Evolutionary stability of de jure and de facto standards

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Abstract. This paper is intended as a step towards a theoretical economic analysis of standards. It is desired that economists can understand the standards more completely, and discussions about the standard-setting policy can be more refined. First, there are two conditions for a unanimously chosen standard, which is called a de jure standard, to be established: the critical mass and the path dependence. Second, a de facto standard depends on the degree of difficulty agreeing on compatibility: the de facto standard can be established even in the case where the de jure standard cannot be expected to be established.

Key words: Standards - Critical mass - Path dependence

JEL-classification: C72; L13

1 Introduction

Most economists would accept the idea that a problem of *standards* is the most important element for understanding the modern economic environment. All over the world certain specifications provide us with dominant standards, for example: DOS/V in the field of personal computers, Windows in the field of operating software, Pentium in the field of micro processor units, and Netscape in the field of WWW browsers. The phenomenon of *uniformity* embodied in the standards seems to be paradoxical in this diverse present age. Nevertheless, *standardization* is a field where big firms compete intensely with each other, especially in network industries. Each firm puts forth much effort to make its own specification the industrial standard. The DVD (Digital Video Disk or Digital Versatile Disk) standard, which will be discussed as a case study in Section 5, is another good example because two different specifications rivaled for a unified standard. The reason is that the firm whose specification becomes the standard can benefit greatly from earning license fees and leading the way in the research and development (R&D) of manufactured products.

This paper is intended as a step towards theoretical economic analysis of the *standards problematic*. Several interesting characteristics of standards and standardization are examined by the model. Although the approach is theoretical and abstract, new points of view towards the standards are given through the discussions in this paper. As a result, economists can understand the standards more completely, and discussions about the standard-setting policy can be refined more. It is useful to describe the knowledge and technique applied in this paper before moving on to the main task. Hawkins et al. (1995) presented some clues for an economic analysis of the standards. Katz and Shapiro (1985, 1986) and Farrell and Saloner (1985, 1986) discussed the limited private incentive to establish compatibility and the first mover advantage. Furthermore, Farrell and Saloner (1988), whose subject is closely related to mine, also argued that the committee system in adopting standards was superior to the market system in respect to social welfare, while the market system was superior to the committee system in respect to speed. Foray (1995) also analyzed the same problem of standardization as analyzed in this paper from a viewpoint of uniform multi-person prisoner's dilemma. Arthur (1989) discussed that an economic system under increasing returns was often thought to be connected to complexity or positive feed back and was remarkably different from those under decreasing returns. Kandori and Rob (1995) demonstrated that an evolutionary process was driven by two forces, namely players switching to the best response against the present strategy and players experimenting with a new strategy from a viewpoint of evolutionary game[1].

The organization of this paper is as follows: Section 1 has provided an introduction. Section 2 prepares a preliminary examination of the standards and presents concise definitions of a de jure standard and a de facto standard. Section 3 investigates theoretically the de jure standard and explains a critical mass condition and a historical path dependence condition. Section 4 investigates theoretically the de facto standard and demonstrates that difficulty in reaching an agreement between the standards results in the phenomenon of complexity. Section 5 presents a case study of the formative process of the DVD standard. Finally, Section 6 presents the main conclusions of this paper.

2 A brief overview of standards

It will be useful to pause here to take a brief look at definitions of standards before turning to the main analysis. According to David (1995), the term *standards* is defined as follows: "a set of technical specifications that may be adhered to by a producer, either tacitly or as a result of a formal agreement (

1 An exhaustive survey on the evolutionary game theory was made by Weibull (1995), who discussed the selective criterion of multiple Nash-equilibria in full detail. It was Krugman (1991) and Fujita (1995) who combined models of monopolistic competition and evolutionary game to investigate the cumulative process through endogenous agglomeration forces. Their studies made it clear that nonlinear forces are possible sources of the complexity in economic systems.

ex. reference, minimum quality, interface, or compatibility)." Furthermore, David considered the definitions of standards under the following headings: 1) *unsponsored standard*, a set of specifications that has no identified originator holding a proprietary interest nor any subsequent sponsoring agency, but nevertheless exists in a well-documented form in the public domain, 2) *sponsored standard*, where one or more sponsors holding a direct or indirect proprietary interest (ex. suppliers, users, or private cooperative ventures into which firms may enter) creates inducements for other firms to adopt particular sets of technical specifications, 3) *agreement standard*, which is arrived at and published by voluntary standard-writing institutions, such as the organizations belonging to the American National Standards Institute (ANSI), and 4) *mandated standard*, which is promulgated by governmental agencies that have some regulatory authority.

Let us attempt to extend the definitions given by David a little more. Type 1 and Type 2 are formed through market processes and are called *de facto standards*.   Keyboard arrangement *QWERTY* is a notable example of Type 1. The VHS system of VTR and WINDOWS provide examples of Type 2. On the other hand, Type 3 and Type 4 are formed through the discretion of a committee which imposes technical standards and are called *de jure standards*. An example of Type 3 is MPEG 2 of the moving picture compressing technique. An example of Type 4 is a legislation which prohibits factories from expelling a harmful substance such as nitrogen monoxide. In addition, Type 1 to Type 3 are established as agreements inside each industry and are called *voluntary standards*. There are many organizations aimed at supporting voluntary standards. The International Organization for Standardization (ISO), which debates global standards and functions as an international forum for adjusting standardization, consists of organizations which set standards in more than 120 countries[2]. For example, the American National Standards Institute (ANSI), the

[2] There may not be much difference between standards that are determined by industrial agreements and by public committees (such as the ISO). The ISO decisions are not always mandatory, so they really resemble industrial decisions.

British Standards Institute (BSI), and the Japan Industrial Standards Committee (JISC) are representative signatories of those countries. Finally, Type 4, so-called *technical regulation*, is primarily regulated by laws. There are two kinds of technical regulations: a *treaty* which has binding force and a *recommendation* which has no binding force. The discussion above is summarized in Table 1.

