Partial agglomeration or dispersion in spatial Cournot competition*

Toshihiro Matsumura†
Institute of Social Science, University of Tokyo

Takao Ohkawa
Department of Economics, Ritsumeikan University

and

Daisuke Shimizu
Graduate School of Economics, University of Tokyo

April 28, 2004

Abstract

We revisit Pal (1998) and Matsushima (2001a), which present different equilibrium locations respectively. We consider nonlinear transport costs and show that Pal’s result (dispersion) is more robust than Matsushima’s (partial agglomeration). Pal’s result holds true for any transport cost function, while Matsushima’s does not hold true under strong concavity or convexity of the transport cost function. If we consider sequential move of location, Pal’s result holds for any transport costs. On the other hand, Matsushima’s does not hold except for the linear transport costs. We also discuss welfare and show that non-linearity of the transport cost function yields rich welfare implications.

JEL classification numbers: R32, D43, L13

Key words: shipping model, spatial Cournot competition, circular markets, equidistance location, partial agglomeration

*We are grateful to participants of the seminars at the University of Tokyo and Japan Economic Association Annual Meeting 2003 for their helpful comments and suggestions. We are also indebted to two anonymous referees for their valuable and constructive suggestions. Needless to say, we are responsible for any remaining errors. The financial supports from the Grant-in-Aid of the Japanese Ministry of Education, Science and Culture and of ISS project “Lost decades?:re-appraising contemporary Japan” are greatly appreciated.

†Correspondence author: Toshihiro Matsumura, Institute of Social Science, University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan. Phone: (81)-3-5841-4932. Fax: (81)-3-5841-4905. E-mail: matsumur@iss.u-tokyo.ac.jp
1 Introduction

Since the seminal work of Hotelling (1929), a rich and diverse literature on spatial competition has emerged. Location models fall into two categories: those in which firms bear the transport costs are shipping or spatial price discrimination models; those in which consumers pay for transport are shopping or mill pricing models. In each type, one can have either Bertrand-type price setting or Cournot-type quantity-setting.\(^1\)

Most papers on location theory use shopping (mill pricing) models with Bertrand competition. Although Cournot-type and Bertrand-type non-spatial models are equally popular, the body of literature on spatial competition that uses Cournot-type models is relatively small. Economists have recently considered shipping models with Cournot competition. Hamilton, Thisse, and Weskamp (1989) and Anderson and Neven (1991) carry out pioneering works on location models.\(^2\) They use linear-city models and show that all firms agglomerate at the central point. Pal (1998) shows that their result is crucially dependent on the assumption of a linear city. He investigates a circular city duopoly model\(^3\) and finds that an equidistant location pattern appears in equilibrium, that is, locational dispersion appears.\(^4\) Matsushima (2001a) extends Pal’s model to an n-firm oligopoly and shows another equilibrium where half of the firms locate at one point and the other half locate at the opposite side (partial agglomeration). These results indicate the multiplicity of equilibria in spatial

---

\(^1\) Whether a price-setting or quantity-setting model is appropriate depends on the nature of the market being analyzed. If the quantity of each firm is more (less) flexible than the price, a price-setting (quantity-setting) model is more appropriate. See Eaton and Lipsey (1989) and Friedman (1983, 1988). For empirical works on this problem, see, among others, Brander and Zhang (1990).

\(^2\) Greenhut and Greenhut (1975) and Norman (1981) have already examined Cournot competition in spatial models, but they discuss the equilibrium price pattern rather than the equilibrium pattern of location. Recently, the literature on location-quantity models has become richer and more diverse. For example, Gupta, Pal, and Sarkar (1997) introduce spatial heterogeneity in the distribution of consumers. Mayer (2000) considers a model in which different locations offer firms different production costs. Chamorro-Rivas (2000) and Pal and Sarkar (2002) consider spatial Cournot competition among multi-plant firms. Shimizu (2002) introduces product differentiation. These studies show that agglomeration of firms does not always emerge. For a discussion of the shopping model with quantity-setting competition, see Hamilton, Klein, Sheshinski, and Slutsky (1994).

\(^3\) For the pioneering works on circular city models, see Vickrey (1964) and Salop (1979).

