Equilibrium Number of Firms and Economic Welfare in a Spatial Price Discrimination Model

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Abstract

We examine the relationship between the equilibrium number of the firms entering the market and socially efficient one. Salop (1979) investigates a mill pricing model and shows that the former is larger than the latter (excess entry theorem). We find that, in contrast to Salop (1979), the equilibrium number of the firms can be either larger or smaller than the efficient one. If the entry cost is small and the resulting equilibrium number of the firms large, it is excessive from the normative viewpoint. On the contrary, if the entry cost is small and the resulting equilibrium number of the firms is large, it can be insufficient.

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1 Introduction

Since the seminal work of Hotelling (1929), the model of spatial competition has been seen by many researchers as an attractive framework for analyzing oligopoly markets. The Hotelling linear-city spatial model has become one of the most important models of product differentiation. Salop (1979) formulates an alternative approach, namely, a circular-city model. Since the circular-city is symmetric at any point, in contrast to the linear-city, we can focus on a symmetric outcome as a possible equilibrium outcome in oligopoly models. Thus, in the circular-city model, we can analyze cases of more than two firms more easily than in the linear-city model. Because of this advantage, this model has also become popular as a useful analytical tool in many fields, including Industrial Organization and Regional Science.  

Salop (1979) shows that the equilibrium number of entering firms is larger than the optimal number of firms. In other words, the number of entering firms is excessive from the viewpoint of social welfare. Later, Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) show that the number of entering firms is excessive in the non-spatial Cournot model as well. This result is known as the excess entry theorem. In this paper, we also examine the relationship between the equilibrium number of firms and socially efficient one. In the Salop (1979), mill pricing model is adopted. We use another standard spatial model, spatial price discrimination (delivered pricing) model.

Spatial models fall into two categories: shipping (spatial price discrimination, or delivered pricing) models are those in which firms bear the transport costs; shopping (or mill pricing) models are those in which consumers pay for transport. There are three reasons why we

\footnote{For applications of circular-city models, see, e.g., Schmalensee (1978), Matsushima (2001), Ishibashi (2003) and Matsumura (2003).}

\footnote{See also Lahiri and Ono (1988), Konishi, Okuno-Fujiwara and Suzumura (1990), Konishi (1990, 1999), and Matsumura (2000).}

\footnote{For non-spatial interpretation of the spatial price discrimination model, see Eaton and Schmitt (1994), and}
investigate the spatial price discrimination model. First, as Thisse and Vives (1988) show, firms usually choose delivered pricing policy rather than mill pricing policy if they choose pricing policy non-cooperatively. Thus, it is natural to use a delivered pricing model when we discuss competitive situation. Second, as Greenhut (1981) reports, the delivered pricing model well describes the price pattern in real world such as in German and Japanese markets. Furthermore, in the context of international trade, firms often set different prices for each country (a phenomenon referred to as “market segmentation”). The delivered pricing (spatial price discrimination) model describes this situation well. Third, it is not always realistic that consumers directly pay the transport costs.

We investigate a circular-city model like Salop (1979). We consider a free entry market, where zero profit condition determined the equilibrium number of the firms. We compare the equilibrium number of the firms and the socially efficient number of the firms. We find that if the entry cost is small and the resulting equilibrium number of the firms is large, it is excessive from the normative viewpoint. On the contrary, if the entry cost is large and the resulting equilibrium number of the firms is small, it can be insufficient. In short, the excess entry theorem does not always hold in the delivered pricing model.

In the spatial price discrimination model, new entries reduce the prices of all markets, but the degree of the effect is not uniform among markets. The new entries significantly reduce the prices at the markets where price-cost margins are large, while the effect is weak at the markets where price-cost margins are small. The reduction of the prices significantly improve welfare at the market where margins are large. Thus, the new entries effectively improve welfare in spatial price discrimination model, and it yields the anti-excess entry theorem.

The paper is organized as follows. Section 2 formulates a model, and Section 3 and 4 analyze equilibrium and optimal number of firms. Section 5 presents our results. Section 6

Norman and Thisse (1999).
concludes the paper.

2 The model

Let there be a circular city of length 1, where a continuum of markets lies uniformly throughout. Firms are also located around the circle, and all travel occurs along the circle. Each firm is allowed to locate in only one location. In order to address the issue of the number of firms, we introduce a fixed cost of entry, $F$.

