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Price Reaction to Momentum Trading and Market Equilibrium

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Abstract. This paper analyzes a financial market with momentum trading, and shows how momentum trading affects the market equilibrium. When momentum traders dominate the market, the private information owned by the rational traders may not be incorporated into the price, whence the price becomes less informative. If trend-chasing traders trade intensively in the market, a market bubble may occur and the price rises even though the fundamental value of the asset is low. It is shown that the reliability of the firm's public announcement such as an earnings forecast is a key factor to avert the market bubble.

Keywords: Market microstructure, rational expectations model, momentum trading, market bubble.

1 Introduction

In Milgrom and Stokey (1982), it was proved that if the allocation before trading is ex-ante Pareto optimum and all traders in the market are rational and fully exploit their information available to them, i.e., if there is no noise trading in the market, no trade occurs even after traders receive different signals on the fundamental value of the asset. This *No Trade Theorem* holds because the revealed price is the sufficient statistic of the fundamental value, and there can be no trade motivation under the Pareto optimum allocation. The theorem implies that in actual financial markets, stocks and bonds are actively traded thanks to so-called noise traders, who trade for exogenous or irrational reasons that cannot be explained by the traditional economic theory. Black Black (1986) emphasized the role of noise traders by saying that “Noise makes trading in financial markets possible, and thus allows us to observe prices for financial assets.” Thus, it is of great importance for financial researchers and practitioners to analyze the market with irrational traders.

In the market microstructure theory, a market with many kinds of traders has been studied. However, in most of the literature, irrational trading is simply modeled as a normal distribution with mean zero, independent of all other random variables. Under this assumption, the price becomes an imperfect signal on the true value of the asset (hereafter we call it the liquidation value), and noise traders merely stand at the opposite trading position to rational traders so that the market clears all orders. Noise trading in these studies only plays a passive role in the market.

In actual markets, however, there are other kinds of traders whose trading strategies seem contradictory to the standard theory of finance, but have a certain impact on the market equilibrium. Barberis and Thaler (2003) survey the literature and provide some examples that cannot be explained by traditional economic theory.

In this paper, we study the effect of *momentum trading* on the market equilibrium. Momentum trading is the trading strategy that trades according to the past price process. Typical examples are “trend-chasing strategy” and “contrarian strategy.” Even professional traders such as technical analysts follow these strategies, which can cause a big price impact. The aim of this paper is to investigate how the momentum trading affects the price formation and the market equilibrium.

The study of momentum trading is not new in the literature. For example, De Long et al. (1990) studied a market with momentum trading. Their results are very similar to ours, and this paper can be thought of as their extension in the following sense. In De Long et al. (1990), informed traders privately observe a noisy signal on the liquidation value. They assume that noise term in the signal is a discrete random variable, while the liquidation value of the asset follows a continuous random variable. We model the liquidation value and all noise terms to be continuous random variables, more specifically (independent) normals. Thanks to the assumption that all random variables follow normals, we can discuss how to avoid a market bubble as we see in Section 4. De Long et al. (1990) only discuss how a market bubble occurs in their model, but do not provide its countermeasures. This is one of advantages in this paper.

Hong and Stein (1999) also analyzed a market with momentum trading, and showed that the prices underreact in the short run and overreact in the long run, as suggested in empirical papers. In their model, however, newswatchers, who can observe private signals on the liquidation value as rational traders in this paper, only maximize their one-period profit although they trade at multiple periods. Due to the assumption of their myopic behavior, newswatchers lose money by momentum traders in some parameter setting. But it is too restrictive to suppose that traders with superior information trade in order to maximize locally. Moreover, they only analyze the stationary price dynamics. Therefore, bubble phenomena due to momentum trading cannot be studied in their setting.

In this paper, we construct a noisy rational expectations market model with 2 periods as Kim and Verrecchia (1991a) or Wang (1993). Thus, we study a more general setting than Hong and Stein (1999). Rational traders, who observe a private signal on the liquidation value, maximize their conditional expected utility given all available information. Each rational trader has the exponential utility function with a constant risk tolerance coefficient. This kind of setting is called a CARA-Normal model, and popular in noisy rational expectations models for its tractability.

In addition to rational traders, we incorporate momentum traders into our model. Momentum traders are assumed to trade an asset according to the past price path. More specifically, we assume that the aggregated order amount from momentum traders at period 2, q_2 say, is given by

$$q_2 = \phi(P_1 - P_0),$$

where P_t denotes the market price at period t. When $\phi > 0$, the price increase at the previous period leads to buy orders by momentum traders.[1] Therefore, a positive ϕ assumes the situation in which so-called trend-chasing traders actively trade in the market. Similarly, a negative ϕ means that there are a significantly many contrarian

[1] We follow the convention that a positive order amount means a net buy order, and vice versa.

traders who submit sell orders when the price has increased participate in the market.[2]

Our main results are the followings. First, momentum trading affects the market price only through the ratio ϕ/r, where r is the average of the absolute risk tolerance coefficient of all rational traders. Intuitively, the coefficient r represents how aggressively rational traders trade an asset. When r is large, rational traders trade actively based on their private information, and the effect of momentum traders on the market equilibrium is diminishing. On the other hand, when the magnitude of ϕ is large in comparison with the risk aversion of rational traders, the impact of momentum trading is big, and the market can be unstable and informationally inefficient.

Second, if ϕ/r is large enough, the price at the earlier period behaves pathologically. For example, when ϕ/r is large, the price falls when the liquidation value rises. In a usual situation, the market price should be increasing as the fundamental value of the asset takes a higher value. This is because the market price incorporates some private information owned by rational traders through their trades. This is not the case when there are many trend-chasing traders in the market.

Third, as ϕ diverges to positive or negative infinity, the price at period 1 converges to the price at period 0, the unconditional expectations. This means that, when momentum traders dominate the market, no matter whether they are trend-chasing or contrarian, the market price has little information about the private signals, i.e. it is informationally inefficient.

Fourth, the price at the earlier period is dependent on the precision of a public signal at the later period. More precisely, if the precision of public signal at period 2 is high, then rational traders trade based on their private signals, and the market price at period 1 becomes stable and informationally efficient, even though rational traders do not observe the realized public signal. This observation indicates that if market participants believe that future earning forecasts are reliable, the present price reflects enough private information available to rational traders. This finding has a very important economic implication. If financial authorities consider the stability of the market, one of effective measures is to assure the transparency and reliability of accounting information or other IR activities. If the public information in the future is reliable, rational traders trade actively based on their private signals before the announcement, and so the private information is properly incorporated into the market price.

This paper is organized as follows. Section 2 describes the model, and Section 3

[2] Hong and Stein (1999) consider the momentum traders' optimization problem with respect to ϕ. In our model, however, we set the value of ϕ exogenously and analyze how ϕ affects the market equilibrium.

provides main results. In Section 4, we discuss in detail about our results and their implications with numerical examples. Section 5 provides the conclusion of this paper. The detailed proof of Theorem 1 is given in Appendix A.

