Vertical Integration, Bundling, and Welfare

Masayoshi Maruyama* and Kazumitsu Minamikawa**

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Abstract

We provide a framework to analyze the effect of vertical integration and mixed bundling in the vertically related markets. We consider a two-stage game. In the first stage, firms choose market structures and pricing schemes. In the second stage, the firms choose prices. We then compare prices, demands, consumer surplus, and social welfare under alternative market structures. If the firms do not use mixed bundling, the component prices of integrated firms become lower than those of independent firms. When mixed bundling is possible, integrated firms set higher prices for individual components and lower prices for bundled goods. Although these have opposite effects on consumer welfare, mixed bundling always reduces consumer surplus. We also show that mixed bundling is a dominant strategy for all firms. Whereas, except for the composite goods that are very close substitutes, the firms are better off when they commit not to use mixed bundling. Our model suggests that, for a wide range of parameters, the restriction against mixed bundling is beneficial to consumers and firms.

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(*) Masayoshi Maruyama, Professor,
Graduate School of Business Administration, Kobe University, JAPAN
Email: mmaru@kobe-u.ac.jp
(**) Kazumitsu Minamikawa, Associate Professor
Department of Business Administration, Nanzan University, JAPAN

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1. Introduction

Consumers combine complementary components to form a composite good such as computer and software, DVD player and content, and cellular phone and communication service etc. For these goods there is a horizontal competition among substitutable components, and a system competition among composite goods. Recently, much interest has been shown in the structure of vertically related market and pricing strategies for complementary components.

There are a variety of market structures for complementary components; independent ownership, parallel vertical integration, and asymmetric structure etc. Independent ownership is a market structure where each complementary component is independently owned by separate firms. Parallel vertical integration is a market structure where there is joint ownership of pairs of complementary components, along with continued competition among substitute components. Asymmetric structure is a market structure where there is an integrated firm offering pairs of complementary components and there are specialist firms supplying only each single component. Economides and Salop [1992] analyze the effect of competition and integration among complementary components on the equilibrium prices by examining a variety of alternative market structures. Among the results we must note that prices are always lower in the case of parallel vertical integration than in independent ownership. It is a generalized result of Cournot’s duopoly of complementary products.

Firms can sell their components separately (pure component pricing), and sell them only as a system (pure bundling), or sell both (mixed bundling). Economides and Salop [1992] assume that integrated firms practice pure component pricing. But, bundling is a common practice for many complementary products.

There are several papers that consider the strategic effects of bundling in a duopoly where both firms produce the two components of a system (i.e., parallel vertical integration). Matutes and Regibeau [1992] investigated a two-stage pricing game based on the Hotelling model. Economides [1993] examined a two-stage game of pricing using the same linear demand model as Economides and Salop [1992]. These models assume that in the first stage firms commit to a certain pricing strategy. On the other hand, Anderson and Leruth [1993] and Liao and Tauman [2002] investigated the case where firms cannot commit to a pricing strategy before deciding on
price levels.

There are also papers that consider the effects of bundling under different market structures related to parallel vertical integration. Ida [2002] considers the issue of mixed bundling by Internet service providers under different market structures of one-way and two-way connections. As related to the proposed GE/Honeywell merger, Choi [2003] compares the prices, profits, and social welfare under the two market structures of pre-merger situation of independent ownership and the post-merger situation of asymmetric structure where an integrated firm is able to engage in mixed bundling. In all of those models, the market structures are given.

In this paper we will examine the choices of market structures and pricing schemes by different firms. We consider a two-stage game. In the first stage, firms choose market structures and pricing schemes. In the second stage, the firms choose prices. We compare prices, demands, consumer surplus, and social welfare under alternative market structures.

As Economides and Salop [1992] have shown, if firms do not use mixed bundling, then component prices of integrated firms become lower than those of independent firms. When mixed bundling is possible, we show that integrated firms set higher prices for individual components than the level the independent firms set. Then the hybrid goods tend to be less attractive to consumers because of their higher prices, and the integrated firms switch customers from rivals to their own bundled goods by setting low bundled prices.

Although there are opposite effects on consumer welfare, it is shown that mixed bundling always reduces both consumer surplus and total surplus.

We will also show that mixed bundling is a dominant strategy for all firms. Whereas, except for the composite goods that are very close substitutes, the firms are better off when they commit not to use mixed bundling. This is a prisoners’ dilemma situation. Our model suggests that the restrictions preventing mixed bundling are beneficial to consumers and firms for a wide range of parameters.

The rest of paper is organized as follows. The basic model is given in section 2. In section 3, we will discuss the pricing subgame that is played under several market structures. In section 4 we compare equilibrium prices and demands. In section 5 we consider a merger game and establish a subgame perfect equilibrium. In section 6 welfare analyses are given for alternative market structures. Section 7 contains our concluding remarks.
2. The Basic Model

The basic model used in this paper is similar to that developed by Economides and Salop [1992]. Suppose that there are two complementary components, $A$ and $B$, which are only used together. Component $A$ is assumed to be able to combine only with component $B$ on a one-to-one basis, to form a composite good $AB$. There are two differentiated brands, each with two components $A_i$ ($i = 1, 2$) and $B_j$ ($j = 1, 2$). We assume full compatibility among the respective components, and then consumers can combine brands of each component to form four composite goods $A_1B_1, A_1B_2, A_2B_1,$ and $A_2B_2$. Let $p_i$ and $q_j$ denote the prices of components $A_i$ and $B_j$. The price of composite good $AB_j$ is the total prices $s_j = p_i + q_j$. We denote the demand for composite good $AB_j$ by $D_j$.