<Insert Table 1. here.>

For the present, it is useful to look more closely at some features of the standards from the viewpoint of economics. In the contemporary network industries, the de jure standard, especially the mandated standard (or the technical regulation), is gradually declining in importance because its formation requires a long and tedious process and because it cannot maintain neutrality for some specific members' interests[3]. As Naemura (1995) acutely points out, "As computing, telecommunication, broadcasting, and so on converge, and we enter an era of multimedia and competition on a global scale, the value of all standards depends more and more on their market acceptance rather than the result of voting in a committee. The market changes very rapidly and often in an unexpected manner, and it does not usually care whether the applied standards are official or de facto as long as they exist and meet the needs" (p.94). On the other hand, the voluntary standard, especially the de facto standard, is increasing in importance gradually. However, it does not necessarily imply that every standardization should be at the mercy of a market process. As Kindleberger (1983) demonstrated, the standardization in itself possesses the properties that usually are associated with *public goods*: the indivisibility of whatever benefits the goods provides for all the

3 It does not mean that the mandated standard is useless. On the contrary, Leveque (1995) seems right in thinking that a public role is important in determining *social standards*, "Those designed to protect individuals, for example those related to health, form another category which serve to reduce negative social externalities. . . . Social standards are not supported by market mechanisms. As a consequence, self-regulation cannot be expected to emerge. However, public and government pressure which pose threats to industrial activities may then provide incentives to firms to adopt social standards" (pp.106-107).

different members of the group and the condition that every member of the group has equal access to the total quantity of the goods available. Since standards are some kind of public goods, market failures could arise. Besen (1995) described three points of possible market failure in standardization: 1) competition to become a de facto standard may ensue for many years before a system of one sponsor grows to the point at which it achieves dominance, 2) without a centralized standards-setting some users might be stranded as orphans with a non-standard technology, and 3) the market process could adopt the wrong technology as a standard because adopters do not take into account the effects of their choices on the well-being of others. Thus, the government could formulate some policies to some extent in order to remedy the market failures of standards[4].

In this study, the main stress falls on the voluntary standard, where much attention is focused these days. A problem may arise. The standards which cannot be classified easily into either de jure standard or de facto standard have increased gradually. Nonetheless, we will follow the conventional usage: the standards in this paper are divided into two types. First, players join a committee and hold a deliberation on establishing the standard. Only when they reach a unanimous consensus, or absolute majority, on the standard can the de jure standard be set up. Second, if players fail to reach a unanimous agreement to establish the de jure standard, a part of them may try to fix the de facto standard, whose compatibility can be valid within their subgroup, and then try to gain influence in the market share.

3 Theoretical analysis of de jure standard

4 For example, Repussard (1995) took several indirect support systems which a government could adopt in setting up a volunteer standard: 1) participation of the government experts in the standards drafting groups, 2) support (usually financial) for particular groups for their participation in standards drafting (ex. consumer groups, trade unions, or small and medium-sized enterprises), 3) education and promotion, 4) allocation of R&D funds to help laboratories prepare technical input standardization, and 5) reference to standards in technical legislation, along with setting specific requirements for such standards.

This section will consider a formative process for the *de jure standard*, which I define as a condition where all players agree on one unanimous standard. First, the setting and the timing of the model are discussed. Second, other details, such as the utility functions, the income constraints, and the maximization problems, are defined. Third, the model is formulated as a noncooperative two-player-and-two-strategy game. Fourth, the result of the model, especially the evolutionary stability of the equilibrium, is investigated. Finally, the implications of the model are discussed. It shall be concluded that the necessary conditions for the de jure standards to be in equilibrium are the *critical mass condition* and the *path dependence condition*.

3.1 Setting and timing

A simple model with n players, who are both producers and consumers, such as an independent *yeoman*, shall be investigated[5]. Each player, $i \in [1,2,\cdots,n]$, belongs to one *network* or has one *software*, which is specified individually by i. The specification can be called a *system*. What has to be noticed is that there are two kinds of systems: a *compatible system* and an *incompatible system*. Under the incompatible system the network or software of i can work only on the system of i, whereas under the compatible system not only the network or software of i but also other network or software of $j(\neq i)$ can operate on the system for i (see Fig. 1).

<Insert Fig. 1. here>

5 The assumption that firms and consumers are the same agents, and hence that the number of potential standards equals the number of consumers/producers may be odd. The distinction between producers (who are the standard setters) and consumers (who merely choose one standard) should be maintained. However, if we assume the distinction between them, the theoretical analysis would be much more complicated. I think that my simple model is just a first approximation to describe the formation process of standards in a way.

The timing of the game is given as a two-period model (see Fig. 2). In the 1st period, each player chooses one of the alternative strategies, [compatibility, incompatibility][6]. In general, it takes a lot of time and cost to establish compatibility. Whether a standard is established or not, the player who chooses compatibility must pay the cost of compatibility. The following assumption of unanimous agreement is imposed: the de jure standard is established only when all players choose compatibility in the 1st period. Next, in the 2nd Period, each player maximizes his own utility subject to an income constraint. The following assumption of utility function form is imposed: in a monopolistic competition model, the utility function between software/network-goods and other goods is given according to the Cobb-Douglas type, and the sub-utility function of software/network-goods is given as the Constant-Elasticity-Substitution type. Such a model has the following convenient characteristics: 1) it deals with a discrimination of specification and represents clearly a preference for a variety of software/network-goods, 2) it abstracts a price cross effect and leads to a clear conclusion, and 3) its optimum conditions are presented simply and suited for a numerical calculation.

<Insert Fig. 2. here.>

3.2 Other details of the model

The utility functions, the income constraints, and the maximization problems are discussed here. The case where both players choose compatibility is called a *compatibility case*, while the case where at least one player chooses incompatibility is called an *incompatibility case*. The utility functions, the income constraints, and the maximization problems are explained first in the compatibility case and next in the incompatibility case.

6 Here, *two-way* compatibility is assumed. If firm *i* makes its brand compatible with brand *j*, it means that brand *j* is also compatible with brand *i*.

First, the utility function in the incompatibility case is given as follows:

$$U_i^I = (C_{Si})^{\mu} (C_{Hi})^{1-\mu}, \quad (1)$$

given that

U_i^I means the utility of the player *i* in the case of incompatibility *(I)*,

C_{Si} means the software/network-goods consumption *(S)* of the player *i* ,

C_{Hi} means the other goods consumption *(H)* of the player *i* ,

$\mu \in (0,1)$ means the ratio of expenditure of the software/network consumption *(S)* to the total expenditure[7].

The income constraint in the incompatibility case is given as follows:

$$Y_i = D_i F_i + P_{Si} C_{Si} + P_{Hi} C_{Hi}, \quad (2)$$

given that

Y_i means the income of the player *i*,

$D_i \in [0,1]$ means the dummy variable for the strategy of the player *i* at the 1st period: $D_i=0$ represents that the player chooses the strategy of incompatibility while $D_i=1$, the strategy of compatibility[8].

F_i means the cost of compatibility which the player *i* pays,

P_{Si} means the price of software *i*,

7 For our purpose, the distinction between software/network-goods and other goods may not be important. Therefore, we can ignore it without any loss of generality in the following analysis by setting $\mu=1$.

8 Here, it should be noted that even in the incompatibility case, where at least one player chooses the strategy of incompatibility ($D_i=0$), one player might choose the strategy of compatibility ($D_j=1$). Once he chooses the strategy of compatibility ($D_j=1$), he must pay the cost of compatibility (or make some efforts), namely F_j.

P_{Hi} means the price of other goods *i*.

