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Ryunosuke Sonoda and Hiroaki Sasaki

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Research Project Center
Graduate School of Economics
Kyoto University
Yoshida-Hommachi, Sakyo-ku
Kyoto City, 606-8501, Japan

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Differences in Wage-Determination Systems between Regular and Non-Regular Employment in a Kaleckian Model*

Ryunosuke Sonoda† and Hiroaki Sasaki‡

Abstract

In this study, we build a Kaleckian model incorporating institutional differences between the wage determination of regular employment and that of non-regular employment. Using this model, we investigate how an employment shift toward regular workers affects the capacity utilization rate and income distribution. Our results show that while such shift in employment decreases the capacity utilization rate and increases the wage share of regular workers, it either increases or decreases the wage share of non-regular workers. An increase in the flexibility of the labor market, as seen in an employment shift toward non-regular workers, increases the amplitude of business cycles. However, the introduction of a minimum wage for

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† Graduate School of Economics, Saga University

‡ Corresponding author. Graduate School of Economics, Kyoto University. Yoshida-Honmachi, Sakyo-ku, Kyoto 606-8501, Japan. E-mail: sasaki@econ.kyoto-u.ac.jp

non-regular workers stabilizes the economy.

Key words: wage gap; regular and non-regular employment; demand-led growth model

JEL Classifications: E12; E25; J31

1. Introduction

The purpose of this study is to build a Kaleckian model that explicitly considers institutional differences in wage–labor determinations between regular workers and non-regular workers and to investigate how labor market institutions affect the dynamics of income distribution and output.

Post Keynesian economists, especially Kaleckian economists, have developed macroeconomic theory that focuses on the relationships among income distribution, demand, and growth.[1] However, in many Kaleckian models, labor is treated as a homogeneous class, with qualitative differences in employment being ignored. In capitalist economies in the real world, sizable differences exist between regular and non-regular workers in terms of the flexibility of labor adjustment and the wage-determination processes involved. In Japan, for example, the labor union participation rate of non-regular workers is low, and the principle of "equal pay for equal work" is not legally established, and hence, such differences can be large. To investigate the effect of labor market institutions on the dynamics of income distribution and demand formation, we need a Kaleckian model that explicitly distinguishes regular and non-regular workers.

To the best of our knowledge, only a few studies using Kaleckian models consider two types of employment. For Kaleckian models that explicitly consider fixed and variable labor, we can examine those of Rowthorn (1981) and Raghavendra (2006).[2] In their models, fixed labor is tied to potential output, while variable labor is tied to actual output. However, in Rowthorn (1981), the wage share is exogenously given, and the mechanism of income distribution determination is not specified. In contrast, in Raghavendra (2006), income distribution is endogenously determined. However, both fixed and variable workers earn the same real wage; the difference between the two types of labor is reflected in differences in the flexibility of employment adjustment. However, in reality, there is a wage gap between these two types of labor, and it is this gap that we consider in our model.

Based on the above observations, Lavoie (2009), Sasaki et al. (2013), and Sasaki (2015) present Kaleckian models that consider the wage gap between two types of labor. Lavoie (2009) assumes that the wages of fixed labor are higher than those of variable labor. Sasaki et al. (2013) and Sasaki (2015) each interpret fixed and variable labor as regular and non-regular employment, respectively, and assume that the real wage rate of regular workers is higher than that of non-regular workers. However, in these models, only the wage-determination mechanism of regular workers is specified, and the wages of non-regular workers are determined by multiplying the real wage rate of regular workers by an exogenous parameter. However, the real wage rate of regular workers and that of non-regular workers differ not only in their magnitude but also in terms of their wage-determination systems. The wage determination of regular workers is influenced by the collective bargaining of labor unions whereas that of non-regular workers is directly influenced by supply–demand conditions within the labor market.

Tavani and Vasudevan (2014) and Palley (2014) present Kaleckian models while considering differences in patterns of wage evolution between two types of labor markets.[3] They build models that consider three classes—namely, capitalists, managers, and workers. Tavani and Vasudevan (2014) specify the wage-gap dynamic between managers and workers, and Palley (2014) specifies the dynamics of the ratio of workers' wage income to total national income. Both studies consider differences in two types of labor markets; however, they do not explicitly consider changes in the wage of *each* type of labor.

Based on the above discussion, we present a Kaleckian model in which the wage-determination mechanism of regular workers is different from that of non-regular workers and investigate the dynamics of income distribution and demand formation from an institutional perspective. In this model, changes in income distribution affect the capacity utilization rate through the demand structure, which again changes income distribution through the labor market. In this process—in which demand fluctuation affects income distribution—institutional differences appear between regular and non-regular employment.

First, we assume that regular workers undertake a collective wage bargaining. Their priority is to secure employment rather than to earn higher wages. Accordingly, in times of recession when the capacity utilization rate decreases, labor unions resist flexible employment adjustment, and instead choose to compromise on wage claims. As a result, with regard to regular workers in a recession, a decrease in labor productivity through labor hoarding and a restraint of real wage through compromise during wage bargaining can be factors that change income distribution.

Second, we assume that non-regular workers do not have the collective-bargaining powers that regular workers do and that their employment is flexibly adjusted through changes in demand fluctuations. Accordingly, in recession, the labor productivity of non-regular workers does not fall, unlike that of regular workers. However, the real wage rate of non-regular workers falls because of reduced labor demand, which can change income distribution. Moreover, the effect of wage bargaining differs, depending on how labor unions behave in society. If labor unions consist of regular workers, the fruits of collective bargaining do not affect wage of non-regular workers. In contrast, if labor unions include non-regular workers, changes in the wages of regular workers affect the wages of non-regular workers.

As such, the wages and labor productivity of regular workers and non-regular workers change under different institutional conditions, and hence, changes in the total wage share of all working classes are determined by those wages and labor productivity levels. By using our model, we can investigate the dynamics of income distribution and output in an economy that has regular and non-regular workers whose mechanisms of employment and wage determination differ.

The remainder of this paper is organized as follows. Section 2 presents our model, in which the wage share of regular workers and that of non-regular workers change independently; the section also formulates the demand regime and the distributive regime. Section 3 obtains a system of differential equations with regard to the capacity utilization rate and the wage shares of regular and non-regular workers; it then investigates the local stability of the steady state equilibrium. The results reveal that the stability of the dynamic system depends on whether collective wage bargaining includes a determination for non-regular workers. Section 4 conducts a comparative static analysis and examines the effect of an employment shift toward regular workers on the steady state equilibrium values. Section 5 conducts numerical simulations to investigate the implications of flexicurity policy. Section 6 concludes.

2. The model

We consider an economy in which one good is produced by oligopolistic firms. We abstract a government sector and an export–import sector. Additionally, there are three classes—namely, capitalists, regular workers, and non-regular workers.

A fraction of the total output Y is produced by regular workers (θY) and the rest is produced by non-regular workers ($(1-\theta)Y$),[4] where $0<\theta<1$. In this case, the production function can be written as follows:

$$Y=\min\left\{\frac{\lambda_r L_r}{\theta},\frac{\lambda_{nr} L_{nr}}{1-\theta},uK\right\}, \tag{1}$$

where λ_r denotes the labor productivity of regular workers; λ_{nr}, the labor productivity of non-regular workers; L_r, the employment of regular workers; L_{nr}, the employment of non-regular workers; u, the capacity utilization rate; and K, the capital stock.

Let w_r and w_{nr} denote the real wage rate of regular workers and that of non-regular workers, respectively. Let ψ denote the fraction of all wage incomes to national income. Then, we have

$$\psi=\frac{w_r L_r + w_{nr} L_{nr}}{Y}=\theta\cdot\frac{w_r L_r}{\theta Y}+(1-\theta)\cdot\frac{w_{nr} L_{nr}}{(1-\theta)Y}. \tag{2}$$

Let $\psi_r = w_r L_r / \theta Y$ and $\psi_{nr} = w_{nr} L_{nr} / (1-\theta)Y$. Then, equation (2) can be written as follows:

$$\psi=\theta\psi_r+(1-\theta)\psi_{nr}. \tag{3}$$

That is, the total wage share ψ is a weighted average of ψ_r and ψ_{nr}, with θ being a weight. Therefore, the wage share (i.e., the ratio of each labor type's wage to national income) of regular workers is given by $\theta\psi_r$ and that of non-regular workers by $(1-\theta)\psi_{nr}$.

