AN EXCURSION-THEORETIC APPROACH TO REGULATOR'S BANK REORGANIZATION PROBLEM

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ABSTRACT. The importance of the global financial system cannot be exaggerated. When a large financial institution becomes problematic and is bailed out, that bank is often claimed as "too big to fail". On the other hand, to prevent bank's failure, regulatory authorities adopt the Prompt Corrective Action (PCA) against a bank that violates certain criteria, often measured by its leverage ratio. In this article, we provide a framework where one can analyze the cost and effect of PCA's. We model a large bank that has deteriorating assets and regulatory actions attempting to prevent the bank's failure. The model uses the excursion theory of Lévy processes and finds an optimal leverage ratio that triggers a PCA. A nice feature includes that it incorporates the fact that social cost associated with PCA's are greatly affected by the size of banks subject to PCA's. In other words, one can see the cost of rescuing a bank which is "too big to fail".

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1. INTRODUCTION

For a description of the Prompt Corrective Action (PCA, thereafter) we first quote from [Shibut et al., 2003]: "The Prompt Corrective Action (PCA) provisions in Federal Deposit Insurance Corporation Improvement Act of 1991 (FDICIA) require that regulators set a *threshold* for critically undercapitalized institutions, and that regulators promptly close institutions that breach the threshold unless they quickly recapitalize or merge with a healthier institution. Many economists expected these provisions to result in dramatically reduced loss rates, or even zero loss rates, for bank failures." In short, the PCA provides a set of mandatory and discretionary actions to be taken by banking supervisors when the bank's capital ratio is declining. In many countries, as the above says, regulatory authorities set minimum capital ratios and intervene bank operations once the bank's capital falls short of the minimum requirement.

There are only a few studies available on PCA's. [Kocherlakota and Shim, 2009] and [Shim, 2011] develop dynamic contract models to analyze under what conditions regulators should subsidize or liquidate a problematic bank. While liquidation is one alternative in PCA's, one needs to analyze a broader spectrum of actions including recapitalization, cash infusion, and changes of risk-return profile of the bank's asset. Considering the catastrophic damage on the global financial system caused by the crisis in 2008, a comprehensive analytical framework for

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regulators' interventions is very much needed. Our model has, among other things, the following features that well capture the real-life experience:

- we directly deal with leverage ratio threshold that triggers PCA's;
- we describe the situation where even a large bank (with high leverage) can *easily* fail;
- we use spectrally negative Lévy processes for modeling sudden declines in the value of bank assets;
- we include cash infusions (at the beginning of PCA's) that are often used for preventing outright insolvency, change the bank's risk-return profile, and consider the possibilities whether the bank comes back to the normal operation or goes to liquidation;
- we consider various costs associated with PCA's and compute the optimal threshold level that triggers PCA's and that minimizes the associated total cost; and
- we obtain some results, among other things, where the bank size has a crucial impact on the cost involved.

In this paper, we describe deterioration of leverage ratio as the excursion from the running maximum. It should be best to explain through an example. We shall define everything rigourously in the next section. Let $Y = e^X$ be the bank's total asset value, where X is a spectrally negative Lévy process and represents the fluctuation rate of the asset. (See [Carr and Wu, 2003] and [Madan and Schoutens, 2008] that use Lévy processes for financial asset values.) Let S be the running maximum of X. We assume that the bank increases its asset base as long as it maintains the predetermined *leverage ratio*, defined as

$$\text{Leverage ratio} := \frac{\text{Debt}}{\text{Total Asset}}.$$

Let us denote the aforementioned leverage ratio by e^{-b}. For example, if the bank has the initial asset of $Y_0 = e^{X_0} = 100$ with $e^{-b} = 0.8$, it has total asset of 100 financed by debt 80 and equity 20. We can think of this ratio as the maximum leverage ratio that is allowed by the banking regulations. We assume that the bank increases its asset base as long as $X = S$ and that the bank's leverage ratio is maintained at 0.8. Hence if the asset value appreciates to 120, then the bank can increase its assets since the equity value is now 40 and the leverage ratio has improved to $40/120$. With this new equity level, the bank increases its leverage up to 0.8. In other words, the total asset increases to 200 financed by debt 160 and equity 40. Note that $e^S = e^X = 200$ and the debt level is $e^{-b}(e^S) = e^{S-b} = 160$. Now if the bank's asset deteriorates due to defaults in the lending portfolio, we would have $S - X > 0$. In other words, there appears an *excursion from the running maximum* S. Since the asset level has stayed at $e^S = 200$, the bank's equity would be wiped out when $e^{S-b} = e^X$. That is, the process is absorbed at $t = T_b$, i.e, the first time X goes below the level of $S - b$.

Moreover, note that this model can incorporate the regulatory requirements that the bank, when experiencing asset deterioration, needs to sell the assets in order to reduce the leverage. For example, assume that when the bank loses one dollar of asset, the bank loses its equity by γ and reduces its debt by $1-\gamma$, where $\gamma \in (1-e^{-b}, 1]$. Then, at the time the equity is wiped out, we have

$$e^X \le e^S \left(1 - \frac{1-e^{-b}}{\gamma}\right),$$

that is, the process is absorbed when the excursion $S - X$ reaches $-\log\left(1 - \frac{1-e^{-b}}{\gamma}\right)$. In this paper, however, we assume $\gamma = 1$ for making our presentation simpler. (We are thankful to Nan Chen for pointing out this requirement.)

For *spectrally negative Lévy processes*, or Lévy processes with only negative jumps, a number of authors have succeeded in solving interesting stochastic optimization problems and in extending certain classical results by using the *scale functions*, which we shall review briefly later. We just name a few here : [Baurdoux and Kyprianou, 2008, Baurdoux and Kyprianou, 2009] for stochastic games, [Avram et al., 2007, Kyprianou and Palmowski, 2007, Loeffen, 2008] for the optimal dividend problem, [Alili and Kyprianou, 2004, Avram et al., 2004] for American and Russian options, and [Egami and Yamazaki, 2013, Kyprianou and Surya, 2007] for credit risk.

An excursion theory for spectrally negative Lévy processes has been developed and advanced recently. See [Bertoin, 1996] as a general reference. More specifically, an exit problem of the reflected process Y has been studied by [Avram et al., 2004], [Pistorius, 2004] [Pistorius, 2007] and [Doney, 2005]. Two-dimensional optimal stopping problems whose stopping region involves both the original process and its running maximum are studied in [Ott, 2013], [Guo and Zervos, 2007] and [Egami and Oryu, 2014].

The rest of the paper is organized as follows. In Section 2, we formulate a mathematical model to express the PCA program and then find an optimal trigger level in Section 3. We shall illustrate the solution through a numerical example and perform comparative statics in Section 4.1. In addition, we present another example by using parameters estimated from the data of an existing bank in Section 4.2. Furthermore, we shall consider the situation where, after the bank successfully emerges from the intervention (i.e., an PCA ends), it again becomes problematic and subject to another PCA. This is in Section 5. We present a brief summary about *scale functions* associated with spectrally negative Lévy processes in Section 6.

2. Mathematical Model

As explained in Section 1, the PCA (prompt corrective action) is a supervisory framework where the regulators enforce actions to banks with inadequate capital. See The Federal Deposit Insurance Corporation Improvement Act of 1991 (https://www.fdic.gov/regulations/laws/rules/1000-4000.html). It says "Each appropriate Federal banking agency and the Corporation (acting in the Corporation's capacity as the insurer of depository institutions under this Act) shall carry out the purpose of this section by taking prompt corrective action to resolve the problems of insured depository institutions" (Sec. 38 (a) (2)). The purpose is to detect banks of capital inadequacy at the early stage. If the regulators assign to the bank one of the following categories: undercapitalized, significantly undercapitalized, or critically undercapitalized, the bank is subject to corrective actions under close monitoring by the regulators. For example, the bank is required to submit and implement plans to restore capital and to restrict asset growth. Based on the above description, in modeling PCA's, one should use bank's leverage ratio for the capital adequacy test. One should also change parameters associated with bank's assets because once a PCA is in force, bank's asset growth should be checked. While the bank is required to raise capital to restore its leverage ratio, as we know from the past experiences in the financial crises, the bank does not have enough creditworthiness to raise capital and hence taxpayers' money may have to be used. Let us now present our model that incorporates these facts.

Let the spectrally negative Lévy processes $X^i = \{X^i_t; t \geq 0\}$ $(i = 0, 1)$ represent the state variable defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where Ω is the set of all possible realizations of the stochastic economy, and $\mathbb{P}$ is a probability measure defined on $\mathcal{F}$. We denote by $\mathbb{F} = \{\mathcal{F}_t\}_{t\geq 0}$ the filtration with respect to which X^0 and X^1 are adapted and with the usual conditions being satisfied. The Laplace exponent ψ_i of X^i, $i = 0, 1$, is given by

$$\psi_i(\lambda) = \mu_i\lambda + \frac{1}{2}\sigma_i^2\lambda^2 + \int_{(-\infty,0)} (e^{\lambda x} - 1 - \lambda x \mathbb{1}_{(x>-1)})\Pi_i(\mathrm{d}x), \tag{2.1}$$

where $\mu_i \geq 0$, $\sigma_i > 0$, and Π_i is a measure concentrated on $\mathbb{R}\backslash\{0\}$ satisfying $\int_{\mathbb{R}}(1 \wedge x^2)\Pi_i(\mathrm{d}x) < \infty$. It is well-known that ψ_i is zero at the origin and convex on $\mathbb{R}_+$.

We define the process $X = \{X_t; t \geq 0\}$ as the solution to the stochastic differential equation

$$\mathrm{d}X_t = \mathrm{d}X_t^{I(t)} \quad \text{and} \quad X_0 = x,$$

where $I = \{I(t); t \geq 0\}$ is the right-continuous switching process which satisfies $I(t) \in \{0, 1\}$ for every $t \in \mathbb{R}_+$. We postpone (see (2.2)) the rigorous mathematical definition of the process I to make the explanation of our model smoother.

