Kyoto University,
Graduate School of Economics
Research Project Center Discussion Paper Series

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Discussion Paper No. E-12-002

Research Project Center
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April, 2012 (revised March, 2013)

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March 25, 2013

Abstract

We incorporate the banks, defined as maturity-mismatching financial intermediaries by Diamond and Rajan (2001a, 2012), into an overlapping-generations model where capital good is reproducible. We show that, in our model, the laissez-faire banks take on undue risks, compared to the social optimum, owing to the pecuniary externalities. Further, the model replicates "rare but severe crises" without assuming any large exogenous shocks because systemic bank runs take place endogenously followed by sharp contractions in output. We also make policy assessments based on the model. The assessment favors some macro-prudential measures over pre-committed bank bailouts.

JEL Classification: E3, G01, G21

Keywords: Financial crisis, Liquidity shortage, Maturity mismatch, Credit externalities, Financial regulation

*We thank Kosuke Aoki, Xavier Freixas, Shinichi Fukuda, Gary Gorton, Fumio Hayashi, Keiichiro Kobayashi, Pok-sang Lam, Hidehiko Matsumoto, Kiyohiko G. Nishimura, Maurice Obstfeld, Akihiko Senoo, Alp Simsek, Frank Smets, Javier Suarez, and Aleh Tsyvinski for comments and discussions. In addition, we received helpful comments from participants at numerous conferences and seminars. We are also indebted to Daisuke Oyama and other members of the Banking Theory Study Group for helpful discussion. Takayuki Tsuruga gratefully acknowledges the financial support of the Grant-in-aid for Scientific Research and the Japan Securities Scholarship Foundation. Views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

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1 Introduction

Since the global financial crisis for 2007-08, one of the challenges posed for macroeconomists has been how to replicate systemic financial crises and ensuing sharp contraction in macroeconomic activity in dynamic stochastic general equilibrium (DSGE) models. To generate such large declines in output, many preceding macroeconomic models with financial frictions assume relatively large shocks to economic fundamentals as a rare event and examine how such a "rare but large" shock amplifies macroeconomic fluctuations.[1] From this viewpoint, the 2007-08 systemic financial crisis could be interpreted as an unavoidable and unfortunate accident arising from tail risks. In the meantime, Bernanke (2012) argues that the triggers of the 2007-08 crisis were "quite modest" in size while heavy dependence on short-term funding, high leverage, and inadequate risk management in the (shadow) banking sector that engaged in maturity transformation was key vulnerability of the system for the devastating outcomes of the crisis. In a related context of the underlying vulnerabilities, others argue that there might have been erosion of discipline owing to the anticipated bank bailouts.[2]

This paper develops a dynamic general equilibrium model with maturity-mismatching banks and explores how individual banks could take on excessive systemic risks that could result in a devastating contraction in macroeconomic activity. We incorporate the banks modeled by Diamond and Rajan (2012, DR) into a two-good overlapping-generations (OLG) model where the capital good is reproducible. In this model, bank overleverage arises due to pecuniary externalities. Recent studies on the pecuniary externalities have reached a broad consensus that pecuniary externalities, or credit externalities, tend to prevent the laissez-faire (LF) economy from achieving the social optimum (e.g., Benigno, Chen, Otrok, Rebucci, and Young 2012, Bianchi 2010, 2011, Bianchi and Mendoza 2010, Jeanne and Korinek 2012a, Korinek 2010, Lorenzoni 2008, Mendoza 2010, Nikolov 2010, and Stein 2012). In line with these previous studies, our result of the bank overleverage is juxtaposed with the second-best allocations which a social planning (SP) agent achieves subject to the same financial constraint as the LF economy. In our model, however, the source of the pecuniary externalities differs from those of the earlier studies. Many of the previous studies include borrowers' collateral constraints affected by price changes that the agents in the LF economy do not internalize. By contrast, the pecuniary externalities in our model operate on the

[1] A few examples include: Gertler and Karadi (2011), Gertler and Kiyotaki (2011), and Christiano, Motto and Rostagno (2010).

[2] See Rajan (2010), Diamond and Rajan (2012), and Fahri and Tirole (2012). They argue the idea using the widely acknowledged term, "the Greenspan put."

solvency constraint affected by changes in the price of the banks' illiquid assets (i.e., the profitability of entrepreneurs' capital investment). Notably, the solvency constraint arises in our model as a natural consequence of incorporating the maturity-mismatching banks that issue non-state-contingent short-term debt and invest them in illiquid asset.[3] Based on the model, we discuss how the bank overleverage leads to inefficiently high crisis probabilities and sharp declines in economic activity after a crisis. We also explore how existing policy measures to enhance the resilience of banking systems affect crisis probabilities.

Key elements of the model are (i) issuance of non-state-contingent debt by banks and (ii) the banks' risk exposure to capital prices which affects the banks' solvency. The first element enables banks to raise funds and to promote liquidity creation in the absence of the complete markets, as discussed in Diamond and Rajan (2001a, b). But this benefit of non-state-contingent debts comes with the cost of potential insolvency of banks, and in extremis of financial crises. Facing this risk of insolvency, banks need to strike the right balance between marginal cost and benefit of increasing their leverage. When a financial crisis is precipitated, banks are required to liquidate all illiquid loans to entrepreneurs to repay depositors, which discontinues the capital goods production and subsequently results in large contraction in output. The second element is incorporated with our OLG framework that explicitly includes factor markets. In our model, banks' balance sheet depends on the capital price for which entrepreneurs sell their own produced capital in the capital good market. This dependence of banks' balance sheet on capital price implies capital-price-dependent banks' solvency which leads to excessive risk-taking in the banking sector as a whole.

The intuition behind banks' excessive risk-taking can be understood as follows. Financial crises in our model are precipitated only if a liquidity shock exceeds a certain threshold because of the non-state-contingency of their debt. As in DR, a highly leveraged bank holding illiquid assets is more vulnerable to liquidity shocks. If a single bank finds it optimal to increase their leverage, the bank needs to keep more illiquid asset until maturity to consolidate their balance sheet. In the absence of heterogeneity, however, every other bank takes the same action based on the same idea, holding on to illiquid asset in the economy as a whole. Therefore, the systemic risk-taking based on the same idea promotes entrepreneurs to continue long-term capital producing projects. As a result, a larger supply of capital

[3] Our model assume that banks issue demand deposits following DR. The banks in our paper, however, can be broadly interpreted as financial intermediaries that raise funds via short-term debt such as repo and commercial paper, and transform maturities on their balance sheet. Demand deposits are an extreme case of short-term debt.

reduces the capital price and, accordingly, the returns of illiquid assets in a banking system as a whole. This reduction in the capital price further erodes the individual banks' solvency.

Even though all the LF banks understand a further erosion of their solvency due to the capital price declines, they do not internalize this pecuniary externality. Each atomic LF bank is simply selfish and lack incentives to coordinate with each other to affect the capital prices. Put another way, pecuniary externalities create the wedge between the social and private marginal cost of increasing the leverage and ill-incentivizes banks to take on excessive risks systemically. In our benchmark simulations, the crisis probability in the LF economy is 6.6 percent, compared to 4.5 percent in the constrained social optimum.

We further suggest several policy implications from our model. Given that a variety of policy measures to enhance the resilience of banking systems are currently being explored in the real world, we assess the realistic policy measures in terms of crisis probabilities. One highlight is that pre-announced bank bailouts may ill-incentivize banks to take on even higher risks, adding to the even higher crisis probabilities. A similar argument is likely to be applied to pre-commitment to a low interest rate policy that aims to prop up a banking system near crisis for the purpose of emergency liquidity provision (ELP). In principle, if banks are informed in advance that they would be bailed out by, for example, publicly supplied liquidity at a time of elevated market distress, they have little reason to be better prepared for an extremity through precautionary deleveraging. While public supply of liquidity *per se* has the effect of curbing the crisis probability, banks' anticipation of such a bailout can reverse the expected outcome, resulting in even more frequent crises. Finally, we argue that banks' capital requirement with prompt corrective action (PCA) may be less exposed to such risks of exacerbation of excessive risk-taking in the banking system.

Our paper is related to previous studies in at least three strands of literature. First, our model relies on the theory of banking in terms of the micro-foundation of banks. Our model is a straightforward extension of Allen and Gale (1998) and DR in terms of the basic modeling approach of a banking system. In our model, however, the socially optimal allocation cannot be achieved in the LF economy in contrast to their model. Their model includes a single liquidity market where the banks can internalize all the effects through the single price changes. In contrast, our model includes multiple markets (e.g., the capital market), in which banks take the price as given and fail to internalize the general equilibrium effect of the price.

Second, our paper contributes to the literature on macroeconomic models with banks. Macroeconomic models with banks have primarily focused on how financial frictions amplify business cycles (e.g., Gertler and Karadi 2011, Gertler and Kiyotaki 2011, Meh and Moran 2010) rather than how and why devastating financial crises could take place sporadically beyond the business cycle frequency. The focus of our model starkly contrasts with these studies in the following ways: we aim to (i) explore a rationale for government intervention and (ii) account for the vast standstill in financial intermediations and subsequent sharp declines in output and investment, both of which the global economy recently experienced. We also stress that our model with banks investigates how frequently financial crises are precipitated as a result of banks' insolvency. Angeloni and Faia (2012) incorporate banks à la Diamond and Rajan (2000, 2001a) into the DSGE model. Whereas their model has the endogenous probability of bank insolvency, the probability of the bank insolvency can broadly be interpreted as a measure of individual bank fragility rather than the probability of financial crises in which the great majority of the banking system comes to a standstill.

Finally, as we mentioned, our paper is closely related to the literature on pecuniary externalities which explores over-borrowing or over-credit based on the borrowers' collateral constraint. Our model is based on the theory of banking and emphasizes that the solvency constraint plays a crucial role in generating overleverage of the LF banking sector. In this regard, Stein (2012) introduces the macroeconomic model with banks that are faced with the pecuniary externalities. While he focuses more on how to contain risks of fire sales, we assess the probability of inefficient financial crises by explicitly modeling banks that are subject to risks of systemic financial crises.

Perhaps, the models that are the closest to ours is Gertler and Kiyotaki (2012) and Boissay, Collard and Smets (2012). In Gertler and Kiyotaki (2012), whether an equilibrium with bank runs exists depends on macroeconomic fundamentals, but financial crises *per se* are precipitated by self-fulfilling expectations as in Diamond and Dybvig (1983). In our model, financial crises are precipitated by fundamental shocks (liquidity preferences) rather than self-fulfilling expectations. In this regard, our model seems to be broadly in line with empirical findings of the business cycle view by Gorton (1988) and Allen and Gale (1998). In other words, financial crises in our model are not unpredictable, entirely random events, but the consequence of excessive risk-taking of banking systems. Boissay, Collard and Smets (2012) focus on the moral hazard in the interbank markets and successfully replicate systemic financial crises followed by deep recessions. On the other hand, we stress the fragility of the maturity mismatching banks as highlighted by the Gorton and Metrick's (2012) "run-on-repo" view.