2.1 Demand regime

Based on the above assumptions, we specify a demand regime that shows the effect of changes in the income distribution of demand formation.

In the Kaleckian model, investment demand is determined independent of saving, and the disequilibrium of the goods market is adjusted through changes in the capacity utilization rate u. Let K denote the capital stock. Then, we have $u = Y/K$.[5] The capital stock K is assumed to be constant in the short run.

Following Marglin and Bhaduri (1990), we assume that investment demand is an increasing function of the capacity utilization rate and the profit share.

$$g^d = \alpha u + \beta(1-\psi) + \gamma, \quad \alpha > 0, \beta > 0, \gamma > 0. \tag{4}$$

In addition, we assume that capitalists save a constant fraction s_c of profit income Π and regular workers save a constant fraction s_r of their wage income W_r.[6] Since capitalists' saving rate is higher than that of regular workers, we have $s_c > s_r$. Non-regular workers obtain wage income $w_{nr}L_{nr}$. Since non-regular workers' wages are lower than regular workers' wages, non-regular workers cannot afford to save: they spend all wage income on consumption. Then, total real saving normalized by capital stock is given by

$$g^s = \frac{s_c \Pi + s_r W_r}{K} = u\{s_c(1-\psi) + s_r \theta \psi_r\}. \tag{5}$$

The capacity utilization rate u changes to adjust the excess demand in the goods market $g^d - g^s$; it is given by

$$\dot{u} = \phi(g^d - g^s), \quad \phi > 0, \tag{6}$$

where ϕ is a positive parameter that denotes the adjustment speed of the goods market— that is, the capacity utilization rate increases with excess demand and decreases with excess supply.

Let u^* denote a capacity utilization such that $\dot{u} = 0$, which is given by

$$u^* = \frac{\beta(1-\psi) + \gamma}{s_c(1-\psi) + s_r\theta\psi_r - \alpha}. \tag{7}$$

From equations (1)–(4), we obtain the partial derivative of $\dot{u}$ with respect to u, as follows:

$$\frac{\partial \dot{u}}{\partial u} = \phi[\alpha - s_c + (s_c - s_r)\theta\psi_r + s_c(1-\theta)\psi_{nr}]. \tag{8}$$

For the quantity adjustment in the goods market to be stable, the right-hand side of equation (8) must be negative: this is the so-called Keynesian stability condition. In the following analysis, we assume the Keynesian stability condition, which is

$$A \equiv s_c - (s_c - s_r)\theta\psi_r^* - s_c(1-\theta)\psi_{nr}^* - \alpha > 0. \tag{9}$$

Then, we obtain

$$\frac{\partial \dot{u}}{\partial u} = -A\phi < 0. \tag{10}$$

Similarly, the partial derivative of $\dot{u}$ with respect to ψ_r is given by

$$\frac{\partial \dot{u}}{\partial \psi_r} = \phi\theta(-\beta + s_c u^* - s_r u^*). \tag{11}$$

With $(-\beta + s_c u^* - s_r u^*) = B_r$, we have

$$\frac{\partial \dot{u}}{\partial \psi_r} = \phi\theta B_r. \tag{12}$$

Here, B_r denotes the effect of changes in ψ_r on effective demand, and θB_r denotes the ultimate effect on the level of demand. Since the sign of B_r can be positive or negative, the sign of the right-hand side of equation (12) can also be positive or negative. This property derives from the dual nature of wage within the Kaleckian model. On one hand, wage has a cost aspect: an increase in ψ_r reduces the profit share, which has a negative effect on investment demand. On the other hand, wage is a source of consumption demand: an increase in income distribution among regular workers who have a saving rate lower than that of the capitalists has a positive effect on investment demand, through increases in consumption demand and capacity utilization rate. Therefore, whether or not an increase in ψ_r raises the capacity utilization rate depends on the size of the parameters.

Next, the partial derivative of $\dot{u}$ with respect to ψ_{nr} is given by

$$\frac{\partial \dot{u}}{\partial \psi_{nr}} = \phi(1-\theta)(-\beta + s_c u^*) . \tag{13}$$

With $(-\beta + s_c u^*) = B_{nr}$, we have

$$\frac{\partial \dot{u}}{\partial \psi_{nr}} = \phi(1-\theta)B_{nr} . \tag{14}$$

The sign of B_{nr} can be positive or negative. Since we have $B_r = B_{nr} - s_r u^*$ and $s_r u^* > 0$, we necessarily obtain

$$B_r < B_{nr} . \tag{15}$$

This property derives from the assumption that non-regular workers do not save. Increases in ψ_r and ψ_{nr} reduce the profit share, and this has a negative effect on investment.

However, a positive effect on the capacity utilization rate that is driven by an increase in consumption demand is necessarily larger for ψ_{nr} than for ψ_r; this is because non-regular workers spend all wage income on consumption, whereas regular workers spend a fraction of wage income on saving. Accordingly, one of the following three situations arises: (1) both an increase in ψ_r and an increase in ψ_{nr} have positive effects on demand, and the effect of ψ_{nr} is larger than that of ψ_r; (2) an increase in ψ_{nr} has a positive effect on demand, whereas an increase in ψ_r has a negative effect on demand; and (3) both an increase in ψ_r and an increase in ψ_{nr} have negative effects on demand and the effect of ψ_{nr} is smaller than that of ψ_r.

In line with the signs of B_r and B_{nr}, we can classify the demand regime type as follows.

(I) When $B_r < B_{nr} < 0$, we have $\partial \dot{u} / \partial \psi_r < 0$ and $\partial \dot{u} / \partial \psi_{nr} < 0$. Accordingly, increases in ψ_r and ψ_{nr} have negative effects on changes in the capacity utilization rate. Therefore, we define this regime as a "profit-led demand regime."

(II) When $B_r < 0 < B_{nr}$, we have $\partial \dot{u} / \partial \psi_r < 0$ and $\partial \dot{u} / \partial \psi_{nr} > 0$. Accordingly, an increase in ψ_r has a negative effect on changes in the capacity utilization rate, while an increase in ψ_{nr} has a positive effect on changes in the capacity utilization rate. Whether ψ_r or ψ_{nr} increases has an opposite effect on demand; therefore, we define this regime as a "regular–non-regular conflicting regime."

(III) When $0 < B_r < B_{nr}$, we have $\partial \dot{u} / \partial \psi_r > 0$ and $\partial \dot{u} / \partial \psi_{nr} > 0$. Accordingly, increases in

ψ_r and ψ_{nr} have negative effects on changes in the capacity utilization rate. Therefore, we define this regime as a "wage-led demand regime."

2.2 Distributive regime

Thus far, we have specified demand regimes that show the effect of distributive change on the capacity utilization rate. Here, we specify the opposite channel—that is, a distributive regime that shows the effect of changes in the capacity utilization rate on income distribution through the labor market.

First, we assume that regular workers conduct collective wage bargaining to secure employment. From this, even when output falls and the capacity utilization rate decreases, employment adjustment is difficult. In this case, firms attempt to adjust working hours by reducing the amount of overtime work. However, firms cannot reduce labor inputs in perfect proportion to the output reduction. Accordingly, the labor productivity of regular workers λ_r falls in a recession but increases in a boom. Such a mechanism is given by the following equation:

$$\hat{\lambda}_r = \varsigma(u - \bar{u}), \quad \varsigma > 0, \tag{16}$$

where the hat over λ_r denotes its rate of change, and $\bar{u}$ is a positive constant.

Second, we specify the rate of change in the real wage rate of regular workers w_r as follows:

$$\hat{w}_r = -\varepsilon(\psi - \overline{\psi}), \quad \varepsilon > 0, \overline{\psi} > 0. \tag{17}$$

Equation (17) shows wage determination based on collective wage bargaining that emphasizes employment security. Since $\psi_r = w_r / \lambda_r$, from equation (16), ψ_r tends to increase during a recession in line with a decrease in labor productivity λ_r due to the labor hoarding effect. However, a labor union that aims to secure employment is likely to undertake corporative wage bargaining. An increase in the wage share—that is, a decrease in the profit share—is likely to be a menace to the continuation of firms and the security of employment; hence, labor unions will accept wage cuts if they assess the wage share as greatly exceeding an appropriate level.[7] In contrast, if labor unions consider the wage share too low and perceive firms as being able to afford to pay higher wages, they will demand the distribution of profits as real wages. A distinctive feature of our model is the wage bargaining of regular workers who look to maintain the wage share.

