Flextime, traffic congestion and urban productivity*

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Abstract:

How many firms choose to adopt flextime without any policy intervention? Does promoting flextime improve social welfare? This paper addresses these two questions. We extend the model of bottleneck congestion to describe the case in which some firms in a city adopt flextime. The model also incorporates effects on urban productivity via agglomeration economy. Each firm chooses whether to adopt flextime or not, taking into account the trade-off between productivity and congestion. Equilibrium determines the number of firms adopting flextime and commuters' departure patterns. We investigate the conditions in which flextime is adopted in equilibrium. Moreover, we demonstrate that multiple equilibria with respect to the number of firms adopting flextime may arise. The less efficient solution, the one without flextime, is likely to persist. We also examine the effect of a congestion toll on social welfare.

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1. Introduction

Most firms in cities adopt a fixed work schedule under which all workers start working at the same time, typically at 9:00 a.m. This situation causes concentration of travel demand around the work start time and consequent heavy congestion. Flextime has been advocated as a measure to mitigate traffic congestion during the morning and evening rush hours. In a typical flextime firm, an individual works the same number of hours in a standard workday, but his or her start and finish times may differ from those of other employees. If firms adopt flextime, departure patterns are more dispersed and peak congestion is flattened. The number of firms adopting flextime has been increasing in recent years[1], but its use is still too limited to markedly reduce the level of congestion. This is because firms have little incentive to deviate from the conventional fixed work schedule: they experience decreased productivity by adopting flextime. Note that firms are agglomerated in a central business district (CBD) to allow frequent mutual communication for information exchange, transactions, etc. Such communications are impeded if firms adopt flextime: a worker may fail to contact a specific person at a flextime firm when that worker has not started working[2].

Henderson (1981) incorporated this productivity effect to analyze the equilibrium and optimum solutions with staggered work hours in which each firm starts work at a different time. He described the peak-period congestion using a flow congestion model, in which trip cost (travel time) increases with departure rate (the number of commuters departing at each time). He showed that, in equilibrium, work start times of firms are distributed continuously. Consequently, wages are differentiated continuously according to firms' work start times. This property is not consistent with real world observations: it was reported that work start times are clustered at several points, such as 8:30, 9:00, and 9:30 (see e.g., Wilson (1988)). Moreover, Henderson focused only on the case in which work hours are totally staggered; there was no comparison between situations with and without staggered hours. For this reason, the benefits and costs of shifting from conventional fixed work hours to

[1] In Japan, the percentage of workers employed by firms adopting flextime increased from 2.3% in 1989 to 7.7% in 1998 (Ministry of Labor, Japan (1999)). The situation in the U. S. A. is described by Beers (2000): from information collected in the 1997 supplement to the Current Population Survey, 27.6 % of all full time workers varied their work hours to some degree; other data (1994-1997 the Bureau of Labor Statistics Employment Benefits Survey) show that less than 6 % of employees have a *formal* flexible work schedule arrangement.

[2] On the other hand, it has been argued that flextime increases productivity: workers may increase effort; firms may attract better workers, etc. See, e.g., Shepard, Clifton and Kruse (1996), Gariety and Shaffer (2001). Our model implicitly incorporates the latter effect in that flextime firms can employ equally productive workers at a lower wage.

staggered work hours remain unknown.

The present study focuses on flextime rather than staggered work hours[3]. In this case, each firm is faced with the choice between flextime and the conventional fixed work schedule (9 to 5). This situation is a *discrete choice*, unlike the continuous choice of work start time under staggered work hours[4]. Firms in the city are classified into two types: those adopting flextime (Group 1) and those adopting a fixed work schedule (Group 2). Workers in Group 1 firms can choose their work start times freely; thereby, they can avoid peak-period traffic congestion. We formulate the peak period congestion based on the bottleneck model, which was originally developed by Vickrey (1969) and elaborated by Arnott, de Palma and Lindsey (1990). This formulation for dealing with the problem of peak period congestion is more appropriate than the naive flow congestion model adopted by Henderson (1981): the bottleneck model describes the dynamic process of congestion such that the traffic situation at a given time depends not only on concurrent road users, but also on those entering the road at different times[5]. We extend the standard model of bottleneck congestion to incorporate the case in which only some firms in a city adopt flextime. We formulate the productivity effect as in Henderson (1981): firm productivity at a certain instant depends on the total number of workers who are on duty in the city. With this setting, the productivity of Group 1 firms should be lower than that of Group 2 firms because some flextime workers may start working when the number of workers on duty in other firms is small. The equilibrium number of workers in Groups 1 and 2 are determined endogenously by choices of firms and workers facing a trade-off between congestion and productivity. Our formulation with discrete choice (i.e., to be Groups 1 or Group 2) produces results that differ from those obtained by the continuous choice approach of Henderson (1981). For example, multiple equilibria with respect to group composition of firms in the city may arise: one equilibrium solution involves some firms adopting flextime; the other solution involves no firms adopting flextime. In such a case, the latter solution is likely to persist even though it is less efficient. This persistence

[3] Moss and Curtis (1985) analyzed the effects of flextime on workers' behavior. However, they did not explicitly address the effects on traffic congestion and urban productivity.

[4] Okumura, Kobayashi and Tanaka (1999) recently studied a similar problem of discrete choice in the context of staggered work hours. Unlike Henderson, they assume the existence of only two work start-time alternatives.

[5] In a dynamic setting where traffic flow rate varies continuously over time, travel time depends not only on the number of trips departing at the same time but also those departing at earlier times. The naïve flow congestion model merely takes into account the former effect. In fact, the latter effect is much more important in the dynamic context. Instead, this type of bottleneck model neglects flow congestion, and it also ignores space occupied by the queue. Mun (1999) relaxed the above assumptions and showed that the bottleneck model is derived as a special case of a more general model based on traffic flow dynamics.

pertains because the benefits and costs of adopting flextime for an individual firm depend on the choices of other firms, the number of firms that choose to adopt flextime. We further examine the effects of a peak-load toll to eliminate bottleneck congestion. We demonstrate that the ranking of equilibrium solutions may change: under peak-load toll, the equilibrium that is attainable without flextime may be more efficient than that with flextime.

The paper is organized as follows. Section 2 presents a model framework and describes conditions to determine the equilibrium numbers of firms (workers) in two groups. In equilibrium, firms and workers choose flextime if the private net benefit of Group 1 is larger than that for Group 2. The private net benefit is defined as the output minus commuting costs, which are obtained in Sections 3 and 4. Section 3 formulates a model of traffic congestion and the departure choice of commuters. The commuting cost is obtained by solving the equilibrium departure patterns subject to bottleneck congestion. Section 4 derives formulas to calculate outputs of firms in the two groups. Section 5 investigates properties of equilibrium solutions and evaluates their efficiencies. The effects of peak-load toll are also examined. Section 6 concludes the paper.