The bank's total asset value is represented by the process $Y = \{e^{X_t}; t \geq 0\}$. Therefore, X represents the fluctuation rate of the bank's total asset value. When $I(t) = 0$, the bank is well capitalized with satisfactory leverage ratio and thus is not subject to the regulator's PCA. Our $\mathrm{d}X^0$ corresponds to the dynamics while the bank not being controlled. On the other hand, when $I(t) = 1$, a PCA is applied, the bank is taken into strict supervision by the regulator and the asset dynamics follow $\mathrm{d}X^1$. In general, it may be often the case that

$$\mu_1 < \mu_0 \quad \text{and} \quad \sigma_1 < \sigma_0$$

to reflect more conservative risk-return profile during the PCA period. We introduce $\mathbb{F}$-stopping times τ_c^+ and τ_c^- $(c \geq 0)$ defined by

$$\tau_c^+ = \inf\{t \geq 0 : X_t^1 \geq c\}, \quad \text{and} \quad \tau_c^- = \inf\{t \geq 0 : X_t^1 \leq c\}.$$

In addition, let $S = \{S_t; t \geq 0\}$ be defined by $S_t = \sup_{u\in[0,t]} X_u \vee s$ with $s = S_0$. We introduce the $\mathbb{F}$-stopping times T_c $(c > 0)$ defined by

$$T_c = \inf\{t \geq 0 : S_t - X_t \geq c\}.$$

We assume that a PCA is applied (i.e. the process I changes form 0 to 1) at $t = T_{b'}$, where $b \geq b' \geq 0$. Note that this is the time when the bank's leverage ratio e^{S-b}/e^X exceeds the level $e^{b'-b}$ (not the level $e^{-b'}$). Indeed, since $X \leq S - b'$, the leverage ratio is

$$\frac{e^{S-b}}{e^X} \geq \frac{e^{S-b}}{e^{S-b'}} = e^{b'-b}.$$

This threshold b' should be determined by the regulator and we call it the *PCA trigger level*. When the bank undergoes a PCA, one of the two scenarios is possible: the bank becomes insolvent (i.e., $S - X \geq b$), or the bank successfully improves its leverage ratio to e^{-b} (i.e., $S - X = 0$).

In this case, we assume that when a PCA is applied, the bank's asset is pushed up to the target level

$$e^{S_{T_{b'}}-a},$$

where $a \in [0, b)$ is some constant. This is done by injecting funds (taxpayers' money) to improve the leverage ratio to some predetermined level e^{a-b}. More specifically, since this action moves the asset level to $\exp(S_{T_{b'}} - a)$ and the debt level at that time is $\exp(S_{T_{b'}} - b)$, the new leverage ratio shall become

$$\frac{e^{S_{T_{b'}}-b}}{e^{S_{T_{b'}}-a}} = e^{a-b}.$$

The amount of public funds to be injected (as equity) is thereby the difference between the asset values before and after PCA's are applied.

To compute the initial cost to be paid, we need to record both of the asset values before and after the cash injection. For this purpose, we define the random variable $\underline{X}$ as $\underline{X} = X_{T_{b'}}$ and, afterwards, redefine $X_{T_{b'}}$ by $X_{T_{b'}} = S_{T_{b'}} - a$. In other words, $e^{\underline{X}}$ represents the asset value *before* the cash infusion is made, and $e^{X_{T_{b'}}}$ indicates the asset value *after* the bank receives fresh money. This way, e^X remains representing the asset value of the bank, and the amount of the cash infusion can be written as

$$e^{S_{T_{b'}}-a} - e^{\underline{X}}.$$

Let us emphasize that this value is large when the size of the bank is large (since the asset value e^S is large). Note that while we use this form of initial cost throughout this paper, the initial cost can be generalized to the form of $c(\underline{X}, S_{T_{b'}})$, where $c : \mathbb{R}_+^2 \mapsto \mathbb{R}_+$, and in those cases, this cost may include, for example, the cash to be set aside in case of the bank's insolvency (the bank's depositors will be bailed out by the FDIC), or the present value of administration costs to alter bank's risk-return profile from ψ_0 to ψ_1.

Let an $\mathbb{F}$-stopping time τ be the time the PCA ends, then τ should be represented by $\tau = T_{b'} + (\tau_a^+ \wedge \tau_{a-b}^-) \circ \theta_{T_{b'}}$, where $(\theta_t)_{t \in \mathbb{R}_+}$ is the shift operator, and the process I can be defined as

$$I(t) = \begin{cases} 0 & \text{for} \quad t < T_{b'}, \tau \leq t \\ 1 & \text{for} \quad t \in [T_{b'}, \tau). \end{cases} \tag{2.2}$$

In summary, at time $t = T_{b'}$, the bank's leverage ratio becomes worse than the PCA trigger level, then a PCA starts and the bank goes under the regulator's control. The corresponding excursion height is $S_{T_{b'}} - \underline{X}$. Then the regulatory authority injects cash in the amount of $e^{S_{T_{b'}}-a} - e^{\underline{X}}$ and the bank's leverage ratio is improved to e^{a-b}. To recover its leverage ratio to e^{-b}, X^1 must go up in the amount of a, and the time is denoted by τ_a^+. However, if X^1 goes down in the amount of $b - a$ earlier, the bank becomes insolvent, and the corresponding time is τ_{a-b}^-.

Total Cost Function:

In addition to the initial cost, there will be certain running cost over time while PCA continues and the social cost (the penalty) when the bank becomes insolvent. We assume that (1) the running cost will be incurred in proportion to the duration of PCA being in place and the ratio is $\alpha \geq 0$, and (2) the social cost to the economy caused by the bank's final insolvency is $\beta e^{S_{T_b}-b}$ ($\beta \geq 0$), that is, some parameter β times the bank's asset when it becomes insolvent. This reflects the fact that the cost of bank's failure becomes greater as the size of the bank becomes larger. Summing up these, the expected total cost C_1 associated with a PCA can be represented as

$$C_1(x, s; b') = \mathbb{E}^{x,s}\left[e^{-qT_{b'}}\left(e^{S_{T_{b'}}-a} - e^{\underline{X}}\right) + \alpha \int_{T_{b'}}^{\tau} e^{-qt}\mathrm{d}t + e^{-q\tau}\left(\beta e^{S_{T_{b'}}-b}\right) 1\!\!1_{\{\tau_a^+ \circ \theta_{T_{b'}} > \tau_{a-b}^- \circ \theta_{T_{b'}}\}}\right], \tag{2.3}$$

which we shall calculate in the next section. Moreover, for a given value of a, we analyze the cost-minimizing PCA trigger level denoted by b^*. It should be noted that the computation is numerical and is for a specific model.

To solve this problem, we shall use the *scale function* associated with every spectrally negative Lévy process. We explain some facts in Section 6. For a comprehensive account of the scale function, see [Bertoin, 1996, Bertoin, 1997, Kyprianou, 2006, Kyprianou and Surya, 2007]. See [Egami and Yamazaki, 2014, Surya, 2008] for numerical methods for computing the scale function.

There exists a (q-)scale function

$$W_i^{(q)} : \mathbb{R} \mapsto \mathbb{R}; \quad q \geq 0, \quad i = 0, 1,$$

which has the following properties: it is continuous and strictly increasing on $[0, \infty)$, and is zero on $(-\infty, 0)$. Moreover, the (q-)scale function is uniquely determined by

$$\int_0^\infty e^{-\beta x} W_i^{(q)}(x) \mathrm{d}x = \frac{1}{\psi_i(\beta) - q}, \qquad \beta > \Phi_i(q), \quad i = 0, 1,$$

where

$$\Phi_i(q) = \sup \{c > 0 : \psi_i(c) = q\}, \quad q \geq 0, \quad i = 0, 1.$$

It is known that when $\sigma_i > 0$, $W_i^{(q)}$ is twice continuously differentiable on $(0, \infty)$, which we assume in this paper.

3. SOLUTION

To solve the problem, we divide the cost function C_1 into three blocks;

$$C_1(x, s; b') = \begin{cases} C_1^0(x, s; b') & \text{if } s - x > b', \\ C_1^1(x, s; b') & \text{if } s - x = 0, \\ C_1^2(x, s; b') & \text{if } s - x \in (0, b'). \end{cases}$$

As the first step, we calculate $C_1^0(x, s; b')$, the cost involved in the PCA when $S_0 - X_0 \geq b'$; in other words, the PCA is applied at time $t = 0$. In particular, this is always the case when $b' = 0$. Note that the scale functions of X^0 and X^1 are explicitly known in some processes including the case that the Lévy process has no jumps (see [Hubalek and Kyprianou, 2011] for example).

Proposition 3.1. *If $S_0 - X_0 \geq b'$, then*

$$\begin{aligned} C_1^0(x, s; b') \quad = \quad & e^{s-a} - e^x + \frac{1}{q}\left(1 - Z_1^{(q)}(b-a) - (1 - Z_1^{(q)}(b))\frac{W_1^{(q)}(b-a)}{W_1^{(q)}(b)}\right) \\ & + \beta e^{s-b}\left(Z_1^{(q)}(b-a) - Z_1^{(q)}(b)\frac{W_1^{(q)}(b-a)}{W_1^{(q)}(b)}\right), \end{aligned} \tag{3.1}$$

where $W_i^{(q)}$, $i = 0, 1$, is q-scale function of X^i, and

$$Z_i^{(q)}(x) = 1 + q \int_0^x W_i^{(q)}(y) \mathrm{d}y.$$

Proof. Since $S_0 - X_0 \geq b'$, we have $T_{b'} = 0$ and

$$\begin{aligned}
\mathbb{E}^{x,s}\left[\int_{T_{b'}}^{\tau} e^{-qt}\mathrm{d}t\right] &= \mathbb{E}^{s-a,s}\left[\int_0^{\tau_a^+\wedge\tau_{a-b}^-} e^{-qt}\mathrm{d}t\right] \\
&= \frac{1}{q}\left(1-\mathbb{E}^{s-a,s}\left[e^{-q(\tau_a^+\wedge\tau_{a-b}^-)}\right]\right) \\
&= \frac{1}{q}\left(1-\mathbb{E}^{s-a,s}\left[\mathbb{1}_{\{\tau_a^+<\tau_{a-b}^-\}}e^{-q\tau_a^+}\right]-\mathbb{E}^{s-a,s}\left[\mathbb{1}_{\{\tau_a^+>\tau_{a-b}^-\}}e^{-q\tau_{a-b}^-}\right]\right).
\end{aligned}$$