Note that we use $\overline{\psi}$—that is, the target value of the wage share of all working classes—rather than the target value of ψ_r. This bears some implications, depending on the type of collective wage bargaining involved. As will be discussed below, in this study, we consider two cases: the case where wage bargaining influences both the real wage rate of regular workers and that of non-regular workers, and the case where wage bargaining influences only the real wage rate of regular workers.

In the first case, labor unions consider the income distribution of both non-regular and regular workers, and so they set the target value of the wage share of all working classes.

In the second case, labor unions that comprise only regular workers set the target value of ψ without considering the interests of non-regular workers. Labor unions actually set the target value of the profit share needed to maintain employment among regular workers; this value is represented by $\overline{\psi}$. Accordingly, if, for example, the wage share of non-regular workers decreases and as a result, firms restore their profit share, then regular workers will demand a real wage increase. Therefore, the assumption that regular workers set the target value of the wage share of all working classes, ψ, reflects a situation in which the labor market of non-regular workers is fully independent of the labor market of regular workers.

Since $\hat{\psi}_r = \hat{w}_r - \hat{\lambda}_r$, from equations (3), (16), and (17), the rate of change in ψ_r is given by

$$\hat{\psi}_r = -\varepsilon\{\theta\psi_r + (1-\theta)\psi_{nr} - \overline{\psi}\} - \varsigma(u - \overline{u}). \tag{18}$$

From equation (18), we obtain the following partial derivatives:

$$\frac{\partial \dot{\psi}_r}{\partial u} = -\varsigma\psi_r^*, \tag{19}$$

$$\frac{\partial \dot{\psi}_r}{\partial \psi_r} = -\varepsilon\theta\psi_r^*, \tag{20}$$

$$\frac{\partial \dot{\psi}_r}{\partial \psi_{nr}} = -\varepsilon(1-\theta)\psi_r^*,. \tag{21}$$

Next, we specify the labor market of non-regular workers.

We assume that firms can flexibly adjust non-regular employment according to changes in output. For this reason, in contrast to the case of regular workers, changes in labor productivity due to the labor hoarding effect do not apply to non-regular workers, and hence λ_{nr} stays constant through time. Therefore, we have

$$\hat{\lambda}_{nr} = 0. \tag{22}$$

In addition, the rate of change in the real wage rate of non-regular workers w_{nr} is given by

$$\hat{w}_{nr} = \eta(u - \overline{u}) + \delta(\psi_r - \overline{\psi}_r), \quad \eta > 0, \delta \geq 0. \tag{23}$$

The first term in the right-hand side of equation (23) shows a reserve army effect. Since we assume flexible employment adjustment for non-regular workers, the level of their real wage reflects supply–demand conditions in the labor market. Hence, the real wage rate increases during a boom, when the capacity utilization rate increases and the labor market tightens; on the other hand, it decreases during a recession, when both the capacity utilization rate and labor demand decrease.

The second term in the right-hand side of equation (23) shows the effect of collective wage bargaining on the wage determination of non-regular workers. In a society where many non-regular workers belong to labor unions, changes in ψ_r as determined by wage bargaining will have some effects on the wage level of non-regular workers. Accordingly, in this case, we have $\delta > 0$, and hence, the real wage of non-regular workers will to some extent change along with the real wage of regular workers. However, in a society where labor unions conduct wage bargaining that reflects only the interests of regular workers—and hence where the fruits of wage bargaining do not influence the real wage of non-regular workers—we have $\delta = 0$. Then, the labor market of non-regular workers will be fully isolated from the labor market of regular workers, and the real wage rate of non-regular workers will therefore change only on account of a reserve army effect. We can interpret $\overline{\psi}_r$ as a reference value such that the fruits of wage bargaining reflect in the real wage rate of non-regular workers. When regular workers obtain a higher real wage in relation to labor productivity, the real wage rate of non-regular workers tends to increase.

From equations (22) and (23), the rate of change in ψ_{nr} is given by

$$\hat{\psi}_{nr} = \hat{w}_{nr} - \hat{\lambda}_{nr} = \eta(u - \overline{u}) + \delta(\psi_r - \overline{\psi}_r). \tag{24}$$

From equation (24), we obtain the following partial derivatives:

$$\frac{\partial \dot{\psi}_{nr}}{\partial u} = \eta \psi^*_{nr}, \tag{25}$$

$$\frac{\partial \dot{\psi}_{nr}}{\partial \psi_r} = \delta \psi^*_{nr}, \tag{26}$$

$$\frac{\partial \dot{\psi}_{nr}}{\partial \psi_{nr}} = 0. \tag{27}$$

2.3 Steady state equilibirum

In summarizing the above discussion, we obtain the following system of differential equations.

$$\dot{u} = \phi\{\alpha u + \beta(1-\psi) + \gamma - [s_c(1-\psi) + s_r \theta \psi_r]u\}, \quad \psi \equiv \theta\psi_r + (1-\theta)\psi_{nr}, \tag{28}$$

$$\dot{\psi}_r = -\varepsilon(\psi - \overline{\psi}) - \varsigma(u - \overline{u}), \tag{29}$$

$$\dot{\psi}_{nr} = \eta(u - \bar{u}) + \delta(\psi_r - \bar{\psi}_r). \tag{30}$$

Let us examine the steady state equilibrium values. By setting $\dot{\psi}_{nr} = 0$ in equation (30), we obtain $\eta(u^* - \bar{u}) = -\delta(\psi_r^* - \bar{\psi}_r)$. From this, it follows that if $u^* > \bar{u}$, then $\psi_r^* < \bar{\psi}_r$, whereas if $u^* < \bar{u}$, then $\psi_r^* > \bar{\psi}_r$. However, if we assume that the growth rate of the labor productivity of regular workers is positive—in other words, the growth rate of the real wage rate of regular workers is positive—we need $u^* > \bar{u}$ from equation (16). Accordingly, the combination of $u^* < \bar{u}$ and $\psi_r^* > \bar{\psi}_r$ is excluded, and the combination of $u^* > \bar{u}$ and $\psi_r^* < \bar{\psi}_r$ is obtained. In this case, setting $\dot{\psi}_r = 0$ in equation (29), we obtain $-\varepsilon(\psi^* - \bar{\psi}) = \varsigma(u^* - \bar{u})$. Here, since $u^* > \bar{u}$, we obtain $\psi^* < \bar{\psi}$. Hence, in the steady state, we obtain

$$u^* > \bar{u}, \ \psi_r^* < \bar{\psi}_r, \ \psi^* < \bar{\psi}. \tag{31}$$

That is, the equilibrium value of the capacity utilization rate exceeds the normal capacity utilization rate, the equilibrium value of ψ_r is smaller than the target wage share of regular workers, and the equilibrium value of the wage share of the whole economy is smaller than the target wage share of the whole economy.

By deleting ψ_r and ψ_{nr} from the equilibrium conditions, we obtain a quadratic equation of u, as follows:

$$\left(\frac{s_c\varsigma}{\varepsilon} - \frac{s_r\theta\eta}{\delta}\right)u^2 + \left[s_c\left(1 - \frac{\varsigma\bar{u}}{\varepsilon} - \bar{\psi}\right) + s_r\theta\left(\bar{\psi}_r + \frac{\eta\bar{u}}{\delta}\right) - \alpha - \frac{\beta\varsigma}{\varepsilon}\right]u + \left[\beta\left(\frac{\varsigma\bar{u}}{\varepsilon} + \bar{\psi} - 1\right) - \gamma\right] = 0. \tag{32}$$

From this, we obtain the steady state equilibrium value of the capacity utilization rate.[8] Using this value, we obtain the steady state equilibrium values of ψ_r and ψ_{nr}.