\(^4\) For applications of this model, see, e.g., Matsushima (2001b) and Matsumura (2003).
Cournot models with a circular city. The multiplicity of equilibria restricts the applicability of the model because the model does not give us a distinct prediction. In this paper we take a close look at Pal (1998) and Matsushima (2001a). We try to solve this problem by extending their linear transport cost model to one with non-linear transport cost. We discuss a simplified model. The numbers of firms and markets (and so possible locations) are 4. Four firms choose either location A, B, C, or D (See Figure 1). If each of the four firms chooses different location, we call this outcome ‘Pal-type’. If two firms locate at A and the other two locate at C, we call this outcome ‘Matsushima-type’. We find that Pal-type equilibrium is much more robust because (i) Pal-type equilibrium always exists as long as transport cost is increasing in the distance, while Matsushima-type equilibrium fails to exist when the transport cost function is significantly convex or concave (Proposition 1), (ii) if firms choose their locations sequentially, the unique equilibrium outcome is Pal-type under non-linear transport cost functions (Proposition 2), and (iii) the profit of each firm in Pal-type is never smaller than that in Matsushima-type, and the former is strictly larger if the transport cost function is non-linear (Proposition 3).

We also discuss welfare comparison between Pal-type and Matsushima-type. If the transport cost function is linear, two outcomes yield exactly the same profits, consumer surplus, and total social surplus. However, this equivalency does not hold if the transport cost function is non-linear, either convex or concave. In this case, Pal-type yields greater total social surplus and profit of each firm, while Matsushima-type yields greater consumer surplus. These results indicate that the welfare implications are sensitive to whether or not the transport cost function is linear.

Anderson and Neven (1991) shows that, in the linear city model, a strong concavity in transport cost changes the equilibrium outcome. This result is related to our results, but we emphasize that our results are different from theirs. First, we show that both concavity and convexity of the transport cost function affect the results while in Anderson and Neven (1991) only the strong convexity changes the result. Next, in our Propositions 2 and 3, even a slight non-linearity changes the results.

Both Pal (1998) and Matsushima (2002a) assume that the transport costs are proportional to distance and volume. We also assume that transport costs are proportional to volume and allow non-linearity of transport cost in the distance space only.
We make a remark on the applicability of the shipping spatial model. Most natural interpretation of the model is that each firm chooses where it builds a plant in the model. There is another important interpretation. We can interpret “space” as product varieties. Each firm’s location indicates the product or sector in which it has an advantage. Distant locations are the products the firm is in a disadvantage and to produce them it incurs additional costs. In short, the location choice corresponds to the technology choice and transportation costs correspond to the additional production costs. Hence, a shipping model is a suitable analytical tool for both spatial and non-spatial competitions.\footnote{For similar discussions, see Anderson and de Palma (1988), Eaton and Schmitt (1994), and Norman and Thisse (1999).} When we use circular city shipping model in this context, multiplicity of equilibria might yield difficulty in analysis. Our result indicates that we should focus on Pal-type in such a case because the Pal-type equilibrium is quite robust. Our result also indicates that, when we discuss welfare implication in this model, the results can be crucially dependent on the shape of the transport cost function.

The paper is organized as follows. Section 2 formulates the model. Section 3 investigates equilibrium outcomes and presents our main results. Section 4 discusses welfare. Section 5 concludes the paper. All proofs are presented in Appendix.

2 Model

There are four markets, A, B, C, and D, which are drawn in Figure 1. Each market generates a demand \( p = a - Q \), where \( p \) is the price of the homogeneous products and \( Q \) is the total quantity supplied by four firms. We assume that \( a \) is sufficiently large so as to ensure that all markets are served by all four firms. When shipping their products, the firms transfer them along the perimeter of the circle. The unit transport cost of shipping between neighboring markets (such as between A and B) is \( t \) and between the opposite markets (such as between A and C) is \( T \), where \( t \) and \( T \) are positive constants. We assume that \( 0 < t < T \) (transport cost is increasing in the distance). We normalize the transport cost within the market to zero. We can measure the degree of concavity (or convexity) by \( T/t \). If transport cost is proportional to the distance (i.e. the transport cost function is
linear in the distance), $T/t = 2$. If the transport cost function is convex (concave) with respect to the distance, $T/t > (<) 2$. A sufficient large (small) $T/t$ implies a large degree of concavity (convexity) of the transport costs.

Each firm $i$ chooses its location $x_i \in \{A, B, C, D\}$. After observing the rivals’ locations, firms compete à la Cournot. Each firm incurs a symmetric constant marginal cost of production, which we normalize to zero. The consumers are assumed to have a prohibitively costly transport cost, preventing arbitrage.\(^7\) These assumptions are standard in the literature.

### 3 Equilibrium Location Pattern

#### 3.1 Cournot competition

The equilibrium concept used is subgame perfect Nash equilibrium. We solve the game by backward induction. Thus, the local competition in the final stage subgames is examined first.

