

**Uneven Development of the World Economy:
from Krugman to Lenin**

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The collapse of the American economy has revealed the fundamental weight shift of the world economy. In my understanding, it is the ‘uneven development’ that Lenin’s *Imperialism* defined, and proved its rightness. To be clearer, firstly, this paper reviews Krugman’s model of ‘uneven development’ between South and North. Secondly, we show that he has misunderstood Lenin’s theory, and Lenin’s theory is more realistic rather than Krugman.

I. Krugman’s ‘Uneven Development’

1. Krugman’s ‘Uneven Development’

In the Western economics, North-South divergence of the world economy has been explained by Ricardo’s theory of comparative costs between industrial countries and agricultural countries, or explained by Heckscher-Ohlin-Samuelson model with capital-intensive countries and labor-intensive countries, or sometimes by Dutt’s learning-by-doing model with much more experienced countries in production and less experienced countries (Dutt (1986)). Besides them, however, here we focus on Krugman (1981) because he also refers Lenin model. He has shown that much more industrialized countries cumulatively accumulate capital than less-industrialized countries under the assumption of increasing return of technology. First, we show his model.

Krugman’s model has two countries, that is ‘North’ which is expressed by suffix ‘N’ and ‘South’ which is expressed by suffix ‘S’. Here, he assumed both countries have same amount of labor force $\bar{L} = L_N = L_S$, and produce two goods, a manufacturing good M and an agricultural product A . He also assumed a single world price of manufacturing goods in terms of agricultural products, P_m . In other words, a single world price of agricultural products was set to unit.

Manufacturing production was assumed as a function of capital input and labor

input, and its technology is increasing return, that is

$$\begin{aligned} c_N &= c(K_N) & c_S &= c(K_S) & c' &< 0 \\ v_N &= v(K_N) & v_S &= v(K_S) & v' &< 0 \end{aligned}$$

, where letting c , v be the unit capital and labor requirements respectively, and them, resulting the following relationship.

$$M_N = K_N / c_N \quad M_S = K_S / c_S \quad (1)$$

Agricultural products were assumed to be produced by labor alone, and he chose units so that one unit of labor produces one unit of agricultural goods. If so, an additional assumption of full employment led him to the following agricultural production functions;

$$A_N = \bar{L} - v_N M_N \quad A_S = \bar{L} - v_S M_S$$

In addition, labor forces are assumed to consume agricultural goods alone, and their saving ratios are zero which means unit labor cost to be one.

Under above mentioned setting-up, profit rates of the manufacturing sectors became

$$\begin{aligned} \rho_N &= (P_M M_N - v_N M_N) / K_N = (P_M - v_N) / c_N \\ \rho_S &= (P_M M_S - v_S M_S) / K_S = (P_M - v_S) / c_S \end{aligned}$$

Because c and v are functions of the capital stocks, these equations can be rewritten as follows;

$$\rho_N = \rho(P_M, K_N) \quad \rho_S = \rho(P_M, K_S) \quad (2)$$

where $\frac{\partial \rho}{\partial P_M} > 0$, $\frac{\partial \rho}{\partial K} > 0$, because $c' < 0$, and $\rho_N > \rho_S$, because $K_N = K_S$. Krugman

investigated its North-South relationship by using above setting up. His first case is with international trade but no international capital movement.

In this case, each country's capital accumulation can be shown as

$$\dot{K}_N / K_N = \rho_N \quad \dot{K}_S / K_S = \rho_S \quad (3)$$

where $\dot{X} = dX / dt$, and because $\rho_N > \rho_S$,

$$\dot{K}_N / K_N > \dot{K}_S / K_S.$$

It means capital accumulation in the North is faster than in the South, but to draw dynamics in the K_N - K_S diagram, he continues to analyze. For this purpose, he changes assumption to that 100μ percent of workers' income is spent for manufacturing goods. In this case, supply – demand balance of manufacturing goods becomes

$$P_M(M_N + M_S) = \mu(L_N + L_S) = 2\mu\bar{L}$$

which was rewritten as

$$P_M = \frac{2\mu\bar{L}}{\left(\frac{K_N}{c_N} + \frac{K_S}{c_S}\right)} \quad (4)$$

Finally Krugman combined (2), (3) and (4), and introduced

$$\dot{K}_N / K_N = g(K_N, K_S)(= \rho_N) \quad \dot{K}_S / K_S = g(K_N, K_S)(= \rho_S) \quad (5)$$

