggregate Kaleckian model, and such a case can be approximated by $i = n = 1$ in the current model. Consequently, the share of saving and investment is always unity, and $\nu = 1$ and $\mu = 1$ hold. Furthermore, as we do not have to consider the sectoral differences concerning the elasticity parameters, these parameters can be replaced with macroeconomic ones, that is, $\alpha_i = \alpha, \beta_i = \beta, \gamma_i = \gamma, \theta_i = \theta_i$, and $\hat{A}_i = \hat{A}$. Then, Equation (10) becomes

$$\hat{X} = \hat{A} - \frac{[\alpha - \beta\delta + \gamma\delta]\dot{\hat{\sigma}}}{\alpha + \beta - (\gamma + \theta)},$$

(25)

which is very similar to the growth and distribution of Bhaduri and Marglin (1990) or the demand regime model shown in Storm and Naastepad (2012). As long as the Keynesian stability condition is assured, the impact of an increase in autonomous investment demand on the output growth rate is positive.\textsuperscript{8} The effect of an increase in the rate of change in the wage share has an ambiguous sign on the output growth rate. In this regard, the model reproduces a Bhaduri

\textsuperscript{8}Moreover, when we add additional assumptions in Equation (25) that workers consume all their wages, capitalists save a constant fraction of their profits (i.e. $\alpha = 0$), and capital accumulation is exogenous (i.e. $\theta = 0$ and $\gamma = 0$), the equation is reduced to an expression similar to the so-called ‘Cambridge equation’ (Pasinetti (1974)) that connects growth and distribution. Using these assumptions and rearranging Equation (25), we get $\beta(\hat{X} - \delta\dot{\sigma}) = \hat{A}$, where $-\delta\dot{\sigma}$ represents the growth rate of the profit share. The LHS of this represents the growth of saving, and the RHS represents that of autonomous capital accumulation. The causality runs from RHS to LHS, and thus, the current model still involves a property of Keynesian demand-led growth.
and Marglin (1990)-type implication. On one hand, an increase in the growth of the wage share reduces saving growth (i.e. increases consumption demand) over time because it redistributes income from profit earners (who have a higher saving elasticity) to wage earners (who have lower saving elasticity). On the other hand, profit squeeze caused by this reduces the investment growth over time through the profit-share elasticity. The establishment of a growth regime depends on the relative size of these effects on each demand component. If a rise in the wage-share growth increases consumption growth faster and offsets its negative impact on investment growth, output growth consequently increases. Such growth corresponds to a typical wage-led regime. However, when a rise in profit share growth has a strong impact on investment growth and offsets its negative impact on consumption growth, output growth also increases, corresponding to a typical profit-led regime.

Thus, in an aggregate Kaleckian model, only elasticity or propensity parameters play an important role in establishing the growth regime. However, the current multi-sectoral growth model derived using the extension of the Kaleckian model shows that sectoral composition also conditions the establishment of the growth regime. The investigation so far indicates that different sectors may have different growth regimes and that growth regimes at the aggregate level do not always correspond to those established in each sector. More precisely, even if the wage-led growth condition is dominant for the output growth rate that realizes an IS balance of a sector, it does not always lead to the implication that an increase in wage share growth also increases overall economic growth. Thus, one of the most important implications derived from this section is that even if an economic phenomenon holds at the sectoral level, it may not do so at the macroeconomic level. This is an example of analytical expression of what Keynesians have emphasized as the fallacy of composition. If we assume that the wage-led growth is true of the macroeconomy just because it is true of a sector in the macroeconomy, we may fall into the fallacy of composition.

The IS balance in the standard Kaleckian model is setup using propensity parameters (Bhaduri and Marglin (1990); Taylor (2004)). However, some Kaleckian models such as Blecker (2002), Naastepad and Storm (2007), and Sasaki (2014) include elasticity parameters in the IS balance. In both formalizations, the meaning of the stability condition for the output and growth rate adjustment is similar and known as the ‘Keynesian stability condition’. This depicts that saving responds more to the change in output than investment does.
4 Conclusion

