International Trade and Industrialization with Negative

Population Growth

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Abstract

This paper builds a small-open-economy, non-scale-growth model with negative population growth and investigates the relationship between trade patterns and per capita consumption growth. Under free trade, if the population growth rate is negative and its absolute value is small, the home country becomes an agricultural country. Then, the long-run growth rate of per capita consumption is positive and depends on the world population growth rate. On other hand, if the population growth rate is negative and its absolute value is large, the home country becomes a manufacturing country. Then, the long-run growth rate of per capita consumption is positive and depends on both the home country and the world population growth rate. Moreover, the home country is better off under free trade than under autarky in terms of per capita consumption growth irrespective of whether the population growth is positive or negative.

Keywords: non-scale-growth model; negative population growth; trade patterns; per capita growth

JEL Classification: F10; F43; O11; O41

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1 Introduction

This paper develops a small-open-economy, two-sector (manufacturing and agriculture), non-scale-growth model in which the growth rate of population is negative and examines the relationship between trade patterns and the long-run growth rate of per capita consumption.

In many developed countries, population growth is stagnant, and in some countries, population growth is negative.[1] The existing economic growth theories assume positive population growth. However, given that population growth can be negative in reality, we need to consider this case as well.

At first, it may seem easy to include negative population growth in economic growth theory, but this is not the case.[2] As Ferrara (2011) and Christiaans (2011) show, incorporating negative population growth in growth models is more complicated than replacing a positive population growth rate with a negative population growth rate.

For example, Christiaans (2011) shows the importance of negative population growth using a simple model. Consider a Solow growth model with a production function that exhibits increasing, but relatively small, returns to scale.[3] When the population growth rate is positive, per capita income growth is positive and increasing in the population growth rate. On the other hand, when the population growth rate is negative, contrary to expectations, per capita income growth is positive and decreasing in the population growth rate.

In this paper, we focus on the so-called non-scale-growth model in which population growth plays an important role in determining the growth rates of per capita income and consumption. The non-scale-growth model can overcome the shortcomings of the endogenous growth model with scale effects. In the endogenous growth model with scale effects, long-run per capita income growth depends positively on the population level (Romer, 1990). That is, population size positively affects per capita growth. However, this assumption seems counterfactual. Jones (1995) attempts to remove the scale effects and presents a non-scale-growth model in which the growth rate of output per capita depends positively on the population growth rate and not on the population size.[4] That is, the higher the population

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[1]For example, according to the Ministry of Internal Affairs and Communications, as of March 2012, Japan has experienced its largest-ever decline in population.

[2]Ritschel (1985) argues that in the standard Solow growth model, a negative savings rate is necessary for the existence of a steady state equilibrium with negative population growth. See also Felderer (1998).

[3]In the Solow model with a constant returns to scale production function, per capita income growth is zero when population growth is positive, whereas it is positive when population growth is negative. For details, see Christiaans (2011).

[4]Strulik (2005) introduces human capital accumulation into a growth model with endogenous technical progress due to R&D and shows that the growth rate of per capita consumption along the balanced growth path

growth rate, the faster the country grows.[5]

Christiaans (2008) is an example of a study that investigates international trade and growth using a non-scale-growth model.[6] He extends Wong and Yip's (1999) small-open-economy endogenous growth model to a non-scale-growth one. Wong and Yip's model includes scale effects, whereas Christiaans' model does not. Using a non-scale-growth model, Christiaans shows that the relationship between the trade patterns and per capita income growth of the home country is determined by whether population growth is larger in the home country or in the world.

Based on Christiaans' (2008) model, we investigate the case where population growth is negative. From our analysis, we obtain the following results.

First, when population growth is negative and its absolute value is small, the home country asymptotically completely specializes in agriculture in the long run. Per capita consumption growth depends positively on the world population growth rate, does not depend on the home country population growth rate, and is larger than that under autarky.

Second, when population growth is negative and its absolute value is large, the home country completely specializes in manufacturing in the long run. Per capita consumption growth depends on both the home country and the world population growth rates and is larger than those under autarky and in the above case.

The rest of the paper is organized as follows. Section 2 derives the equilibrium under autarky. Section 3 derives the equilibrium under free trade. In this analysis, we investigate the case where the home country diversifies and produces both agricultural and manufactured goods, the case where the home country completely specializes in manufacturing, and the case where the home country asymptotically completely specializes in agriculture. Section 4 investigates the transitional dynamics with regard to whether the home country approaches complete specialization in manufacturing or in agriculture. Section 5 concludes the paper.