While this is the basic relations to draw K_N - K_S dynamics, we need the division lines which divide areas into K_N and K_S increasing areas and decreasing areas. These lines are $\rho_N = 0$ and $\rho_S = 0$, and by the appendix A, Krugman introduced following relations;

$$\left. \frac{\partial K_N}{\partial K_S} \right|_{\rho=0} < 0$$

$$\left| \left. \frac{\partial K_N}{\partial K_S} \right|_{\rho_S=0} \right| < \left| \left. \frac{\partial K_N}{\partial K_S} \right|_{\rho_N=0} \right|$$

They show that both of lines $\rho_N = 0$ and $\rho_S = 0$ are downward declining and the line $\rho_N = 0$ is steeper than the line $\rho_S = 0$ as shown in figure 1.

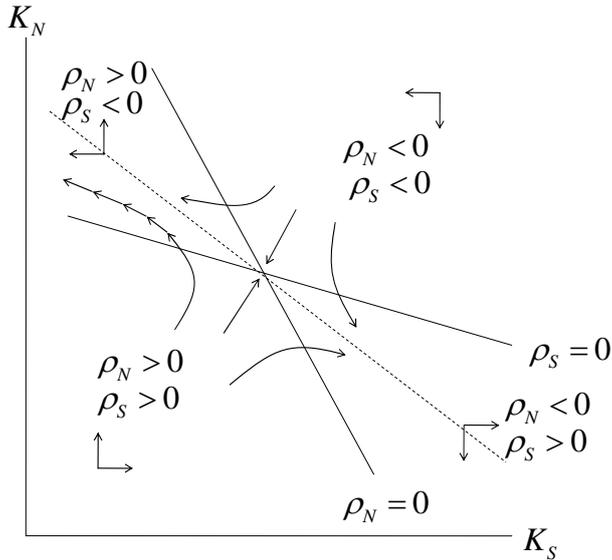


Figure 1

Therefore, K_N - K_S dynamics becomes as shown in figure 1, where the intersection of

both of lines $\rho_N = 0$ and $\rho_S = 0$ is on the 45 degree line. It illustrates that if North-South relation starts from A – where Northern capital stock is larger than Southern capital stock –, situation moves to upper-right wards and then turn to B. It implies that Northern manufacture grows faster and finally South will be specialized in agriculture. That is the Krugman's theory of uneven development which international trade leads to even if there is no international capital movement.

2. Krugman's 'Lenin Model'

In the Western economics, Krugman type of dynamics which increases North-South difference is called as 'divergence', but reality of the world economy at least in the East Asia is opposite. Lowest growth is in Japan, and the highest is China. The world economy is also similar in the sense that the US and European economy have collapsed and the center of gravity of the world economy is going to shift to the biggest developing country, that is China. However, Krugman cannot understand such reality.

Furthermore, Krugman also misunderstood Lenin's theory of *Imperialism*. In his understanding, Lenin and Hobson wanted to illustrate North-South relations when capital moves internationally in the second stage of internationalization of the world economy. He understood correctly to here, but mistook from here because he wanted to understand Lenin by using his model which now we saw.

Because Krugman changed his model to the model with perfect capital mobility, the orbit of K_N and K_S sticks to the vertical axis as shown as 'A→→→B' in figure 2. It is because all the capital move quickly to the higher profit countries under the assumption of perfect capital mobility, and then K_N reaches at K_{MAX} which implies that North specializes in manufacturing goods. In K_{MAX} , which is shown as B, reserve army in agricultural sector disappears (it means reaching at turning point in terms of development economics), and wages rise to push out capital to South by pressuring Northern profit rates to lower level than in South. Therefore, from here, world situation starts to move to C from b as shown in figure 2 by the international capital movement. In other words, first 'A→→→B' process proceeds keeping specialization in manufacturing sector pushed by free trade, and then in 'B→→→C' process, international capital movement occurs to South. In this way, Krugman's model has been understood to express two stages of internationalization; the stage of free trade and 'imperialist' stage.

Furthermore, Krugman illustrated Lenin's theory of 'labor aristocracy' in 'B→→→C' process in the sense that Northern wages are higher than Southern in this

process.