The maximization problem is "MAX (1) s.t. (2)", and its first-order conditions in the incompatibility case are given as follows:

$$C_{Hi} = \frac{(1-\mu)(Y_i - D_i F_i)}{P_{Hi}}, \tag{3}$$

$$C_{Si} = \frac{\mu(Y_i - D_i F_i)}{P_{Si}}. \tag{4}$$

Next, the utility function in the compatibility case is given as follows:

$$U_i^C = \left(\int_0^n (C_{Si})^\rho di\right)^{\frac{\mu}{\rho}} (C_{Hi})^{1-\mu}, \tag{5}$$

given that

U_i^C means the utility of player *i* in the case of compatibility *(C)*,

n means the number of players (or the variety of compatible software/networks),

$\rho \in (0,1)$ means the substitution parameter of the sub-utility function.

If we prefer a discrete form to a continuous form, the utility function can be represented as follows:

$$U_i^C = \left(\sum_{i=1}^n (C_{Si})^\rho di\right)^{\frac{\mu}{\rho}} (C_{Hi})^{1-\mu}. \tag{5'}$$

In either case, the first-order conditions are identical.

The income constraint in the compatibility case is given as follows:

$$Y_i = F_i + \int_0^n P_{Si} C_{Si} di + P_{Hi} C_{Hi} \tag{6}$$

The maximization problem is "MAX (5) s.t. (6)", and its first-order conditions in the compatibility case are given as follows:

$$C_{Hi} = \frac{(1-\mu)(Y_i - F_i)}{P_{Hi}}, \tag{7}$$

$$C_{Si} = \frac{\mu(Y_i - F_i)}{P_{Si}} \frac{(P_{Si})^{-\frac{\rho}{1-\rho}}}{\int_0^n (P_{Si})^{-\frac{\rho}{1-\rho}} di}. \tag{8}$$

For simplicity purposes, an assumption of symmetry is imposed[9]: the functional form of each player is assumed to be symmetrical; $P_{Si}=P_S$, $C_{Si}=C_S$, $C_{Hi}=C_H$, $Y_i=Y$ and $F_i=F/m$ for any i, where F is the gross cost of compatibility, and m is the number of the players who choose compatibility in the 1st period. One may notice that from the assumption of unanimous agreement, the compatibility can be attained only at $m=n$. From the assumption of symmetry, for example, (8) can be rewritten as follows:

$$C_S = \frac{\mu(Y - \frac{F}{n})}{nP_S}. \tag{8'}$$

3.3 Two-strategy game: compatibility and incompatibility

In this subsection, the model is formulated as a noncooperative two-player-and-two-strategy game. The player i, whose payoff matrix is presented in Table 2, negotiates with any representative player $j \in n$ $(j \neq i)$. With no loss of generality, the utility of the other goods is assumed to be zero, $\mu=1$, and the following is defined:

<Insert Table 2. here.>

9 This assumption of symmetry is just concerning parameters. Hence, it does not mean that every player chooses the same strategy. However, the assumption of symmetry certainly takes away the interesting strategic flavor of the model.

$$U_i^{11} = n^{\frac{1-\rho}{\rho}} \frac{Y - \frac{F}{n}}{P_s},\ U_i^{12} = \frac{Y - F}{P_s},\ U_i^{21} = \frac{Y}{P_s},\ U_i^{22} = \frac{Y}{P_s}. \tag{9}$$

Only when all players agree to choose compatibility can the de jure standard be established. While $U_i^{12} < U_i^{22}$ always holds, the relative magnitude of U_i^{11} or U_i^{21} depends on the parameters, ρ, Y, n, and F. When $\Delta^{11\text{-}21} = P_s(U_i^{11} - U_i^{21})$ is defined, the following is obtained:

$$\Delta^{11\text{-}21} = n^{\frac{1-\rho}{\rho}} \left(Y - \frac{F}{n}\right) - Y. \tag{10}$$

Then, the following partial derivatives are obtained:

$$\frac{\partial \Delta^{11-21}}{\partial \rho} < 0,\ \frac{\partial \Delta^{11-21}}{\partial Y} > 0,\ \frac{\partial \Delta^{11-21}}{\partial n} > 0,\ \frac{\partial \Delta^{11-21}}{\partial F} < 0. \tag{11}$$

Given that the rival player *j* chooses compatibility, the incentive for the player *i* to choose compatibility is represented by $\Delta^{11\text{-}21}$, which increases as the initial income Y and the number of players n increase and decreases as the substitution parameter ρ and the cost of compatibility F increase.

3.4 Evolutionary stability of equilibrium

The result of the model, especially the evolutionary stability of the equilibrium, shall be investigated here. The parameter n is defined as a function of time t, namely $n(t)$. If the initial number of players is small enough, U_i^{11} is smaller than U_i^{21}:

$$n(0) \quad 1 \rightarrow \Delta^{11\text{-}21} < 0. \tag{12}$$

It is assumed that the number of players increases as time passes. Since $\Delta^{11\text{-}21}$ is an increasing function of the number of players *n*, there must be some number *n** which satisfies $\Delta^{11-21}=0$. The number *n** is called the *critical mass*, and the time τ* which corresponds to *n** is called the *critical time.* For example, the values specified in (13) are assigned to (10):

$$n(0)=0,\ \rho=0.25,\ \Upsilon=4,\ F=1,\ dn/dt=0.1. \quad (13)$$

Then, it follows that the critical mass $n^*=1.07$, and the critical time $t^*=10.7$.

The equilibrium pattern of the game can be classified into three categories: before the critical time, at the critical time, and after the critical time. The equilibrium pattern in each category shall be analyzed from the perspective of evolutionary stability, which implies the dynamic stability of agreement on the system compatibility among players. According to the evolutionary game theory[10], it is assumed that a dynamic process follows *replicator dynamics*[11]. To sum up, if the difference of the expected payoff between a pure strategy *[0,1]* and a mixed strategy $[1-x, x]$ is positive (negative), given that an opponent chooses a mixed strategy $[1-x, x]$, *x* would increase (decrease):

10 Evolutionary game theory is summarized as follows by Weibull (1995), "A key concept in evolutionary game theory is that of an evolutionarily stable strategy. Such a strategy is robust to evolutionary selection pressures in an exact sense. Suppose that individuals are repeatedly drawn at random from a large population to play a symmetric two-person game, and suppose that initially all individuals are genetically or otherwise 'programmed' to play a certain pure or mixed strategy in this game. Now inject a small population share of individuals who are likewise programmed to play some other pure or mixed strategy. The incumbent strategy is said to be evolutionarily stable if, for each such mutant strategy, there exists a positive invasion barrier such that if the population share of individuals playing the mutant strategy falls below this barrier, then the incumbent strategy earns a higher payoff than the mutant strategy," (p.33).