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depends positively or negatively on the growth rate of population.

[5]For a systematic exposition of scale effects and non-scale growth, see Jones (1999). For more sophisticated non-scale-growth models, see also Kortum (1997), Dinopoulos and Thompson (1998), Peretto (1998), Segerstrom (1998), Young (1998), and Howitt (1999). In contrast, by estimating the population of the world from one million B.C. to 1990, Kremer (1993) shows that the world population is positively correlated with the growth rate of per capita income. This empirical fact supports the existence of scale effects.

[6]For other studies that examine international trade and economic growth using non-scale-growth models, see Sasaki (2008, 2011a, 2011b, 2012).

2 Autarky

Consider an economy with manufacturing and agricultural sectors. Let good 1 denote goods produced in the manufacturing sector and good 2 denote goods produced in the agricultural sector. Good 1 is used for both consumption and investment, and good 2 is used only for consumption. Manufactured goods are produced with labor L_1 and capital stock K, whereas agricultural goods are produced with labor L_2. Both production functions are specified as follows:

$$X_1 = AK^\alpha L_1^{1-\alpha}, \quad A = K^\beta \tag{1}$$

$$= K^{\alpha+\beta} L_1^{1-\alpha}, \quad 0 < \alpha < 1,\ 0 < \beta < 1,\ \alpha + \beta < 1, \tag{2}$$

$$X_2 = L_2, \tag{3}$$

where X_1 and X_2 denote the production of goods 1 and 2, respectively. Here, $A = K^\beta$ captures the externality due to capital accumulation and implies that manufacturing production has increasing returns to scale. The assumption $\alpha + \beta < 1$ means that the extent of the increasing returns is not very large, which means our model exhibits non-scale growth. Moreover, the Marshallian externality is imposed so that profit-maximizing firms regard A as exogenously given. Therefore, increasing returns to scale are consistent with perfect competition.

Suppose that labor supply is equal to the population L, which grows at a constant rate n, and that labor is fully employed. Therefore, we have $L_1 + L_2 = L$. Moreover, we assume that labor is free to move across the two sectors. Accordingly, wages in both sectors are equal.

Let the agricultural goods be the numéraire. Moreover, let the wage rate, the profit rate, and the price of manufactured goods be w, r, and p, respectively. From profit maximization, we obtain the following equations:

$$w = p \frac{\partial X_1}{\partial L_1} = p(1-\alpha)K^{\alpha+\beta}L_1^{-\alpha} = 1, \tag{4}$$

$$r = \left.\frac{\partial X_1}{\partial K}\right|_{A:\text{given}} = \alpha K^{\alpha+\beta-1}L_1^{1-\alpha}. \tag{5}$$

As long as agricultural goods are produced, the wage rate is $w = 1$.

For simplification, we make the classical assumption that wage income wL and capital income rpK are entirely devoted to consumption and saving, respectively. In the canonical one-sector Solow model, under the golden rule steady state where per capita consumption is maximized, consumption is equal to the real wage, and all profits are saved and invested. Hence, our assumption has some rationality and can be interpreted as a simple rule of thumb for consumers with dynamic optimization (Christiaans, 2008). In addition, we assume that

consumers' preferences take the Cobb-Douglas form.

$$\max_{C_1, C_2} U = C_1^{\gamma} C_2^{1-\gamma}, \quad 0 < \gamma < 1, \tag{6}$$

$$\text{s.t. } pC_1 + C_2 = wL. \tag{7}$$

Solving the above utility-maximizing problem, we find that a fraction γ of total wage income is spent on good 1 and the rest $1 - \gamma$ on good 2.

$$pC_1 = \gamma L, \tag{8}$$

$$C_2 = (1 - \gamma)L. \tag{9}$$

In addition, solving the expenditure-minimizing problem, we find that the consumer price index p_c is given by $p_c = p^{\gamma}$.[7]

We assume that all savings are spent on investments. From our assumption, savings are equal to profits. Then the following equation is satisfied.

$$\frac{\dot{K}}{K} = r. \tag{10}$$

That is, the rate of capital accumulation is determined by the rate of profit.

The market clearing conditions for both goods are given by

$$X_1 = C_1 + I, \tag{11}$$

$$X_2 = C_2. \tag{12}$$

By Walars' law, one of these two equations is redundant.