In the same way, we have

$$\begin{aligned}
\mathbb{E}^{x,s}\left[e^{-q\tau}\left(\beta e^{S_{T_{b'}}-b}\right)\mathbb{1}_{\{\tau_a^+\circ\theta_{T_{b'}}>\tau_{a-b}^-\circ\theta_{T_{b'}}\}}\right] &= \beta e^{s-b}\mathbb{E}^{s-a,s}\left[\mathbb{1}_{\{\tau_a^+>\tau_{a-b}^-\}}e^{-q(\tau_a^+\wedge\tau_{a-b}^-)}\right] \\
&= \beta e^{s-b}\mathbb{E}^{s-a,s}\left[\mathbb{1}_{\{\tau_a^+>\tau_{a-b}^-\}}e^{-q\tau_{a-b}^-}\right]
\end{aligned}$$

Then we can write

$$\begin{aligned}
C_1^0(x,s;b') &= e^{s-a}-e^x+\frac{1}{q}\left(1-\mathbb{E}^{s-a,s}\left[\mathbb{1}_{\{\tau_a^+<\tau_{a-b}^-\}}e^{-q\tau_a^+}\right]-\mathbb{E}^{s-a,s}\left[\mathbb{1}_{\{\tau_a^+>\tau_{a-b}^-\}}e^{-q\tau_{a-b}^-}\right]\right) \\
&\quad +\beta e^{s-b}\mathbb{E}^{s-a,s}\left[\mathbb{1}_{\{\tau_a^+>\tau_{a-b}^-\}}e^{-q\tau_{a-b}^-}\right].
\end{aligned} \tag{3.2}$$

It is well known (see [Kyprianou, 2006] and [Doney, 2005]) that

$$\begin{aligned}
\mathbb{E}^{s-a,s}[\mathbb{1}_{\{\tau_a^+<\tau_{a-b}^-\}}e^{-q\tau_a^+}] &= \frac{W_1^{(q)}(b-a)}{W_1^{(q)}(b)}, \quad \text{and} \\
\mathbb{E}^{s-a,s}[\mathbb{1}_{\{\tau_a^+>\tau_{a-b}^-\}}e^{-q\tau_{a-b}^-}] &= Z_1^{(q)}(b-a)-Z_1^{(q)}(b)\frac{W_1^{(q)}(b-a)}{W_1^{(q)}(b)}.
\end{aligned}$$

Hence we have (3.1). ☐

Now we calculate the cost in the case that $S_0 = X_0 = s$ by using Proposition 3.1.

Proposition 3.2. *If $b' > 0$ and $S_0 = X_0 = s$, then*

$$\begin{aligned}
C_1^1(s,s;b') &= \frac{\sigma^2}{2}\left(\frac{(W_0^{(q)\prime}(b'))^2}{W_0^{(q)}(b')}-W_0^{(q)\prime\prime}(b')\right)\int_s^\infty \mathrm{d}m\exp\left(-(m-s)\frac{W_0^{(q)\prime}(b')}{W_0^{(q)}(b')}\right)C_1^0(m-b',m;b') \\
&\quad +\iint_E \Pi(\mathrm{d}h)\mathrm{d}y\left(W_0^{(q)\prime}(y)-\frac{W_0^{(q)\prime}(b')}{W_0^{(q)}(b')}W_0^{(q)}(y)\right) \\
&\quad \times\left(\int_s^\infty \mathrm{d}m\exp\left(-(m-s)\frac{W_0^{(q)\prime}(b')}{W_0^{(q)}(b')}\right)C_1^0(m-y+h,m;b')\right),
\end{aligned} \tag{3.3}$$

where $E = \{(y,h)\in\mathbb{R}^2 : 0\leq y<b', y-b<h<y-b'\}$.

Proof. As for the initial cost part of (2.3), we have, by splitting into the case where PCA trigger level b' is continuously crossed and the case where it is overshot by a downward jump,

$$\begin{aligned}
&\mathbb{E}^{s,s}\left[e^{-qT_{b'}}\left(e^{S_{T_{b'}}-a}-e^{S_{T_{b'}}-\underline{X}}\right)\right]\\
=\;&\int_s^\infty \mathrm{d}m\mathbb{E}^{s,s}\left[e^{-qT_{b'}}\mathbb{1}_{\{S_{T_{b'}}-\underline{X}=b',S_{T_{b'}}\in\mathrm{d}m\}}\right]\left(e^{m-a}-e^{m-b'}\right)\\
&+\iiint_D \mathbb{E}^{s,s}\left[e^{-qT_{b'}}\mathbb{1}_{\{S_{T_{b'}-}-X_{T_{b'}-}\in\mathrm{d}y,\underline{X}-X_{T_{b'}-}\in\mathrm{d}h,S_{T_{b'}-}\in\mathrm{d}m\}}\right]\left(e^{m-a}-e^{m-y+h}\right),
\end{aligned}$$

where $D=\{(m,y,h)\in\mathbb{R}^3 : 0\le y<b', y-b<h<y-b', m>s\}$. Note that $X_{T_{b'}-}$ is, as usual, the pre-jump position of X at time $T_{b'}$. Because of the Markov property of X^0 and X^1, we have

$$\begin{aligned}
\mathbb{E}^{s,s}\left[\int_{T_{b'}}^\tau e^{-qt}\mathrm{d}t\right] &= \mathbb{E}^{s,s}\left[e^{-qT_{b'}}\int_0^{(\tau_a^+\wedge\tau_{a-b}^-)\circ\theta_{T_{b'}}} e^{-qt}\mathrm{d}t\right]\\
&= \mathbb{E}^{s,s}\left[e^{-qT_{b'}}\right]\mathbb{E}^{s,s}\left[\int_0^{(\tau_a^+\wedge\tau_{a-b}^-)\circ\theta_{T_{b'}}} e^{-qt}\mathrm{d}t\middle|\mathcal{F}_{T_{b'}}\right]\\
&= \mathbb{E}^{s,s}\left[e^{-qT_{b'}}\right]\mathbb{E}^{s-a,s}\left[\int_0^{\tau_a^+\wedge\tau_{a-b}^-} e^{-qt}\mathrm{d}t\right]\\
&= \mathbb{E}^{s,s}\left[e^{-qT_{b'}}\right]\frac{1}{q}\left(1-\mathbb{E}^{s-a,s}\left[e^{-q(\tau_a^+\wedge\tau_{a-b}^-)}\right]\right)\\
&= \mathbb{E}^{s,s}\left[e^{-qT_{b'}}\right]\frac{1}{q}\left(1-\mathbb{E}^{s-a,s}\left[\mathbb{1}_{\{\tau_a^+<\tau_{a-b}^-\}}e^{-q\tau_a^+}\right]-\mathbb{E}^{s-a,s}\left[\mathbb{1}_{\{\tau_a^+>\tau_{a-b}^-\}}e^{-q\tau_{a-b}^-}\right]\right)\\
&= \left(\int_s^\infty \mathbb{E}^{s,s}\left[e^{-qT_{b'}}\mathbb{1}_{\{S_{T_{b'}}-\underline{X}=b',S_{T_{b'}}\in\mathrm{d}m\}}\right]\right.\\
&\quad\left.+\iiint_D \mathbb{E}^{s,s}\left[e^{-qT_{b'}}\mathbb{1}_{\{S_{T_{b'}-}-X_{T_{b'}-}\in\mathrm{d}y,\underline{X}-X_{T_{b'}-}\in\mathrm{d}h,S_{T_{b'}-}\in\mathrm{d}m\}}\right]\right)\\
&\quad\times\frac{1}{q}\left(1-\mathbb{E}^{m-a,m}\left[\mathbb{1}_{\{\tau_a^+<\tau_{a-b}^-\}}e^{-q\tau_a^+}\right]-\mathbb{E}^{m-a,m}\left[\mathbb{1}_{\{\tau_a^+>\tau_{a-b}^-\}}e^{-q\tau_{a-b}^-}\right]\right).
\end{aligned}$$

On the penalty part, we have

$$\begin{aligned}
&\mathbb{E}^{s,s}\left[e^{-q\tau}\left(\beta e^{S_{T_{b'}}-b}\right)\mathbb{1}_{\{\tau_a^+\circ\theta_{T_{b'}}>\tau_{a-b}^-\circ\theta_{T_{b'}}\}}\right]\\
=\;&\mathbb{E}^{s,s}\left[e^{-q(T_{b'}+\tau_{a-b}^-\circ\theta_{T_{b'}}}\left(\beta e^{S_{T_{b'}}-b}\right)\mathbb{1}_{\{\tau_a^+\circ\theta_{T_{b'}}>\tau_{a-b}^-\circ\theta_{T_{b'}}\}}\right]\\
=\;&\int_s^\infty \mathbb{E}^{s,s}\left[e^{-qT_{b'}}\mathbb{1}_{\{S_{T_{b'}}-\underline{X}=b',S_{T_{b'}}\in\mathrm{d}m\}}\right]\mathbb{E}^{m-a,m}\left[\beta e^{m-b}\mathbb{1}_{\{\tau_a^+>\tau_{a-b}^-\}}e^{-q\tau_{a-b}^-}\right]\\
&+\iiint_D \mathbb{E}^{s,s}\left[e^{-qT_{b'}}\mathbb{1}_{\{S_{T_{b'}-}-X_{T_{b'}-}\in\mathrm{d}y,\underline{X}-X_{T_{b'}-}\in\mathrm{d}h,S_{T_{b'}-}\in\mathrm{d}m\}}\right]\mathbb{E}^{m-a,m}\left[\beta e^{m-b}\mathbb{1}_{\{\tau_a^+>\tau_{a-b}^-\}}e^{-q\tau_{a-b}^-}\right].
\end{aligned}$$

Then, by summing those three parts, we can write in view of (3.2)

$$\begin{aligned}
C_1^1(s,s;b') &= \int_s^\infty \mathbb{E}^{s,s}\left[e^{-qT_{b'}}\mathbb{1}_{\{S_{T_{b'}}-\underline{X}=b',S_{T_{b'}}\in\mathrm{d}m\}}\right]C_1^0(m-b',m;b')\\
&\quad+\iiint_D \mathbb{E}^{s,s}\left[e^{-qT_{b'}}\mathbb{1}_{\{S_{T_{b'}-}-X_{T_{b'}-}\in\mathrm{d}y,\underline{X}-X_{T_{b'}-}\in\mathrm{d}h,S_{T_{b'}-}\in\mathrm{d}m\}}\right]C_1^0(m-y+h,m;b').
\end{aligned}\tag{3.4}$$