The demand functions for the components are derived from the demand functions of composite goods. Since component $A_i$ is sold as a part of composite goods $A_1B_i$ and $A_2B_i$, the demand for component $A_i$ is given by

$$D_{A_i} = D_{1i} + D_{2i} \quad (i = 1, 2).$$

Similarly, the demand for component $B_j$ is given by

$$D_{B_j} = D_{1j} + D_{2j} \quad (j = 1, 2).$$

We assume that the four composite goods are substitutes for one another, and the demand $D_j$ for composite good $AB_j$ is decreasing in its own price and increasing in the prices of the other three substitutable composite goods. Following Economides and Salop [1992], we assume demand functions are linear and symmetric for prices, that is, we assume that the demand functions are given by

$$D_{1j}(s_{11}, s_{12}, s_{21}, s_{22}) = a \prod b s_{1j} + c s_{1j} + d s_{2j} + e s_{22},$$
$$D_{12}(s_{11}, s_{12}, s_{21}, s_{22}) = a \prod b s_{12} + c s_{11} + d s_{22} + e s_{21},$$
$$D_{2j}(s_{11}, s_{12}, s_{21}, s_{22}) = a \prod b s_{2j} + c s_{22} + d s_{11} + e s_{12},$$
$$D_{22}(s_{11}, s_{12}, s_{21}, s_{22}) = a \prod b s_{22} + c s_{21} + d s_{12} + e s_{11},$$

where $a, b, c, d, e > 0$. We assume that $b > c + d + e$, because composite goods are gross substitutes. For the sake of simplicity, we assume that all four composite goods are equally substitutable, that is $c = d = e$ with the restriction of $b > 3c$. 

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3. Pricing Subgame

We will consider three market structures, independent ownership, parallel vertical integration, and asymmetric structure. In this section we characterize the equilibria in the pricing subgame for all the market structures. For the sake of simplicity, we assume marginal costs are zero in this paper.

3. 1. Independent Ownership

First we will consider the market structure where four brands of components are independently owned and supplied by four separate firms. This market structure is the case of independent ownership in Economides and Salop [1992], as illustrated in Figure 1.

![Diagram](image)

Figure 1

Under independent ownership, the four firms independently choose prices to maximize profits, given by

\[ \partial a_1 = p_1 D_{a1} = p_1 (D_{a1} + D_{a2}), \quad \partial b_1 = q_1 D_{b1} = q_1 (D_{b1} + D_{b2}) \]

\[ \partial a_2 = p_2 D_{a2} = p_2 (D_{a2} + D_{a1}), \quad \partial b_2 = q_2 D_{b2} = q_2 (D_{b2} + D_{b1}) \]

assuming here that the other prices of the complementary components are given. The demand functions for composite goods are given by

\[ D_{a1} = a \partial b (p_1 + q_1) + c (p_1 + q_2) + c (p_2 + q_1) + c (p_2 + q_2) \]

\[ D_{a2} = a \partial b (p_2 + q_2) + c (p_2 + q_1) + c (p_2 + q_2) + c (p_2 + q_1) \]

\[ D_{b1} = a \partial b (p_2 + q_2) + c (p_2 + q_1) + c (p_2 + q_2) + c (p_2 + q_1) \]

\[ D_{b2} = a \partial b (p_2 + q_2) + c (p_2 + q_1) + c (p_2 + q_2) + c (p_2 + q_1) \]

Differentiating the profit functions with respect to each price, we have the four first order conditions for profit maximization

\[ \partial a_1 / \partial p_1 = 0, \quad \partial a_1 / \partial q_1 = 0, \quad \partial a_2 / \partial p_2 = 0, \quad \partial a_2 / \partial q_2 = 0, \]
which are solved for the Nash equilibrium prices of components,

\[ p_1 = p_2 = q_1 = q_2 = \frac{a}{3b \cdot c} \]  \hspace{1cm} (1)

The equilibrium prices for the composite goods, the demand for each component, and the profit for each firm are given by

\[ s_{1i} = s_{2i} = s_{21} = s_{22} = \frac{2a}{3b \cdot c}, \quad D_{1i} = D_{12} = D_{21} = D_{22} = \frac{a(b \cdot c)}{3b \cdot c}, \quad \]  \hspace{1cm} (2)

\[ \Pi_{AI} = \Pi_{A2} = \Pi_{B1} = \Pi_{B2} = \frac{2a^2(b \cdot c)}{(3b \cdot c)^2}. \]

3.2. Parallel Vertical Integration with Mixed Bundling

Next, we will consider parallel vertical integration where components \( A_i \) and \( B_i \) are supplied by the same integrated firm \( i, \quad (i = 1, 2), \) as illustrated in Figure 2.

![Figure 2](image)

In this section, we suppose that both firms use mixed bundling and are selling the individual components separately as well as selling the composite goods. Each firm sets three prices, the bundled price of composite good \( s_i \), and the respective prices of the two individual components, \( p_i \) and \( q_i \), where the composite good is offered at a discount, \( s_i < p_i + q_i \).