2. Model framework

This city consists of a CBD and a residential area. All production takes place in the CBD, to which all workers commute from the residential area using a road that is subject to congestion. It is assumed that firms produce homogenous goods with constant returns to scale technology, and labor is the only input for production. Firms face a competitive labor market and are price takers in the output market. There exist two types of firms in the city: those adopting flextime (Group 1) and those adopting a fixed work schedule (Group 2). All workers have identical skills and preferences and they must work H hours per day. Each firm chooses whether to adopt flextime or a fixed work schedule. Each firm seeks to maximize profit per worker, which is defined as the output per worker minus the wage, as

$$\pi_i = Y_i - w_i , \tag{1}$$

where $i\ (=1,2)$ indicates the group. Y_i, w_i are the daily output per worker and the wage rate in Group i, respectively.

We assume that workers can move freely between firms. Therefore, workers individually choose the type of firm to which they supply labor. Each worker’s objective is to maximize net income, which is defined as wage minus commuting cost, i.e., $w_i - C_i$, where C_i is commuting cost for a worker employed by the Group i firm[6].

Let us denote the numbers of workers in Groups 1 (flextime) and 2 (fixed schedule) by N_1 and N_2, respectively. In addition, $N_1 + N_2 = N$ holds, where N is the total number of workers, given exogenously, in the city[7]. It is useful to classify the equilibrium solutions according to the group composition of firms in the CBD, as follows:

[Case A] All firms adopt a fixed work schedule;

[Case B] All firms adopt flextime;

[Case C] Some firms have a fixed schedule and others have flextime.

Case C is the interior solution; Cases A and B are the corner solutions.

In equilibrium, workers have no incentive to change the type of firm at which they are employed; moreover, no firm has an incentive to change the type of work schedule. For an interior solution in which the number of firms (workers) in both groups is strictly positive, equilibrium requires that the net incomes of workers in both groups are the same, and the profits of firms in both groups are the same. That is,

$$N_1{*} > 0,\ N_2{*} > 0 \ \Rightarrow\ Y_1 - w_1 = Y_2 - w_2 \text{ and } w_1 - C_1 = w_2 - C_2 = w^*, \tag{2}$$

where $N_i{*}$ is the equilibrium number of workers in Group i. The two equations in (2) can be reduced to a single equation as

$$N_1{*} > 0,\ N_2{*} > 0 \ \Rightarrow\ Y_1 - C_1 = Y_2 - C_2\,. \tag{3a}$$

Similarly, we obtain the equilibrium conditions for corner solutions as follows.

$$N_1{*} = N,\ N_2{*} = 0 \ \Rightarrow\ Y_1 - C_1 \geq Y_2 - C_2 \tag{3b}$$

$$N_1{*} = 0,\ N_2{*} = N \ \Rightarrow\ Y_1 - C_1 \leq Y_2 - C_2 \tag{3c}$$

Hereafter, we call $Y_i - C_i$ $(i = 1, 2)$ the ***Private net benefit***. Note that (3a), (3b) and (3c) correspond respectively to Cases C, B and A defined above. Detailed expressions of commuting costs C_i and outputs Y_i are described respectively in Sections 3 and 4 below.

3. Congestion and departure patterns of commuters

[6] C_i is the commuting cost in equilibrium. Note that it is independent of departure time since commuting cost is equalized in equilibrium regardless of departure time, as discussed in Section 3.

[7] N represents the size of the city. If the private net benefit is increased by policies, such as promoting flextime, firms and households migrate into this city from the rest of the world. Consequently the city sizes may differ with and without policy. Allowing variable N requires modeling migration behavior as above, which is beyond the scope of this paper

Suppose a single road connects a residential area and the CBD; the road has a bottleneck just before the CBD. Vehicles are assumed to drive at constant speed from a home to the bottleneck point: travel time for this portion of the trip, T_f, is constant. A queue develops when the traffic flow rate (= departure rate from the residential area) exceeds the bottleneck capacity, *k*. Travel time for a vehicle departing at *t*, $T(t)$, is formulated as

$$T(t) = T_f + \frac{Q(t)}{k} , \tag{4}$$

where $Q(t)$ is the queue length. The second term represents the waiting time within the queue behind the bottleneck. Following Arnott et al. (1990) we set $T_f = 0$, hereafter. This setting does not affect the qualitative results.

The queue length that the trip maker departing at *t* encounters is calculated as

$$Q(t) = \int_{t_q}^{t} [F(s) - k] ds , \tag{5}$$

where t_q is the time when a queue starts to develop, and $F(t)$ is the departure rate at *t*.

Suppose that every morning, *N* individuals (= workers) commute from their residences to offices located in the CBD, driving along the road as stated above[8]. The morning and evening peaks are symmetrical from the assumption that working hours are fixed and identical.

Workers who are employed by firms in Group 2 are assumed to arrive before a specified work start time; late arrivals are not allowed. The commuting cost for these workers comprises the travel time cost and scheduling cost. The scheduling cost is incurred by arriving at the office earlier than the specified work start time; it is the opportunity cost of the waiting time before work[9]. On the other

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[8] We assume that all traffic during the rush hour is commuting, and other types of trips are ignored. In reality, a substantial share of traffic is non-commuting even during peak hours. However, if we assume that the volume of non-commuting traffic is constant during morning rush hours, the results of our model are not affected. Adding constant non-commuting traffic is equivalent to a decrease in traffic capacity. Incorporating departure choice and time variation of non-commuting traffic would make the analysis extremely complicated. To the best of our knowledge, there has been no theoretical model describing the time variation of non-commuting traffic during rush hour, which is an important topic for future research.

[9] Henderson (1981) considered the scheduling cost in a somewhat broader sense: it may be associated with inconveniences engendered by deviations from the best schedule for family activities, etc. The present model is still applicable if $\tilde{t}$ is interpreted as the best time for personal activities other than work. In reality, a fixed work start time and the best time for personal activities do not coincide. It is possible to formulate a model that includes both aspects of scheduling costs, but such a task would require new parameters that would complicate the analysis

hand, each worker in a Group 1 firm (i.e., adopting flextime) can choose the work start time. This implies that the scheduling costs for flextime workers are zero; they incur only travel time cost. Each commuter chooses a departure time so as to minimize commuting cost.

