Investor Confidence, Short-Sales Constraints, and the Behavior of Security Prices

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Abstract

Investor confidence affects financial markets. Information, noise, market frictions cause investor confidence to influence security prices, leading to a price different from the rational expectations value. This paper presents a simple theoretical model of asset prices where investor confidence is allowed to differ across traders, and across time – depending on observed outcomes. The presence of short-sales constraints causes asset prices to behave asymmetrically: short-run returns display reversal after good news, but momentum after bad news. This can change somewhat if investor confidence varies because of biased self-attribution: good news causes returns to exhibit short-run momentum and long-run reversal. We also investigate the extent to which asset supply affects the price paths. In case of zero net supply the equilibrium prices are set unilaterally by either of two investor groups. Yet if we allow for positive asset supply we find an intermediate range of information signals and resulting prices consistent with both trader types being active in the market – this happens if news events are moderate. We further identify the shrinkage effect: the range of signals where both groups of traders have positive asset demands may shrink in subsequent trading due to biased self-attribution.

Keywords: behavioral finance, overconfidence, underconfidence, overreaction, momentum

When rational agents interact with quasi-rational agents, the rational agents cannot be expected either to take all the quasi’s money, or to set prices unilaterally.

Richard H. Thaler

*Preliminary and incomplete. First version: January 2003. This draft: April 2008. We thank Ryoko Wada and participants of the fist Meeting of the Association of Behavioral Economics and Finance (Osaka 2007) for useful comments. Financial support from the Japan Society for the Promotion of Science is gratefully acknowledged. Comments welcome. E-mail: greg@fc.ritsumei.ac.jp
1 Introduction

Behavioral finance has undergone rapid development in recent years. A number of new asset pricing models have been proposed, most of them highly stylized, yet elegant and insightful. Some of the most celebrated among these include Barberis, Huang, and Santos (2001), Barberis, Shleifer, and Vishny (1998), Hong and Stein (1999), Shleifer and Vishny (1997). Economists once thought that behavior was either rational or impossible to formalize. It is now becoming increasingly obvious that models of quasi-rationality are not only possible but also can provide much more accurate descriptions of behavior than purely rational models. The assertion of behavioral finance is that security prices and expected returns are based both on risk and investor misvaluation, thus leading to mispricing. Equilibrium prices thus reflect a weighted average of the expectations of both rational and irrational traders. In an environment characterized by complex and/or asymmetric information structures, investors’ decisions will not in general be optimal. Rationality, as understood by economic theory, requires impossible powers of calculation. Full rationality – the one employed by most of contemporary asset pricing theories – is at best a significant special case; it can serve as a benchmark for evaluating and comparing more general models. It may be even the case that in some respects, all investors are imperfectly rational. After all nobody really possesses infinite amount of time needed to analyze a given problem, nobody can observe every piece of information that might be relevant, and nobody has unlimited calculation powers. As a result, widespread decision biases can arise that lead to outcomes different from those in the rational expectations paradigm.

One of the most widely discussed behavioral phenomena concerned with financial asset markets is that of overconfidence, i.e. a tendency for people to put too much weight in their own judgments, to believe their own point of view to be more accurate than it actually is when considered objectively. While in this paper it is mainly individual investors who are thought of as the acting economic agents, it has been found that experts, armed with sophisticated models, actually exhibit more overconfidence than laymen! Economist Robert J. Shiller (2000a) provides a comprehensive account of investor opinion surveys carried out over a period spanning some two decades, indicating a pervasive tendency for both individual and institutional investors to be overconfident. Overconfidence is usually considered as a static phenomenon; for example, an investor is assumed to perceive the variance of an information signal he obtains to be lower than it truly is. In this paper, a plausible dynamic interpretation of overconfidence is also considered: an investor confident in his prior valuation of a risky asset (based, say, on a research done by herself on a
company) is liable to update his valuation differently depending on the type of new information. Overconfidence makes him overreact to news confirming his estimate and underreact to news that contradicts his valuation. Such a behavioral pattern is called in the cognitive psychology literature *biased self-attribution*. It causes individuals to attribute successes to their own qualities and failures to chance. Overconfidence and biased self-attribution are static and dynamic counterparts; self-attribution causes individuals to learn to be overconfident rather than converging to an accurate self-assessment. Shiller (2000b)\[13\] presents evidence that investor confidence does vary through time, based on investor opinion polls in the US and Japan.

In this article, interaction between overconfident and underconfident traders in a financial market setting is analyzed. While overconfidence is a well-documented pervasive pattern of human behavior, underconfidence seems to be a less salient phenomenon. However, it may become more understandable why underconfidence, and generally heterogeneous confidence among different traders, is employed in our model, if we observe its connection with noise. In his influential essay, Black (1986)\[3\] remarks that “people sometimes trade on noise as if it were information”. He further asserts that “people who trade on noise are willing to trade even though from an objective point of view they would be better off not trading”. In today’s world of information flooding, distinguishing noise from valuable information is an extremely difficult task. This observation motivates the approach in this paper based on investor psychology. The investors in possession of a piece of information can never be sure that they are actually trading on information rather than noise. It is possible the information has already been reflected in prices. Trading on that sort of information would be just like trading on noise. It is therefore plausible to say that investors who are uncertain as to the quality and relevance of their information are underconfident.