Under parallel vertical integration with mixed bundling, the two integrated firms independently choose prices to maximize profits, given by

\[ \Pi_{AI} = \Pi_{A2} = s_i D_{1i} + p_i D_{12} + q_i D_{21}, \]
\[ \Pi_{B1} = \Pi_{B2} = s_i D_{21} + p_i D_{22} + q_i D_{21}, \]

assuming the others prices to be given. The demand functions for composite goods are given by
\[ D_{11} = a \left[ b s_1 + c (p_1 + q_1) + c (p_2 + q_1) + cs_2, \right. \]
\[ D_{12} = a \left[ b (p_1 + q_2) + cs_1 + cs_2 + c (p_2 + q_1), \right. \]
\[ D_{21} = a \left[ b (p_2 + q_1) + cs_2 + cs_1 + c (p_1 + q_2), \right. \]
\[ D_{22} = a \left[ b s_2 + c (p_2 + q_1) + c (p_1 + q_2) + cs_1. \right. \]

Differentiating the profit functions with respect to each price, we have the six first order conditions for profit maximization

\[ \frac{\partial \Pi_{11}}{\partial s_1} = 0, \quad \frac{\partial \Pi_{11}}{\partial p_1} = 0, \quad \frac{\partial \Pi_{11}}{\partial q_1} = 0, \]
\[ \frac{\partial \Pi_{22}}{\partial s_2} = 0, \quad \frac{\partial \Pi_{22}}{\partial p_2} = 0, \quad \frac{\partial \Pi_{22}}{\partial q_2} = 0, \]

which are solved for the Nash equilibrium prices

\[ s_1 = s_2 = \frac{a}{2b \sqrt[3]{5c}}, \quad p_1 = p_2 = q_1 = q_2 = \frac{2a}{3(2b \sqrt[3]{5c})}. \]  

(3)

The demand for each component, and the profit for each firm are given by

\[ D_{11} = D_{22} = \frac{a(3b \sqrt[3]{4c})}{3(2b \sqrt[3]{5c})}, \quad D_{12} = D_{21} = \frac{a}{3}, \quad \Pi_{11} = \Pi_{22} = \frac{a^2(17b \sqrt[3]{32c})}{9(2b \sqrt[3]{5c})}. \]

(4)

3.3. Parallel Vertical Integration with Non-Mixed Bundling

In this section, we suppose that both firms don’t use mixed bundling. They sell the individual components separately and each firm sets the two respective prices of its individual components, \( p_i \) and \( q_i \).

The integrated firms independently choose prices to maximize profits, given by

\[ \Pi_i = \Pi_{i1} + \Pi_{i2} = p_i (D_{11} + D_{12}) + q_i (D_{11} + D_{21}), \]
\[ \Pi_{22} = \Pi_{i2} + \Pi_{i2} = p_2 (D_{21} + D_{22}) + q_2 (D_{12} + D_{22}), \]

assuming the other prices of the complementary components to be given. Differentiating the profit functions with respect to each price, we have the four first order conditions for profit maximization

\[ \frac{\partial \Pi_{11}}{\partial p_1} = 0, \quad \frac{\partial \Pi_{11}}{\partial q_1} = 0, \]
\[ \frac{\partial \Pi_{22}}{\partial p_2} = 0, \quad \frac{\partial \Pi_{22}}{\partial q_2} = 0, \]

which are solved for the Nash equilibrium prices

\[ p_i = p_2 = q_1 = q_2 = \frac{2a}{7b \sqrt[3]{17c}}, \]

\[ s_{i1} = s_{i2} = s_{i1} = s_{i2} = \frac{4a}{7b \sqrt[3]{17c}}. \]

(5)
The demand for each component, and the profit for each firm are given by

\[ D_{11} = D_{12} = D_{21} = D_{22} = \frac{a(3b \cdot 5c)}{7b \cdot 17c}, \quad \Gamma_{11} = \Gamma_{22} = \frac{8a^2(3b \cdot 5c)}{7b \cdot 17c^2}. \]  

(6)

3. 4. Parallel Vertical Integration with Mixed Bundling by Only One Firm

In this section, we assume that only one firm; say firm 1 uses mixed bundling. The integrated firm 1 sets the prices \( s_1, p_1, q_1 \) and the integrated firm 2 sets the prices of the individual components \( p_2, q_2 \).

The integrated firms independently choose prices to maximize profits, given by

\[ \Gamma_{a1} = \Gamma_{a1} + \Gamma_{b1} = s_1D_{11} + p_1D_{12} + q_1D_{21}, \]
\[ \Gamma_{a2} = \Gamma_{a2} + \Gamma_{b2} = p_2(D_{22} + D_{21}) + q_2(D_{22} + D_{12}), \]

assuming the prices of the others prices to be given. The demand functions for composite goods are given by

\[ D_{11} = a \cdot b \cdot a + c(p_1 + q_2) + e(p_2 + q_1) + f(p_2 + q_2), \]
\[ D_{12} = a \cdot b \cdot a + c(p_1 + q_2) + e(p_2 + q_1) + f(p_2 + q_2), \]
\[ D_{21} = a \cdot b \cdot a + c(p_2 + q_1) + e(p_2 + q_2) + f(p_2 + q_2), \]
\[ D_{22} = a \cdot b \cdot a + c(p_2 + q_1) + e(p_2 + q_2) + f(p_2 + q_2) + c. \]

