A Marxist = Neo-classical Modeling of Capitalism as An Optimal Roundabout Production System

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Abstract

This paper intends to reinterpret Marxian economics within a framework of neoclassical optimal growth theory. The theory of optimal economic growth not only provides the optimal capital-labor ratio but also illustrates the optimal path for the capital-labor ratio to reach the optimal level. Indeed, this is tantamount to elucidating the historical role of capitalism as the mechanism of machinery, capital accumulation, and capital multiplication. This paper attempts to discuss the whole conception of the origination, advancement and extinction of the society under this framework.

Keywords: Marxist economics; neo-classical economics; capitalism; the Industrial Revolution; roundabout production system

1 Introduction

The purpose of this paper is to resolve the mutual noncommunication between modern economics and Marxian economics, and reinterpret Marxian economics in terms of modern economics. In this field, "Fundamental Marxian Theorem" by Okishio (1957) was highly regarded internationally. But this theorem has now become the subject of criticism by analytical Marxism, its theoretical successor. Okishio (1967) also built a bridge across the chasm between Marxian economics and modern economics by interpreting Marxism from the standpoint closer to Keynesian economics, while Mizuchi (1984) later reversely reinterpreted the Marxist theory of exploitation on the basis of the neoclassical marginal productivity theory. Contrary to these approaches, this paper attempts to interpret Marxist theories on the basis of the neoclassical optimal growth theory.

Another characteristic of the interpretation of Marxist theories in this paper is the formulation of a labor theory of value and theory of surplus value, from the standpoint of historical materialism which claims that surplus value exists only in a certain historical stage. Engels (1880) defined historical materialism as the core substance of Marxist theories along with the theory of surplus value. But the significance of historical materialism appears to have been generally downplayed partly because Marx did not produce a systematic series of books on the subject. For example, the aforementioned "Fundamental Marxian Theorem" has a theoretical structure that is imprecise in its argument that exploitation is the inherent trait of capitalism. This paper also seeks to overcome these weaknesses in established theories.

2 Basic Model

The basic approach to understanding capitalism by Marxian economics was set forth by Ohnishi (2000). In that approach, capitalism is depicted in the social context of post-Industrial Revolution society and its mode of production is portrayed as a collaborative work of human power and machinery, rather than production performed by human power alone. In other words, such production activities can be more efficient in the sense that the same amount of production can be performed with less labor, viz., the same amount of labor can produce more. However, the optimal ratio of capital to labor must be understood as a dynamic being in the process of long-term growth, rather than a static being at a specific point in time. In line with that approach, this paper addresses the modeling of the Marxist interpretation of capitalism as optimal growth theory over an infinite period and examines its characteristics.

The ultimate purpose of social production is primarily aimed at its consumption. However, for the sake of the aforementioned social efficiency, society also has to make production goods to produce consumer goods. It is, therefore, assumed here that there are two sectors of production—the consumption goods and the production goods sectors. In this society the allocation of labor L of the whole society is divided into these two sectors at the ratio of s:1-s (0<s≤1). Now, it is assumed that L is constant over time. Furthermore, if the production function for the consumption goods is assumed as a Cobb-Douglas type with constant return to scale, then,

$$Y(t) = [s(t)L]^{1-\alpha} K(t)^{\alpha} \tag{1}$$

For the production goods sector, the following linear homogeneous function is also formulated simply, disregarding depreciation.

$$\dot{K}(t) = [1 - s(t)]L \tag{2}$$

$K(t)$ is capital at time point t, and $\dot{K}(t)$, which is a derivative with respective to time, represents an amount of capital increase or decrease at time point t. In other words, $\dot{K}(t)$ denotes investment in capital used for production of consumption goods. In this formula, although it may appear that the factor to produce production goods is employed by one and the same labor, such a condition is unnecessary. Total labor left to produce production goods, (1-s)L, can be divided into "directive labor left to produce production goods (for the production of consumption goods)" and "labor to produce production goods for the production of production goods." The case in point here is that, as a consequence, these production goods are produced only with total labor (1-s)L left for that purpose.

The marginal rate of substitution is constant for both factors of production in the production sector concerned[1].

