Abstract

Electronic commerce ("e-commerce") has prevailed in our economy at a rapid speed since second half of 1990’s, and has changed not only the retail market but also the marketing channel structure. In this paper, we construct a model of a manufacturer’s marketing channel choice in an Internet economy. Specifically, using a variant of Hotelling (1929) type linear city model, we examine under what condition the manufacturer deals with electronic retailers as well as with conventional retailers in equilibrium. Then, we investigate welfare properties of its choice. Our main findings are as follows: When the manufacturer trades with electronic retailers, the equilibrium retail prices always fall and consumer surplus increases necessarily. However, we also show that total surplus does not always increase with the start of Internet marketing channel. This is because the manufacturer’s marketing channel choice is not socially optimal and efficiency loss of production and distribution accrues.

Keywords: E-commerce, linear city model, marketing channel choice

JEL classification: L13
1 Introduction

Electronic commerce (“e-commerce”) has prevailed in our economy at a rapid speed since second half of 1990’s. METI (2006) reports that the Japanese market sales of Business to Consumers e-commerce in 2005 was about 3.5 trillion yen (1.2% of total sales), while the US figure was about 144.3 billion dollar (2.4% of total sales).

Confronting these facts, the literature of economics and marketing accumulates various empirical studies.\(^1\) Theoretical papers also appear gradually.\(^2\) However, most existing research assumes the marketing channel structure as given, although e-commerce changes it as well as the retail market. Thus, it is urgent to investigate the impact of e-commerce on an economy by starting from a manufacturer’s marketing channel choice.\(^3\)

In this paper, we construct a model of a manufacturer’s marketing channel choice in an Internet economy. Specifically, using a variant of Hotelling (1929) type linear city model where one manufacturer, two conventional retailers and potentially many electronic retailers play a multi-stage game, we examine under what condition the manufacturer deals with electronic retailers as well as with conventional retailers in equilibrium. Then, we investigate welfare properties of its choice and find that, when the manufacturer trades with electronic retailers, the equilibrium retail prices always fall and consumer surplus increases necessarily. However, we also show that total surplus does not always increase with the start of Internet marketing channel because the manufacturer’s marketing channel choice is not socially optimal and efficiency loss of production and distribution accrues.\(^4\)

The remainder of this paper is organized as follows. In section 2, we describe a model.

\(^{1}\)For example, see Goolsbee (2000), Morton, Zettelmeyer, and Risso (2001), and Brynjolfsson and Smith (2000). Smith, Bailey and Brynjolfsson (2000) contain a comprehensive list of papers on e-commerce at that time.

\(^{2}\)For example, see Balasubramanian (1998), Bouckaert (2000) and Nakayama (2003).

\(^{3}\)Recently, Chiang, Chhajed and Hess (2003), Kumar and Ruan (2006) and Aiura (2006) analyze this issue from different viewpoints.

\(^{4}\)The similar results are obtained in a different context by Chen (2003) who examines the countervailing-power of dominant retailers.
Then, in section 3, we characterize an equilibrium of the model. In section 4, we examine welfare properties of the equilibrium and present main results. Concluding remarks are given in Section 5. All proofs of Lemmas and Propositions are presented in the Appendix.

2 The Model

Consider a linear city of length 1. Consumers are uniformly distributed with unit density along this interval $[0, 1]$. Each consumer buys at most one unit of a product, whose reservation price is $v$.\(^5\)

For supply side, there are one manufacturer, two conventional retailers and potentially many electronic retailers. The two conventional retailers (hereafter $C$-retailers 1 and 2) set up their physical store on the linear city, while the electronic retailers (hereafter $E$-retailers), who do not have such store, but ship products from their warehouse to consumers. For simplicity, we assume that $C$-retailer 1 ($C$-retailer 2) builds its store on the left (right) end of the city.

We analyze a three-stage game. In the first stage, the manufacturer chooses retailers to sell the product. There are three choices for the manufacturer: it sells the product to (0) only two $C$-retailers, (1) two $C$-retailers and one $E$-retailer, and (2) two $C$-retailers and all $E$-retailers. Choice (0) is a benchmark. Choice (1) means that the Internet channel is selective\(^6\) (Balasubramanian (1998) and Bouckaert (2000)), while choice (2) implies that it is open (Nishimura (1995)). We assume that the manufacturer does not choose only $E$-retailers to deal with.

In the second stage, the manufacturer determines its wholesale prices. We assume that wholesale price to $C$-retailers is exogenously given by $w$. Thus, when choice (0) is made in

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\(^5\)We assume that $v$ is sufficiently large so that each consumer buys one unit in equilibrium.

\(^6\)More precisely, choice (1) is divided into two choices depending on whether the manufacturer levies the franchise fee on the $E$-retailer that it trades the product with. See section 3.2 for detail.
the first stage, the manufacturer has no choice in the second stage. When the manufacturer makes choice (1) or (2), it determines wholesale price to E-retailers, \( w^* \), given \( w \).

In the third stage, given the wholesale prices, each retailer sets its retail price simultaneously. C-retailer \( i \)'s price is denoted by \( p_i \) (\( i = 1, 2 \)). E-retailer \( j \)'s price is denoted by \( p_j^* \) (\( j = 1, 2, \ldots, n \)), where \( n \) is the number of E-retailers to deal with the manufacturer. However, since E-retailers are not differentiated each other, if the manufacturer makes choice (2) in the first stage, pure Bertrand competition occurs among E-retailers, and it holds that \( p_j^* = w^* \) (\( j = 1, 2, \ldots, n \)). Thus, hereafter, we denote E-retailers’ price as \( p^* \) for notational simplicity.

Next, we describe consumers’ demand. Since we have assumed that each consumer buys one unit of the product, each consumer minimizes the total cost to buy the product. If a consumer buys the product from C-retailer \( i \) located at a distance \( x \) from the consumer, she incurs \( p_i + cx \) where \( c (> 0) \) is a unit cost per distance. On the other hand, if she buys the product from an E-retailer, she incurs \( p^* + c^* \) where \( c^* (\geq 0) \) is a delivery fee which is the same to all consumers.\(^7\) This setting about the cost that consumers bear is the same as the literature (Nishimura (1995), Balasubramanian (1998) and Bouckaert (2000)). Since consumers do not take care of \( p^* \) and \( c^* \) independently, but they care about the total cost, \( p^* + c^* \), henceforth we denote \( \bar{p}^* = p^* + c^* \) and assume that E-retailers choose \( \bar{p}^* \) to maximize their profit.