Differentiating the profit functions with respect to each price, we have the five first order conditions for profit maximization

\[ \frac{\partial \Gamma_{a1}}{\partial s_1} = 0, \quad \frac{\partial \Gamma_{a1}}{\partial p_1} = 0, \quad \frac{\partial \Gamma_{a1}}{\partial q_1} = 0 \]
\[ \frac{\partial \Gamma_{a2}}{\partial p_2} = 0, \quad \frac{\partial \Gamma_{a2}}{\partial q_2} = 0, \]

which are solved for the Nash equilibrium prices

\[ s_1 = \frac{a(11b \cdot 9c)}{2(11b^2 \cdot 37bc + 24c^2)}, \quad p_1 = q_1 = \frac{a(4b \cdot 3c)}{11b^2 \cdot 37bc + 24c^2}, \quad p_2 = q_2 = \frac{3a(b \cdot c)}{11b^2 \cdot 37bc + 24c^2}. \]  

(7)

The demand for each component, and the profit for each firm are given by

\[ D_{11} = \frac{a(11b^2 \cdot 25bc + 12c^2)}{2(11b^2 \cdot 37bc + 24c^2)}, \quad D_{12} = \frac{a(8b^2 \cdot 25bc + 15c^2)}{2(11b^2 \cdot 37bc + 24c^2)}, \]
\[ D_{21} = \frac{a(10b^2 \cdot 23bc + 15c^2)}{2(11b^2 \cdot 37bc + 24c^2)}, \quad D_{22} = \frac{a(11b^2 \cdot 25bc + 12c^2)}{2(11b^2 \cdot 37bc + 24c^2)}, \]  

(8)
and the profits for firms are given by

\[
\mathcal{D}_{11} = \frac{3a^2(83b^3 \Box 290b^2c + 299bc^2 \Box 96c^3)}{4(11b^2 \Box 37bc + 24c^2)^2}, \quad \mathcal{D}_{22} = \frac{18a^2(b \Box c)^2(3b \Box 5c)}{(11b^2 \Box 37bc + 24c^2)^2}.
\]

(9)

### 3. 5. Asymmetric Structure with Mixed Bundling

We will consider the asymmetric structure where there is an integrated firm, say firm 1, offering pairs of complementary components and there are specialist firms \( A_1 \) and \( B_2 \) both supplying only one component, as illustrated in Figure 3.

![Figure 3](image)

In this section, we assume that the integrated firm uses mixed bundling. The integrated firm 1 sets the prices \( s_1, p_1, q_1 \) and the specialist firms set the prices of the individual components \( p_2, q_2 \). Then the integrated firm chooses prices \( s_1, p_1 \) and \( q_1 \) to maximize profits, given by

\[
\mathcal{D}_{11} = \mathcal{D}_{11} + \mathcal{D}_{b1} = s_1D_{11} + p_1D_{12} + q_1D_{21},
\]

and the two specialist firms independently choose prices to maximize profits

\[
\mathcal{D}_{a2} = p_2D_{a2} = p_2(D_{21} + D_{22}), \quad \mathcal{D}_{b2} = q_2D_{b2} = q_2(D_{12} + D_{22}),
\]

assuming others prices to be given. The demand functions for composite goods are given by

\[
D_{11} = a \Box bs + c(p_1 + q_1) + c(p_2 + q_1) + c(p_2 + q_2),
\]

\[
D_{12} = a \Box b(p_1 + q_2) + cs + c(p_2 + q_2) + c(p_2 + q_1),
\]

\[
D_{21} = a \Box b(p_2 + q_1) + c(p_2 + q_2) + cs + c(p_1 + q_2),
\]

\[
D_{22} = a \Box b(p_2 + q_2) + c(p_2 + q_1) + c(p_1 + q_2) + cs.
\]

Differentiating the profit functions with respect to each price, we have the five first order
conditions for profit maximization
\[
\frac{\partial \Pi_{1}}{\partial s_{i}} = 0, \quad \frac{\partial \Pi_{1}}{\partial p_{1}} = 0, \quad \frac{\partial \Pi_{1}}{\partial q_{1}} = 0, \\
\frac{\partial \Pi_{2}}{\partial p_{2}} = 0, \quad \frac{\partial \Pi_{2}}{\partial q_{2}} = 0,
\]
which are solved for the Nash equilibrium prices
\[
s_{i} = \frac{a(3b \square c)}{2(3b^2 \square 9bc + 4c^2)}, \quad p_{1} = q_{1} = \frac{ab}{3b^2 \square 9bc + 4c^2}, \tag{10}
\]
\[
p_{2} = q_{2} = \frac{a(b \square c)}{3b^2 \square 9bc + 4c^2}.
\]
The demand for each component, and the profit for each firm are given by
\[
D_{11} = \frac{ab(3b \square 5c)}{2(3b^2 \square 9bc + 4c^2)}, \quad D_{22} = \frac{a(2b^2 \square 3bc + 3c^2)}{2(3b^2 \square 9bc + 4c^2)}, \\
D_{12} = D_{21} = \frac{a(2b^2 \square 5bc + c^2)}{2(3b^2 \square 9bc + 4c^2)} \tag{11}
\]
and the profits for firms are given by
\[
\Pi_{11} = \frac{a^2 b^2 (17b^2 \square 9bc + 3c^2)}{4(3b^2 \square 9bc + 4c^2)^2}, \quad \Pi_{12} = \Pi_{21} = \frac{2a^2 (b \square c)^3}{(3b^2 \square 9bc + 4c^2)^2}. \tag{12}
\]