2 Model Setup

Our market model is based on Kim and Verrecchia (1991a). We consider a securities market where one risky asset is traded. Trades take place at $t = 1$ and 2. After period 2, the liquidation value is realized, all positions are cleared, and consumptions occur. The liquidation value, denoted by v, is normally distributed with mean P_0 and precision (reciprocal of the variance) ρ_{v_0}. The distribution of the liquidation value is known to all market participants. The price of the asset at period t is denoted by P_t.[3] In the market, there are two types of traders: rational traders and momentum traders.

The assumptions of rational traders are exactly the same as in Kim and Verrecchia (1991a). We assume a $[0, 1]$ continuum of rational traders indexed by $j \in [0, 1]$. We denote the position of rational trader j by x_{jt}. The initial position x_{j0} is exogenously and randomly given to rational trader j. As in Kim and Verrecchia (1991a), the aggregated endowment of the risky asset, denoted by

$$x := \int_{j\in[0,1]} x_{j0}\mathrm{d}j,$$

is not known to individual traders, and all traders believe that x is normally distributed with mean 0 and precision ρ_x, and independent of all random variables. The randomness of the risky asset supply captures the fact that securities markets are generally subject to random demand and supply fluctuations due to changing liquidity needs, weather, political situations, etc.

Before period 1, trader j observes his/her private signal on the liquidation value, s_j say, before the market opens. The signal s_j is a noisy unbiased signal, i.e.,

$$s_j = v + \epsilon_j,$$

where ϵ_j is normally distributed with mean 0 and precision ρ_{ϵ_j}, and independent of v.

In addition to the private signal, all traders observe a public announcement about the liquidation value at each period. We assume that the announcement is of the form

$$y_t = v + \eta_t, \quad t = 1, 2,$$

[3]We implicitly assume that the unconditional mean P_0 is the price before period 1, and that the price P_0 reflects all available information at period 0. This assumption is also adopted by De Long et al. (1990)

where η_t is normally distributed with mean 0 and precision ρ_{η_t}, and independent of all other random variables. The public signal can be thought of as a kind of earnings forecasts. After the announcement of the public signal, rational traders update their conditional distribution of the liquidation value v, and submit an order based on their updated belief.

Rational traders trade a risky asset to maximize their conditional expected utilities, a negative exponential function with risk tolerance coefficient r_j. More formally, the strategy of rational trader j is given by

$$\max \mathrm{E}\left[-\mathrm{e}^{-\frac{1}{r_j}[(P_2-P_1)x_{j1}+(v-P_2)x_{j2}]}\middle|\mathcal{I}_{jt}\right], \quad t=1,2,$$

where $\mathcal{I}_{jt}$ is all available information of rational trader j at period t. Note that each rational trader can utilize the value of P_t as a signal on the liquidation value v when they trade at period t.

Momentum traders trade at period 2 according to the price movement from period 0 to 1. The amount of the aggregated orders by momentum traders, denoted by q_2, is given by

$$q_2 = \phi(P_1 - P_0).$$

The parameter ϕ represents how much momentum traders respond to the past price movement. If $\phi > 0$, we assume that there are many trend-chasing traders in the market, while if $\phi < 0$, the order from contrarian traders dominates the momentum trading. The modeling of momentum trading is the same as Hong and Stein (1999) except that ϕ is exogenously given.

Finally, we define the equilibrium of this market model.

Definition 1 *In the equilibrium, the following conditions are satisfied:*

(i) Each rational trader maximizes his/her conditional expected utility given all available information.

(ii) The amount of total demand is equal to the supply of the asset:

$$x = \begin{cases} x_1 & \textit{for } t = 1 \\ x_2 + q_2 & \textit{for } t = 2. \end{cases} \tag{2.1}$$

Definition 1 is standard and adopted by most of noisy rational expectation models such as Admati (1985), Kim and Verrecchia (1991a). The only difference is q_2 at period 2, the amount of momentum trading.

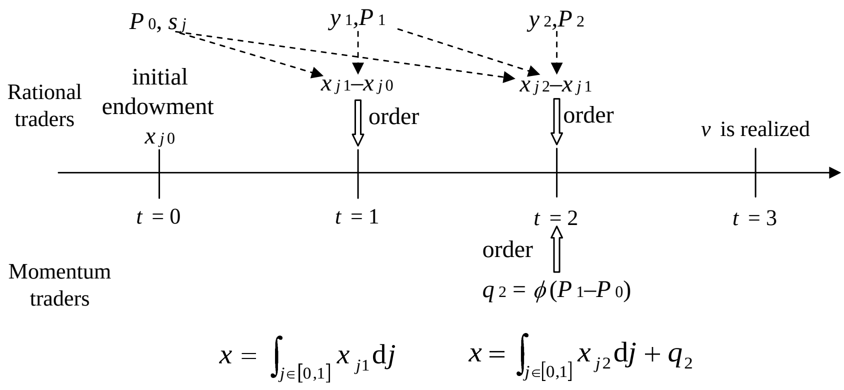

Figure 1: Time line of the model. The dotted arrows indicate the available information to rational traders. Note that the price at each period has some signal on the liquidation value.

Figure 1 concisely illustrates the time line of the model. Note again that the price at each period has some signal on the liquidation value. This is because through orders by rational traders, the privates signals owned by rational traders is partially incorporated.

3 Results

In this section, we derive the equilibrium price formulae, and analyze the properties of the market. The procedure to derive the equilibrium prices is similar to most of noisy rational expectations models, e.g., Kim and Verrecchia (1991a).

Theorem 1 *Define*

$$r := \int_j r_j \mathrm{d}j, \quad \rho_\epsilon := \frac{1}{r} \int_j r_j \rho_{\epsilon_j} \mathrm{d}j,$$

$$\rho_{v_1} := \rho_{v_0} + \rho_{\eta_1} + \rho_\epsilon + (r\rho_\epsilon)^2 \rho_x, \quad \rho_{v_2} := \rho_{v_1} + \rho_{\eta_2}$$

and

$$\kappa := \frac{\phi}{r\rho_{v_2}}.$$

(i) Suppose $\kappa \neq 1$. Then, the equilibrium price at period 1 is given by

$$P_1 = \frac{\rho_{v_0} - \kappa\rho_{v_1}}{(1-\kappa)\rho_{v_1}} P_0 + \frac{\rho_{\eta_1}}{(1-\kappa)\rho_{v_1}} y_1 + \frac{\rho_\epsilon(1+r^2\rho_\epsilon\rho_x)}{(1-\kappa)\rho_{v_1}} v - \frac{1+r^2\rho_\epsilon\rho_x}{(1-\kappa)r\rho_{v_1}} x, \tag{3.1}$$