The effect of short-sales constraints is also examined in a behavioral finance context. Selling short can be expensive. In order to sell short, one must borrow the stock from a current owner, and this stock lender may charge a fee to the short seller. In addition to these direct costs, there are other costs and risks associated with shorting, such as the risk that the short position will have to be involuntarily closed at a loss due to a recall of the stock loan. In addition, legal and institutional constraints inhibit or prevent investors from selling short. Finally, some market participants seem to behave as though they were facing considerable shorting costs, even though they are not. In financial economics, these impediments and costs are collectively referred to as “short sales constraints”. We attempt to analyze the impact of short sales constraints on the equilibrium prices in presence of confidence-biased investors.

The purpose of the analysis is to show how the interplay of the two factors, namely investor
confidence and short-sales constraints, affects the behavior of security prices, in particular short- and long-run return autocorrelation. First finding of our analyzes is that if the market is populated by confidence-biased investors, with two groups – either under- or overconfident, the resulting equilibrium price will not be equal to its corresponding rational expectations counterpart if the population fraction of overconfident traders is larger than their own bias. We then show that, in the presence of short-sales constraints, the equilibrium price movements will be asymmetric with respect to new information. Accordingly, short-run returns will exhibit negative auto-correlation after good news, and positive auto-correlation after bad news. We proceed to show that if investor confidence can change depending on the outcome of another information signal, then after favorable news, prices exhibit short-run momentum, but long-run reversal.

The formal analysis is contained in section 3; it commences with a simple analytical model, where the possibility of equilibrium prices being equal to their rational values is investigated in a market populated by less than rationally behaving investors. The overall flow of this paper is organized as follows: section 2 presents a review of some recent models related to the one considered here; section 3 contains the basic model with some extensions; we discuss the relevance of our findings and some extensions in section 4 and offer a few concluding remarks in section 5.

2 Related Research

In this section, some notable previous work that also focuses on the role of investor psychology, in particular investor confidence, and/or short-sales constraints in asset pricing is discussed.

In one of few models applying both static and dynamic aspects of investor confidence, Daniel, Hirshleifer, and Subrahmanyam (1998) stress biases in the interpretation of private, rather than public information. Imagine that the investor does some research on his own to try to determine a firm’s future cashflows. DHS assume that she is overconfident about this information; in particular, they argue that investors are more likely to be overconfident about private information they have worked hard to generate than about public information. If the private information is positive, overconfidence means that investors will push prices up too far relative to fundamentals. Future public information will slowly pull prices back to their intrinsic value, thus generating long-term reversals. To get momentum and a post-earnings announcement effect, DHS assume that the public information alters the investor’s confidence in her original private informa-

\[^{1}\] Some time after the completion of the first version of this paper in January 2003, we found out that similar results were derived by Xu (2005).
tion in an asymmetric fashion, a phenomenon known as self-attribution bias: public news which confirms the investor’s research strongly increases the confidence she has in that research. Disconfirming public news, though, is given less attention and the investor’s confidence in the private information remains unchanged. This asymmetric response means that initial overconfidence is on average followed by even greater overconfidence, generating momentum. An extension of the basic model of DHS with short-sales constraints is one of the building blocks of this article.

Gervais and Odean (2001) provide a model that accommodates analytical solution for the learning process under biased self-attribution. In a multiperiod one-asset market model, the authors describe, assuming an adjusted Bayesian updating algorithm, both the process by which traders learn about their ability and how a bias in this learning can create overconfident traders. Traders in their model initially do not know their ability. They learn through experience. Investors who successfully forecast next period dividends improperly update their beliefs: they overweight the possibility that their success was due to superior ability. In so doing they become overconfident. Such traders are also, as a result of success, wealthy. Overconfidence does not make traders wealthy, but the process of becoming wealthy can make them overconfident. As traders become overconfident, trading volume and market return volatility increase. Since equity is in positive net supply, the model also predicts that trading volume will be higher after market rises than market falls. The paper’s principal conclusion is that a simple and prevalent bias in evaluating one’s own performance is sufficient to create markets in which investors are, on average, overconfident.

In an arguably most comprehensive account of the effects of (static) overconfidence on securities markets, Odean (1998) examines how the effects of investor confidence depend on who, in a market, is overconfident and on how information in that market is disseminated. He studies the statics of overconfidence when there is a single risky security in three different settings: price-taking competitive traders, a strategically trading insider, and risk-averse market makers. When price-taking investors think the signal is more accurate than it really is, the market price overreacts to the signal. Eventually, when the true state of the world resolves, the price corrects. This pattern of overreaction and reversal causes excess price volatility, and negative long-run return autocorrelation. Odean (1998) asserts the most robust effect of overconfidence to be increased trading volume – it results in all three considered cases; furthermore, overconfidence can facilitate orderly trading even in the absence of noise traders. Overconfident traders can cause markets to underreact to the information of rational traders, leading to positive serially correlated returns. Moreover, returns are also positively serially correlated when traders underweight new information and negatively serially correlated when they overweight it. The degree of this under- and overreaction depends on
the fraction of all traders who under- or overweight the information. The author also finds that overconfidence reduces traders’ expected utility, mainly because of them holding underdiversified portfolios; yet, when information is costly and traders are overconfident, informed traders do worse than uninformed traders. Another finding is that overconfidence increases market depth, hence market liquidity. Overconfident insiders can improve the quality of prices (as measured by the variance of the difference between price and the underlying asset value), but overconfident price-takers worsen it.