3. Stability analysis

In section 2, we formulated the demand regime and the distributive regime in the two types of labor market. In this section, we investigate the stability of the dynamic system consisting of three variables, u, ψ_r, and ψ_{nr}.

From equations (28), (29), and (30) shown in the previous section, the Jacobian matrix that is evaluated at the steady state equilibrium values is given by

$$\mathbf{J} = \begin{pmatrix} \frac{\partial \dot{u}}{\partial u} & \frac{\partial \dot{u}}{\partial \psi_r} & \frac{\partial \dot{u}}{\partial \psi_{nr}} \\ \frac{\partial \dot{\psi}_r}{\partial u} & \frac{\partial \dot{\psi}_r}{\partial \psi_r} & \frac{\partial \dot{\psi}_r}{\partial \psi_{nr}} \\ \frac{\partial \dot{\psi}_{nr}}{\partial u} & \frac{\partial \dot{\psi}_{nr}}{\partial \psi_r} & \frac{\partial \dot{\psi}_{nr}}{\partial \psi_{nr}} \end{pmatrix} = \begin{pmatrix} -A\phi & \phi\theta B_r & \phi(1-\theta)B_{nr} \\ -\varsigma\psi_r^* & -\varepsilon\theta\psi_r^* & -\varepsilon(1-\theta)\psi_r^* \\ \eta\psi_{nr}^* & \delta\psi_{nr}^* & 0 \end{pmatrix}. \tag{33}$$

From the Routh-Hurwitz stability criterion, when the following four values are all positive, they constitute the necessary and sufficient conditions for the local stability of the three-dimensional dynamic system

$$a_1 = -\mathrm{tr}\mathbf{J} = A\phi + \varepsilon\theta\psi_r^*, \tag{34}$$

$$\begin{aligned} a_2 &= \begin{vmatrix} -A\phi & \phi\theta B_r \\ -\varsigma\psi_r^* & -\varepsilon\theta\psi_r^* \end{vmatrix} + \begin{vmatrix} -A\phi & \phi(1-\theta)B_{nr} \\ \eta\psi_{nr}^* & 0 \end{vmatrix} + \begin{vmatrix} -\varepsilon\theta\psi_r^* & -\varepsilon(1-\theta)\psi_r^* \\ \delta\psi_{nr}^* & 0 \end{vmatrix} \\ &= \phi[\theta\psi_r^*(\varepsilon A + \varsigma B_r) - (1-\theta)\psi_{nr}^*\eta B_{nr}] + (1-\theta)\delta\varepsilon\psi_r^*\psi_{nr}^*, \end{aligned} \tag{35}$$

$$a_3 = -\det\mathbf{J} = \phi(1-\theta)\psi_r^*\psi_{nr}^*[\theta\varepsilon\eta(B_r - B_{nr}) + \delta(\varsigma B_{nr} + \varepsilon A)], \tag{36}$$

$$a_1 a_2 - a_3 . \tag{37}$$

We can express a_1, a_2, a_3, and $a_1 a_2 - a_3$ as functions of parameter ϕ that denotes the adjustment speed of the goods market, as follows:

$$a_1 = A\phi + C , \tag{38}$$

$$a_2 = D\phi + E , \tag{39}$$

$$a_3 = F\phi , \tag{40}$$

$$a_1 a_2 - a_3 \equiv f(\phi) = AD\phi^2 + (AE + CD - F)\phi + CE . \tag{41}$$

We have $A > 0$ and $C = \varepsilon\theta\psi_r^* > 0$ from our assumptions, and therefore $a_1 > 0$ necessarily holds. $D = \theta\psi_r^*(\varepsilon A + \varsigma B_r) - (1-\theta)\psi_{nr}^*\eta B_{nr}$, and so the sign of D is ambiguous. Additionally, we can know that $E = (1-\theta)\delta\varepsilon\psi_r^*\psi_{nr}^* > 0$. From these signs, the sign of a_2 is ambiguous.

$F = (1-\theta)\psi_r^*\psi_{nr}^*\{\theta\varepsilon\eta(B_r - B_{nr}) + \delta(\varsigma B_{nr} + \varepsilon A)\}$, and so the sign of F is ambiguous; therefore, the sign of a_3 is also ambiguous. However, in the case of $\delta = 0$, because $B_r - B_{nr} < 0$ holds from equation (15), $F < 0$ is necessarily satisfied and we can obtain $a_3 < 0$. In this case, this dynamic system is locally unstable, because one of the local stability conditions is not satisfied. From this analysis, we obtain the following proposition.

Proposition 1. *If the wage of non-regular workers is not at all affected by the wage of regular workers, the steady state equilibrium is locally unstable.*

We can explain as follows the mechanism that destabilizes the dynamic system. We can consider the case where a wage-led demand regime holds, as an example.[9] Both ψ_r of regular workers and ψ_{nr} of non-regular workers have a positive influence on changes in the capacity utilization rate; however, ψ_{nr} is more strongly affected than ψ_r. The reason is that non-regular workers who spend all their wage income more greatly increase aggregate demand when the wage share increases than do regular workers who spend a fraction of their wage income on saving, as mentioned above.

Now we suppose that a kind of shock occurs and the capacity utilization u falls below the equilibrium level u^*. Then, from equation (16), ψ_r increases because of a fall in labor productivity owing to a labor hoarding effect. In contrast, from equation (23), ψ_{nr} decreases because of a reserve army effect. The increase in ψ_r has a positive effect on demand; on the other hand, the decrease in ψ_{nr} has a negative effect on it. Because the latter effect is necessarily larger than the former, the total effect on demand is negative, and so demand decreases. As a result, the capacity utilization rate falls further and cumulatively moves away from equilibrium.

Therefore, we can derive the proposition as follows. Suppose that there are two labor markets, that is, one that consists of regular workers who strongly secure employment and spend a fraction of wage income on saving and the other that consists of non-regular workers whose real wage is affected only by supply and demand in the labor market and who spend all wage income. If there is no channel that connects these two labor markets through comprehensive collective bargaining, then the dynamic system of income distribution and effective demand is unstable.

However, if $\delta(\varsigma B_{nr} + \varepsilon A)$ that constitutes F is positive and its absolute value is sufficiently large, then $F > 0$ is satisfied—that is, $a_3 > 0$ holds. Therefore, by making collective bargaining include non-regular workers and connecting the two labor markets to some extent, a possibility exists that the dynamic system will be stabilized.

In addition, from $\delta > 0$, $\varsigma > 0$, and $\varepsilon A > 0$, we know that this stability condition is more likely to be satisfied when B_{nr} is positive and its absolute value is sufficiently large. Furthermore, because $B_r = B_{nr} - s_r u^*$ necessarily holds, if $B_r > 0$ is satisfied, B_{nr} is positive and its absolute value is likely to be sufficiently large. Therefore, when a demand regime is wage-led—that is, in the case of $0 < B_r < B_{nr}$—this dynamic system is most likely to be stable.

Even though $F > 0$ is satisfied, $a_1 a_2 - a_3 > 0$ may not hold. We must therefore investigate the sign of $f(\phi) = a_1 a_2 - a_3$. In the following, we analyze the stability of dynamics in the case of $D > 0$ and in that of $D < 0$. From these analyses, we obtain the following proposition:

Proposition 2. *Both in the case of $D > 0$ and in that of $D < 0$, when the speed of adjustment of the goods market lies within some range, a limit cycle occurs.*

Proof. See the Appendix.

In the following numerical simulations, we show that there is actually a significant equilibrium and that it is possible for a limit cycle to occur, in the case where a wage-led demand regime and $D > 0$ holds.[10]

4. Effect of an increase in regular employment

The important parameter in our model is θ, the ratio of output produced by regular workers to total output.[11]

Now we consider how equilibrium values are affected when the parameter θ increases and an employment shift toward regular workers progresses. Because the steady state equilibrium must be stable for comparative static analysis, in the following we assume that all necessary and sufficient conditions for stability are satisfied.