3. 6. Asymmetric Structure with Non-Mixed Bundling

The integrated firm chooses prices \( p_{1} \) and \( q_{1} \) to maximize profits, given by
\[
\Pi_{1} = \Pi_{11} + \Pi_{12} = p_{1}(D_{11} + D_{12}) + q_{1}(D_{11} + D_{21})
\]
and the two specialist firms independently choose prices to maximize profits, given by
\[
\Pi_{12} = p_{2}(D_{12} + D_{22}), \\
\Pi_{21} = q_{2}(D_{21} + D_{22}),
\]
assuming others prices to be given. Differentiating the profit functions with respect to each price, we have the four first order conditions for profit maximization
\[
\frac{\partial \Pi_{11}}{\partial p_{1}} = 0, \quad \frac{\partial \Pi_{11}}{\partial q_{1}} = 0, \quad \frac{\partial \Pi_{12}}{\partial p_{2}} = 0, \quad \frac{\partial \Pi_{21}}{\partial q_{2}} = 0
\]
which are solved for the Nash equilibrium prices
\[ p_1 = q_1 = \frac{8ab}{29b^2 \cdot 78bc + 21c^2}, \quad p_2 = q_2 = \frac{2a(5b \cdot 3c)}{29b^2 \cdot 78bc + 21c^2}, \]
\[ s_{11} = \frac{16ab}{29b^2 \cdot 78bc + 21c^2}, \quad s_{12} = s_{21} = \frac{6a(3b \cdot c)}{29b^2 \cdot 78bc + 21c^2}, \quad s_{22} = \frac{4a(5b \cdot 3c)}{29b^2 \cdot 78bc + 21c^2} \]

The demand for each component, and the profit for each firm are given by
\[ D_{11} = \frac{a(13b^2 \cdot 22bc \cdot 3c^2)}{29b^2 \cdot 78bc + 21c^2}, \quad D_{12} = D_{21} = \frac{a(11b^2 \cdot 18bc + 3c^2)}{29b^2 \cdot 78bc + 21c^2}, \]
\[ D_{22} = \frac{a(9b^2 \cdot 14bc + 9c^2)}{29b^2 \cdot 78bc + 21c^2}, \]
and the profits for firms are given by
\[ \Pi_1 = \frac{128a^2b^2(3b \cdot 5c)}{29b^2 \cdot 78bc + 21c^2}, \quad \Pi_{A2} = \Pi_{B2} = \frac{8a^2(b \cdot c)(5b \cdot 3c)^2}{(29b^2 \cdot 78bc + 21c^2)^2}. \]

4. The Comparison of Prices and Demands

We now compare the equilibrium prices of the three symmetric market structures. The individual prices of components are given by
\[ p_i^s = q_j^s = \frac{a}{3b \cdot 7c}, \quad p_i^N = q_j^N = \frac{2a}{7b \cdot 17c}, \quad p_i^b = q_j^b = \frac{2a}{3(2b \cdot 5c)} \quad (i, j = 1, 2), \]
where superscript \( S \) denotes a separate firm (independent ownership), \( N \) denotes an integrated firm with non-mixed bundling, and \( B \) denotes an integrated firm with mixed bundling. The bundled price set by the integrated firm and the sum of component prices are given by
\[ s_i^b = \frac{a}{2b \cdot 5c}, \quad s_j^S = p_i^s + q_j^S = \frac{2a}{3b \cdot 7c}, \quad s_j^N = p_i^N + q_j^N = \frac{4a}{7b \cdot 17c} \quad (i, j = 1, 2). \]
Comparing prices, we have the following results.

**Proposition 1**

1. If integrated firms do not use mixed bundling, then the component prices of integrated firms are lower than those of independent firms,
\[ p_i^N = q_j^N < p_i^S = q_j^S \quad (i, j = 1, 2). \]
2. If mixed bundling is used, the prices of individual components are higher than those of any
other component, and the bundled price set by the integrated firm is lower than the sum of any component prices,

\[ s_i^B < s_i^N < s_j^S < s_j^B \quad (i, j = 1, 2). \]

Intuitively, the result is as follows. If any firm supplying component A decreases its component price, then the demand for its complementary component \( B_i \) and \( B_2 \) increase. However, an independent firm that maximizes his own profit, does not take into account of these externalities; hence they have less incentive to decrease prices. The integrated firms internalize externalities and set the prices lower than the level of the two independent firms and would choose, \( p_i^N = q_i^N < p_i^S = q_i^S \). This is a result shown by Economides and Salop [1992]. But, if mixed bundling is possible, the integrated firm will have an incentive to set higher prices for its component than the level the independent firms choose. This makes the hybrid goods using its component and another firm’s component less attractive to the buyers. Then, the integrated firm switches customers from rivals to its own bundled good by setting a low bundled price.