In the following, we describe Consumers’ demand given the choice that the manufacturer makes at the first stage. First, we explain the case without Internet channel. Then, we proceed the case with Internet channel.

**The Case without Internet Channel**

Suppose that a consumer located at a point, \( x (\in [0, 1]) \) is indifferent whether she pur-

\(^7\) When the manufacturer sell an information product, \( c^* \) can be interpreted as decreased utility of the information product. For example when consumers buy a digital music from an E-retailer, they do not get a card of lyrics, which causes their utility to decrease.
chases the product from C-retailer 1 or from C-retailer 2. Then, it holds that

\[ p_1 + cx = p_2 + c(1-x) \iff x = \frac{c + p_2 - p_1}{2c}, \quad (1) \]

which provides the demand for C-retailer \( i \ (i = 1, 2) \) given \( p_j \ (j \neq i) \) as follows

\[
D_i = \begin{cases} 
0 & \text{if } p_i > p_j + c, \\
\frac{c + p_j - p_i}{2c} & \text{if } p_j - c \leq p_i \leq p_j + c, \\
1 & \text{if } p_i < p_j - c. 
\end{cases}
\quad (2)
\]

**The Case with Internet Channel**

We describe the demand for C-retailer \( i \), given that \( p_j < \overline{p}^* \). Suppose that a consumer located at a distance \( x \) from C-retailer \( i \) is indifferent whether she purchases the product from this C-retailer or from E-retailers. Then, it holds that

\[ p_i + cx = \overline{p}^* \iff x = \frac{\overline{p}^* - p_i}{c}. \quad (3) \]

Some consumers purchase the product from E-retailers if and only if \( x < x \ (\iff 2\overline{p}^* - (p_j + c) < p_i) \). Thus, the the demand for C-retailer \( i \) given \( p_j < \overline{p}^* \ (j \neq i) \) is as follows

\[
D_i = \begin{cases} 
0 & \text{if } \overline{p}^* < p_i, \\
\frac{\overline{p}^* - p_i}{c} & \text{if } 2\overline{p}^* - (p_j + c) \leq p_i \leq \overline{p}^*, \\
\frac{p_j + c - p_i}{2c} & \text{if } p_j - c \leq p_i < 2\overline{p}^* - (p_j + c), \\
1 & \text{if } p_i < p_j - c. 
\end{cases}
\quad (4)
\]

Next, we describe the demand for E-retailers, \( D^* = 1 - \sum_{i=1,2} D_i \). Note that, if \( \frac{p_1 + p_2 + c}{2} < \overline{p}^* \), E-retailers obtain no demand.\(^8\). Thus, the demand for E-retailers given \( p_i \ (i = 1, 2) \) is

\(^8(p_1 + p_2 + c)/2 \) is the total cost that the consumer located at \( x \) incurs. Substitute \( x = (c + p_2 - p_1) / (2c) \)
represented by

\[
D^* = \begin{cases} 
0 & \text{if } \frac{p_1 + p_2 + c}{2} < \bar{p}, \\
\frac{c + p_1 + p_2 - 2 \bar{p}}{c} & \text{if } \max(p_1, p_2) \leq \bar{p}^* \leq \frac{p_1 + p_2 + c}{2}, \\
1 - \max\left(\frac{\bar{p} - p_1}{c}, \frac{\bar{p} - p_2}{c}\right) & \text{if } \min(p_1, p_2) \leq \bar{p}^* < \max(p_1, p_2), \\
1 & \text{if } \bar{p}^* < \min(p_1, p_2).
\end{cases}
\]

(5)

3 Equilibrium

In this section, we analyze a subgame perfect equilibrium of the model by backward induction. Therefore, first we examine equilibrium outcomes in the third-stage subgames.

3.1 The Third Stage: Retail Competition

The Case without Internet Channel

In this subgame, only two C-retailers are in operation in the retail market. C-retailer \(i\) maximizes its profit

\[
\Pi_i^{C} = (p_i - w) D_i, \quad i = 1, 2,
\]

(6)

given the wholesale price \(w\) and C-retailer \(j\)'s price \(p_j\). We set \(p_j = p_0\) ( \(j \neq i\)). From (2) and (6), the profit maximization price is

\[
\hat{p}_i = \frac{1}{2} c + \frac{1}{2} p_0 + \frac{1}{2} w
\]

(7)

if it holds that

\[
w - c \leq p_0 \leq w + 3c
\]

(8)

into \(p_1 + cx\) or \(p_2 + c (1 - x)\), then we obtain this.
to satisfy $p_0 - c \leq \hat{p}_i \leq p_0 + c$. If $w > p_0 + c$ ($\Leftrightarrow \hat{p}_i > p_0 + c$), it is optimal to set $\hat{p}_i = w$.

On the other hand, if $w < p_0 - 3c$ ($\Leftrightarrow \hat{p}_i < p_0 - c$), $\hat{p}_i = p_0 - c$ is low enough to obtain all consumers, thus is the optimal price.

To achieve a symmetric equilibrium where both C-retailers set the same price, substitute $\hat{p}_i = p_0$ into (7), we obtain

$$p_0 = w + c, \quad i = 1, 2,$$

which satisfies (8). Thus, this is an equilibrium price of this subgame.

**The Case with Selective Internet Channel**

In this subgame, two C-retailers and one E-retailer are in operation in the retail market. The profits of C-retailer $i$ and that of the E-retailer, given the wholesale prices, $w$ and $w^*$, are represented as

$$\Pi^C_i = (p_i - w) D_i \quad i = 1, 2,$$

$$\Pi^E = [p^* - (w^* + c^*)] D^*,$$

respectively, where $D_i$ and $D^*$ are the demand for C-retailer $i$ and that for the E-retailer : (4) and (5). Hereafter we denote $\overline{w^*} = w^* + c^*$.