The rest of the paper proceeds as follows. Section 2 illustrates the macroeconomic model with maturity-mismatching banks and characterizes the banks' optimal leverage in the competitive equilibrium. In Section 3, we compare the LF equilibrium in Section 2 with the allocation achieved by the SP banks and explain why a competitive banking sector tends to be overleveraged. Section 4 discusses numerical results. Section 5 assesses policy measures aimed at reducing crisis probabilities and discusses policy implications. Section 6 concludes.

2 A Macroeconomy with Banks

2.1 Agents, Endowment, Preferences, and Technology

We consider an infinite-horizon OLG model incorporating banks with a maturity mismatch. Each generation of agents consists of households, entrepreneurs, and bankers. Each period, generation t is born at the beginning of period t and lives for two periods, t and $t+1$. Each agent is identical and constant in the population. Furthermore, an initial old generation lives for one period and the subsequent generations live for two periods.

Households are risk averse and subject to a liquidity shock that affects their preference for consumption over the two periods. The liquidity shock is an aggregate shock and the only source of the uncertainty in the model. The households aim to smooth their consumption intertemporally. Following DR, households are endowed with a unit of consumption goods at birth and do not consume the initially endowed consumption goods at the beginning of period t. The households deposit all initial endowments at banks operating in the same generation.[4] They receive wages w_t in the competitive labor market by supplying one unit of labor in both periods, t and $t+1$.

Entrepreneurs are risk neutral and have access to capital-producing technology. They launch long-term investment projects at the beginning of period t, by borrowing households' endowments via the banks in the same generation. The investment project requires one period for gestation, and capital goods are produced in period $t+1$. We call this capital producing technology a "project." Entrepreneurs sell the capital goods in the competitive market for the capital goods price q_{t+1}.

Banks raise funds from households and lend them to entrepreneurs at the beginning of period t.[5] In principle, we follow Diamond and Rajan (2001a) to model banks. Banks

[4] We implicitly assume intra-period perishability of endowments. More precisely, all endowments perish before the realization of the liquidity shock.

[5] We assume intra-generational banking, which effectively means that all bankers of generation t die out at the end of period $t+1$.

are risk neutral and competitive at raising and lending funds in the markets. They issue demand deposits (short-term debt) and commit to repaying the households. In the nature of demand deposits, banks can provide insurance against depositors' liquidity shocks. However, when households demand repayment before the completion of the entrepreneurs' projects, banks must liquidate premature projects to meet the demand for repayment. This maturity mismatch, represented by the combination of long-term assets and short-term liabilities, leaves banks exposed to risks of a default because, depending on the amount of withdrawals in the interim period, the banks' solvency is endangered.

The technology to produce consumption goods Y_t is represented by a standard constant-returns-to-scale Cobb-Douglas production function:

$$Y_t = F\left(K_t, H_t\right) = K_t^{\alpha} H_t^{1-\alpha},$$

where K_t and H_t denote the capital stock and hours worked, respectively. Demand for labor and capital satisfies

$$w_t = F_{H,t} = (1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} \tag{1}$$

$$q_t = F_{K,t} = \alpha\left(\frac{K_t}{H_t}\right)^{\alpha-1}. \tag{2}$$

Accordingly, the second derivatives are denoted by $F_{KK,t}$, $F_{HH,t}$, and $F_{HK,t}$.

In what follows, we describe each agent's decisions (consumption, withdrawal, and liquidation of the entrepreneurs' projects) *after* the liquidity shock is realized. Then, we move on to the bank's decision on its leverage *before* the realization of the liquidity shock. Table 1 summarizes the sequence of events in each generation.

2.2 Households

Under the competitive banking sector, each household accepts the banks' offer on deposit face value D_t at the beginning of period t, and observes the liquidity shock θ_t in the middle of period t. The liquidity shock is common across all households in the same generation and has the probability density function $f(\theta_t)$ with a support of $[0, 1]$. This shock represents households' preference for consumption when young and signals the need for liquidity in period t.[6]

After the realization of θ_t, households make their decisions for consumption smoothing without uncertainty. Given that a crisis does not take place, households then choose withdrawal amount g_t to maximize

$$\begin{aligned} U\left(C_{1,t}, C_{2,t+1}\right) &= \theta_t \log C_{1,t} + \left(1-\theta_t\right) \log C_{2,t+1} \\ \text{s.t. } C_{1,t} &= w_t + g_t \\ C_{2,t+1} &= w_{t+1} + R_t\left(D_t - g_t\right), \end{aligned} \tag{3}$$

where $C_{1,t}$ and $C_{2,t+1}$ denote the consumption of households born in period t when young and old, respectively. Each household supplies a unit of labor in each period and receives wage income w_t in period t and w_{t+1} in period $t+1$. Here R_t denotes the one-period gross interest rate from period t to $t+1$.

In our model, a financial crisis takes place with the endogenous probability π_t, depending on the realization of θ_t. With the probability $1-\pi_t$, a financial crisis is not taking place and households can withdraw g_t in period t and all the remaining deposits in period $t+1$.[7] With the probability π_t, however, a financial crisis arises and households' withdrawals amount to the liquidation value of premature projects, X (<1), in period t and nothing is left in period $t+1$. In the case of a crisis, households fail to smooth out their consumption and end up with $C_{1,t} = w_t + X$ and $C_{2,t+1} = w_{t+1}$.

When the households can smooth out their consumption, the intertemporal first-order condition for consumption is satisfied:

$$\frac{\theta_t}{1-\theta_t}\left(\frac{C_{1,t}}{C_{2,t+1}}\right)^{-1} = R_t. \tag{4}$$

Given the Euler equation (4), the withdrawals in the absence of a crisis can be written as

$$g_t = \theta_t\left(\frac{w_{t+1}}{R_t} + D_t\right) - \left(1-\theta_t\right) w_t. \tag{5}$$

[6] Although in fact all households are subject to the same aggregate shock, we assume that an infinitesimally small number of households are believed to face a different θ_t from other households. This assumption ensures the existence of a Nash equilibrium, in which all households run to the banks when households believe that the banks are insolvent under the observed θ_t.

[7] In the maximization problem of households, we assume that wage income in period t is low relative to the initial endowment, ensuring a non-negative withdrawal g_t in the equilibrium.

The withdrawal function implies that large θ_t and D_t are likely to precipitate a financial crisis.

2.3 Entrepreneurs

Entrepreneurs are risk neural and maximize their expected lifetime utility represented by $E\left(C^e_{1,t} + C^e_{2,t+1}\right)$, where $C^e_{1,t}$ and $C^e_{2,t+1}$ denote entrepreneurs' consumption when young and old. They use a unit of consumption goods financed from banks for their capital goods production, and this production technology takes one period for gestation before its completion. In period $t+1$, the project yields a random capital goods output $\breve{\omega}$, which is uniformly distributed over $[\omega_L, \omega_H]$ with the probability density function $h(\breve{\omega})$.[8] If this project is prematurely liquidated in period t, the transformation from the consumption goods into capital is incomplete. As a result, the output is reduced to X units of consumption goods and is repaid fully to banks in period t. When the project is completed in period $t+1$, however, entrepreneurs can sell their output in the capital goods market for the capital price q_{t+1}.

Each entrepreneur can borrow from a bank who has, or can learn until the project to mature, knowledge about an alternative, but less profitable, method to operate the project. The bank's specific knowledge allows it to generate $\gamma q_{t+1}\breve{\omega}$ from a project outcome with $\gamma < 1$. Once a bank has lent, no one else (including other banks) can learn this alternative way to operate the project. As a result, entrepreneurs accept the financing contract with each bank and repay $\gamma q_{t+1}\breve{\omega}$. They are left with $1-\gamma$ of the share of their profit and enjoy their own consumption based on their linear utility. We assume that entrepreneurs are endowed with $\underline{I}$ units of capital goods at the beginning of period $t+1$.[9] They sell this endowment capital together with the newly created capital made from the consumption goods.

2.4 Banks

Banks are also risk neutral and maximize their expected lifetime utility $E\left(C^b_{1,t} + C^b_{2,t+1}\right)$, where $C^b_{1,t}$ and $C^b_{2,t+1}$ denote consumption of banks when young and old. We borrow the micro-foundation of the banking business from Diamond and Rajan (2001a, b, 2012). Banks have no initial endowment at birth and thus they need to raise funds from households. As

[8] Following the literature, we take the assumption that there is no aggregate uncertainty in the project outcome.

[9] For simplicity, we assume a 100 percent depreciation rate in the law of motion for capital. The introduction of the endowment of capital goods here guarantees a finite capital price in the aftermath of a financial crisis in which all projects are scrapped due to full liquidation.

the relationship lender, a bank has the knowledge to operate the entrepreneurs' project that cannot be transferred to households. Banks issue demand deposits (short-term debt) as a commitment device to compensate for the lack of transferability of their knowledge (i.e., collection skills).[10] As discussed by a number of early studies, a collective action problem for depositors is created by this demandable nature of deposit contracts: depositors run to the banks whenever depositors anticipate that the banks cannot honor the debt. In the theory of banking, the deposit contract is predetermined before observing the liquidity shock. In our model, D_t is predetermined at the beginning of period t, and a liquidity shock is realized in the middle of period t.

Each bank attracts many entrepreneurs through a competitive offer on the loan, resulting in an identical portfolio shared by all the symmetric banks. This setup effectively leads to a convenient outcome in the model: each bank and the aggregate economy face an identical distribution of entrepreneurs. In period t, the banks receive signals ω that perfectly predict the realized value of $\tilde{\omega}$ in period $t+1$. With this information ω and the households' liquidity demand observed in period t, each bank chooses one of the options: (i) to liquidate projects in period t, obtaining X of consumption goods per project; or (ii) to collect a fraction $\gamma q_{t+1}\omega$ from a completed project in period $t+1$. The bank liquidates the project if the outcome of a project falls short of $\tilde{\omega}_{t+1}$, defined as a function of R_t/q_{t+1}:[11]

$$\tilde{\omega}_{t+1} = \frac{X}{\gamma}\frac{R_t}{q_{t+1}}. \tag{6}$$

Otherwise, the bank continues the project, and then receives repayment of $\gamma q_{t+1}\omega$ and entrepreneurs consume the remaining fraction of outcome, $(1-\gamma)\, q_{t+1}\omega$, per project. After repaying the full amount of the households' withdrawals, the banks consume their own capital.

Let the banks' asset be $A\,(R_t/q_{t+1})$. The banks' asset at the beginning of period t (i.e.,

[10] Diamond and Rajan (2001b) discuss the micro-foundation of maturity mismatching banks and explain why this demand deposits can promote liquidity creation under the lack of transferability of their collection skill.