From these equations, the short-run equilibrium is given by

$$L_1 = \gamma L, \tag{13}$$

$$L_2 = (1 - \gamma)L, \tag{14}$$

$$p = \frac{(\gamma L)^{\alpha}}{(1 - \alpha)K^{\alpha + \beta}}. \tag{15}$$

First, we consider the case where $n > 0$.

Along the balanced growth path (BGP, hereafter), the main variables grow at constant rates. The growth rate of capital stock is constant when the profit rate is constant. Calculating the growth rate of the profit rate from equation (5) and letting the resulting expression

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[7]Strictly speaking, the consumer price index leads to $p_c = \gamma^{-\gamma}(1 - \gamma)^{-(1-\gamma)}p^{\gamma}$. However, for simplicity, we use $p_c = p^{\gamma}$ because the constant term does not affect the growth rate of p_c that is needed for later analysis.

equal zero, we obtain the following equation: $(\alpha + \beta - 1)g_K + (1 - \alpha)n = 0$. From this, the BGP growth rate of capital stock g_K^* is given by

$$g_K^* = \phi n > 0, \quad \text{where } \phi \equiv \frac{1 - \alpha}{1 - \alpha - \beta} > 1. \tag{16}$$

In the following analysis, $g_x = \dot{x}/x$ denotes the growth rate of a variable x, and an asterisk "$*$" denotes the long-run value of a variable.

Other BGP growth rates are given by

$$g_{X_1}^* = g_K = \phi n > 0, \tag{17}$$

$$g_{X_2}^* = n, \tag{18}$$

$$g_p^* = -(\phi - 1)n < 0, \tag{19}$$

$$g_c^* = g_w^* - \gamma g_p^* = -\gamma g_p^* = \gamma(\phi - 1)n > 0. \tag{20}$$

The relative price of manufactured goods is decreasing at a constant rate along the BGP. When deriving g_c, we use the fact that per capita real consumption c is equal to the real wage w/p^γ.

Considering the BGP growth rate of capital stock, we introduce a new state variable $k = K/L^\phi$. When K grows at its BGP rate, k is constant. From equations (5) and (10), the dynamics of the scale-adjusted capital stock lead to

$$\dot{k} = \alpha\gamma^{1-\alpha}k^{\alpha+\beta} - \phi nk. \tag{21}$$

The steady state is where $\dot{k} = 0$. When population growth is positive, there exists a $k > 0$ such that $\dot{k} = 0$, given by

$$k^* = \left(\frac{\alpha\gamma^{1-\alpha}}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}. \tag{22}$$

Moreover, this steady state is stable because

$$\left.\frac{d\dot{k}}{dk}\right|_{k=k^*} = -k^*[(1 - \alpha - \beta)\alpha\gamma^{1-\alpha}(k^*)^{\alpha+\beta-2} + \phi n] < 0. \tag{23}$$

Next, we consider the case where $n < 0$.

If $n < 0$, there never exists a $k > 0$ such that $\dot{k} = 0$, and we have $\dot{k} > 0$ for $k > 0$. That is, if $k > 0$, then k diverges to infinity.

In this case, we examine the growth rate of per capita consumption. Considering that per capita consumption is equal to the real wage measured in terms of the consumer price index, we obtain

$$g_c = \alpha(\alpha + \beta)\gamma^{2-\alpha}k^{\alpha+\beta-1} - \gamma\alpha n. \tag{24}$$

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From this, we have

$$\lim_{k \to +\infty} g_c = -\gamma \alpha n > 0. \tag{25}$$

From the above analysis, we obtain the following proposition.

Proposition 1. *Suppose that the population growth rate is negative. Then, g_c^* is decreasing in n, and $g_c^* > 0$ even though $n < 0$.*

3 Free trade

We consider free trade between the home country and the world. Suppose that the home country is a small open economy. Suppose also that the world's parameters are the same as those of the home country except for the population growth rate and that the world is located on its BGP. If we denote $n_w > 0$ as the average population growth rate of the world, then the growth rate of the terms of trade determined by the world market is given by

$$g_p^* = -(\phi - 1)n_w < 0. \tag{26}$$

We investigate the existence of a steady state when the home country diversifies, when it asymptotically completely specializes in agriculture, and when it completely specializes in manufacturing.