11 Weibull (1995) explained, "We here address the question how stability in the replicator dynamics is related to the criteria of evolutionary and neutral stability of the associated mixed strategy. It turns out that every population state in the subset Δ^{ESS} is asymptotically stable in the replicator dynamics, and every population state in Δ^{NSS} is Lyapunov stable. In this sense evolutionary stability implies asymptotic stability, and neutral stability implies Lyapunov stability," (p.95).

$$dx/d\tau = [EU_i([0,1],[1-x, x]) - EU_i([1-x, x],[1-x, x])]x$$
$$= [(U_i^{22} - U_i^{12})x + (U_i^{21} - U_i^{11})(1-x)]x(1-x). \quad (14)$$

Before the critical time *(t<t*)*, both $U_i^{11} < U_i^{21}$ and $U_i^{12} < U_i^{22}$ hold. Since the incompatibility strategy strictly dominates the compatibility strategy for the player *i*, it is not only a symmetric Nash equilibrium but also an evolutionarily stable equilibrium for each player to choose incompatibility. The evolutionary stability in this case is represented as Fig. 3a. It is assumed that the ratio *x* of players choose incompatibility, whereas the ratio *(1-x)* of players choose compatibility. The term τ is used to represent the adjustment time of agreement formation, and also the parameter *x* is assumed to be a function of τ. In this case, it can be concluded that all players finally come to choose incompatibility for any initial ratio *x(0)*:

<Insert Fig. 3a here.>

$$\lim_{\tau \to \infty} x(\tau) = 1. \quad (15)$$

At the critical time *(t=t*)*, both $U_i^{11} = U_i^{21}$ and $U_i^{12} < U_i^{22}$ hold. Since the incompatibility strategy weakly dominates the compatibility strategy for the player *i*, it is a symmetric Nash equilibrium for each player to choose either compatibility or incompatibility unanimously. The two pure-strategy Nash equilibria, nevertheless, have quite different characteristics. Incompatibility is an evolutionarily stable strategy, but compatibility is an evolutionarily unstable strategy. The evolutionary stability in this case is represented as Fig. 3b. It can be concluded that all players finally come to choose incompatibility for any initial ratio *x(0)*, except for *x(0)=0*:

<Insert Fig. 3b here.>

$$\lim_{\tau \to \infty} x(\tau) = 1 \text{ if } x(0) \neq 0,$$
$$\lim_{\tau \to \infty} x(\tau) = 0 \text{ if } x(0) = 0. \quad (16)$$

After the critical time *(t>t*)*, both $U_i^{11} > U_i^{21}$ and $U_i^{12} < U_i^{22}$ hold. There are two pure-strategy Nash equilibria and one mixed-strategy Nash equilibrium. Generally speaking, this kind of game is called a *coordination game*. The former, the pure-strategy Nash equilibria, are evolutionarily stable, whereas the latter, the mixed-strategy Nash equilibrium, is evolutionarily unstable. Since $U_i^{11} > U_i^{22}$ holds, the symmetric compatibility equilibrium dominates the symmetric incompatibility equilibrium in the *Pareto optimum* sense. Now, the term x^* is defined as the critical ratio where the mixed-strategy Nash equilibrium exists:

$$x = \frac{\text{™}^{11-21}}{\text{™}^{22-12} + \text{™}^{11-21}}\ , \text{ where } \Delta^{ab\text{-}cd} = P\,s(U_i^{ab} - U_i^{cd}). \qquad (17)$$

It may be noted that the critical ratio x^* is an increasing function of the number of players n. The evolutionary stability in this case is represented as Fig. 3c. It can be concluded that in the case where the initial ratio is smaller than the critical ratio, $x(0) < x^*$, all players choose compatibility, whereas in the case where the initial ratio is bigger than the critical ratio, $x(0) > x^*$, all players choose incompatibility:

<Insert Fig. 3c here.>

$$\lim_{\tau\to\infty} x(\tau)=0 \ \text{ if } \ x(0)<x^*,$$
$$\lim_{\tau\to\infty} x(\tau)=x^* \ \text{ if } \ x(0)=x^*,$$
$$\lim_{\tau\to\infty} x(\tau)=1 \ \text{ if } \ x(0)>x^*. \qquad (18)$$

The results indicated in Fig. 4. are based on the parameters specified in (13). Before the critical time, only the incompatibility can be in equilibrium given any initial value of x. The critical time is given $t^*=10.7$, and the critical mass is given as $n^*=1.07$. After the critical time, the number of players reaches critical mass, if and only if the initial ratio x is smaller than the critical ratio x^*. Since the critical ratio x^* increases gradually after the critical time t^*, then the scope of where the compatibility can be in equilibrium also increases.

<Insert Fig. 4. here.>

3.5 Implications of model

It will be useful to discuss the implications of the model. Two concepts of time have been introduced in the above analysis: t represents the one with respect to the number of players n, and τ represents the one with respect to the evolutionary stability of equilibrium. Now, they are put together, $t=\tau$, and the term T is used to illustrate the formative time of the de jure standards. The implications of the model are discussed in four points. First, when time T is small enough, the number of players, or the variety of compatible software/networks, does not attain the critical mass n^*. Therefore, it is not an equilibrium strategy for all players to choose compatibility. Second, as the time T passes, the two factors, which are contrary to each other, begin to be effective: one is an increasing function of n, and the other is an increasing function of x. While the former means that the benefits which each player receives from agreeing on compatibility and hence developing the de jure standard increase, the latter means that the number of the players who choose incompatibility likewise increases gradually. Third, at the critical time T^*, the number of players attain the critical mass n^*. Then, the *mature market*, where it can be in equilibrium for all players to agree on compatibility and hence develop the de jure standard, appears. This does not necessarily mean, however, that all players choose compatibility, because incompatibility is also an evolutionarily stable strategy. Fourth, if the ratio of the players who choose incompatibility at the critical time T^* is smaller than the critical ratio where the mixed strategy becomes an equilibrium after the critical time, $x < x^*$, the ratio of the players who choose compatibility $(1-x)$ increases, and consequently the de jure standard is formed. Even if $x > x^*$ holds at the critical time, the ratio of the players who choose compatibility $(1-x)$ might increase. The reason is that the critical ratio x^* increases because of the process of evolutionary stability, whereas the ratio *x* by itself increases also. Therefore, if the rate of increase of x^* is more rapid than that of *x*, the rate of change in *x* might turn to decrease, and then the ratio of the players who choose compatibility $(1-x)$ would increase.

An example of the formation of the de jure standard is given in Fig. 5. The ratio of players choosing compatibility is given as $(1-x)$. Before the critical times, the number of players does not attain the critical mass. Therefore, since each player does not have the incentive to agree on compatibility, the ratio of players choosing compatibility decreases. However, at the initial critical time T^*, there is the incentive for all players to agree on compatibility. After the second critical time T^{**}, when the ratio of players agreeing on compatibility is larger than the critical level, $(1-x)>(1-x^*)$, the ratio of players choosing compatibility increases gradually thus leading to the formation of the de jure standard, namely a unanimous agreement on compatibility. The main point that has been made in this section is summarized as Proposition 1.

