Overconfidence in Economic Contests*

Munetomo Ando†
National Graduate Institute for Policy Studies

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Abstract

This paper studies an economic contest with two participants, who have overconfidence in their probability of winning in the contest. We examine two different sources of overconfidence and compare the behavioral consequences of them. The main result is that the overconfidence from an overestimation of own type increases in agents’ efforts, while that from an underestimation of the type distribution decreases in agents’ expected efforts.

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*Very preliminary and incomplete draft.
†Address: 2-2 Wakamatsu-cho, Shinjuku-ku Tokyo, 162-8677 Japan, TEL. +81-3-3341-0748, FAX. +81-3-3341-9220, E-mail: m.ando@grips.ac.jp (M. Ando).
1 Introduction

Overconfidence is one of the famous stylized facts about human behavior. Several studies indicate that humans are overconfident about their own (relative) abilities.\footnote{See, for example, Camerer (2003).} In addition, tendencies to over-confide in some kinds of economic events are also acknowledged.\footnote{Some studies on the behavioral finance are examples. Behavioral finance encompasses research that drops the traditional assumptions of expected utility maximization with rational investors in efficient markets. See, for example, Barberis and Thaler (2003).} Thus, the researches on the overconfidence have some directions.

This paper considers an economic contest with two participants. In this contest, each participant has overconfidence in his probability of winning. We examine two different sources of overconfidence and compare the behavioral consequences in these situations. As the sources of overconfidence, we consider “the overestimation of own type” and “the underestimation of the type distribution.”

We consider the following situation. Different agent has different type, which is equal to his monetary value of winning in the contest. The types are independently and identically distributed. He knows prior belief about his type and the type distribution, and his belief may or may not be correct. That is, he may have a misinformation about his type and/or the type distribution. Each agent chooses his effort to maximize his expected profit. The effort level of each agent is observable by all players at the end when the agents already chosen their effort levels. For the principal, larger efforts are profitable, so that she maximizes the expected effort per agent.

The main result in this paper is as follows. An overestimation of own type always increases agents’ efforts, and therefore, it is profitable for the principal. On the other hand, an underestimation of the type distribution decreases or does not change the agents’ expected efforts, and therefore, it is not profitable for the principal. This result tells us that the principal prefers to employ an agent who has overestimation of own type.

The reason why an overestimation of own type increases the agents’ efforts is as follows. Under an overestimation of own type, an agent behaves as if he has a higher type. Since the equilibrium strategy is increasing in types, he chooses a higher effort compared to the correct-estimated case. Thus, overconfidence from an overestimation of own type is profitable for the principal.

An underestimation of type distribution brings about increases in lower type agents’ efforts in exchange for decreases in other agents’ efforts. The reason why an underestimation of type distribution changes agents’ efforts is that the underestimation changes each type’s recognized probability of winning. The expected value of increases in the lower types’ efforts is smaller or equal to that of decreases in the higher types’ efforts. Consequently, an
underestimation decreases or does not change the agents’ expected efforts.

There are two strands of theoretical works related to the present paper. The first class is analyses of economic contests as a variation of the all-pay auction.\(^3\) Recently, the contest design problems from the viewpoint of the contest designer have attracted much attention. Examples include Singh and Wittman (1998, 2001), Moldovanu and Sela (2001), and Ando (2003). However, these studies usually assume that the prior belief about own type and the type distribution are correct. This is different from the present paper.


The rest of this paper proceeds as follows. Section 2 describes the environment of our model. Section 3 examines the benchmark case, the behavioral consequence of an overestimation of own type, and the behavioral consequence of an underestimation of the type distribution. Section 4 concludes.

2 Model

We consider an economic contest with two risk neutral agents, agents 1 and 2. They compete for winning the contest. Each agent \(i\) decides his effort \(e_i\). Efforts are outlaid simultaneously and independently. The agents’ effort levels are observable by the principal and the agents at the end when the agents have already chosen their effort level.