Unlike the neoclassical growth model (e.g., Ramsey Model) that makes the savings rate endogenous, the current model does not take the action of saving consumption goods into account, and produces production goods at the expense of the production of consumption goods[2]. In this sense, 1-*s* can be understood as the broadly defined savings rate, or the ratio of roundabout production, considering that the ultimate purpose is not to produce production goods. Within the framework of Marxist theories, however, it can imply the "rate of exploitation," if the issue of depreciation is disregarded, which will be discussed later. According to

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[1] "The sector of production for production goods to produce consumption goods" and "the sector of production goods for such production goods" are now expressed in the following two equations.

$$\dot{K} = \alpha(1-s)L + bI_I \cdots\cdots\cdots\cdots \quad ①$$

$$I_i = (1-a)(1-s) + (1-b)I_i \cdots\cdots\cdots\cdots ②$$

Here, the contribution to the production of production goods in these two sectors is expressed in the form of flows, in accordance with the Marxian reproduction schema. Production goods I_I for the sector ① use I_I for production, and the total I_I produced is used separately both for the sector ① and sector ②. The ratio for this use is expressed as *b*: (1-*b*) in the above equation. Total labor (1-s)L that is available for both sectors is also divided into the two sectors at the ratio of a: (1-a). When I_I is gathered to the left-hand side of the equation ②,

$$bI_I = (1-a)(1-s)L$$

When this is assigned to equation ①,

$$\dot{K} = a(1-s)L + (1-a)(1-s)L = (1-s)L$$

Production goods $\dot{K}$ for the production of consumption goods in this society can be understood to be produced with the direct or indirect use of labor (1-*s*)L left unused in the production of consumption goods. This is suggested by Equation (2).

[2] The neoclassical growth model (Ramsey Model) is the optimal growth model for a single sector. The choice between the consumption and saving of the sole product is made at each point of time. For details, see Barro and Sala-i-Martin (1995) Chapter 2.

the traditional Marxist theories, workers consume all their income as a class, and only capitalists accumulate, to put it reversely, the role played by the capitalist class has both historical and social significance. How much income is accumulated means, therefore, how much income is shared by capitalists. In this model, the ratio of the capitalists' share is represented in proportion to the total labor utilized to produce production goods for the future production of consumption goods. Construed this way, $1-s$, in fact, can be the "rate of exploitation," an acceptable definition from the perspective of the labor theory of value that equates total labor with total value. From now on, this rate is expressed as the "rate of savings" in some cases, and as the "rate of exploitation" in others.

What has been described so far is no more than technical conditions for production. Under these conditions, it is further assumed that society allocates resources—to the two sectors of production—over time in a manner that seeks to maximize efficiency over an infinite period. This does not presuppose that neutral social planners are capable of controlling the economy, but suggests the notion that accords with historical materialism, such as "the need for production determines the superstructure" and "the need of 'society' (ultimately) determines the modality of 'society.'" The same conclusion could be drawn from the model that assumes all individuals in society are uniform and homogeneous representatives; they own K/L capital, receive the allocation of profits from that capital, and have the equilibrium discount rate for utilities. However, it is not appropriate to make such assumptions about the Marxist model. Even if the tradition of modern economics is pursued, people's tastes for consumption, at present and for the future, are not identical. Nor are their preferences for risky behavior. These differences would divide people into owners and non-owners of K, and this division in turn, would produce differences in the actual roles they play in the social accumulation of capital; the relationship under which the allocation to the owners of K would lead to greater contribution to the accumulation than to the non-owner. To put it reversely, if the representative individual model, just mentioned above, is adopted, individual decisions in the market would not dictate trends. It is understood either as ① "optimization" under the representative individual model, or as ② the optimization model from the perspective of society as a whole, if not under the representative individual model. Bearing this in mind, the issue of optimization of "society" over time is formulated as follows[3].

$$\max \quad U = \int_0^{\infty} e^{-\rho t} \log Y(t) dt \tag{3}$$

$$\text{s.t} \quad \dot{K}(t) = [1 - s(t)]L \tag{4}$$

U stands for utility over time, log (Y) for instantaneous utility time at point t, and p for the rate of time preference. The current value Hamiltonian H is formulated as in the following[4].