Because constant marginal cost is assumed, each local market can be analyzed independently. Let $q_i(j)$ and $\pi_i(j)$ ($i \in \{1, 2, 3, 4\}, j \in \{A, B, C, D\}$) denote the equilibrium output and the equilibrium profit of firm $i$ at market $j$ in the second stage respectively. The standard analysis yields

$$q_i(j) = \frac{1}{5}(a + c_1(j) + c_2(j) + c_3(j) + c_4(j) - 5c_i(j)),$$

$$\pi_i(j) = \frac{1}{25}(a + c_1(j) + c_2(j) + c_3(j) + c_4(j) - 5c_i(j))^2 = (q_i(j))^2,$$

where $c_i(j)$ denotes firm $i$’s unit transport cost between its location and market $j$. The total profit of firm $i$ is

$$\Pi_i = \sum_{j \in \{A, B, C, D\}} \pi_i(j).$$

#### 3.2 Simultaneous location choice

First we discuss a model with simultaneous location choice. The game runs as follows. In the first stage, each firm chooses its location simultaneously. After observing all firms’ location, four firms face Cournot competition described in Section 3.1.

\(^7\) This assumption is not essential. Unless transport costs for consumers are strictly smaller than those of firms, consumer arbitrage plays no role in our model. For this discussion, see Hamilton, Thisse, and Weskamp (1989).
There are 8 possible location patterns, as shown in Table 1. For example, pattern #5 has two firms located at A, one at B, and one at C. These eight cover all the possible patterns, as others are different only by symmetry to one of the patterns. Note that #7 is the Matsushima-type outcome (two firms locate at A and the others locate at C) and #8 is the Pal-type outcome (each firm chooses different location).

As noted above we use subgame perfection as equilibrium concept. A location pattern is an equilibrium outcome if no firm is willing to deviate from its current location, considering how its location affects the second stage output of each firm and resulting profits. In the following proposition, we show that only 2 out of the 8 patterns, Pal-type and Matsushima-type, can be an equilibrium location outcome.

**Proposition 1:** (i) Pal-type equilibrium always exists. (ii) Matsushima-type equilibrium exists if and only if \( \frac{3 - \sqrt{3}}{2} \leq \frac{T}{t} \leq \frac{3 + \sqrt{3}}{2} \). (iii) No other location pattern constitutes a subgame perfect equilibrium outcome.

As mentioned, if transport cost is linear (convex, concave), \( \frac{T}{t} = (>, <)2 \). In contrast to the robustness of Pal-type equilibrium, neither significant concavity nor convexity of the transport cost function supports Matsushima-type equilibrium.\(^8\) We explain the intuition behind Proposition 1 (ii). Consider a Matsushima-type outcome where firm 1 and firm 2 locate at A and firm 3 and firm 4 locate at C. We explain that the deviation (choosing B) by firm 4 improves its payoff given significant convexity or concavity of the transport cost function.

Suppose that \( t \) is sufficiently small (i.e., the unit transport cost function is sufficiently convex). Suppose that firm 4 deviates from the above Matsushima-type equilibrium and moves from C to B. The deviation reduces the costs firm 4 must incur for market A (\( T \rightarrow t \)) and market B (\( t \rightarrow 0 \)). Since

\(^8\) Increasing the number of the markets from 4 makes Matsushima-type equilibrium more difficult to be supported. For example, if Matsushima-type equilibrium fails to exist in the 4-market case, it also fails to exist in the 4n equidistant market case, with \( n \) a positive integer. Meanwhile, if we increase the number of firms and consider the case where it is larger than that of markets, it is obvious that (at least partial) agglomeration must appear in equilibrium. We believe that such a case is quite unnatural for analyzing whether Pal’s or Matsushima’s equilibrium is robust. Note that in the original Pal’s model (as well as Matsushima’s) the number of firms is finite, while that of markets is uncountable.
is small, the latter effect is negligible. The deviation increases the costs for market D \((t \rightarrow T)\) and market C \((0 \rightarrow t)\) and the latter effect is negligible. In market D (A) the total costs of firms 1–3 are \(3t \ (T)\), so the competition in market D is more severe than in market A when \(t\) is small, and the output of firm 4 is larger in market A than in market D. Thus, the cost-reducing effect for market A dominates the cost-raising effect for market D and the deviation increases firm 4’s profit.

Suppose that \(t\) is sufficiently close to \(T\) (i.e., the transport cost function is sufficiently concave). Suppose that firm 4 deviates from the above Matsushima-type equilibrium and moves from C to B. The deviation reduces the costs for market A \((T \rightarrow t)\) and market B \((t \rightarrow 0)\). Since \(t\) is close to \(T\), the former effect is negligible. The deviation increases the costs for market D \((t \rightarrow T)\) and market C \((0 \rightarrow t)\) and the former effect is negligible. In market B (C) the total costs of firms 1–3 are \(3t \ (2T)\), so the competition in market C is more severe than in market B when \(t\) is close to \(T\), and the output of firm 4 is larger in market B than in market C. Thus, the cost-reducing effect for market B dominates the cost-raising effect for market C and the deviation increases firm 4’s profit.