Some calculations yields the following lemma:

**Lemma 3.1** *In the subgame with selective Internet channel, Nash equilibrium retail prices*
\[(p_S(w), \bar{p}_S(w)) = \begin{cases} (w + c, \bar{w}) & \text{if } \bar{w} > w + \frac{3}{2}c, \\ (\bar{w} - \frac{1}{2}c, \bar{w}) & \text{if } w + c < \bar{w} \leq w + \frac{3}{2}c, \\ \left(\frac{c}{6} + \frac{2w + \bar{w}}{3}, \frac{c + w + 2\bar{w}}{3}\right) & \text{if } w - \frac{c}{2} < \bar{w} \leq w + c, \\ (w, w) & \text{if } \bar{w} \leq w - \frac{1}{2}c. \end{cases}\]

The Case with Open Internet Channel

In this case, we can prove the next lemma:

**Lemma 3.2** In the subgame with open Internet channel, Nash equilibrium retail prices are given as

\[(p_O(\bar{w}), \bar{p}_O(\bar{w})) = \begin{cases} (w + c, \bar{w}) & \text{if } \bar{w} > w + \frac{3}{2}c, \\ (\bar{w} - \frac{1}{2}c, \bar{w}) & \text{if } w + c < \bar{w} \leq w + \frac{3}{2}c, \\ (\frac{\bar{w} + \frac{w}{2}}{\bar{w}}), \bar{w}) & \text{if } w < \bar{w} \leq w + c, \\ (w, \bar{w}) & \text{if } \bar{w} \leq w. \end{cases}\]

From Lemmas 3.1 and 3.2, if \(w < \bar{w} < w + c\) which implies that each retailer has positive demand in both cases, we find that

\[p^S(\bar{w}) - p^O(\bar{w}) = \frac{w + c - \bar{w}}{6} > 0.\]

That is, given \(\bar{w}\) (and \(w\)), the C-retailer’s price in the case with selective Internet channel is higher than in the case with open Internet channel. This is because the retail market is more competitive in the latter case with many E-retailers.
3.2 The Second Stage: Wholesale Price Choice

We analyze the manufacturer’s choice of wholesale price to Internet channel given that it launches this channel in the first stage. We assume that the manufacturer chooses $\bar{w}^*$ to maximize its profit.

**The Case with Selective Internet Channel**

Consider the case with selective Internet channel. From (4), (5) and Lemma 3.1, the demand from each C-retailer and that from the E-retailer which the manufacturer faces are respectively as follows:

$$D_S(\bar{w}^*) = \begin{cases} 1/2 & \text{if } \bar{w}^* > w + c, \\ \frac{1}{6c}(c - 2w + 2\bar{w}^*) & \text{if } w - \frac{c}{2} < \bar{w}^* \leq w + c, \\ 0 & \text{if } \bar{w}^* \leq w - \frac{1}{2}c, \\
\end{cases}$$

(12)

$$D'_S(\bar{w}^*) = \begin{cases} 0 & \text{if } \bar{w}^* > w + c, \\ \frac{2}{3c}(c + w - \bar{w}^*) & \text{if } w - \frac{c}{2} < \bar{w}^* \leq w + c, \\ 1 & \text{if } \bar{w}^* \leq w - \frac{1}{2}c. \\
\end{cases}$$

(13)

We consider two sub-cases depending on whether the manufacturer collects a franchise fee from the E-retailer or not. We assume that the manufacturer does not collect a fee from C-retailers, to make the cases with Internet channel comparable with the benchmark case without Internet channel.\(^9\)

First, we consider the subgame of a selective Internet channel without the franchise fee. Hereafter we denote this one by subgame $SNF$. The manufacturer’s profit is given as

$$\Pi^M_{SNF} = (w - m)[2D_S(\bar{w}^*)] + [\bar{w}^* - (m^* + c^*)] D'_S(\bar{w}^*),$$

(14)

\(^9\)The benchmark case, where the manufacturer sets the wholesale price to C-retailers higher than the marginal cost without levying the fee on them, does not cause inefficiency, since we focus on the situation where each consumer buys one unit of the product.
where \( m \) ( \( m^* \) ) is the marginal cost to produce and sell the products for C-retailer (E-retailer). If the manufacturer produces one physical product, it is plausible that the marginal cost is the same for the two channel (\( m^* = m \)). However, if it produces an information product for Internet channel, it is likely that the marginal cost for the Internet channel is lower (possibly zero) than that for the conventional channel (\( m^* < m \)). Below, we distinguish the marginal cost for the two channel when calculating the model and denote \( m^* + c^* \) as \( \overline{m}^* \).

Maximization of (14) with respect to \( \overline{w}^* \) given \( w \) yields the following lemma:

**Lemma 3.3** In the subgame SNF, the manufacturer chooses the wholesale price to the Internet channel as follows:

\[
\overline{w}_{SNF}^* = \begin{cases} 
  w + c & \text{if } \overline{m}^* > m + c, \\
  w + \frac{(m-c)}{2} & \text{if } m - 2c < \overline{m}^* \leq m + c, \\
  w - \frac{c}{2} & \text{if } \overline{m}^* \leq m - 2c.
\end{cases}
\]

Then, we consider the subgame of a selective Internet channel with the franchise fee. Hereafter we denote this by subgame SF. We assume that the manufacturer can collect the E-retailer’s profit as the fee. Then, the manufacturer’s profit is given as

\[
\Pi^M_{SF} = (w - m) [2D_S(\overline{w}^*)] + (\overline{w}^* - \overline{m}^*) D_S^* (\overline{w}^*) + \hat{\Pi}^E (\overline{w}^*), \tag{15}
\]

where \( \hat{\Pi}^E (\overline{w}^*) \) is the fee from the E-retailer:

\[
\hat{\Pi}^E (\overline{w}^*) = \begin{cases} 
  0 & \text{if } \overline{w}^* > w + c, \\
  \frac{2[(w+c)-\overline{w}^*]^2}{9c} & \text{if } w - \frac{c}{2} < \overline{w}^* \leq w + c, \\
  w - \overline{w}^* & \text{if } \overline{w}^* \leq w - \frac{1}{2}c.
\end{cases}
\]

Maximization of (15) leads to the next lemma:
Lemma 3.4 In the subgame $SF$, the manufacturer chooses the wholesale price to the Internet channel as follows:

$$
\overline{w}_{SF}^* = \begin{cases} 
  w + c & \text{if } \overline{m} > m + c, \\
  w + \frac{3m - (3m - c)}{4} & \text{if } m - c < \overline{m} \leq m + c, \\
  w - \frac{c}{2} & \text{if } \overline{m} \leq m - c.
\end{cases}
$$

The Case with Open Internet Channel

Next, consider the subgame of an open Internet channel. Hereafter we denote this one by subgame $O$.