Departure patterns are derived below for three cases – A, B, and C – as classified in Section 2.

[Case A] All firms adopt fixed work schedule.

In this case, the situation is the same as that analyzed by Arnott et al. (1990). It is assumed that all firms adopting the fixed work schedule start working at the same time, $\tilde{t}$. The commuting cost for a worker departing at time *t* is written as

$$C(t) = \alpha T(t) + \beta(\tilde{t} - t - T(t)) \qquad \textit{for } t + T(t) \leq \tilde{t}\ , \tag{6}$$

where α is the monetary value of unit travel time, β is the monetary value of the unit waiting time due to early arrival. We assume $\alpha > \beta$ to obtain well-behaved solutions. This assumption implies that workers prefer to wait at the office rather than spend time in the car, because they can spend their time more usefully in the office than in the car.

Equilibrium is attained when commuters have no incentive to change their departure times. Since commuters are homogeneous, commuting costs must be the same at all times when departures occur. Hence, the equilibrium condition is $\frac{\partial C(t)}{\partial t} = 0$. Applying this condition to (6) and using (4), (5), we have

$$F(t) = \frac{\alpha k}{\alpha - \beta} \qquad (t_1 \leq t \leq t_2), \tag{7}$$

where t_1 and t_2 are the departure times of the first and last commuters, respectively[10]. The first commuter departing at t_1 does not encounter a queue. That commuter incurs only a scheduling cost, so that individual's commuting cost is equal to

$$C(t_1) = \beta(\tilde{t} - t_1)\ . \tag{8}$$

Recalling that equilibrium requires $C(t_1) = C(t)$ for all times, the travel time for an individual departing at *t* is obtained as

$$T(t) = \frac{\beta}{\alpha - \beta}(t - t_1) \qquad (t_1 \leq t \leq t_2), \tag{9}$$

where t_1, t_2 are obtained by solving the following two equations.

without affecting the main results.

[10] In fact, t_1 is equal to t_q in Eq. (2), i.e., the time when a queue starts to develop.

$$N = \int_{t_1}^{t_2} F(t)dt \quad (10)$$

$$t_2 + T(t_2) = \tilde{t} \quad (11)$$

Equation (10) states that total number of commuters who depart between t_1 and t_2 must be equal to *N*. Equation (11) states that the last commuter departing at t_2 must arrive at the work start time, $\tilde{t}$. Solving (10) and (11) yields the following.

$$t_1 = \tilde{t} - \frac{N}{k} \quad (12)$$

$$t_2 = \tilde{t} - \frac{\beta N}{\alpha k} \quad (13)$$

From (12) and (8) we obtain the equilibrium commuting cost as

$$C^A = \frac{\beta N}{k} . \quad (14)$$

Figure 1 illustrates the relation among time, cumulative departures from home, and arrivals at the CBD. Because the departure rate, $\frac{\alpha k}{\alpha - \beta}$, is greater than the arrival rate, *k*, a queue develops behind the bottleneck. The queue length is represented by the vertical distance between the two curves.

Figure 1

[Case B] All firms adopt flextime

Each employee of a firm that adopts flextime chooses a work start time on a day-to-day basis, but they work the same number of hours (= *H*) every day. Commonly, firms specify “core hours” during which all employees must be at work. We assume that all firms adopt the same core hour period, beginning at $\bar{t}'$ and ending at $\bar{t} + H$. Figure 2 illustrates a working schedule in a flextime firm. Note that an employee arriving at the office between $\bar{t}$ and $\bar{t}'$ incurs no scheduling cost. Hereafter, we call that period, [$\bar{t}$, $\bar{t}'$], ***flex-commuting hours***. During flex-commuting hours, the equilibrium condition is that travel time is equalized regardless of the arrival time at the office. On the other hand, an employee who arrives at a time *t* that is earlier than $\bar{t}$ incurs a scheduling cost that is equal to $\beta(\bar{t} - t - T(t))$. For this period, the equilibrium departure rate is identical to that in Case A.

Figure 2

Forms of departure distributions depend on the total number of workers (=commuters), N, road capacity, k, and the length of flex-commuting hours, $(\bar{t}'-\bar{t})$. When flex-commuting hours are sufficiently long for all commuters to pass through a bottleneck within the time interval [$\bar{t}$, $\bar{t}'$], i.e., $N \le k(\bar{t}'-\bar{t})$ is satisfied, a queue does not exist in equilibrium: all commuters can find departure times that avoid queuing and scheduling costs. On the other hand, when $N > k(\bar{t}'-\bar{t})$, it is infeasible for all commuters to pass through the bottleneck within [$\bar{t}$, $\bar{t}'$]. Since late arrivals are not allowed, some workers should arrive at the office before $\bar{t}$. Those workers must wait to start working until $\bar{t}$; thereby, they incur the scheduling cost. Consequently, those workers arriving between $\bar{t}$ and $\bar{t}'$ must encounter a queue so that equilibrium attains.

This study specifically addresses only the former case, in which a queue is not formed while flextime workers commute[11]. In this case, both travel time cost and scheduling cost are equal to zero. Consequently, the equilibrium commuting cost is zero: $C^B = 0$. Infinite possibilities of departure distributions exist such that all commuters arrive within [$\bar{t}$, $\bar{t}'$] and departure rates do not exceed k throughout the period. Although the departure pattern of a particular day is indeterminate, we assume a uniform distribution of departures throughout flex-commuting hours, as

$$F(t) = \frac{N}{\bar{t}'-\bar{t}}. \tag{15}$$

The above specification is not unrealistic: the uniform departure distribution is actually attained in equilibrium if flow congestion is introduced. Flow congestion is defined as the situation in which travel time depends solely on the flow rate (=departure rate) as long as the flow rate does not exceed the bottleneck capacity. Therefore, uniform departure distribution under flow congestion is compatible with the equilibrium condition that the travel time cost is constant regardless of departure time. Alternatively, (15) can be interpreted as the average of infinitely many departure distributions. We do not require specification of a departure distribution at this stage because the equilibrium commuting cost is zero in all cases. Nevertheless, we will use this specification in Section 4 to calculate the productivity of firms.

[Case C] Some firms have flextime and others have a fixed schedule.

We derive the equilibrium departure patterns when both groups of workers use a road during the morning rush hour.