3.3 Sequential choice of location

We now discuss what happens if each firm chooses its location sequentially. The game runs as follows. Firm 1 chooses its location. After observing firm 1’s location, firm 2 chooses its location. After observing these two firms’ locations, firm 3 chooses its location. After observing these three firms’ locations firm 4 chooses its location. Finally, after observing all firms’ locations, four firms face Cournot competition described in Section 3.1.

By solving the game by backward induction, we find that the unique equilibrium outcome is Pal-type as long as \(T/t \neq 2\). The procedure of solving this problem is presented in Appendix (see Proof of Proposition 2).

**Proposition 2:** Let the four firms sequentially enter the market. Pal-type location pattern always constitutes a subgame perfect equilibrium outcome and it is the unique equilibrium location pattern if \(T/t \neq 2\).

As we show in the next section (Proposition 3 (ii)), unless the unit transport cost function is linear
in the distance, each firm prefers Pal-type to Matsushima-type. As is well known, in the pure coordination game, the sequential-move model chooses efficient equilibrium among multiple equilibria in the simultaneous-move model. This model also has that sequential-move game yields Pal-type rather than Matsushima-type outcome.

4 Welfare

We compare the welfare levels for Pal-type and Matsushima-type results. First we present one supplementary result, which helps us to understand the main result (Proposition 3).

**Lemma 1:** (i) The average price of four markets is \((a + 2t + T)/5\) regardless of the locations of firms; (ii) in Pal-type, prices are the same among four markets; (iii) in Matsushima-type, the prices of markets A and C are higher than (lower than, equal to) those of markets B and D if \(T/t > (\leq, =)2\); and (iv) the total transport cost of each firm is higher in Matsushima-type than in Pal-type if \(T/t \neq 2\).

The total output depends on the total cost of four firms. Regardless of the location of the firm, its marginal cost is zero for the market where it located, \(t\) for two neighboring markets, and \(T\) for the farthest market. Thus total cost is \(2t + T\) for each firm, and total costs for all four firms is \(8t + 4T\) regardless of the locations of firms. Thus, the total output does not depend on the location pattern, so the average price does not depend on the location pattern, either. Since Pal-type is symmetric at all four markets, no price dispersion exists. On the other hand, in Matsushima-type there is asymmetry between markets A and B, so price dispersion naturally appears. These yield Lemma 1 (ii) and (iii). The intuition behind Lemma 1 (iv) is explained after Proposition 3.

We now present Proposition 3. We let consumer surplus be denoted by

\[
CS = \frac{1}{2} \left[ \sum_{j \in \{A,B,C,D\}} \left( \sum_{i \in \{1,2,3,4\}} q_i(j) \right)^2 \right],
\]

and welfare be

\[
W = CS + \sum_{i \in \{1,2,3,4\}} \Pi_i.
\]
If the transport cost function is linear, both Pal-type and Matsushima-type yield exactly the same consumer surplus, profit of each firm, and total social surplus. Even a slight concavity or convexity of transport cost breaks this equivalence result.

**Proposition 3:** (i) If \( T/t = 2 \) (transport cost function is linear), consumer surplus, profit of each firm, and total social surplus in Matsushima-type location is equal to those in Pal-type location; and (ii) if \( T/t \neq 2 \), the social welfare level and profit of each firm in Pal-type location are greater than those in Matsushima-type location, and consumer surplus in Matsushima-type location is greater than that in Pal-type location.

We discuss the intuition behind Proposition 3. First, we discuss consumer surplus. Consider a Pal-type equilibrium where firm 1 (2,3,4) locates at A (B,C,D) and a Matsushima-type equilibrium where firms 1 and 2 (3 and 4) locate at A (C). From Lemma 1, we can see that both Pal-type and Matsushima-type yield the same average price of four markets. In Pal-type location the equilibrium price is the same in all four markets. On the other hand, in Matsushima-type, the price in market A is equal to that in market B only if \( T/t = 2 \). If \( T/t > 2 \) (convex transport cost function), the price in market A is higher than that in market B. If \( T/t < 2 \) (concave transport cost function), the price in market A is lower than that in market B. In short, if \( T/t \neq 2 \), the prices are more dispersed in Matsushima-type. Since price dispersion increases average consumer surplus, Matsushima-type yields greater consumer surplus.

Next, we discuss the profit of each firm. When \( T/t = 2 \), \( q_1(j)(j = A, B, C, D) \) does not depend on whether the location is of Matsushima-type or Pal-type, so from (2) we have that both Matsushima-type and Pal-type yield the same profit for firm 1. Now consider the cases in which \( T/t \neq 2 \). The revenue of each firm is higher in Pal-type than in Matsushima-type. And total cost of each firm is higher in Matsushima-type than in Pal-type (Lemma 1 (iv)). Both yield Proposition 3 (ii).