and the equilibrium price at period 2 is given by

$$\begin{aligned} P_2 = &\left[\frac{\rho_{v_0}}{\rho_{v_2}} - \frac{\kappa\rho_{\eta_1}}{(1-\kappa)\rho_{v_1}}\right] P_0 + \frac{\rho_\epsilon(1+r^2\rho_\epsilon\rho_x)}{\rho_{v_2}} \frac{\rho_{v_1}+\kappa\rho_{\eta_2}}{(1-\kappa)\rho_{v_1}} v \\ &+ \frac{\rho_{\eta_1}}{\rho_{v_2}} \frac{\rho_{v_1}+\kappa\rho_{\eta_2}}{(1-\kappa)\rho_{v_1}} y_1 + \frac{\rho_{\eta_2}}{\rho_{v_2}} y_2 - \frac{1+r^2\rho_\epsilon\rho_x}{r\rho_{v_2}} \frac{\rho_{v_1}+\kappa\rho_{\eta_2}}{(1-\kappa)\rho_{v_1}} x. \end{aligned} \tag{3.2}$$

When $\kappa = 1$, no equilibrium exists.

(ii) Let

$$\theta := \frac{(1-\kappa)\rho_{v_1}}{\rho_\epsilon(1+r^2\rho_\epsilon\rho_x)} P_1 - \frac{\rho_{v_0} - \kappa\rho_{v_1}}{\rho_\epsilon(1+r^2\rho_\epsilon\rho_x)} P_0 - \frac{\rho_{\eta_1}}{\rho_\epsilon(1+r^2\rho_\epsilon\rho_x)} y_1.$$

The demand functions of rational trader j at periods 1 and 2 are given by

$$\begin{aligned} x_{j1}^* = r_j \Bigg[&\frac{\rho_{v_2}}{\rho_{\eta_2}}(\rho_{v_0} - \kappa\rho_{v_1})P_0 + \rho_{\epsilon_j} s_j + \left(\frac{\rho_{v_2}}{\rho_{\eta_2}}(\rho_\epsilon + r^2\rho_\epsilon^2\rho_x) - \rho_\epsilon\right)\theta \\ &- \left(\frac{\rho_{v_2}}{\rho_{\eta_2}}(1-\kappa)\rho_{v_1} - (\rho_{\epsilon_j} - \rho_\epsilon)\right) P_1 \Bigg] \end{aligned} \tag{3.3}$$

and

$$x_{j2}^* = r_j \left[\rho_{v_0} P_0 + \rho_{\epsilon_j} s_j + \rho_{\eta_1} y_1 + \rho_{\eta_2} y_2 + r^2\rho_\epsilon\rho_x\theta - \rho_{j2} P_2\right], \tag{3.4}$$

respectively. Here, $\rho_{j2} := \rho_{v_0} + \rho_{\epsilon_j} + \rho_{\eta_1} + \rho_{\eta_2} + r^2\rho_\epsilon\rho_x$.

Note from (3.1) and (3.2) that momentum trades only affect the equilibrium prices through the term $\kappa := \phi/r\rho_{v_2}$. Equation (3.2) can be rewritten as

$$P_2 = \left(\frac{\rho_{v_0}}{\rho_{v_2}} - \kappa\right) P_0 + \left(\frac{\rho_\eta}{\rho_{v_2}} - \kappa\right) y_1 + \frac{\rho_\eta}{\rho_{v_2}} y_2 + \frac{\rho_\epsilon(1+r^2\rho_\epsilon\rho_x)}{\rho_{v_2}} v - \frac{1+r^2\rho_\epsilon\rho_x}{r\rho_{v_2}} x + \kappa P_1. \tag{3.5}$$

See Appendix A for details. The last term of (3.5) indicates that κ is the sensitivity of P_2 with respect to P_1. Therefore, κ is a parameter that connects the prices of the two periods through momentum trading.

When $\kappa = 1$, the equilibrium does not exist. To see this, the demand function of all rational traders is given by

$$x_1^* = r\left[\frac{\rho_{v_0}\rho_{v_2}(1-\kappa\rho_{v_1})}{\rho_{\eta_2}} P_0 + \frac{\rho_{\eta_1}\rho_{v_2}}{\rho_{\eta_2}} y_1 + \frac{\rho_{v_1}\rho_\epsilon + \rho_{v_2}\rho_\theta}{\rho_{\eta_2}}\theta + \rho_\epsilon s_j - \frac{\rho_{v_1}}{r\rho_{\eta_2}} x - \frac{\rho_{v_1}\rho_{v_2}(1-\kappa)}{\rho_{\eta_2}} P_1\right]. \tag{3.6}$$

where $\rho_\theta := r^2\rho_\epsilon^2\rho_x$. From the last term in (3.6), we observe that the demand of rational traders is independent of P_1 if $\kappa = 1$. Therefore, the price at period 1 becomes indeterminate, and no equilibrium exists.

Next, we take a closer look at the proprieties of the market equilibrium.

Proposition 1 *(i) The price change at period 2 is expressed as*

$$P_2 - P_1 = \frac{\rho_{\eta 2}}{\rho_{v_2}}\left[y_2 - \int_{j\in[0,1]} \mathrm{E}[v|\mathcal{I}_{j1}]\mathrm{d}j + \frac{x}{r\rho_{v_1}}\right]. \tag{3.7}$$

(ii) The trading amount of trader j at period 2 is expressed as

$$x_{j2} - x_{j1} = -r_j(\rho_{\epsilon_j} - \rho_\epsilon)(P_2 - P_1) - \frac{r_j}{r}q_2 \tag{3.8}$$

The first part of Proposition 1 is exactly the same as the result obtained in Kim and Verrecchia (1991a). The price reaction to a public announcement is proportional to the importance of the announced information relative to the average posterior beliefs of rational traders and the surprise contained in the announced information plus noise. That is

$$P_2 - P_1 = \text{Surprise}+\text{Noise},$$

where

$$\text{Surprise} := \frac{\rho_{\eta 2}}{\rho_{v_2}}\left(y_2 - \int_{j\in[0,1]} \mathrm{E}[v|\mathcal{I}_{j1}]\mathrm{d}j\right)$$

and

$$\text{Noise} := \frac{\rho_{\eta 2}}{\rho_{v_2}}\frac{x}{r\rho_{v_1}}.$$

Note that the price movement does not include the effect of momentum trading. This finding indicates that the price at period 1 fully reflects the impact of momentum trading, although momentum trading only appears in period 2. Even if market participants believe that there are momentum trading only in the future, the price effectively contains the impact before momentum traders arrive at the market.