[11] Equation (6) can be reinterpreted as follows: $\gamma q_{t+1}\omega/X$ corresponds to the marginal rate of transformation (MRT) between the period-t consumption goods (i.e., liquidation) and the period-$t+1$ consumption goods (i.e., continuation of projects). The MRT is here compared with the marginal rate of substitution of the households that is observed as the interest rate, R_t.

prior to the withdrawals) can be expressed as

$$A\left(\frac{R_t}{q_{t+1}}\right) = \int_{\omega_L}^{\tilde{\omega}_{t+1}} Xh(\omega)\,d\omega + \frac{\gamma q_{t+1}}{R_t}\int_{\tilde{\omega}_{t+1}}^{\omega_H} \omega h(\omega)\,d\omega$$
$$= L\left(\frac{R_t}{q_{t+1}}\right) + \frac{\gamma q_{t+1}}{R_t} I\left(\frac{R_t}{q_{t+1}}\right). \tag{7}$$

Note that $h(\omega)$ is interchangeable with $h(\breve{\omega})$ owing to the perfect signaling. The banks' asset denoted in (7) can be decomposed into two components: the values of the prematurely liquidated projects denoted as $L_t = L(R_t/q_{t+1}) \equiv \int_{\omega_L}^{\tilde{\omega}_{t+1}} Xh(\omega)\,d\omega$, which is used to meet the liquidity demand (i.e., withdrawals) from the households, and the banks' share of the investment output (measured in the present value of consumption goods) denoted as $\gamma q_{t+1} I_t/R_t$, where $I_t = I(R_t/q_{t+1}) = \int_{\tilde{\omega}_{t+1}}^{\omega_H} \omega h(\omega)\,d\omega$. The banks are subject to the *solvency constraint* $D_t \leq A(R_t/q_{t+1})$. Owing to the uniform distribution assumption for ω, it can be easily shown that $A(\cdot)$ monotonically decreases with R_t/q_{t+1}. We can then define the relative price R_t^*/q_{t+1}^* that satisfies the solvency constraint with equality

$$D_t = A\left(\frac{R_t^*}{q_{t+1}^*}\right). \tag{8}$$

We refer to R_t^* and q_{t+1}^* as the *threshold* interest rate and capital price, respectively. Hereafter, we denote a variable with an asterisk as the variable on the threshold. For the purpose of subsequent discussion, we note that given $A(R_t/q_{t+1})$, the bank leverage $D_t/(A_t - D_t)$ is uniquely determined once D_t is chosen, and hence we refer to D_t as leverage hereafter. We will discuss this issue in Section 4 in terms of numerical interpretation.

2.5 Market Clearing Conditions

Four markets need to clear in the competitive equilibrium: (i) liquidity; (ii) consumption goods; (iii) capital goods; and (iv) labor. The liquidity market clearing condition is given by

$$L\left(\frac{R_t}{q_{t+1}}\right) = \theta_t\left(\frac{w_{t+1}}{R_t} + D_t\right) - (1-\theta_t)\,w_t. \tag{9}$$

Next, the market clearing condition for consumption goods is

$$Y_t + L\left(\frac{R_t}{q_{t+1}}\right) = C_{1,t} + C_{2,t} + C_{2,t}^e + C_{2,t}^b. \tag{10}$$

The left-hand side of (10) includes the supply of goods from the liquidated projects. On the right-hand side of (10), $C_{2,t}$, $C_{2,t}^{e}$, and $C_{2,t}^{b}$ are consumption when generation $t-1$ is old.

The capital goods market clearing condition is

$$K_{t+1} = \begin{cases} \underline{I} + I\left(R_t/q_{t+1}\right) & \text{at normal times} \\ \underline{I} & \text{at crises.} \end{cases} \tag{11}$$

Here the equation suggests that the capital goods supply sharply declines, in the aftermath of a crisis. Throughout the paper, we use $\underline{w}$ and $\underline{F}_H$ to denote the wage rate and the marginal product of labor evaluated at $K_{t+1} = \underline{I}$.

Finally, both young and old generations supply a unit of labor in each period. Therefore, H_t equals two for all t.

2.6 Optimal Bank Leverage

We now consider the banks' optimal leverage, which is chosen before the realization of the liquidity shock. We focus on the laissez-faire (LF) banks in this subsection, and will discuss the social planning (SP) banks in Section 3.

The banks are competitive at issuing demand deposits, and we assume that households' endowments are scarce in comparison to entrepreneurs' projects. As a result of competition, the banks make a competitive offer of deposits for households, aiming to maximize the household welfare (Allen and Gale 1998, 2007), while in fact they are maximizing their own profits. Maximizing the household utility via the deposit offers means that banks internalize the liquidity market clearing condition in determining the offer. Through this internalization, the banks take into account possible changes in the crisis probability π_t. On the other hand, outside the liquidity market, they take the capital prices and wages as given.

To understand how the banks' choice of D_t affects π_t, we take three steps. First, we define a function R_{LF}^{*} as

$$R_{LF}^{*}\left(D_t\right) = q_{t+1}^{*} A^{-1}\left(D_t\right), \tag{12}$$

from (8). We emphasize that, in $R_{LF}^{*}\left(D_t\right)$, the threshold capital price q_{t+1}^{*} is treated as a parameter, reflecting the price-taking behavior of the LF banks. Second, using (9), we define a function θ_{LF}^{*} as

$$\theta_{LF}^{*}\left(D_t\right) = \frac{L\left(R_t^{*}/q_{t+1}^{*}\right) + w_t}{w_t + D_t + w_{t+1}^{*}/R_t^{*}}, \tag{13}$$

where $R_t^{*} = R_{LF}^{*}\left(D_t\right)$ while w_t, w_{t+1}^{*}, and q_{t+1}^{*} are given parameters for the LF banks. We reemphasize that these prices are not constant parameters but in fact vary according to (1) and (2). But for the LF banks, when they determine the optimal D_t, they just take them as given. With $R_t^* = R_{LF}^*(D_t)$ and other threshold variables, the threshold level of the liquidity shock $\theta_t^* = \theta_{LF}^*(D_t)$ clears the liquidity market with the LF banks as shown in (13).

The final step is to connect θ_t^* to the crisis probability π_t. The above-defined $\theta_{LF}^*(D_t)$ means that any changes in D_t always give rise to changes in θ_t^* for the liquidity market to clear. By the solvency constraint with equality (8), any level of D_t, once chosen, determines the threshold relative price, R_t^*/q_{t+1}^*. Hence, θ_t^* can be interpreted as the liquidity shock on the brink of a financial crisis. Namely, when θ_t is strictly greater than θ_t^*, the banks turn out to be insolvent and a crisis is precipitated. Thus, the crisis probability π_t has a one-to-one relationship to θ_t^* via the probability density function $f(\theta_t)$:

$$\pi_t = \int_{\theta_t^*}^{1} f(\theta_t)\, d\theta_t. \tag{14}$$

In sum, the banks' choice of the leverage specifies R_t^*/q_{t+1}^* and this threshold relative price determines the threshold level of the liquidity shock θ_t^*, completing the link between the bank leverage and the crisis probability.

We are now ready to set up the optimization problem for the banks to determine the size of their leverage. In the problem, as discussed, banks take into account the endogenously changing θ_t^*.

Problem LF *In a laissez-faire economy, banks maximize the household expected utility*

$$\begin{aligned}
&\max_{D_t} \int_0^{\theta_t^*} \{\theta_t \ln(w_t + L_t) + (1-\theta_t)\ln[w_{t+1} + R_t(D_t - L_t)]\} f(\theta_t)\, d\theta_t \\
&+ \int_{\theta_t^*}^{1} [\theta_t \ln(w_t + X) + (1-\theta_t)\ln(\underline{w})] f(\theta_t)\, d\theta_t,
\end{aligned} \tag{15}$$

subject to (9) and (13).

The banks choose their leverage according to the following first-order condition:

$$\begin{aligned}
&\left[\theta_t^* \log\left(\frac{\theta_t^* m_t^*}{w_t + X}\right) + (1-\theta_t^*)\log\left(\frac{R_t^*(1-\theta_t^*) m_t^*}{\underline{w}}\right)\right] \frac{d\pi_t}{d\theta_t^*} \theta_{LF}^{*\prime}(D_t) \\
= &\int_0^{\theta_t^*} \left[\frac{1}{m_t}\left(1 - \frac{w_{t+1}}{R_t^2} R_{LF}'(D_t, \theta_t)\right) + (1-\theta_t)\frac{R_{LF}'(D_t, \theta_t)}{R_t}\right] f(\theta_t)\, d\theta_t,
\end{aligned} \tag{16}$$

where $m_t \equiv w_t + D_t + w_{t+1}/R_t$ is the lifetime income of households and $m_t^* \equiv w_t + D_t + w_{t+1}^*/R_t^*$, accordingly. More importantly, $\theta_{LF}^{*\prime}(D_t)$ is calculated from (13), taking capital prices and wages as given in line with the behavior of the LF banks. Likewise, with a slight abuse of notation, $R_{LF}^{\prime}(D_t, \theta_t)$ denotes marginal changes in R_t with respect to D_t, given θ_t. By (9),

$$R_{LF}^{\prime}(D_t, \theta_t) \equiv \frac{\theta_t}{L^{\prime}/q_{t+1} + \theta_t w_{t+1}/R_t^2} > 0 \tag{17}$$

can be obtained where $L^{\prime}$ is the derivative of L_t with respect to R_t/q_{t+1}.

Equation (16) provides an economic interpretation that is in line with broad intuition. The terms in brackets on the left-hand side of (16) represent the loss of utility in a crisis compared to the threshold. From (14), the term outside the brackets indicates the marginal changes in a crisis probability with respect to bank leverage. The left-hand side of the equation consists of the expected loss of utility and the marginal change in the crisis probability. Simply put: the left-hand side of (16) is the marginal cost of increasing D_t.

The right-hand side of (16) consists of the effects of increasing leverage on the expected households' utility through their lifetime income. On the one hand, the increase in D_t has an outright positive effect on the households' income: the higher the leverage, the larger the withdrawal, allowing households to enjoy more consumption. On the other hand, the increase in D_t leads to a higher interest rate via liquidity shortage, discounting the households' labor income in period $t+1$ and reducing returns on forgoing withdrawal until period $t+1$. Hence, as far as the outright effect on the lifetime income exceeds the effect on the interest rate, the higher leverage is beneficial to households. Simply put: the right-hand side of (16) is the marginal benefit of increasing D_t.

We finally define the equilibrium in the LF economy as follows.

Definition (Laissez-faire economy) *A competitive equilibrium consists of allocations and prices* $\{g_t, D_t, L_t, K_t, I_t, H_t, R_t, q_t, w_t\}_{t=0}^{\infty}$ *such that (i) withdrawal decisions are given by (5) for* $\theta_t \leq \theta_t^*$*; (ii) banks' leverage satisfies (16); (iii) banks' liquidity supply is determined by (6); and (iv) all markets clear.*

3 Systemic Risks and Welfare

3.1 Social Planning Banks

This section introduces the SP banks which choose their leverage as the constrained social planner. We characterize the allocations for the SP banks as the constrained social optimum and compare the allocations with those in the LF economy. To lead off the analysis, we clarify the constraint to which the SP banks are subject. We *assume* that the SP banks must make all their decisions before observing θ_t. After realizing θ_t, they are left with no options. In other words, the SP banks are subject to the constraint that they can neither control households' behaviors nor choose their outright consumption levels because households can react to any realized value of θ_t. The allocations chosen by the SP banks must be distinguished from the unconstrained, first-best optimum. Under the first-best optimum, the Arrow securities that pay off contingent on all possible realizations of θ_t are available and hence households can enjoy the maximum utility without experiencing any financial crisis. But we cannot disregard a maturity mismatch and resulting financial crises. As such, we assume that the SP banks are entities engaged in a maturity mismatch and pre-commit to payment on their debt regardless of the states realized following their commitment. The extra ability given to the SP banks compared to the price-taking competitive banks is that the former can internalize all price effects in all markets when they make decisions regarding their leverage.