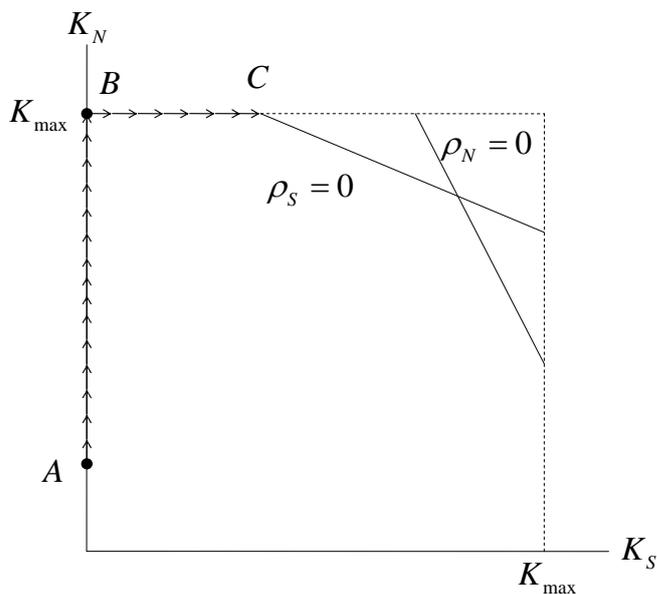


Figure 2

II. Lenin's 'Uneven Development'

1. Lenin's 'Uneven Development'

As we saw, some points of Lenin's theory was illustrated by Krugman's model. However, I am skeptical that Krugman could understand and express Lenin's basic idea. We must know what Lenin said. Following sentences are from his book *Imperialism*:

As long as capitalism remains what it is, surplus capital will be utilized not for the purpose of raising the standard of living of the masses in a given country, for this would mean a decline in profits for the capitalists, but for the purpose of increasing profits by exporting capital abroad to the backward countries. In these backward countries profits are usually high, for capital is scarce, the price of land is relatively low, wages are low, raw materials are cheap. The export of capital is made possible by a number of

backward countries having already been drawn into world capitalist intercourse; main railways have either been or are being built in those countries, elementary conditions for industrial development have been created, etc. The need to export capital arises from the fact that in a few countries capitalism has become "overripe" and (owing to the backward state of agriculture and the poverty of the masses) capital cannot find a field for profitable" investment. ...

(Lenin (1917))

Therefore, we can identify two points which are different Krugman's understanding, That is;

- 1) In Krugman's model, South cannot be industrialized after reaching at C in figure 2, but Lenin did not have such a limitation. Lenin said that backward capitalism can overtook advanced capitalism, and claims that it also has a right to be redistributed the colonies. Of course, the advanced capitalism does not agree with this claim, and therefore this conflict between the backward capitalism and advanced capitalism becomes a world war. That is, in the theory of Lenin's imperialist war, backward capitalism's overtaking advanced capitalism is critical.
- 2) Lenin said that the reason why backward capitalism's profit rate is low is its scarceness of capital. It means counter-correlation between profit rates and capital stock, that is $\frac{\partial \rho}{\partial K} < 0$. Therefore, we must assume diminishing return of capital to build Lenin's model. It is completely opposite to Krugman's model setting.

2. International Trade and Capital Movement under Diminishing Return of Capital

Therefore, we investigate international trade and capital movement under diminishing return of capital.

First, we consider the case with no international capital movement but trade like in

figure 1. That is, on the line $\rho_S=0$ and the line $\rho_N=0$, we can introduce $\frac{\partial K_N}{\partial K_S}|_{\rho=0} < 0$ which means both of lines $\rho=0$ are downward-sloping like in figure 1. However, steepness of both lines become opposite to the Krugman case (see appendix b). That is,

$$\left| \frac{\partial K_N}{\partial K_S} \Big|_{\rho_S=0} \right| > \left| \frac{\partial K_N}{\partial K_S} \Big|_{\rho_N=0} \right|.$$

Therefore, here we can draw figure 3 which shows two countries' situation tend to be equalized at the intersection of both lines (E). It means that international trade makes backward countries to catch up advanced capitalism under diminishing return of capital.

Second, we turn to consider the next case with international movement, but because here we assume diminishing return of capital, c' becomes positive and then $\rho_N < \rho_S$ when $K_N > K_S$. Therefore, capital move from North to South until K_N will be equalized to K_S . If the speed of this capital movement is unlimited, situation of both countries always on the forty-five degree line in figure 3. However, if this speed is limited, dynamics path becomes speeded up from the case without capital movement, but that path is basically same as figure 3. Anyway, our new assumption of diminishing return of capital leads the world to convergence; that is decrease in disparity between South and North.