From (4), (5) and Lemma 3.2, the demand from each C-retailer and that from E-retailers which the manufacturer obtains are respectively as follows:

$$
D_O^*(\overline{w}) = \begin{cases} 
  \frac{1}{2} & \text{if } \overline{w} > w + c, \\
  \frac{1}{2c}(\overline{w} - w) & \text{if } w < \overline{w} \leq w + c, \\
  0 & \text{if } \overline{w} \leq w,
\end{cases}
$$

$$
D_O^*(\overline{w}) = \begin{cases} 
  0 & \text{if } \overline{w} > w + c, \\
  \frac{1}{c}[(w + c) - \overline{w}] & \text{if } w < \overline{w} \leq w + c, \\
  1 & \text{if } \overline{w} \leq w.
\end{cases}
$$

(16)

(17)

Since each E-retailer’s profit is zero in this subgame, the manufacturer cannot collect the franchise fee from them. Thus, the manufacturer’s profit is given as

$$
\Pi_{O}^M = (w - m) [2D_O^*(\overline{w})] + (\overline{w} - \overline{m}^*) D_O^*(\overline{w}).
$$

(18)

Maximization of (18) yields the next lemma:

Lemma 3.5 In the subgame $O$, the manufacturer chooses the wholesale price to the Internet
channel as follows:

\[
    \overline{w}_O = \begin{cases} 
    w + c & \text{if } \overline{m} > m + c, \\
    w + \frac{(m-c)}{2} & \text{if } m - c < \overline{m} \leq m + c, \\
    w & \text{if } \overline{m} \leq m - c.
    \end{cases}
\]

Using what we have derived above, we summarize the equilibrium retail prices and demands depending on the manufacturer’s channel choice in Table 1.\(^{10}\) Note that the equilibrium retail prices \((p_O, p^*_O)\) and demands \((D_O, D^*_O)\) in the subgame \(O\) are the same as \((p_{SF}, p^*_{SF})\) and \((D_{SF}, D^*_{SF})\), respectively. This is because, in open Internet channel where the retail price equals to the wholesale price, the manufacturer virtually owns its Internet store and obtains the same profit as in selective Internet channel with franchise fee.

From (9) and Table 1, we obtain the following proposition:

**Proposition 3.1** If the manufacturer launches the Internet channel, the equilibrium retail price of each C-retailer always decreases irrespective of the type of this channel.

That is, the introduction of the Internet channel forces the C-retailers to lower their price since the retail market is more competitive.

In addition, from Table 1, we obtain the following proposition:

**Proposition 3.2** The equilibrium retail prices have the next relationship:

\[
p_{SF} = p_O < p_{SNF}, \quad p^*_{SF} = p^*_O < p^*_{SNF},
\]

when \(m - c < \overline{m} < m + c\) (i.e. the demand for each retailer is positive).

\(^{10}\)Note that, if in the subgame \(SNF\) the manufacturer increases \(w^*_{SNF}\) higher than \(w + c\) when \(\overline{m} > m + c\), then \(p_{SNF}\) increases, too. However, we suppose that the manufacturer does not do because its profit does not increase. The similar supposition also applies to the subgames \(SN\) and \(O\).
That is, the equilibrium retail prices in the subgame $SNF \ (p_{SNF}, p^*_{SNF})$ are is higher than those in the other subgames. This is because the manufacturer collecting no franchise fee sets higher wholesale price to the E-retailer, which loosens the retail competition and leads all retailers to set higher prices.

### 3.3 The First Stage: Marketing Channel Choice

Now, we analyze the manufacturer’s marketing channel choice. For this purpose, we derive the profit of the manufacturer depending on its channel choice.

In the subgame without Internet channel, the manufacturer’s profit is as follows:

$$\Pi^M_0 = w - m. \quad (19)$$

Next, from (12), (13), (14) and Lemma 3.3, the profit of the manufacturer in the subgame $SNF$ is given as:

$$\Pi^M_{SNF} = \begin{cases} 
  w - m & \text{if } \overline{m}^* > m + c, \\
  (w - m) + \frac{\left[\left(m+c-\overline{m}\right)^2\right]}{6c} & \text{if } m - 2c < \overline{m}^* \leq m + c, \\
  (w - m) + \left(m - \frac{c}{2} - \overline{m}^*\right) & \text{if } \overline{m}^* \leq m - 2c.
\end{cases} \quad (20)$$

Finally, from (12), (13), (15) and Lemma 3.4 (or (16), (17), (18) and Lemma 3.5), the profit of the manufacturer in the subgames $SF$ and $O$ is given as:

$$\Pi^M_{SF} = \Pi^M_{O} = \begin{cases} 
  w - m & \text{if } \overline{m}^* > m + c, \\
  (w - m) + \frac{\left[\left(m+c-\overline{m}\right)^2\right]}{4c} & \text{if } m - c < \overline{m}^* \leq m + c, \\
  (w - m) + \left(m - \overline{m}^*\right) & \text{if } \overline{m}^* \leq m - c.
\end{cases} \quad (21)$$
Using (19), (20) and (21), we find that

\[
\Pi_0^M = \Pi_{SNF}^M = \Pi_{SF}^M = \Pi_O^M \quad \text{if } \overline{m^*} \geq m + c,
\]
\[
\Pi_0^M < \Pi_{SNF}^M < \Pi_{SF}^M = \Pi_O^M \quad \text{if } \overline{m^*} < m + c.
\]

When \( \overline{m^*} \geq m + c \), the manufacturer’s profit with any type of Internet channel is the same as that without this channel, because, from Lemmas 3.3, 3.4 and 3.5, it sets \( \overline{m} \) high enough to make the demand for E-retailers zero.

Thus, the channel choice by the manufacturer is summarized as the following proposition:

**Proposition 3.3** If \( \overline{m^*} \geq m + c \), the manufacturer has no incentive to launch any type of Internet channel. If \( \overline{m^*} < m + c \), then it is optimal for the manufacturer to launch a selective Internet channel with the franchise fee or an open Internet channel.

This proposition shows that, when \( \overline{m^*} < m + c \), if the manufacturer has the difficulty to levy the fee from one E-retailer (possibly due to the E-retailer’s large bargaining power), it simply should make the Internet channel open to obtain the same profit. However, the openness of the Internet channel may damage the value of the manufacturer’s brand in the future. To cope with this, the manufacturer can choose a selective Internet channel without the fee as the second best, since such a choice brings the manufacturer larger profit than without the Internet channel.