Biased self-attribution can cause conservatism, which makes people update their beliefs too slowly in face of new evidence, because an individual who has explicitly adopted a belief may be reluctant to admit to himself that he had made a mistake. Conservatism and representativeness heuristic (a tendency for people to see patterns in truly random sequences) are employed by Barberis, Shleifer, and Vishny (1998) to offer an explanation for under- and overreactions based on a model in which actual earnings for a risky asset follow a random walk, but investors do not understand this. They mistakenly believe that the earnings process stochastically fluctuates between a regime with mean-reverting earnings, and a regime with expected earnings growth. If recent earning changes reverse, investors erroneously believe the firm is in a mean-reverting state, and underreact to recent news, consistent with conservatism. If investors see a sequence of growing earnings, they tend to wrongly conclude that the firm is in a growth regime, and overextrapolate trends, which is suggestive of representativeness. Overreaction to a long enough trend implies subsequent low returns during the process of correction. Thus, there can be long-term overreaction and correction, implying negative long-lag return autocorrelation. Yet the average response to an initial impulse can be smooth, implying positive short lag autocorrelation. Similarly, the model can accommodate a positive short-term correlation between the asset return and an earnings change, and a negative long-term correlation. If sporadic events such as dividend initiations are viewed as isolated from earnings patterns, a single event version of the model implies, under appropriate parameter values, underreaction.

None of the above articles considers the effects of short-sales constraints, presence of which is shown to play a significant role in a recent theoretical paper. In Hong and Stein (2003), the focus is on the problem of why stock markets may be vulnerable to crashes. Two short-sales constrained investors with differing private signals about a single asset’s value, overconfident in these signals, interact with rational arbitrageurs in an auction setting. The arbitrageurs (who know the true asset value to be the arithmetical average of the two private signals), though, may not always get to see all the signals. Because of short-sales constraints, bearish investors do not initially participate
in the market and their information is not revealed in prices. However, if other, ex ante bullish investors enter the market at a later date and their signals turn out to be low, the originally bearish group may become active in the market, and hence more will be learned about their signals. Thus previously hidden information tends to come out during market declines. The authors analyze skewness of returns in a stylized auction market model. Sufficiently large differences of opinions between investors are found to cause negative skewness in the distribution of market returns, which is further predicted to be most pronounced conditional on high trading volume. Also, the model is capable to accommodate an explanation for large movements in prices unaccompanied by significant news about fundamentals and increased correlation among stocks in a falling market (“contagion”).

3 Investor Confidence and Short-Sales Constraints

This section presents the formal analysis of asset prices with emphasis on investor confidence, short-sales constraints and the interplay of these two factors. The basic structure is presented in section 3.1. It covers the derivation of investors’ demand functions, equilibrium prices, as well as a simple examination of the properties of prices, namely its relationship with a rational expectations equilibrium price. Section 3.2 introduces short-sales constraints and presents the consequences of their presence for the short-run behavior of asset returns. Outcome-dependent confidence, based on biased self-attribution, is then introduced in section 3.3 along with some implications for the performance of asset prices. This is an extension of one of the models from Daniel, Hirshleifer, and Subrahmanyam (1998)\textsuperscript{4}, with the addition of short-sales constraints.

3.1 Two Types of Traders with Opposing Biases: Security Price Behavior

We begin the formal analysis with a simple model of asset prices with two types of boundedly rational investors present in the market. The first type overestimates the accuracy of their information whilst the second type of investors underestimates the precision of theirs. Let us start the analysis with the characterization of the model structure. There is one risk-free asset with constant payoff equal to unity and one risky stock with net supply normalized to zero.\textsuperscript{2} There are three dates: \( t = 0, t = 1, t = 2 \). At \( t = 2 \), the stock pays a terminal dividend equal to \( F \). It is assumed to be normally distributed according to \( F \sim \mathcal{N}(0, \sigma_F^2) \). At \( t = 1 \), all traders receive a noisy signal about

\textsuperscript{2}Analysis of the effect of a positive net supply of the risky asset is the subject of section 3.3.
the risky asset’s intrinsic value; the signal may thus be regarded as a public one:

\[ s_1 = F + \epsilon, \]

where \( \epsilon \sim \mathcal{N}(0, \sigma^2) \). The signal precision is given by the reciprocal of its variance, \( 1/\sigma^2 \). Random variables \( F \) and \( \epsilon \) are independently distributed. At \( t = 0 \), the price is simply equal to its prior mean, \( P_0 = 0 \). Investors misperceive the precision of the information they receive; such a bias is modeled in the following way: an investor indexed with \( k \) holds a belief about the distribution of her signal summarized by \( s_k = F + B_k \epsilon \), where \( k = c, d \). Overconfident investors, indicated with the subscript “\( c \)” believe that their signal is more accurate than it actually is, i.e. \( 0 < B_c < 1 \); thus they believe its distribution to be “too tight”. On the other hand, underconfident investors, denoted by “\( d \)” believe the distribution of their signal to be “too loose”, i.e. \( B_d > 1 \). We also allow the case of fully rational expectations, so that \( B_d \geq 1 \). A possible justification for this structure is a situation where different investors get their information – which is the same from an objective point of view – from different sources: one reliable and the other questionable, or “untrustworthy”. Then one group will believe the precision of their signal to be more accurate than it truly is, whilst the other group will think their signal contains too much noise. Such an arrangement results effectively in a structure that is possible to be characterized by overconfidence and underconfidence. Let the subset of investor population consisting of overconfident traders be denoted by \( \lambda \), so that the number of underconfident traders is \( 1 - \lambda \), where \( 0 < \lambda < 1 \).