The second statement in Proposition 1 says that the trading volume at period 2 is decomposed into three parts: surprise term, noise effect, and momentum term. That is

$$x_{j2} - x_{j1} = -(\text{Surprise}+\text{Noise})+\text{Momentum},$$

where

$$\text{Momentum} := \frac{r_j}{r} q_2.$$

This decomposition implies that there are two kinds of motivations for each rational trader to trade the asset. The first one is the information effect appeared in first term of (3.8). Each rational trader updates their belief based on the information available to him/her. If the private information of rational trader j is not accurate in comparison with those of other traders, i.e. ρ_{ϵ_j} is smaller than ρ_ϵ, he/she updates their belief, and buys the asset if the price moves upward. If $\rho_{\epsilon_j} > \rho_\epsilon$, trader j will think that his/her own private signal is reliable, and that the price process does not contain much information on the liquidation value. In this case, trader j buys when the price moves downward, and vice versa.

The second one is the momentum trading effect. In this setting, rational traders know the existence of momentum traders in advance. Hence, rational traders take the effect of momentum trading at period 2 into account when they trade. When $\phi > 0$ and $P_1 > P_0$, rational traders know that the demand-supply balance at period 2 becomes tight because of momentum traders. Then, the price is likely to go up regardless of the liquidation value. Therefore, rational traders can earn a profit by buying at period 1 and selling at period 2. The last term of (3.8) explains this effect.

Next, we analyze the effect of momentum trading on the price at period 1. The following proposition can easily be obtained by taking partial derivatives.

Proposition 2 *When $\kappa > 1$, we have*

$$\frac{\partial P_1}{\partial v} < 0, \quad \frac{\partial P_1}{\partial y_1} < 0, \quad \frac{\partial P_1}{\partial x} > 0. \tag{3.9}$$

Proposition 2 implies that if ϕ is large enough, the price at the earlier period behaves pathologically. For example, the first inequality of (3.9) indicates that when a higher value of v is realized, the price at period 1 becomes smaller. This contradicts a natural intuition that the price partially reflects private information owned by rational traders. The reason of this behavior is that if trend-chasing traders dominate the market, then rational traders know that momentum traders sell at period 2 when P_1 falls, and they manipulate the market and earn profits at the expense of momentum traders.

The second inequality of (3.9) says that when market participants believe that the liquidation value is expected to be higher, the market price becomes lower in the case of positive momentum trading. This means that in a market with trend-chasing momentum traders, when there is a public belief that the business performance is likely to be good ex ante, the price decrease in spite of the good news. In most of noisy rational expectations modes with multiple periods, the price gradually converges from the initial price to the liquidation value. In the case of positive momentum trading, the gradual convergence of the price does not happen. In a market with many trend-chasers, the effect of the public signal is the opposite to the usual one.

The third inequality of (3.9) is also contrary to the ordinary situation. A high value of x means a large supply of the risky asset. Then, the balance of demand-supply becomes loose, and the market price falls in a normal setting. When trend-chasers actively trade in the market, a large supply of the asset may lead to a higher price at the earlier period.

The next proposition shows that positive momentum trading also yields a peculiar price formulation at period 2.

Proposition 3 *For $\kappa < 1$, we have*

$$\frac{\partial P_2}{\partial v} \leq 0, \quad \frac{\partial P_2}{\partial y_1} < 0, \quad \frac{\partial P_2}{\partial x} \leq 0. \tag{3.10}$$

When positive momentum traders trade intensively, the market price at period 2 also behaves pathologically. This is because if momentum traders dominate the market, rational traders can guess the price at period 2 by observing the price at 1, and make a bigger profit by taking the effect of momentum trading into account and not utilizing the private signals. Therefore, the prices do not contain the private information and the market becomes unstable. Proposition 3 implies that the dominance of positive momentum traders makes the market unusual and unstable.

Next, we investigate the impact of momentum trading on the uncertainty of the price movement.

Proposition 4 *The variances of the price movement are given by*

$$\mathrm{Var}[P_1 - P_0] = \left(\frac{\rho_\epsilon (1 + r^2 \rho_\epsilon \rho_x)}{\rho_{v_1} (1 - \frac{\phi}{r\rho_{v_2}})} \right)^2 \frac{1}{\rho_{v_0}} + \left(\frac{1 + r^2 \rho_\epsilon \rho_x}{r\rho_{v_1} (1 - \frac{\phi}{r\rho_{v_2}})} \right)^2 \frac{1}{\rho_x}$$

and

$$\mathrm{Var}[P_2 - P_1] = \left(\frac{\rho_\eta \rho_{v_0}}{\rho_{v_1} \rho_{v_2}} \right)^2 \frac{1}{\rho_{v_0}} + \frac{\rho_\eta}{\rho_{v_2}^2} + \left(\frac{\rho_\eta}{\rho_{v_2}} \right)^2 \frac{1}{\rho_x}.$$

In particular, when $\phi \to \pm\infty$, we have $\mathrm{Var}[P_1 - P_0] \to 0$ and $P_1 \to P_0$.

Proposition 4 indicates that momentum trading affects the uncertainty of the price movement at the earlier period, not at the later period. This observation is consistent with Proposition 1. The price at period 1 fully reflects the impact of momentum trading, although momentum traders trade only in period 2. In other words, the market price at period 1 incorporates all available information, including the structure of momentum trading at period 2. This is because all rational traders take the existence of momentum traders into account and fully exploit their irrational strategy at period 1.

When $\phi \to \pm\infty$, the price at period 1 converges to the unconditional expectation, which means that price has no new information on the liquidation value. When momentum trader dominates the market, the price becomes less informative. In the case of $\phi \to \pm\infty$, all rational traders trade based on the motivation of momentum trading effect, and do not trade according to their private signal. Equation (3.8), the trading volume at period 2, confirms this finding.

Finally, we look at the informational efficiencies of the market price. We define the informational efficiency at period t to be $\mathrm{E}[(v - P_t)^2]$.[4]

Proposition 5 *The variances of the difference between the liquidation value and the market price are given by*

$$\mathrm{E}[(v - P_1)^2] = \left[\frac{\kappa \rho_{v_1}}{\rho_{v_1}(1-\kappa)}\right]^2 \frac{1}{\rho_{v0}} + \left[\frac{1 + r^2 \rho_\epsilon \rho_x}{r \rho_{v_1}(1-\kappa)}\right]^2 \frac{1}{\rho_x}$$

and

$$\begin{aligned}\mathrm{E}[(v - P_2)^2] = {} & \left(\frac{\kappa}{1-\kappa}\right)^2 \frac{1}{\rho_{v0}} + \left(\frac{\rho_{v0}}{\rho_{v_2}}\right)^2 P_0^2 + \left(\frac{(1 + r^2 \rho_\epsilon \rho_x)}{r \rho_{v_2}} \frac{\rho_{v_1} + \kappa \rho_{\eta 2}}{\rho_{v_1}(1-\kappa)}\right)^2 \frac{1}{\rho_x} \\ & + \left(\frac{\rho_{\eta 1}}{\rho_{v_2}} \frac{\rho_{v_1} + \kappa \rho_{\eta 2}}{\rho_{v_1}(1-\kappa)}\right)^2 \frac{1}{\rho_{\eta 1}} + \left(\frac{\rho_{\eta 2}}{\rho_{v_2}}\right)^2 \frac{1}{\rho_{\eta 2}}.\end{aligned}$$

Proposition 5 says that if momentum traders dominate the market, i.e., $|\kappa|$ is large enough, the price at period 1 have little private information observed by each rational trader. This observation reconfirms the results obtained in Propositions 1 and 4.