By totally differentiating the three equilibrium conditions, we obtain the following equation:

$$\frac{du^*}{d\theta} = \frac{-\delta\varepsilon s_r \psi_r^* u^*}{\varepsilon\eta\theta(B_r - B_{nr}) + \delta(\varsigma B_{nr} + \varepsilon A)}. \tag{42}$$

As shown in the stability analysis in section 3, $F > 0$ must hold so that the equilibrium is stable; in this case, $\varepsilon\eta\theta(B_r - B_{nr}) + \delta(\varsigma B_{nr} + \varepsilon A) > 0$ necessarily holds, and so the denominator of the right-hand side of this equation is positive. In addition, the numerator of the right-hand side of this equation is always negative, and so $du^* / d\theta < 0$ necessarily holds.

Therefore, when an employment shift toward regular workers progresses, regardless of the demand regime, the equilibrium capacity utilization rate will fall. The reason is that an increase in the ratio of regular workers with a greater propensity to save has a negative effect on effective demand.

Totally differentiating the equilibrium conditions, we obtain:

$$\frac{d\psi_r^*}{d\theta} = \frac{\varepsilon\eta s_r \psi_r^* u^*}{\theta\varepsilon\eta(B_r - B_{nr}) + \delta(\varsigma B_{nr} + \varepsilon A)}. \tag{43}$$

The numerator and the denominator of the right-hand side of this equation are both positive, and so this differential coefficient is unambiguously positive. Therefore, an employment shift toward regular workers increases ψ_r. The reason is that an increase in θ has a negative effect on the labor productivity of regular workers λ_r through a decrease in the equilibrium capacity utilization rate u^* and a labor hoarding effect; that increase will prompt a subsequent increase in ψ_r. From this result, we can show the effect of a change in θ on $\theta\psi_r$, as follows:

$$\frac{d\theta\psi_r}{d\theta} = \psi_r + \theta\frac{d\psi_r}{d\theta} > 0. \tag{44}$$

Therefore, an employment shift toward regular workers increases the wage share of regular workers $\theta\psi_r$.

We can also show that the total wage share ψ and ψ_r change in the same direction, as $d\psi = (\delta\varsigma / \eta\varepsilon)d\psi_r$ holds. Thus, an increase in parameter θ increases ψ^*:

$$\frac{d\psi^*}{d\theta} > 0. \tag{45}$$

Therefore, an employment shift toward regular workers increases the total wage share. The result is that this employment shift will reduce the profit share.

Next, we show the effect of a change in θ on ψ_{nr} as follows:

$$\frac{d\psi_{nr}}{d\theta} = \frac{1}{1-\theta}\left[\frac{(\varsigma\delta - \theta\varepsilon\eta)s_r\psi_r u}{\theta\varepsilon\eta(B_r - B_{nr}) + \delta(\varsigma B_{nr} + \varepsilon A)} - (\psi_r - \psi_{nr})\right]. \tag{46}$$

The sign of this differential coefficient is ambiguous. Therefore, it is ambiguous whether an employment shift toward regular workers increases or reduces ψ_{nr}. We can explain the reasoning as follows. If a reserve army effect η is large and a ripple effect from ψ_r to ψ_{nr} is small, because an increase in θ will always reduce u^*, an employment shift toward regular workers will reduce ψ_{nr} through a reserve army effect. However, if a reserve army effect η is small and a ripple effect from ψ_r to ψ_{nr} is large, an increase in θ may increase ψ_{nr} with ψ_{nr}.

From this, we can show the effect of a change in θ on the wage share of non-regular workers $(1-\theta)\psi_{nr}$ as follows:

$$\frac{d(1-\theta)\psi_{nr}}{d\theta} = \frac{(\varsigma\delta - \theta\varepsilon\eta)s_r\psi_r u}{\theta\varepsilon\eta(B_r - B_{nr}) + \delta(\varsigma B_{nr} + \varepsilon A)} - \psi_r . \quad (47)$$

In the case of $\varsigma\delta - \theta\varepsilon\eta < 0$, an employment shift toward regular worker reduces the wage share of non-regular workers. On the other hand, in the case of $\varsigma\delta - \theta\varepsilon\eta > 0$, the effect of an employment shift toward regular workers on the wage share of non-regular workers is ambiguous. The reason is the same as that mentioned above.

5. Numerical simulations

In this section, using numerical simulations, we show that business cycles actually occur; we also analyze the effect of an increase in the flexibility of the labor market—that is, an employment shift toward non-regular workers—on steady state equilibrium values and the dynamics of our model.

First, we set the parameters as follows:

$$\theta = 0.5, \quad \alpha = 0.1, \quad \beta = 0.15, \quad \gamma = 0.1, \quad s_c = 0.7, \quad s_r = 0.2, \quad \phi = 0.76, \quad \varepsilon = 0.2, \quad \varsigma = 1,$$

$$\overline{\psi} = 0.75, \quad \overline{u} = 0.8, \quad \eta = 0.3, \quad \delta = 0.1, \quad \overline{\psi}_r = 0.65 .$$

This numerical example corresponds to the case where a wage-led demand regime and $D > 0$ hold. In this case, the equilibrium values are $u^* = 0.812808$, $\psi_r^* = 0.611577$, and $\psi_{nr}^* = 0.760346$. Next, we set the initial conditions to $u(0) = 0.8$, $\psi_r(0) = 0.6$, and

$\psi_{nr}(0) = 0.76$. We also set the speed of adjustment of the goods market, $\phi = 0.76$.[12] Figures 1–3 show the results of our numerical simulation. Through these figures, we can show that the variables converge to the equilibrium with vibration.

This case is an example where the limit cycle is unstable—that is, if the initial value is close to the steady state equilibrium, the variables converge to the equilibrium with circulation; however, if the initial value is away from the steady state equilibrium, the variables diverge with circulation. This corresponds to the concept of corridor stability, as proposed by Leijonhufvud (1973).

5.1 Implications for flexicurity

Recently, the concept known as "flexicurity" has been proposed. This is a labor market policy by which employment is made more flexible, and thus, it improves social security. In this subsection, we analyze how flexicurity affects an economy.[13]

First, we analyze the effect of an increase in mobility in employment on an economy. In this study, we consider a decrease of θ as an increase in mobility in employment. That is, we consider an employment shift toward non-regular workers as an increase in mobility within the labor market. Now, we reduce the value of θ from 0.5 to 0.49, but do not change any of the other parameters; Figures 4–6 show the results of this numerical simulation. The new steady state equilibrium values are $u^* = 0.813295$, $\psi_r^* = 0.6101127$, and $\psi_{nr}^* = 0.754050$;

these values indicate that u increases, ψ_r decreases, and ψ_{nr} decreases. Therefore, an increase in mobility in employment has a positive effect on output. However, in this numerical example, the variables diverge with vibration when the value of θ decreases.

We can explain this mechanism as follows. As shown in section 3, when the capacity utilization rate falls below the equilibrium level in the case where a wage-led demand regime holds, ψ_r increases and has a positive effect on demand; in contrast, ψ_{nr} decreases and has a negative effect on demand. Because the negative effect of the latter is necessarily larger than the positive one of the former, if the two labor markets have no connection, the total effect on demand will be negative, reduce the capacity utilization rate more, and make the dynamic system unstable. However, if δ is positive and its absolute value is sufficiently large, an increase in ψ_r affects the labor market of non-regular workers and weakens a decrease in ψ_{nr}, and therefore the positive effect of ψ_r on demand will exceed the negative one of ψ_{nr}, that is, the total effect turns from negative to positive and stabilizes the dynamic system. The numerical simulation shown in Figures 1–3 corresponds to this case. However, if an employment shift toward non-regular workers occurs, the negative effect of ψ_{nr} on demand will again exceed the positive one of ψ_r, and as a result, the capacity utilization rate will continue to fall. Therefore, an increase in mobility in employment might destabilize an economy.

Now, we introduce social security to our model. In our model, social security is a policy that introduces a minimum wage with regard to non-regular workers. To set a lower limit for the wage of non-regular workers means that we make the lower limit $\psi_{nr}^{\min}$ for ψ_{nr}.

Previous studies try to introduce a minimum wage to business cycle models, such as Flaschel and Greiner (2009, 2011), and Sasaki et al. (2013). These studies claim that the introduction of a minimum wage has the effect of stabilizing an economy, in the sense that it narrows the width of fluctuations in the business cycles. We shall now investigate how the introduction of a minimum wage affects the economy in our model.