All investors have CARA (Constant Absolute Risk Aversion) utility functions with equal risk tolerance coefficient \( \gamma \):

\[ E[U(W_k)] = E[-\exp(-\frac{W_k}{\gamma})] \quad \text{for} \quad k = c, d. \]

With normal distributions, this implies in effect a mean-variance utility function. In a multiperiod model, wealth depends on investor decisions in all the periods. Unfortunately, the general solution for such a problem is quite complex; however, for our purposes it will be sufficient to focus on “myopic” behavior: traders are assumed to focus only on the immediate period, and so decisions are independent across periods. This effectively ignores any inter-period linkages but does allow the problem to be analyzed tractably.

The wealth at the final date for each trader is the sum of the initial wealth \( W_0 \) and the gain derived from the two types of assets. Since the payoff of the risk-free asset is always equal to unity, it follows that for trader \( k \) (\( k = c, d \)): \( W_k = W_0 + D_k(F - P) \), where \( D_k \) is trader \( k \)'s demand for
the risky asset and $P$ is its price. The trader $k$’s maximization problem is therefore:

$$
\max_{D_k} E_k[W_k|s] - \frac{\text{Var}[W_k|s]}{2\gamma} \quad \text{s.t.} \quad W_k = W_0 + D_k(F - P) \quad \text{for} \quad k = c, d. \tag{3}
$$

By standard properties of normal variables it follows that:

$$
E[F|s] = E[F] + \frac{\text{Cov}[F, s]}{\text{Var}[s]}(F + \epsilon)
$$

and

$$
\text{Var}[F|s] = \text{Var}[F] - \frac{\text{Cov}[F, s]\text{Cov}[F, s]}{\text{Var}[s]},
$$

which in the present model becomes:

$$
E_k[F|s] = \frac{\sigma^2_F}{\sigma_k^2 + B_k^2 \sigma^2_\epsilon}(F + \epsilon) \quad \text{for} \quad k = c, d; \tag{4}
$$

$$
\text{Var}_k[F|s] = \frac{\sigma^2_F B_k^2 \sigma^2_\epsilon}{\sigma^2_F + B_k^2 \sigma^2_\epsilon} \quad \text{for} \quad k = c, d, \tag{5}
$$

where $B_k = B_c$ for overconfident traders and $B_k = B_d$ for underconfident traders. Furthermore, solving the above maximization problem yields the following demand functions:

$$
D_k = \gamma \frac{[\alpha_k(F + \epsilon) - P]}{\beta_k}, \tag{6}
$$

where $\alpha_k = \frac{\sigma^2_F}{\sigma^2_F + B_k^2 \sigma^2_\epsilon}$ and $\beta_k = \text{Var}_k[F|s] = \frac{\sigma^2_F B_k^2 \sigma^2_\epsilon}{\sigma^2_F + B_k^2 \sigma^2_\epsilon}$.

It can be seen that $\alpha_c > \alpha_d$ and $\beta_c < \beta_d$, resulting in $|D_c| > |D_d|$. To put it another way, overconfident traders’ higher conditional mean and lower conditional variance result in them taking larger positions in the risky stock. The assumption of differing signal distributions leads to an equilibrium, which is not a rational expectations equilibrium. The model’s equilibrium is thus characterized by the investors’ optimal demands as given in (6) and by the market clearing condition equating total demand with total supply:

$$
\lambda D_c + (1 - \lambda) D_d = 0. \tag{7}
$$

Substituting appropriate demand functions into the last equation results in the equilibrium price being equal to:

$$
P_1 = \frac{\lambda \alpha_c \beta_d + (1 - \lambda) \alpha_d \beta_c}{\lambda \beta_d + (1 - \lambda) \beta_c}(F + \epsilon). \tag{8}
$$

9
Having obtained the equilibrium price, we now turn our attention to the analysis of how this price relates to one that would be obtained under proper, neutral confidence; in doing so, the emphasis is applied at the issue of whether the above price can be equal to the “rational” price. In other words, the goal is to check if differing biases might cancel each other out leading to prices being close or equal to those in case of only rational investors being present. To examine the possibility of the price being equal to its corresponding rational expectations value, let us first observe that this price would be equal to

\[ P^r = \alpha_r(F + \epsilon) = \frac{\sigma^2_F}{\sigma^2_F + \sigma^2_\epsilon}(F + \epsilon), \]  

where the superscript “\( r \)” indicates the rational price. This would be the case if all the investors correctly estimated the precision of their signals, that is, if \( B_c = B_d = 1 \). Equating the last two formulas, \( P = P^r \), yields after some algebra a condition necessary for the two biases to cancel each other out, leading to a rational equilibrium asset price:

\[ P = P^r \iff 1 = \lambda \frac{1}{B_c} + (1 - \lambda) \frac{1}{B_d}. \]  

(10)

The above equation has some intuitive properties. Whether the price can attain its rational expectations value, will depend on the interplay of the three parameters: \( \lambda \), \( B_c \), and \( B_d \). It can be seen that it is only in a special case that the price can attain its rational value. If the first term on the right-hand side of equation (10) prevails, overconfident investors’ dominance will show in the price being above its absolute rational level; the opposite happens when the underconfident investors dominate the market. The asset price depends on the extent of investor confidence and the relative fractions of the two trader types in the whole population.