Furthermore, in the last part of the path shown as *, K_N decreases while K_S increases. It means the deindustrialization in North which is the present world situation; rapid and radical weight shift of the world economy. Krugman could not explain this phenomenon, but this Lenin model can explain completely.

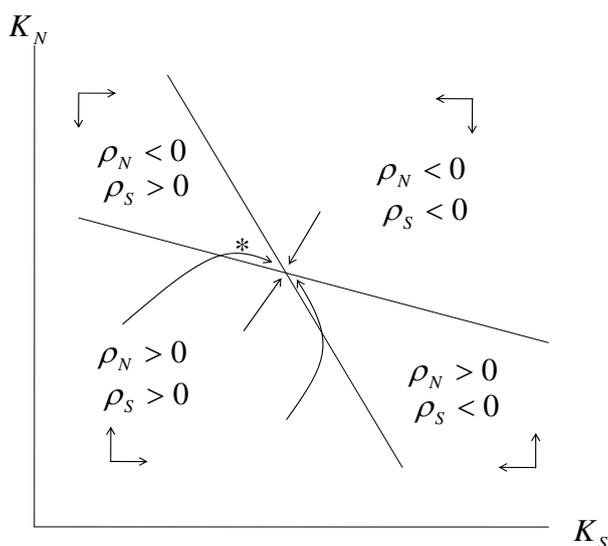


Figure 3

III. Historical Change from Krugman Case to Lenin Case

1. From Divergence to Convergence by the Change of Technological Characteristics

However, such catching-up process of the backward countries were not general before 1970s even in Asian NIES, and before 1985 even in Southeast Asian countries. Lenin said “The export of capital is made possible by a number of backward countries having already been drawn into world capitalist intercourse; main railways have either been or are being built in those countries, elementary conditions for industrial development have been created, etc”.

Therefore, the completion of the social infrastructure is the precondition for the economic development, and without it, backward countries cannot start to catch up. A technological characteristic of such infrastructures is generally increasing return of capital, and it is assumed by Krugman model. However, after the completion of such infrastructure technological characteristics can turn to diminishing return of capital.

In this sense, here we could understand that uneven development of the post-war world among the US, Japan and other Asian countries must be explained first by increasing return of capital and then by diminishing return of capital. Maybe, every countries experience increasing return of capital until a certain stage of development, and then go to the stage of diminishing return of capital. Such kind of technological change is widely known as ‘S-shaped production function’. In this

sense, such kind of change of technological assumption is reasonable.¹

2. S-shaped Production Function in Asian-Pacific Countries and the Change of Technology

Furthermore, this kind of technological change can be introduced, if we assume that the general path of the economic development has three phases; that is 1) low economic growth due to capital constraint, 2) high economic growth by dissolution of the constraint, 3) low economic growth due to labor constraint. Under this assumption, these three phases can be illustrated A→B phase, B→C phase and C→D phase in figure 4 respectively, and then marginal productivity of capital rise in the first phase, highly stable in the second phase and fall in the third phase, if this economy has stable Cobb-Douglas production function $Y=AK^\alpha L^\beta$ and $\alpha + \beta \doteq 1$, $\alpha < 1$ (see figure 5).

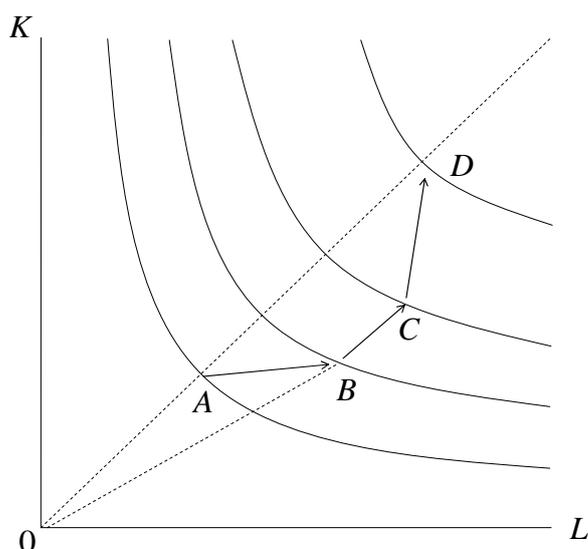


Figure 4

¹ Krugman does not always claim divergence. In the second chapter, Krugman(1991a) said “there is a kind of product cycle, in which emergent new industries initially flourish in localized industrial districts, then disperse as they mature ” on the industrial sector. However, its reason to say so is too much agglomeration of labor force, too specialized intermediate goods, spillover of the knowledge. All of them are not technological characteristics. It is same in its third chapter in which decrease in transportation cost makes agglomeration cycle of industries, that is agglomeration → dispersion → agglomeration.