3.1 Diversification

As long as agricultural goods are produced, we have $w = 1$. From the goods market clearing condition, we obtain the growth rate of capital stock.

$$\frac{\dot{K}}{K} = \alpha(1 - \alpha)^{\frac{1-\alpha}{\alpha}} K^{\frac{\beta}{\alpha}} p^{\frac{1-\alpha}{\alpha}}. \tag{27}$$

Along the BGP, we have

$$g_K^* = -\frac{1 - \alpha}{\beta} g_p = \phi n_w > 0. \tag{28}$$

Considering the BGP growth rate of capital stock, we introduce a new variable $\tilde{k} = Kp^{\frac{1-\alpha}{\beta}}$. The dynamic equation of $\tilde{k}$ is given by

$$\dot{\tilde{k}} = \alpha(1 - \alpha)^{\frac{1-\alpha}{\alpha}} \tilde{k}^{\frac{\alpha+\beta}{\alpha}} - \phi n_w \tilde{k}. \tag{29}$$

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There exists a steady state value $\tilde{k} > 0$ such that $\dot{\tilde{k}} = 0$.

$$\tilde{k}_e = \left[\frac{\phi n_w}{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}} \right]^{\frac{\alpha}{\beta}}. \tag{30}$$

However, this steady state is unstable because $d\dot{\tilde{k}}/d\tilde{k}\big|_{k=\tilde{k}_e} > 0$, and hence, $\tilde{k}$ converges to zero or diverges to infinity.

3.2 Specialization in agriculture

As long as agricultural goods are produced, we have $w = 1$. In the above diversification case, if $\tilde{k}_0 < \tilde{k}_e$, then $\tilde{k}$ approaches zero. Then, we can prove that X_1 approaches zero when $\tilde{k}$ approaches zero. Manufacturing production can be expressed by $\tilde{k}$ and p as follows:

$$X_1 = (1-\alpha)^{\frac{1-\alpha}{\alpha}} \tilde{k}^{\frac{\alpha+\beta}{\alpha}} p^{-\frac{1-\alpha}{\beta}}. \tag{31}$$

Here, we investigate $\lim_{t\to+\infty} X_1$. The rate of change in X_1 is given by

$$\frac{\dot{X}_1}{X_1} = \frac{\alpha+\beta}{\alpha} \frac{\dot{\tilde{k}}}{\tilde{k}} - \frac{1-\alpha}{\beta} \frac{\dot{p}}{p} = (\alpha+\beta)(1-\alpha)^{\frac{1-\alpha}{\alpha}} \tilde{k}^{\frac{\beta}{\alpha}} - \frac{\beta\phi}{\alpha} n_w. \tag{32}$$

When $\tilde{k} \to 0$, $\dot{X}_1/X_1$ approaches $-(\beta\phi/\alpha)n_w$, which is negative. Accordingly, in the long run, we obtain

$$\lim_{t\to+\infty} X_1 = 0. \tag{33}$$

Therefore, if $\tilde{k}_0 < \tilde{k}_e$, manufacturing production approaches zero, and consequently, the home country asymptotically completely specializes in agriculture.

In this case, the home country becomes entirely agricultural, and capital stock is not accumulated. Nevertheless, per capita consumption growth is not zero because consumption of manufactured goods is met by imports and the relative price of these goods is decreasing at the constant rate g_p^*. Therefore, the BGP growth rate of per capita consumption approaches

$$g_c^* = -\gamma g_p^* = \gamma(\phi - 1)n_w > 0. \tag{34}$$

3.3 Specialization in manufacturing

When only manufactured goods are produced, the wage rate is given by $w = p(1-\alpha)K^{\alpha+\beta}L^{-\alpha}$. From the goods market clearing condition, the growth rate of capital stock is given by

$$\frac{\dot{K}}{K} = \alpha K^{\alpha+\beta-1} L^{1-\alpha}. \tag{35}$$

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In this case, the dynamics are described by k as in autarky and given by

$$\dot{k} = \alpha k^{\alpha+\beta} - \phi n k. \tag{36}$$

The same arguments as in autarky are applicable. If $n > 0$, there exists a steady state value of k, which is given by

$$k_I = \left(\frac{\alpha}{\phi n}\right)^{\frac{1}{1-\alpha-\beta}}, \tag{37}$$

and the steady state is stable because $d\dot{k}/dk\big|_{k=k_I} < 0$. On the other hand, if $n < 0$, there never exists a steady state value of $k > 0$ such that $\dot{k} = 0$. Then, we have $\dot{k} > 0$ for $k > 0$.