**4 Social Welfare**

In this section, we examine the introduction of an Internet channel by the manufacturer in the standpoint of social welfare. As a measure of the social welfare, we adopt consumer surplus and total surplus. From Proposition 3.3, we know that, when \( \overline{m^*} < m + c \), the manufacturer has an incentive to launch an Internet channel. Thus, we only consider the case that \( \overline{m^*} \)
satisfies this inequality. Moreover, since the equilibrium outcome of the subgame $SF$ is the same as that of the subgame $O$, welfare properties of the two subgames are identical. Thus, we omit to describe welfare properties of the latter.

4.1 Consumer Surplus

The consumer surplus without an Internet channel is as follows

$$CS_0 = 2 \int_0^{\frac{1}{2}} (v - p_0 - cx) \, dx = v - p_0 - \frac{1}{4}c,$$

(22)

where $p_0$ is (9). The consumer surplus with an Internet channel is given as

$$CS(D, p, p^*) = 2 \int_0^D (v - p - cx) \, dx + \int_{1-2D}^{0} (v - \overline{p}) \, dx$$

$$= v - (2D)p - (1 - 2D)\overline{p}^* - c(D)^2,$$

(23)

where $D$, $p$ and $\overline{p}^*$ are the demand for each C-retailer, prices of C-retailer and E-retailer in equilibrium respectively (the latter includes the delivery fee). These values are different depending on the manufacturer’s choice.

From Propositions 3.1 and 3.2, we know that the introduction of an Internet channel always decreases retail prices and that the retail prices in the subgame $SF$ are lower than those in the subgame $SNF$. Thus, we can prove the following proposition.\(^{11}\)

**Proposition 4.1** The consumer surpluses associated with the manufacturer’s channel choice have the following relationship:

$$CS_{SF} > CS_{SNF} > CS_0 \quad \text{if} \quad m - 2c < \overline{m}^* < m + c,$$

$$CS_{SF} = CS_{SNF} > CS_0 \quad \text{if} \quad \overline{m}^* \leq m - 2c.$$

\(^{11}\)In what follows, we abbreviate $CS(D_{SNF}, p_{SNF}, p^*_{SNF})$ and $CS(D_{SF}, p_{SF}, p^*_{SNF})$ as $CS_{SNF}$ and $CS_{SF}$ respectively.
This proposition shows that the introduction of any type of Internet channel is desirable for consumers, since it enhances the competition among retailers and increases the consumer surplus. Moreover it shows that, when $m - 2c < \overline{m} < m + c$, a selective Internet channel with the franchise fee is more desirable for them than one without the fee, because the retail prices of the former are lower than those of the latter.

If social welfare is defined as the consumer surplus, the introduction of an Internet channel by the manufacturer is always desirable for an economy. Whether this result still holds if we use the total surplus as a measure of social welfare is an intriguing question, which we will examine subsequently.

### 4.2 Total Surplus

The total surplus without an Internet channel is given as

$$TS_0 = 2 \int_0^{\frac{v}{2}} (v - m - cx) \, dx = v - m - \frac{1}{4}c,$$

while that with an Internet channel is as follows

$$TS(D) = 2 \int_0^{D} (v - m - cx) \, dx + \int_{0}^{1-2D} (v - \overline{m}) \, dx$$

$$= v - (2D) m - (1 - 2D) \overline{m} - c(D)^2,$$

where $D$ is the demand for each C-retailer in equilibrium, the value of which is different depending on the manufacturer’s choice.

First, we derive the conditions under which the introduction of an Internet channel increases the total surplus. We can prove the following two lemmas.\textsuperscript{12}

\textsuperscript{12}In what follows, we abbreviate $TS(D_{SNF})$ and $TS(D_{SF})$ as $TS_{SNF}$ and $TS_{SF}$ respectively.
Lemma 4.1  The total surplus with an selective Internet channel without the fee is larger than that without an Internet channel ($TS_{SNF} > TS_0$), if and only if $\overline{m}^* < m + \frac{5}{11}c$.

Lemma 4.2  The total surplus with an selective Internet channel with the fee is larger than that without an Internet channel ($TS_{SF} > TS_0$), if and only if $\overline{m}^* < m + \frac{3}{11}c$.

Next, we prove the lemma to compare the total surplus of the subgames $SNF$ with that of $SF$:

Lemma 4.3  The two total surpluses of the subgames $SNF$ and $SF$ have the following relationship:

$$TS_{SF} < TS_{SNF} \quad \text{if} \quad m + \frac{7}{19}c < \overline{m}^* < m + c,$$

$$TS_{SF} \geq TS_{SNF} \quad \text{if} \quad m - 2c < \overline{m}^* \leq m + \frac{7}{19}c,$$

$$TS_{SF} = TS_{SNF} \quad \text{if} \quad \overline{m}^* \leq m - 2c.$$

Finally, summarizing Lemmas 4.1, 4.2 and 4.3, we obtain the next proposition:

Proposition 4.2  The Total Surpluses associated with the manufacturer’s channel choice have the following relationship:

$$TS_{SF} < TS_{SNF} \leq TS_0 \quad \text{if} \quad m + \frac{5}{11}c \leq \overline{m}^* < m + c,$$

$$TS_{SF} \leq TS_0 < TS_{SNF} \quad \text{if} \quad m + \frac{3}{11}c \leq \overline{m}^* < m + \frac{5}{11}c,$$

$$TS_0 < TS_{SF} \leq TS_{SNF} \quad \text{if} \quad m + \frac{7}{19}c \leq \overline{m}^* < m + \frac{3}{11}c,$$

$$TS_0 < TS_{SNF} < TS_{SF} \quad \text{if} \quad m - 2c < \overline{m}^* < m + \frac{7}{19}c,$$

$$TS_0 < TS_{SF} = TS_{SNF} \quad \text{if} \quad \overline{m}^* \leq m - 2c.$$

We know that, when $\overline{m}^* < m + c$, the manufacturer has an incentive to launch an Internet channel from Proposition 3.3. However, Lemmas 4.1 and 4.2 show that the manufacturer’s decision to launch an Internet channel may decrease the total surplus. Specifically, it is
noteworthy that when \( m + \frac{c}{2} < \bar{m}^* < m + c \), the manufacturer sets up the Internet channel although there is no room to increase the social welfare. Moreover, from Proposition 4.2, we find that, when \( m + \frac{7}{19}c < \bar{m}^* < m + c \), the total surplus of the subgame SNF is larger than that of the subgame SF, although it is optimal for the manufacturer to choose the channel SF. This means that the manufacturer’s channel choice is not desirable in the standpoint of the total surplus.