Furthermore, Krugman(1991b) assumed international free movement of labor force, and studied the effects of transportation cost and ratio of interregionally movable industries. However, this analysis does not have historical perspective.

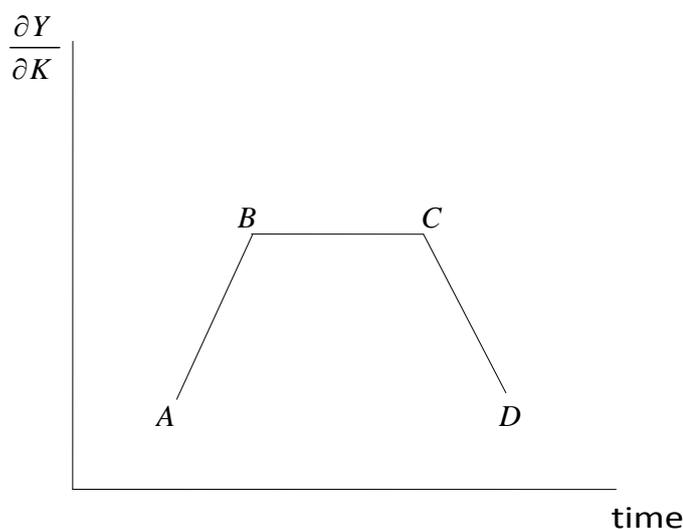


Figure 5

To show this relationship, first, let me introduce

$$\text{profit rate} = \frac{\partial Y}{\partial K} = \alpha A \left(\frac{K}{L} \right)^{\alpha-1} L^{\alpha+\beta-1} \doteq \alpha A \left(\frac{K}{L} \right)^{\alpha-1}.$$

It means that marginal productivity of capital is a decreasing function of capital-labor ratio (K/L) and can be shown as the slopes of the lines which connect the origin and each point in figure 4. ² Therefore, we can understand $A \rightarrow B$ process as a rising phase of profit rate where lower speed of capital accumulation than labor supply makes capital-labor ratio to decrease in figure 4,³ and then makes the profit rate to rise. Similarly, we can understand $C \rightarrow D$ process as a falling phase of profit rate where lower speed of population growth rate than capital accumulation makes capital-labor ratio to increase, and then makes the profit rate to fall. In the $B \rightarrow C$ process, because capital-labor ratio is stable, profit rate is also stable. Such relationship is completely same with S-shaped production function which could be drawn in Y-K diagram.⁴

² Barro (1991) also noted that lower capital-labor ratio in developing countries makes higher marginal productivity of capital, and then leads higher growth rate than in developed countries.

³ An estimation of marginal productivity of capital in pre-war period in Japan by the chapter seven in Inada, Sekiguchi & Shoda (1992) was negative in light industries, and non-negative in heavy industries. Therefore, relatively higher weight of heavy industries rather than light industries might lead to increasing return of capital in this period.

⁴ Strictly speaking, to regard this $A \rightarrow B$ process as a period of diminishing return of

Therefore, we must note that the characteristics of the technology determine the world system. This idea is surely materialistic, and completely consistent with Lenin's theory which claims amount of accumulated capital compared with labor force (i.e. capital-labor ratio) determines profit rate and then leads international capital movement. Lenin's claim that wage level is lower in backward countries also can be understood as a result of capital-labor ratio by introducing the following equation:

$$\text{wage rate} = \frac{\partial Y}{\partial L} = \beta A \left(\frac{K}{L} \right)^{1-\beta} K^{\alpha+\beta-1} \doteq \beta A \left(\frac{K}{L} \right)^{1-\beta}.$$

This equation shows that scarceness of capital results lower wage rate. Therefore, Lenin's theory of uneven development can be understood as a theory of return of capital, but more precisely as a theory of long trend of capital-labor ratio.