$$\mathrm{H} \equiv \log Y(t) + \mu(t)[1 - s(t)]L \tag{5}$$

$$= (1-a)\log s(t) + (1-a)\log L + \alpha \log K(t) + \mu(t)[1 - s(t)]L \tag{6}$$

Thus, the first-order conditions of optimization are as follows.

$$\frac{\partial H}{\partial \mathrm{s}} = 0 \Leftrightarrow \frac{1-\alpha}{s} - \mu L = 0 \tag{7}$$

$$\frac{\partial H}{\partial K} = -\dot{\mu} + \rho\mu \Leftrightarrow \frac{\alpha}{K} = -\dot{\mu} + \rho\mu \tag{8}$$

The following can be derived from (7).

$$\frac{\dot{\mu}}{\mu} = -\frac{\dot{s}}{s}, \mu = \frac{1-\alpha}{sL} \tag{9}$$

When these are assigned to (8), the following is obtained.

$$\frac{\alpha}{K} \cdot \frac{sL}{1-\alpha} = \frac{\dot{s}}{s} + \rho \tag{10}$$

If this is converted, the following is obtained.

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[3] With regards to optimization over an infinite period below, see Barro and Sala-i-Martin (1995), Appendix on Mathematical Methods.

[4] Variables that change with time are $Y(t)$, $s(t)$, $K(t)$, and $\mu(t)$. In the following, t is omitted.

$$\dot{s} = \frac{L}{K}\frac{1-\alpha}{\alpha}s^2 - \rho^s \tag{11}$$

$$= s\left(\frac{L}{K}\frac{\alpha}{1-\alpha}s - \rho\right) \tag{12}$$

Next, $\dot{s}=0$ in stationary equilibrium. Since 0<s≤1 is assumed, when the solution of s=0 is disregarded, the following is obtained.

$$s = \frac{\rho(1-\alpha)}{\alpha L}K \tag{13}$$

Further, since $\dot{K}=0$ is also assumed in stationary equilibrium,

$$(1-s)L = 0 \tag{14}$$

In other words,

$$s = 1 \tag{15}$$

is obtained. When simultaneous equations (13) and (15) are solved, the long-term equilibrium solution is obtained. Namely,

$$K^* = \frac{\alpha L}{(1-\alpha)\rho}, s^* = 1 \tag{16}$$

Also, by the same token, the optimal capital-labor ratio is as follows.

$$\left(\frac{K}{L}\right)^* = \frac{\alpha}{(1-\alpha)\rho} \tag{17}$$

Since $\dot{K}=0$, s=1 is obtained from equation (2). In other words, all labor is employed to produce consumption goods and, as long as depreciation is disregarded, there is no production of production goods. For this reason, society does not require accumulation other than that for depreciation, nor does it need for exploitation by the capitalist class. Communist society recognizes no social class or exploitation constructed this way. The equations (14) and (15) illustrated on the s-K plane surface yields the following phase diagram (see **Figure 1**).

3 Characteristics of the Transition Path and Primitive Accumulation

It is not only the destination mapped for the future, but also the path to reach that destination that is especially interesting and significant.

Firstly, the path is monotonically increasing. Since K can be generally considered as less than K^* under historical conditions, the figure in the parenthesis on the right-hand side of equation (12) is positive. Thus, $\dot{s} > 0$ is obtained. This indicates that s increases monotonically from the initial point to the stationary equilibrium point. To put it reversely, the savings rate as rate of exploitation is led to decrease monotonically. This characteristic is called "path attribute ①."

Secondly, since $\dot{s} > 0$ is assumed, for the duration of this transition path, s must always stay on the upper side of $\dot{s} = 0$ line. This is called "path attribute ②."

Thirdly, the shape of this $\dot{s} = 0$ line is the straight line that goes through the original point on the s-K plain surface, on the basis of equation (13), when L, α, and p are constant. This is called "path attribute ③." The path with these attributes is shown by the thick dotted line on Figure 1. Here, the starting point A lies on the s axis because the economy before the Industrial Revolution is interpreted as simple; there was no need for the accumulation of machinery due of its non-existence and everything was done with manual labor. Explained by signs in equation (1), the multiplier of K, as α, is zero; therefore, K^*, derived from equation (16), is also zero. Such an economy is considered to have existed, in its infancy, in the era before the Industrial Revolution.

