Abstract:
This paper presents a simple spatial model of traffic congestion for a monocentric city, to investigate the effects of cordon pricing on trip-making and congestion level in each location. Optimal cordon pricing is obtained as a combination of the cordon location (i.e. distance of the cordon from the CBD) and the amount of toll charged there that maximizes the total social surplus in a city. Under optimal cordon pricing, trips originating from locations inside the cordon are under-priced, those just outside the cordon are over-priced, and those near the urban fringe are under-priced. Numerical simulations using the parameter values based on Japanese data suggest that cordon pricing attains an economic welfare level very close to the first-best optimum.

Running head: Cordon Pricing
1. INTRODUCTION

Cordon pricing is adopted in most practices of road pricing to control area-wide congestion in a city (e.g. Singapore, Hong Kong, Oslo, etc.)\(^1\). A typical cordon pricing system is designed as follows: each vehicle is charged a fixed toll when it passes through the specified cordon surrounding the central area of a city where traffic is most congested. Although this scheme is not the first best pricing rule for congestion management, the system is simple and easy to implement. However, it is not an easy task for a traffic control authority to rationally determine the toll level and the cordon location, since it should take into account the distortions in the market that are commonly present in the second best world. For this reason, it is unclear whether particular toll levels in the actual cases are too high or low, or whether the sizes of cordoned areas are too large or small.

This paper presents a formal economic analysis dealing with the following issues of cordon pricing: where the cordon line should be located; at what level the toll should be set. Recently, an increasing number of researchers have approached this problem using network models. May et al. [8] computed the effects of alternative road pricing schemes including cordon pricing by applying a network simulation model to the city of Cambridge, U. K. They merely examined the consequences of exogenously specified cordon locations and toll levels. Santos, et al. [10] used the same network simulation model to obtain the optimal cordon tolls for eight English towns. They did not discuss the optimal locations of cordons. The above studies were chiefly concerned with empirical estimates of toll levels and social welfare, so theoretical analysis on qualitative properties of traffic pattern and resource allocation under cordon pricing has not been presented. On the other hand, Verhoef [15], Zhang and Yang [16] discussed the mathematical problem to obtain the optimal choices of toll levels and locations of toll collection in a network. They mainly focused on methods to compute the optimal solutions, and presented the numerical results for hypothetical example networks\(^2\). Although network models are useful for practical applications, they are not suitable for investigating the general properties of the problem, since the results depend on the network structures specified for calculations. We need a model that explicitly deals with the spatial patterns of trip making behavior and traffic congestion in an idealized setting, such as the continuous space model developed in urban economics literature. Such an approach may provide common insights into the design of cordon pricing.

Urban economists have developed urban spatial models incorporating congestion effects and

\(^{1}\) Small and Gomez-Ibañez [11] provide an overview of various practices of road pricing across the world.

\(^{2}\) Recent paper by de Palma and Lindsey [2] considers the problem of finding the optimal number and locations of a tolled road in the radial network. Although they provide some interesting policy implications, the analysis is based on artificial setting in that all residents are located on a single circumference (same distance from the center).
discussed the discrepancy between equilibrium and optimal land use (e.g., Anas and Xu [1], Fujita [3], Kanemoto [4], Sullivan [12]). They showed that a congestion toll (or location tax) internalizing congestion externalities should be charged to each resident to achieve the first-best optimal allocation. Congestion externalities vary depending on locations: the levels of tolls should be differentiated by residential location. Obviously, the implementation of such a tolling policy is practically infeasible, so second best policies should be considered instead. In this direction, Kanemoto [4] focused on the problem of how road capacity at each location should be determined in the absence of a toll. Most of the earlier works in urban economics, however, consider two extreme pricing schemes, rigorous first best toll and no toll. In other words, the second-best pricing policies in the context of urban space have not been sufficiently explored. Sullivan [13] and Kraus [5] are exceptions. Sullivan examined a second best policy in which the toll is proportional to the distance traveled (this is effectively the same as fuel tax), based on the general equilibrium simulation model of urban land use. Kraus numerically calculated the welfare gains from various pricing regimes including cordon pricing. To our knowledge, Kraus’s paper is the only work that examines the effect of cordon pricing based on the urban spatial model. In the present paper, unlike Kraus, we explicitly solve the spatial patterns of trip generation, and this enables us to describe the situations of resource allocations at different locations in a city.

Another limitation in the literature on urban economics is that trip demand is assumed to be inelastic. In this case, tolling policies have no effect in the short-run where land use is unchanged. Instead, we focus on the roles of tolling policies as instruments to control trip demand generation and its spatial distribution. So, our model allows elastic trip demand while land use is assumed fixed. Each resident chooses how often to travel; the choice is affected by trip cost that varies with residential location.