If the home country continues to completely specialize in manufacturing, the growth rate of per capita consumption in the long run is given by

$$\begin{aligned}
g_c^* &= g_p + (\alpha + \beta)g_K - \alpha n - \gamma g_p \\
&= -\alpha n - (1 - \gamma)(\phi - 1)n_w. \tag{38}
\end{aligned}$$

4 Transitional dynamics

The analysis in the preceding section assumes that when switching from autarky to free trade, one of the three trade patterns is realized. In this section, by contrast, using a phase diagram, we investigate how trade patterns evolve through time by assuming that when switching from autarky to free trade, the home country diversifies.

As the foregoing analysis shows, the dynamics of our model are described by $\tilde{k}$ and k, so we investigate the dynamics on the $(k, \tilde{k})$-plain. Considering the definitions of k and $\tilde{k}$, we obtain the following relationship between k and $\tilde{k}$.

$$k = \tilde{k} L^{-\phi} p^{\frac{\alpha-1}{\beta}}. \tag{39}$$

Log-differentiating both sides, we obtain

$$\frac{\dot{k}}{k} = \frac{\dot{\tilde{k}}}{\tilde{k}} + \phi(n_w - n). \tag{40}$$

If we want to express the dynamics under diversification using k, we can substitute the equation for $\dot{\tilde{k}}$ (i.e., equation (29)) into the right-hand side of equation (40). Similarly, if we want to express the dynamics under complete specialization in manufacturing by using $\tilde{k}$, we can substitute the equation for $\dot{k}$ (i.e., equation (36)) into the left-hand side of equation (40).

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From the above reasoning, we obtain the dynamic systems for diversification and for complete specialization in manufacturing.

$$\text{Diversification}: \begin{cases} \dot{k} = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\tilde{k}^{\frac{\beta}{\alpha}}k - \phi nk \\ \dot{\tilde{k}} = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\tilde{k}^{\frac{\alpha+\beta}{\alpha}} - \phi n_w\tilde{k} \end{cases} \tag{41}$$

$$\text{Specialization in M}: \begin{cases} \dot{k} = \alpha k^{\alpha+\beta} - \phi nk \\ \dot{\tilde{k}} = \alpha k^{\alpha+\beta-1}\tilde{k} - \phi n_w\tilde{k} \end{cases} \tag{42}$$

Next, we derive the borderline that divides the $(k, \tilde{k})$-plain into the diversification region and the complete specialization region. This borderline corresponds to the combination of k and $\tilde{k}$ such that $X_2 = 0$. Expressing $X_2 = 0$ using k and $\tilde{k}$, we have

$$\tilde{k} = \left[\frac{1}{(1-\alpha)^{1-\alpha}k^{\alpha(1-\alpha-\beta)}}\right]^{\frac{1}{\beta}}. \tag{43}$$

This is a downward sloping curve on the $(k, \tilde{k})$-plain. The region above this curve corresponds to complete specialization in manufacturing, whereas the region below this curve corresponds to diversification ($X_2 > 0$).

We first consider diversification. When $\dot{k} = 0$, from the first part of equation (41), the following relation holds.

$$\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\tilde{k}^{\frac{\beta}{\alpha}} - \phi n = 0. \tag{44}$$

However, no $\tilde{k} > 0$ satisfies equation (44). On the other hand, when $\dot{\tilde{k}} = 0$, we have $\tilde{k} = \tilde{k}_e$ from the second part of equation (41).

Next, we consider complete specialization in manufacturing. When $\dot{k} = 0$, from the first part of equation (42), the following relation holds.

$$\alpha k^{\alpha+\beta-1} - \phi n = 0. \tag{45}$$

However, no $k > 0$ satisfies equation (45). On the other hand, when $\dot{\tilde{k}} = 0$, from the second equation of equation (42), the following relation holds.

$$k_e = \left(\frac{\alpha}{\phi n_w}\right)^{\frac{1}{1-\alpha-\beta}}. \tag{46}$$

Summarizing the above results, we obtain figure 1.

[Figure 1 around here]

The transitional dynamics that start at point S_1 in figure 1 are interesting, and we obtain two results. First, starting from diversification, the home country then completely specializes in manufacturing, sooner or later begins to diversify, and finally asymptotically completely specializes in agriculture. Second, starting from diversification, the home country completely specializes in manufacturing and continues to do so for all time.