Here, we summarize the results obtained in the above analysis:

- Rational traders have two trade motivations. One is from the informational effect, and the other is from the effect of momentum trading.

[4] In a Kyle-type model, the informational efficiency is define by $\mathrm{Var}[v|\mathcal{I}_t]$, where $\mathcal{I}_t$ denotes the public information at period t. If the price satisfies $P_t = \mathrm{E}[v|\mathcal{I}_t]$, then

$$\mathrm{E}[(v - P_t)^2] = \mathrm{E}[(v - \mathrm{E}[v|\mathcal{I}_t])^2] = \mathrm{E}[\mathrm{E}[(v - \mathrm{E}[v|\mathcal{I}_t])^2|\mathcal{I}_t]] = \mathrm{Var}[v|\mathcal{I}_t],$$

where we have used in the last equation the fact that the conditional variance is independent of the realization of v. Hence, our definition is consistent with other studies.

- Even if momentum trading is only at period 2, the price fully reflect the impact of momentum trading at period 1.
- When trend-chasing momentum traders dominate in the market, the price behaves pathologically at both periods.
- When the impact of momentum trading is large, the price at period 1 reflect less private information observed by rational traders.

4 Discussion

In Propositions 1 through 5, we observe that the momentum trading affects the market equilibrium only through the ratio $\phi/r\rho_{v_2}$. This implies that when we consider the influence of momentum trading on the market, we should take into account not only the magnitude of momentum trading, but also rational traders' risk tolerance and the precision of information. As r is large enough relative to ϕ, rational traders trade aggressively based on their private information, and the impact of momentum traders decreasing. This finding is of great importance. When you belong to the financial authority and are in charge of the market design, you must consider the momentum trading in comparison with the rational traders' aggressiveness for the effective and stable market.

In a market with large $\phi > 0$ and small r, the market bubble may happen. For (3.1), when ϕ is large and r is small, rational traders trade so as to maximize his global expected profit through momentum trading as we discussed above. This can be understood that there is a kind of market manipulation in the market. the prices at both periods 1 and 2 are camouflaged by momentum trading, and do not reflect the private information.

Here, we provide some numerical examples. The following two examples confirm the fact that the market price is definitely dependent on the ratio of ϕ and $r\rho_{v_2}$. The common parameters and variables are presented in Table 1.

parameter	r	ρ_{v_0}	ρ_ϵ	ρ_x	ρ_η	P_0	$P_3 = v$	x	y_1	y_2
value	1	1	1	1	1	1	-1	0	-1	-1

Table 1: Base case parameters

Note that P_0, the initial price, is 1 and the liquidation value is -1. Moreover, the public signal at both periods are equal to the true liquidation value. Hence, the price should be decreasing gradually if the price reflects the information privately owned by rational traders.

Example 1 In Example 1, we set $\phi = 0$. Under this parameter setting, there is no momentum trading. Figure 2 depicts the price path from period 0 to period 3.

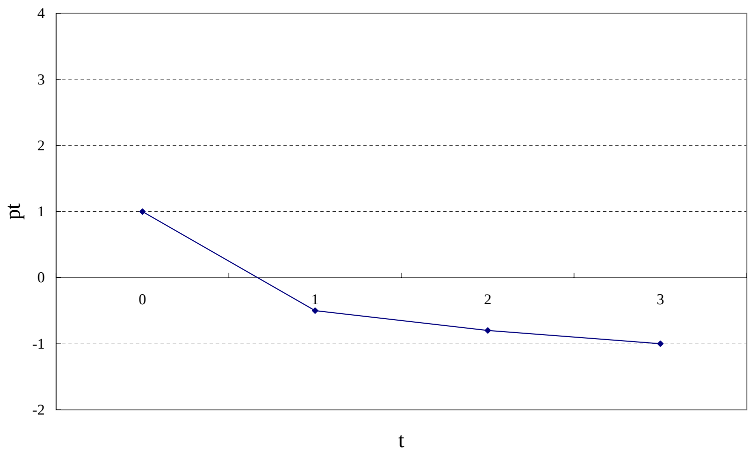

Figure 2: Price path when there is no momentum trading. n this case, the price are gradually decreasing because private information is incorporated into price through the trades by rational traders.

We observe from Figure 2 that if there is no momentum trading, the market price is gradually decreasing over time as we expected. This is because public and private information observed by rational traders is gradually incorporated into the price through their trades.

Example 2 In Example 2, we set $\phi = 8$. The risk tolerance of rational trader is the same as in Example 1, but there is a prominent momentum trading in this case.

In Example 2, the price at period 1 rises sharply, and the price at period 2 is still higher than the initial price P_0. Finally the liquidation value reveals, and the price sharply falls. This result indicates that if trend-chasing strategy is popular in the market, it causes a bubble market and makes the market unstable. We know that rational traders' profit opportunity lies in the two resources: the public and private signals on the liquidation value, and the information of momentum traders' strategy. When r is small or ϕ is large, rational traders think more of the behavior of momentum traders than their private signals, and manipulate the market in order to induce the opposite trades by momentum

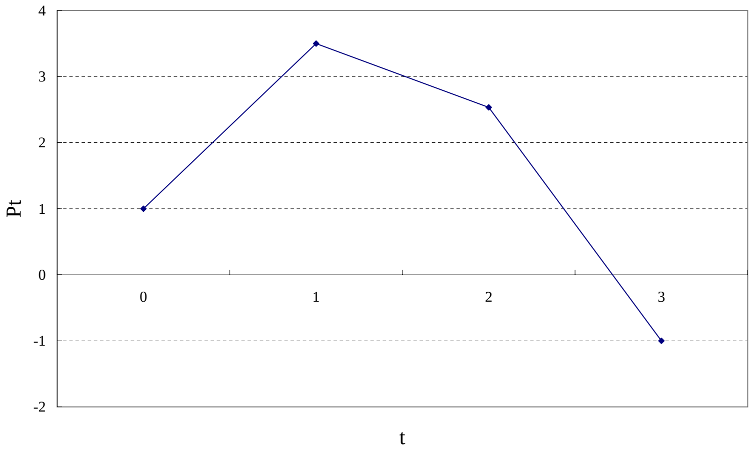

Figure 3: Price path when there is momentum trading. In this case, the price fluctuates around P_0 at period 1 and 2, and sharply falls at period 3. This observation indicates that the price process does not reflect private signals observed by rational traders. Rational traders try to earn profit by utilizing momentum trades.

traders in the later period. Rational traders do not trade based on their private signal, and so the market price becomes unstable. Finally, after the market is closed, the liquidation value reveals, and all traders learn that the price does not reflect the private information.