This paper discusses the optimal combination of cordon location and toll level in a monocentric city. We investigate how cordon pricing affects trip demand and resource allocation at each location in a city, and evaluate the performance of the optimal cordon pricing in terms of social welfare by comparison with the welfare levels under no-toll equilibrium, or the first-best optimum toll. We further examine the effects of parameter changes by numerical simulations.

2. THE MODEL

2-1 Trip demand and cost

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3 On the other hand, there exists extensive literature on second best pricing in non-spatial setting (e.g., Liu and McDonald [6], Marchand [7], Verhoef, et al. [14]).
Suppose a linear city where CBD is located at the center (origin of coordinate) and residential areas are developed around the CBD, the land area of the CBD is negligible and the characteristic of each residential location is represented solely by distance from the CBD. It is assumed that all residents in the city are homogenous and population density is unity throughout the urban area. Each individual makes trips for some purpose from his/her residence to the CBD by car; other types of trip are neglected. Trip demand is elastic; depending on the cost of car trips, each individual chooses the number of car trips within a certain period (sometimes he/she gives up a trip or uses an alternative mode of transportation such as public transit). Let $x$ be the distance from the CBD, and $q(x)$ the trip demand of a resident located at $x$, then the (inverse) trip demand function is given as,

$$
p(q(x)) = a - bq(x),
$$

where $a, b$ are positive constants. $p(q(x))$ represents the private marginal benefit of a trip.

We assume that the cost for driving the unit distance around $x$ is an increasing function of the traffic volume there, $Q(x)$, which is denoted by $t(Q(x))$. This implies that the road width is constant at all locations. Then, the cost for a trip from $x$, $C(x)$, is given as,

$$
C(x) = \int_0^x t(Q(y))dy;
$$

Since all trips are destined for the CBD and population density is unity, traffic volume at $x$, $Q(x)$ is defined as,

$$
Q(x) = \int_x^B q(y)dy,
$$

where $B$ is the distance from the CBD to the edge of the urban area.

In this paper, the functional form of $t(Q(y))$ is specified as follows;

$$
t(Q(y)) = f + cQ(y),
$$

where $f$ and $c$ represent, respectively, the cost for driving the unit distance when the traffic volume is zero (= at free speed), and the marginal cost with respect to traffic volume.

### 2-2 No-toll equilibrium

Each individual decides to make a trip as long as the private marginal benefit exceeds the private cost. In equilibrium, the following relation holds at all locations,

$$
p(q(x)) = C(x) \quad \text{for all } x, \ 0 \leq x \leq B,
$$

together with Eqs. (2) and (3).

By substituting Eqs. (1) and (2) into Eq. (5), and differentiating both sides of the obtained equation with respect to $x$, we have,

$$
-bq'(x) - t(Q(x)) = 0,
$$
where \( q'(x) \) is the first derivative of \( q(x) \). Next, differentiating both sides of Eq. (3) with respect to \( x \) yields the following,

\[
Q'(x) = -q(x) .
\]  (7)

Differentiating Eq. (6) once more and incorporating the relations of Eqs. (4) and (7), we obtain the following differential equation,

\[
-bq''(x) + cq(x) = 0 ,
\]  (8)

where \( q''(x) \) is the second derivative of \( q(x) \). It turns out that \( q'(x) < 0 \) from (6), and \( q''(x) > 0 \) from (8)\(^4\). In other words, the number of trips per person at each location decreases with the distance from the CBD, and the rate of decrease diminishes.

The equilibrium number of trips generated at each location, \( q^*(x) \), is obtained by solving the differential equation (8), as follows:

\[
q^*(x) = \lambda_1 \exp(\alpha x) + \lambda_2 \exp(-\alpha x) ,
\]  (9)

where \( \alpha = \sqrt{\frac{c}{b}} \), and \( \lambda_1, \lambda_2 \) are unknown constants to be determined by boundary conditions.