The SP banks do not take the factor prices as given, but take into account their changes reflecting the marginal product. Formally, we replace q_t and w_t with $F_{K,t}$ and $F_{H,t}$, respectively. Note that, nonetheless, the SP banks take the households' behaviors as given, as they cannot make their contract contingent on θ_t. In other words, households always choose their consumption and withdrawal given D_t pre-committed by the SP banks.

To specify the problem for the SP banks, we clarify the solvency constraint with which the SP banks are faced, $D_t \leq A\left(R_t/F_{K,t+1}\right)$. We note that the constraint effectively remains the same as in Problem LF because $q_{t+1} = F_{K,t+1}$ from (2). The newly introduced solvency constraint for the SP banks, however, has different effects on the threshold because (8) and (12) are now replaced with

$$D_t = A\left(\frac{R_t^*}{F_{K,t+1}^*}\right) \quad (18)$$

$$R_{SP}^*\left(D_t\right) = F_{K,t+1}^* A^{-1}\left(D_t\right), \quad (19)$$

respectively, in the problem for the SP banks where the SP banks can internalize general equilibrium effects of the factor prices.

We summarize the SP banks' problem as follows:

Problem SP *The social planning banks maximize the household expected utility,*

$$\max_{D_t} \int_0^{\theta_t^*} \{\theta_t \ln (F_{H,t} + L_t) + (1-\theta_t) \ln [F_{H,t+1} + R_t (D_t - L_t)]\} f(\theta_t) d\theta_t$$
$$+ \int_{\theta_t^*}^1 [\theta_t \ln (F_{H,t} + X) + (1-\theta_t) \ln \underline{F}_H] f(\theta_t) d\theta_t,$$

subject to

$$L\left(\frac{R_t}{F_{K,t+1}}\right) = \theta_t \left(\frac{F_{H,t+1}}{R_t} + D_t\right) - (1-\theta_t) F_{H,t} \tag{20}$$

$$\theta_{SP}^* (D_t) \equiv \frac{L\left(R_t^*/F_{K,t+1}^*\right) + F_{H,t}}{F_{H,t} + D_t + F_{H,t+1}^*/R_t^*}, \tag{21}$$

where $R_t^* = R_{SP}^* (D_t)$ *from (19) and* $\theta_t^* = \theta_{SP}^* (D_t)$ *from (21).*

Note that all the factor prices, including q_{t+1}^* and w_{t+1}^* in Problem LF, are replaced with marginal products in Problem SP. More importantly, because the SP banks factor in all general equilibrium effects, $\theta_{SP}^{*\prime} (D_t)$ can be denoted as $d\theta_t^*/dD_t$. The solution of Problem SP is conceptually comparable to the constrained optimum as discussed in Allen and Gale (1998).

In Problem SP, all the factor prices in period $t+1$ are functions of K_{t+1} or K_{t+1}^*. In this context, the capital goods market clearing condition in Problem SP needs extra attention when we replace the capital price in (11) with the marginal product. We note that K_{t+1} depends solely on the market interest rate R_t and this relationship is denoted by a function $\Phi(R_t)$. Provided that a crisis does not take place, K_{t+1} evolves according to

$$\begin{aligned} K_{t+1} &= \underline{I} + I\left(\frac{R_t}{F_{K,t+1}}\right) \\ &\equiv \Phi(R_t), \end{aligned} \tag{22}$$

where $\Phi' < 0$ represents the derivative of K_{t+1} with respect to R_t.[12] To move on, along with the labor market clearing condition $H_t = H = 2$ for all t, we reaffirm that $F_{K,t+1}^*$ and $F_{H,t+1}^*$

[12] Solving $\Phi' = (1 - \Phi' R_t F_{KK,t+1}/F_{K,t+1}) I'/F_{K,t+1}$ for Φ' ensures that Φ' is negative.

in (18), (19), and (21) can be written as

$$F^*_{K,t+1} = F_K\left(K^*_{t+1}, H\right) = F_K\left[\Phi\left(R^*_t\right)\right]$$
$$F^*_{H,t+1} = F_H\left(K^*_{t+1}, H\right) = F_H\left[\Phi\left(R^*_t\right)\right].$$

Our first main result is as follows. With the factor prices replaced by marginal products in Problem SP, the allocations that the SP banks achieve differ from those achieved by the LF banks because of the extra ability given to the SP banks. Comparison between Problems LF and SP confirms that the two problems are subject to exactly the same constraints. With identical constraints, any discrepancy in the first-order conditions generally results in different allocations across the two problems. To see this, for example, we focus on $\theta^{*\prime}_{SP} = d\theta^*_t/dD_t$ and $\theta^{*\prime}_{LF}$, both of which are the part of the first-order conditions in each problem:

$$\theta^{*\prime}_{SP}\left(D_t\right) = \frac{1}{m^*_t}\left[\frac{\partial}{\partial R^*_t}\left(L^*_t - g^*_t\right) - \zeta^*_\varepsilon\right]\frac{dR^*_t}{dD_t} - \frac{\theta^*_t}{m^*_t}, \tag{23}$$
$$\theta^{*\prime}_{LF}\left(D_t\right) \equiv \frac{1}{m^*_t}\left[\frac{\partial}{\partial R^*_t}\left(L^*_t - g^*_t\right)\right]R^{*\prime}_{LF}\left(D_t\right) - \frac{\theta^*_t}{m^*_t}, \tag{24}$$

where $\zeta^*_\varepsilon \equiv \left(\Phi^{*\prime}/R^*_t\right)\left[\left(R^*_t/F^*_{K,t+1}\right)^2 L^{*\prime}F^*_{KK,t+1} + \theta^*_t F^*_{HK,t+1}\right]$ and $dR^*_t/dD_t = R^{*\prime}_{SP}\left(D_t\right)$. Alongside, $\Phi^{*\prime}$ and $L^{*\prime}$ in ζ^*_ε represent $\Phi'\left(R^*_t\right)$ and $L'\left(R^*_t/q^*_{t+1}\right)$, respectively.

In general, non-zero ζ^*_ε ensures the difference between the two equilibria, while in fact comparison of the two first-order conditions reveals further discrepancies in addition to ζ^*_ε. The next subsection focuses on the key discrepancy between $R^{*\prime}_{LF}\left(D_t\right)$ and $dR^*_t/dD_t = R^{*\prime}_{SP}\left(D_t\right)$, which provides a clear economic interpretation.

3.2 Crisis Probabilities and Marginal Systemic Risk (MSR)

To facilitate assessment of the systemic risk of an economy, we define *marginal systemic risk* (MSR) as the marginal increase in the crisis probability against a unit change in bank leverage at and around the equilibrium.[13] Specifically, let $D_{LF,t}$ be the level of bank leverage

[13] MSR can be applied in broad models where a financial crisis takes place as a non-zero probability event. Depending on the focus of studies, MSR can be defined vis-à-vis bank leverage, aggregate credit, bank lending, or potentially, asset prices.

chosen in the LF economy. Then,

$$\begin{aligned} MSR_{k,t} &= \frac{d\pi_t}{d\theta_t^*}\theta_k^{*\prime}(D_{LF,t}) \\ &= -f(\theta_t^*)\,\theta_k^{*\prime}(D_{LF,t}) \quad \text{for } k = LF, SP. \end{aligned} \tag{25}$$

Recall that a bank in the LF economy takes other banks' decisions as given, but in fact the crisis probability is affected by the synchronized decisions by the banking sector as a whole. In this regard, $\theta_{LF}^{*\prime}(D_{LF,t})\, d\pi_t/d\theta_t^*$ can be interpreted as the marginal risks perceived by the individual price-taking banks, which can be contrasted with the true marginal risk, $\theta_{SP}^{*\prime}(D_{LF,t})\, d\pi_t/d\theta_t^*$ at the chosen $D_t = D_{LF,t}$. Technically, MSR is applicable for any level of D_t. We utilize MSRs evaluated at $D_t = D_{LF,t}$ because this allows us to directly compare the MSRs across different problems under the same allocations and prices.

With this interpretation in mind, we stress that, if $MSR_{LF,t}$ is smaller than $MSR_{SP,t}$, the gap indicates that the banking sector in the LF economy underestimates the marginal cost of higher leverage by not taking into account the systemic risks. As a result, the LF banks are likely to be overleveraged at (and around) the LF equilibrium. Instead, if $MSR_{LF,t}$ takes a larger value than $MSR_{SP,t}$, the gap points to underleverage in the LF economy. In principle, an undervaluation of $MSR_{LF,t}$ compared to $MSR_{SP,t}$ would provide the ground for government intervention to rein in excessive leverage of banks, because such regulatory risk reduction can improve welfare.

Our second main result is that $MSR_{SP,t}$ exceeds $MSR_{LF,t}$, relying on the concept of the MSR. For the formal proof, our second main result requires one technical condition:

Condition 1 *$\alpha\varepsilon_{g,t}^* \leq (1-\alpha)\,\varepsilon_{L,t}^*$ for $D_t = D_{LF,t}$, where $\varepsilon_{g,t}^* = -\partial \log g_t^*/\partial \log R_t^*$ and $\varepsilon_{L,t}^* = \partial \log L_t^*/\partial \log R_t^*$ are the elasticity of the liquidity demand and supply with respect to R_t^* in the liquidity market, respectively.*

Condition 1 ensures that $\zeta_\varepsilon^* \geq 0$ for a chosen D_t. This condition is likely to be satisfied because, in general, the capital share α takes a substantially smaller value than the labor share (e.g., $\alpha = 1/3$). With the Cobb-Douglas production function, these shares, α and $1-\alpha$, can also be interpreted as the elasticities of wages and the capital price with respect to K_{t+1}^*, respectively, both of which translate into demand and supply in the liquidity market. The SP banks internalize factor price changes via the production technology but the LF banks do not. Due to this internalization by the SP banks, a discrepancy ζ_ε^* arises. This ζ_ε^* is positive under Condition 1 since we assume that the capital price is more sensitive to changes in capital than wages and/or the supply curve is relatively flatter than the demand curve with respect to R_t^* in the liquidity market. Now, we are ready to state Proposition 1.

Proposition 1 *Under Condition 1, $MSR_{SP,t}$ is strictly larger than $MSR_{LF,t}$.*

Proof. See Appendix A.1. ■

Proposition 1 provides a foundation for understanding why the crisis probability is higher in the LF economy than in the social optimum. From (23) and (24),

$$\begin{aligned} MSR_{SP,t} - MSR_{LF,t} &= f(\theta_t^*)\left[\theta_{LF}^{*\prime}(D_{LF,t}) - \theta_{SP}^{*\prime}(D_{LF,t})\right] \\ &= \frac{f(\theta_t^*)}{m_t^*}\left\{\frac{\partial}{\partial R_t^*}(L_t^* - g_t^*)\left[R_{LF}^{*\prime}(D_{LF,t}) - R_{SP}^{*\prime}(D_{LF,t})\right] + \zeta_\varepsilon^*\right\} \quad (26) \end{aligned}$$

Note that all functions are evaluated at the LF equilibrium (i.e., $D_t = D_{LF,t}$). In (26), $f(\theta_t^*)/m_t^*$ and the slope of the excess liquidity supply function denoted as $\partial(L_t^* - g_t^*)/\partial R_t^*$ are both positive. Condition 1 ensures that we are left with the deviation of the changes in R_t^* with respect to D_t. From (12) and (19), the inverse function theorem yields

$$\begin{aligned} R_{LF}^{*\prime}(D_{LF,t}) - R_{SP}^{*\prime}(D_{LF,t}) &= \left(1 - \frac{1}{1 - F_{KK,t+1}^*\Phi^{*\prime}R_t^*/q_{t+1}^*}\right)\frac{q_{t+1}^*}{A^{*\prime}} \\ &= -\frac{R_t^*}{q_{t+1}^*}F_{KK,t+1}^*\Phi^{*\prime}\frac{dR_t^*}{dD_t} > 0, \quad (27) \end{aligned}$$

where $A^{*\prime} = A'\left(R_t^*/q_{t+1}^*\right)$. The sign of $dR_t^*/dD_t = R_{SP}^{*\prime}(D_{LF,t})$ is ensured to be negative as shown in Appendix A.1.