<Insert Fig. 5. here.>

Proposition 1: Two formative conditions of the de jure standard:

1) The Critical Mass Condition -- The de jure standard is formed only when the number of players or the variety of compatible software/networks reaches the critical level.

2) The Path Dependence Condition -- The ratio of players who agree on compatibility must be bigger than the critical level of the players even after the critical time when the critical mass condition is satisfied.

If the above two conditions are satisfied, all players eventually agree on compatibility, and consequently the de jure standard can be formed as a result of unanimous consensus.

4 Theoretical analysis of de facto standard

In the preceding section, the formative process of the de jure standard was discussed, and in this section, the formative process of the de facto standard will be considered. According to the definition, the term *de jure standard* refers to the case in which all *n* players come to agree on compatibility unanimously, and the term *de facto standard* refers to the case in which only some *m* players come to agree on compatibility, as Fig. 6 shows. In this section, first, the refinement of the model is discussed, second, the numerical analysis of the model is investigated, and finally, the implication of the model is examined. It shall be concluded that the de facto standard can be established even in the case where the de jure standard cannot be expected to be established.

<Insert Fig. 6. here.>

4.1 Refinement of the model

Several points, mainly about the utility function, are refined, and the *potential function* is introduced to examine the formative process of the de facto standard although the basic model proposed in the previous section is also maintained here. Among *n* players, the utility in the case where *m* players develop the de facto standard is indicated as $U_i(m)$:

$$U_i(m) = m^{\frac{1-\rho}{\rho}} \frac{Y - \frac{F}{m}}{P_S}. \qquad (19)$$

The cost of agreement is defined as a function of the number of players who participate in the de facto standard, $F(m)$. The agreement incentive for the *m* players who participate in the de facto standard is specified as $\Pi_i(m)$, which we will call the *potential function*[12]. If the potential function is positive,

12 About the notion of the potential function, see Krugman (1991) and Fujita (1995).

$\Pi_i(m)>0$, the positive incentive for *m* players to develop the de facto standard dominates. However, if the potential function is negative, $\Pi_i(m)<0$, the negative incentive for *m* players to develop the de facto standard dominates:

$$\Pi_i(m)=U_i(m)-U_i^I, \quad \text{given that } U_i^I=Y/P_S. \qquad \textbf{(20)}$$

Equation (20) is nonlinear and has a complicated pattern of solutions depending on the cost of agreement *F(m)*. For simplicity of analysis, we assume a simple cost function in Equation (19):

$$F(m)=a+bm^c, \quad \text{given that } a>0,\ b>0, \text{ and } c\geqq 0. \qquad (21)$$

Equation (21) is not specific, although seemingly, because the parameter *c* can represents a scale economy, a scale neutrality, and a scale diseconomy of the standardization process. If the parameter *c* is zero, *c=0, F(m)* is a fixed cost which is insensitive to the number of players *m*. In general, the cost of agreement is thought to be an increasing function of *m*, and therefore $c>0$ should be assumed. The larger *c*, the higher the marginal cost of agreement. If *c=1*, the marginal cost of agreement is constant. If the parameter *c* is bigger than 1, $c>1$, the marginal cost of agreement is increasing. By substituting Equation (21) into Equation (20), the next equation is obtained:

$$\Pi_i(m)=m^{\frac{1-\rho}{\rho}}\frac{Y-\frac{a+bm^c}{m}}{P_S}-\frac{Y}{P_S}. \qquad (22)$$

The critical value *m**, which satisfies $\Pi_i(m)=0$, is the equilibrium number of players whose incentive to develop the de facto standard is zero. As described in Equation (22), it depends on the parameter *c* whether an interior solution *m** may exist or not.

4.2 Numerical analysis of the equilibrium pattern

Now, the parameters are assigned as specified in Equation (23), and an interesting equilibrium pattern is analyzed numerically without serious loss of generality:

$$\rho=0.25,\ \gamma=4,\ \text{and}\ P_s=A=B=1. \tag{23}$$

By substituting Equation (23) into Equation (22), the next equation is obtained:

$$4m^3-m^{2+c}-m^2-4=0. \tag{24}$$

In case of c=0,1,2, and 3, the interior solutions are presented in Table 3..

<Insert Table 3. here.>

As described in Fig. 7, equilibrium patterns can be classified into four categories depending on the parameter *c*, which represents the magnitude of increase in marginal cost: below constant marginal cost ($c \leqq 1$), weakly increasing marginal cost ($1<c<2.9$), critically increasing marginal cost ($c \quad 2.9$), and strongly increasing marginal cost ($c>2.9$). In Fig. 7, the vertical axis represents the potential function Π_i, and the horizontal axis represents the number of members $m \in [0, n]$. Let us define a dynamic process $dm/d\tau = k\Pi_i$, where the parameter *k* is a positive constant. If Π_i is positive (negative), the number of members increases (decreases).

<Insert Fig. 7. here.>

In (i) of Fig. 7, where the marginal cost of agreeing on compatibility is below constant, there is one interior solution, $m \quad 1.2$, as well as two corner solutions, $m=0, n$. Both left and right corner solutions are dynamically stable while the interior solution is dynamically unstable because of $d\Pi_i/dm > 0$. Therefore, there are three equilibrium patterns: no-standard, de facto standard, and de jure standard, the second of which is removed from the point of dynamic stability. In (ii), where the marginal cost of agreeing on compatibility is weakly increasing, there are two interior solutions, $m \quad 1.3$ and 3.6, as well as two corner solutions, $m=0, n$. Because of $d\Pi_i/dm>0$ near $m \quad 1.3$ and $d\Pi$

$_i/dm<0$ near $m \quad 3.6$, the right corner solution and the small interior solution are dynamically unstable while the left corner solution and the large interior solution are dynamically stable. Therefore, there are four equilibrium patterns: no-standard, small de facto standard, large de facto standard, and de jure standard, the second and the last of which are removed from the point of dynamic stability. In (iii), where the marginal cost of agreeing on compatibility is critically increasing, there is one interior solution, $m \quad 1.5$, as well as two corner solutions, $m=0, n$. Because of $d\Pi_i/dm>0$ at $m<1.5$, $d\Pi_i/dm=0$ at $m \quad 1.5$ and $d\Pi_i/dm<0$ at $m>1.5$, the interior solution is a saddle point, left unstable and right stable, while the left corner solution is dynamically stable, and the right corner solution is dynamically unstable. Therefore, there are three equilibrium patterns: no-standard, de facto standard, and de jure standard, the second on the left and the last of which are removed from the point of dynamic stability. In (iv), where the marginal cost of agreeing on compatibility is strongly increasing, there is no interior solution but two corner solutions, $m=0, n$. The left corner solution is dynamically stable while the right corner solution is dynamically unstable because of $d\Pi_i/dm<0$. Therefore, there are two equilibrium patterns: no-standard and de jure standard, the latter of which is removed from the point of dynamic stability.