Let $\tilde{t}$ be the work start time for workers in Group 2. We assume that $\bar{t} < \tilde{t} < \bar{t}'$. As in Case B,

[11] Equilibrium departure patterns when a queue forms during flex-commuting hours are presented in Mun and Yonekawa (2004).

we focus on the case in which flextime workers do not encounter a queue. This implies that either $(N_1 + N_2) \le k(\bar{t}' - \bar{t})$ or $N_1 \le k(\bar{t}' - \tilde{t})$ is satisfied. In this setting, the two patterns depicted in Fig. 3 are possible in equilibrium[12]. In Case C1, Group 1 workers use the road for $\bar{t} \le t \le t_1$ and $\tilde{t} \le t \le \bar{t}'$, whereas Group 1 workers use the road only for $\tilde{t} \le t \le \bar{t}'$ in Case C1. In both cases, workers in Groups 1 and 2 never use the road simultaneously because equilibrium conditions for the two groups concerning travel time variations are mutually incompatible. As explained for Case B, the equilibrium condition for Group 1 workers is that travel time costs should be constant regardless of the departure time because they do not incur the scheduling cost. On the other hand, because Group 2 workers must incur the scheduling cost, travel time cost must increase with time to attain the equilibrium. Moreover, as shown in Fig. 3, Group 2 workers use the road exclusively for a period just prior to $\tilde{t}$. Otherwise, both groups of workers have incentives to change their departure times, which is incompatible with equilibrium.

Figure 3

Whether Case C1 or C2 emerges depends on the relation between the earliest departure times of Group 1 and Group 2 workers, $\bar{t}$ and t_1, respectively. Case C1 (Case C2) emerges if $\bar{t}$ is earlier (later) than t_1. The earliest and latest departure times of Group 2 workers, t_1, and t_2, are obtained as

$$t_1 = \tilde{t} - \frac{N_2}{k}, \text{ and} \tag{16a}$$

$$t_2 = \tilde{t} - \frac{\beta N_2}{\alpha k}, \tag{16b}$$

which correspond to Eqs. (12) and (13) in Case A. The conditions in which Cases C1 and C2 emerge in equilibrium are as follows.

$$\text{Case C1: } N_2 \le k(\tilde{t} - \bar{t}) \text{ and } N \le k(\bar{t}' - \bar{t}) \tag{17a}$$

$$\text{Case C2: } N_2 > k(\tilde{t} - \bar{t}) \text{ and } N_1 \le k(\bar{t}' - \tilde{t}) \tag{17b}$$

Based on the above analysis, relations between departure patterns and parameters, k, $\bar{t}$, $\bar{t}'$, $\tilde{t}$, N_1 and N_2 are illustrated in Fig. 4.

Figure 4

[12] Mun and Yonekawa (2004) show that four patterns of equilibrium departure distributions exist under Case C, including the situation where a queue is formed during flex-commuting hours. They describe the conditions under which respective patterns emerge as equilibrium solutions.

Group 1 workers incur neither the scheduling cost nor travel time cost: they do not encounter the queue. In other words, the equilibrium commuting cost is equal to zero, i.e., $C_1^C = 0$.

On the other hand, the equilibrium commuting cost for Group 2 is

$$C_2^C = \frac{\beta N_2}{k} \quad . \tag{18}$$

It is immediately apparent that $C_1^C < C_2^C$. In other words, the commuting cost for flextime workers is lower than that of fixed schedule workers. Recall the definition, $N_2 = N - N_1$. Then, Eq. (18) implies that the commuting cost for a worker of Group 2 firms decreases as more firms shift from a fixed work schedule to flextime. This is an externality effect: Group 2 workers enjoy lower commuting costs thanks to other firms' adoption of flextime.

Departure rates of Group 1 workers for Cases C1 and C2 are obtained by a similar procedure to that in Case B (Eq. (15)) as

$$\text{Case C1: } F(t) = \frac{N_1}{\bar{t}' - \bar{t} - \dfrac{N_2}{k}}, \text{ for } \bar{t} \le t \le t_1 \text{ or } \tilde{t} \le t \le \bar{t}' \tag{19a}$$

$$\text{Case C2: } F(t) = \frac{N_1}{\bar{t}' - \tilde{t}}, \text{ for } \tilde{t} \le t \le \bar{t}' . \tag{19b}$$

Because these departure rates are lower than k, they coincide with the arrival rate at the CBD.

The departure rate of Group 2 workers is identical to that in Case A; it is obtained by Eq. (7).

4. Effects of flextime on productivity

Firms concentrate in the CBD to exploit agglomeration economies that are associated with communication with numerous firms. Many urban economic models have been built assuming that the productivity of a firm increases with the city size in which it locates. Henderson (1981) extended this type of model to incorporate a time dimension. That model assumes that the productivity of a firm at a given time increases with the number of workers on duty. We apply this approach to analyze the effects of flextime on urban productivity.

We assume that there are many small firms in the CBD and that each firm has identical technology with constant returns to scale. Output per worker at time t is represented as an instantaneous production function of the form

$$y(t) = g(n(t))a , \tag{20}$$

where a is a constant representing technology, $n(t)$ is the total number of workers on duty in the

CBD at time t, and $g(n(t))$ is a shift factor representing agglomeration economy. In that equation, $g' > 0$ are assumed.

As in the previous section, we describe the production of a firm for the three cases – A, B and C – defined in Section 2.

[Case A] All firms adopt a fixed work schedule.

In this case, all workers start working at $\tilde{t}$ and finish at $\tilde{t} + H$: the number of workers on duty in the city is equal to *N* throughout the day. Output per worker for one day, Y^A, is calculated by integrating the instantaneous production function from $\tilde{t}$ to $\tilde{t} + H$.

$$Y^A = \int_{\tilde{t}}^{\tilde{t}+H} a \cdot g(N)dt = aHg(N) \tag{21}$$

[Case B] All firms adopt flextime.

In flextime firm, each employee starts working upon arrival at the office. Therefore, the number of workers on duty, and consequently the output per worker, varies over time. Eq. (20) describes that the productivity of a firm depends on the number of workers in other firms in the city. Introducing flextime has another effect on the productivity that depends on the number of workers that are simultaneously present within the same firm. Eq. (20) is valid if all workers within the firm are present. If some workers are not present during flex-commuting hours, the productivity of a flextime firm is decreased for two reasons: first is the direct effect of the decrease in labor input; second is the indirect effect such as difficulties in collaborative activities, limitation of time for meetings, etc. Let us assume that this intra-firm productivity effect is a function of the proportion of workers on duty at time *t*, which is equal to $\frac{n(t)}{N}$ on average. Therefore, the instantaneous production function in Eq. (20) is modified for a flextime firm as $g(n(t))h\left(\frac{n(t)}{N}\right)a$, where $h(\bullet)$ represents the intra-firm productivity effect, and $h' > 0,\ h(1) = 1$ are assumed.