We now explain the reason why the revenue is smaller in Matsushima-type. As mentioned above, both Pal-type and Matsushima-type yield the same average price of four markets, and the price is more dispersed in Matsushima-type. In Matsushima-type, the output quantity sold at markets A and
C is different from that at markets B and D. Obviously, the output sold at the lower price markets must be larger than that at the higher price markets. Since the total output is the same in both cases, the total revenue is smaller in Matsushima-type than in Pal-type.

We then explain the reason why the cost is higher in Matsushima-type. Suppose that $T$ becomes $2t + 5\varepsilon$ where $\varepsilon$ is a small and positive constant. In Matsushima-type, the increase in $T$ increases $q_1(A)$ by $2\varepsilon$ and reduces $q_1(C)$ by $3\varepsilon$. In Pal-type, it increases $q_1(A), q_1(B), q_1(D)$ by $\varepsilon$ and reduces $q_1(C)$ by $4\varepsilon$. Since firm 1’s cost for market C increases while its costs for other markets remain unchanged, a smaller output for market C substantially reduces total transport costs. Since a reduction of output for market C is larger in Pal-type than in Matsushima-type, Pal-type yields a smaller total transport cost.

Finally, we explain the reason why total social surplus is greater in Pal-type. As mentioned above, the cost is smaller in Pal-type. Since the total output remains unchanged, the change from Matsushima-type to Pal-type saves costs through reallocation of the production among four firms and it improves welfare.\footnote{There are many papers discussing welfare-improving reallocation of production among firms in oligopoly markets. See, among others, Brander (1981), Lahiri and Ono (1988, 1998, 1999), Matsumura (1998) Ono (1990), Riordan (1998), and Ushio (2000).}

5 Concluding Remarks

In this paper we reexamine the spatial Cournot competition discussed by Pal (1998) and Matsushima (2001a). Both Pal and Matsushima assume the linear transport cost function (unit transport cost is proportional to the distance). Pal shows that an equidistant location pattern appears in an equilibrium, and Matsushima shows that partial agglomeration in which half of the firms agglomerate at one point and the others agglomerate at the opposite point appears in another equilibrium. We call the
former location pattern ‘Pal-type’ and the latter ‘Matsushima-type’. We investigate the robustness of these two types of equilibria by introducing non-linearity to transport cost. We find that Pal-type is much more robust than Matsushima-type. First, we show that Pal-type equilibrium exists, while neither significant concavity nor convexity of transport costs yields Matsushima-type equilibrium. Second, if we consider sequential-move of location choice, Matsushima-type fails to be an equilibrium even under slight concavity or convexity of transport costs. On the contrary, Pal-type always becomes an equilibrium and it is a unique equilibrium if the transport cost function is non-linear. Third, the profit of each firm in Pal-type is never smaller than that in Matsushima-type, and the former is strictly larger if the transport cost function is non-linear. Thus, firms choose Pal-type if they can coordinate their locations. The multiplicity of equilibria yields difficulty in analysis. Our result might indicate that we sold avoid this problem by choosing Pal-type rather than Matsushima-type. Our result also shows the importance of non-linearity of the transport cost.

We also compare welfare of the two types. If the transport cost function is linear with respect to the distance, both types yield exactly the same welfare implications, while non-linearity of the transport cost function yields the differences in the two types. Pal-type yields a larger total surplus than Matsushima-type. In other words, dispersion is better than partial agglomeration from the viewpoint of social welfare.

Locations of plants were severely regulated until recently in Japan, and firms could not build new plants in Tokyo or Osaka area. Such regulations of plant locations so as to avoid excessive concentration widely exist. Most theoretical supporting for such regulations are based on technological externalities such as congestion effects. Our paper gives another rationale of such regulations without assuming any congestion effects.

---

10 Myerson (1991) discusses that the efficient equilibrium among multiple equilibria becomes focal point.
APPENDIX

Proof of Proposition 1 (i): Suppose that $x_1 = A, x_2 = B,$ and $x_3 = C.$ We show that $x_4 = D$ is firm 4’s best reply. The profit of firm 4 when $x_4 = A, x_4 = B,$ and $x_4 = D$ are given respectively

\[
\frac{1}{25} \left( \left( a + t + T \right)^2 + (a - 2t)^2 + (a + t - 3T)^2 + (a - 2t + T)^2 \right) \quad (x_4 = A, C),
\]

\[
\frac{1}{25} \left( \left( a - 3t + T \right)^2 + (a + 2t)^2 + (a - 3t + T)^2 + (a + 2t - 3T)^2 \right) \quad (x_4 = B),
\]

\[
\frac{1}{25} \left( \left( a - 3t + T \right)^2 + (a + 2t - 4T)^2 + (a - 3t + T)^2 + (a + 2t + T)^2 \right) \quad (x_4 = D).
\]

From the symmetry, the profit of firm 4 when $x_4 = C$ is also given by (4). (4) − (6) = −8((T − t)^2 + t^2)/25 < 0 and (5) − (6) = −8T^2/25 < 0. Thus, firm 4’s best reply is $x_4 = D$. Q.E.D.