We observe from the numerical examples that the dominance of momentum investors is the main cause of bubble economy. De Long et al. (1990) also obtained a similar result. However, they do not provide its countermeasures. Do we have a way to avert such bubble?

Note that in Propositions 2 and 3, what determines the occurrence of bubble is $\phi/r\rho_{v_2}$. We easily see that if ρ_{v_2} takes a higher value, the conditions of bubbles given in Propositions 2 and 3 does not hold, and we expect that the price behaves normally.

Note also from (3.2) that $\frac{\partial P_2}{\partial y_2}$ is always positive irrespective of the parameter setting. We propose that one way to solve the problem is to make the public information reliable, i.e., take a large ρ_{η_2}. This means that if we know that the future public announcement such as an earning forecast is precise enough, then prices at both periods behave appropriately even when ϕ is relatively large.

In the same setting of Example 2, if we set $\rho_{\eta_2} = 30$, the behavior of the price path becomes normal and the market price reflects both public and private information.[5] Figure

[5]However, when ρ_{η_2} is set so that ϕ is around $r\rho_{v_2}$, the demand of rational traders is insensitive to P_1 as we saw in the previous section. Therefore, halfway large ρ_η can destabilize the market equilibrium.

4 illustrates that the price path reflects private information owned by rational traders. The prices at period 1 and 2 are at around v, the liquidation value. This means that when market participants observe the reliable public announcements at period 2, the market price at period 1 properly reflects the private information owned by rational traders.

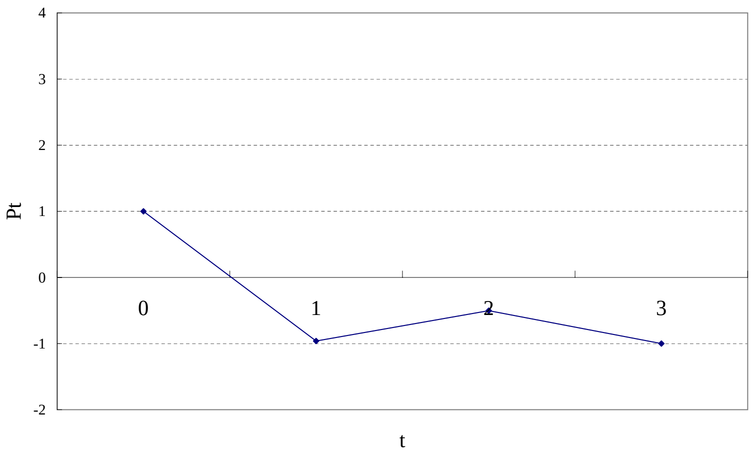

Figure 4: Price path when there is momentum trading ($\phi = 8$), but $\rho_\eta = 30$. The price almost fully reflect private information at period 1, and no bubble occurs in the market. When public announcements are reliable, rational traders trade based on their private signal rather than utilizing momentum trading. Thus, the price behaves naturally.

The precision of the public information affects the trading strategies of rational traders at all periods, through which the price reflects private information. This means that the reliability of public announcements not only affects directly the realized price, but also reflects private signals through trades by rational traders. Thanks to high precision of the information on the liquidation value, rational traders put more emphasis on the information on the liquidation value than the utilization of the effect of momentum trading, and so the pressure of market manipulation is decreasing.

This finding has a important implication that the market transparency is of great importance when we consider the fair and stable market. Financial authorities which are in charge of market design should take into account of the fact that information precision at later period affects the price at the earlier period.

5 Conclusion

This paper studies a market where there are momentum traders who trades according to the past price movement, and analyzed how momentum trading affects the market equilibrium. We showed that the orders from momentum traders have an impact on the market price in relation to the rational traders' risk tolerance.

When there are dominantly many momentum traders in the market, whether trend-chasing or contrarian, the market price becomes informationally inefficient. If there are so many trend-chasing traders in the market, the price behaves pathologically and the market bubble may occur. Our results imply that market participants or financial authorities should pay attention not only to momentum trading, but also to rational traders' trading behavior when they analyze the market. One of the ways to avoid the market instability is to improve the informational transparency.

A Proof of Theorem 1

In Appendix A, we provide the detailed proof of Theorem 1. The procedure is similar to Kim and Verrecchia (1991a).

First, we conjecture that P_1 and P_2 are linear functions of random variables. That is, suppose that

$$P_1 = a_1 P_0 + b_1 v + c_1 x + d_1 y_1 \tag{1.1}$$

and

$$P_2 = a_2 P_0 + b_2 v + c_2 x + d_{21} y_1 + d_{22} y_2 + e_2 q, \tag{1.2}$$

where all coefficients are constants. Define

$$\theta_1 := v + \frac{c_1}{b_1} x = \frac{1}{b_1}(P_1 - a_1 P_0 - d_1 y_1) \tag{1.1'}$$

and

$$\theta_2 := v + \frac{c_2}{b_2} x = \frac{1}{b_2}(P_1 - a_2 P_0 - d_{21} y_1 - d_{22} y_2 - e_2 q). \tag{1.2'}$$

The parameter θ_t is $\mathcal{I}_{jt}$-measurable at period t because all variables in the right-hand side, including the market price P_t, are measurable when rational traders trade in the market. If $c_1/b_1 \neq c_2/b_2$, then the liquidation value v fully reveals at period 2 by solving the simultaneous equations system of (1.1′) and (1.2′). In this paper, we only consider a partially revealing equilibrium in which v does not fully reveal at period 2. Hence, we also conjecture that $c_1/b_1 = c_2/b_2$, and that rational traders cannot know the true value of v. In this case, when $c_1/b_1 = c_2/b_2$, $\theta_1 = \theta_2$ holds. Let $\theta := \theta_1 = \theta_2$.

Now, we solve the optimization problem of rational trader j and derive the market prices backwardly.