Finally, it is known from Theorem 1 and 2 in [Pistorius, 2005] that

$$\mathbb{E}^{s,s}\left[e^{-qT_{b'}}1\!\!1_{\{S_{T_{b'}}-\underline{X}=b',S_{T_{b'}}\in\mathrm{d}m\}}\right]=\frac{\sigma^2}{2}\left(\frac{W_0^{(q)'}(b')}{W_0^{(q)}(b')}-W_0^{(q)''}(b')\right)\exp\left(-(m-s)\frac{W_0^{(q)'}(b')}{W_0^{(q)}(b')}\right)\mathrm{d}m,$$

and

$$\begin{aligned}&\mathbb{E}^{s,s}\left[e^{-qT_{b'}}1\!\!1_{\{S_{T_{b'}-}-X_{T_{b'}-}\in\mathrm{d}y,\underline{X}-X_{T_{b'}-}\in\mathrm{d}h,S_{T_{b'}-}\in\mathrm{d}m\}}\right]\\ =\ &\Pi(\mathrm{d}h)\mathrm{d}y\mathrm{d}m\left(W_0^{(q)'}(y)-\frac{(W_0^{(q)'}(b'))^2}{W_0^{(q)}(b')}W_0^{(q)}(y)\right)\exp\left(-(m-s)\frac{W_0^{(q)'}(b')}{W_0^{(q)}(b')}\right).\end{aligned}$$

Hence we have (3.3). □

Note that some condition is needed for the finiteness of $C_1^1(s,s;b')$, and the following Remark shows it.

Remark 3.1. $C_1^1(s,s;b')<\infty$ if and only if $1-\frac{W_0^{(q)'}(b')}{W_0^{(q)}(b')}<0$.

Proof. By Proposition 3.1, we have

$$\begin{aligned}C_1^0(m-u,m;b')=&\left(e^{-a}-e^{-u}+\beta e^{-b}\left(Z_1^{(q)}(b-a)-Z_1^{(q)}(b)\frac{W_1^{(q)}(b-a)}{W_1^{(q)}(b)}\right)\right)e^m\\&+\frac{1}{q}\left(1-Z_1^{(q)}(b-a)-(1-Z_1^{(q)}(b))\frac{W_1^{(q)}(b-a)}{W_1^{(q)}(b)}\right),\quad \text{for } m\geq s \text{ and } u\in[b',b).\end{aligned}$$

Hence, there are some positive constants $M_1,M_2<\infty$ with which we can write

$$\begin{aligned}&\int_s^\infty \mathrm{d}m\exp\left(-(m-s)\frac{W_0^{(q)'}(b')}{W_0^{(q)}(b')}\right)C_1^0(m-u,m;b')\\=&\int_s^\infty\mathrm{d}m\left(M_1\exp\left(\left(1-\frac{W_0^{(q)'}(b')}{W_0^{(q)}(b')}\right)m\right)+M_2\exp\left(-m\frac{W_0^{(q)'}(b')}{W_0^{(q)}(b')}\right)\right),\quad\text{for } u\in[b',b),\end{aligned}$$

and therefore, the integral above is finite if and only if $1-W_0^{(q)'}(b')/W_0^{(q)}(b')<0$. Since $y-h\in(b',b)$ on E and the other terms in (3.3) satisfy

$$\frac{\sigma^2}{2}\left(\frac{(W_0^{(q)'}(b'))^2}{W_0^{(q)}(b')}-W_0^{(q)''}(b')\right)<\infty,\quad\text{and}$$
$$\iint_E\Pi(\mathrm{d}h)\mathrm{d}y\left(W_0^{(q)'}(y)-\frac{W_0^{(q)'}(b')}{W_0^{(q)}(b')}W_0^{(q)}(y)\right)<\infty\quad\text{for } b'>0,$$

we can say that $C_1^1(s,s;b')<\infty$ if and only if $1-W_0^{(q)'}(b')/W_0^{(q)}(b')<0$. □

Finally, we calculate $C_1^2(x,s;b')$ in the case that $S_0-X_0\in(0,b')$, by using Propositions 3.1 and 3.2.

Proposition 3.3. *If $b' > 0$ and $S_0 - X_0 \in (0, b')$, then*

$$C_1^2(x,s;b') = \frac{\sigma^2}{2}\left(W_0^{(q)}(b'-s+x) - \frac{W_0^{(q)\prime}(b')}{W_0^{(q)}(b')}W_0^{(q)}(b'-s+x)\right)C_1^0(s-b',s;b') \\ + \iint_E \Pi(\mathrm{d}h)\mathrm{d}y\left(\frac{W_0^{(q)}(b'-s+x)}{W_0^{(q)}(b')}W_0^{(q)}(y) - W_0^{(q)}(y-s+x)\right)C_1^0(s-y+h,s;b') \\ + \frac{W_0^{(q)}(b-s+x)}{W_0^{(q)}(b)}C_1^1(s,s;b'). \tag{3.5}$$

Proof. In the case $S_0 - X_0 \in (0, b')$, two scenarios are possible. One is that X reaches to s before PCA is applied, and the other is that PCA applies before reaching s. Mathematically, this means that

$$\begin{aligned} C_1^2(x,s;b') &= \mathbb{E}^{x,s}\left[e^{-q\tau_{s-x}^+}\mathbb{1}_{\{\tau_{s-x}^+ < \tau_{s-x-b'}^-\}}C_1^1(s,s;b')\right] \\ &\quad + \mathbb{E}^{x,s}\left[e^{-q\tau_{s-x-b'}^-}\mathbb{1}_{\{\tau_{s-x}^+ > \tau_{s-x-b'}^-\}}C_1^0(X_{\tau_{s-x-b'}^-}, S_{\tau_{s-x-b'}^-};b')\right] \\ &= \mathbb{E}^{x,s}\left[e^{-q\tau_{s-x}^+}\mathbb{1}_{\{\tau_{s-x}^+ < \tau_{s-x-b'}^-\}}\right]C_1^1(s,s;b') \\ &\quad + \mathbb{E}^{x,s}\left[e^{-qT_{b'}}\mathbb{1}_{\{S_{T_{b'}} - \underline{X} = b', S_{T_{b'}} = s\}}\right]C_1^0(s-b',s;b') \\ &\quad + \iint_E \mathbb{E}^{s,s}\left[e^{-qT_{b'}}\mathbb{1}_{\{S_{T_{b'}-} - X_{T_{b'}-} \in \mathrm{d}y, \underline{X} - X_{T_{b'}-} \in \mathrm{d}h, S_{T_{b'}-} = s\}}\right]C_1^0(s-y+h,s;b'). \end{aligned}$$

From Theorem 1 and 2 in [Pistorius, 2007] again, we have

$$\mathbb{E}^{x,s}\left[e^{-qT_{b'}}\mathbb{1}_{\{S_{T_{b'}} - \underline{X} = b', S_{T_{b'}} = s\}}\right] = \frac{\sigma^2}{2}\left(W_0^{(q)}(b'-s+x) - \frac{W_0^{(q)\prime}(b')}{W_0^{(q)}(b')}W_0^{(q)}(b'-s+x)\right),$$

and

$$\begin{aligned} &\mathbb{E}^{s,s}\left[e^{-qT_{b'}}\mathbb{1}_{\{S_{T_{b'}-} - X_{T_{b'}-} \in \mathrm{d}y, \underline{X} - X_{T_{b'}-} \in \mathrm{d}h, S_{T_{b'}-} = s\}}\right] \\ = \; &\Pi(\mathrm{d}h)\mathrm{d}y\left(\frac{W_0^{(q)}(b'-s+x)}{W_0^{(q)}(b')}W_0^{(q)}(y) - W_0^{(q)}(y-s+x)\right). \end{aligned}$$

Hence we have (3.5). □

Now we have all three parts of C_1.

4. Examples

4.1. **A Numerical Example.** In this section, we analyze a specific example. We assume that $X^i = \{X_t^i; t \geq 0\} (i = 0, 1)$ are Brownian motions with drift and exponentially distributed jumps;

$$X_t^i = \mu_i t + \sigma_i B_t^i - \sum_{j=1}^{N_t^i} \epsilon_j^i, \quad i = 0, 1, \tag{4.1}$$

where $\mu_i \geq 0, \sigma_i > 0$, and ϵ^i are i.i.d. random variables which are exponentially distributed with parameter $\rho_i > 0$. In addition, $N^i := \{N^i_t; t \geq 0\}(i = 0, 1)$ are independent Poisson processes with corresponding intensities $c_i > 0$. Before solving the problem, we introduce the explicit representation of the scale function for the process. The Laplace exponent ψ of X has the following simple form;

$$\psi(\lambda) = \frac{\sigma^2}{2}\lambda^2 + \mu\lambda - \frac{c\lambda}{\rho + \lambda}, \quad \lambda \geq 0.$$

The equation $\psi(\lambda) = q$ ($q > 0$) has three real solutions $\{\Phi(q), \alpha, \beta\}$ ($\Phi(q) > \alpha > \beta$), and the q-scale function $W^{(q)}$ of X is given by

$$W^{(q)}(x) = \frac{e^{\Phi(q)x}}{\psi'(\Phi(q))} + \frac{e^{\alpha x}}{\psi'(\alpha)} + \frac{e^{\beta x}}{\psi'(\beta)}. \tag{4.2}$$

Let ψ_i, $i = 0, 1$ be defined by

$$\psi_i(\lambda) = \frac{\sigma_i^2}{2}\lambda^2 + \mu_i\lambda - \frac{c_i\lambda}{\rho_i + \lambda}, \quad \lambda \geq 0.$$

Let us emphasize here that the cost function $C(x, s; b')$ in equation (3.2) consists of three kinds; (1) initial cost of a PCA, (2) running cost, and (3) penalty in case that the bank fails. Figure 1 shows the graph of $C_1(0, 0; b')$ with parameters $X_0 = S_0 = 0$, $b = 1$, $a = 0.3$, $q = 0.1$, $\alpha = \beta = 1$, $\mu_0 = 0.2$, $\sigma_0 = 0.2$, $\mu_1 = 0.1$, $\sigma_1 = 0.1$, $c_0 = c_1 = 1$, and $\rho_0 = \rho_1 = 10$. Note that the assigned value of $b = 1$ may not be realistic; $b = 1$ means that the bank uses $e^{-1} = 0.3679$ as its leverage ratio (= Debt/Total Asset) during the normal business period. This may be too restrictive for the banks. Nevertheless, we use this number since the function values are too sensitive to conduct comparative statics analysis if we use more realistic values such as $b = -\log 0.8$. Note also that, in the next subsection, we shall use realistic parameter values derived from empirical data.