Proof of Proposition 1 (ii): Suppose that $x_1 = x_2 = A,$ and $x_3 = C.$ We show that $x_4 = C$ is firm 4’s best reply if and only if $(3 - \sqrt{3}) \leq T/t \leq (3 + \sqrt{3}).$ The profit of firm 4 when $x_4 = A, x_4 = B,$ and $x_4 = C$ are given respectively

\[
\frac{1}{25} \left( \left( a + T \right)^2 + (a - t)^2 + (a - 2T)^2 + (a - t)^2 \right) \quad (x_4 = A),
\]

\[
\frac{1}{25} \left( \left( a - 4t + T \right)^2 + (a + 3t)^2 + (a - 4t + 2T)^2 + (a + 3t - 4T)^2 \right) \quad (x_4 = B, D),
\]

\[
\frac{1}{25} \left( \left( a - 3T \right)^2 + (a - t)^2 + (a + 2T)^2 + (a - t)^2 \right) \quad (x_4 = C).
\]

From the symmetry, the profit of firm 4 when $x_4 = D$ is also given by (8).

\[
(7) - (9) = -\frac{8}{25}T^2 < 0,
\]

\[
(8) - (9) = \frac{8}{25}(6t^2 - 6tT + T^2).
\]

(11) is non-positive if and only if $(3 - \sqrt{3}) \leq T/t \leq (3 + \sqrt{3}).$ Q.E.D.

Proof of Proposition 1 (iii): Before we proceed with the proof, we introduce Table 2, which shows the profit levels of firms for each location pattern. For example, $\Pi(#5A)$ indicates the profit for the firms located at $A$ in the pattern #5. This is given by equation (4), which when expanded is equal to $(4a^2 - 4at - 2aT + 10t^2 - 8tT + 11T^2)/25.$ Note that $\Pi(#4)$ indicates the profit level of
Π(#4A) or Π(#4B), as they are equivalent. Similarly we drop the location letter from Π(#7) and Π(#8). Also, we use Table 3, which shows the differences in payoffs for selected pairs of profit levels.

Now, we show that for patterns #1 to #6 there exists at least one firm willing to deviate from its current location. (We just show one example each of a possible deviation.)

For #1, any firm relocating from A to C can increase its profit from Π(#1) to Π(#3C) (See Table 3).

For #2, the firm located at B can relocate to C and increase its profit from Π(#2B) to Π(#3C).
For #3, a firm at A can increase its profit from Π(#3A) to Π(#7) by relocating to C.
For #4, a firm at B relocating to C gives an increase in profit, from Π(#4) to Π(#5C).
For #5, a firm located at A can relocate to D, increasing its profit from Π(#5A) to Π(#8).
For #6, a firm at B can relocate to D, which increases its profit from Π(#6B) to Π(#8).

Thus none of the location patterns #1 to #6 can be a location equilibrium outcome. Q.E.D.

Proof of Proposition 2: We solve for the subgame perfect equilibrium of the game. When the first firm enters, all four possible locations are equivalent. Thus, without loss of generality, we let firm 1 locate at location A. From here on, we look at the subgames with firm 2 locating at A, B (or D), or C, and solve the game using backward induction.

First we look at the subgame where firms 1 and 2 both are located at A. If firm 3 locates at A, then firm 4 would choose C, as Π(#3C) is larger than Π(#1) or Π(#2B) (See Table 3 from here for calculation for the differences in profits). If firm 3 selects B (or D by symmetry), firm 4 again locates at C, as Π(#5C) is greater than Π(#2A), Π(#4), or Π(#6A). If firm 3 chooses C, then firm 4 locates at B if Π(#7) − Π(#5B) = 8(−6t^2 + 6tT − T^2) is smaller than or equal to 0, and locates at C if it is larger than or equal to 0. A is not chosen, as Π(#7) > Π(#3A).