**Proposition 1.** The relationship between the attained equilibrium price and the price in a model with only rational traders present is characterized by the following condition:

\[ 1 \geq \frac{1}{\lambda B_c} + (1 - \lambda) \frac{1}{B_d} \iff P \geq P^r. \]  

(11)

Concentrating for the moment on the overconfident fraction of the trader population, it follows that the price cannot be rational if \( \lambda > B_c \). Observe that the above condition can be rewritten in terms of the relative population fractions of both trader types as

\[ \frac{1 - \lambda}{\lambda} \geq \frac{\frac{1}{B_c} - 1}{1 - \frac{1}{B_d}} \iff P \geq P^r. \]  

(12)

Thus, for any \( B_d > 1 \), we shall have:

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\(^3\)One of leading arguments against behavioral finance is the assertion that different biases of quasi-rational agents cancel each other out in equilibrium and thus have no effect on prices and other variables of interest.
**Corollary 1.** *The price of a risky asset in a market populated by overconfident and underconfident traders is larger than its rational counterpart if the fraction of overconfident traders is larger than the overconfident traders’ bias, that is, if $B_c < \lambda$.\*

It is worthwhile to examine some special cases. First, assume that the two biases are symmetric in the following sense: $B_d = 2 - B_c$, i.e. the overconfident overestimate the precision of their information to the same extent as the underconfident underestimate theirs. In this case, the necessary condition for the price to be rational is $B_c = 2\lambda$. But the price will not be rational if $\lambda > B_c$, and so the price can only attain its rational value if $\lambda < 1/2$.

**Observation 1.** *If the imperfectly rational investors’ biases are symmetric, price cannot be equal to its rational value if there are more overconfident traders than underconfident ones.*

Figure 1 illustrates the possible prices as a function of investor confidence in this symmetric case for a range of values for the overconfident traders’ population fraction $\lambda$.

As a second special case, assume that the only biased group are the overconfident, that is $B_d = 1$. In this case, it follows clearly from the above discussion that the price will not be equal to its rational counterpart, as the necessary condition for this to happen, namely (12), becomes $B_c = 1$, while we have $0 < B_c < 1$.

**Observation 2.** *The price cannot attain its rational value if overconfident investors interact with rational ones – it will always overreact to new information, as long as overconfident traders are present in the market.*

Irrational investors will thus affect prices, similarly to the oft-cited model in De Long, Shleifer, Summers and Waldmann (1990)[5]. However, while in their model irrational investors have first-order erroneous beliefs – with respect to an asset’s expected value, our traders only misperceive precision of their information.

### 3.2 Short-Sales Constraints

We now turn to the next part of the analysis, where the presence of short-sales constraints is assumed. Assume for the moment that the signal at $t = 1$ was positive. Whether fully rational or underconfident in the precision of their signals, the $1 - \lambda$ fraction of such investors will be pushed out of the market by the overconfident investors if short-sale constraints are introduced into the model above. This is because the necessary condition for not overconfident traders to stay out of
Figure 1: Price as a function of investor confidence for different values of the overconfident investors fraction in the trader population. The risky asset’s equilibrium price as calculated according to equation (8); the parameter values are: $s_1 = 1$, $\sigma^2 = \sigma^2_F = 1$. Investor confidence is symmetric, i.e. $B_d = 2 - B_c$. The price can attain its rational value only if $\lambda < 1/2$.

the market (this being equivalent to their valuation of the asset being lower than the prevailing price), that is $\alpha_d(F + \epsilon) < P_1$, is fulfilled for all allowed parameter values. The price in this case will be set by the overconfident traders, and it will always lie above the rational price level. On the other hand, let us assume the news at $t = 1$ was adverse and the signal was negative. Since the overconfident – by overestimating the precision of their signal – overreact to information, their valuation will now be below the market price and it is them now, it turn, who will be pushed out of the market by the other traders.

In the presence of short-sales constraints, the demand functions of the two types of investors
have to be revised and will now be given by:

$$D_k = \max \left\{ \frac{\gamma [\alpha_k (F + \epsilon) - P]}{\beta_k}, 0 \right\}$$, for $k = c, d$. \hfill (13)

The implications for prices follow immediately; they are summarized in the following statement:

**Proposition 2.** At $t = 1$, the risky asset price can fall in one of two distinct regions:

1. The price is set by the overconfident traders and the underconfident sit out of the market when the signal is positive. It is given by:

$$P_{1c} = \alpha_c (F + \epsilon) = \frac{\sigma^2_c}{\sigma_F^2 + B^2_c \sigma^2_\epsilon} (F + \epsilon).$$ \hfill (14)

2. The price is set by the underconfident traders and the overconfident sit out of the market when the signal is negative. It is given by

$$P_{1d} = \alpha_d (F + \epsilon) = \frac{\sigma^2_d}{\sigma_F^2 + B^2_d \sigma^2_\epsilon} (F + \epsilon).$$ \hfill (15)

The price will be equal to its unbiased value in a special case when $B_d = 1$, i.e. when there was a negative signal and perfectly rational traders push the overconfident traders out of the market.

3. No matter if the signal is positive or negative, the fraction of overconfident or underconfident investors present in the market has no effect on equilibrium prices: the price $P$ does not depend on $\lambda$.