We need two boundary conditions: the first is \( q^*(0) = \frac{a}{b} \), which is derived by evaluating Eq. (5) at \( x = 0 \) and applying \( C(0) = 0 \); the second is \( -bq^{**}(0) = f + cQ(0) \) from Eq. (6) at \( x = 0 \). With these two relations, \( \lambda_1, \lambda_2 \) are determined as follows:

\[
\lambda_1 = \frac{a \exp(-\alpha B) - \frac{f}{\alpha}}{b(\exp(\alpha B) + \exp(-\alpha B))} ,
\]

\[
\lambda_2 = \frac{a \exp(\alpha B) + \frac{f}{\alpha}}{b(\exp(\alpha B) + \exp(-\alpha B))} .
\]

### 2-3 First-best optimum

The first-best optimum is defined as the trip pattern that maximizes total social surplus in a city, formulated as follows:

\[
S = \int_0^B \left[ \int_0^{\alpha(x)} p(q) dq - C(x)q(x) \right] dx_,
\]  (10)

From the optimal conditions, we have the following relation,

\[
p(q(x)) = C(x) + \int_0^t Q'(y)Q(y)dy ,
\]  (11)

---

\(^4\) As shown later, differential equations of trip rate functions for first-best optimum and equilibrium under cordon pricing have the same structure as Eq. (8). Thus, \( q'(x) < 0 \) and \( q''(x) > 0 \) also hold for these schemes.
where the second term of the RHS represents the congestion externalities that an additional trip from \( x \) imposes on all drivers using the road between \( x \) and 0. Therefore Eq. (11) is consistent with the general rule for social efficiency: The social marginal benefit from an additional trip at location \( x \) should be equalized to the social marginal cost. Such socially efficient allocation can be decentralized by levying the toll equal to the congestion externality on each trip generated at each location. Implementing this tolling scheme is not practically feasible since toll levels should be differentiated at each location. This case does not provide an alternative policy but serves as a reference point for evaluating the performance of cordon pricing as a second policy.

Using the specifications of Eqs. (1) and (4), Eq. (11) is rewritten as,

\[
a - bq(x) = fx + 2c \int_0^y Q(y) dy.
\]  

(12)

As in the no-toll equilibrium, differentiating twice the above equation with respect to \( x \) yields,

\[
-bq^*(x) + 2cq(x) = 0
\]

(13)

and following the same procedure as above, optimal trips at each location, \( q^o(x) \), are obtained as,

\[
q^o(x) = \eta_1 \exp(\gamma x) + \eta_2 \exp(-\gamma x),
\]

(14)

where \( \gamma = \sqrt{2c/b}, \eta_1 = \frac{a \exp(-\gamma B) - \int_0^y \gamma}{b(\exp(\gamma B) + \exp(-\gamma B))}, \eta_2 = \frac{a \exp(\gamma B) + \int_0^y \gamma}{b(\exp(\gamma B) + \exp(-\gamma B))} \)

3. EQUILIBRIUM UNDER CORDON PRICING

Suppose that the cordon is located at distance \( x_m \) from the CBD, and a toll equal to \( \tau \) is levied on each vehicle passing the cordon. In this situation, trip cost for a resident living outside the cordon \( (x > x_m) \) is the sum of the travel time cost and toll, while a resident inside the cordon incurs only travel time cost. Let \( q^**(x) \) and \( q^o**(x) \) represent the equilibrium number of trips departing at \( x \) inside and outside the cordon, respectively. Equilibrium requires that the following relations hold,

\[
\begin{cases}
   p(q^**(x)) = C(x) \\
   C(x) = \int_{x_m}^x t(Q_i(y)) dy \\
   Q_i(y) = \int_y^{x_m} q_i**(z) dz + \int_{x_m}^x q^o**(z) dz
\end{cases}
\]

for \( 0 \leq x \leq x_m \)  

(15)
Eqs. (15) and (16) give two differential equations describing the spatial variation of trips, but these are connected to each other by Eq. (17), which states that traffic volume function should be continuous at $x_m$.

As in the previous section, we use specifications $t(Q) = f + cQ$ and $p(q) = a - bq$, to solve the differential equations. Then, the equilibrium trip rate functions under cordon pricing are derived as,

\begin{align*}
q_i^{**}(x) &= \mu_1 e^{\alpha x} + \mu_2 e^{\alpha x}, \quad 0 \leq x \leq x_m, \\
q_o^{**}(x) &= \mu_3 e^{\alpha x} + \mu_4 e^{\alpha x}, \quad x_m \leq x \leq B,
\end{align*}

where $\alpha$ has been defined in the previous section, and $\mu_1, \mu_2, \mu_3, \mu_4$ are unknown constants to be determined by boundary conditions as follows.