In the following, we introduce the minimum wage, $\psi_{nr}^{\min} = 0.75$. Figures 7–9 show the results of a numerical simulation in the case of $\theta = 0.49$ and $\psi_{nr}^{\min} = 0.75$. As shown in these figures, in this case, the variables converge to constant values, almost without vibration. Therefore, in our model, the introduction of a minimum wage has a stabilizing effect, in the sense that it narrows the width of the business cycles; thus, this finding is the same as that of previous studies.

In the case where we introduce a minimum wage, the values at which the variables converge are $u^* = 0.8117457$, $\psi_r^* = 0.61476266$, and $\psi_{nr}^* = 0.75$. We can obtain the values of u^* and ψ_r^* by solving the simultaneous equations where we substitute $\psi_{nr}^{\min} = 0.75$ for the equation of $\dot{u} = 0$ and $\dot{\psi}_r = 0$. We can assess this numerical example as appropriate, because the minimum wage $\psi_{nr}^{\min}$ is smaller than the equilibrium value of ψ_{nr} in the case of $\theta = 0.49$. The reason is that a minimum wage level is inappropriate if it exceeds the equilibrium value. From this result, we can show that the capacity utilization rate in the long term is minimized in the case where we try to make a labor market more flexible and introduce a minimum wage. In other words, if we make employment more flexible in order to increase the capacity utilization rate, it will widen the width of the vibrations among the business cycles, and if we introduce a minimum wage to moderate the effects of destabilization, it will make the capacity utilization rate lower than that in the case where we do not implement any policy—that is, in the case of $\theta = 0.5$. Therefore, flexicurity policies that try concurrently to make employment more flexible and improve social security may not bring about the desired results.

6. Concluding remarks

In this study, we build a Kaleckian model in which there are institutional differences as regards employment adjustment and wage determination between regular workers and non-regular workers and analyze the stability of the dynamic system. Our conclusions are as follows.

If collective bargaining does not include the wage determination of non-regular workers—that is, if the two labor markets are completely divided—the dynamic systems of income distribution and demand formation will be unstable. The reason is that with a fluctuation in the capacity utilization rate, there is a difference in the changes in income distribution between regular workers and non-regular workers, given the institutional differences between the labor markets; this makes demand fluctuations unstable, as they are combined with differences in the propensity to save.

However, if wage bargaining by a labor union reflects the interests of non-regular workers and there is an institution by which the fruits of the wage bargaining influence not only the wages of regular workers but also those of non-regular workers, the dynamic system could be stabilized. Depending on the parameter values, limit cycles may occur. With regard to the relationship between demand regime and dynamics stability, we show that the dynamics are most likely to be stable in the case where a wage-led demand regime holds.

We also conduct a comparative static analysis, and find that an employment shift toward regular workers reduces the capacity utilization rate, increases the wage share of regular workers, and reduces the profit share. However, the wage share of non-regular workers does not necessarily decrease with an employment shift towards regular workers—that is, it may instead increase on the condition.

In addition, by using numerical simulations, we investigate the effect of flexicurity policies that seek to both make employment more flexible and improve social security. We consider an employment shift towards non-regular workers as being synonymous with an increase in employment flexibility, and show that an increase in employment flexibility increases the equilibrium value of the capacity utilization rate; however, it also widens the width of vibration among business cycles. By introducing a minimum wage among non-regular workers, the width of this vibration can be narrowed. However, in this case, the point to which the variables converge may differ from the steady state equilibrium values, and under such circumstances, the implications of flexicurity policies are not clear.

This study contributes to the literature by explicitly introducing the institutional differences in labor markets between regular workers and non-regular workers, thus showing the relationship between the formation of collective bargaining and the stability of a dynamic system. In so doing, this study clarifies which demand regime is most likely to stabilize the economy, and it analyzes the effects of changes to institutional parameters on equilibrium values.

Despite its contributions, this study does have some limitations, and leaves some problems unresolved.

First, this study does not analyze fluctuations in the employment rate. In this study, we focus only on the change in income shares and do not explicitly deal with labor supply and employment level. As a result, we cannot analyze how the employment rate fluctuates. In reality however, as a matter of course, the unemployment rate necessarily increases when firms adjust the employment of non-regular workers in times of recession. In order to formulate a fluctuation of wage through the reserve army effect more appropriately, we should explicitly introduce a change in unemployment to our model.

Second, we intend to undertake empirical analyses based on this model. Our “two labor markets” model may be useful, in particular, in undertaking an empirical study of the Japanese economy. We need to resolve the problems to inherent in current labor markets by developing and applying both model analyses and empirical research.

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Appendix

(I) The case of $D > 0$

In this case, we can obtain $a_1 > 0$, $a_2 > 0$, and $a_3 > 0$. Additionally, we can know that $AD > 0$ and $CE > 0$, and that the sign of $AE + CD - F$ and of $a_1 a_2 - a_3$ is ambiguous.

First, because $f(\phi)$ is a parabola, convex downward, and $f(0) > 0$, if the discriminant of $f(\phi) = 0$ is negative, for $\phi > 0$, $f(\phi) = a_1 a_2 - a_3 > 0$ holds, and therefore, all the necessary and sufficient conditions that the equilibrium is locally stable are satisfied.

Next, if the discriminant of $f(\phi) = 0$ is positive, from $f(0) > 0$, about ϕ_1 and ϕ_2 that are two values of ϕ that satisfy $f(\phi) = 0$, we can demonstrate that ϕ_1 and ϕ_2 are either both negative or both positive. In the case where ϕ_1 and ϕ_2 are both negative, $f(\phi) = a_1 a_2 - a_3 > 0$ holds; for $\phi > 0$, and therefore all the necessary and sufficient conditions that the equilibrium is locally stable are satisfied. In the case where ϕ_1 and ϕ_2 are both positive, the sign of $f(\phi) = a_1 a_2 - a_3$ alternates; for $\phi > 0$. In other words,

$f(\phi) = a_1 a_2 - a_3 > 0$ holds; for $\phi \in (0, \phi_1)$ and $\phi \in (\phi_2, +\infty)$, on the other hand, $f(\phi) = a_1 a_2 - a_3 < 0$ holds; for $\phi \in (\phi_1, \phi_2)$. In addition, because of $f'(\phi_1) \neq 0$ and $f'(\phi_2) \neq 0$, we can prove that ϕ_1 and ϕ_2 are both Hopf bifurcation points.

(II) The case of $D < 0$

In this case, $a_1 > 0$ and $a_3 > 0$ necessarily hold. Now, we define the value of ϕ that satisfies $a_2 = D\phi + E = 0$ as $\bar{\phi} \equiv -E / D > 0$; in such a case, $a_2 = D\phi + E > 0$ is satisfied; for $\phi \in (0, \bar{\phi})$. Because $f(\phi)$ is a parabola, convex upward, $f(0) > 0$, and $f(\bar{\phi}) = EF / D < 0$, we can confirm that there is $\phi_3 > 0$ that satisfies $f(\phi_3) = 0$ within $\phi \in (0, \bar{\phi})$. In other words, $f(\phi) = a_1 a_2 - a_3 > 0$ and $a_2 = D\phi + E > 0$ hold; for $\phi \in (0, \phi_3)$, $f(\phi) = a_1 a_2 - a_3 < 0$ and $a_2 = D\phi + E > 0$ hold; for $\phi \in (\phi_3, \bar{\phi})$, $f(\phi) = a_1 a_2 - a_3 < 0$ and $a_2 = D\phi + E < 0$ hold; for $\phi \in (\bar{\phi}, +\infty)$. In addition, because $f'(\phi_3) \neq 0$, we can prove that ϕ_3 is the Hopf bifurcation point. Therefore, as in case (I), we can demonstrate two patterns where variables converge to the equilibrium, and a limit cycle occurs in case (II).