Considering this reaction by firm 4, firm 3 gets Π(#3A) if it chooses A, Π(#5B) if it chooses B, Π(#5C) if it chooses C and (−6t^2 + 6tT − T^2) ≤ 0, and Π(#7) if it chooses C and (−6t^2 + 6tT − T^2) ≥ 0. First, Π(#5B) is greater than Π(#3A), so A is not chosen. Also, Π(#5C) > Π(#5B) holds. Finally, Π(#7) − Π(#5B) = 8(−6t^2 + 6tT − T^2) ≥ 0 in the subgame where firm 4 chooses C after firm 3 chooses C allowing Π(#7). Consequently, firm 3 chooses C in this subgame.
Now, we consider the subgame where firm 1 chooses $A$ and firm 2 chooses $B$. Firm 3 can locate at $A$ (or $B$ by symmetry) or $C$ (or $D$). If $A$ is chosen, then firm 4 selects $C$, as in the case firms 1, 2, and 3 chose $A$, $A$, and $B$ in the last subgame. If $C$ is chosen, then firm 4 locates at $D$, since $\Pi(#8)$ is larger than $\Pi(#5A)$ or $\Pi(#6B)$. Considering this, firm 3 chooses to locate at $C$, as $\Pi(#8)$ is greater than $\Pi(#5A)$. Thus firm 3 locates at $C$ and firm 4 locates at $D$ in this subgame.

Finally, we consider the subgame where firm 1 is at $A$ and firm 2 then locates at $C$. Firm 3 can locate at $A$ (or $C$) or at $B$ (or $D$). If $A$ is selected, then firm 4 chooses $B$ if $(-6t^2 + 6tT - T^2) \leq 0$ and $C$ if $(-6t^2 + 6tT - T^2) \geq 0$ just as one of the cases above. If $B$ is selected, then firm 4 locates at $D$, since $\Pi(#8)$ is larger than $\Pi(#5A)$ or $\Pi(#6B)$. Considering this, firm 3 chooses $B$, as $\Pi(#8) > \Pi(#5A)$ and $\Pi(#8) - \Pi(#7) = 6(2t - T)^2 \geq 0$. It can also choose $A$, getting a payoff $\Pi(#7)$ when $\Pi(#7) = \Pi(#8)$. Here, it must be the case that $T/t = 2$ (or, a linear transport), which implies that $(-6t^2 + 6tT - T^2) \geq 0$ so that firm 4 chooses $C$.

Now we have examined the reactions by firms 3 and 4. The final step is to look at firm 2. If it locates at $A$, its payoff is $\Pi(#5A)$ if $(-6t^2 + 6tT - T^2) \leq 0$ and $\Pi(#7)$ if $(-6t^2 + 6tT - T^2) \geq 0$. If firm 2 locates at $B$, it obtains $\Pi(#8)$. If firm 2 locates at $C$, it gets $\Pi(#8)$, or $\Pi(#7)$ if $T/t = 2$ (which again implies $(-6t^2 + 6tT - T^2) \geq 0$ is satisfied). Since $\Pi(#8) \geq \Pi(#7)$ and $\Pi(#8) > \Pi(#5A)$, firm 2 can choose either $B$ or $C$ to achieve Pal-type outcome profit. In the case $T/t = 2$, firm 2 can choose any location and get the profit level $\Pi(#7) = \Pi(#8)$. Thus, $#8$, or the Pal-type outcome, is the subgame perfect equilibrium outcome, and $#7$, or the Matsushima-type outcome, is also that if $T/t = 2$ holds. \[\text{Q.E.D.}\]

**Proof of Lemma 1:** Let $p(j)$ denote the price in market $j$.

(i) From (1), the total output equals

$$\sum_{j \in \{A,B,C,D\}} \sum_{i \in \{1,2,3,4\}} q_i(j) = \sum_{j \in \{A,B,C,D\}} \frac{1}{5}(4a - c_1(j) - c_2(j) - c_3(j) - c_4(j)) = \frac{4}{5}(4a - 2t - T).$$

Since $p(j) = a - \sum_{i \in \{1,2,3,4\}} q_i(j)$, the average price is

$$\frac{1}{4} \sum_{j \in \{A,B,C,D\}} p(j) = \frac{1}{4}(4a - \frac{4}{5}(4a - 2t - T)) = \frac{1}{5}(a + 2t + T).$$
(ii) Obvious, since the markets and locations are all symmetric.

(iii) By symmetry, the prices in markets A and C are the same and those in markets B and D are the same. We compare the prices in markets A and B.

\[ p(A) = a - \sum_{i \in \{1,2,3,4\}} q_i(A) = a - \frac{1}{5}(4a - c_1(A) - c_2(A) - c_3(A) - c_4(A)) = \frac{1}{5}(a + 2T), \]

\[ p(B) = a - \sum_{i \in \{1,2,3,4\}} q_i(B) = a - \frac{1}{5}(4a - c_1(B) - c_2(B) - c_3(B) - c_4(B)) = \frac{1}{5}(a + 4t), \]

\[ p(A) - p(B) = \frac{2}{5}(-2t + T). \]

The last equation implies (iii).