Next, we compare the equilibrium demands of the three symmetric market structures. The demands for the composite goods are given by

\[
D_i^S = \frac{a(b \square c)}{3b \square 7c}, \quad D_i^N = \frac{a(3b \square 5c)}{7b \square 17c}, \quad D_i^B = \frac{a(3b \square 4c)}{3(2b \square 5c)},
\]

\[
D_j^S = D_j^N = \frac{a(b \square c)}{3b \square 7c}, \quad D_j^N = \frac{a(3b \square 5c)}{7b \square 17c}, \quad D_j^N = \frac{a}{3} \quad (i, j = 1, 2),
\]

and the total demands are given by

\[
D^S = \frac{4a(b \square c)}{3b \square 7c}, \quad D^N = \frac{4a(3b \square 5c)}{7b \square 17c}, \quad D^B = \frac{2a(5b \square 9c)}{3(2b \square 5c)}.
\]

Comparing demands, we have the following results.

**Proposition 2**

1. The demand for hybrid goods is lower for integration with mixed bundling,

\[ D_{ij}^B < D_{ij}^S < D_{ij}^N \quad (i, j = 1, 2). \]

2. The demand for bundled goods is higher for integration with mixed bundling,

\[ D_{ii}^S < D_{ii}^N < D_{ii}^B \quad (i = 1, 2). \]

3. In the case where the integrated firms do not use mixed bundling, the total demand is larger
for integrated firms, \( N \), than for separate firms, \( S \), (independent ownership). If the integrated firms use mixed bundling, and the composite goods are close substitutes, i.e., \( 3/11 < c/b < 1/3 \), then total demand is smaller for integrated firms, \( B \), than for separate firms, \( S \). That is,

\[
D^S < D^B < D^N \quad \text{if} \quad 0 < \frac{c}{b} < \frac{3}{11},
\]

\[
D^B < D^S < D^N \quad \text{if} \quad \frac{3}{11} < \frac{c}{b} < \frac{1}{3},
\]

where \( D \) denotes total demand for composite goods.

The integrated firms internalize externalities and set the prices lower than the levels the two independent firms would choose. Hence the demand tends to be larger for integrated firms than for independent ownership firms. If mixed bundling is possible, the integrated firms set high prices for individual components and low prices for bundled goods, which lead to lower demands for hybrid goods and higher demands for bundled goods. There are basically two forces working in the opposite direction. If composite goods are close substitutes, i.e., for \( 3/11 < c/b < 1/3 \), then the total demand decreases because of a large reduction of demand for hybrid goods.

5. Merger Game

We consider a complementary merger between firms \( A_1 \) and \( B_1 \), and /or a merger between firms \( A_2 \) and \( B_2 \). Suppose that there are three possible configurations: independent ownership, parallel vertical integration with mixed bundling, and parallel vertical integration with non-mixed bundling. We suppose a two-stage game. In the first stage, the firm \( A_1 \) joins in a paring with the firm \( B_1 \), and they collectively choose the structure that maximizes their joint profits. In the second stage, firms independently choose prices. In each stage, we assume that firms move simultaneously, and each assumes others’ decisions are given.

In section 3 we characterized the equilibrium in all types of subgames. The equilibrium joint profits for all respective subgames can be summarized in Table 1.

In this table, the strategies \( B, N, \) and \( S \) denote integration with mixed bundling, integration with non-mixed bundling, and independent ownership, respectively. Let \( \pi_i \) denote the joint profits of the firm \( A_i \) and \( B_i \). Then it must hold that

\[
\pi_{1IB} = \pi_{2IB} \quad \text{and} \quad \pi_{2IB} = \pi_{1IB}, \quad \pi_{1SB} = \pi_{2SB} \quad \text{and} \quad \pi_{2SB} = \pi_{1SB}, \quad \pi_{1SN} = \pi_{2SN} \quad \text{and} \quad \pi_{2SN} = \pi_{1SN}.
\]
Table 1: Equilibrium Joint Profits

<table>
<thead>
<tr>
<th>Pair of firms $A_1$ and $B_1$</th>
<th>$B$</th>
<th>$N$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>$(\Pi_{i}^B, \Pi_{2}^B)$</td>
<td>$(\Pi_{i}^{BN}, \Pi_{2}^{BN})$</td>
<td>$(\Pi_{i}^{BS}, \Pi_{2}^{BS})$</td>
</tr>
<tr>
<td>$N$</td>
<td>$(\Pi_{i}^{NB}, \Pi_{2}^{NB})$</td>
<td>$(\Pi_{i}^{N}, \Pi_{2}^{N})$</td>
<td>$(\Pi_{i}^{NS}, \Pi_{2}^{NS})$</td>
</tr>
<tr>
<td>$S$</td>
<td>$(\Pi_{i}^{SB}, \Pi_{2}^{SB})$</td>
<td>$(\Pi_{i}^{SN}, \Pi_{2}^{SN})$</td>
<td>$(\Pi_{i}^{S}, \Pi_{2}^{S})$</td>
</tr>
</tbody>
</table>