It follows from what has been said that there are two kinds of critical parameters: $c=1$ and $c \quad 2.9$. Table 4 indicates the relationship between the magnitude of increase in the marginal cost and the equilibrium patterns. The no-standard pattern is always in equilibrium and moreover dynamically stable. The de facto standard pattern can be in equilibrium as long as the magnitude of increase in the marginal cost is not too large, $c \leqq 2.9$. When c is intermediate, $1<c<2.9$, the large de facto standard pattern can be a dynamically stable equilibrium. Although the de jure standard pattern is always in equilibrium, it can be stable only when the magnitude of increase in the marginal cost is below constant, $c \leqq 1$.

<Insert Table 4. here.>

4.3 Implication of the model

There are several implications that can be obtained from the model. Since the equilibrium pattern changes violently near the two critical times t_1* and t_2*, a slight change in the parameters causes the standardization process to change greatly. For example, we shall discuss the case where the awareness of the importance of standards among members improves over time, so that the magnitude of increases in the marginal cost declines over time. Next, let us define $c(0)=3.5$, $dc/dt=-0.1$, and $n=5$ without serious loss of generality. The equilibrium number of members and the evolutionary stability over time can be indicated in Fig. 8. At $t=0$, the no-standard pattern is a stable equilibrium, and the de jure standard pattern is an unstable equilibrium. As time passes, it becomes easier to agree on compatibility. At $t=6$ and $m=1.6$, the de facto standard, in which 36% of the members agree, reaches a saddle point, which is an above-stable and below-unstable equilibrium. After $t=6$, the de facto standard pattern bifurcates; the equilibrium number of the stable de facto standard pattern keeps rising while that of unstable one keeps falling. At $t=16.7$, the stable de facto standard pattern reaches the 100% de jure standard and therefore corresponds to the de jure standard pattern substantially. The main point that has been made in this section is summarized in Proposition 2.

<Insert Fig. 8. here.>

Proposition 2: Effect of agreement cost on de facto standard:

When the increase in members does not lead to a large increase in the agreement cost, the de facto standard can become an equilibrium. In particular, it is interesting to note the case in which the increase in members leads to an intermediate increase in the agreement cost, the de jure standard is not a

dynamically stable equilibrium, but the de facto standard which is relatively large-scale can be dynamically stable. In this respect, the de facto standard can be established even in the case where the de jure standard cannot be expected to be established.

5 Simple case study of unification of DVD standard

This section focuses on the formation process of the standard for the DVD (Digital Video Disk or Digital Versatile Disk), which is the most interesting case of recent standardization. Further, it discusses that the models and propositions in this paper explain well the recent phenomenon of the DVD standardization. The DVD applications, which are expected to sell "more than one hundred twenty million by the year 2000" according to Toshiba, Inc. and to develop "a seven hundred billion yen (per year) market in the near future" according to Matsushita Inc., are the most promising commodities in the audio and video (AV) industry. Several kinds of DVD, such as DVD-Video, DVD-ROM, DVD-RAM, and so on, will replace the current CD, CD-ROM, LD, and even VCR because of their gigabyte information storage capacity, which will be utilized not only for AV players but also for personal computers. A single-layer DVD has 4.7 gigabytes of memory capacity (seven times that of a 12 cm CD-ROM) and can play back more than two hours of moving images with razor-sharp resolution and high-quality sound in digital format. However, there has been a fight between the two DVD standards in the 1990s similar to that of the VCR standards in the 1980s.

To discuss the formation process of the DVD standard, one must start with the *dream of Ross*. Mr. Ross, the former chairman of Time-Warner, Inc., wished strongly that movies could be enjoyed at a reasonable price at home, which raised the question of whether the 12cm optical disk could maintain a high level of picture quality. Almost all the major electronics makers denied this technological possibility. It was an ironical thing that Time-Warner never looked to Toshiba, which later became the head of an alliance of the SD standard, because it was not a major firm in the consumer electronics industry at that time. It was after Toshiba developed the standard of MPEG2 (Moving Picture Coding Experts Group 2), intended for all fields of broadcasting, communication, and computers that Toshiba and Time-Warner allied with each other for developing the DVD standard.

Originally, there were two kinds of the DVD standards, and these demonstrated the intense competition between both rivals aiming for the industrial standard: Sony and Philips advocated the standard of MMCD (Multi Media CD), which was single-sided with one-layer and had 3.7 gigabytes of memory capacity, and Toshiba advocated the standard of SD (Super Density), which combined two 0.6 mm thick palates together and had 5 gigabytes of memory capacity. Sony and Philips, which even now have a basic patent on the optical disk, planned a strategy to follow their CD format and said that their design would be less costly because manufacturing facilities used to produce the audio CD could also be used for DVD. As a result, other electronic makers were afraid they would have to pay Sony and Philips expensive license fees all the time. On the other hand, Toshiba endeavored to establish a majority against Sony and Philips by joining forces with Time-Warner and eventually won over major Hollywood studios, such as MGM/UA and Paramount Pictures, to their side. It was Matsushita that broke the deadlock. Although Sony had expected Matsushita to be at the center of its group, Matsushita stood by the Toshiba group at the last moment so that the group of seven electronics makers led by Toshiba and Matsushita gained predominance. The Sony group attempted to recover the dominant position by releasing a single-sided DVD with two layers. However, they failed because the Toshiba group promptly released a double-sided DVD with two layers.

Only three strategies were left to the outnumbered Sony group at that time: 1) to surrender to the Toshiba group and abide by the SD standard and to let it become the de jure standard, 2) to negotiate and arrange a compromise between the MMCD and the HD standard, or 3) to keep its own standard and rush into market competition for the de facto standard. At first, Sony seemed to take strong measures toward the third strategy, as they had done in the case of VCR. However, the Technical Working Group (TWG) organized by the American computer firms, such as IBM, Microsoft, and Apple, entered the picture to mediate. IBM, which had the most influential voice, leaned toward the Toshiba group and its standard. In compliance with the demands of the information industries, including the computer industry and the AV software industry in the U.S.A., Toshiba and Sony finally reached an agreement. Both considered it a top priority to take the last opportunity of unifying the DVD standard, before the summer of 1996 when DVD was going to be put on the market. However, this industrial standard was mainly based on the SD standard and only slightly based on the MMCD standard; for example, the two-layer disc system of the SD standard was adopted as the standard. We might reasonably conclude, therefore, that Sony lost through this decision. Nevertheless, the possibility that the Toshiba group and the Sony group will be divided again remains, and it is very difficult to predict if the development of the DVD standard will revert to a competition of de facto standards in the future.

6 Conclusion

In the process of this paper on formation of standards, one has met the following propositions based on the theoretical analysis. First, there are two conditions for a unanimously chosen standard, which is called the de jure standard, to be established: the critical mass and the path dependence. Second, the de facto standard depends on the degree of difficulty in agreeing on compatibility: in the case of an easy agreement, no-standard and the de jure standard become long-run equilibria; in the case of an intermediate agreement, no-standard and the de facto standard become long-run equilibria; and in the case of a difficult agreement, only no-standard becomes the long-run equilibrium.