Then, our last problem is whether there is such kind of historical trend of marginal productivity of capital, and in fact, we could observe it in the Asian Pacific seven countries shown in figure 6 and 7. These two figures are based on our own estimation of capital stock which is shown in Ohnishi (1998) and Resaca (1996), and extended. As shown, the Philippines, Thailand and Indonesia have A→B phase and B→C phase, while USA, Japan, Korea and Australia basically have C→D phase.⁵ In this sense, above-mentioned hypothesis could be regarded as realistic.

IV. Conclusion

This paper has first reviewed Krugman's understanding on the effect of international trade, capital movement and then Lenin's theory. However, we could see Krugman misunderstood Lenin's basic point, and this misunderstanding came from his technological assumption of increasing return of capital. Therefore, we has changed this assumption to diminishing return of capital, and introduced following implications:

- 1) Lenin's theory of uneven development could be modeled by the assumption

capital, we should assume K as a function of time (t) and $K'(t) > 0$. In this case, because

$$c \equiv \frac{K}{AK^\alpha L^\beta} = A^{-1} \left(\frac{K}{L} \right)^\beta K^{1-\alpha-\beta} \doteq A^{-1} \left(\frac{K}{L} \right)^\beta, \text{ c is decreasing as time proceeds}$$

$\left(\frac{dc}{dt} < 0 \right)$. Therefore, $\frac{dc}{dK} \bullet \frac{dK}{dt} \equiv \frac{dc}{dt} < 0$, and because of the above assumption

$(K'(t) > 0)$, $\frac{dc}{dK} \equiv c'$ must be negative. It means increasing return of capital.

⁵ Strictly speaking, Japan, Korea and Australia also have short A→B phase and B→C phase in the early stage of post-war period.

- of diminishing return of capital.
- 2) Technologies of the Asian Pacific countries are moving from increasing return of capital to diminishing return of capital, and by this transition, this area has turned toward convergence.
 - 3) All of these analyses proved that technology is the key factor to determine the world. It is a part of historical materialism.

Reference

- Barro,R.J.(1991), "Economic Growth in a Cross Section of Countries," *Quarterly Journal of Economics*, vol.106, pp.407-443.
- Dutt,A.K.(1986), "Vertical Trading and Uneven Development," *Journal of Development Economics*, No.20, pp.339-359.
- Heckscher,E.(1950), "The Effect of Foreign Trade on the Distribution of Income," in *Readings in the Theory of International Trade*, ed. By American Economic Association, pp.272-300.
- Inada, Y., Sekiguchi,S. and Shoda,Y.(1992), *The Mechanism of Economic Development: Growth in the Japanese and East Asian Economies*, Clarenton Press, Oxford.
- Krugman,P.(1981), "Trade, Accumulation, and Uneven Development," *Journal of Development Economics*, No.8, pp.149-161.
- Krugman,P.(1991a), *Geography and Trade*, MIT Press.
- Krugman,P.(1991b), "Increasing Returns and Economic Geography," *Journal of Political Economy*, vol.99, no.3, pp.483-499.
- Lenin,V.I.(1917), *Imperialism, the Highest Stage of Capitalism*, First published in 1917 in pamphlet form, but here cited from Lenin Internet Archive 2005 (<http://www.marxists.org/archive/lenin/works/1916/imp-hsc/>)
- Ohnishi,H.(1998), *Rise-and -fall of the Pacific Rim Countries: Structure and Simulations of the Kyoto University Pacific Rim Model*, Kyoto University Press, in Japanese.
- Urasaka,J.(1996), "Estimation of Capital Stocks and Other Variables in Japan, United States and ASEAN for the KYPAC Model ", *The Research and Study*, No.10, Kyoto University Economic Society, pp.50-85, in Japanese.

APPENDIX A Krugman Case

- 1) First, we check the sign of the slope of the line $\rho_S = 0$. Here, $\rho_S = (P_M - v_S)/c_S = 0$ means $P_M = v_S$. By combining this equation with equation

(4); then we have

$$2\mu\bar{L} = v_s(K_s) \left\{ \frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right\}. \quad (6)$$

Then, by totally differentiating the both sides of this equation, we have

$$0 = v_s'(K_s) \left\{ \frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right\} dK_s + v_s(K_s) \left[\frac{c(K_s) - K_s c'(K_s)}{c(K_s)^2} \right] dK_s \\ + v_s(K_s) \left[\frac{c(K_N) - K_N c'(K_N)}{c(K_N)^2} \right] dK_N$$