Based on the above discussion, the output per worker for one day, Y^B, is obtained as

$$Y^B = \int_{\bar{t}}^{\bar{t}'} \left\{ g(n(t))h\left(\frac{n(t)}{N}\right)a \right\} dt + (\bar{t} + H - \bar{t}')g(N)a + \int_{\bar{t}+H}^{\bar{t}'+H} \left\{ g(n(t))h\left(\frac{n(t)}{N}\right)a \right\} dt \,. \tag{22}$$

The first term of the RHS is output during morning rush hours when workers are arriving. The second term is output during core hours when all workers are on duty. The third term is output during evening hours when workers are leaving the office to go home.

As stated in Section 3, we only consider the situation: $N \le k(\bar{t}' - \bar{t})$. In this case, during flex-commuting hours, the arrival rate of employees at the office is equal to the departure rate given by Eq. (15). The number of workers on duty for the morning rush hours is calculated by integrating the departure rate given as (15), that is

$$n(t) = \int_{\bar{t}}^{t} F(t)dt = \frac{N}{\bar{t}'-\bar{t}}(t-\bar{t}), \quad \text{for } \bar{t} \le t \le \bar{t}'. \tag{23a}$$

In the evening, when workers are leaving from offices,

$$n(t) = N - n(t-H) = \frac{N}{\bar{t}'-\bar{t}}(\bar{t}'+H-t), \quad \text{for } \bar{t}+H \le t \le \bar{t}'+H. \tag{23b}$$

[Case C] Some firms are fixed schedule and others are flextime.

Let us denote by $n_1(t),\ n_2(t)$ the total number of workers of Groups 1, 2, respectively, who are on duty at time *t*. The values of output per worker for two groups of firms are calculated by

$$Y_1^C = \int_{\bar{t}}^{\bar{t}'+H} \left[g\left(n_1(t)+n_2(t)\right) h\left(\frac{n_1(t)}{N_1}\right) a \right] dt, \tag{24a}$$

$$Y_2^C = \int_{\tilde{t}}^{\tilde{t}+H} \left[g\left(n_1(t)+n_2(t)\right) a \right] dt. \tag{24b}$$

where $Y_1^C,\ Y_2^C$ are daily output in a firm of Group 1, 2, respectively. Note that the integral intervals of (24a) and (24b) are different. In Group 1 firms (flextime), employees start working as soon as they arrive at their offices. Because they arrive at offices between $\bar{t}$ and $\bar{t}'$ in the morning and leave their offices between $\bar{t}+H$ and $\bar{t}'+H$ in the evening, the integral interval for Group 1 should be between $\bar{t}$ and $\bar{t}'+H$. On the other hand, all employees of Group 2 firms (fixed schedule) are on duty between $\tilde{t}$ and $\tilde{t}+H$, even though they arrive at offices before $\tilde{t}$ and stay after $\tilde{t}+H$.

Forms of $n_1(t),\ n_2(t)$ depend on the departure patterns, which are derived as Cases C1 and C2 in the last section. Details are described in Appendix A.

5. Analysis of equilibrium solutions

5-1 Corner solutions

We first investigate the corner solutions, Cases A and B, defined by Eqs. (3c) and (3b), respectively, to examine whether the situation in which firms adopt flextime is realized as an equilibrium solution. We obtain some analytical results for these corner solutions.

We assume $\tilde{t} = \frac{\bar{t}'+\bar{t}}{2}$, implying that flex-commuting hours spread symmetrically around the starting times of Group 2 firms. We observe that this assumption applies to many flextime practices in the real world. A typical example is that Group 1 (flextime) workers start working at any time between 8:00 and 10:00, whereas conventional Group 2 workers start at 9:00. Furthermore, we specify the forms of functions representing the productivity effects, as follows[13]

$$g(x) = x^{\gamma} \qquad 0 < \gamma \leq 1. \tag{25a}$$

$$h(x) = x \tag{25b}$$

Applying these specifications to (21), (22), we obtain output levels for Cases A and B, as

$$Y^{A} = a \cdot N^{\gamma} H \tag{26a}$$

$$Y^{B} = a \cdot N^{\gamma}(H + \bar{t} - \bar{t}') + \frac{2a \cdot N^{\gamma}(\bar{t}' - \bar{t})}{(2+\gamma)}. \tag{26b}$$

It follows immediately that

$$Y^{A} - Y^{B} = \frac{a \cdot \gamma \cdot N^{\gamma} \cdot (\bar{t}' - \bar{t})}{2+\gamma} > 0.$$

In other words, the output level under flextime is lower than that under a fixed work schedule.

We first examine the situation of Case A, in which all firms adopt a fixed work schedule. Evaluating $(Y_1 - C_1) - (Y_2 - C_2)$ at $N_1 = 0,\ N_2 = N$, we obtain the following relations (see Appendix B for its derivation):

if $N < k(\tilde{t} - \bar{t})$,

$$(Y_1 - C_1) - (Y_2 - C_2) < 0 \quad \Leftrightarrow \quad \frac{\beta N}{k} - \frac{a(\bar{t}' - \bar{t})(\bar{t}' - \bar{t} - 2N/k)}{4(\bar{t}' - \bar{t} - N/k)} N^{\gamma} < 0 \tag{27a}$$

if $N \geq k(\tilde{t} - \bar{t})$,

$$(Y_1 - C_1) - (Y_2 - C_2) < 0 \quad \Leftrightarrow \quad \frac{\beta N}{k} - \frac{1}{2} a(\bar{t}' - \tilde{t}) N^{\gamma} < 0 \tag{27b}$$

$(Y_1 - C_1) - (Y_2 - C_2) < 0$ is the condition that situation $N_1 = 0,\ N_2 = N$ is sustained as an equilibrium solution. Conversely, private net benefit is increased by deviating from this situation (i.e., by adopting flextime) if parameters values are such that the inequality $(Y_1 - C_1) - (Y_2 - C_2) > 0$ holds. In this case, $N_1 = 0,\ N_2 = N$ is no longer in equilibrium. Therefore, firms have an incentive to change voluntarily from a fixed schedule to flextime. We explored the possibility of this case and obtained the following proposition.

[13] The specification of $h(x)$ as (25b) implies that we only consider the direct effect of input changes.