Figures

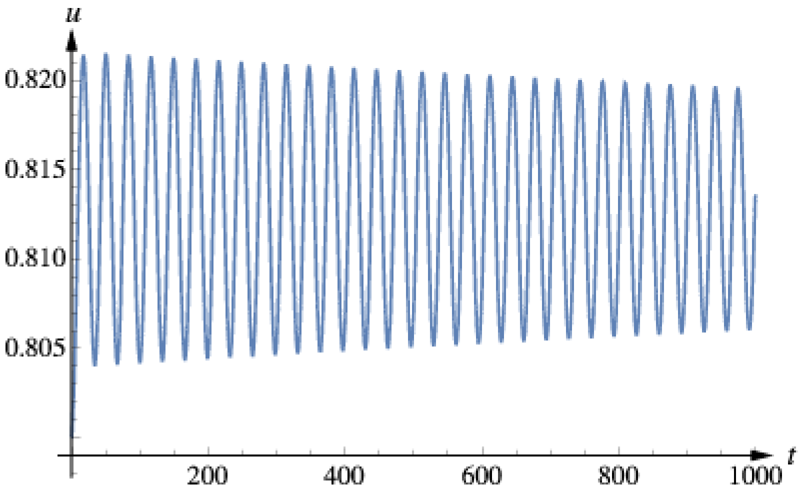

Figure 1. Time series of u under a wage-led demand regime, with $\theta = 0.5$

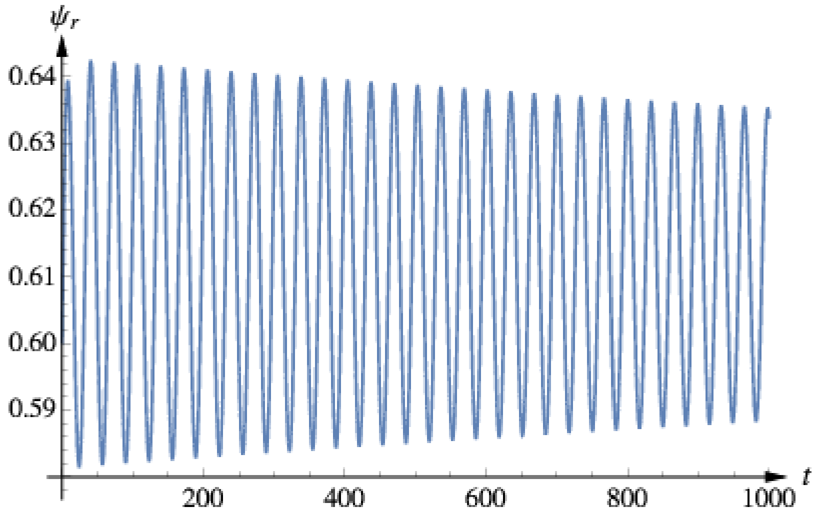

Figure 2. Time series of ψ_r under a wage-led demand regime, with $\theta = 0.5$

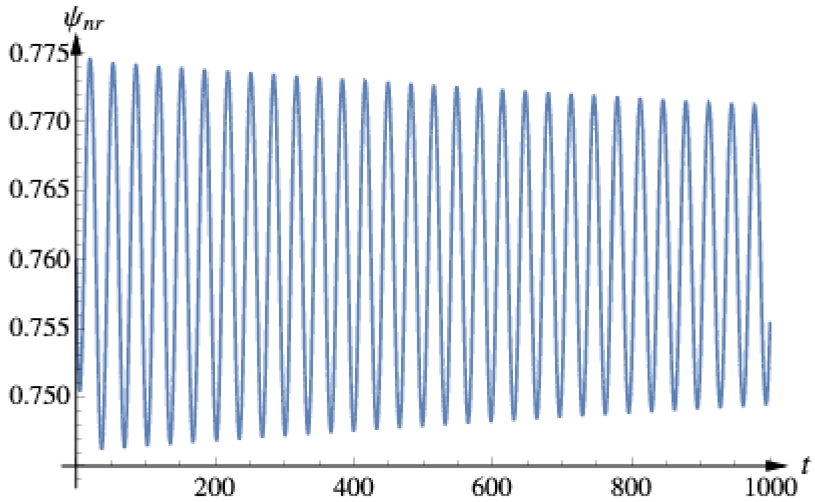

Figure 3. Time series of ψ_{nr} under a wage-led demand regime, with $\theta = 0.5$

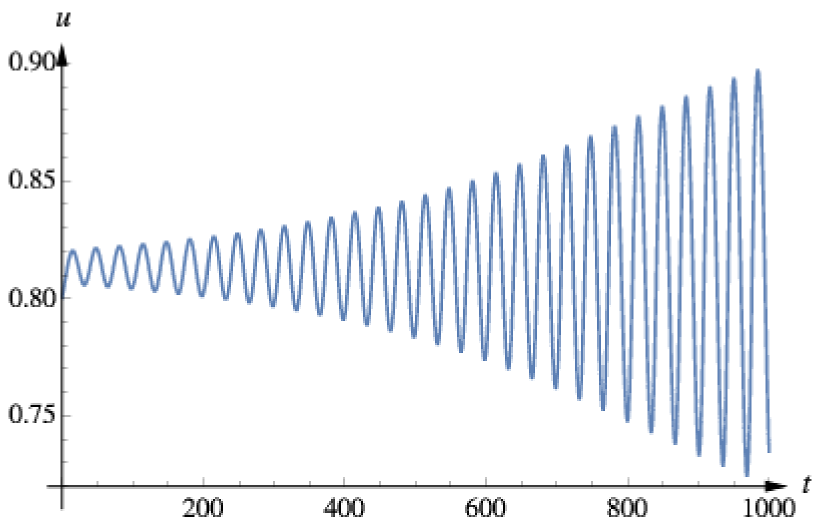

Figure 4. Time series of u under a wage-led demand regime, with $\theta = 0.49$

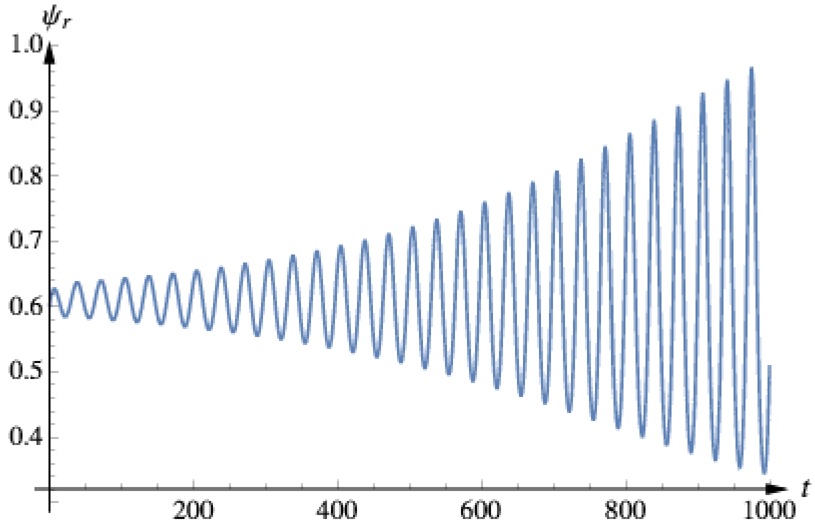

Figure 5. Time series of ψ_r under a wage-led demand regime, with $\theta = 0.49$

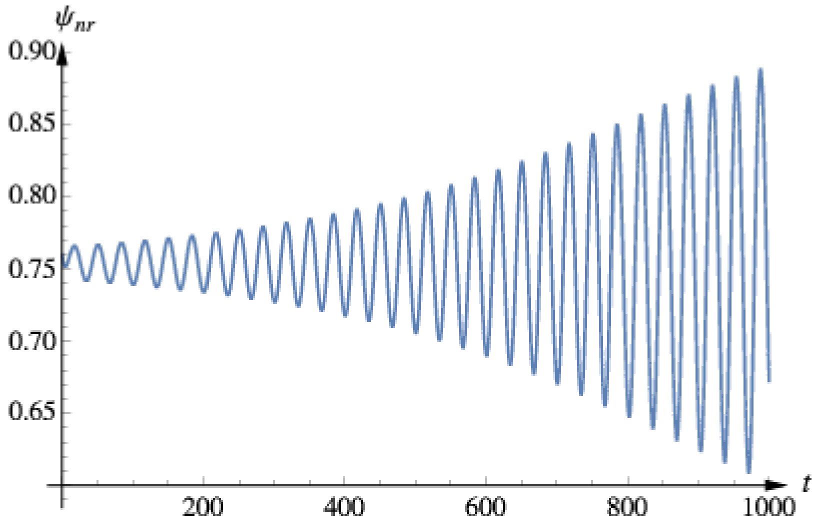

Figure 6. Time series of ψ_{nr} under a wage-led demand regime, with $\theta = 0.49$