It is clear that at $t = 1$, $|P_c| > |P_d|$. Prices display asymmetry – they overreact to good signals and underreact to bad ones. This differs from Daniel, Hirshleifer, and Subrahmanyam (1998)[4], where the equilibrium price is symmetric in that it always overreacts to private signals, no matter if good or bad. Figure[2] presents this situation in a simple graphical example.

Let us recall that at $t = 0$, when investors had identical prior beliefs, the price was equal to zero, its prior mean, and notice further that at $t = 2$ the asset value becomes commonly known and thus will trivially be equal to $F$. This, combined with the results of the last proposition, allows us to draw conclusions regarding the short-term behavior of asset prices under short-sales constraints. We thus have the following corollary.

**Corollary 2.** Prices exhibit momentum in the bad news region: $\text{Cov}[(\Delta P_2, \Delta P_1)|\epsilon < 0] > 0$, and reversal in the good news region: $\text{Cov}[(\Delta P_2, \Delta P_1)|\epsilon > 0] < 0$, where $\Delta P_t = P_t - P_{t-1}$ for $t = 1, 2$.\hfill 13
Figure 2: Price behavior as a function of time with differing investor confidence and short-sales constraints. At time $t = 1$, equilibrium price is seen to display overreaction to good signals and underreaction to bad signals. As a consequence, short-run returns are positively autocorrelated in the bad news region, negatively autocorrelated in the good news region.

### 3.3 Outcome-Dependent Confidence

Thus far, investor confidence only mattered at a single date. If there are more trading dates and information events in between the initial time $t = 0$ and the terminal announcement of the asset’s true value, investor confidence might change as a result of actions and their outcomes. In this section, the dynamic counterpart of overconfidence, termed biased self-attribution is assumed to affect investor behavior. To analyze the effects of such behavior, a version of the model of Daniel, Hirshleifer, and Subrahmanyam (1998) is considered, with the major divergence being the presence of short-sales constraints. A few adjustments to the previous model are thus introduced. Assume that the payout of the terminal dividend is postponed until a new date $t = 3$. There is a new public signal $s_2$ at $t = 2$; as in Daniel, Hirshleifer, and Subrahmanyam (1998), it is assumed...
to be pure noise (This modeling device is for tractability purposes and should be understood as the limiting case of the signal being in fact correlated to the fundamental, when the correlation approaches zero). It can be equal to either $-1$ or $+1$; the probability of $s_2 = +1$ is exogenously given. All the traders are short-sales constrained. The underconfident investors do not react to new information. However, if the overconfident investors see a favorable signal at $t = 2$ after they previously received also a positive signal, they update their perceived precision of their $t = 1$ signal further, to $s_1c = F + (B_c - G)\epsilon$, where $0 < G < B_c$. Otherwise nothing changes and the price stays at its $t = 1$ level. If the above happens however, it goes up further to:

$$P_2^c = \frac{\sigma_F^2}{\sigma_F^2 + (B_c - G)^2\sigma_\epsilon^2}(F + \epsilon).$$  \hspace{1cm} (16)$$

In this case the behavior of prices can be illustrated as in Figure 3

![Figure 3: Price behavior as a function of time with outcome-dependent investor confidence and short-sales constraints. Confirming public news causes overconfident investors to update their time-2 valuations of the asset’s value further upward, resulting in short-run positive return autocorrelation in the good news region.](image)

This simple extension of the model considered previously allows us to make some inferences
about the short-run versus long-run behavior of asset prices in case of investor confidence boosted by biased self-attribution. The role of short-sales in producing an asymmetry in prices is again significant. Investigation of the price behavior in the positive news region leads to the discovery of initial positive autocorrelation, followed by a correction caused by the revelation of the asset’s value at \( t = 3 \). As illustrated in Figure 3, the initial \( t = 1 \) overreaction to a favorable private signal is followed by yet more overreaction at \( t = 2 \) brought about by another positive news event. Short-sales constraints thus cause prices to exhibit asymmetry in that they initially overreact to good news increasing too much, but then they converge to the true value in a correction phase; on the other hand, prices underreact to bad news and converge to the true value at the final announcement date. Formally, we summarize the above in the following statement.

**Proposition 3.** If investor confidence changes because of biased self-attribution, prices after good news exhibit initial short-lag positive autocorrelation (“momentum”): \( \text{Cov}[(\Delta P_2, \Delta P_1)] > 0 \), followed by a correction phase: \( \text{Cov}[(\Delta P_3, \Delta P_2)] < 0 \), and long-lag negative autocorrelation (“reversal”): \( \text{Cov}[(\Delta P_3, \Delta P_1)] < 0 \). In the bad news region, prices display long-lag positive autocorrelation: \( \text{Cov}[(\Delta P_3, \Delta P_1)] > 0 \).