Proposition 1 *Suppose initially that all firms adopt a fixed work schedule:* ($N_1 = 0,\ N_2 = N$). *Firms have more of an incentive to deviate from this initial situation by shifting to flextime, if (a) the opportunity cost of early arrival,* β, *is larger, (b) the bottleneck capacity, k, is smaller, (c) the agglomeration effect,* γ, *is smaller, and (d) the total number of workers, N, is larger.*

We next address another corner solution: Case B, in which all firms adopt flextime. We examine whether the equilibrium condition (3b) is satisfied or not. It turns out that (see Appendix C for its derivation), when $N_1 = N,\ N_2 = 0$,

$$(Y_1 - C_1) - (Y_2 - C_2) = \frac{a \cdot N^{\gamma} (\bar{t}' - \bar{t})\ (2 + \gamma - 2^{\gamma+1})}{2^{\gamma-1}(2+\gamma)(1+\gamma)} < 0 . \tag{28}$$

The above inequality implies the following.

Proposition 2 *Under the assumption,* $\tilde{t} = \dfrac{\bar{t}' + \bar{t}}{2}$, *the situation in which all firms adopt flextime,* ($N_1 = N,\ N_2 = 0$), *is never realized as an equilibrium solution.*

Combining Propositions 1 and 2, we conclude that, if inequality (27) does not hold, an interior solution exists in which some firms adopt flextime.

5-2 Patterns of equilibrium solutions

In the case of interior solutions, such as Cases C1 or C2, as shown in Sections 3 and 4, formulas to calculate the outputs and commuting costs of the two groups are too complicated to solve analytically. Therefore, we investigate patterns of interior solutions by means of numerical simulations.

Parameter values for numerical analyses are given as follows.

β=1900, $\tilde{t}$ =9, $H = 8$, $\bar{t} = 8.5$, $\bar{t}' = 9.5$, $k = 3000$, $a = 100$

The above figures imply that each employee works 8 hours per day, firms adopting a fixed work schedule start at 9:00, and flex-commuting hours are from 8:30 to 9:30.

Figures 5(a)–5(c) show possible patterns of equilibrium solutions for $N = 2000$. In those figures, the number of workers in Group 1 (Group 2) is measured from the right (left) end of the horizontal axis. Two curves in each figure are the loci of private net benefits of Groups 1 and 2, for each combination of $(N_1,\ N_2)$. Note that the private net benefit curve for Group 1 kinks at $N_2 = 1500$. This occurs because the commuting pattern changes at that point. Under the setting of the parameter values shown above, Case C1 emerges for $N_2 \le 1500$, whereas Case C2 emerges for $N_2 > 1500$ (see

(17) in Section 2). Arrows indicate the directions of adjustments toward equilibrium[14]. In Pattern 1 (Fig. 5(a)), the private net benefit of Group 2 exceeds that of Group 1 for the entire range of $(N_1,\ N_2)$. For that reason, the equilibrium obtains at the right end, where all firms adopt a fixed work schedule. Two stable equilibrium points exist in Pattern 2 (Fig. 5(b)). The interior solution attains a higher benefit level than the corner solution at the right end. In other words, welfare is increased by inducing firms to adopt flextime. Market forces adjust themselves toward this interior solution if the initial point is located on the left hand side of point S in Fig. 5(b). Compelling government offices to adopt flextime may be an effective policy measure to move the initial point toward the left. In Pattern 3 (Fig. 5(c)), a unique equilibrium is obtained at the interior point where both Group 1 and 2 workers coexist. Note that equilibrium is never attained at the left end, as shown by Proposition 2.

Economic efficiencies of equilibrium solutions are evaluated by the social surplus, which is defined as the sum of the net benefit, as

$$S\left(N_1, N_2\right) = N_1(Y_1 - C_1) + N_2(Y_2 - C_2)\,. \tag{29}$$

Note that $N_1 = N - N_2$. Let us differentiate $S(N - N_2, N_2)$ with respect to N_2 and evaluate at $N_2 = 0$, then we have

$$\left.\frac{dS}{dN_2}\right|_{N_2=0} = \frac{2^{-\gamma}(2^{1+\gamma} - 2 - \gamma)\left\{k(\bar{t}\,' - \bar{t}) - N\right\} a N^{1+\gamma}}{k(2+\gamma)} > 0$$

The above inequality implies the following.

Proposition 3 *The value of social surplus is never maximized in the situation in which all firms adopt flextime*, ($N_1 = N,\ N_2 = 0$).

Proposition 3 shows that there are three patterns of social surplus variation for each combination of $(N_1,\ N_2)$ as in Figure 6, each of which is described as follows:

Pattern R: social surplus is maximized when all firms adopt a fixed work schedule

Pattern M: social surplus is maximized at $N_2 = \dfrac{k(\bar{t}\,' - \bar{t})}{2}$ where the commuting pattern changes,

Pattern L: social surplus is maximized at an intermediate point between $N_2 = 0$ and $N_2 = \dfrac{k(\bar{t}\,' - \bar{t})}{2}$.

[14] $\dot{N}_1 > 0$ (and $\dot{N}_2 < 0$), if $(Y_1 - C_1) - (Y_2 - C_2) > 0$, and vice versa.

Figure 5

Figure 6

Figure 7 depicts relations between parameters and patterns of equilibrium solutions. For example, within the area shown as 1-R, the equilibrium solution corresponds to Pattern 1, and the form of social surplus is Pattern R. In other words, equilibrium is attained at the point where all firms adopt a fixed work schedule, and the social surplus is maximized in that situation[15]. This pattern emerges when the agglomeration effect, γ, is larger and the total number of workers, N, is smaller. This result is consistent with Proposition 1. Outside area 1-R, equilibrium solutions do not coincide with those maximizing the social surplus. Note that Patten M is never realized if $N < \frac{k(\bar{t}' - \bar{t})}{2}$ (=1500, in Figure 7), since only Case C1 emerges: changes in commuting patterns do not occur. In most cases, adoption levels of flextime in equilibrium tend to be smaller than the efficient levels. Exceptions are 2-M and 3-M, where the share of flextime workers in equilibrium is larger than those maximizing the social surplus.