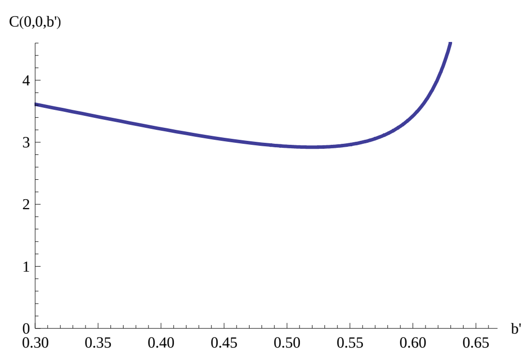


FIGURE 1. Graph of the value of cost function $C_1(0, 0; b')$ against various values of b'. The parameters are $X_0 = S_0 = 0, b = 1, a = 0.3, q = 0.1, \alpha = \beta = 1, \mu_0 = 0.2, \mu_1 = 0.1, \sigma_0 = 0.2, \sigma_1 = 0.1, c_0 = c_1 = 1$ and $\rho_0 = \rho_1 = 10$. The cost is minimized at $b' = 0.5201$.

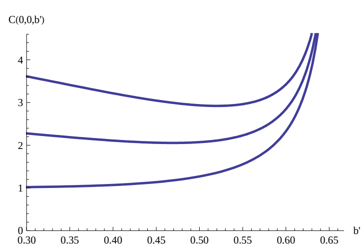
In Figure 1, when we fix a at 0.3, we obtain $b^* = 0.5201$ as the optimal PCA trigger level at which $C_1(0, 0; b')$ is minimized, and the corresponding cost is $C_1(0, 0; b^*) = 2.921$. As indicated in Remark 3.1, at $b' = 0.6672$, we have $W_0^{(q)'}(b')/W_0^{(q)}(b') = 1$ and $C_1(0, 0; b')$ diverges to infinity on $b' > 0.6672$. This can be interpreted as follows: with a large b', since it is not likely that the PCA is initiated at an early stage, X shall be at very high level when the PCA is indeed applied, and therefore, the initial cost part and the penalty part become too large. This result suggests that one should make a discrete choice of PCA trigger level in order to prevent the bank from becoming too costly to rescue. On the other hand, the cost is relatively high when b' is near to a. It is because, in this situation, the PCA may be applied too early so that the running cost to be incurred would become large.

Comparative Statics

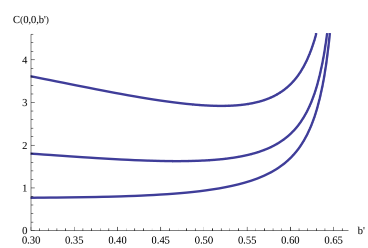
We conduct the following comparative statics.

4.1.1. *Drift and volatility:* First, we change the values of $\sigma_1 = 0.1, 0.2, 0.4$ and $\mu_1 = 0.1, 0.2, 0.4$ (while the other parameters remain the same). Recall that σ_1 and μ_1 are the drift and volatility parameters while the PCA is in place. These parameters affect the cost in various ways after a PCA starts. For instance, once a PCA is initiated, the bank manages assets with these new drift and volatility parameters and may reach the level of S (at which the bank recovers from the PCA) or $S - b$ (at which the bank fails and the economy shall lose a large amount), whichever comes first. The final penalty to be paid at the failure depends on the size of the insolvent bank. An important thing is that the size of the bank (i.e., the level S) will still fluctuate during the PCA period and is influenced by these μ_1 and σ_1.

Figure 2 (i) and (ii) show some results. These graphs show that a higher σ_1 and a higher μ_1 would lead to smaller optimal values of b^* (that is, PCA starts earlier) and the associated total cost $C_1(0, 0; b^*)$ becomes smaller. With a larger σ_1, the time to reach either S or $S - b$ shall become smaller and hence the running cost shall be smaller. The reason for the early PCA with a higher σ_1 is not so simple since there are several factors involved. Nevertheless, it could be interpreted this way: since higher volatility may increase the danger of becoming insolvent and ending up paying penalty (i.e., reaching $S - b$ earlier rather than S), it would be safer to start the PCA earlier in the hope that the bank would not become too large.



(i) comparative statics with various σ_1.

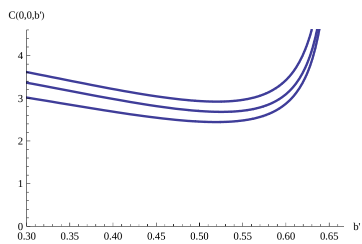


(ii) comparative statics with various μ_1

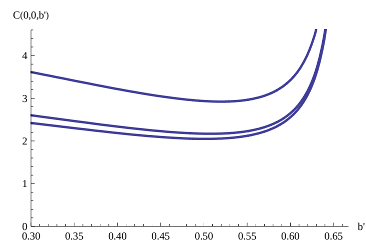
FIGURE 2. The plot on the left (i) shows the effects of volatility parameters during the PCA period: $\sigma_1 = 0.1, 0.2, 0.4$ from top to bottom. With greater asset volatility, the total cost becomes smaller and PCA's should be initiated earlier. The plot on the right (ii) shows the effects of drift parameters during the PCA period: $\mu_1 = 0.1, 0.2, 0.4$ from top to bottom. With greater asset growth rate, the total cost becomes smaller and PCA's should be initiated earlier.

We obtain a similar result when μ_1 becomes larger. That is, the total cost shall be smaller and the starting time of the PCA becomes earlier. To interpret this properly, we need to bear in mind the two things. One is that a higher μ_1 implies that X^1 is more likely to reach S earlier than $S-b$, and the other is that no matter when and where the PCA is initiated, X is always raised to the same level $S-a$. It follows that (1) when μ_1 is large, by initiating the PCA earlier, one can save on the cost of initial cash injection, and (2) greater possibility to come back to the normal leverage ratio (i.e., reaching S) means that it is less likely to pay the penalty (in the event of insolvency). From this analysis, while the conclusion may become different with different sets of parameters, we should note that more conservative risk-return profile (after PCA starts), that is a small μ_1 with a small σ_1, does not necessarily make the total cost associated with PCA's smaller if we take the duration of a PCA period and the probability of recovery into account.

4.1.2. *Frequency and average jump size:* Figure 3 (i) and (ii) show the results with different jump size parameters $\rho_1 = 10, 15, 20$ and intensities $c_1 = 1, \frac{2}{3}, 0.5$. Recall that the average jump size is given by $1/\rho_1$. We have to keep in mind that ρ_1 affects the distribution of overshooting the boundaries $S-b'$ or $S-b$.



(i) comparative statics with various ρ_1



(ii) comparative statics with various c_1

FIGURE 3. The plot on the left (i) shows the effect of jump size parameters during the PCA period: $\rho_1 = 10, 15, 20$ from top to bottom. With smaller average jump size $(1/\rho_1)$, the total cost becomes smaller but PCA trigger level does not change monotonically. The plot on the right (ii) shows the effect of jump frequency parameters during the PCA period: $c_1 = 1, 0.6667, 0.5$ from top to bottom. With lower jump frequency rate, the total costs becomes smaller and PCA's should be inititated earlier.

When $\rho_1 = 10, 15, 20$, we obtained the optimal thresholds $b^* = 0.5201, 0.5252, 0.5193$, respectively, which do not monotonously decrease as ρ_1 increases. This result indicates that, when $\rho_1 = 10$, PCA's should be started earlier than in the case $\rho_1 = 15$. With the smaller ρ_1 (i.e., larger average jump size), it is more likely that X^1 will reach $S-b$ before reaching S, so there may be greater motivation to start the PCA earlier and to make possible penalty payment smaller (by avoiding the bank becoming too large). Note that this is similar to the effects of larger σ_1's on the level of b^*. On the other hand, the optimal threshold b^* in the case $\rho_1 = 20$ is also smaller than that in the case of $\rho_1 = 15$. This is because, with the higher ρ_1 (i.e., smaller average jump size), X^1 is more likely to come back to S rather than being absorbed to $S-b$, and hence one does not have to worry too much about the penalty payable at insolvency. In this case, it would be more important to lower the initial cost by initiating the PCA earlier.

As for the cases $c_1 = 1, \frac{2}{3}$ and 0.5, the values are $b^* = 0.5201, 0.5184$ and 0.5129, respectively. When the jump intensity increases, b^* becomes larger. This may be explained as follows: c_1 does not affect overshoot size at the time X^1 reaches $S - b$. Hence c_1 has smaller effect on the initial and penalty costs than the average jump size $1/\rho_1$ does, so even when c_1 becomes larger, there is relatively small incentive to have a earlier PCA (in order to prevent the bank from becoming too large). Thus, a small c_1 seems to have a similar effect to a large μ_1.

4.1.3. *The existence of jumps:* We consider the Brownian motion with drift $Z_t := ut + \sigma B_t$ with $\sigma = 0.2 = \sigma_0$. We set $u = \mu_0 - \frac{c_0}{\rho_0} = 0.1$, so that Z's drift u is the same as the overall drift of X. For simplicity, we set $\mu_1 = \mu_0$, $\sigma_1 = \sigma_0$, $c_1 = c_0$ and $\rho_1 = \rho_0$. The result is shown in Figure 4, where we see that the minimizer b^* for the original model is 0.441, while the one for the no-jump model is 0.459. The corresponding minimum costs are 1.44250 and 1.19319, respectively. Hence the existence of jumps has significant impact on the total cost. When jumps exist, PCA's should be started earlier. Even by doing so, we have to pay a larger amount of cost due to the possibility of overshooting the boundary. In practical terms, the bank should have a well-diversified portfolio because impacts of defaults in its lending portfolio on the asset value would be small and hence it can reduce the possibility of sizable overshooting.