This paper also addresses the problems of standardization discussed by others. Whether the de jure standard can be established or not depends on the critical mass and the path dependence. When the market is mature enough, every player has a positive incentive to agree on compatibility since its benefit is larger than its cost. However, even when the compatibility strategy is socially desirable, incompatibility strategy embodies Arthur and Foray's "lock-in" tendencies. Further, it is important to recognize that a small historical "chance" could cause the formation process of de jure standard to "tip" dramatically. If there are free riders, the de jure standard is even more unlikely to be established. Therefore, it could be a very important factor that government standardization policies set up a mechanism to collect license fees corresponding to each member's benefit from compatibility in the case of establishment of the de jure standard. Since an establishment of the de jure standard does need a lot of restrictions, the de facto standard has an advantage with respect to speed, cost, and consensus formation.

This paper is intended as a step towards a theoretical economic analysis of standards. It is expected that economists can understand the standards more completely and discussions about the standard-setting policy can be more refined. While the variety of software often related to a network externality and increasing returns on the consumption side cause complexity in system compatibility, it is the coordination failure among competing corporations that causes the absence of the de jure standard. In addition, since incompatibility creates no-standard and has the strongest lock-in effect, a laissez-faire policy cannot be recommended, and further, since it takes a lot of effort to agree on a mandatory standard, government technical regulation cannot be recommended as a process of formation of standardization. Therefore, it is most important to set up a voluntary (via corporate agreement) standardization mechanism with fair and transparent social rules.

It should also be noted that this paper leaves much room for the developments that are necessary in respects to theory and evidence. The models analyzed in this paper are not always general, and the case study of the DVD standard is not complete. However, the amount of research which has objects very similar to that in this paper has been increasing gradually. Accordingly, it will be important to compare and integrate the conclusions of this research and these of other research.

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Table 1. Types of standards

Unsponsored Standard	De facto Standard	Voluntary Standard
Sponsored Standard		
Agreement Standard	De jure Standard	
Mandated Standard		Technical Regulation

Fig. 1. System incompatibility and compatibility

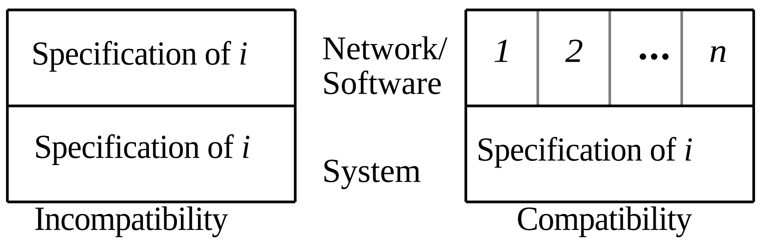


Fig. 2. Timing of game

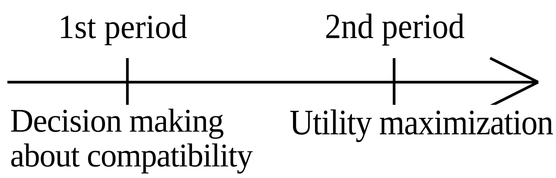
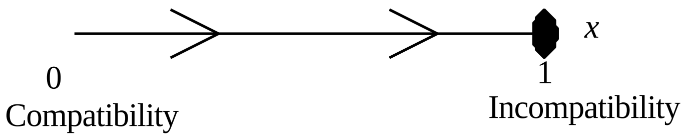


Table 2. Payoff matrix of compatibility/incompatibility game

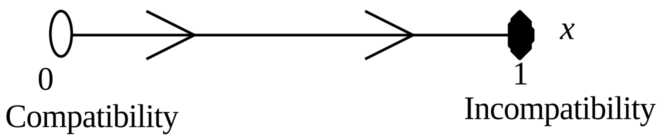
		Player *j*	
		Compatibility	Incompatibility
Player *i*	Compatibility	$\boldsymbol{U}_i^{11}$	$\boldsymbol{U}_i^{12}$
	Incompatibility	$\boldsymbol{U}_i^{21}$	$\boldsymbol{U}_i^{22}$

Fig. 3a-c.