Finally, the speed of transition on this path needs to be examined. In Figure 1, when A-B (the left-hand side of the path) and B-C (the right-hand side of the path) are compared, appearing low absolute level of K and high savings rate, $\dot{K}/K$ or the growth of K for A-B is commonly considered to be higher than for B-C. Put another way, since the arrival at the final stationary equilibrium point is technically far beyond, initially the economy approaches K^* at a visible speed and then slows down. In Figure 2, this progress is illustrated as the path on the time axis, showing that the savings rate declined sharply after the initial discontinuous jumps due to the Industrial Revolution, and eventually settled at a lower level after some point. In the economic frame of reference, this means that any society, during a special period immediately after the Industrial Revolution, has to go through a fairly severe exploitation, or a period of “primitive accumulation”. Nonetheless, such society can ultimately break away from this accumulation as it occurs only during a certain special period. The “primitive accumulation” after the Industrial Revolution can be explained in this way.

4 Depreciation and C in Value Composition

So far, the economic model has been formulated on the assumption that production goods are not depleted at all. In reality, however, depletion of production goods is inevitable at a certain ratio δ. Taking this into account, a certain ratio of production goods should be left in order to maintain the K^* level in the coming stationary equilibrium. From the standpoint of the Marxist theory of value, this specific point relates to the problem of C found in the value composition of C+V+S. This problem will be addressed in the end. Some of the formulae accounted above should be reformulated and that is discussed first. The production function of the production goods sector is the following.

$$\dot{K} = (1-s)L - \delta K \tag{18}$$

The stationary equilibrium point can be obtained through similar procedures previously employed. The $\dot{s} = 0$ line can be conduced as below.

$$s = \frac{(\rho+\delta)(1-\alpha)}{\alpha L} K \tag{19}$$

Also, taking account of $\dot{K} = 0$, the following formula is derived.

$$(1-s)L = \delta K \tag{20}$$

When the above two simultaneous equations are solved, K and s as stationary equilibrium are obtained as follows.

$$K^{*} = \frac{\alpha L}{\delta + (1-\alpha)\rho}, s^{*} = 1 - \frac{\delta}{L} K* = 1 - \frac{\alpha\delta}{\delta + (1-\alpha)\rho} \tag{21}$$

The savings, as much as $(\delta/L)K^{*}$, must continue to be made even at the final stationary equilibrium point. This is illustrated in the distance between E^{*} point and the s=1 line on Figure 3. The $(\delta/L)K$ portion can be interpreted as the C portion of the Marxist theory of value composition, which means that the rest of the value composition constitutes V+S.

To take an example, assume that an economy is at the D point on the path. The s axis represents the allocation of total labor for the production of consumption goods, which is used directly for consumption in the period concerned as 1-savings rate. Thus, this portion is equivalent to the V portion in the Marxist theory of value composition, and in this line of reasoning, the portion that is neither the C portion nor the V portion should be the S portion. In terms of its original meaning, this portion shows the extent to which the production of consumption goods as the consumption of consumption goods is restrained in order to acquire the K^{*} stock of production goods in the future. In other words, it means how much consumption is restrained—restrains on the allocation of workers under the assumption that workers do not accumulate at all—compared with the future society. If only capitalists make such accumulation, as assumed by this paper, it represents the acquisition of production resources by capitalists for such a purpose, an increase in K, rather than depreciation. In this sense, our model conveys the "acquisition of surplus value" as "exploitation."

The "C portion" of the production of production goods must continue in order to cover the depreciation portion. Without doing so, the society discussed here would not be able to maintain K^{*}, and for that reason, could not maximize consumption over an infinite period. In that sense, instead of allocating the final production factor as labor existing to produce consumption goods, it is necessary to establish a social system under which a certain ratio of labor is allocated to produce production goods. In such a case, if workers, being shortsighted, resist the allocation of resources to produce production goods, some sort of forced mechanism is required in order to suppress their resistance. All violence cannot be eliminated from society. To eradicate all forms of violence in society, with the state being the case in point, the "need for the S portion" must disappear and, at the same time, workers as a class need to acquire a long-term perspective.