Each agent has different type, which represents his monetary value of winning. The true type of agent \(i\) is denoted by \(\theta_i\) and the exact value may be perceptible when he wins. We assume that each agent has his prior belief about his type, \(a_i\), and this is his private information. Note that the prior belief is not necessarily equal to the true type. Here, high \(\theta_i\) (\(a_i\), respectively) means high true valuation of winning (prior belief, respectively). The true type of agent \(i\) is drawn from an interval \([0, 1]\) according to the distribution function \(F_i\) that has a continuous and everywhere strictly positive density function \(f_i\).\(^4\) In the standard model of economic contests, the distribution functions are assumed to be common knowledge for the principal and both agents. However, in this paper we relax this assumption. Agent \(i\) believes that his type is drawn from an interval \([0, 1]\) according to the distribution

\(^3\)Amann and Leininger (1996) and Krishna and Morgan (1997) are examples of studies on all-pay auctions.

\(^4\)We assume that the type space is \([0, 1]\). This restriction is only for the analytical convenience. We can preserve all our result in any non-negative types with bounded support case.
function $G^i_i$, and the rival’s type is drawn according to the distribution function $G^i_j$.

In this contest, the payoff of agent $i$ is, $\theta_i - e_i$ if he wins, and $-e_i$ if he does not. Each agent chooses his effort in order to maximize his expected payoff, given his belief on own type and the knowledge about the type distributions, $G^i_i$ and $G^i_j$. For the principal, larger efforts are profitable, so that she maximizes the expected effort per agent.

3 Analysis

3.1 The benchmark case

In this subsection, we describe the benchmark case. We consider the situation where $a_i = \theta_i$ and $G^i_i = F_i, G^i_j = F_j$, $i = 1, 2$. In addition, types are i.i.d. (this means $F_i = F_j = F$) and the distribution function $F$ is common knowledge.

This contest game is a variation of the standard all-pay auction, and therefore, the symmetric equilibrium effort strategy exists and is described as follows.

**Proposition 1.** In a symmetric equilibrium, the effort strategy is

$$\beta(\theta, F) = \int_0^\theta yf(y)dy,$$

and the symmetric equilibrium is unique.

*Proof.* The equilibrium strategy can be easily derived with the standard method in auction literature. The uniqueness can be shown in the process of derivation. Hence the proof is omitted here.

In the symmetric equilibrium, since $\beta(\cdot)$ is strictly increasing, agent $i$’s probability of winning (hereafter $p(\theta_i, F)$) is equivalent to the probability that his type is no lower than the rival’s one, that is, $p(\theta_i, F) = F(\theta_i)$.

In this contest, the expected effort per agent is

$$E(e, F) = \int_0^1 \beta(\theta)f(\theta)d\theta.$$

By using integration by parts with expression (1), we obtain the following.

**Proposition 2.** The expected effort per agent is

$$E(e, F) = \int_0^1 (1 - F(\theta))yf(y)dy.$$

We provide some examples.
Example 3.1. 1. If $F(\theta) = \theta^2$, $\beta(\theta, F) = 2\theta^3/3$ and $E(e, F) = 4/15$.

2. If $F(\theta) = \theta$, $\beta(\theta, F) = \theta^2/2$ and $E(e, F) = 1/6$.

3. If $F(\theta) = 2\theta - \theta^2$, $\beta(\theta, F) = \theta^2 - 2\theta^3/3$ and $E(e, F) = 1/10$.

3.2 An overestimation of own type

In this subsection, we examine the consequence of an overestimation of own type in economic contests. To simplify the analysis, we assume that the belief structure is the following simple one. The prior knowledge about the type distribution is correct (i.e., $G_i = F_i$ and $G_j = F_j$, $i = 1, 2$) and types are i.i.d. (i.e., $F_1 = F_2 = F$). In addition, this belief structure is common knowledge. However, they are misinformed about their types. We assume that they overestimate their types in a systematic way, for all $\mu_i \in [0, 1)$, $a_i > \mu_i$, $i = 1, 2$.