a. Evolutionary stability of equilibrium before critical time



b. Evolutionary stability of equilibrium at critical time



c. Evolutionary stability of equilibrium after critical time

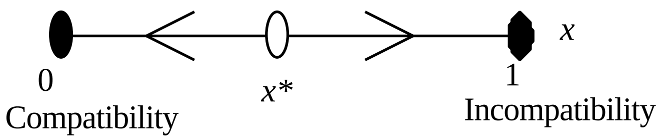


Fig. 4. Evolutionarily stable equilibrium pattern of de jure standard

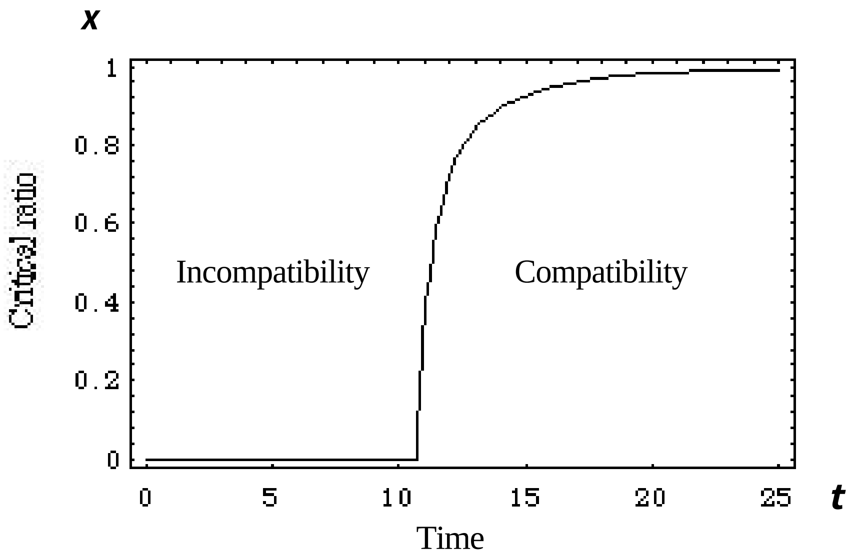


Fig. 5. Formation of de jure standard

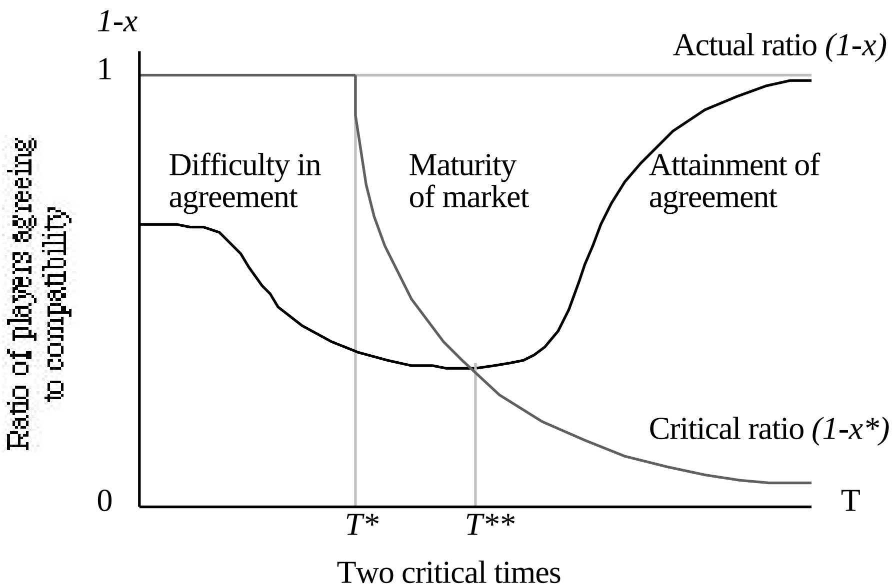


Fig. 6. De jure standard and de facto standard

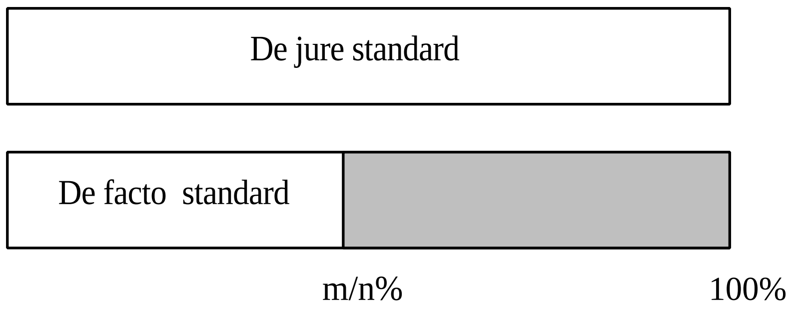
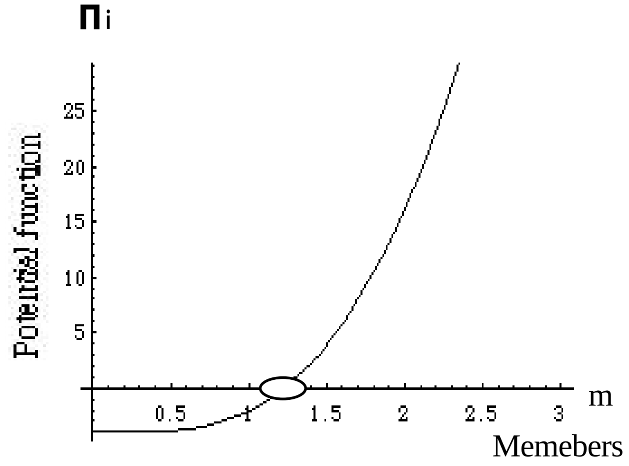


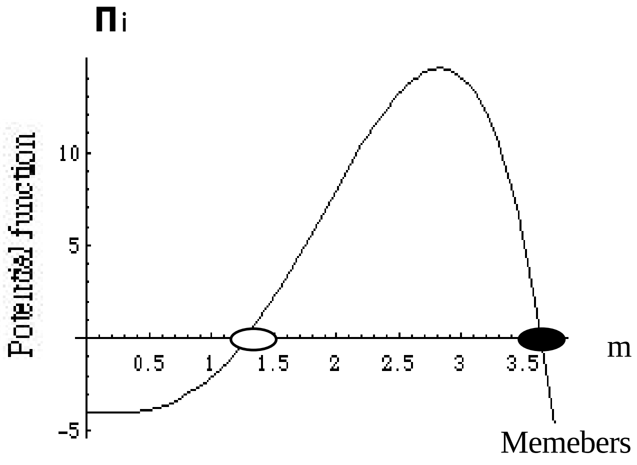
Table 3. Magnitude of increasing marginal cost and equilibrium members

	c=0	c=1	c=2	c=3
Interior equilibrium number	one	one	two	zero
Interior solution	m*=1.2	m*=1.2	m*=1.3, 3.6	–

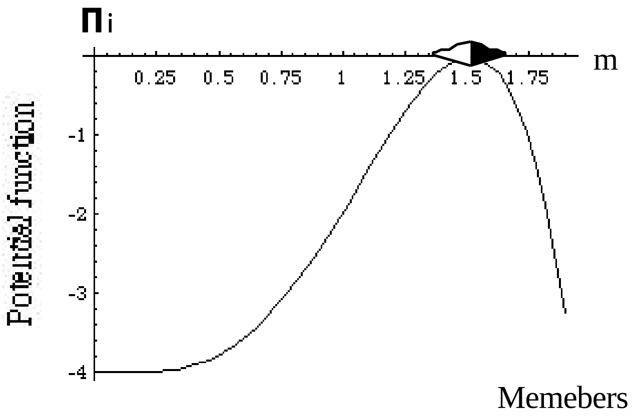
Fig. 7. Agreement cost and equilibrium pattern



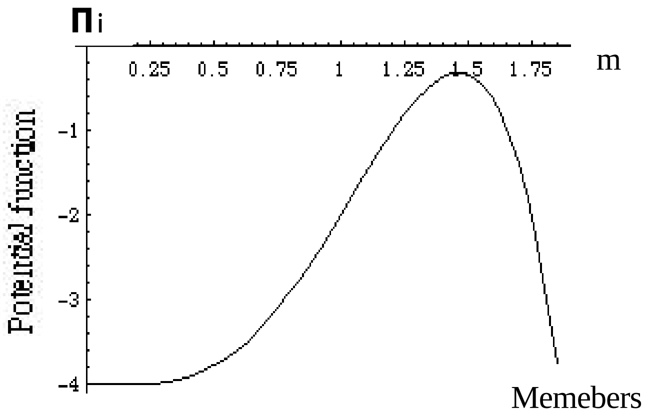
(i) Below constant marginal cost (ex. c=0.9)



(ii) Weakly increasing marginal cost (**ex.** c=2)



(iii) Critically increasing marginal cost (c 2.9)



(iv) Strongly increasing marginal cost (ex. c=3)

Table 4. Magnitude of increase in marginal cost and equilibrium pattern

	c≦1	1<c<2.9	c 2.9	2.9<c
No-standard	stable	stable	stable	stable
De facto standard	unstable	small: unstable	saddle point	none
		large: stable	(left: unstable, right: stable)	
De jure standard	stable	unstable	unstable	unstable

Fig. 8. Equilibrium number of members and evolutionary stability