On the other hand, when \( \bar{m}^* < m + \frac{1}{2}c \), the introduction of an Internet channel has the possibility that increases the total surplus to serve consumers most far away from C-retailers. However, Proposition 4.2 shows that, when \( m + \frac{5}{11}c < \bar{m}^* < m + \frac{1}{2}c \), the launch of any type of Internet channel decreases the total surplus. This is because the equilibrium market division between the two types of retailers by the manufacturer’s wholesale price choice is not necessarily the optimal one that maximizes the total surplus, or minimize the social cost. For example, when \( \bar{m}^* \) satisfies the above inequalities, the market of E-retailers is excessively large from a viewpoint of the social cost. On the other hand, when \( \bar{m}^* < m \), it is socially efficient to use only the Internet channel. However, when \( \bar{m}^* > m - c \) in the subgame SF (\( \bar{m}^* > m - 2c \) in the subgame SNF), the C-retailers obtain some markets. In these subgames, if the manufacturer lower the wholesale price to E-retailers, the C-retailers do not obtain any positive demand. However, the manufacturer does not do for its profit maximization. That is, in equilibrium, there is no dead weight loss due to too few consumption, but there exists dead weight loss due to too much social cost.

5 Concluding Remarks

In this paper, we construct a model of a manufacturer’s marketing channel choice in an economy with Internet. Specifically, using a variant of Hotelling (1929) type linear city model, we examine under what condition the manufacturer deals with retailers based on
Internet (E-retailers) as well as with conventional retailers having physical stores (C-retailers) in equilibrium. Then, we investigate welfare properties of its choice.

We find the followings. The manufacturer has an incentive to launch an Internet channel of selective type with franchise fee or open type, when the sum of the manufacturer’s cost to produce and sell the product to E-retailers and the delivery cost to consumers are sufficiently small. Introducing an Internet channel strengthens the retail competition, lowers retail prices, and increases the consumer surplus. However, it does not necessarily increase the total surplus, due to efficiency loss in production and distribution. Note that all consumers buy one product in equilibrium of the model, thus there is no dead weight loss of few consumption caused by double marginalization.

It should be noted that our model concerns only with the trade-diverting effect of an Internet channel, and it does not consider possible demand-creation effects. Thus, if an Internet channel creates entirely new demand, then the conclusion of this paper may be reversed. It is important to examine under what condition an introduction of an Internet channel increases the total surplus in equilibrium when total demand increases. It is also important to investigate the effect of the government’s infrastructure policy on an economy with Internet because it can affect the welfare of consumers by accumulating various public capital which reduces trip costs when consumers go shopping at conventional retail outlets and/or communication costs when they go shopping at electronic outlets in the long run. These are important research agenda for future research.
Appendix

Proof of Lemma 3.1

First, we consider the profit maximization of C-retailer $i$ given $p_j$ ($j \neq i$), $\bar{p}^*$ and $w$. Below, we denote $p_j = p$ for simplicity. Suppose that

$$p < \bar{p}^* < p + c \quad \text{(A.1)}$$

in order to analyze whether there exists an equilibrium outcome where each retailer obtains positive demand.

Using (4) and (10), we find the following is the profit maximization price of C-retailer $i$:

$$p_{S,i} = \frac{\bar{p}^* + w}{2}, \quad \text{(A.2)}$$

when

$$3\bar{p}^* - 2(p + c) \leq w \leq \bar{p}^* \quad \text{(A.3)}$$

to satisfy $2\bar{p}^* - (p + c) \leq p_{S,i} \leq \bar{p}^*$. From (A.2), if $w = \bar{p}^*$, then $p_{S,i} = w$. When the wholesale price is high enough to satisfy $w > \bar{p}^*$, C-retailer $i$ chooses the same price. If it holds that

$$p - 3c \leq w < 4\bar{p}^* - 3(p + c), \quad \text{(A.4)}$$

then

$$p'_{S,i} = \frac{c + w + p}{2} \quad \text{(A.5)}$$

is the optimal price since it holds that $p - c \leq p'_{S,i} < 2\bar{p}^* - (p + c)$ and two C-retailers’ markets adjoin as in the case without Internet channel. From (A.5), if $w = p - 3c$, then
\( p'_{S,i} = p - c \). This price is low enough to obtain the overall market. When \( w < p - 3c \), the same price is optimal because lower price than \( p - c \) does not yield the additional demand.

Note that \( 3p^* - 2(p + c) - [4p^* - 3(p + c)] = (p + c) - p^* > 0 \) given (A.1). Thus, there remains the range that

\[
4p^* - 3(p + c) \leq w < 3p^* - 2(p + c),
\]

which is not covered by with (A.3) or (A.4). This is due to the kink of the demand function (4) when \( p_i \) equals

\[
p'_{S,i} = 2p^* - (p + c), \tag{A.7}
\]

If (A.6) holds, then (A.7) is optimal.

Next, we proceed to the maximization problem by the E-retailer given \( p_i = p \ (i = 1, 2) \) and \( w^* \). From (5) and (11), the profit maximization price is

\[
\overline{p}_S = \frac{c}{4} + \frac{p + \overline{w}^*}{2}, \tag{A.8}
\]

if it holds that

\[
p - \frac{c}{2} \leq \overline{w}^* \leq p + \frac{c}{2} \tag{A.9}
\]

to satisfy \( p \leq \overline{p}_S \leq p + \frac{c}{2} \). From (A.8), \( \overline{p}_S = \overline{w}^* \) when \( \overline{w}^* = p + \frac{c}{2} \). Also, when \( \overline{w}^* > p + \frac{c}{2} \), the same price is optimal. On the other hand, if \( \overline{w}^* = p - \frac{c}{2} \), then \( \overline{p}_S = p \), which is low enough to obtain the all market. Thus, the same price is optimal when \( \overline{w}^* < p - \frac{c}{2} \).