In this situation, each agent derives the equilibrium strategy $\bar{\beta}(\cdot, F)$ by $F$. Then he chooses his action by calculation with $\bar{\beta}(\cdot, F)$ and $a_i$. By the above facts, we obtain the following.

Proposition 3. For an agent $i$ with true type $\mu_i \in [0, 1)$, an overestimation of own type increases his effort (i.e., $\beta(a_i, F) > \beta(\mu_i, F)$ for all $\mu_i \in [0, 1)$ and for all $a_i > \mu_i$).

Proof. The effort strategy $\beta(\cdot, F)$ is constructed by $F$, and therefore, $\beta(a, F) = \int_0^a yf(y)\text{dy}$. Since $yf(y)$ is positive for all $y \in (0, 1]$, $\int_0^a yf(y)\text{dy} > \int_0^\theta yf(y)\text{dy}$ for all $a > \theta$. Thus, we obtain $\beta(a, F) > \beta(\mu, F)$. 

This proposition implies that, under an overestimation of own type, an agent who has true type $\theta_i$ behaves as if he has a higher type, $a_i$. Since the equilibrium strategy is increasing in types, he chooses a higher effort compared to the benchmark case. Thus, overconfidence from an overestimation of own type is profitable for the principal.

We provide an example. In this example, each type $\theta \in (0, 1)$ has an overestimation of his type and the misinformation is constructed by the following simple rule, $a = a(\theta) = \sqrt{\theta}$. This formulation permits us the direct calculation of the expected efforts.

Example 3.2. Suppose that $F(\theta) = \theta$ and $a_i = a(\theta_i) = \sqrt{\theta_i}$. In this situation, $\beta(a_i, F) = \theta_i/2$ and $E(\beta(a_i, F)) = 1/4$; $\beta(\mu_i, F) = \theta_i^2/2$ and $E(\beta(\mu_i)) = 1/6$.

The equilibrium strategies of the above example are depicted in Figure 1. Horizontal axis is types and vertical axis is effort levels.
3.3 An underestimation of the type distribution

In this subsection, we examine the consequence of an underestimation of the type distribution. We assume that each agent $i$ knows his true type, that is, $a_i = \theta_i$, $i = 1, 2$. However, both agents have misinformation about the type distributions. We assume that, $F_1 = F_2 = F$, $G_i^1 = G_j^1 = G$ for $i = 1, 2$, and $G \neq F$. In addition, both agents believe that the distribution function $G$ is common knowledge.

Now, we turn our attention to the characteristics of the misinformed type distribution $G$. We define the following.

**Definition 1.** *Both agents under estimate the type distribution if*

$$\forall \theta \in (0, 1), G(\theta) > F(\theta).$$

In the rest of this subsection, we consider the situation where the agents underestimate the type distribution. The next proposition shows the effort strategy in this situation.

**Proposition 4.** *In this contest game, there exist a symmetric equilibrium effort strategy,

$$\beta(\theta, G) = \int_0^\theta yg(y)dy,$$

and the symmetric equilibrium is unique.*

**Proof.** The proof is omitted here.
In this contest, the expected effort per agent is

\[ E(e, G) = \int_0^1 \beta(\theta, G) f(\theta) d\theta. \]

By using integration by parts with expression (3), we obtain the following.

**Proposition 5.** The expected effort per agent is

\[ E(e, G) = \int_0^1 (1 - F(\theta)) g(y) dy. \] (4)

Note that the expected effort is based on the following facts. Each type’s effort level is constructed by \( G \) and \( \theta_i \). However, the expectation is based on the true distribution of types, \( F \).

In the rest of the analysis, to simplify the exposition, we restrict our attention to the situations where the distribution function is \( \theta, \theta^2 \), or \( 2\theta - \theta^2 \).

Now, we describe how an underestimation affects each type’s effort.

**Proposition 6.** An underestimation of type distribution changes agents’ efforts. There exists \( \hat{\theta} \in (0, 1) \) such that, for all \( \theta \in (0, \hat{\theta}) \), \( \beta(\theta, G) > \beta(\theta, F) \) and for all \( \theta \in (\hat{\theta}, 1) \), \( \beta(\theta, G) < \beta(\theta, F) \).