Figure 7

Figure 8(a) depicts the private net benefits of two groups for various values of scheduling cost parameters, β. Positions of equilibrium points are indicated by ● in the figure. We postulate that the value of β increases as time becomes increasingly valuable in human life, such as productivity increase in the process of economic growth. Presume that the value of β is very low initially, say β=1500. At this stage, equilibrium attains when all firms adopt the conventional fixed work schedule. As β increases, the private net benefit curve of Group 2 tilts clockwise, whereas that of Group 1 is unchanged. Group 1 workers incur no scheduling cost in equilibrium. When β exceeds a certain level (some value between 1600 and 1800), another equilibrium point emerges around $N_2 = 1400$ (indicated by ◌). Figure 8(b) shows that the social surplus for the newly emerged interior equilibrium point is larger than that for the initial equilibrium point at the right end. However, this interior equilibrium point is not realized until β reaches a sufficiently large value exceeding 2300

[15] The share of flextime workers maximizing social surplus as shown in Figure 6 does not necessarily coincide with the socially optimal level, since congestion externality and agglomeration economy are not internalized.

because the initial equilibrium is at the right end. This fact suggests the necessity of intervening to induce the move of the initial solution leftward. Note that the jump of equilibrium occurs from the right end to the interior point when the value of parameter β exceeds the threshold level.

Figure 8

5-3 Effects of a peak-load toll

When some firms adopt flextime, those firms' employees adjust their departure (work start time) to avoid congestion. As a consequence, traffic congestion decreases. However, from Proposition 2, equilibrium never attains a situation in which all firms adopt flextime. Moreover, queuing remains when workers of Group 2 firms commute. Therefore, we require congestion pricing to eliminate the welfare loss of queuing. Below, we examine the effects of a peak-load toll on economic welfare and its implications.

An optimal peak-load toll induces a departure rate equal to the bottleneck capacity, *k*, to prevent queue formation. As in Arnott, de Palma and Lindsey (1990), the optimal toll at time *t*, $\tau(t)$ should be

$$\tau(t) = \beta(t - t_1), \quad (t_1 \le t \le \tilde{t}), \tag{30}$$

where t_1 is obtained by (16), i.e., the time at which the first commuter of Group 2 departs. Under the above toll schedule, the rate of arrivals to the CBD is equal to the bottleneck capacity, *k*, which is the same as the rate without the toll. This equality implies that the tolls do not affect productivity. Furthermore, as Arnott et al. showed, commuting costs are also unchanged by introduction of the toll. Therefore, a peak-load toll affects neither the private net benefits of both groups nor the equilibrium solutions. It only affects the social surplus: the society gains toll revenue that is equal to the welfare loss of congestion in the case without the toll. The total toll revenue, ρ, is obtained as

$$\rho = \int_{t_1}^{\tilde{t}} \tau(t) \cdot k dt = \int_{t_1}^{\tilde{t}} \beta\,(t - t_1) k dt = \frac{\beta N_2^2}{2k}. \tag{31}$$

Figure 9 describes the effects of peak-load toll on the social surplus for three patterns of equilibrium. In each figure, ρ is equal to the vertical difference between the two curves. The positions of equilibrium solutions are indicated by ●.

Figure 9

From (31), the amounts of social surplus gains are larger as the number of Group 2 workers, N_2, increases. Therefore, gains from a peak-load toll are relatively large if the number of flextime workers in equilibrium is small, such as in Fig. 9(a). In cases of multiple equilibria, such as in Fig. 9(b), the ranking of solutions in terms of social welfare differs between cases with and without a toll. With a peak-load toll, the equilibrium solution at the right end, the situation in which no firm adopts flextime, attains a higher level of social welfare than the situation in which some firms adopt flextime. This result implies that policies to promote the adoption of flextime may be unnecessary under a peak-load toll. In other words, flextime and a peak-load toll are substitute policies in some circumstances. In the case of Fig. 9(c), in which the adoption rate of flextime is high, the peak-load toll improves the value of social surplus only slightly. This result is reasonable because the high adoption of flextime reduces congestion level. In this case, introduction of the peak-load toll may reduce the social welfare if the administrative costs for toll collection are great[16].

Finally, it should be noted that, as stated in footnote 15, internalizing agglomeration economy is required to attain the social optimum.

6. Conclusions

This paper presents a formal economic analysis of flextime. We develop a model that describes the choices of firms and workers concerning the adoption of flextime, taking into account the trade-off between traffic congestion and the negative effect on productivity. The main results are summarized as follows.

(1) There exist equilibrium in which some firms adopt flextime, when the cost of traffic congestion is large relative to firms' dependence on agglomeration economies. A situation in which all firms adopt flextime never emerges as equilibrium.

(2) The number of firms adopting flextime in equilibrium tends to be smaller than the socially efficient level, although there may be exceptional circumstances where adoptions of flextime are excessive.

(3) Multiple equilibria may arise: one with a positive number of firms adopting flextime and another without adoption. Although the former solution attains higher social welfare, it is not realized unless the proportion of flextime workers exceeds a threshold level. This fact suggests the necessity of policy intervention to increase the proportion of flextime workers to a certain level,

[16] Seila and Wilson (1991) analyze the effect of the collection costs that arise when vehicles must stop to pay the toll at the gates. They show that peak-load toll may reduce the social welfare.

such as introducing flextime in government offices, or subsidizing firms that adopt flextime, etc.

(4) The peak-load toll to eliminate congestion may be a substitute policy of flextime: the peak-load tolls are more effective when the proportion of flextime workers is lower.

We introduce a number of assumptions to simplify the analysis. Let us discuss below some possibilities of relaxing assumptions in future works. First, each firm may choose its own core hours, instead of assuming the same core hours. Casual observations suggest that core hours do not vary among firms. If the length of core hours is fixed, it is conjectured that firms would choose the same core hours in equilibrium. Since we suppose the situation that flextime workers do not experience congestion, there is no reason that firms would choose different core hours: agglomeration economy is maximized when all firms choose the same core hours. However, if the length of core hours is variable, it is difficult to predict the consequences. Next, we primarily focus on the productivity effect associated with agglomeration economy, so do not sufficiently take into account the productivity effect within a firm. Flextime also negatively affects the productivity within the firm, e.g., teamwork is hampered, possibilities of meetings are limited, etc. Firm size should be considered in dealing with activities within the firm. So the assumption of constant returns to scale is to be relaxed. A related point is as follows. We should address the discussions in the literature of labor economics (Shepard, Clifton, Kruse (1996), Gariety, Shaffer (2001)) suggesting that the positive effect of flextime on firm's productivity exceeds the negative effect. If this is true, there must be no bad news for firms adopting flextime. So why is the number of firms' adopting flextime so small? Investigating the activities within a firm in more detail may help to solve this puzzle.