5 Concerning Problems in the Period to the Stationary State

The approach taken in this paper has to deal with the basis of Marxist economics and is also an enterprise to "reconstruct" Marxist economics. Its arguments have basic theoretical differences with established theories. As such, without full and detailed discussions, there is no prospect for consensus within the academic community of Marxist economics, or the academic community of modern economics. Being fully aware of that and for that particular reason, the authors of this paper would like to address several problems that need to be examined for future debate.

The first of such problems is the time required for an economy to converge to a stationary state. This problem directly concerns whether our model can really explain the whole aspects of "capitalism." The authors first calculated the time required for the economy to reach 95% of the stationary state value under the parameter of $(\alpha, p, \delta)=(0.3, 0.02, 0.05)$ assuming the existence of depreciation cited in the preceding section. The time required to reach the 95% point was used because this level can be considered to represent the state where most necessary capital has been accumulated. The calculation produced a result of about 20 years, which appeared to be a very short period. If necessary capital can be accumulated in such a short period of time, then capitalism would reach the stage of zero economic growth a shortly after the Industrial Revolution, or put another way, would reach communism as an "era where the labored accumulation of capital is not necessary." This is not realistic, to say the least.

Another calculation was attempted, therefore, with the different value for α as in (α, p, δ)=(0.7, 0.02, 0.05) by reinterpreting capital K to include human capital in accordance with Chapter 2 of Barro=Sara-i-Martin (1995). In this case, a more realistic calculation result was obtained, with the time required to reach 95% of the stationary state value coming to about 50 years. The similar period of time was calculated when the logarithmic instantaneous utility function adopted in this paper was replaced with the more common utility function with constant relative risk aversion (CRRA)[5]. Yet, these calculation results still show that the required time appears too short to be practical. In other words, the calculation results raise the possibility that this paper's model responds only to short-term or medium-term problems in a period "from the invention of a specific technology to its diffusion," and has limitations as a model to explain the total picture of capitalism.

That said, there remain several points of references to explain the problem. Firstly, the technological jump did not occur only once at the time of the Industrial Revolution, rather technological innovations occurred without a break, providing evidence such as the second Industrial Revolution in Europe at the end of the 19th century, the technological revolution in the 20th century that created the automobile and oil culture, and then the information technology revolution. Given these historical circumstances, it is possible that the arrival at the 95% point is being pushed back successively.

It may also be conceivable that the economy is not always following the "optimal" path. For example, the labor distribution rate did not fall sharply in the non-industrial sector immediately after the Industrial Revolution, and in fact it is highly likely that it had exceeded the optimal labor distribution rate in macroeconomic terms. This distance between the possible "optimal value" and the

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[5] The calculation was made with the parameter of (α, p, δ, θ)=(0.7, 0.02, 0.05, 3) after setting the following utility function over time.

$U = \int_0^\infty e^{-\rho t} \frac{Y(t)^{1-\theta} - 1}{1-\theta} dt (\theta \geq 0, \theta \neq 1)$ For the CRRA utility function, see Chapter 2, Blanchard and Fischer (1989).

“reality,” or the occurrence of this gap due to the “adjustment cost” associated with behavioral changes, including the cost of the mechanism of violence necessary for the forcible lowering of the labor distribution rate, have to be taken into account. This is not an issue that can be dealt with simply by setting up the parameter of (a, p, δ,θ), it will require research into the long-term historical process from the perspectives of econometrics as well as cliometrics. The authors would like to simply address that these issues are left undiscussed.

6 Other Points for Discussion

There are points of discussions other than those cited above.

Firstly, as noted in the text of this paper, our model does not present a solution derived from responsible optimal behaviors of individuals in society with the assumption that each individual and each class has different tastes. When a society consists of individuals who have different tastes, some individuals become owners of K (capitalist), while other individuals become workers. Because of this class division or bipolarization seen more generally, what change should the uniform representative individual model for individuals have to undergo? On the assumption of the class division in this sense, what consequences would be brought about by individual optimization behaviors of respective classes? This paper has yet to shed light on these issues. For example, as a consequence of individual decisions in a class society like this, it is possible that the actual labor distribution rate is higher than the socially optimal labor distribution rate in the “period of primitive accumulation,” and as a result, the “social planner” may have to adopt various policies for the suppression of workers. Even when the optimal capital-labor ratio is nearly reached, if the capitalist obtains the capital share (α) technically determined under perfect competition, then it may be higher than the socially optimal capital share at the time. (The labor share resulting from individual decisions solely based on the market can be lower than the socially optimal labor share.) In that case, the profit squeeze or the redistribution to workers by the "social planner" may become the social necessity. These issues are presented as possibilities but still need to be clarified.