### 3.4 Positive Asset Supply

So far we have assumed that the risky asset was in zero net supply. In this section we relax this assumption and allow positive net supply \( X > 0 \). In effect, this will allow both investor types to be present in the market, even in presence of short-sales constraints. The resulting market equilibrium condition thus becomes:

\[
\lambda \frac{\gamma (\alpha_c s - P)}{\beta_c} + (1 - \lambda) \frac{\gamma (\alpha_d s - P)}{\beta_d} = X. \tag{17}
\]

The resulting price can be now written as

\[
P_1 = \left( \hat{\lambda} \alpha_c + (1 - \hat{\lambda}) \alpha_d \right) (F + \epsilon) - \hat{X}, \tag{18}
\]

where

\[
\hat{\lambda} = \frac{\lambda \beta_d}{\lambda \beta_d + (1 - \lambda) \beta_c}
\]

is the adjusted proportion of overconfident investors with \( \hat{\lambda} > \lambda \) and

\[
\hat{X} = \frac{\beta_c \beta_d}{\gamma (\lambda \beta_d + (1 - \lambda) \beta_c)} X.
\]
We can see that \( \hat{\lambda} \) is related to \( \lambda \) in a straightforward manner when we observe that
\[
\frac{\hat{\lambda}}{1 - \hat{\lambda}} = \frac{\lambda}{\beta_c/\beta_d}.
\]  
(19)

Thus while in our benchmark zero-supply model the relative proportions of both confidence-biased types of traders were independent of that investors’ biases (represented by \( \beta_c \) for the overconfident and \( \beta_d \) for the underconfident), in this setting the relative adjusted proportions are a ratio of the actual proportions to the relationship between the confidence biases.

With positive supply \( X \), one can derive the condition necessary for both types of traders to submit positive demands. For the case of a positive \( t = 1 \) signal, the underconfident will not be pushed out of the market if \( \alpha_d s > (\hat{\lambda}\alpha_c + (1 - \hat{\lambda}\alpha_d))s - \hat{X} \), which can be rewritten in terms of the condition on the signal realization as
\[
s < \frac{\hat{X}}{\lambda(\alpha_c - \alpha_d)} = \frac{\beta_c}{\gamma\lambda(\alpha_c - \alpha_d)}X.
\]  
(20)

In the same manner, for the case of a negative signal \( s_1 \), the overconfident traders will stay in the market as long as \( \alpha_c s > (\hat{\lambda}\alpha_c + (1 - \hat{\lambda}\alpha_d))s - \hat{X} \), or
\[
s > -\frac{\hat{X}}{(1 - \hat{\lambda})(\alpha_c - \alpha_d)} = -\frac{\beta_d}{\gamma(1 - \hat{\lambda})(\alpha_c - \alpha_d)}X.
\]  
(21)

Combining the two inequalities above yields a condition necessary for both types of investors to be present in the market. With both trader types present, there will now be a region of the price range, where prices will be symmetric for both negative and positive news: when both the overconfident and the underconfident submit positive demands; and a region with asymmetric prices, where either group of traders occupies the market: the overconfident for the case of a large positive signal, the underconfident for a large negative signal.

**Proposition 4.** With positive asset supply \( X > 0 \) at \( t = 1 \), the risky asset price can fall in one of three distinct regions:

1. The region of signal realizations where two types of traders coexist in the market is characterized by
\[
s^d \equiv -\frac{1}{\gamma(1 - \hat{\lambda})(\alpha_c - \alpha_d)}X < s < \frac{\beta_c}{\gamma\lambda(\alpha_c - \alpha_d)}X \equiv s^u.
\]  
(22)

The risky asset’s price in this case will be given by formula (18), or
\[
P_1 = \frac{\lambda\beta_d\alpha_c + (1 - \lambda)\beta_c\alpha_d}{\lambda\beta_c + (1 - \lambda)\beta_d}(F + \epsilon) - \frac{\beta_c\beta_d}{\gamma(\lambda\beta_d + (1 - \lambda)\beta_c)}X.
\]  
(23)
2. For sufficiently positive news the overconfident investors set the price to

\[ P_1^c = \alpha_c (F + \epsilon) - \frac{X \beta_c}{\lambda \gamma}. \]  \hspace{1cm} (24)

3. For sufficiently negative news the underconfident traders set the price to

\[ P_1^d = \alpha_d (F + \epsilon) - \frac{X \beta_d}{(1 - \lambda) \gamma}. \]  \hspace{1cm} (25)

Note that in contrast to the case with zero asset supply, both the proportion of either type of traders \( \lambda \) as well as the risk tolerance parameter \( \gamma \) now enter the equilibrium price equation.

As before, we also analyze what happens if overconfident investors become even more confident because of a confirming new signal at \( t = 2 \). The price path is similar to the case examined before when we assumed zero asset supply, but only for significant news events. Prices will be symmetric in the intermediate region where short-sales constraints are not binding. Moreover, even if biases in confidence cancel out in the first trading period and the resulting price is equal to the rational case, i.e. if \( \lambda = \beta_c (\beta_d - 1) / (\beta_c - \beta_d) \) holds, biased self-attribution still bites in the second trading period resulting in price overshooting. Figure 4 illustrates all the possible price paths.

The intermediate region of price range with both investor types present will now be narrower than at \( t = 1 \), when no self-attribution bias was present. To see this, note that if investors receive two consecutive positive signals, i.e. \( s_1 > 0 \) and \( s_2 = 1 \), then the range of possible first period signals for which both investors are present in the market at \( t = 2 \) shrinks to

\[ s'_{d} \equiv -\frac{1}{\gamma (1 - \lambda)(\alpha'_c - \alpha_d)} X < s < \frac{1}{\gamma \lambda (\alpha'_c - \alpha_d)} X \equiv s'_{u}, \]  \hspace{1cm} (26)

since the overconfident investors have updated their signal precisions to \( s_{1c} = F + (B_c - G) \epsilon \), with \( 0 < G < B_c \), and this results in \( \alpha'_c > \alpha_c \) and \( \beta'_c < \beta_c \).