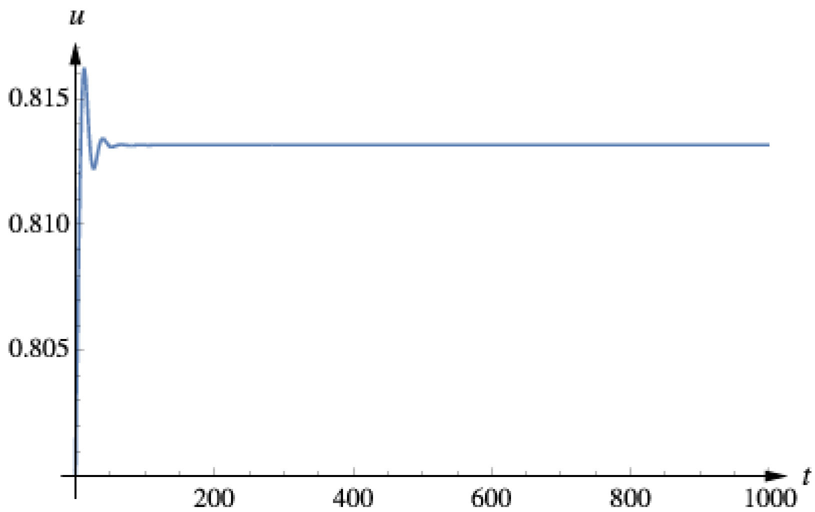

Figure 7. Time series of u with a minimum wage, with $\theta = 0.49$ and $\psi_{nr}^{\min} = 0.75$

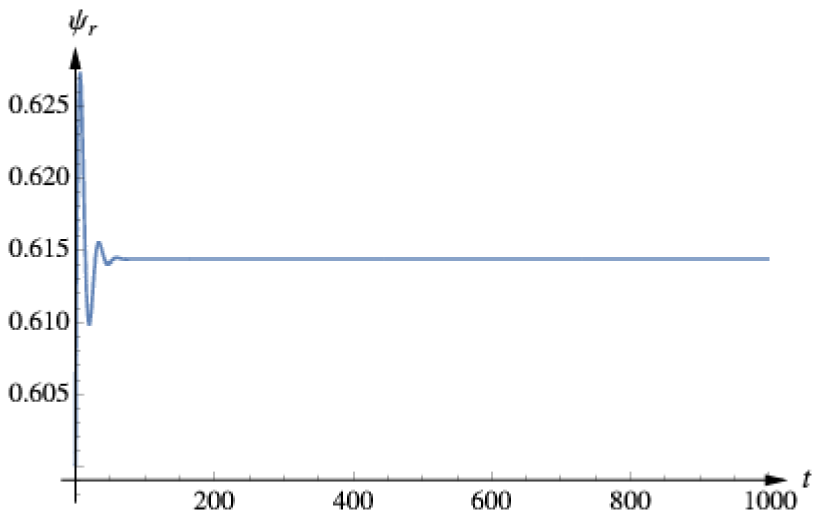

Figure 8. Time series of ψ_r with a minimum wage, with $\theta = 0.49$ and $\psi_{nr}^{\min} = 0.75$

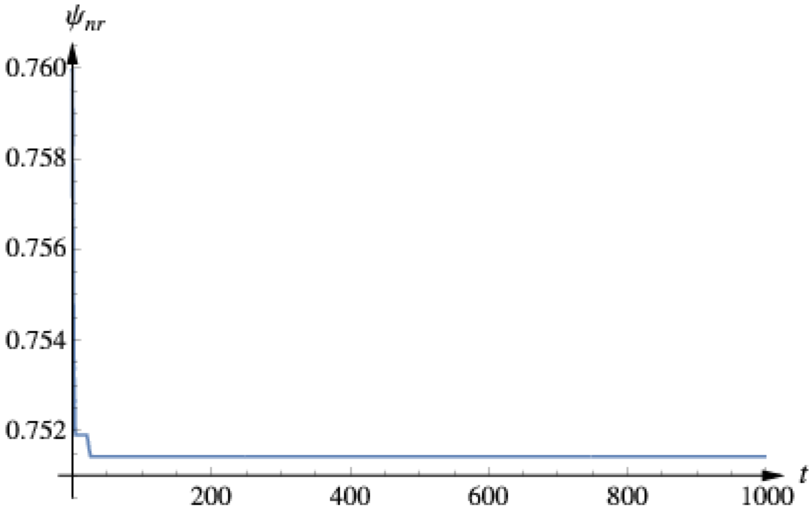

Figure 9. Time series of ψ_{nr} with a minimum wage, with $\theta = 0.49$ and $\psi_{nr}^{\min} = 0.75$

Notes

[1] For the basic framework of the Kaleckian model, see Lavoie (2014).

[2] Charpe et al. (2014) investigate a Keynesian macrodynamics model that consider two types of labor. Like Kaleckian models, Keynesian macrodynamics models also emphasize effective demand. However, unlike Kaleckian models, Keynesian macrodynamics models do not emphasize income distribution.

[3] Similar to our model, Flaschel and Greiner (2011) and Flaschel et al. (2012a) consider two types of labor markets and investigate the dynamics of the employment rates, wage rates, and income shares. Their analyses are based on the Goodwin model.

[4] Our model assumes that regular workers and non-regular workers produce the same good. Rowthorn (1981) adopts a specification such that the output of fixed labor (regular employment) is tied to the level of potential output and not affected by changes in demand whereas the output of variable labor (non-regular employment) is tied to the level of actual output. However, since in this present study, the output of regular workers varies with changes in demand and labor productivity changes due to the labor hoarding effect, the output of regular workers will change in line with variations in the capacity utilization rate.

[5] We assume that the technological ratio of the potential output to capital stock is constant. In this case, the output–capital ratio becomes a proxy variable for the capacity utilization rate.

[6] As Pasinetti (1962) rightly points out, if regular workers save, they indirectly own capital stock. However, for simplicity, we disregard this fact.

[7] To flexibly lower a basic wage is difficult in reality. However, if the fraction of income and bonus associated with business results is large—as it often is in Japanese firms—such a flexible wage adjustment is possible.

[8] In the setting of the numerical simulations introduced below, the left-hand side of this quadratic equation becomes a parabola, convex upward. By solving this equation, we obtain two positive capacity utilization rates, the larger of which corresponds to the steady state equilibrium rate of the

capacity utilization rate. If we use a smaller one, both ψ_r and ψ_{nr} become larger than unity, and the wage share of the whole economy ψ becomes larger than unity, which is inappropriate.

[9] We can explain it in the same way even the case where a profit-led demand regime or regular–non-regular conflicting regime holds; however, we use a wage-led demand regime as an example, because the steady state equilibrium is most likely to be stable when this regime holds, as mentioned below.

[10] We can also show a numerical example, wherein a limit cycle occurs in the case where a wage-led demand regime and $D < 0$ hold.

[11] The ratio of the employment of regular workers to that of non-regular workers can be expressed as $L_r / L_{nr} = [\theta/(1-\theta)] \cdot (\lambda_{nr} / \lambda_r)$. In this equation, because λ_{nr} is constant, when θ increases—as shown in the following comparative static analysis results—u^* necessarily falls; as a result, the growth rate of λ_r falls and (as long as $u^* - \bar{u}$ remains constant) λ_r also falls. We can show from this analysis that an increase in θ increases both $\theta/(1-\theta)$ and λ_{nr} / λ_r—and that, as a result, it will necessarily increase L_r / L_{nr}. Therefore, an increase in θ leads to an employment shift toward regular workers.

[12] In this numerical example, the Hopf bifurcation points are $\phi_1 = 0.050$ and $\phi_2 = 0.752$.

[13] For flexicurity, see also Flaschel et al. (2012b).