Equation (27) indicates that an individual price-taking bank underestimates the marginal changes in R_t^* due to the pecuniary externalities. An increase in leverage reduces R_t^*, because, in general, highly leveraged banks would be more likely to default under a lower threshold interest rate. But the perceived reduction in R_t^* differs between the LF and SP banks and this gap creates the wedge in the two MSRs in our model. In this regard, this wedge reflects the LF banks' underestimation of the marginal cost of higher leverage. As a result of the underestimation of the marginal cost, the LF banking system finds itself insolvent more frequently than expected.

To better understand the gap in the MSRs, we can focus on $F_{KK,t+1}^*\Phi^{*\prime}$, included in (27). This term points to a side effect arising from higher leverage: in general equilibrium, the reduction in R_t^* increases $K_{t+1}^* = \Phi(R_t^*)$. That is, the lower R_t^* stimulates capital supply on the threshold and this increase in K^*_{t+1} triggers the decline in the threshold capital price q^*_{t+1}via the lower marginal product of capital.[14] With this side effect, the lower capital price further undermines the bank's solvency, compared to the case without the side effect of increasing the leverage. Because the atomistic banks do not take into account this side effect, the lower-than-expected capital price and the undermined banks' solvency raise the probability of a financial crisis compared with the economy with the SP banks.

Looking at the real-world experience of past financial crises, it may be pointed out that, with hindsight, outlooks regarding asset prices frequently tended to be overly optimistic in the run-ups to crises. Some argue that such over-optimism arises from irrationality. While we do not claim that irrational behavior is irrelevant, our model suggests that despite the full rationality, pecuniary externalities can result in seemingly irrational over-optimism. The key to understanding the externalities lies in the synchronized decisions made by the individual banks in a competitive sector. For each bank, capital prices are given but the given prices affect the solvency of the banking system as a whole. In general, maturity-mismatching banks face solvency constraints because they issue non-state-contingent debt. As long as the effects of the asset prices on their solvency are not internalized, distortion arises. In our model, the distortion shows up as the overleveraged banking sector with a higher crisis risk because of the side effect as shown in (27), which can be interpreted broadly in line with the real-world observations.

4 Numerical Results

4.1 Solving the Model

We provide numerical solutions of the model in this section to address the following quantitative questions: (i) How frequently does a financial crisis arise? (ii) To what extent do the LF banks deviate from the social optimum? And (iii) How can we compare the numerical results with existing empirical studies on the probability of crises?

Our calibration mostly follows DR. We set the value of prematurely liquidated project X at 0.95. The parameters for the distribution of ω are set to $[\omega_L, \omega_H] = [0.5, 3.5]$, similar to

[14]Here, the actual capital supply K_{t+1} observed for normal times and the threshold level capital K^*_{t+1} need to be distinguished. When the LF banks are about to choose on D_t, they fail to take into account the oversupply of capital represented by K^*_{t+1}. Once D_t is chosen at a higher level in the LF economy, however, the LF economy overconsume and underinvest compared to the SP economy because the higher leverage allows households higher consumption on average for normal times.

the original calibration of DR. The degree of banks' special collection skills γ is set at 0.9. In addition to parameterization of DR, we need to set several other parameters. We calibrate the capital share in the production function, α, to be 1/3. The capital goods endowment received by entrepreneurs, $\underline{I}$, and the level of total factor productivity, which is suppressed in (1) and (2), together effectively determine the size of the scarring effect of a financial crisis. We parameterize them so that the post-crisis contraction in output matches the estimated size of scarring effects from past empirical studies.[15] More importantly, we assume that the liquidity shock θ_t follows the beta distribution with a mean of 0.50 and a standard deviation of 0.07. This parameterization indicates a symmetric bell-shaped distribution. To simulate the model, we numerically solve the nonlinear system of the equations consisting of the first-order conditions and resource constraints.

Before interpreting the numerical results, we reconfirm the economic interpretations of D_t. In the context of our model, D_t represents the pre-committed gross return on bank deposits. On the other hand, D_t cannot be translated into an annual percentage rate or an interest rate *per annum*, because the model does not specify the length of each period of time (e.g., one year or one quarter). To focus more clearly on the economic interpretations, D_t needs to be translated into a timeless measure such as the bank leverage. It is the exact reasoning that we have relied on in this interpretation of D_t, that is, bank leverage.

4.2 The Endogenous Crises Probabilities and Model Dynamics

Our benchmark results are summarized in Table 2. The upper panel of the table reports that the LF banks take on more risks than the SP banks, indicating a higher crisis probability. We note that the two economies share the same state variable (i.e., the initial capital stock K_t) in our comparisons. Our calibration points to a 6.59 percent crisis probability in the LF economy compared to 4.50 percent in the social optimum. Hence, the results suggest that in an arbitrary period out of 1,000 simultaneous attempts, about 66 attempts would trigger crises in the LF economy. The overleverage can be confirmed by the values of D_t in the upper panel of the table. In fact, D_t is 1.2 percentage points higher in the LF economy than in the social optimum.

The prediction that the LF banking sector is overleveraged implies that the LF economy undergoes crises created by pecuniary externalities, some of which could be avoided under the social optimum. To illustrate this, we run the model over 100 periods by generating

[15] See Barro (2009) as discussed in Section 4.5.

liquidity shocks randomly. The upper panel of Figure 1 plots the dynamic paths of output Y_t generated by the same random shock sequences across the two economies. The red lines correspond to the case of the LF economy, while the blue lines point to the case of the social optimum.

Notably, this random shock follows an i.i.d. process and no "rare but large shock" is prompting a crisis as confirmed by the lower panel of Figure 1. Comparisons between the two economies reveal that the dynamic paths of the output are almost identical except that the production under the LF economy sharply declines more frequently. Under the LF economy, crises take place in periods 5, 16, and 94, and output falls sharply in each subsequent period. This simulation result indicates that the latter two crises could have been prevented if the banks had taken the risks at the optimal level, implying a need for government intervention to forestall the crises. However, the first crisis takes place even in an economy in which the SP banks strike the right balance between the costs and benefits of increasing the leverage. Therefore, this crisis should not be avoided, as discussed in Allen and Gale (1998) in the context of the optimal financial crises.

The overleveraged LF banking system, which results in inefficient financial crises, remains a robust outcome across a range of calibrated parameters (e.g., X and distributions of ω and θ_t).[16] In addition to the robustness, the sensitivity analysis regarding the volatility of θ_t has noteworthy implications for the inherent fragility of the banking system in our model. Table 3 examines how bank leverage and crisis probabilities are affected by the volatility of θ_t. In each simulation, the standard deviation of θ_t changes from the benchmark value of 0.07 to either 0.02 or 0.10 with other parameters held unchanged at the benchmark calibration. The table, together with the benchmark results, confirms that the crisis probability rises monotonically along with the increase in the standard deviation of θ_t. We emphasize that any extremely small volatility of the liquidity shock does not fully wipe out financial crisis because the banks raise their leverage if they find liquidity demand less volatile. As a result, a financial crisis always remains a non-zero probability event.

4.3 The Size of Distortions

We next examine the extent to which the LF banks deviate from the allocation achieved by the SP banks. More broadly, we discuss the quantitative implications of the higher leverage in the LF economy for welfare.

[16]The tables of the sensitivity analysis are available upon request.

The upper panel of Table 2 reports the MSRs evaluated under the allocation in the LF economy. The deviation of the two MSRs is 0.45, indicating a higher risk in the LF economy and the extra increase in the crisis probability arises solely from pecuniary externalities. In particular, if leverage is increased by one percent from the allocation in the LF economy, each LF bank expects that the crisis probability increases up to 8.22 percent, but they are, in fact, exposed to a higher crisis probability of 8.70 percent.[17]

The lower panel of Table 2 compares the bank capital ratio defined as $(A_t - D_t)/A_t$ and the output of the consumption goods Y_{t+1} under the LF economy and the social optimum, when the realized value of θ_t takes the mean of 0.5. The LF banks are undercapitalized by 1.1 percentage points compared to the social optimum. Nevertheless, it may be surprising that the production does not substantially differ across the two allocations, provided that a financial crisis does not take place. We also compute the levels of consumption for households in a generation. The household's consumption is $(C_{1,t}, C_{2,t+1}) = (2.21, 2.61)$ in the LF economy in comparison to $(C_{1,t}, C_{2,t+1}) = (2.21, 2.59)$ in the social optimum.

The above exercise indicates that the welfare loss primarily arises from the inefficiently elevated crisis probability. Given that crises are considered rare events that we cannot observe frequently, the inefficiency or welfare loss may not be easily detected by looking at the volatility of consumption or output in normal times. In this sense, assessing the inefficiency with crisis probabilities or MSRs appears to be more appropriate than using the volatility of consumption or output.

4.4 Comparison with Empirical Studies

The numbers included in Table 2 may be compared to some recent empirical studies on crises. Among a number of works on catastrophe risks, Barro (2009) provides a comparable benchmark. He sets the disaster probability at two percent per year, arguing that a disaster could reduce GDP by 30 percent on average. Another notable example is Reinhart and Rogoff (2008, 2009). Based on their extensive data going back to the 1800s, they report that the frequencies are 7.2 percent for the advanced economies. In a similar context, the Basel Committee on Banking Supervision (2010, BCBS) provides a broader perspective on the frequency of banking crises based on multiple datasets, such as Laeven and Valencia (2008) and Reinhart and Rogoff (2008), and summarizes that the frequency lies in the range of 3.6 to 6.8 percent.

[17] These increases in the probability are obtained by transforming the MSRs into the semi-elasticity of the probability: $MSR_{k,t} \times D_{LF,t}$ for $k = LF, SP$.

MSR can be applied in line with a broad empirical exercise as demonstrated by the BCBS. They report empirical measures that are comparable to the MSR in assessing the impact of changes in bank capital on the probability of systemic banking crises. Using reduced-form econometric models, they estimate that a one percent increase in bank capital from the pre-reform cross-country average level could reduce the crisis probability by 1.0-1.6 percentage points. Their estimates roughly match our simulated value ($8.22 - 6.59 = 1.63$) of the extra increase in the crisis probability from raising the leverage computed from the MSR for the LF economy. We underscore the proximity of the BCBS estimates and the numerical results included in Table 2.[18]

5 Policy Intervention

This section discusses a variety of policy measures that have been implemented and are about to be enacted by both national and international regulatory bodies. In this regard, rather than examining the maximization of the social welfare, whose function is not specified, we focus on assessing the policies that aim to reduce the crisis probability by curbing bank leverage D_t.