Now, we examine a Nash equilibrium of this subgame. We restrict our attention to the equilibrium where each C-retailer sets the same price \( (p_{S,i} = p_S, \ i = 1, 2) \). The pair \( (p_S, \overline{p}_S) \)
satisfying both (A.2) and (A.8) is as follows

\[
\begin{align*}
    p_S &= \frac{c}{6} + \frac{2w + \overline{w^*}}{3}, \\
    \overline{p}_S &= \frac{c + w + 2\overline{w^*}}{3}.
\end{align*}
\]  

(A.10)  

(A.11)

Substitute \( p = p_S \) and \( \overline{p}^* = \overline{p}_S \) into (A.3) (or (A.9)), we obtain

\[
w - \frac{c}{2} \leq \overline{w^*} \leq w + c.
\]

That is, (A.10) and (A.11) are Nash equilibrium prices of this subgame when (A.12) holds.

Substitute \( \overline{w^*} = w + c \) into (A.10) and (A.11), we get

\[
\begin{align*}
    p_S &= w + \frac{1}{2}c, \\
    \overline{p}_S &= w + c (= \overline{w^*}).
\end{align*}
\]

In this case, the demand for the E-retailer is zero. When \( \overline{w^*} > w + c \), the E-retailer sets the same price and its demand is zero. Taking this into account, we examine the condition under which (9) and \( \overline{p}_S = \overline{w^*} \) are equilibrium prices. Substitute \( p = w + c \) and \( \overline{p}^* = \overline{w^*} \) into (A.4), then we find that \( \overline{w^*} > w + \frac{3}{2}c \). In this range, \( (p_S, \overline{p}_S) = (w + c, \overline{w^*}) \) are equilibrium prices. In the range that \( w + c < \overline{w^*} \leq w + \frac{3}{2}c \), the E-retailer sets \( \overline{p}_S = \overline{w^*} \) and from (A.7) each C-retailer sets \( p_S = \overline{w^*} - \frac{1}{2}c \).

13 Substitute \( \overline{w^*} = w - \frac{1}{2}c \) into (A.10) and (A.11), then we get

\[
\begin{align*}
    p_S &= w, \\
    \overline{p}_S &= w.
\end{align*}
\]

When \( \overline{w^*} < w - \frac{1}{2}c \), each C-retailer sets the same price, \( p_S = w \). Thus, the E-retailer has no incentive to change its price and maintains \( \overline{p}_S = w \).

\[\text{13That is, the C-retailer’s price does not respond to the change of its cost } w. \text{ This phenomenon is similar to Salop (1979) due to the kink of the demand.}\]
Proof of Lemma 3.2

The behavior of C-retailer $i$ is the same as in the case with selective Internet channel. Thus, we omit to derive it. It is enough to replace the subscript $S$ in (A.2), (A.7) and (A.7) with $O$. On the other hand, many E-retailers are active in the market and they are not differentiated. Thus, from the competition among them, each E-retailer sets the same price:

$$p^*_O = w^*. \quad (A.13)$$

Then, we examine a Nash equilibrium of this subgame. We restrict our attention to the equilibrium where each C-retailer sets the same price ($p_{O,i} = p_O, i = 1, 2$). Substitute (A.13) into (A.2), we obtain

$$p_O = \frac{w^* + w}{2}. \quad (A.14)$$

Substitute (A.13) and (A.14) into (A.3), we obtain

$$w \leq w^* \leq w + c. \quad (A.15)$$

That is, (A.13) and (A.14) are Nash equilibrium prices if (A.15) holds. When $w^* > w + c$, each C-retailer sets the same price as in the case with Selective Internet Channel. Substitute $w^* = w$ into (A.14), we obtain $p_O = w$. When $w^* < w$, each C-retailer sets this price. On the other hand, Bertrand competition among E-retailers forces them to maintain (A.13) even if $w^*$ does not satisfy (A.15).

Proof of Lemma 3.3
Suppose \( w - \frac{c}{2} \leq \overline{w} \leq w + c \). Then, differentiate (14) with respect to \( \overline{w} \), we obtain

\[
\overline{w}_{SNF} = w + \overline{m} - \frac{(m - c)}{2}.
\]

This is the optimal price when \( m - 2c \leq \overline{m} \leq m + c \) in order to satisfy \( w - \frac{c}{2} \leq \overline{w}_{SNF} \leq w + c \).

When \( \overline{m} = m + c \) (\( \overline{m} = m - 2c \)), (A.16) becomes \( \overline{w}_{SNF} = w + c \) (\( \overline{w}_{SNF} = w - \frac{c}{2} \)) and the C-retailers (the E-retailer) obtain all the market. When \( \overline{m} > m + c \) (\( \overline{m} < m - 2c \)), the manufacturer chooses the same prices: \( \overline{w}_{SNF} = w + c \) (\( \overline{w}_{SNF} = w - \frac{c}{2} \)), because, even if it raises (lowers) the price, its profit is unchanged (decreased).

**Proof of Lemmas 3.4 and 3.5**

Since it is similar as the proof of Lemma 3.3, the derivation is omitted.

**Proof of Proposition 3.1**

Since we immediately find the result from (9) and Table 1, the derivation is omitted.

**Proof of Proposition 3.2**

From Table 1, we know that, when \( m - c < \overline{m} < m + c \),

\[
\begin{align*}
    p_{SNF} - p_{SF} &= \frac{m + c - \overline{m}}{12}, \\
    \overline{p}_{SNF} - \overline{p}_{SF} &= \frac{m + c - \overline{m}}{6},
\end{align*}
\]

both of which are positive. Note that \((p_O, \overline{p}_O) = (p_{SF}, \overline{p}_{SF})\) for any \( \overline{m} \). Thus, the proposition is proved.

**Proof of Proposition 4.1**

First, we show that, if the manufacturer launches an Internet channel, then the consumer
surplus always increases. Some calculations of (23) yields

\[
CS(D, p, p^*) = v - p + [p + c(D) - \overline{p}^*](1 - 2D) - c(D) + c(D)^2.
\]

Since it always holds that \( p + c(D) = \overline{p}^* \) in equilibrium, we simplify the above equation as follows

\[
CS(D, p) = v - p - c(D) + c(D)^2. \tag{A.17}
\]

From (22) and (A.17), we obtain the consumer surplus differential

\[
CS(D, p) - CS_0 = (p_0 - p) + c(D - \frac{1}{2})^2,
\]

which is positive since we know that the introduction of an Internet channel always decreases retail prices from Proposition 3.1.