Secondly, as a matter relating to the above, what might emerge when technological advances in terms of productivity are examined? It is possible to assume a type of technological innovation—an increase in total factor productivity—where Y increases even when there is no change in α in equation (1) and L and K are constant[6]. Likewise, it is possible to assume a type of technological innovation where the constant L produces more of K. Furthermore, due attention needs to be paid to a possible difference that may arise between the sudden technological innovation at a given time and technological innovation taking place on an ongoing basis. It may be relevant to consider what might arise in such a case. If the conclusions in the area of growth theory in modern economics are amplified, it is possible to assume the stationary state where Y/L and K/L may not be constant but grow at a constant speed. However, since our model does not only assume labor-augmenting technological change, the calculation is relatively complicated and the existence of those solutions cannot be assumed. If that is the case, should it be reasonable to conclude there is no optimal capital-labor ratio in the stationary state? Does capital accumulation remain as a social issue in the distant future? These matters need to be examined in detail separately.

Thirdly, in the context of developing technical mathematical formulae, it is worth considering what happens if the consumer goods production function is not constant return to scale? Also, what happens if the assumed production function for the production goods is changed from the linear homogeneous function to the Cobb-Douglas type ? These matters concern the universality of the model and influence the length of the transition period more than by parameter values. They, too, need to be investigated.

Finally, we would like to suggest some possible arguments regarding the idea of an ultimate stationary state. When we distributed a draft of this paper as a

[6] They include Hicks neutral technological change, Harrod neutral (labor-augmenting) technological change, and Solow neutral (capital-augmenting) technological change. See Barro=Sala-i-Martin(1995) Chapter 1.

discussion paper, the following two questions were raised: in the stationary state as "communist society," ① is maximum labor productivity not achieved? and ② is it appropriate to describe the "communist society" as a stagnant society with zero economic growth, apart from technological change? Both questions are related to the fact that the objective function of the model of this paper is the maximization of utility, not the "maximization of labor productivity." Therefore, accumulating capital beyond the accumulation of capital in a stationary state runs counter to the above-mentioned "utility maximum." While it was necessary to restrain consumption in favor of accumulation to achieve the optimal capital-labor ratio, once the stationary state is attained, it is no longer necessary to keep restraining consumption. Since this is the kind of society assumed in this paper, "zero growth" itself does not necessarily mean a negative state but rather an ideal state. In this sense, a society with "zero growth" can be understood to represent an ideal state. Avoiding "excessive accumulation," as it were, becomes a necessity at that point of time. It is, however, important to distinguish between such a state and the present "zero growth" observed in Japanese society as a process of international imbalance correction. In this sense, there should a clear recognition of the problem of discriminating the "zero growth" as an ideal state from the "zero growth" as a result of the failure to adopt policies fit for the times, "zero growth as a failure".