Which of the two results is realized depends on whether the home country crosses the borderline. The second is obtained if the home country remains in the complete specialization region. For example, we consider point P in figure 2. The home country continues to move southeast through time, and it either begins to cross the borderline and enter the diversification region or stays in the complete specialization region. For the home country to always stay in the complete specialization region, from equation (43), the following condition must continue to be satisfied.

$$\tilde{k} > \left[\frac{1}{(1-\alpha)^{1-\alpha} k^{\alpha(1-\alpha-\beta)}} \right]^{\frac{1}{\beta}}. \tag{47}$$

If we take the growth rates of both sides, we obtain

$$g_{\tilde{k}} > -\frac{\alpha}{\phi - 1} g_k. \tag{48}$$

If complete specialization in manufacturing continues to hold, then from equation (42), we have

$$g_k = \alpha k^{\alpha+\beta-1} - \phi n > 0, \tag{49}$$

$$g_{\tilde{k}} = \alpha k^{\alpha+\beta-1} - \phi n_w. \tag{50}$$

Since k diverges to infinity in figure 2, at the limit, we obtain

$$\lim_{k \to +\infty} g_k = -\phi n > 0, \tag{51}$$

$$\lim_{k \to +\infty} g_{\tilde{k}} = -\phi n_w < 0. \tag{52}$$

Substituting these expressions into equation (48), we obtain

$$-\alpha n - (\phi - 1)n_w > 0 \implies n < -\frac{\phi - 1}{\alpha} n_w \equiv \bar{n}. \tag{53}$$

As long as this condition is satisfied, once the home country specializes in manufacturing, it continues to do throughout time.

The long-run growth rate of per capita consumption is given by equation (38). Since $0 < \gamma < 1$, equation (38) is positive if equation (53) is satisfied. That is, the long-run growth rate of per capita consumption is positive. Moreover, it depends on both n and n_w.

[Figure 2 around here]

We can summarize the above results as the following propositions.

Proposition 2. *Suppose that $\bar{n} < n < 0$. Then, the home country asymptotically completely specializes in agriculture and per capita consumption growth depends on n_w. On the other hand, suppose that $n < \bar{n}$. Then, the home country completely specializes in manufacturing and per capita consumption growth depends on both n and n_w.*

Figure 3 summarizes the relationships between population growth and per capita consumption growth under autarky and free trade. The case where $n > 0$ under free trade is obtained from Christiaans (2008).

[Figure 3 around here]

The solid line corresponds to autarky, and the dashed line corresponds to free trade. When $n < \bar{n}$ under free trade, the home country completely specializes in manufacturing in the long run, and the growth rate of per capita consumption under free trade is larger than that under autarky. When $\bar{n} \leq n < n_w$ under free trade, the home country asymptotically completely specializes in agriculture, and the growth rate of per capita consumption under free trade is larger than that under autarky. When $n_w \leq n$ under free trade, the home country completely specializes in manufacturing, and the growth rate of per capita consumption under free trade is equal to that under autarky.[8]

Summarizing the above results, we obtain the following proposition.

Proposition 3. *The growth rate of per capita consumption under free trade is equal to or larger than that under autarky irrespective of the size and the sign of the growth rate of population.*

5 Conclusions

This paper has built a small-open-economy, two-sector, non-scale-growth model and investigated the relationship between trade patterns and per capita consumption growth. Importantly, we have assumed that the growth rate of population is negative. The main results are as follows.

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[8]Both $n_w \leq n$ and $\tilde{k}_0 > \tilde{k}_e$ are the necessary and sufficient conditions for the home country to completely specialize in manufacturing in the long run. Therefore, $n_w \leq n$ is a necessary condition for complete specialization. For details, see Christiaans (2008).

Under autarky, if the population growth rate is negative, then the long-run growth rate of per capita consumption is positive and decreasing in the population growth rate.

Under free trade, if the population growth rate is negative and its absolute value is small, then the home country becomes an agricultural country. Then, the long-run growth rate of per capita consumption is positive and depends on the world population growth rate. On other hand, if the population growth rate is negative and its absolute value is large, then the home country becomes a manufacturing country. Then, the long-run growth rate of per capita consumption is positive and depends on both the home country and the world population growth rate. Moreover, the long-run growth rate of per capita consumption is larger in the case where the home country becomes a manufacturing country than in the case where it becomes an agricultural country.