Recall that \( \beta_c \) is the perceived variance of the overconfident conditional on observing a signal; it decreases with boosted confidence.

This shrinkage effect implies that some investors with positive holdings in previous trading period will liquidate their positions due to increased confidence of investors in the other group. Moreover, they will do so at the best possible time: one period before the fundamental value is revealed, exactly when the price departs from it the most. This obviously relates to the trading volume as well as to the distribution of profits among various investor groups.
Figure 4: Price behavior as a function of time with outcome-dependent investor confidence and short-sales constraints and positive asset supply. We assume here that condition (11) is satisfied with equality, i.e. the parameters of the model are such that the price is equal to its rational counterpart at $t = 1$ when both investor types submit nonzero demands.

4 Discussion

The main factor behind variability of price paths in our model is the presence of short-sales constraints. While data on which particular stocks are easily available for short-selling is limited, it is a fact that some stocks are difficult to short. For such stocks it is plausible to conclude that their prices are dictated by a limited number of investors, who may not be acting rationally. Shiller (2003, p. 97) in support of this point argues, while recalling the infamous example of 3com sale of Palm:

"Some stocks could be in a situation where zealots have bought into a stock so much that only zealots own shares, and trade is only among zealots, and so the zealots alone determine the price of the stock."

We have shown that obstacles to trading in the form of short-sales constraints can indeed cause stock prices to be determined by such zealots and thereby significantly deviate from fundamental
Our basic models presented in the previous section may serve as a starting point for the analysis of various financial variables of interest, notably market volatility and trading volume.

Since in a few cases that we study above there is only one group of traders present in the market and all of the traders misvalue their (effectively public) signal equally, no trade takes place even though price changes are possible. However, when there are traders with different risky asset valuations present in the market simultaneously, trade does take place and we can analyze the resulting volume. The latter case is possible only in case of positive asset supply in our model. The emergence of the shrinkage effect in the second trading period intuitively suggests some possibly interesting dynamic patterns of volume. On the one hand, some investors with $t = 1$ positive holdings are pushed out of the market at $t = 2$. On the other hand, traders affected by biased self-attribution increase their demands at $t = 2$, but only if confirming news are observed. In any event, positive volume will be observed in the second trading period, in absence of any news directly related to the asset’s fundamentals.

As the price will also change in the example discussed above, it will be more variable than the underlying fundamentals. Thus our model based on differences of opinion due to biases in confidence can possibly shed light on the excess volatility puzzle.

We have emphasized that it is only a special case when prices in our model can be equal to its counterpart in case when rational traders set it. It is nevertheless instructive to investigate the quality of prices and their role in information revelation. While the investors in our model are not fully rational, they do possess legitimate private information and it is thus meaningful to ask to what extent that information is reflected in market prices. Obviously, in presence of short-sales constraints, one immediately notices that some negative information might remain hidden. The asymmetry in prices resulting from different confidence biases suggests that positive news events will be transmitted with amplification, extreme negative information will be incorporated in prices only gradually.

5 Concluding Remarks

Systematic investor irrationality affects asset price behavior in important ways. Complexity and variable quality of information along with market frictions make bounded rationality exert a significant influence on the functionality and viability of financial markets. This paper attempts to
shed some light on these issues. Simple analysis presented in the previous section brings about a few conclusions, summarized below.

- In a market populated by investors affected by biases in confidence, rational risky asset price is a special case, in which the biases of investors – in the form of heterogeneous confidence – cancel each other out. In any other case, prices display either overreaction or underreaction to information. Investors’ misvaluation of a stock is a notable determinant of its price.

- The presence of market frictions in the form of short-sales constraints amplifies the effects of behavioral biases on asset prices: with zero asset supply, the equilibrium price is set unilaterally by either of two groups of investors. Short-run returns display reversal after good news and momentum after bad news. Short-sales constraints thus cause asset prices to display asymmetry in reaction to different information.

- If investor confidence changes because of biased self-attribution, favorable information produces a short-run momentum and long-run reversal in asset returns; negative information causes asset returns to exhibit long-run momentum, gradually bringing the price to its true intrinsic value.

- With positive net asset supply, moderate news events result in both types of traders being active in the market whilst extreme news result in either group of traders being pushed out of the market. The intermediate price region where both investor types are present in the market is subject to the shrinkage effect in subsequent trading period brought about by biased self-attribution.

The models presented in this paper, although admittedly simplistic, serve to convey a simple message that in today’s highly complicated economic environment, the way people form their beliefs and the process of financial decision making cannot be ignored in asset pricing modeling. Overconfidence about abilities to acquire or process information, being the most widely agreed upon behavioral bias among economists, leads investors to believe they can profit from superior information they have. To the extent overconfidence takes the form of biased self-attribution, investors who have profited through a fortunate turn of events will mistakenly attribute that to their own skillfulness and be overconfident about subsequent investments. Investor psychology and the manner in which information is disseminated within the economy are the two chief factors that need to be the focus of behavioral asset pricing research in the coming years; little is as of now
known about the social processes by which people form and transmit ideas about markets and securities. Perhaps with time, and with more understanding of systematic behavioral patterns that creep in when people make investment decisions, individual decision-makers will learn their way out of overconfidence and other psychological biases. For the time being though, asset pricing without modeling individual behavior is doomed to be incomplete.

References