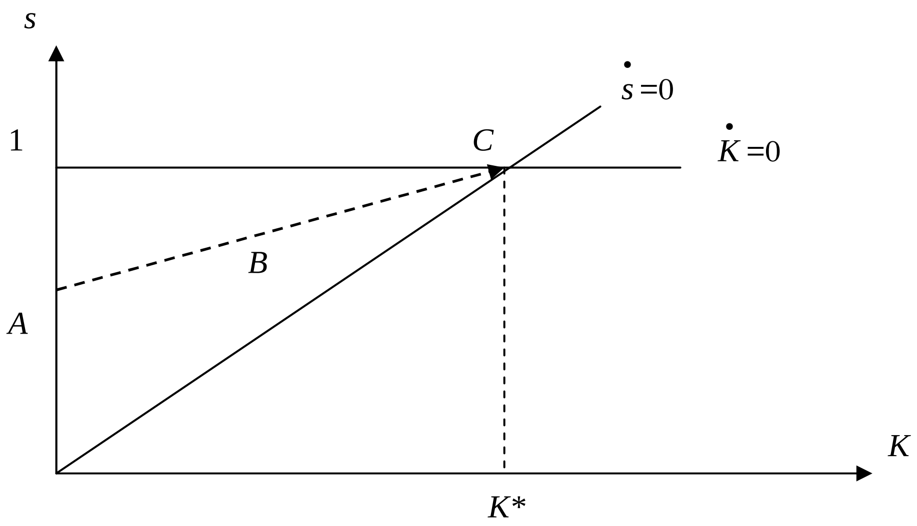

Figure 1

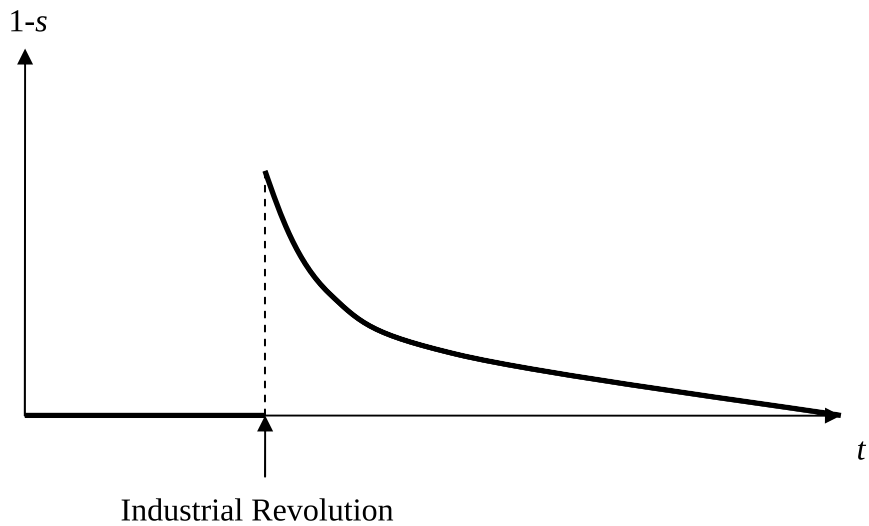

Figure 2

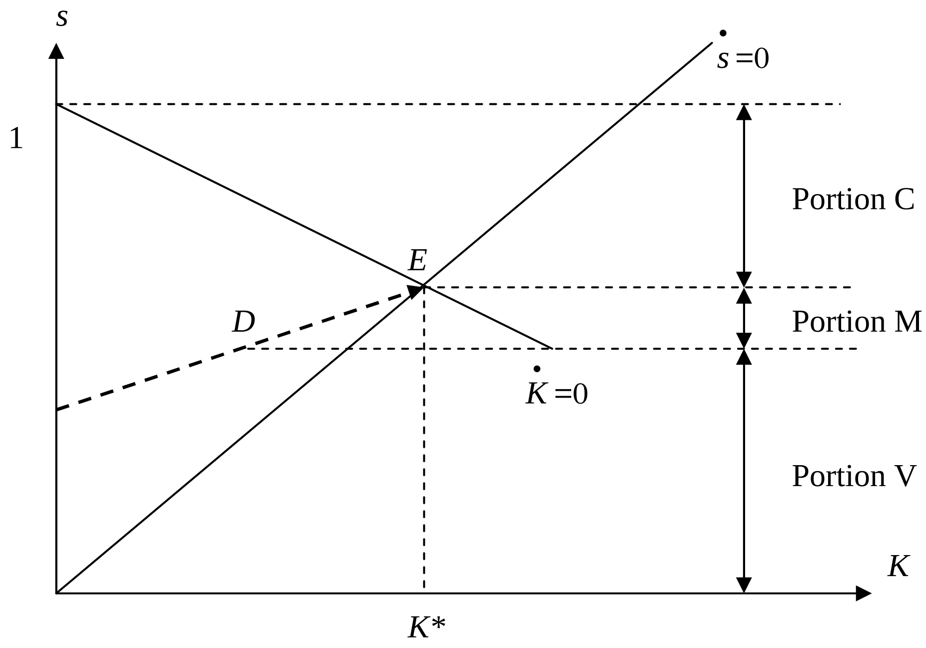

Figure 3

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