5.1 Bank Levy

Suppose that, in an attempt to decrease the crisis probability, a government/central bank (GC) introduces a levy on bank size measured by its liability (D_t).[19] In practice, the levy can be used to rescue troubled banks by means of a bailout and, in fact, in the next subsection we will consider such interventions with the intentions of a bank bailout. For the moment, in this subsection we do not specify the purpose for which the GC spends the funds earned from the levy, but we simply assume that the GC consumes the levy (i.e., they just waste it) to focus on the impact of such a levy *per se* on banks' risk-taking behavior.

[18] In addition to the evident proximity of the marginal changes in the crisis probabilities, the level of bank capital reported in BCBS (2010) does not substantially differ from that in our model. BCBS (2010) argues that the pre-reform cross-country average of the TCE/RWA (tangible common equity divided by Basel II risk-weighted assets) ratio is 7 percent. The TCE is an extremely narrow definition of bank capital that, by and large, could be doubled or even tripled (i.e., to 14-21 percent) if measured in more conventional measures for bank capital, such as the Tier I ratio.

[19] For example, the U.K. government enacted a bank levy as of January 2011 in an attempt to "encourage banks to move away from risky funding models" and to share the burden of financial crises with the banking sector. See HM Revenue and Customs (2010). Another example is the Volcker plan, which includes a proposal to restrict the size of banks' liabilities.

The GC imposes τ percent of the levy on banks' liabilities. Under the levy in place, τD_t of the tax burden, which is measured by period-t consumption goods, falls on the banks of generation t. In terms of the operation, we assume that the GC collects $R_t \tau D_t$ of consumption goods at $t+1$ from banks. In period $t+1$, the GC consumes the collected consumption goods for itself, which is denoted by $C^g_{t+1}= R_t \tau D_t$. Because there is no uncertainty in the economy after the realization of θ_t, both banks and households correctly recognize that the banks' solvency is undermined by τD_t as of period t. Under the bank levy, the solvency constraint is written as

$$(1+\tau) D_t \leq A (R_t/q_{t+1}). \tag{28}$$

Accordingly, we replace (8) with

$$(1+\tau) D_t = A \left(R^*_t/q^*_{t+1}\right), \tag{29}$$

where all variables with an asterisk are redefined in line with the new constraint (28). From (29), we can define the threshold interest rate function R^*_{BL},

$$R^*_{BL}(D_t) = q^*_{t+1} A^{-1} [(1+\tau) D_t]. \tag{30}$$

In a similar manner to Problem LF, note that by the inverse function theorem, $R^{*\prime}_{BL}(D_t) = q^*_{t+1}(1+\tau)/A^{*\prime}$. Using (9), a function θ^*_{BL} can be defined as

$$\theta^*_{BL}(D_t) = \frac{L\left(R^*_t/q^*_{t+1}\right) + w_t}{w_t + D_t + w^*_{t+1}/R^*_t}, \tag{31}$$

where $R^*_t = R^*_{BL}(D_t)$ from (30) while w_t, w^*_{t+1}, and q^*_{t+1} are given parameters for the competitive banks.

We now formally state the banks' problem with the levy as follows:

Problem BL *Let $\theta^*_t = \theta^*_{BL}(D_t)$. In an economy with a bank levy (BL), banks maximize (15) subject to (9) and (31).*

Table 4 reports the allocation and the crisis probabilities under a reasonable range of $\tau = 0.01, 0.02$, and 0.03. In comparison to the leverage of 1.061 in the LF economy (Table 2), banks' overleverage is reined in to 1.052, 1.044, and 1.036 for each case as confirmed in the first column ("Bank levy") in Table 4. But the crisis probabilities rise, rather than decline, contrary to the expected outcome. The results can be interpreted easily. The effects of the levy act via two channels: (i) the banks deleverage in response to a levy and this channel in fact reduces crisis probabilities because the bank's active, intentional risk-taking subsides; and (ii) on the other hand, the levy erodes the banks' profitability and capital, which are tabulated in the fourth row of each panel. By law, the banks pay out money to their depositors or creditors out of the after-tax profit and assets. This simply exposes the banks to a higher risk of insolvency, because they are left with fewer resources that can be paid out to their creditors. On balance, the latter effect dominates the former one, resulting in the higher crisis probabilities despite the banks' lower leverage. The point is that the GC does not rescue banks near crisis by using the collected tax, and this assumption regarding how the levy is spent is admittedly less realistic in the light of past experience and current practice.

5.2 Public Liquidity Provision as a Bank Bailout

5.2.1 Characterization

In the previous case, we assumed that the GC consumes the collected tax for its own benefit. But, presumably and more realistically, the bank levy would be used for the particular purpose of bailing out troubled banks. As assumed in the previous case (BL), likewise, the GC collects $R_t \tau D_t$ of the levy in period $t+1$. Furthermore, in this case, the GC pre-commits to stepping into the liquidity market to provide the liquidity M_t to rescue a banking system near crisis in period t when such intervention is possible and needed. This emergency liquidity provision (ELP) can prop up the banking system near crisis if properly designed with feasible financing.

In this case, we assume that the ELP by the GC is funded by tax collected from households after the realization of θ_t in period t. We further assume that the tax burden on households at t will be compensated by the income transfer from the GC at $t+1$ together with the interest payment.[20] Because, as noted, the GC collects $R_t \tau D_t$ of consumption goods from banks as the bank levy in period $t+1$, the GC can make the income transfer that keeps the lifetime income of households unchanged as long as the total supply of public liquidity does not exceed the amount of the bank levy, that is, $\tau D_t \geq M_t$. Put differently, as long as the funding resource of the GC is ensured, the income transfer can increase the total supply of liquidity by the amount of M_t in the period-t liquidity market.[21]

[20] This assumption rules out inter-generational income transfers from the GC.

[21] An alternative way to validate the same operation is to simply assume that the GC has a storage

5.2.2 Implementation: commitment to a low interest rate policy

Because this intervention helps increase liquidity supply in the market, the interest rate would decline to a lower level than would have been the case otherwise. Consequently, the ELP at a time of elevated financial tension can be reinterpreted as a commitment to a low interest rate policy by a central bank.

The sequence is as follows. First, the GC announces its commitment to forestalling banks' insolvency by stepping into the period-t liquidity market to provide public liquidity in a case where the banking system cannot remain solvent without such an operation. Then, by fully recognizing the GC's commitment, the banks determine their leverage. Subsequently, the GC (and everyone else) recognizes the maximum interest rate (R_t^*) in line with (29) above which the banking system fails to remain solvent. Then, θ_t is realized. Suppose that the materialized θ_t exceeds a certain level θ_t^c. The high θ_t accordingly would give rise to a high interest rate, $R_t > R_t^*$, if the intervention did not take place. In line with the pre-announced commitment, however, the GC provides liquidity to prop up a banking system near crisis by cutting the market interest rate.[22] Such ELP can effectively be reinterpreted as placing a cap on the market interest rate at $R_t \leq R_t^*$. At normal times, the liquidity market clearing condition in period t is given by (9) for $\theta_t \leq \theta_t^c$, where θ_t^c is formally defined as the level of liquidity preference shock that requires the GC to intervene in the market to supply extra liquidity and keep the interest rate at R_t^*:

$$\theta_t^c = \frac{L\left(R_t^*/q_{t+1}^*\right) + w_t}{w_t + D_t + w_{t+1}^*/R_t^*}. \tag{32}$$

By contrast, similar to the BL economy, we define $\theta_t^* = \theta_{BB}^*\left(D_t\right)$ that precipitates a financial crisis:

$$\theta_{BB}^*\left(D_t\right) = \frac{L\left(R_t^*/q_{t+1}^*\right) + w_t + \tau D_t}{w_t + D_t + w_{t+1}^*/R_t^*}, \tag{33}$$

where $R_t^* = R_{BB}^*\left(D_t\right)$. Here, $R_{BB}^*\left(D_t\right)$ is identical to $R_{BL}^*\left(D_t\right)$ because the banks' solvency constraint remains the same as (28). Note that if $\theta_t > \theta_t^*$, a financial crisis cannot be fore-

technology for consumption goods between before and after the realization of θ_t. If this is the case, the GC can collect τD_t of consumption goods from banks of generation t before θ_t is realized and uses the resource to finance the intervention when possible and needed after θ_t is realized. If a high θ_t is realized, the GC provides $M_t\left(\leq \tau D_t\right)$ of liquidity out of the τD_t of funds that is wasted.

[22]From the viewpoint of the implementation, the GC makes a commitment to a level of R_t^* while the commitment is effectively equivalent to a level of the relative price R_t^*/q_{t+1}^*. This is because q_{t+1}^* has a one-to-one relationship with R_t^*, through $q_{t+1}^* = F_K\left[\Phi\left(R_t^*\right)\right]$.

stalled by the GC's intervention. For $\theta_t \in (\theta_t^c, \theta_t^*]$, the GC steps in and rescues a banking system near crisis by ELP of M_t. Accordingly, the liquidity market clearing condition when the GC's intervention is underway is given by

$$\theta_t \left(\frac{w_{t+1}^*}{R_t^*} + D_t \right) - (1 - \theta_t) w_t = L \left(R_t^* / q_{t+1}^* \right) + M_t. \tag{34}$$

In other words, the GC's liquidity provision is performed subject to the response function:

$$M_t = g_t^* - L \left(R_t^* / q_{t+1}^* \right), \tag{35}$$

with $M_t \leq \tau D_t$, because the GC can finance this intervention only with the funds raised via the bank levy.

We formally state the banks' problem under this policy intervention:

Problem BB *Let $\theta_t^* = \theta_{BB}^* (D_t)$. In an economy with the bank bailout (BB), banks maximize*

$$\max_{D_t} \int_0^{\theta_t^c} \left\{ \theta_t \ln (w_t + L_t) + (1 - \theta_t) \ln [w_{t+1} + R_t (D_t - L_t)] \right\} f(\theta_t) \, d\theta_t$$
$$+ \int_{\theta_t^c}^{\theta_t^*} \left\{ \theta_t \ln (w_t + L_t^*) + (1 - \theta_t) \ln \left[w_{t+1}^* + R_t^* (D_t - L_t^*) \right] \right\} f(\theta_t) \, d\theta_t$$
$$+ \int_{\theta_t^*}^{1} \left[\theta_t \ln (w_t + X) + (1 - \theta_t) \ln (\underline{w}) \right] f(\theta_t) \, d\theta_t, \tag{36}$$

subject to (9), (34), (32), and (33).

In this problem, the competitive banking sector takes capital prices, wages, and the GC's response function (35) as given. The objective function includes three terms. The newly included term reflects the household's expected utility with a banking system near crisis for $\theta_t \in (\theta_t^c, \theta_t^*]$ while the GC successfully bails out the system with the ELP. Reflecting the new term in the objective function, the efficiency condition for the banks with respect to D_t accordingly has the new term for the range of θ_t.[23]

Before moving on to the main results of this section, we emphasize that if $\tau = 0$, the allocations and prices both in the BL and BB economies are identical to the LF equilibrium. Bearing this fact in mind, the following proposition summarizes the main results regarding

[23] The detail of the derivation is available upon request.

crisis probabilities across the BL and BB economies.