In the first stage, potential entrants simultaneously choose whether or not to enter. Let $n$ denote the number of entering firms. Following Salop (1979), those firms do not choose their location, but rather are automatically located equidistant from one another on the circle. Thus, maximal differentiation is exogenously imposed. In the second stage, firms compete in prices, given these locations. Each firm simultaneously chooses its price at every market in the continuum $[0, 1]$ so that its total profit is maximized. Let $x$ be a point on the circle located at a distance from 0, measured clockwise. The demand function at each market is set to be linear, i.e., $p(x) = A - Q(x)$, where $A$ is a positive constant, and $p(x)$ and $Q(x)$ are the price and the total quantity supplied at market $x$, respectively. Each firm incurs a symmetric constant marginal cost of production, which we normalize to zero without loss of generality.

The firms must pay transport costs for the shipment of its products to the markets. We adopt a linear transport cost function. Transport costs are propositional to both the output quantity and the distance. To ship a unit of the product from its plant $x_i$ to a market at point $x$, firm $i$ pays a transport cost $T_i = t|x - x_i|$, where $t$ is a positive constant and $|x - x_i|$ is the distance between $x$ and $x_i$. The norm signifies the shorter distance of the two possible ways to transfer the goods along the perimeter. The consumers are assumed to have a prohibitively

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4 Lederer and Hurter (1986) and Hamilton et al. (1989) discuss equilibrium location pattern in the spatial price discrimination models with Bertrand competition.
costly transport cost, preventing arbitrage. Finally, we assume that the equilibrium price at each market is strictly smaller than the monopoly price, so the rival serves as a potential competitor. The above assumptions are standard in the literature.

3 Equilibrium

We restrict our attention to pure strategy equilibria. We use subgame perfection as the equilibrium concept. The game is solved by backward induction.

First, we consider competition in the second stage. Suppose that \( n \) firms enter the market. Because of the symmetry, we discuss the market \( x \in [0, 1/n] \), where firm 1 compete against firm 2. Since the marginal costs are constant, prices set at different points by the same firm are strategically independent. The second-stage Bertrand equilibrium of this game can then be characterized by a set of independent Bertrand equilibria, one for each point \( x \). As is well known, under the homogeneous goods Bertrand competition, the firm with a lower cost obtains whole share of the market and the equilibrium price is equal to the rival’s cost. Firm 1 serves all markets \( x \leq 1/(2n) \) and firm 2 serves markets \( x \geq 1/(2n) \). Thus, firm 1’s profit at market \( x \in [0, 1/(2n)] \) is \( (T_2 - T_1)(A - T_2) = (t(1/n - x) - tx)(A - t(1/n - x)) \).

Firm 1 also obtains profits from market \( x \in [(2n - 1)/2n, 1] \) and the resulting total profit of firm 1 is

\[
\Pi_1(n) = 2 \int_0^{1/(2n)} (t(1/n - x) - tx)(A - t(1/n - x))dx - F = g(n) - F, \tag{1}
\]

where

\[
g(n) \equiv -\frac{5t^2}{12n^3} + \frac{At}{2n^2}. \tag{2}
\]

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5 This assumption is not essential. Unless transport costs for consumers are strictly smaller than those of firms, consumer arbitrage plays no role in our model. For this discussion, see Hamilton, Thisse, and Weskamp (1989).
Second, we consider the equilibrium number of the firms. Let \( n^E \) denote the equilibrium number of the firms. It is given by the following zero profit condition,

\[
g(n) = F. \tag{3}
\]

Solution exists if and only if \( F \leq 8A^3/(75t) \). We assume that \( F \leq 8A^3/(75t) \). If \( F < 8A^3/(75t) \), this equation has two solutions. One is smaller than \( 5t/4 \) and the other is larger than \( 5t/4 \). We concentrate on the latter solution. There are two reasons for this. One is that only the latter solution is stable for the standard adjustment process. The other is that we can show that the former never satisfies the assumption that the equilibrium price at each market is strictly smaller than the monopoly price. The assumption requires \( n^E A > 2t \). If \( n^E < 5t/(4A) \), it is impossible that \( n^E A > 2t \).

4 Second best problem

We consider total social surplus when the number of firms is given exogenously. The second stage competition (price competition) is the same as in the previous section.