(iv) By symmetry the total transport cost for all four firms is the same for a given location type. Thus, we need only consider one firm for each type. The total transport cost of firm 1 is as follows:

Pal: \[ CS_P = \frac{2}{25}(4a - 2t - T)^2 \]

\[ \pi_P = \frac{1}{25}[2(a - 3t + T)^2 + (a + 2t - 4T)^2 + (a + 2t + T)^2] \]

Matsushima: \[ CS_M = \frac{1}{25}(4a - 2T)^2 + (4a - 4t)^2 \]

\[ \pi_M = \frac{1}{25}[(a - 3T)^2 + (a + 2T)^2 + 2(a - t)^2], \]

where the superscripts denote Pal and Matsushima type results. From this, we have

\[ CS_P - CS_M = -\frac{2}{25}(2t - T)^2 \leq 0 \]

\[ \pi_P - \pi_M = \frac{6}{25}(2t - T)^2 \geq 0 \]

\[ W_P - W_M = \frac{22}{25}(2t - T)^2 \geq 0, \]

and the equalities are satisfied if and only if \( T/t = 2 \). Q.E.D.
References


T: Unit transport cost between A and C (B and D)
t: Unit transport cost between neighboring markets

Figure 1
<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>#2</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>#3</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>#4</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>#5</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>#6</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>#7</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>#8</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: Eight location patterns

| Π(#1) | (4a^2 - 4at - 2aT + 2t^2 + T^2)/25 |
| Π(#2A) | (4a^2 - 4at - 2aT + 10t^2 - 10t + 5T^2)/5 |
| Π(#2B) | (4a^2 - 4at - 2aT + 50t^2 - 48tT + 25T^2)/25 |
| Π(#3A) | (4a^2 - 4at - 2aT + 2t^2 + 5T^2)/25 |
| Π(#3C) | (4a^2 - 4at - 2aT + 2t^2 + 25T^2)/25 |
| Π(#4) | (4a^2 - 4at - 2aT + 26t^2 - 24tT + 13T^2)/25 |
| Π(#5A) | (4a^2 - 4at - 2aT + 10t^2 - 8tT + 21T^2)/25 |
| Π(#5B) | (4a^2 - 4at - 2aT + 50t^2 - 48tT + 11T^2)/25 |
| Π(#5C) | (4a^2 - 4at - 2aT + 26t^2 - 24tT + 19T^2)/25 |

Table 2: Profit levels for each location
<table>
<thead>
<tr>
<th>First profit</th>
<th>Second profit</th>
<th>Difference $\times 25$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pi(#3C)$</td>
<td>$\Pi(#1)$</td>
<td>$24T^2$</td>
</tr>
<tr>
<td>$\Pi(#3C)$</td>
<td>$\Pi(#2B)$</td>
<td>$-48t^2 + 48tT &gt; 0$</td>
</tr>
<tr>
<td>$\Pi(#5C)$</td>
<td>$\Pi(#2A)$</td>
<td>$16T^2 &gt; 0$</td>
</tr>
<tr>
<td>$\Pi(#5C)$</td>
<td>$\Pi(#4)$</td>
<td>$-16t^2 + 16tT + 8T^2 &gt; 0$</td>
</tr>
<tr>
<td>$\Pi(#5C)$</td>
<td>$\Pi(#6A)$</td>
<td>$-16t^2 + 16tT &gt; 0$</td>
</tr>
<tr>
<td>$\Pi(#7)$</td>
<td>$\Pi(#3A)$</td>
<td>$8T^2 &gt; 0$</td>
</tr>
<tr>
<td>$\Pi(#7)$</td>
<td>$\Pi(#5B)$</td>
<td>$-48t^2 + 48tT - 8T^2$</td>
</tr>
<tr>
<td>$\Pi(#5B)$</td>
<td>$\Pi(#3A)$</td>
<td>$48t^2 - 48tT + 16T^2 &gt; 0$</td>
</tr>
<tr>
<td>$\Pi(#5C)$</td>
<td>$\Pi(#5B)$</td>
<td>$-40t^2 + 40tT &gt; 0$</td>
</tr>
<tr>
<td>$\Pi(#8)$</td>
<td>$\Pi(#5A)$</td>
<td>$16t^2 - 16tT + 8T^2 &gt; 0$</td>
</tr>
<tr>
<td>$\Pi(#8)$</td>
<td>$\Pi(#6B)$</td>
<td>$8T^2 &gt; 0$</td>
</tr>
<tr>
<td>$\Pi(#8)$</td>
<td>$\Pi(#7)$</td>
<td>$24t^2 - 24tT + 6T^2 \geq 0$</td>
</tr>
</tbody>
</table>

Table 3: Selected differences in profit levels