We need four boundary conditions. At $x = 0$, the following relations hold,

\begin{align*}
p(q_i^{**}(0)) &= C(0) = 0, \\
p \frac{dq_i^{**}(0)}{dx} - t(Q_i(0)) &= 0.
\end{align*}

The next condition is derived by incorporating Eq. (17) into Eqs. (15) and (16) at $x = x_m$; that is,

\begin{equation}
p(q_i^{**}(x_m)) = p(q_o^{**}(x_m)) - \tau. \tag{19c}
\end{equation}

Finally, at the edge of the urban area, $x = B$,

\begin{equation}
p \frac{dq_o^{**}(B)}{dx} - t(Q_o(B)) = 0. \tag{19d}
\end{equation}

From the above conditions (19a)-(19d), unknown constants are determined as follows:

\begin{align*}
\mu_1 &= \frac{-2 \frac{f}{\alpha} + 2a e^{-\alpha B} + \tau \left(e^{\alpha (B-x_m)} - e^{-\alpha (B-x_m)}\right)}{2b \left(e^{\alpha B} + e^{-\alpha B}\right)}, \tag{20a} \\
\mu_2 &= \frac{2 \frac{f}{\alpha} + 2a e^{\alpha B} - \tau \left(e^{-\alpha (B-x_m)} - e^{-\alpha (B-x_m)}\right)}{2b \left(e^{\alpha B} + e^{-\alpha B}\right)}, \tag{20b} \\
\mu_3 &= \frac{-2 \frac{f}{\alpha} + 2a e^{-\alpha B} - \tau \left(e^{-\alpha (B-x_m)} + e^{-\alpha (B+x_m)}\right)}{2b \left(e^{\alpha B} + e^{-\alpha B}\right)}, \tag{20c}
\end{align*}
The effects of exogenous changes in cordon location and toll level on the equilibrium number of trips in each location are given as follows:

\[
\frac{\partial q_*^m(x)}{\partial x_m} = -\tau (ae^{-\alpha x_m} + \alpha e^{-\alpha(x+\alpha x_m)}) \left( e^{\alpha x_m} - e^{-\alpha x_m} \right) < 0, \quad \text{for } 0 \leq x \leq x_m, \tag{21a}
\]

\[
\frac{\partial q_*^m(x)}{\partial x_m} = -\tau \alpha e^{-\alpha(x+x_m)} \left( e^{-2\alpha x_m} + e^{-2\alpha(x+\alpha x_m)} \right) - e^{-2\alpha x_m} + e^{-2\alpha(x+\alpha x_m)} < 0, \quad \text{for } x_m \leq x \leq B, \tag{21b}
\]

\[
\frac{\partial q_*^m(x)}{\partial \tau} = \left( e^{\alpha x_m} + e^{-\alpha x_m} \right) \left( e^{\alpha(x+\alpha x_m)} - e^{-\alpha(x+\alpha x_m)} \right) > 0, \quad \text{for } 0 \leq x \leq x_m, \tag{22a}
\]

\[
\frac{\partial q_*^m(x)}{\partial \tau} = -\frac{e^{-\alpha(x+x_m)} \left( e^{2\alpha x_m} + e^{2\alpha x_m} \right) (1 + e^{2\alpha x_m})}{2b(1 + e^{2\alpha x_m})} < 0, \quad \text{for } x_m \leq x \leq B. \tag{22b}
\]

(21a) and (21b) state that, as the cordon location moves outward, the equilibrium number of trips decreases in all locations\(^5\). (22a) and (22b) state that, as the toll level rises, the number of trips increases in locations inside the cordon but decreases outside the cordon. This also implies that in inner (outer) locations, the numbers of trips under cordon pricing are larger (smaller) than those under no-toll equilibrium. This is illustrated in Fig. 1. Trip demand decreases in outer locations due to additional toll burden, causing a reduction in traffic volume (in other words, congestion level) in inner locations. Trip makers in inner locations enjoy congestion relief with no charge, then respond by increasing trips. In other words, the cordon pricing induces increase in consumer surplus for residents in inner locations and decrease in consumer surplus for those in outer locations.

(21a) implies that outward move of the cordon location causes aggravation of traffic congestion in inner locations, while trip demand decreases in all locations. To see this, we examine the change in traffic volume at location \(x\), \(0 \leq x \leq x_m\) caused by an infinitesimal change in the cordon location from \(x_m\) to \(x_m + dx_m\).

\[
\frac{\partial Q_m(x)}{\partial x_m} = \int_{x_m}^{x_m + dx_m} \frac{\partial q_*^m(y)}{\partial x_m} dy + \int_{x_m}^{x_m + dx_m} \frac{\partial q_*^m(y)}{\partial x_m} dy + q_*^m(x_m) - q_*^m(x_m) \tag{23}
\]

Although the first and second terms of the RHS are negative from (21), the third term is larger than the fourth term. Trip makers located between \(x_m\) and \(x_m + dx_m\) increase trips from \(q_*^m(x_m)\) to \(q_*^m(x_m)\) because they are exempt from the toll after the move. This increase in trip demand exceeds

\(^5\) Rigorously speaking, there is an exception: as shown later, the equilibrium number of trips at \(x_m\) increases because it switches from \(q_o(x_m)\) to \(q_i(x_m)\), and \(q_o(x_m) < q_i(x_m)\) as shown by (22).
the sum of the trip decrease over all locations. When the cordon location moves outward, those who are exempt from the toll due to this change are better off, while the others are worse off.