Therefore,

$$\frac{dK_N}{dK_S} = - \frac{v_s' \left(\frac{K_N}{c_N} + \frac{K_S}{c_S} \right) + v_s \frac{c_S - K_S c_S'}{c_S^2}}{v_s \frac{c_N - K_N c_N'}{c_N^2}}$$

where this denominator and the second member of this numerator are positive

because $c' < 0$. Therefore, if $|v'|$ is not so large⁶, $\frac{\partial K_N}{\partial K_S} \Big|_{\rho_s=0} < 0$. It means that

⁶ Krugman (1991) assumed $\frac{\partial \rho}{\partial K} < \left(\frac{\partial \rho}{\partial P_M} \right) \left(\frac{\partial P_M}{\partial K} \right)$ instead of our assumption

$$v_N' > v_N \left[\frac{v_N \frac{c_N - K_N c_N'}{c_N^2}}{\frac{K_N}{c_N} + \frac{K_S}{c_S}} \right], \quad v_S' > v_S \left[\frac{v_S \frac{c_S - K_S c_S'}{c_S^2}}{\frac{K_N}{c_N} + \frac{K_S}{c_S}} \right], \text{ and made the lines}$$

$\rho_s = 0$ and $\rho_N = 0$ downward-sloping. These two assumptions are basically same.

The right side of Krugman's inequality is positive, because $\frac{\partial \rho}{\partial P_M} = \frac{1}{c} > 0$,

$$\frac{\partial P_M}{\partial K_N} = \left[\frac{-2\mu\bar{L} \frac{c_N - K_N c_N'}{c_N^2}}{\frac{K_N}{c_N} + \frac{K_S}{c_S}} \right] < 0, \text{ and } \frac{\partial P_M}{\partial K_S} = \left[\frac{-2\mu\bar{L} \frac{c_S - K_S c_S'}{c_S^2}}{\frac{K_N}{c_N} + \frac{K_S}{c_S}} \right] < 0. \text{ On}$$

the other hand, the left side of Krugman's inequality is $\frac{-v'c - c'(P_M - v)}{c^2}$.

Therefore, Krugman's inequality becomes

$$-v'c - c(P_M - v) < c^2 \left\{ \left(\frac{\partial \rho}{\partial P_M} \right) \left(\frac{\partial P_M}{\partial K} \right) \right\}.$$

the line $\rho_S = 0$ is downward-sloping.

2) Then, we turn to check the slope of the line $\rho_N = 0$. Same as the former case,

$$0 = v_N'(K_N) \left\{ \frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right\} dK_N + v_N(K_N) \left[\frac{c(K_S) - K_S c'(K_S)}{c(K_S)^2} \right] dK_S \\ + v_N(K_N) \left[\frac{c(K_N) - K_N c'(K_N)}{c(K_N)^2} \right] dK_N$$

Therefore,

$$\frac{dK_N}{dK_S} = - \frac{v_N \frac{c_N - K_S c_S'}{c_S^2}}{v_N' \left\{ \frac{K_N}{c_N} + \frac{K_S}{c_S} \right\} + v_N \frac{c_N - K_N c_N'}{c_N^2}}$$

Here, this denominator can be positive, if we again assume that $|v'|$ is not so large. In this case, $\frac{\partial K_N}{\partial K_S} \Big|_{\rho_N=0} < 0$. It means that the line $\rho_N = 0$ is also downward-sloping.

3) Then, we compare the slopes of the lines $\rho_S = 0$ and $\rho_N = 0$.

$$\frac{dK_N}{dK_S} \Big|_{\rho_N=0} - \frac{dK_N}{dK_S} \Big|_{\rho_S=0} = - \frac{v_N \frac{c_N - K_S c_S'}{c_S^2}}{v_N' Z + v_N \frac{c_N - K_N c_N'}{c_N^2}} + \frac{v_S' Z + v_S \frac{c_S - K_S c_S'}{c_S^2}}{v_S \frac{c_N - K_N c_N'}{c_N^2}} \\ = \frac{v_S' Z}{v_S \frac{c_N - K_N c_N'}{c_N^2}} + \frac{v_N' Z v_S \frac{c_S - K_S c_S'}{c_S^2}}{\left(v_N Z + v_N \frac{c_N - K_N c_N'}{c_N^2} \right) \left(v_S \frac{c_N - K_N c_N'}{c_N^2} \right)} \quad (7)$$

where $Z = \frac{K_N}{c_N} + \frac{K_S}{c_S}$, and in this case, the first member of the right side of the

above equation is negative, because its denominator is positive and its numerator is negative. The second member is also negative, because its denominator is positive and its numerator is negative due to the negative sign of