In addition, from the results of Christiaans (2008), we know that when population growth is positive, the long-run growth rate of per capita consumption under free trade is larger than or equal to that under autarky.

Therefore, the home country is better off under free trade than under autarky in terms of per capita consumption growth when population growth is both positive and negative.

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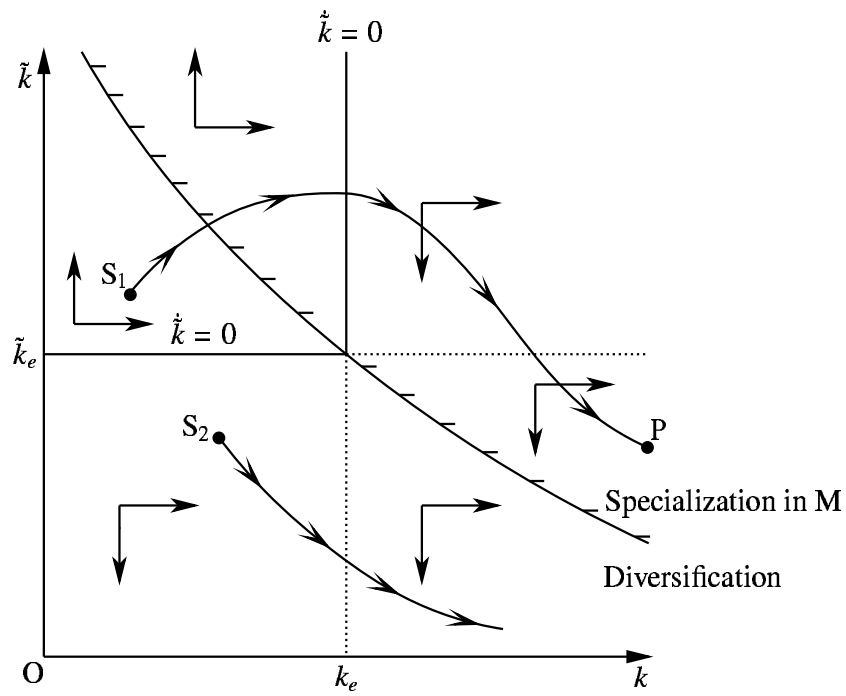Young, A. (1998) "Growth without scale effects," *Journal of Political Economy* 106 (1), pp. 41–63.
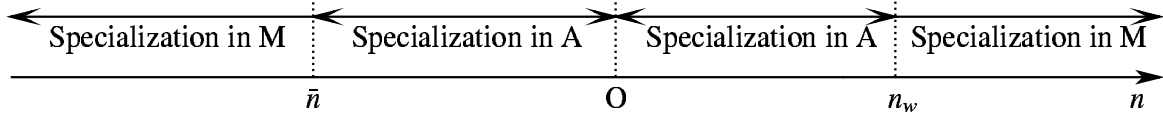
Figure 1: Phase diagram when $n < 0$

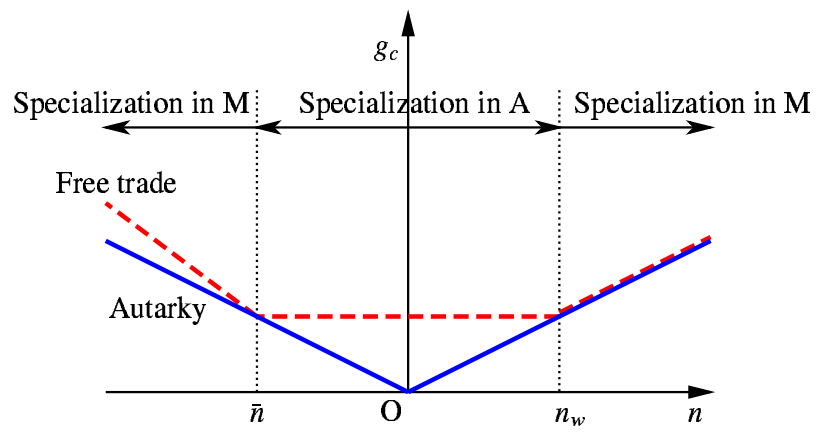Figure 2: Long-run relationship between population growth and trade patterns

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Figure 3: Long-run relationship between population growth and per capita consumption growth