We now compare equilibrium joint profits. First, given that the pair of firms $A_2$ and $B_2$ integrates with mixed bundling, we can show that the pair of firms $A_i$ and $B_i$ has higher joint profits if it integrates with mixed bundling as well,

$$\Pi_{i}^B > \Pi_{i}^{NB} \text{ and } \Pi_{i}^B > \Pi_{i}^{SB}. \quad (16)$$

Secondly, given that the pair of firms $A_2$ and $B_2$ integrates with non-mixed bundling, we can also show that the pair of firms $A_i$ and $B_i$ has higher joint profits if it integrates with mixed bundling,

$$\Pi_{i}^{BN} > \Pi_{i}^{N} \text{ and } \Pi_{i}^{BN} > \Pi_{i}^{SN}. \quad (17)$$

Thirdly, given that the pair of firms $A_2$ and $B_2$ chooses independent ownership, the pair of firms $A_i$ and $B_i$ has higher joint profits if it integrates with mixed bundling,

$$\Pi_{i}^{BS} > \Pi_{i}^{NS} \text{ and } \Pi_{i}^{B} > \Pi_{i}^{S}. \quad (18)$$

The same results hold for the pair of firms $A_2$ and $B_2$. Thus, the integration with mixed bundling is a dominant strategy for each pair of firms in the first stage of the game.

**Proposition 3**

The situation in which both pairs of firms choose integration with mixed bundling is a dominant strategy equilibrium in the merger game.

Next, we compare the equilibrium joint profits in three symmetric structures. This yields the following results.
Proposition 4
When the composite goods are very close substitutes, i.e., $0.3209 < c/b < 1/3$, firms realize maximum profits under integration with mixed bundling. But when the composite goods are close substitutes, i.e., $0.17817 < c/b < 0.3209$, firms realize maximum profits under individual ownership. When the composite goods are substitutes but not close substitutes, i.e., $0 < c/b < 0.17817$, firms realize maximum profits under integration with non-mixed bundling$^1$.

1. $\mathcal{Q}_1^b > \mathcal{Q}_1^s > \mathcal{Q}_1^N$ if $0.3209 < c/b < \frac{1}{3}$,
2. $\mathcal{Q}_1^s > \mathcal{Q}_1^b > \mathcal{Q}_1^N$ if $\frac{177\sqrt{577}}{496} = 0.3084 < c/b < 0.3209$,
3. $\mathcal{Q}_1^b > \mathcal{Q}_1^N > \mathcal{Q}_1^s$ if $\frac{20\sqrt{65}}{67} = 0.17817 < c/b < \frac{177\sqrt{577}}{496} = 0.3084$,
4. $\mathcal{Q}_1^s > \mathcal{Q}_1^b > \mathcal{Q}_1^N$ if $0.102 < c/b < \frac{20\sqrt{65}}{67} = 0.17817$,
5. $\mathcal{Q}_1^N > \mathcal{Q}_1^s > \mathcal{Q}_1^b$ if $0 < c/b < 0.102$.

We showed that mixed bundling is a dominant strategy for all firms. Whereas, except for when the composite goods are very close substitutes, Proposition 4 shows the firms are better off when they commit not to use mixed bundling. This is a prisoners’ dilemma situation.

6. Welfare Analysis

It is interesting to compare consumer surplus and total surplus for the several market structures. Suppose that the utility function of the representative consumer is given by

$$U = \frac{a}{b \sqrt{3c}} (D_{11} + D_{12} + D_{21} + D_{22}) \frac{b \sqrt{2c}}{2(b \sqrt{3c})(b + c)} \left( \frac{D_{11} \cdot D_{21}^2 + D_{12}^2 + D_{21} \cdot D_{22}^2 + D_{22} \cdot D_{12}^2}{2(b \sqrt{3c})(b + c)} \right)$$

(19)

This utility function is symmetric in the demands four composite goods, i.e., $D_{11}, D_{12}, D_{21}, D_{22}$.

The consumer maximizes his/her utility function with respect to $D_{11}, D_{12}, D_{21}, D_{22}$, subject to the budget constraint. Then the demand functions for composite goods can be derived (see
appendix).

We suppose that consumer surplus (CS) is given by

\[ CS = U \left[ (s_{11}D_{11} + s_{12}D_{12} + s_{21}D_{21} + s_{22}D_{22}) \right] \]  \hspace{1cm} (20)

and total surplus \( W \) is given by

\[ W = CS + (s_{11}D_{11} + s_{12}D_{12} + s_{21}D_{21} + s_{22}D_{22}) \]  \hspace{1cm} (21)

Comparing consumer surplus and total surplus in three symmetric structures, we have the following results.

**Proposition 5**

Both consumer surplus and total surplus are always at the maximum under integration with non-mixed bundling. When the composite goods are very close substitutes, i.e., \( 0.29598 < c/b < 1/3 \), consumer surplus and total surplus are at the minimum under integration with mixed bundling. When the composite goods are substitutes but not close substitutes, i.e., \( 0 < c/b < 0.23637 \), consumer surplus and total surplus is at the minimum under individual ownership \(^3\).