Equilibrium at $t = 2$. Note that $\mathcal{I}_{j2}$, the information available to trader j at period 2, is the σ-field generated by s_j, y_1, y_2, P_1 and P_2. In our setting, observing P_1 and P_2 with y_1 and y_2 is equivalent to observing θ. Hence, we can rewrite

$$\mathcal{I}_{j2} = \sigma(s_j, y_1, y_2, \theta).$$

Also note from (1.1′) and (1.2′) that θ is a noisy unbiased signal of v with noise term $\frac{c_2}{b_2}x$. Then, we obtain from the projection theorem of normal distributions that

$$\mathrm{E}[v|\mathcal{I}_{j2}] = \frac{\rho_{v_0}P_0 + \rho_{\epsilon_j}s_j + \rho_{\eta_1}y_1 + \rho_{\eta_2}y_2 + \rho_\theta\theta}{\rho_{v_0} + \rho_{\epsilon_j} + \rho_{\eta_1} + \rho_{\eta_2} + \rho_\theta} \tag{1.3}$$

and

$$\mathrm{Var}[v|\mathcal{I}_{j2}] = \frac{1}{\rho_{v_0} + \rho_{\epsilon_j} + \rho_{\eta_1} + \rho_{\eta_2} + \rho_\theta} =: \frac{1}{\rho_{j2}}, \tag{1.4}$$

where $\rho_\theta = \left(\frac{b_2}{c_2}\right)^2 \rho_x$, the precision of $\frac{c_2}{b_2}x$.

From (1.3), (1.4), and the moment generating function of normal distributions, the demand function of rational trader j at period 2 is calculated as

$$\begin{aligned} x^*_{j2} :=& \frac{r_j}{\mathrm{Var}[v|\mathcal{I}_{j2}]}(\mathrm{E}[v|\mathcal{I}_{j2}] - P_2) \\ =& r_j(\rho_{v_0}P_0 + \rho_{\epsilon_j}s_j + \rho_{\eta_1}y_1 + \rho_{\eta_2}y_2 + \rho_\theta\theta - \rho_{j2}P_2). \end{aligned}$$

Hence, by the demand-supply equality (2.1), we obtain

$$\begin{aligned} x &= \int_0^1 r_j(\rho_{v_0}P_0 + \rho_{\epsilon_j}s_j + \rho_{\eta_1}y_1 + \rho_{\eta_2}y_2 + \rho_\theta\theta - \rho_{j2}P_2)\mathrm{d}j + q \\ &= r\rho_{v_0}P_0 + r\rho_\epsilon v + r\rho_{\eta_1}y_1 + r\rho_{\eta_2}y_2 + r\rho_\theta\theta - r\rho_{v_2}P_2 + q, \end{aligned} \tag{1.5}$$

where we have used $\int_j r_j\rho_{\epsilon_j}\epsilon_i\mathrm{d}j = 0$ by the law of large numbers as in Admati (1985). Solving (1.5) with respect to P_2 and substituting $\theta = v + \frac{c_2}{b_2}x$ into (1.5) leads to

$$\begin{aligned} P_2 =& \frac{1}{\rho_{v_2}}\left(\rho_{v_0}P_0 + \rho_\epsilon v + \rho_{\eta_1}y_1 + \rho_{\eta_2}y_2 + \rho_\theta\theta - \frac{1}{r}x + \frac{1}{r}q\right) \\ =& \frac{\rho_{v_0}}{\rho_{v_2}}P_0 + \frac{\rho_\epsilon + \rho_\theta}{\rho_{v_2}}v + \frac{r\rho_\theta\frac{c_2}{b_2} - 1}{r\rho_{v_2}}x + \frac{\rho_{\eta_1}}{\rho_{v_2}}y_1 + \frac{\rho_{\eta_1}}{\rho_{v_2}}y_2 + \frac{1}{r\rho_{v_2}}q. \end{aligned} \tag{1.6}$$

Comparing (1.6) with (1.2), we have

$$a_2 = \frac{\rho_{v_0}}{\rho_{v_2}}, \quad b_2 = \frac{\rho_\epsilon + \rho_\theta}{\rho_{v_2}}, \quad c_2 = \frac{r\rho_\theta \frac{c_2}{b_2} - 1}{r\rho_{v_2}},$$
$$d_{21} = \frac{\rho_{\eta_1}}{\rho_{v_2}}, \quad d_{22} = \frac{\rho_{\eta_2}}{\rho_{v_2}}, \quad e_2 = \frac{1}{r\rho_{v_2}}.$$

The relation $\frac{c_2}{b_2} = \frac{r\rho_\theta \frac{c_2}{b_2} - 1}{r(\rho_\epsilon + \rho_\theta)}$ implies $\frac{b_2}{c_2} = -r\rho_\epsilon$. Then, we get $\rho_\theta = \left(\frac{b_2}{c_2}\right)^2 \rho_x = r^2\rho_\epsilon^2\rho_x$. Substituting the coefficients and $q = \phi(P_1 - P_0)$ into (1.2), we finally have the price formula at period t as

$$\begin{aligned} P_2 =& \frac{\rho_{v_0}}{\rho_{v_2}} P_0 + \frac{\rho_\epsilon(1 + r^2\rho_\epsilon\rho_x)}{\rho_{v_2}} v - \frac{1 + r^2\rho_\epsilon\rho_x}{r\rho_{v_2}} x + \frac{\rho_{\eta_1}}{\rho_{v_2}} y_1 + \frac{\rho_{\eta_2}}{\rho_{v_2}} y_2 + \frac{1}{r\rho_{v_2}} q \\ =& \left(\frac{\rho_{v_0}}{\rho_{v_2}} - \kappa\right) P_0 + \frac{\rho_\epsilon(1 + r^2\rho_\epsilon\rho_x)}{\rho_{v_2}} v - \frac{1 + r^2\rho_\epsilon\rho_x}{r\rho_{v_2}} x + \frac{\rho_{\eta_1}}{\rho_{v_2}} y_1 + \frac{\rho_{\eta_2}}{\rho_{v_2}} y_2 + \kappa P_1. \end{aligned} \tag{1.7}$$

Equilibrium at $t = 1$. Using the law of iterated expectations, the objective function at period 1 becomes

$$\begin{aligned} & \mathrm{E}\left[\mathrm{e}^{-\frac{1}{r_j}[(P_2 - P_1)x_{j1} + (v - P_2)x_{j2}^*]} \middle| \mathcal{I}_{j1} \right] \\ =& \mathrm{E}\left[\mathrm{E}\left[\mathrm{e}^{-\frac{1}{r_j}[(P_2 - P_1)x_{j1} + (v - P_2)x_{j2}^*]} \middle| \mathcal{I}_{j2} \right] \middle| \mathcal{I}_{j1} \right] \\ =& \mathrm{E}\left[\mathrm{e}^{-\frac{1}{r_j}(P_2 - P_1)x_{j1} - \frac{(E[v|\mathcal{I}_{j2}] - P_2)^2}{2\mathrm{Var}[v|\mathcal{I}_{j2}]}} \middle| \mathcal{I}_{j1} \right]. \end{aligned} \tag{1.8}$$