Figure 1

4. THE OPTIMAL CORDON PRICING

Optimal cordon pricing is the combination of the cordon location \( x_m \) and toll \( \tau \) that maximizes the social surplus defined as follows:

\[
S = \int_0^{x_m} \left[ \int_0^{x_m} p(q)dq - C(x)q_{i_m}(x) \right] dx + \int_{x_m}^{B} \left[ \int_0^{x_m} p(q)dq - C(x)q_{o_m}(x) \right] dx
\]

Constraints to this problem are Eqs. (15)-(17), i.e., the equilibrium conditions under cordon pricing. Note that the equilibrium number of trips under cordon pricing is solved explicitly as \( q^{**}(x) \) in the last section. Hence, we can treat this problem as optimization without constraints by substituting (18) and (19) into the objective function above.

The optimal conditions with respect to \( x_m \) and \( \tau \) are given respectively as follows:

\[
\int_0^{x_m} \left[ p(q_i^{**}(x)) - C(x) - E(x) \right] \frac{d q_i^{**}(x)}{d x_m} dx + \int_{x_m}^{B} \left[ p(q_o^{**}(x)) - C(x) - E(x) \right] \frac{d q_o^{**}(x)}{d x_m} dx = 0,
\]

\[
\int_0^{x_m} \left[ p(q_i^{**}(x)) - C(x) - E(x) \right] \frac{d q_i^{**}(x)}{d \tau} dx + \int_{x_m}^{B} \left[ p(q_o^{**}(x)) - C(x) - E(x) \right] \frac{d q_o^{**}(x)}{d \tau} dx = 0,
\]

where

\[
E(x) = \begin{cases} 
\int_0^x t(Q_i(y))Q_i(y)dy, & \text{for } 0 < x \leq x_m \\
\int_0^{x_m} t(Q_i(y))Q_i(y)dy + \int_{x_m}^{x} t(Q_o(y))Q_o(y)dy, & \text{for } x_m < x \leq B
\end{cases}
\]

\( E(x) \) is the sum of the congestion externality that an additional trip from \( x \) imposes on all drivers

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6 This is verified by expanding the RHS of Eq. (23), as follows:

\[
\frac{\partial Q_i(x)}{\partial x_m} = \frac{\alpha \left( e^{\alpha(x+\delta)} - e^{\alpha(x-\delta)} + e^{\alpha(x+\delta)} - e^{\alpha(x-\delta)} \right)}{2(1 + e^{2\alpha\delta})} > 0
\]
using the road between \( x \) and 0.

The first line of Eq. (24a) represents the direct effect on social surplus caused by outward move of the cordon location \( x_m \): Increase in consumers’ surplus for those who are exempt from the toll minus decrease in toll revenue\(^7\). It is easily seen that the sum of the terms on the first line has positive value. Integral terms on the second line of (24a) represent the sum of indirect effects on social surplus through changes in the number of trips caused by outward move of the cordon location. In other words, these effects are considered as changes in the amount of dead weight losses present in the second-best situation. The first integral describes the effects in locations inside the cordon, which has positive value in view of Eqs. (15) and (21a)\(^8\). This positive effect on social surplus implies that the sum of the dead weight losses inside the cordon decreases as the cordon moves outward. Based on the discussion so far, the second integral term should be negative for Eq. (24a) to hold. Recalling \( \frac{\partial q^o}{\partial x_m} < 0 \) from Eq. (21b), \( \{ p(q^o(x)) - C(x) - E(x) \} \) should have positive values for, at least, some locations between \( x_m \) and \( B \).

Likewise, the first integral term on the LHS of Eq. (24b) has negative value from (22a) and (15): Dead weight losses increase in locations inside the cordon as the toll rises. For this condition to hold, it is necessary that \( \{ p(q^o(x)) - C(x) - E(x) \} \) in the second integral term must have negative values for, at least, some locations between \( x_m \) and \( B \).

Synthesizing the above discussions, it turns out that, in locations outside the cordon, \( p(q^o(x)) - C(x) - E(x) \) has negative values in some locations and positive values in other locations. Note that \( p(q^o(x)) - C(x) - E(x) = \tau - E(x) \) from Eq. (16), and \( E(x) \) is monotonously increasing with \( x \). Thus, only the configuration as illustrated in Figure 2 is possible: There exists some point \( \overline{x} \) such that \( \tau - E(x) \) is positive between \( x_m \) and \( \overline{x} \) but negative outside \( \overline{x} \).