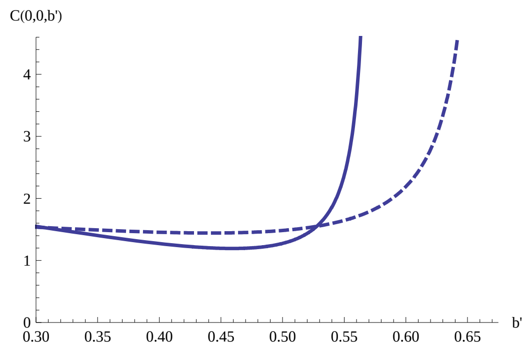


FIGURE 4. Graphs of $C_1(0, 0; b')$ of no jump (solid) and with jumps (dashed). While we set the same overall drifts, the cost is greater and the PCA starts earlier in the jump model. The minimizer b^* for the dashed graph is given at 0.441, while the one for the solid graph is given at 0.459.

4.1.4. *The asset position upon cash infusion:* We change levels of a to which the asset value is raised upon cash infusion. Table 1 shows b^* and $C_1(0, 0; b^*)$ with $a = 0.1, 0.2, \ldots, 0.6$, and Figure 5 shows the graphs of $C_1(0, 0; b')$ with $a = 0.1, 0.3$, and 0.5. It may be appropriate to analyze the difference $b^* - a$, rather than the values of b^* themselves, since $b^* - a$ is concerned with the amount of cash infusion (i.e., initial cost). When the value of a is small, the bank's leverage ratio significantly improves. That is, $X_{T_{b^*}}$ (after receiving public funds) gets closer to $S_{T_b^*}$. In this case, the bank has a better chance to come back to the normal leverage ratio successfully. This fact justifies the large initial payment (i.e., the large value of $b^* - a$ as shown in Table 1).

On the other hand, the cost $C_1(0, 0; b^*)$ does not change monotonically. According to the table, around $a = 0.4$, the cost has a local maximum. One of interesting results is that when $a = 0.6$, b^* is a boundary solution 0.6.

This means that if X continuously crosses the level $S - b'$, the PCA is indeed applied, but there would be no cash infusion.

a	0.1	0.2	0.3	0.4	0.5	0.6
b^*	0.3918	0.4841	0.5201	0.5410	0.5544	0.6
$b^* - a$	0.2918	0.2841	0.2201	0.1410	0.0544	0
$C_1(0, 0; b^*)$	1.887	2.591	2.921	3.027	2.977	2.939

TABLE 1. Changes of b^* and $C_1(0, 0; b^*)$ for different a's. The minimum cost is attained at $a = 0.1$ with the corresponding $b^* = 0.3918$, while the worst cost is attained at $a = 0.4$ with the corresponding $b^* = 0.5410$.

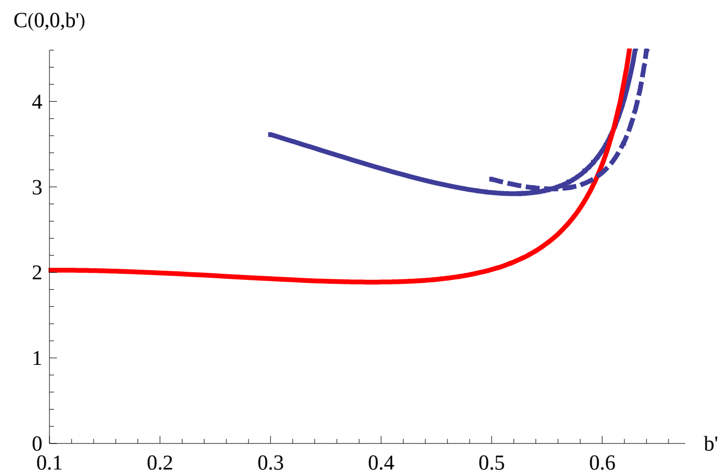


FIGURE 5. Graphs of $C_1(0, 0; b')$ with $a = 0.1$ (red), 0.3 (solid), 0.5 (dashed). A graphical illustration of Table 1.

4.2. **Another Example with Realistic Parameters.** In this section, we provide another example with more realistic set of parameters.

4.2.1. *Assumption:* To simplify the exposition and parameter estimation, we assume that X^0 and X^1 have no jumps and the Laplace exponents ψ_0 and ψ_1 are identical: that is, the parameters related to the dynamics of X do not change before and after PCA's are applied. The "no jump" assumption is justifiable when the size of the bank is very large and its asset is well diversified: losses caused by defaults in its credit portfolio should be small relative to the total asset size. Under these assumptions, the process X is Brownian motion with drift, and accordingly, the asset price process $V = (e^X)_{t\in\mathbb{R}}$ is geometric Brownian motion. Since $\psi_0 = \psi_1$, we will omit the subscripts on the parameters in this section. As for the other parameters, we set $a = 0.05$, $b = 0.1054$, $\alpha = 0.05V_0$, and $\beta = 1$. The choice of $b = 0.1054$ is because the operating leverage ratios e^{-b} of the major U.S. banks lie around 0.9 (i.e., $b = -\log 0.9$). For example, according to the data from Captal IQ via Yahoo Finance on the web (http://finance.yahoo.com/q/ks?s=JPM+Key+Statistics), J.P. Morgan Chase (JPM) has ROE 7.40% and ROA 0.65% as of December 2013, which give (Asset)/(Equity)= 11.38 and hence the leverage ratio (Debt)/(Asset) is 0.89. This number is almost identical across the major banks: Bank of America (BAC), Wells Fargo (WFC), and Citigroup (C). Our assumption of $\alpha = 0.05V_0$ means that the running cost (of PCA's) increases at the rate of 5% of the asset value per unit time. This assumption seems to be natural since the administration cost of banks under the PCA provisions depends on the size of the bank.

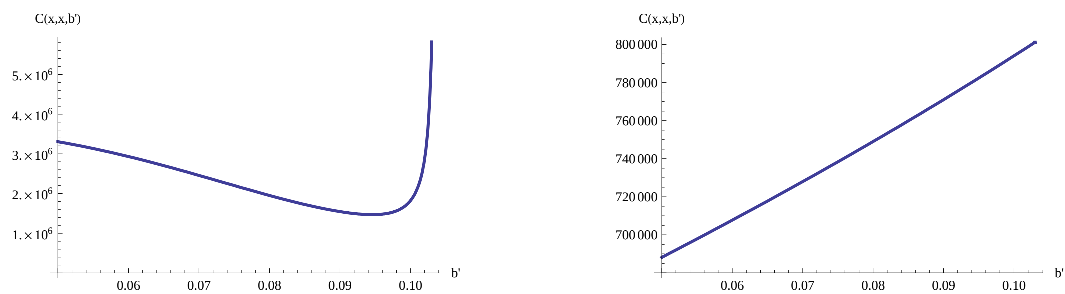
4.2.2. *Parameter estimations:* We need to estimate the drift μ and volatility σ of the bank asset. Since the market value of the asset is not observable, we use the option-theoretic approach: corporate equity can be seen as a contingent claim (call option) written on the asset (see [Black and Scholes, 1973], [Merton, 1974] and the list of papers on page 27 in [Bielecki and Rutkowski, 2002]). Moody's KMV offers default probabilities estimation based on this approach. By viewing the equity as a call option on the asset with the strike price being the future notional value of the bank's debt, [Duan, 1994], [Duan, 2000] and [Lehar, 2005] provide the maximum likelihood estimation of μ and σ of unobservable asset values. We use JP Morgan's equity price on a weekly basis taken from Thomson Reuter's Datastream and its year-end debt amounts from the annual reports. For the debt amounts, we follow Moody's KMV and use the bank's short-term debt amount plus a half of its long-term debt. Since the bank's μ and σ may fluctuate over time, we split the data period (January 2006- December 2013) into four blocks; (1) Jan. 2006- Dec. 2007, (2) Jan. 2008- Dec. 2009, (3) Jan. 2010- Dec. 2011, and (4) Jan. 2012- Dec. 2013. Table 2 summarizes some of the data and parameters. We estimated μ, σ, and V_0, the last being the asset value at the beginning of each sub-period. It is interesting to observe that the parameters μ and σ have become much smaller after the financial crisis.

	2006-2007	2008-2009	2010-2011	2012-2013
Equity	148,567	161,156	130,936	219,190
Debt	1,223,360	1,566,970	1,684,960	1,826,460
μ	0.002781	0.002086	0.0003978	0.001180
σ	0.02925	0.04975	0.009950	0.006575
V_0	1,299,890	1,449,900	1,804,570	2,045,390
$x = \log V_0$	14.08	14.19	14.41	14.53

TABLE 2. The estimated parameters of JP Morgan Chase. The units are in millions except μ and σ, which are the bank asset's drift and volatility over the weekly intervals. We used $q = 0.001$ for a weekly discount rate.

4.2.3. *Results:* We compute the total cost and optimal threshold level b^* of initiating a PCA. Figure 6 (i) shows the graph of $C_1(x, x; b')$ for the period of 2012-2013. The minimal cost is $C_1(x, x; b^*) = 1,469,060$ millions and obtained at $b^* = 0.09457$, which is one-tenth of $b = 0.9$. It is advisable to start a PCA promptly since if we wait until $b' \simeq 0.1$, the bank would become too large and the cost associated with the PCA might become extraordinary. Interestingly, b^* is independent of the initial value x, and $C_1(x, x; b^*)$ is proportionate to e^x. These facts stem from our assumption about α. With different setting on α, we may obtain different results, but setting α as a specific rate of the asset value seems to be natural. See Remark 4.1 below for a proof of this matter.