**Proof.** \( \beta(\theta, G) - \beta(\theta, F) = \int_0^\theta y(g(y) - f(y)) dy \). This is positive at the type just by zero, since \( g(y) > f(y) \) at \( y = 0 \). Moreover, \( g(y) \) and \( f(y) \) are crossing at a certain \( y \in (0, 1) \) only once if \( F, G = \theta, \theta^2 \), or \( 2\theta - \theta^2 \). At the left hand side, \( g(y) > f(y) \), and at the right hand side, \( g(y) < f(y) \). Since \( \int_0^1 (g(y) - f(y)) dy = 0 \) and \( y \) is strictly increasing function, we obtain \( \int_0^1 (g(y) - f(y)) dy > 0 \). Consequently, we can conclude that there exists \( \hat{\theta} \in (0, 1) \) such that, for all \( \theta \in (0, \hat{\theta}) \), \( \beta(\theta, G) > \beta(\theta, F) \) and for all \( \theta \in (\hat{\theta}, 1) \), \( \beta(\theta, G) < \beta(\theta, F) \). \qed

We briefly describe the reason why an underestimation of the type distribution changes agents’ efforts as described in Proposition 6. When agents under estimate the type distribution, each type’s probability of winning in the equilibrium is changed from \( F(\theta) \) to \( G(\theta) \). In the underestimated situation, since the probability of winning at the left tail increases faster compared to the benchmark case, an underestimation induces more aggressive effort at the left tail. For the higher types, an underestimation decreases the gradient of the probability of winning function. This reflects decreases in the gradient of the effort strategy function for the higher types.

We provide an example.

**Example 3.3.** Suppose that \( a_i = \theta_i, i = 1, 2 \), and \( F(\theta) = \theta \) and \( G(\theta) = 2\theta - \theta_i^2 \). In this situation, \( \beta(\theta, F) = \theta^2 / 2 \) and \( \beta(\theta, G) = \theta^2 - 2\theta^3 / 3 \).
Figure 2: The equilibrium strategies of the above example are depicted in Figure 2. Horizontal axis is types and vertical axis is effort levels.

Finally, we describe changes in the expected effort from an underestimation of type distribution.

**Proposition 7.** Overconfidence from an underestimation of the type distribution decreases or does not change the expected effort per agent.

**Proof.** There are three possibility of the pair of prior belief and true type distribution: (1) the true distribution is \( \theta^2 \) and prior belief is \( \theta \), (2) the true distribution is \( \theta^2 \) and prior belief is \( 2\theta - \theta^2 \), and (3) the true distribution is \( \theta \) and prior belief is \( 2\theta - \theta^2 \). Since the expected effort level is \( \int_{0}^{1} (1 - F(\theta))yg(y)dy \), by the direct computation, we can conclude that an underestimation of the type distribution decreases the expected effort in cases (1) and (2), while does not change in case (3).

## 4 Concluding Remarks

In this paper, we examined two different sources of overconfidence and compared the behavioral consequences of these two situations. Both an overestimation of own type and an underestimation of the type distribution increase each type’s probability of winning in the contest. Consider an agent with true type 1/2 in the situation where \( F(\theta) = \theta \). In the equilibrium, his true probability of winning is .5. However, in the situation of overestimation of own type \( a_i = a_i(\theta_i) = \sqrt{\theta_i} \), his expected probability of winning is \( \sqrt{1/2} \) that is strictly larger than the true value, 1/2. In the situation where \( G(\theta) = 2\theta - \theta^2 \), his expected probability of winning is 3/4 that is
strictly larger than the true value $1/2$. In both situations, he has overconfidence about the probability of winning in the contest. However, these two situations yielded different consequences. The former is profitable for the principal and the latter is not.

This paper considered two specific situations, an overestimation of own type and an underestimation of the type distribution. Examinations of more general situations are relegated to future research.
References