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\(^7\) Note that outward move of cordon location reduces the private cost for those located at \( x_m \) from \( C(x_m) + \tau \) to \( C(x_m) \). The first line of (24a) can be rewritten as follows:

\[
\int_0^{\tau(x_m)} p(q) dq - C(x_m) q^o(x_m) \quad - \quad \int_0^{\tau(x_m)} p(q) dq - \{ C(x_m) + \tau \} q^o(x_m) \quad - \quad \tau q^o(x_m).
\]

Two bracketed terms are consumer surplus for trip makers located just inside and outside of the cordon, respectively. Thus, they represent increase in consumer surplus for those located at \( x_m \). The third term is the amount of toll charged to a trip maker located just outside the cordon, which is foregone revenue due to move of the cordon location.

\(^8\) Since \( p(q^o(x)) - C(x) = 0 \) from (15), the bracketed term in the first integral on the second line of (24a) becomes \( - E(x) \), which is negative. And \( \frac{\partial q^o}{\partial x_m} < 0 \) from (21a). Therefore, the first integral has positive value.
Based on the above discussion, the situations of resource allocation under cordon pricing for three typical locations are illustrated in Fig. 3. \( C(x) + E(x) \) represents the social marginal cost, i.e., increment of social cost due to marginal increase in trips from \( x \). In locations inside the cordon (\( x < x_m \)), the social marginal cost exceeds the social marginal benefit, \( p(q''(x)) \), in other words, trips are under-priced, or the number of trips is larger than the efficient level. Similarly, trips are over-priced for \( x_m < x < \tilde{x} \), and under-priced for \( \tilde{x} < x < B \). The amount of dead weight losses due to inefficient trip making as discussed above is equal to the shaded areas in the figure. As the cordon location or toll level is changed, dead weight loss is increased in some locations and decreased in other locations.

Table 1 summarizes the directions of changes in dead weight losses due to changes in the cordon location and toll at their optimal levels. In the table, sign + (-) means that dead weight loss is increased (decreased) by increasing the corresponding control variable (i.e., \( x_m \) or \( \tau \)) from their optimal levels. For example, at locations within \( 0 \leq x < x_m \), dead weight loss is decreased as \( x_m \) increases (the cordon location moves outward). Optimal cordon pricing is designed so that the gains from the decrease in dead weight loss offset the losses from the increase in dead weight loss.

5. EVALUATION OF ECONOMIC WELFARE

This section numerically examines the effects of cordon pricing on economic welfare, by comparing the values of social surplus under optimal cordon pricing, no-toll equilibrium and first-best optimum. To obtain quantitative insights, we calibrated parameter values of the model by using the actual data for Osaka Prefecture, Japan, as follows:

\( B=50, \quad a = 130, \quad b = 498, \quad c = 0.52, \quad f = 1.2 \).

Sources of data and details of procedure to calibrate parameter values are given in the Appendix.
Table 2 summarizes the result for these parameter values. Optimal cordon location is 7.54 km from the CBD, and the time-equivalent value of the optimal toll is 29.42 minutes. The amount of the toll is 980.7 Yen, if we adopt 2000 Yen/hour for the value of travel time, which is the estimate from an empirical study in Japan (Ohta [9]). The table also shows values of social surplus under three schemes, which are measured in time equivalent units\(^\text{10}\). From the table, social surplus for cordon pricing is larger than that for no-toll equilibrium by 12\%, and smaller than the first-best optimum by only 0.7\%. Although the cordon pricing is a very simple system, the performance is almost as good as the first-best optimum that requires prohibitive effort for information processing.

**Table 2**

Does the above result depend on the specific parameter values used here? We carried out simulations for various parameter values to check the robustness of the results against the parameter changes. Table 2 shows the results for different values of parameters in demand and cost functions. Larger \(b\) implies that trip demand is less elastic. Smaller \(c\) means that the congestion level is less sensitive to an increase in traffic volume, which is interpreted as a situation whereby road capacities are larger at all locations. The table shows that, as trip demand is less elastic, the optimal cordon location becomes farther from the center and the toll level is lower. On the other hand, smaller \(c\) causes a larger-sized cordoned area and lower toll level. Figures 4 and 5 plot the social surpluses under no-toll equilibrium, optimal cordon pricing and first-best optimum for various values of \(b\) and \(c\).

**Figure 4**

**Figure 5**

Figures show that the values of social surpluses for optimal cordon pricing are very close to those for the first-best optimum regardless of parameter values. This suggests that cordon pricing attains an economic welfare level nearly as good as the first-best optimum for a wide range of parameter values.