This study revealed the relationship of the rate of change in income distribution and economic growth in a multi-sectoral Kaleckian model. Here, we investigated sectoral properties that induce aggregate output growth within the Kaleckian model. In doing so, we emphasized the importance of sectoral heterogeneities, such as sectorally different elasticities of saving and investment and sectorally different shares of these volumes. It is a simple closed model but a new one in that it includes sectoral properties of an economy that have not been sufficiently investigated by the existing literature. Using this framework, we intend to reveal how such properties affect the stability and steady-state values of the aggregate output growth rate. In summary, heterogeneous properties among sectors act to both stabilize and destabilize the aggregate output growth rate adjustment, and they determine the type of growth regime at the sectoral and aggregate levels. Moreover, the current Kaleckian multi-sectoral model is a more comprehensive model than standard aggregate Kaleckian models such as those of Bhaduri and Marglin (1990), Blecker (2002), Taylor (2004), and Lavoie (2006), because it presents implications similar to those of these models and those that can never be obtained from these models. To be more precise, the main results are summarized as follows.

First, there are differences between the standard aggregate Kaleckian model and a multi-sectoral Kaleckian model with regard to the stability conditions concerning output growth. In the aggregate Kaleckian growth model, propensity or elasticity parameters of saving and investment are critical to the stability of aggregate output growth. For the stability of output growth, the response of saving to the change in the output variables has to be larger than that of investment. This is known as the Keynesian stability condition, and it is often imposed in the Kaleckian model in an aggregate form. By contrast, the current multi-sectoral model reveals that in addition to the elasticity parameters of saving and investment of each sector, the structure of the economy is also important to the realization of stable output growth. In this paper, the term economic structure is used to especially indicate the sectoral composition of saving and investment. Even if stabilization conditions similar to those of the aggregate model are not satisfied in some sectors, stabilization of output growth at the aggregate level can be realized depending on the share of saving and investment in each sector. Therefore, not only changes in saving and investment behaviour in each sector but also sectoral shifts of saving and investment shares matter to the stability of the economic growth rate. In other words, the multi-sectoral model clarifies that their
sectoral reallocation is another source of stable economic growth.

Second, there are differences between the standard aggregate Kaleckian model and the multi-sectoral Kaleckian model as to the formation of the type of growth regime. In the aggregate Kaleckian model, in general, when the saving propensity to wages and the profit share effect on investment are relatively large (small) compared with the accelerator effect on investment, the growth regime tends to be profit-led (wage-led). Thus, propensity or elasticity parameters of saving and investment principally determine the type of growth regime in an economy. It should be emphasized that the current multi-sectoral model reveals that the structure of the economy, reflected in the sectoral share of saving and investment volumes, also plays an important role in determining the growth regime. A structural change in sectoral saving and investment composition may transform a growth regime into another one. For example, a sectoral shift of saving share to a sector with a higher difference of saving elasticities to wage and profit income contributes to the establishment of a wage-led growth regime. In addition, a sectoral shift of investment share toward a sector with higher elasticity of investment to profit contributes to the establishment of a profit-led growth regime.

The last important implication, related to the second result, is that the aggregate output growth rate that equilibrates aggregate investment and saving balance over time may differ from the one that equilibrates each sectoral balance over time. Consequently, there is a different mechanism of growth regime formation at the sectoral and macroeconomic levels. Even if a certain regime is dominant in some sectors, it does not necessarily mean that the growth regimes in another sector and at the aggregate level are the same. In other words, the current multi-sectoral Kaleckian model shows that while some sectors are operating in a wage-led regime, other sectors and the aggregate growth regime may be operating in a profit-led regime. This is because the aggregate growth regime is also affected by the sectoral shares of saving and investment and because the sectoral composition of saving and investment as well as the elasticity of saving and investment matter for the growth regime at the aggregate level.

These are the original implications obtained from the current Kaleckian model. The Kaleckian economic growth model in its disaggregate form enables us to consider how sectoral characteristics are concerned with economic growth. We hope that the results in this paper will provide useful foundations for further research, because few works on multi-sectoral Kaleckian models exist. Then, some extensions are required. For instance, the rate of change in income distribu-
tion in the current model is still exogenous. Therefore, an extended model that can dynamically endogenize the income distribution should be presented. Furthermore, in our model, the sectoral composition of saving and investment is assumed to also be constant and exogenous. Such an assumption may be valid as long as the structural change is a long-run process (Isaksson (2010)), and a long period is required to observe a clear change in sectoral composition values. An extended model that captures the changing sectoral structure will also be interesting. These issues are left for future research.

References