The consumer surplus at market \( x \in [0, 1/(2n)] \) is

\[
 cs(x) = \frac{1}{2} \left( A - t(1/n - x) \right)^2
\]

and the total consumer surplus is

\[
 CS(n) = 2n \int_{0}^{1/(2n)} \left( A - t/n + tx \right)^2 dx = \frac{1}{6} \left( \frac{7t^2}{4n^2} - \frac{9At}{2n} + 3A^2 \right).
\]

The total social surplus is given by

\[
 W(n) \equiv CS(n) + n\Pi_1(n) = -\frac{t^2}{8n^2} - \frac{At}{4n} + \frac{A^2}{2} - nF.
\]

The first order condition for optimality is

\[
 W'(n) = 0 \iff h(n) - F = 0, \tag{4}
\]
where

\[ h(n) = \frac{t^2}{4n^3} + \frac{At}{4n^2}. \]

The second order condition \( h' < 0 \) is satisfied. Thus the efficient number of the firms, \( n^* \), is given by (4).

## 5 Result

We now compare \( n^E \) and \( n^* \). Since \( h(n) \) is decreasing, \( n^* > (=,<)n^E \) if \( h(n^E) > (=,<)F \).

Since \( g(n^E) = F \), \( h(n^E) > (=,<)F \) is equivalent to \( h(n^E) > (=,<)g(n^E) \). From (2) and (4) we have

\[ g(n) - h(n) = \frac{t}{12n^2}(-8t + 3An). \] (5)

Thus, \( h(n^E) > (=,<)g(n^E) \) is equivalent to \( n^E > (=,<)8t/(3A) \). This yields the following proposition.

**Proposition 1:** (i) If \( n^E > 8t/(3A) \) (or equivalently \( F < 99A^2/(2048t) \)), then \( n^* > n^E \). (ii) If \( n^E < 8t/(3A) \) (or equivalently \( F > 99A^2/(2048t) \)), then \( n^* > n^E \).

Proposition 1 states that the number of entering firms are insufficient when the entry cost is large, while it is excessive when it is small. Proposition 1 (ii) presents an anti-excess entry theorem. We explain the intuition why the number of entering firms can be insufficient.

We explain the intuition. An additional entry reduces the market shares of existing firms and reduces their profits (business stealing effect). A new entrant neglects this effect, so the private incentive of new entry can be excessive. On the other hand, there is an welfare-improving effect of new entry. Suppose that the number of existing firms is two and two new entries take place. The equilibrium prices at market \( x \in [0,1/4] \) are presented in Figure 1. From this figure we can see that the new entries substantially reduce the prices at the markets.
near to the point 0 and the effect is small for the markets near to the point 1/4. In other words, the new entries reduce the prices unequally.

At the market 0, the marginal cost (unit transport cost) of supplying firm (firm 1) is small, but the equilibrium price is highest. Thus, the margin (price minus marginal cost) is maximized at this market. This implies that an increase of the output at this market increases total surplus most effectively. On the other hand, at the market 1/4, the margin is zero before the entries. The new entries increases the output substantially at the markets where the margin is relatively large, while this effect is weak at the market where the margin is relatively small. Thus, the welfare-improving effect becomes stronger than the case where new entries uniformly reduce the prices (if the average price reduction is the same among two cases).

When $F$ is large and the resulting $n^E$ is small, the margin is relatively large, so this welfare-improving effect becomes further strong and dominate the usual business stealing effect. This is the reason why the number of entering firms can be insufficient.

6 Concluding Remarks

In this paper, we discuss whether or not the efficient number of firms exceeds the equilibrium one. We find that the equilibrium number of firms can be either larger or smaller than the efficient one. New entries affect the equilibrium price unequally. The prices at the markets where price-cost margins are relatively large fall effectively, while the prices at the markets where price-cost margins are relatively large do not. Thus, welfare-improving effect of new entries becomes stronger than the case where prices falls uniformly (such as in mill pricing model). Therefore, as opposed to the result of mill pricing model, the welfare-improving effect of new entries can dominate the well known ‘busyness stealing effect’, which makes the efficient number of firms to exceed the equilibrium one.

In this paper we consider symmetric firm model and do not consider the problem of en-
try deterrence. In the context of spatial competition, this problem is intensively discussed, especially in the mill pricing model. Extending our model to this direction remains for future research.\footnote{For the entry deterrence in mill pricing model, see e.g., Schmalensee (1978), Shaked and Sutton (1982), Judd (1985), Ashiya (2000) and Ishibashi (2003).}
References