Why does the cordon pricing produce such a good result as shown above? Let us investigate in more detail the workings of cordon pricing as a device to control congestion externality. Note that, as

\(^{\text{10}}\) Note that parameter values are specified in the context of the model setting: City is monocentric, population density is unity, etc. Therefore, the values of social surpluses are meaningful only for comparison among different tolling schemes.
seen in Eq. (25), congestion externality depends on traffic volume at each location that is integral of trips originating in outer locations. Fig. 6 plots traffic volumes $Q(x)$ for no-toll equilibrium, first-best optimum and cordon pricing. By definition (Eq. (3)), traffic volume at the edge of the urban area, $B$, is equal to zero in all cases, and the (negative) slope of each curve is equal to the trip rate originating at each location, $q(x)$.

**Figure 6**

The figure shows that the traffic volume curve for the cordon pricing closely matches the curve for the first-best optimum, and two curves cross twice at intermediate locations. Recall that, under the optimal cordon pricing, trips are under-priced in locations inside the cordon, over-priced just outside the cordon and under-priced in the fringe of the urban area. Accordingly, the trip rate under cordon pricing tends to be larger (smaller) than that for the first-best optimum in locations where congestion is under-priced (over-priced). This is reflected in the relative steepness of the two curves in the figure; traffic volume curve for the cordon pricing is steeper inside the cordon, flatter just outside the cordon, and steeper in the fringe of the urban area. Although cordon pricing is such a simple system in which tolls are collected at only one point, it divides the urban area into three zones and fine-tunes the trip rate in each zone (=slope of traffic volume curve) to minimize the deviation of traffic volume from the first-best.

6. CONCLUSION

This paper presents a simple spatial model of traffic congestion for a monocentric city, to investigate the effects of cordon pricing on trip-making and congestion level in each location. Optimal cordon pricing is obtained as a combination of the cordon location (i.e. distance of the cordon from the CBD) and the amount of toll charged there that maximizes the total social surplus in a city. Under the optimal cordon pricing, trips from locations inside the cordon are under-priced, those just outside the cordon are over-priced, and those in the fringe of the urban area are under-priced. Numerical simulations using the parameter values based on Japanese data suggest that the cordon pricing attains an economic welfare level very close to the first-best optimum.

This paper introduces a number of assumptions to simplify the analysis. The most restrictive one is that the city is monocentric: All trips are destined to the CBD. If this assumption is relaxed, the result that cordon pricing attains good performance may be modified significantly. In this case, it might be necessary to introduce multiple cordons. We should also consider the land use change to see the
long-run effects. Since trips departing at locations inside the cordon are exempt from tolls, central locations become more attractive under cordon pricing. This induces land use structure with higher density in central locations, which is likely to have positive impacts on efficiency. As shown in the literature of monocentric city models with congestion, efficiency is improved when the physical size of the city becomes more compact (e.g., Fujita [3], Kanemoto [4]). This implies that the centralization of land use caused by cordon pricing improves the efficiency of spatial distribution in a city. When we relax the assumption of monocenter, however, different forms of land use changes may occur. Firms and households may move to locations outside the cordon, thereby completing trips without crossing the cordon, i.e., avoiding tolls. In other words, introducing cordon pricing induces the dispersion of spatial structure unlike centralization in the monocentric case. And such changes may weaken the effectiveness of the policy. This problem is worth investigating in future works. Anas and Xu [1] provides a useful prototype framework for the analysis of this problem, although they do not consider the second-best pricing policies. We have also neglected the costs of setting up the cordon pricing system. It is natural that the set-up costs tend to increase as the cordon moves outward, since the number of road links crossing the cordon increases. Therefore, optimal location of the cordon becomes closer to the CBD than that obtained in this paper. Other important issues to be addressed by future study include investment for road capacity, as Kanemoto [4] discussed, naive benefit-cost criterion is not applicable to the second-best situations. Since all these extensions will make the model structure too complicated, we need to rely on numerical methods.

REFERENCES

5. M. Kraus, The welfare gains from pricing road congestion using automatic vehicle identification

Kraus (1989) takes into account this cost in numerical analysis.


**APPENDIX: Parameters for numerical analysis**

We used the data from “Person Trip Survey for the Keihanshin Area” in 1990, which include information of origin, destination and travel time for each individual trip on a given day. The data are
aggregated by 67 jurisdictions in Osaka prefecture, and then the number of trips from each jurisdiction to the CBD (defined as Kita-ku and Chu-o-ku of Osaka City) is extracted. Aggregated numbers of trips are divided by populations of jurisdictions to obtain the number of trips per person. Distances from the centers of jurisdictions to the CBD are measured on the map.