It can be rewritten as

$$v' > - \frac{\left\{ c^2 \left(\frac{\partial \rho}{\partial P_M} \right) \left(\frac{\partial P_M}{\partial K} \right) + c'(P_M - v) \right\}}{c}$$

It is same with our assumption that $|v'|$ is not so large.

v_N' . Therefore, the right side of the above equation is negative, and it results that the line $\rho_N = 0$ is steeper than the line $\rho_S = 0$. The figure 1 illustrates this relationship.

- 4) Then, we investigate which side of the line $\rho_S = 0$ is $\rho_S > 0$ or $\rho_S < 0$. For this purpose, first we assume $\rho_S > 0$ as follows. Here,

$$P_M > v_S,$$

because $\rho_S = \frac{P_M - v_S}{c_S} > 0$. By combining the above inequality with equation

(4); then we have

$$2\mu\bar{L} > v_S(K_S) \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right). \quad (8)$$

Here, because $2\mu\bar{L}$ is constant, our problem which side of the line $\rho_S = 0$ is $\rho_S > 0$ becomes to which side of the line $\rho_S = 0$ makes the right side of inequality (8) is smaller. To know that, by assuming again that $|v'|$ is not so large, and differentiate the right side of inequality (7); then we have

$$\frac{\partial \left[v_S \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_N} = v_S \frac{c_N - K_N c_N'}{c_N^2} > 0$$

$$\frac{\partial \left[v_S \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_S} = v_S' \left(\frac{K_N}{c_N} + \frac{K_S}{c_S} \right) + v_S \frac{c_S - K_S c_S'}{c_S^2} > 0$$

Therefore, the inequality (8) and $\rho_S > 0$ are satisfied in the area under the line $\rho_S = 0$, and vice versa.

- 5) Same as the former case, we investigate which side of the line $\rho_N = 0$ is $\rho_N > 0$ or $\rho_N < 0$. For this purpose, first we assume $\rho_N > 0$ and then we introduce

$$2\mu\bar{L} > v_N(K_N) \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right). \quad (9)$$

By assuming again that $|v'|$ is not so large, and differentiate the right side of the above inequality; then we have

$$\frac{\partial \left[v_N \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_N} = v_N \left(\frac{K_N}{c_N} + \frac{K_S}{c_S} \right) + v_N \frac{c_N - K_N c_N'}{c_N^2} > 0$$

$$\frac{\partial \left[v_N \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_S} = v_N \frac{c_S - K_S c_S'}{c_S^2} > 0$$

Therefore, also in this case, the inequality (9) and $\rho_N > 0$ are satisfied in the area under the line $\rho_N = 0$, and vice versa.

APPENDIX B Lenin Case

- 1) With respect to the slope of the line $\rho_S = 0$, we replace the former assumption of increasing return of capital ($v' < 0$, $c' < 0$) with the new assumption of diminishing return of capital ($v' > 0$, $c' > 0$), and then we have $\frac{\partial K_N}{\partial K_S} \Big|_{\rho_S=0} < 0$ without any additional assumption. It means that the line $\rho_S = 0$ is downward-sloping.
- 2) Same as the above case, the line $\rho_N = 0$ becomes downward-sloping.
- 3) Then, we compare the slopes of the lines $\rho_S = 0$ and $\rho_N = 0$ in this case. The first member of the right side of the equation (7) is positive, because both of its numerator and denominator are positive, and the second member is also positive, because both of its numerator and denominator are also positive. Therefore, in this case, the line $\rho_S = 0$ is steeper than the line $\rho_N = 0$ differently from the former case. This relationship is illustrated in figure 3.
- 4) In this case, with any signs of c' and v' , we have

$$\frac{\partial \left[v_S \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_N} > 0, \quad \frac{\partial \left[v_S \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_S} > 0$$

$$\frac{\partial \left[v_N \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_N} > 0, \quad \frac{\partial \left[v_N \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_S} > 0.$$

Therefore, under the line $\rho_N = 0$, $\rho_N > 0$, and under the line $\rho_S = 0$, $\rho_S > 0$, and vice versa.