It is instructive to compare with the period that just preceded the financial crisis of 2008. Figure 6 (ii) shows the graph of $C_1(x, x; b')$ of the period of 2006-2007. The cost $C_1(x, x; b')$ is minimized at $b^* = a = 0.05$, which is 5.5% of $b = 0.9$. The associated cost is $C_1(x, x; b^*) = 688, 187$ millions. As indicated in Table 2, the volatility level was much higher, around $0.02925 \sim 0.04975$. Hence a PCA should be (or should have been) implemented very early before the bank becomes too large. See Subsection 4.1.1. We have illustrated so far that, by using the real data, one may obtain useful information about when to start a PCA with a view to maintaining soundness of the financial system.



(i)2012-2013 (ii)2006-2007

FIGURE 6. The left (i) is the total cost function $C_1(0, 0; b')$ based on the the estimated drift and volatility parameters $\mu = 0.001180$ and $\sigma = 0.006575$ over the weekly intervals during the 2012-2013 period. The optimal level is $b^* = 0.09457$. The right (ii) is the total cost function based on the the estimated drift and volatility parameters $\mu = 0.002781$ and $\sigma = 0.02925$ over the weekly intervals during the 2006-2007 period. The optimal level is $b^* = 0.05$.

[Zhang and Hadjiliadis, 2012] provide the following interesting quantity (in terms of Laplace transform). Let us first define

$$J := \sup\{t \in [0, T_{b^*}] : S_t = X_t\}$$

which is the last time of the maximum is visited before time T_{b^*}. Now the difference between between J and T_{b^*} measures how much time is left until a PCA should be started after the bank attains its peak asset level:

$$\mathcal{S} := T_{b^*} - J$$

which is called the *speed of market crash* in [Zhang and Hadjiliadis, 2012]. The Laplace transform of $\mathcal{S}$ given in the paper is

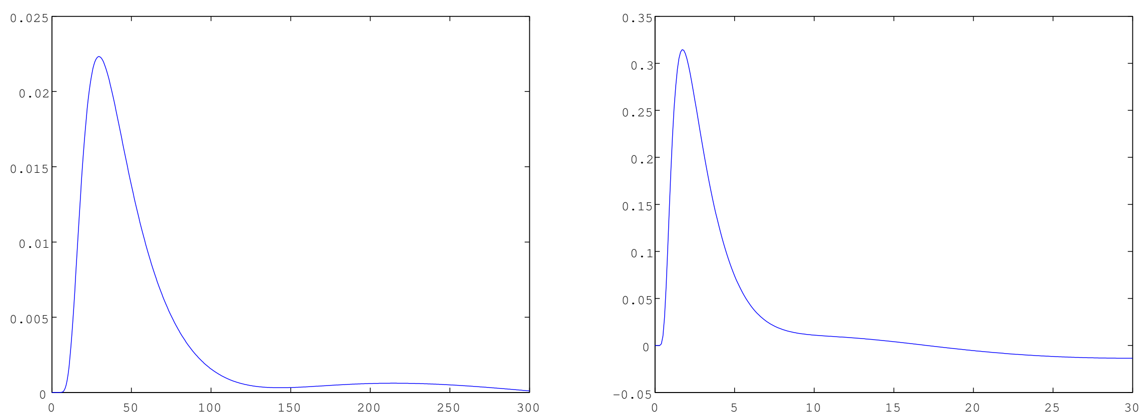
$$\mathbb{E}^x[e^{-\lambda \mathcal{S}} | S_{T_{b^*}} = M] = \frac{B}{A} \cdot \frac{\sinh(A \cdot b^*)}{\sinh(B \cdot b^*)}, \tag{4.3}$$

where

$$A := \frac{\mu}{\sigma^2} \quad \text{and} \quad B := \sqrt{A^2 + \frac{2\lambda}{\sigma^2}},$$

and the Laplace transform is independent of the value of M in the Brownian case.

In Figure 7, we plot the density of $\mathcal{S}$ in the period of 2012-2013 and in the period of 2006-2007. To obtain the density, we convert the Laplace transform (4.3) by the method proposed by Zakian (see [Halsted and Brown, 1972]). For comparison purposes, we fix b' at 0.09 and see the effect of differences in μ and σ on the speed of excursion $S - X$ to reach the same level. During the period of 2012-2013, μ and σ are relatively small (see Table 2). The time for the excursion $S - X$ to reach 0.09 is in the range of $0 \sim 100$ weeks. On the other hand, during the period of 2006-2007, μ and σ are estimated to be relatively large and the time for the excursion to reach the same level is in the range of $0 \sim 10$ weeks. This kind of information is quite useful because it is important to estimate how fast the bank would be in trouble and to avoid situations of being too late.



(i)2012-2013 (ii)2006-2007

FIGURE 7. We compute the density of the time $\mathcal{S}$ of excursion reaching a certain level ($b' = 0.09$). By comparing (i) and (ii) we see that the difference of μ and σ of the asset value process makes a striking impact on $\mathcal{S}$.

Remark 4.1. We show that b^* is independent of x when we set $\alpha = 0.05V_0$. To use Proposition 3.2, we first compute $C_1^0(\cdot, \cdot; b')$ by using Proposition 3.1. Indeed, we have, for $y \geq b'$,

$$\begin{aligned} C_1^0(s-y, s; b') &= e^s \left(e^{-a} - e^{-y} + \frac{0.05}{q} \left(1 - Z_1^{(q)}(b-a) - (1 - Z_1^{(q)}(b)) \frac{W_1^{(q)}(b-a)}{W_1^{(q)}(b)} \right) \right. \\ &\quad \left. + \beta e^{-b} \left(Z_1^{(q)}(b-a) - Z_1^{(q)}(b) \frac{W_1^{(q)}(b-a)}{W_1^{(q)}(b)} \right) \right). \end{aligned}$$

Hence we have (see (3.3))

$$\begin{aligned}
& \int_s^\infty \mathrm{d}m \exp\left(-(m-s)\frac{W_0^{(q)\prime}(b')}{W_0^{(q)}(b')}\right) C_1^0(m-b', m; b') \\
= \quad & \exp\left(\frac{sW_0^{(q)\prime}(b')}{W_0^{(q)}(b')}\right) \int_s^\infty \mathrm{d}m \exp\left(\left(1-\frac{W_0^{(q)\prime}(b')}{W_0^{(q)}(b')}\right) m\right) \\
& \times \left(e^{-a} - e^{-b'} + \frac{0.05}{q}\left(1 - Z_1^{(q)}(b-a) - (1-Z_1^{(q)}(b))\frac{W_1^{(q)}(b-a)}{W_1^{(q)}(b)}\right)\right. \\
& \left. +\beta e^{-b}\left(Z_1^{(q)}(b-a) - Z_1^{(q)}(b)\frac{W_1^{(q)}(b-a)}{W_1^{(q)}(b)}\right)\right) \\
= \quad & e^s \left(1-\frac{W_0^{(q)\prime}(b')}{W_0^{(q)}(b')}\right)^{-1} \left(e^{-a} - e^{-b'} + \frac{0.05}{q}\left(1 - Z_1^{(q)}(b-a) - (1-Z_1^{(q)}(b))\frac{W_1^{(q)}(b-a)}{W_1^{(q)}(b)}\right)\right. \\
& \left. +\beta e^{-b}\left(Z_1^{(q)}(b-a) - Z_1^{(q)}(b)\frac{W_1^{(q)}(b-a)}{W_1^{(q)}(b)}\right)\right),
\end{aligned}$$

which shows that in case of $x = s$, this term is in the form of $e^x \cdot g_1(b')$ where g_1 is a function on $\mathbb{R}_+$ independent of x. Moreover, the term $\int_s^\infty \mathrm{d}m \exp\left(-(m-s)\frac{W_0^{(q)\prime}(b')}{W_0^{(q)}(b')}\right) C_1^0(m-y+h, m; b')$ on the set E in (3.3) can be separated in the same way. Hence we can represent $C_1^1(x, x; b')$ by

$$C_1(x, x; b') = e^x G(b'),$$

where the function $G : \mathbb{R}_+ \mapsto \mathbb{R}$ is independent of x. This shows the independence of b^* from x.

5. Extension to Multiple PCA's

We considered so far that the PCA is applied only once and calculated the cost associated with it. However, after the bank recovers its leverage ratio to e^{-b} thanks to a PCA, it can be under the regulator's control again when the leverage ratio deteriorates to $e^{b'-b}$ or worse ($S - X \geq b'$). Now we incorporate the possibility that PCA's are repeatedly applied until the bank becomes finally insolvent. With the method we shall provide here, while it is not of an explicit form, one can recursively calculate the cost for multiple PCA's. For a mathematical representation, we redefine the process I by

$$I(t) = \mathbb{1}_{\{\tau_1 \leq t < \tau_2\}} + \mathbb{1}_{\{\tau_3 \leq t < \tau_4\}} + \cdots + \mathbb{1}_{\{\tau_{2n-1} \leq t < \tau_{2n}\}} + \cdots, \quad t \in \mathbb{R}_+$$

where τ_n, $n = 1, 2, \ldots$ are $\mathbb{F}$-stopping times defined recursively by $\tau_1 = T_{b'}$,

$$\begin{aligned}
\tau_{2n} &= \tau_{2n-1} + (\tau_a^+ \wedge \tau_{a-b}^-) \circ \theta_{\tau_{2n-1}}, \\
\tau_{2n+1} &= \tau_{2n} + T_{b'} \circ \theta_{\tau_{2n}},
\end{aligned}$$

and $(\theta_t)_{t \in \mathbb{R}_+}$ is shift-operator. This definition means that the bank goes through the nth PCA at time τ_{2n-1}, and recovers or becomes insolvent at time τ_{2n}.

Additionally, we need the asset values at time τ_{2n-1} before pushed up, so in the same way as Section 3, we define $\underline{X}_n = X_{\tau_{2n-1}}$ for $n = 1, 2, \ldots$ and then, redefine $X_{\tau_{2n-1}} = S_{\tau_{2n-1}} - a$.