Parameters to be calibrated are \( B, a, b, c, \) and \( f \).

First, we assume that the distance from the CBD to the edge of the urban area, \( B \), is 50 km, considering that the southern edge of the Osaka area (Misaki Cho) is located 59 km from the CBD while the northern edge (Nose Cho) is 39 km away.

Parameter \( a \) in the demand function is interpreted as the travel time at which one gives up making trips. We set \( a = 130 \), in reference to the fact that the longest travel time among trips to the CBD reported in the trip survey is equal to 120 minutes. Since the demand function is linear, \( b \) is obtained by drawing a line connecting two points: One is the intercept \((0,a)\) on the number of trips - travel time plane, the other one is the observation at the zone where the number of trips per person is largest in the study area. Then, we have \( b = 498 \). Parameter setting in the above manner implies that the trip cost is measured in time units (minutes). Thus, the amount of toll is also computed in time units. The amount of toll in monetary units is obtained by multiplying the value of time.

Parameter \( f \) in the cost function represents the time to drive the unit distance in free-flow traffic, i.e., when traffic volume is equal to zero. Assuming that speed in free-flow is equal to 50 km/h, it follows that \( f = 1.2 \). Parameter \( c \) is determined so that the trip pattern computed by the model best fits the actual pattern, although the model is built on a number of assumptions that are not necessarily compatible with the situation of an actual city, such as monocentricity, unit density, constant road capacity, etc. By substituting already determined parameter values, \( B, a, b, \) and \( f \), to Eq. (9), we have \( q^*(x) \) as a function of \( x \) and \( c \). And substituting this into Eq. (3), \( Q(x) \) is obtained as a function of \( x \) and \( c \). Let \( x_i \) be the distance from zone \( i \) to the CBD, then in the context of the model, the travel time from zone \( i \) to the CBD, \( T(x_i,c) \), is computed by the following formula,

\[
T(x_i,c) = f x_i + c \int_0^{x_i} Q(y) dy.
\]

Finally, \( c \) is determined so that the sum of square errors is

\[
\sum_{i=1}^{67} \left[ T(x_i,c) - \bar{T}_i \right]^2, \]

where \( \bar{T}_i \) is the observed travel time from \( i \). It follows that \( c = 0.52 \).
Table 1  Changes in dead weight losses under optimal cordon pricing

<table>
<thead>
<tr>
<th>locations</th>
<th>$0 \leq x &lt; x_m$</th>
<th>$x_m &lt; x &lt; \bar{x}$</th>
<th>$\bar{x} &lt; x \leq B$</th>
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</thead>
<tbody>
<tr>
<td>$x_m$</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\tau$</td>
<td>+</td>
<td>+</td>
<td>-</td>
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</table>
Table 2  Numerical results

<table>
<thead>
<tr>
<th></th>
<th>Basic case</th>
<th>Larger $b$</th>
<th>Smaller $c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$ in demand function</td>
<td>498</td>
<td>748</td>
<td>498</td>
</tr>
<tr>
<td>$c$ in cost function</td>
<td>0.52</td>
<td>0.52</td>
<td>0.26</td>
</tr>
<tr>
<td>Optimal cordon location</td>
<td>7.54 km</td>
<td>8.32 km</td>
<td>8.78 km</td>
</tr>
<tr>
<td>Optimal toll</td>
<td>29.42 min</td>
<td>25.63 min</td>
<td>22.65 min</td>
</tr>
<tr>
<td>Social surplus:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No-toll equilibrium;</td>
<td>233.5 min (0.887)</td>
<td>184.6 min (0.918)</td>
<td>309.0 min (0.939)</td>
</tr>
<tr>
<td>First-best optimum;</td>
<td>263.4 min (1.000)</td>
<td>201.0 min (1.000)</td>
<td>329.2 min (1.000)</td>
</tr>
<tr>
<td>Cordon pricing;</td>
<td>261.6 min (0.993)</td>
<td>200.0 min (0.995)</td>
<td>327.9 min (0.996)</td>
</tr>
</tbody>
</table>
Figure 1  Spatial variations of trip rates under no-toll equilibrium and cordon pricing
Figure 2  External cost and toll under the optimal cordon pricing
Figure 3  Relations among trip costs, tolls and external costs in three representative locations
Social surplus

![Graph](image)

**Figure 4** Demand elasticity parameter $b$ and economic welfare

Social surplus

![Graph](image)

**Figure 5** Cost parameter $c$ and economic welfare
Figure 6 Profiles of traffic volume