Proposition 2 *Let $MSR_{BL,t}$ and $MSR_{BB,t}$ be the marginal systemic risks in Problems BL and BB, respectively. Then, around the LF allocation, $MSR_{BL,t}$ is strictly larger than $MSR_{BB,t}$.*

Proof. An infinitesimally small $\tau > 0$ affects each MSR as follows:

$$\begin{aligned} MSR_{BL,t} &= -f(\theta_t^*)\,\theta_{BL}^{*\prime}(D_{LF,t}) \\ &= -f(\theta_t^*)\left\{\frac{1}{m_t^*}\left[\frac{\partial}{\partial R_t^*}(L_t^* - g_t^*)\right]R_{BL}^{*\prime}(D_{LF,t}) - \frac{\theta_t^*}{m_t^*}\right\} \\ MSR_{BB,t} &= -f(\theta_t^*)\,\theta_{BB}^{*\prime}(D_{LF,t}) \\ &= -f(\theta_t^*)\left\{\frac{1}{m_t^*}\left[\frac{\partial}{\partial R_t^*}(L_t^* - g_t^*)\right]R_{BB}^{*\prime}(D_{LF,t}) - \frac{\theta_t^* - \tau}{m_t^*}\right\}. \end{aligned}$$

Recall $R_{BL}^{*\prime}(D_{LF,t}) = R_{BB}^{*\prime}(D_{LF,t}) = (1+\tau)\,q_{t+1}^*/A^{*\prime}$. With $D_t = D_{LF,t}$, the allocations L_t^*, g_t^*, m_t^*, and θ_t^* are the same across the two economies. Hence,

$$MSR_{BL,t} - MSR_{BB,t} = \frac{\tau f(\theta_t^*)}{m_t^*} > 0,$$

which proves the proposition.■ ■

As discussed in the comparison between the SP and the LF economies, likewise, a smaller MSR in the BB economy suggests that the crisis probability would be higher than in the BL economy. This can be interpreted in line with economic intuition: the GC intends to forestall a crisis by reining in the otherwise rising interest rate to a low level, that is, R_t^*. If the intervention succeeds, the banking system remains solvent even if it faces a high $\theta_t \in (\theta_t^c, \theta_t^*]$ with the aid of the GC. But this is not the end of the story. When the banks determine their D_t, they fully anticipate that the bailout will be enacted at a time of financial distress. By correctly taking into account the increased safety owing to the bailout (i.e., ELP), the banks take on more risks, resulting in a higher leverage, given the same burden levied on the banks. The ultimate outcome would be an even higher probability of crisis compared to an economy without such a bailout. Public liquidity provision as a bank bailout can thus raise, rather than reduce, the probability of a crisis as articulated in Table 4.[24]

[24] In a similar context, repercussions of the authorities' commitment to a low interest rate policy via liquidity provision are pointed out by DR and Fahri and Tirole (2012) and are discussed from a broader perspective in Rajan (2010).

5.3 Capital Requirement with Prompt Corrective Action

In an attempt to reduce the crisis probability, the GC may choose another option: in fact, the capital requirement with prompt corrective action (PCA) has been up and running as the primary banking sector regulatory tool, as typically represented by Basel II. The regulation requires banks to hold a certain minimum level of capital. In this context, this regulation can be translated into the temporary transfer of the required bank capital from banks to the GC.[25] If a bank fails to maintain the required level of capital, it would be taken into receivership by the GC. Because of the capital requirement, the banks are faced with the constraint, $\phi \leq \left[A\left(R_t/q_{t+1}\right) - D_t\right]/A\left(R_t/q_{t+1}\right)$, where ϕ points to the required minimum capital ratio. The requirement can equivalently be rewritten as the *PCA activation condition,*

$$D_t \leq (1-\phi)\, A\left(R_t/q_{t+1}\right), \tag{37}$$

which appears, on surface, similar to (28) although it is effectively quite different. Basically, (37) is not a solvency constraint for banks because, even if the constraint is violated, banks may still hold positive capital and remain solvent. If a bank fails to hold $\phi A\left(R_t/q_{t+1}\right)$ of capital, it is taken into receivership by the GC. Under the PCA, the bank can continue to operate but it is run by new management, typically appointed by the GC.

An issue that emerges in such cases is that the new management appointed by the GC may have inferior skills in fostering the remaining long-term projects because the new management consists of less experienced bankers who inherited unfamiliar projects. Reflecting the inferior skills, we assume that, once the bank is taken into receivership, the new management of the bank can obtain $\lambda\gamma\omega \leq \gamma\omega$ from the completed project. If λ is equal to one, the bank's ability as a relationship lender is fully retained while, by contrast, if λ is strictly lower than one, it points to a loss of human capital in the banks because of the receivership.

The banks' assets under the PCA can be expressed as

$$\tilde{A}\left(R_t/q_{t+1};\lambda\right) = \int_{\omega_L}^{\omega_{t+1}^a} Xh(\omega)\,d\omega + \frac{\lambda\gamma q_{t+1}}{R_t}\int_{\omega_{t+1}^a}^{\omega_H} \omega h(\omega)\,d\omega,$$

where we denote $\tilde{A}\left(R_t/q_{t+1};1\right) = A\left(R_t/q_{t+1}\right)$ and $\omega_{t+1}^a = XR_t/\left(\lambda\gamma q_{t+1}\right)$. Using the new

[25] While the ownership of the required capital remains with the bankers legally in the real world, our interpretation is compatible with a variety of cases where the required capital is not at the bankers' disposal because of the regulation.

notations, the *bona fide solvency constraint* of the bank is

$$D_t \leq (1-\phi)\,\tilde{A}\,(R_t/q_{t+1};\lambda) + T_{t+1}, \tag{38}$$

where the required bank capital is represented by $T_{t+1} = \phi\tilde{A}\,(R_t/q_{t+1};\lambda)$ that the bank takes as given. The solvency constraint remains broadly unchanged compared to the LF case, except for λ, because the required capital is possessed by the new management of the bank. Although the capital requirement generates the incentives for banks to deleverage, this requirement is much less stringent on the bank's solvency than the bank levy because T_{t+1} is left with banks as usable funds for payout.

In parallel with θ_t^c defined in the previous subsection, a threshold value for the PCA activation needs to be introduced. Suppose that the realized θ_t is larger than a certain level θ_t^a. This condition defines the interest rate and capital price on the brink of the PCA activation such that

$$\frac{1}{1-\phi}D_t = A\left(R_t^a/q_{t+1}^a\right).$$

If the relative price R_t/q_{t+1} exceeds R_t^a/q_{t+1}^a, the PCA is activated. In this case, the banks can remain solvent but are taken into receivership due to undercapitalization. As we define (32) in the BB case, θ_t^a is written as

$$\theta_t^a = \frac{L\left(R_t^a/q_{t+1}^a\right) + w_t}{w_t + D_t + w_{t+1}^a/R_t^a}. \tag{39}$$

By contrast, if $\theta_t > \theta_t^*$, the bona fide solvency constraint (38) is violated and a crisis is precipitated. This condition reintroduces the interest rate and capital price on the brink of financial crises:

$$\frac{1}{1-\phi}D_t = \tilde{A}\left(R_t^*/q_{t+1}^*;\lambda\right) + \frac{1}{1-\phi}T_{t+1}^*. \tag{40}$$

If the relative price R_t/q_{t+1} is greater than R_t^*/q_{t+1}^*, the banking system is precipitated into a crisis, and this is likely to take place for low values of λ. Accordingly, we define a function R_{PCA}^* as

$$R_{PCA}^*(D_t) = q_{t+1}^*\tilde{A}^{-1}\left(\frac{D_t}{1-\phi} - \frac{T_{t+1}^*}{1-\phi};\lambda\right), \tag{41}$$

in parallel with the practice in previous cases. Note that the inverse function theorem assures $R_{PCA}^{*\prime}(D_t) = q_{t+1}^*/\left[(1-\phi)\,\tilde{A}^{*\prime}\right] < 0$, where $\tilde{A}^{*\prime} \equiv \tilde{A}'\left(R_t^*/q_{t+1}^*;\lambda\right)$.

Under the PCA, the threshold level of the liquidity shock for banks' solvency takes a form similar to (13) in the LF economy. We define a function θ^*_{PCA} as

$$\theta^*_{PCA}(D_t) = \frac{\tilde{L}\left(R^*_t/q^*_{t+1};\lambda\right) + w_t}{w_t + D_t + w^*_{t+1}/R^*_t}, \tag{42}$$

where $\tilde{L}_t = \tilde{L}(R_t/q_{t+1};\lambda) = \int_{\omega_L}^{\omega^a_{t+1}} Xh(\omega)\,d\omega$ is the liquidity supply under the PCA, which is also a function of λ and $R^*_t = R^*_{PCA}(D_t)$ from (41). The liquidity market clearing condition under the PCA is

$$\tilde{L}(R_t/q_{t+1};\lambda) = \theta_t\left(\frac{w_{t+1}}{R_t} + D_t\right) - (1-\theta_t)\,w_t. \tag{43}$$

We then state the banks' problem under the capital requirement with the PCA:

Problem PCA *Let* $\theta^*_t = \theta^*_{PCA}(D_t)$*. In an economy with the capital requirement with prompt corrective action (PCA), banks maximize*

$$\begin{aligned}
&\max_{D_t} \int_0^{\theta^a_t} \{\theta_t \ln(w_t + L_t) + (1-\theta_t)\ln[w_{t+1} + R_t(D_t - L_t)]\} f(\theta_t)\,d\theta_t \\
&+ \int_{\theta^a_t}^{\theta^*_t} \left\{\theta_t \ln\left(w_t + \tilde{L}_t\right) + (1-\theta_t)\ln\left[w_{t+1} + R_t\left(D_t - \tilde{L}_t\right)\right]\right\} f(\theta_t)\,d\theta_t \\
&+ \int_{\theta^*_t}^{1} [\theta_t \ln(w_t + X) + (1-\theta_t)\ln(\underline{w})]\, f(\theta_t)\,d\theta_t.
\end{aligned} \tag{44}$$

subject to (9), (43), (39), and (42).

This policy intervention can reduce the crisis probability compared with that in the LF (and BL) economies, as shown in the first column of Table 5. Taking an example of Panel A where the required minimum capital ratio is 4 percent, the probability is 6.32 percent when $\lambda = 1$, lower than 6.58 percent in the LF economy. The capital requirement *per se* discourages banks' risk-taking while keeping unchanged the banks' resources that are payable to creditors. This improved resilience of the banking system, however, may not be achieved under alternative conditions. As the third column in Table 5 ($\lambda = 0.75$) indicates, the crisis probability is 7.30 percent, which is higher than that in the LF economy. If the banks under PCA are run by low-skilled bankers, the risk-reduction effect through deleveraging could be dominated by the perils of lower solvency owing to the loss of resources. The overall assessment regarding the capital requirement with the PCA suggests a straightforward point: the success of this policy option largely depends on how efficiently the GC can manage the troubled banks under receivership.