Then the cost $C_n(x,s;b')$ for the nth PCA can be represented by

$$C_n(x,s,b') = \mathbb{E}^{x,s}\Bigg[\mathbb{1}_{A_n}\Bigg(e^{-q\tau_{2n-1}}\left(e^{S_{\tau_{2n-1}}-a}-e^{\underline{X}_n}\right)+\alpha\int_{\tau_{2n-1}}^{\tau_{2n}}e^{-qt}\mathrm{d}t + e^{-q\tau_{2n}}\left(\beta e^{S_{\tau_{2n-1}}-b}\right)\mathbb{1}_{\{\tau_a^+\circ\theta_{\tau_{2n-1}}>\tau_{a-b}^-\circ\theta_{\tau_{2n-1}}\}}\Bigg)\Bigg],$$

where A_n is the event in which the bank is under regulator's strict supervision more than n times until insolvency; that is, A_n can be written by $A_1=\Omega$ and $A_n=\bigcap_{k=1}^{n-1}\{\tau_a^+\circ\theta_{\tau_{2k-1}}<\tau_{a-b}^-\circ\theta_{\tau_{2k-1}}\}$ for $n=2,3,\ldots$, and the total cost $C(x,s,b')$ is given by

$$C(x,s;b') = \sum_{n=1}^{\infty}C_n(x,s;b').$$

Proposition 5.1. *If $S_0=X_0=s$, then*

(5.1)
$$C_{n+1}(s,s;b') = \frac{W_1^{(q)}(b-a)}{W_1^{(q)}(b)}\int_s^\infty \mathrm{d}m\exp\left(-(m-s)\frac{W_0^{(q)\prime}(b')}{W_0^{(q)}(b')}\right)C_n(m,m;b') \times\left(\frac{\sigma^2}{2}\left(\frac{(W_0^{(q)\prime}(b'))^2}{W_0^{(q)}(b')}-W_0^{(q)\prime\prime}(b')\right)+\iint_E\Pi(\mathrm{d}h)\mathrm{d}y\left(W_0^{(q)\prime}(y)-\frac{W_0^{(q)\prime}(b')}{W_0^{(q)}(b')}W_0^{(q)}(y)\right)\right),$$

for $n=1,2,\ldots$.

Proof. The first-time PCA ends at time $t=\tau_2$. Since X^1 only have negative jumps by definition, we have $S_{\tau_2}=S_{T_{b'}}$. Hence

$$\begin{aligned}
C_{n+1}(s,s;b') &= \mathbb{E}^{s,s}\left[e^{-q\tau_2}\mathbb{1}_{A_2}C_n(S_{\tau_2},S_{\tau_2};b')\right]\\
&= \mathbb{E}^{s,s}\left[e^{-qT_{b'}}C_n(S_{T_{b'}},S_{T_{b'}};b')\left(e^{-q(\tau_a^+\wedge\tau_{a-b}^-)}\mathbb{1}_{\{\tau_a^+<\tau_{a-b}^-\}}\right)\circ\theta_{T_b'}\right]\\
&= \mathbb{E}^{s,s}\left[e^{-qT_{b'}}C_n(S_{T_{b'}},S_{T_{b'}};b')\right]\mathbb{E}^{s,s}\left[\left(e^{-q(\tau_a^+\wedge\tau_{a-b}^-)}\mathbb{1}_{\{\tau_a^+<\tau_{a-b}^-\}}\right)\circ\theta_{T_b'}\Big|\mathcal{F}_{T_b'}\right]\\
&= \mathbb{E}^{s-a,s}\left[e^{-q(\tau_a^+\wedge\tau_{a-b}^-)}\mathbb{1}_{\{\tau_a^+<\tau_{a-b}^-\}}\right]\left(\int_s^\infty\mathbb{E}^{s,s}\left[e^{-qT_{b'}}\mathbb{1}_{\{S_{T_{b'}}-\underline{X}=b',S_{T_{b'}}\in\mathrm{d}m\}}\right]C_1(m,m;b')\right.\\
&\quad\left.+\iiint_D\mathbb{E}^{s,s}\left[e^{-qT_{b'}}\mathbb{1}_{\{S_{T_{b'}-}-X_{T_{b'}-}\in\mathrm{d}y,\underline{X}-X_{T_{b'}-}\in\mathrm{d}h,S_{T_{b'}-}\in\mathrm{d}m\}}\right]C_1(m,m;b')\right).
\end{aligned}$$

From Theorem 1 and 2 in [Pistorius, 2007], we have (5.1). □

As for the finiteness of $C_n(s,s;b')$, the following remark can be shown in the same way as Remark 3.1.

Remark 5.1. If $C_n(m,m;b')<\infty$ for $m\in[s,\infty)$, then $C_{n+1}(s,s;b')<\infty$. Hence by Remark 3.1, if $1-W_0^{(q)\prime}(b')/W_0^{(q)}(b')<0$, then $C_n(s,s;b')<\infty$ for every $n\geq 1$.

Since we already calculated C_1 in the previous subsection, C_n is obtained by repeatedly using this proposition when $S_0=X_0$. The following two propositions are for the other cases; $S_0-X_0\geq b'$ and $S_0-X_0\in(0,b')$. We skip the proofs here since the essential techniques used are the same as in the propositions above. Note that the results of Proposition 5.1 and Proposition 5.2 are needed for the calculation in Proposition 5.3.

Proposition 5.2. *If $S_0 - X_0 \geq b'$, then*

$$C_{n+1}(x, s; b') = \frac{W_1^{(q)}(b-a)}{W_1^{(q)}(b)} C_n(s, s; b'). \tag{5.2}$$

Proposition 5.3. *If $S_0 - X_0 \in (0, b')$, then*

$$\begin{aligned} C_n(x, s; b') = &\frac{\sigma^2}{2}\left(W_0^{(q)}(b'-s+x) - \frac{W_0^{(q)\prime}(b')}{W_0^{(q)}(b')} W_0^{(q)}(b'-s+x)\right) C_n(s-b', s; b') \\ &+ \iint_E \Pi(\mathrm{d}h)\mathrm{d}y \left(\frac{W_0^{(q)}(b'-s+x)}{W_0^{(q)}(b')} W_0^{(q)}(y) - W_0^{(q)}(y-s+x)\right) C_n(s-y+h, s; b') \\ &+ \frac{W_0^{(q)}(b-s+x)}{W_0^{(q)}(b)} C_n(s, s; b'). \end{aligned} \tag{5.3}$$

where $C_n(\cdot, \cdot; b')$'s on the right-hand side can be computed by Propositions 5.1 and 5.2.

6. Appendix

6.1. **Scale functions.** Associated with every spectrally negative Lévy process, there exists a (q-)scale function

$$W^{(q)} : \mathbb{R} \mapsto \mathbb{R}; \quad q \geq 0,$$

that is continuous and strictly increasing on $[0, \infty)$ and is uniquely determined by

$$\int_0^\infty e^{-\beta x} W^{(q)}(x)\mathrm{d}x = \frac{1}{\psi(\beta) - q}, \qquad \beta > \Phi(q).$$

where

$$\Phi(q) = \sup\left\{\lambda > 0 : \psi(\lambda) = q\right\}, \quad q \geq 0.$$

Fix $a > x > 0$. If τ_a^+ is the first time the process goes above a and τ_0 is the first time it goes below zero, then we have

(6.1)

$$\mathbb{E}^x\left[e^{-q\tau_a^+} 1_{\{\tau_a^+ < \tau_0,\, \tau_a^+ < \infty\}}\right] = \frac{W^{(q)}(x)}{W^{(q)}(a)} \quad \text{and} \quad \mathbb{E}^x\left[e^{-q\tau_0} 1_{\{\tau_a^+ > \tau_0,\, \tau_0 < \infty\}}\right] = Z^{(q)}(x) - Z^{(q)}(a)\frac{W^{(q)}(x)}{W^{(q)}(a)},$$

where

$$Z^{(q)}(x) := 1 + q\int_0^x W^{(q)}(y)\mathrm{d}y, \quad x \in \mathbb{R}.$$

Here we have

$$W^{(q)}(x) = 0 \quad \text{on} \quad (-\infty, 0) \quad \text{and} \quad Z^{(q)}(x) = 1 \quad \text{on} \quad (-\infty, 0]. \tag{6.2}$$

We also have

$$\mathbb{E}^x\left[e^{-q\tau_0}\right] = Z^{(q)}(x) - \frac{q}{\Phi(q)} W^{(q)}(x), \quad x > 0. \tag{6.3}$$

In particular, $W^{(q)}$ is continuously differentiable on $(0,\infty)$ if Π does not have atoms and $W^{(q)}$ is twice-differentiable on $(0,\infty)$ if $\sigma > 0$; see, e.g., [Chan et al., 2011].

Fix $q > 0$. The scale function increases exponentially;

$$W^{(q)}(x) \sim \frac{e^{\Phi(q)x}}{\psi'(\Phi(q))} \quad \text{as } x \uparrow \infty. \tag{6.4}$$

There exists a (scaled) version of the scale function $W_{\Phi(q)} = \{W_{\Phi(q)}(x); x \in \mathbb{R}\}$ that satisfies

$$W_{\Phi(q)}(x) = e^{-\Phi(q)x} W^{(q)}(x), \quad x \in \mathbb{R} \tag{6.5}$$

and

$$\int_0^\infty e^{-\beta x} W_{\Phi(q)}(x) \mathrm{d}x = \frac{1}{\psi(\beta + \Phi(q)) - q}, \quad \beta > 0.$$

Moreover $W_{\Phi(q)}(x)$ is increasing, and as is clear from (6.4),

$$W_{\Phi(q)}(x) \uparrow \frac{1}{\psi'(\Phi(q))} \quad \text{as } x \uparrow \infty. \tag{6.6}$$

Regarding its behavior in the neighborhood of zero, it is known that

$$W^{(q)}(0) = \left\{ \begin{array}{ll} 0, & \text{unbounded variation} \\ \frac{1}{d}, & \text{bounded variation} \end{array} \right\} \quad \text{and} \quad W^{(q)\prime}(0+) = \left\{ \begin{array}{ll} \frac{2}{\sigma^2}, & \sigma > 0 \\ \infty, & \sigma = 0 \text{ and } \Pi(0,\infty) = \infty \\ \frac{q+\Pi(0,\infty)}{d^2}, & \text{compound Poisson} \end{array} \right\} \tag{6.7}$$

where $d := \mu - \int_{(-1,0)} x \Pi(\mathrm{d}x)$ and $\Pi(\cdot)$ is the Lévy measure as in (2.1). See Lemmas 4.3-4.4 of [Kyprianou and Surya, 2007].

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