1. \( CS^S < CS^B < CS^N \) \hspace{0.5cm} if \hspace{0.5cm} \( 0 < \frac{c}{b} < 0.29598 \)
2. \( CS^B < CS^S < CS^N \) \hspace{0.5cm} if \hspace{0.5cm} \( 0.29598 < \frac{c}{b} < \frac{1}{3} \)
3. \( W^S < W^B < W^N \) \hspace{0.5cm} if \hspace{0.5cm} \( 0 < \frac{c}{b} < 0.23637 \)
4. \( W^B < W^S < W^N \) \hspace{0.5cm} if \hspace{0.5cm} \( 0.23637 < \frac{c}{b} < \frac{1}{3} \)

**7. Concluding Remarks**

This article provides a framework to analyze the effect of vertical integration and mixed bundling in the vertically related markets. We examined the choice of market structures and pricing schemes by firms. As we have seen, mixed bundling is a dominant strategy for all firms. Whereas, except for the composite goods that are very close substitute, firms are better off when they both commit not to use mixed bundling. This is a generalization based on the results of Economides [1993] that is restricted to the case of parallel vertical integration.

In addition, we show that both consumer surplus and social surplus are always at the
maximum in the case of integration with non-mixed bundling. Our model suggests that the restriction against mixed bundling is almost beneficial for both firms and consumers. The result of this paper has an important implication for many areas of regulations, including, for example, regulation of network industries.

Here, we must note the result shown by Liao and Tauman [2002] that bundling may increase consumer surplus when firms cannot commit a pricing strategy before deciding on price levels. The welfare effects of bundling may differ whether the commitment of pricing strategies is possible or not. The precise analysis of this point waits for a future research.

NOTES

1) Note that the numbers 0.102 and 0.3209 are the numerical solution of the equation \( \sqrt[n]{11} = \sqrt[n]{1} \).

2) Note that the numbers 0.29598 is the numerical solution of the equation \( CS^B = CS^S \) and 0.23637 is the numerical solution of the equation \( W^B = W^S \).
APPENDIX

Suppose that the consumer maximizes the utility function

\[
U = \frac{a}{b \sqrt[3]{3c}} \left(D_{11} + D_{12} + D_{21} + D_{22}\right) \frac{b \sqrt[3]{2c}}{2(b \sqrt[3]{3c}) (b + c)} \left(D_{11}^2 + D_{12}^2 + D_{21}^2 + D_{22}^2\right)
\]

\[
+ \frac{2c}{2(b \sqrt[3]{3c}) (b + c)} \left(D_{11}D_{12} + D_{11}D_{21} + D_{11}D_{22} + D_{12}D_{21} + D_{12}D_{22} + D_{21}D_{22}\right)
\]

with respect to \( D_{11}, D_{12}, D_{21}, D_{22} \) subject to budget constraint

\[
I = s_1D_{11} + s_2D_{12} + s_2D_{21} + s_2D_{22},
\]

where \( I \) is the consumer’s income. Or, equivalently we suppose that consumer maximizes consumer surplus \( CS \), given by

\[
CS = U \left[ (s_1D_{11} + s_2D_{12} + s_2D_{21} + s_2D_{22}) \right].
\]

From the first order conditions for maximization, we can derive the inverse demand functions.

\[
s_{11} = \frac{a}{b \sqrt[3]{3c}} \left(b \sqrt[3]{2c}\right) D_{11} \frac{c}{(b \sqrt[3]{3c}) (b + c)} D_{12} \frac{c}{(b \sqrt[3]{3c}) (b + c)} D_{21} \frac{c}{(b \sqrt[3]{3c}) (b + c)} D_{22}
\]

\[
s_{12} = \frac{a}{b \sqrt[3]{3c}} \left(b \sqrt[3]{2c}\right) D_{11} \frac{c}{(b \sqrt[3]{3c}) (b + c)} D_{12} \frac{c}{(b \sqrt[3]{3c}) (b + c)} D_{21} \frac{c}{(b \sqrt[3]{3c}) (b + c)} D_{22}
\]

\[
s_{21} = \frac{a}{b \sqrt[3]{3c}} \left(b \sqrt[3]{2c}\right) D_{11} \frac{c}{(b \sqrt[3]{3c}) (b + c)} D_{12} \frac{c}{(b \sqrt[3]{3c}) (b + c)} D_{21} \frac{c}{(b \sqrt[3]{3c}) (b + c)} D_{22}
\]

\[
s_{22} = \frac{a}{b \sqrt[3]{3c}} \left(b \sqrt[3]{2c}\right) D_{11} \frac{c}{(b \sqrt[3]{3c}) (b + c)} D_{12} \frac{c}{(b \sqrt[3]{3c}) (b + c)} D_{21} \frac{c}{(b \sqrt[3]{3c}) (b + c)} D_{22}
\]

When we solve these equations for \( D_{11}, D_{12}, D_{21}, D_{22} \), we have the following demand functions.

\[
D_{11}(s_{11}, s_{12}, s_{21}, s_{22}) = a \left[ bs_{11} + cs_{12} + cs_{21} + cs_{22}\right],
\]

\[
D_{12}(s_{12}, s_{11}, s_{22}, s_{21}) = a \left[ bs_{12} + cs_{11} + cs_{22} + cs_{21}\right],
\]

\[
D_{21}(s_{21}, s_{22}, s_{11}, s_{12}) = a \left[ bs_{21} + cs_{22} + cs_{11} + cs_{12}\right],
\]

\[
D_{22}(s_{22}, s_{21}, s_{12}, s_{11}) = a \left[ bs_{22} + cs_{21} + cs_{12} + cs_{11}\right].
\]

These are equivalent to the equations we have used in this paper. Hence we will use the consumer surplus specified by equation (20) for the welfare analysis.
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