Rearranging (1.6) and noting $\theta = v - \frac{1}{r\rho_\epsilon}x$, we get

$$\begin{aligned} P_2 &= \frac{1}{\rho_{v_2}} \left[\rho_{v_0} P_0 + \rho_\epsilon \left(v - \frac{1}{r\rho_\epsilon} x \right) + \rho_\theta\theta + \rho_{\eta_1} y_1 + \rho_{\eta_2} y_2 + \frac{1}{r} q \right] \\ &= \frac{1}{\rho_{v_2}} \left(\rho_{v_0} P_0 + (\rho_\epsilon + \rho_\theta)\theta + \rho_{\eta_1} y_1 + \rho_{\eta_2} y_2 + \frac{1}{r} q \right). \end{aligned} \tag{1.9}$$

From (1.3) and (1.9), we obtain

$$\begin{aligned} \mathrm{E}[v|\mathcal{I}_{j2}] - P_2 =& \frac{\rho_{v_0} P_0 + \rho_{\epsilon_j} s_j + \rho_{\eta_1} y_1 + \rho_{\eta_2} y_2 + \rho_\theta\theta}{\rho_{j2}} \\ & - \frac{\rho_{v_0} P_0 + (\rho_\epsilon + \rho_\theta)\theta + \rho_{\eta_1} y_1 + \rho_{\eta_2} y_2 + \frac{1}{r} q}{\rho_{v_2}} \\ =& \frac{1}{\rho_{j2}} \left[-(\rho_{\epsilon_j} - \rho_\epsilon) P_2 + \rho_{\epsilon_j} s_j - \rho_\epsilon\theta - \frac{1}{r} q \right]. \end{aligned}$$

Using this relation, the objective function (1.8) is expressed as

$$\mathrm{E}\left[\mathrm{e}^{-\frac{1}{r_j}(P_2 - P_1)x_{j1} - \frac{\left[-(\rho_{\epsilon_j} - \rho_\epsilon)P_2 + \rho_{\epsilon_j} s_j - \rho_\epsilon\theta - \frac{1}{r}q\right]^2}{2\rho_{j2}}} \middle| \mathcal{I}_{j1} \right]. \tag{1.10}$$

The following lemma is useful for the calculation of (1.10).

Lemma 1 *Let $X \sim \mathcal{N}(\mu, \sigma^2)$. Then*

$$\mathrm{E}\left[\mathrm{e}^{-aX^2+bX}\right] = \frac{\mathrm{e}^{\frac{b^2\sigma^2-2a\mu^2+2b\mu}{2(1+2a\sigma^2)}}}{\sqrt{1+2a\sigma^2}}$$

for $a > 0$.

In (1.10), P_2 is the only random variable, and follows a normal conditional on $\mathcal{I}_{j1}$ under the assumption of linearity (1.7). Therefore, from Lemma 1, we observe that (1.10) is proportional to

$$-\exp\left\{\frac{P_1}{r_j}x_{j1} + \frac{\left(\frac{(\rho_{\epsilon_j}-\rho_\epsilon)\left(\rho_{\epsilon_j}s_j-\rho_\epsilon\theta-\frac{1}{r}q\right)}{\rho_{j2}} - \frac{x_{j1}}{r_j}\right)\mathrm{E}_{i1}[P_2]}{2\left(1+\frac{(\rho_{\epsilon_j}-\rho_\epsilon)^2}{\rho_{j2}}\mathrm{Var}_{i1}[P_2]\right)} \right.$$
$$\left. + \frac{\left\{\frac{(\rho_{\epsilon_j}-\rho_\epsilon)\left(\rho_{\epsilon_j}s_j-\rho_\epsilon\theta-\frac{1}{r}q\right)}{\rho_{j2}} - \frac{x_{j1}}{r_j}\right\}^2\mathrm{Var}_{i1}[P_2]}{2\left(1+\frac{(\rho_{\epsilon_j}-\rho_\epsilon)^2}{\rho_{j2}}\mathrm{Var}_{i1}[P_2]\right)}\right\}.$$

We obtain the first-order condition as

$$\begin{aligned}
x_{j1} =& r_j\left[\frac{1}{\mathrm{Var}[P_2|\mathcal{I}_{i1}]}\mathrm{E}[P_1] - \frac{1+\frac{(\rho_{\epsilon_j}-\rho_\epsilon)^2}{\rho_{j2}}\mathrm{Var}_{i1}[P_2]}{\mathrm{Var}_{i1}[P_2]}P_1 + \frac{(\rho_{\epsilon_j}-\rho_\epsilon)\left(\rho_{\epsilon_j}s_j-\rho_\epsilon\theta-\frac{1}{r}q\right)}{\rho_{j2}}\right] \\
=& r_j\left[\frac{\rho_{v_2}\rho_{v_0}}{\rho_{\eta_2}}P_0 + \frac{\rho_{v_2}\rho_{\eta_1}}{\rho_{\eta_2}}y_1 + \rho_{\epsilon_j}s_j + \left(\frac{\rho_{v_2}}{\rho_{\eta_2}}(\rho_\epsilon+\rho_\theta)-\rho_\epsilon\right)\theta\right. \\
& \left. + \frac{\rho_{v_1}}{r\rho_{\eta_2}}q - \left(\frac{\rho_{v_1}\rho_{v_2}}{\rho_{\eta_2}} + (\rho_{\epsilon_j}-\rho_\epsilon)\right)P_1\right] \\
=& r_j\left[\left(\frac{\rho_{v_2}\rho_{v_0}}{\rho_{\eta_2}} - \frac{\phi\rho_{v_1}}{r\rho_{\eta_2}}\right)P_0 + \frac{\rho_{v_2}}{\rho_{\eta_2}}\rho_{\eta_1}y_1 + \rho_{\epsilon_j}s_j + \left(\frac{\rho_{v_2}}{\rho_{\eta_2}}(\rho_\epsilon+\rho_\theta)-\rho_\epsilon\right)\theta\right. \\
& \left. - \left(\frac{\rho_{v_1}\rho_{v_2}}{\rho_{\eta_2}} - \frac{\phi\rho_{v_1}}{r\rho_{\eta_2}} + (\rho_{\epsilon_j}-\rho_\epsilon)\right)P_1\right],
\end{aligned}$$

where we have substituted $q = \phi(P_1 - P_0)$ in the last equation.

Now we can obtain the formula of P_1 by taking a similar procedure as that for P_2. Using the demand-supply equality (2.1) and solving the resulting equation with respect to P_1, we obtain (3.1). Substituting (3.1) into (1.7) yields (3.2), the expression of the price formula at period 1 given in Theorem 1. The condition $\frac{c_1}{b_1} = \frac{c_2}{b_2}$ holds and guarantees the partial revealing equilibrium.

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