Finally, we reiterate that this result should not be interpreted as either welfare improvement or deterioration. Identifying the optimal policy design requires a full-fledged assessment of policy alternatives with a well-defined social welfare function.

6 Conclusions

Our model demonstrated that a competitive banking sector cannot always achieve the second-best allocation. The banks fail to internalize the side effects of changes in the illiquid asset prices on their own solvency because of pecuniary externalities. In the light of real-world experience, our model could serve as a foundation for a better understanding of the repeatedly observed financial and economic crises. Further, our policy experiments tend to favor some macro-prudential policy measures over an anticipated bank bailout, because the bailout could ill-incentivize banks to take on even higher risks by undermining discipline in the banking system.

The analysis demonstrated in this paper can be extended in a number of directions. First, it may be necessary to examine how changes in a variety of economic environments (e.g., changes in the stochastic process of the liquidity shock) or newly introduced aggregate shocks (e.g., shocks to the asset side of banks' balance sheets) affect the economy's exposure to crisis risks and macroeconomic fluctuations. Second, the optimal macro-prudential regulations could be fully considered. Third, our framework may be translated into an infinitely-lived agent model for integration with quantitative business cycle studies. All of these directions would provide important avenues for future research.

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A Appendix

A.1 Proof of Proposition 1

We first show Lemma 1 which indicates that $dR_t^*/dD_t < 0$ and then proves the proposition.

Lemma A.1 $dR_t^*/dD_t < 0$**.**

Proof By taking total derivatives of (18) with respect to D_t, we obtain

$$\frac{dR_t^*}{dD_t} = \left(\frac{1}{1 - F_{KK,t+1}^* \Phi^{*\prime} R_t^* / q_{t+1}^*} \right) \frac{q_{t+1}^*}{A^{*\prime}}.$$

Then it suffices to show $F_{KK,t+1}^* \Phi^{*\prime} R_t^* / q_{t+1}^* < 1$. Applying the implicit function theorem to (22) yields

$$\Phi^{*\prime} = \frac{1}{q_{t+1}^* / I^{*\prime} + R_t^* F_{KK,t+1}^* / q_{t+1}^*},$$

where $I^{*\prime} = I' \left(R_t^* / q_{t+1}^* \right) < 0$. Hence,

$$F_{KK,t+1}^* \Phi^{*\prime} \frac{R_t^*}{q_{t+1}^*} = \frac{1}{1 + \left(q_{t+1}^* \right)^2 / \left(R_t^* I^{*\prime} F_{KK,t+1}^* \right)} < 1,$$

which proves Lemma A.1. ■

Recall that the discrepancy between $R_{LF}^{*\prime}(D_{LF,t})$ and $R_{SP}^{*\prime}(D_{LF,t})$ is given by (27)

$$R_{LF}^{*\prime}(D_{LF,t}) - R_{SP}^{*\prime}(D_{LF,t}) = -\frac{R_t^*}{q_{t+1}^*} F_{KK,t+1}^* \Phi^{*\prime} \frac{dR_t^*}{dD_t} > 0.$$

The sign of the discrepancy is ensured by Lemma A.1.

Table 1: Sequence of events for generation t

		Period t
1.		Households receive endowments.
2.		Banks offer deposits to households and loans to entrepreneurs.
3.		Entrepreneurs launch their projects.
4.		Households supply labor and receive wages w_t determined by the labor market conditions along with the old generation's labor supply.
5.		Liquidity shock θ_t is realized, and banks receive signals of project outcomes.
6.		Households decide the withdrawal amount g_t.
7.		Banks decide which projects to discontinue and supply liquidity L_t.
	(i)	If $g_t > L_t$, a financial crisis is precipitated and households receive repayment of X.
	(ii)	Otherwise, the households can transfer their wealth into the period $t+1$.
8.		All agents consume.
		Period $t+1$
1.		Entrepreneurs receive endowments.
2.		Entrepreneurs' projects are completed, and they sell their capital goods for q_{t+1} and make repayment to banks.
3.		Households supply labor and receive wages w_{t+1} determined by the labor market conditions along with the young generation's labor supply.
4.		Households fully withdraw deposits, if any.
5.		All agents consume.

Table 2: Crisis probabilities and allocations
under laissez-faire banking sector and social planning banks

	SP banks	LF banks
Leverage and crisis probabilities		
D_t	1.049	1.061
π_t (%)	4.499	6.585
MSR	1.993	1.544
Bank capital and output		
Bank capital ratio (%)	15.097	13.952
Y_{t+1}	5.459	5.457

Note: Simulation results based on the assumption that the liquidity shock θ_t follows the beta distribution. The level of bank leverage D_t and the probability of a financial crisis π_t are obtained from Problems LF and SP, respectively. The marginal systemic risk, MSR, is given by (25). The bank capital ratio is $(A_t - D_t)/A_t$.

Table 3: Financial crisis probabilities and bank leverage

	SP banks	LF banks	SP banks	LF banks
Leverage and probabilities	std(θ_t) = 0.02		std(θ_t) = 0.10	
D_t	1.129	1.132	1.014	1.030
π_t (%)	1.275	1.703	5.997	9.433

Note: Simulation results based on various values of the standard deviation of the liquidity shock θ_t. The other details can be seen in the note for Table 2.

Table 4: Bank levy and bank bailout

	Bank levy	Bank bailout
A: $\tau = 0.01$		
Leverage and probabilities		
D_t	1.052	1.055
π_t (%)	6.670	6.740
MSR	1.704	1.613
Bank capital and GDP		
Bank capital (%)	13.948	13.660
Y_{t+1}	5.458	5.458
B: $\tau = 0.02$		
Leverage and probabilities		
D_t	1.044	1.050
π_t (%)	6.755	6.890
MSR	1.855	1.673
Bank capital and GDP		
Bank capital (%)	13.943	13.365
Y_{t+1}	5.460	5.459
C: $\tau = 0.03$		
Leverage and probabilities		
D_t	1.036	1.044
π_t (%)	6.840	7.034
MSR	1.996	1.725
Bank capital and GDP		
Bank capital (%)	13.937	13.068
Y_{t+1}	5.462	5.460

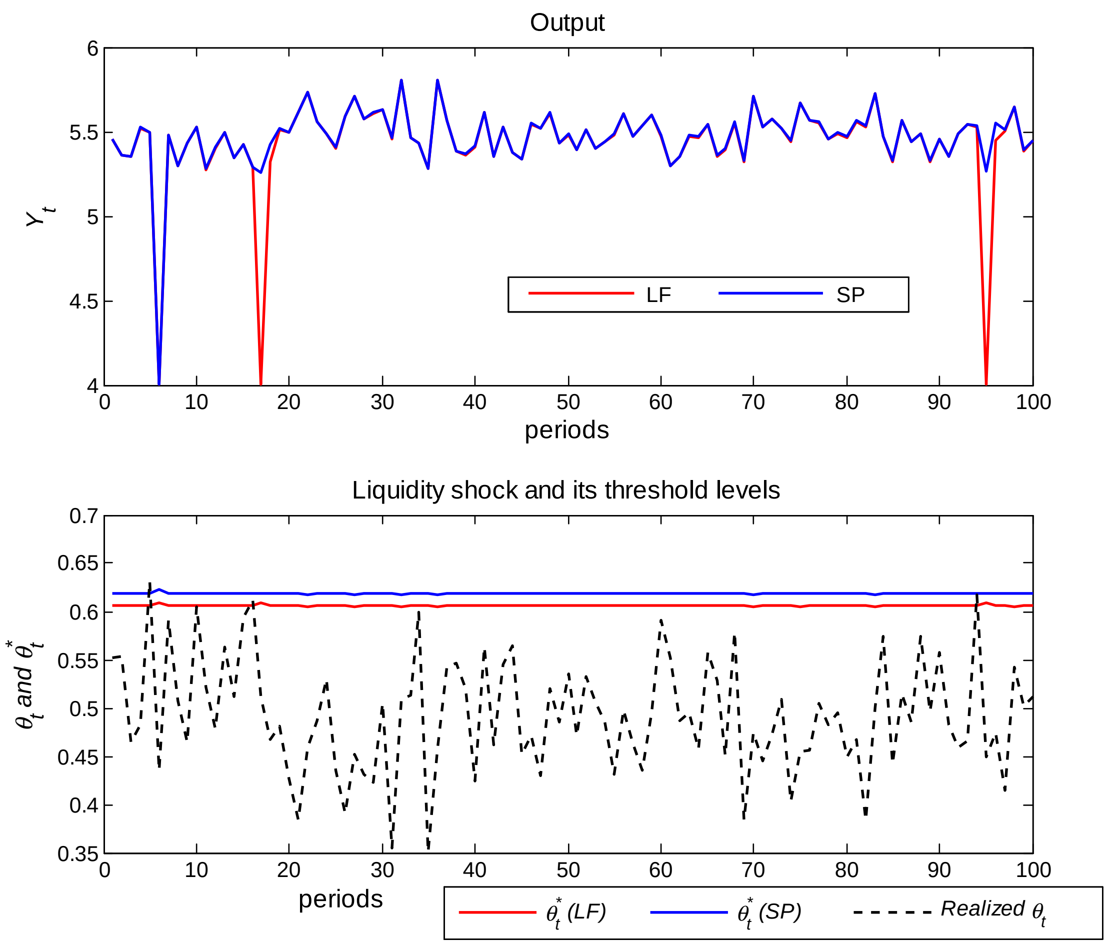
Note: The bank leverage D_t and the probability of a financial crisis π_t are obtained from different values of τ in (28). Panels A, B, and C correspond to the case of $\tau = 0.01$, 0.02, and 0.03, respectively. The bank capital ratio excludes the surcharge and is defined as $[A_t/(1+\tau) - D_t]/[A_t/(1+\tau)]$. The other details can be seen in the note for Table 2.

Table 5: Capital requirement with prompt corrective action

	A: $\phi = 0.04$		
	$\lambda = 1.00$	$\lambda = 0.95$	$\lambda = 0.75$
D_t	1.060	1.057	1.043
π_t (%)	6.321	6.457	7.298
Y_{t+1}	5.457	5.457	5.460
	B: $\phi = 0.06$		
	$\lambda = 1.00$	$\lambda = 0.95$	$\lambda = 0.75$
D_t	1.059	1.056	1.042
π_t (%)	6.188	6.321	7.066
Y_{t+1}	5.457	5.458	5.461

Note: Numbers in the table are obtained under the capital requirement with prompt corrective action across various capital requirement ratios (ϕ) and various degrees of reduction in the value of investment projects (λ) arising from the new management taken by the government/central bank. Panels A and B correspond to the cases of $\phi = 0.04$ and 0.06. Each column shows the degrees of reduction in the value of investment projects (λ). The level of output Y_{t+1} is obtained under the assumption that θ_t takes the mean of 0.5.

Figure 1: Simulated paths of output and the liquidity shock



Note: In both panels, the blue lines are for the economy with the social planning (SP) banks and the red lines are for the economy with the laissez-faire (LF) banks. The upper panel shows the simulated dynamic paths of output Y_t. The liquidity shock plotted as the dashed black line in the lower panel is generated from the beta distribution with a mean of 0.50 and a standard deviation of 0.07. The solid blue and red lines in the lower panel are the threshold level of the liquidity shock that satisfies the solvency constraint with equality.