Next, we examine the consumer surplus differential of the cases with Internet channels. Using (A.17), we obtain

\[
CS_{SF} - CS_{SNF} = (p_{SNF} - p_{SF}) + c(D_{SNF} - D_{SF}) [1 - (D_{SNF} + D_{SF})]. \tag{A.18}
\]

From Table 1, it is evident that \( CS_{SF} = CS_{SNF} \) when \( \overline{m}^* \leq m - 2c \). For other cases, (A.18)
is calculated as

\[ CS_{SF} - CS_{SNF} = \frac{(m + c) - \overline{m}^*}{12} \left\{ 1 + \frac{5}{12c} [(m + c) - \overline{m}^*] \right\} \]

\( (if \; m - c < \overline{m}^* < m + c), \)

\[ CS_{SF} - CS_{SNF} = \frac{\overline{m}^* - (m - 2c)}{6} \left\{ \frac{3}{2} + \frac{1}{6c} [(m + c) - \overline{m}^*] \right\} \]

\( (if \; m - 2c < \overline{m}^* \leq m - c), \)

both of which are positive.

To prove Lemmas 4.1 and 4.2, first we prove the next lemma:

**Lemma A**

\[ \frac{c}{2} \left( \frac{c}{2} \right) \left[ \overline{m}^* - \left( m + \frac{1}{4}c \right) \right] \Leftrightarrow TS(D) > TS_0 \]

**Proof of Lemma A**

Using (24) and (25), we obtain

\[ TS(D) - TS_0 = (1 - 2D) (m - \overline{m}^*) + c \left( \frac{1}{4} - D^2 \right) \]

\[ = \left( \frac{c}{2} \right) (1 - 2D) \left\{ D - \left( \frac{2}{c} \right) \left[ \overline{m}^* - \left( m + \frac{1}{4}c \right) \right] \right\}. \]

Note that \( 1 - 2D (= D^*) \) is positive when \( \overline{m}^* < m + c \). Thus, this lemma is proved.

**Proof of Lemma 4.1**

In the case of a selective Internet channel without the fee (\( D = D_{SNF} \)), from Table 1 (a), it is evident that \( D > 2 \left( \overline{m}^* - \left( m + \frac{1}{4}c \right) \right) /c \) when \( \overline{m}^* < m - 2c \). When \( m - 2c \leq \overline{m}^* < m + c \),
it holds that
\[ \overline{m}^* < m + \frac{5}{11}c \Leftrightarrow D > \frac{2}{c} \left[ \overline{m}^* - \left( m + \frac{1}{4}c \right) \right]. \]

Thus, combined with Lemma A, the lemma is proved.

**Proof of Lemma 4.2**

In the case of a selective Internet channel with the fee (\( D = D_{SF} \)), from Table 1 (b), it is evident that \( D > 2 \left[ \overline{m}^* - (m + \frac{1}{4}c) \right] /c \) when \( \overline{m}^* < m - c \). When \( m - c \leq \overline{m}^* < m + c \), it holds that
\[ \overline{m}^* < m + \frac{3}{7}c \Leftrightarrow D > \frac{2}{c} \left[ \overline{m}^* - \left( m + \frac{1}{4}c \right) \right]. \]

Thus, combined with Lemma A, the lemma is proved.

**Proof of Lemma 4.3**

Using (25), we calculate the total surplus differential such that
\[
TS_{SF} - TS_{SNF} = 2 \left( D_{SNF} - D_{SF} \right) \left[ m - \overline{m}^* + \frac{c}{2} \left( D_{SNF} + D_{SF} \right) \right]. \tag{A.19}
\]

From Table 1 (a) and (b), it is evident that, when \( \overline{m}^* \leq m - 2c \), \( TS_{SNF} = TS_{SF} \). For other cases, (A.19) is given as
\[
TS_{SF} - TS_{SNF} = \frac{19}{144c} \left[ (m + c) - \overline{m}^* \right] \left( m + \frac{7}{19}c - \overline{m}^* \right) \quad (if \ m - c < \overline{m}^* < m + c),
\]
\[
TS_{SF} - TS_{SNF} = \frac{11}{36c} \left[ \overline{m}^* - (m - 2c) \right] \left( m + \frac{2}{11}c - \overline{m}^* \right) \quad (if \ m - 2c < \overline{m}^* \leq m - c).
\]
Thus, it holds that, when \( m - 2c < \overline{m} < m + c \),

\[
\overline{m} > m + \frac{7}{19}c \iff TS_{SF} < TS_{SNF}.
\]

Thus, the lemma is proved.

**Proof of Proposition 4.2**

Straightforward application of Lemmas 4.1, 4.2 and 4.3 proves this proposition.

Table 1: the equilibrium retail prices and demands

(a) the subgame \( SNF \)

\[
(p_{SNF}, p_{SNF}^*) = \begin{cases} 
(w + \frac{1}{2}c, w + c) & \text{if } \overline{m} > m + c \\
\left( w + \frac{m-(m-2c)}{6}w + \frac{m-(m-2c)}{3} \right) & \text{if } m - 2c < \overline{m} \leq m + c \\
(w, w) & \text{if } \overline{m} \leq m - 2c
\end{cases}
\]

\[
(D_{SNF}, D_{SNF}^*) = \begin{cases} 
\left( \frac{1}{2}, 0 \right) & \text{if } \overline{m} > m + c \\
\left( \frac{\overline{m}-(m-2c)}{6c}, \frac{(m+c)-\overline{m}}{3c} \right) & \text{if } m - 2c < \overline{m} \leq m + c \\
(0, 1) & \text{if } \overline{m} \leq m - 2c
\end{cases}
\]

(b) the subgame \( SF \)

\[
(p_{SF}, p_{SF}^*) = \begin{cases} 
(w + \frac{1}{2}c, w + c) & \text{if } \overline{m} > m + c \\
\left( w + \frac{m-(m-c)}{4}w + \frac{m-(m-c)}{2} \right) & \text{if } (m - c < \overline{m} \leq m + c) \\
(w, w) & \text{if } \overline{m} \leq m - c
\end{cases}
\]

\[
(D_{SF}, D_{SF}^*) = \begin{cases} 
\left( \frac{1}{2}, 0 \right) & \text{if } \overline{m} > m + c \\
\left( \frac{\overline{m}-(m-c)}{4c}, \frac{(m+c)-\overline{m}}{2c} \right) & \text{if } m - c < \overline{m} \leq m + c \\
(0, 1) & \text{if } \overline{m} \leq m - c
\end{cases}
\]

(c) the subgame \( O \)

The equilibrium retail prices \((p_O, p_O^*)\) and demands \((D_O, D_O^*)\) in the subgame \( O \) are the same as those in the subgame \( SF \).
References