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Appendix A: Total number of workers on duty in two groups at each time

Case C1:

Figure 3(a) shows that arrivals of Group 1 workers occur during $[\bar{t}, t_1]$ and $[\tilde{t}, \bar{t}']$ at the rate described by Eq. (19a). They start working as soon as they arrive. Therefore, the total number of Group 1 workers on duty at a given time *t* within flex-commuting hours is calculated by integrating the departure rate (=arrival rate) from $\bar{t}$ to *t*, as follows

$$n_1(t) = \int_{\bar{t}}^{t} F(s)ds \,. \tag{A1}$$

Note that $n_1(t)$ is constant (i.e., $F(t) = 0$) for $[t_1, \tilde{t}]$ because Group 2 commuters use the road exclusively during that period. Next, $n_1(t) = N_1$ for core-hours, $[\bar{t}', \bar{t} + H]$. In summary, $n_1(t)$ is obtained by

$$n_1(t) = \begin{cases} \dfrac{N_1}{\bar{t}' - \bar{t} - \frac{N_2}{k}}(t - \bar{t}) & (\bar{t} < t < \tilde{t} - \dfrac{N_2}{k}) \\ \dfrac{N_1}{\bar{t}' - \bar{t} - \frac{N_2}{k}}(\tilde{t} - \dfrac{N_2}{k} - \bar{t}) & (\tilde{t} - \dfrac{N_2}{k} < t < \tilde{t}) \\ \dfrac{N_1}{\bar{t}' - \bar{t} - \frac{N_2}{k}}(t - \dfrac{N_2}{k} - \bar{t}) & (\tilde{t} < t < \bar{t}') \\ N_1 & (\bar{t}' < t < \bar{t} + H) \\ \dfrac{N_1}{\bar{t}' - \bar{t} - \frac{N_2}{k}}(\bar{t}' + H - \dfrac{N_2}{k} - t) & (\bar{t} + H < t < \tilde{t} + H - \dfrac{N_2}{k}) \\ \dfrac{N_1}{\bar{t}' - \bar{t} - \frac{N_2}{k}}(\bar{t}' - \tilde{t}) & (\tilde{t} + H - \dfrac{N_2}{k} < t < \tilde{t} + H) \\ \dfrac{N_1}{\bar{t}' - \bar{t} - \frac{N_2}{k}}(\bar{t}' + H - t) & (\tilde{t} + H < t < \bar{t}' + H) \end{cases} \tag{A2}$$

On the other hand, Group 2 workers arrive during $[t_1, \tilde{t}]$, but start working in unison at $\tilde{t}$.

$$n_2(t) = \begin{cases} 0 & (\bar{t} < t < \tilde{t}) \\ N_2 & (\tilde{t} < t < \tilde{t} + H) \\ 0 & (\tilde{t} + H < t < \bar{t}' + H) \end{cases} \tag{A3}$$

Case C2:

Following the same procedure as in the above case, we obtain the following.

$$n_1(t)=\begin{cases}\dfrac{N_1}{\bar{t}'-\tilde{t}}(t-\tilde{t}), & (\tilde{t}\le t\le \bar{t}')\\ N_1, & (\bar{t}'<t\le \tilde{t}+H)\\ \dfrac{N_1}{\bar{t}'-\tilde{t}}(\bar{t}'+H-t), & (\tilde{t}+H<t\le \bar{t}'+H)\end{cases} \tag{A4}$$

$$n_2(t)=\begin{cases}N_2 & (\tilde{t}<t<\tilde{t}+H)\\ 0 & (\tilde{t}+H<t<\bar{t}'+H)\end{cases} \tag{A5}$$

Appendix B: Derivation of (27)

When all firms adopt a fixed work schedule, the output per worker in a Group 2 firm is given as Y^A in (26a). The commuting cost is C^A in (14). Thus, the Group 2 firm private net benefit is

$$Y_2-C_2=aN^{\gamma}H-\frac{\beta N}{k}.$$

Suppose that one firm deviates from this situation to adopt flextime. We assume that the size of each individual firm is negligible. For that reason, $n_1(t),\ n_2(t)$ is not affected by that deviation. Employees of this firm do not necessarily start working from $\tilde{t}$, but may start earlier or later than $\tilde{t}$. They never use the road while fixed schedule workers do use the road, $\left[\tilde{t}-N/k,\tilde{t}\right]$, because a queue exists during this period. Instead they choose their departure time so that they do not encounter the queue. This implies that $C_1=0$. As explained in Section 3, there are two departure distributions depending on parameter values: Cases C1 and C2. Difference in departure distributions does not affect the commuting cost, but it affects the daily output by influencing the presence of employees.

For Case C1 ($N<k(\tilde{t}-\bar{t})$), substituting (A2) (A3) into (21a) and applying $N_1=0,\ N_2=N$ to the resulting expression, we obtain

$$Y_1=aN^{\gamma}H-\frac{a(\bar{t}'-\bar{t})(\bar{t}'-\bar{t}-2N/k)}{4(\bar{t}'-\bar{t}-N/k)}N^{\gamma}.$$

Substituting the above results to $(Y_1-C_1)-(Y_2-C_2)$ yields (27a).

(27b) for Case C2 can be derived similarly.

Appendix C: Derivation of (28)

When all firms adopt flextime, the output per worker in a Group 1 firm is given by Y^B in (26b). In addition, the commuting cost is zero. Therefore, the private net benefit per worker is obtained as

$$Y_1 - C_1 = Y^B = a \cdot N^{\gamma}(H + \bar{t} - \bar{t}') + \frac{2a \cdot N^{\gamma}(\bar{t}' - \bar{t})}{(2+\gamma)}. \quad \text{(C1)}$$

Presume that one firm deviates from this situation to adopt a fixed schedule. The commuting cost for employees of this firm is still zero because all other firms adopt flextime and departures are dispersed. Employees of this firm start working at $\tilde{t}$ and finish at $\tilde{t} + H$, whereas the number of workers on duty in the city as a whole, $n(t)$, is given by (23a) and (23b). Therefore, the daily output of this firm is calculated as follows.

$$\begin{aligned} Y_2 &= \int_{\tilde{t}}^{\bar{t}'} g(n(t))adt + (\bar{t} + H - \bar{t}')g(N)a + \int_{\bar{t}+H}^{\tilde{t}+H} g(n(t))adt \\ &= a \cdot N^{\gamma}(H + \bar{t} - \bar{t}') + \frac{2aN^{\gamma}(\bar{t}' - \bar{t})\ (1 - 2^{-\gamma-1})}{(1+\gamma)} \end{aligned} \quad \text{(C2)}$$

The last expression is obtained under the assumption that $\tilde{t} = \frac{\bar{t}' + \bar{t}}{2}$. Substituting the above results into $(Y_1 - C_1) - (Y_2 - C_2)$ yields (28).

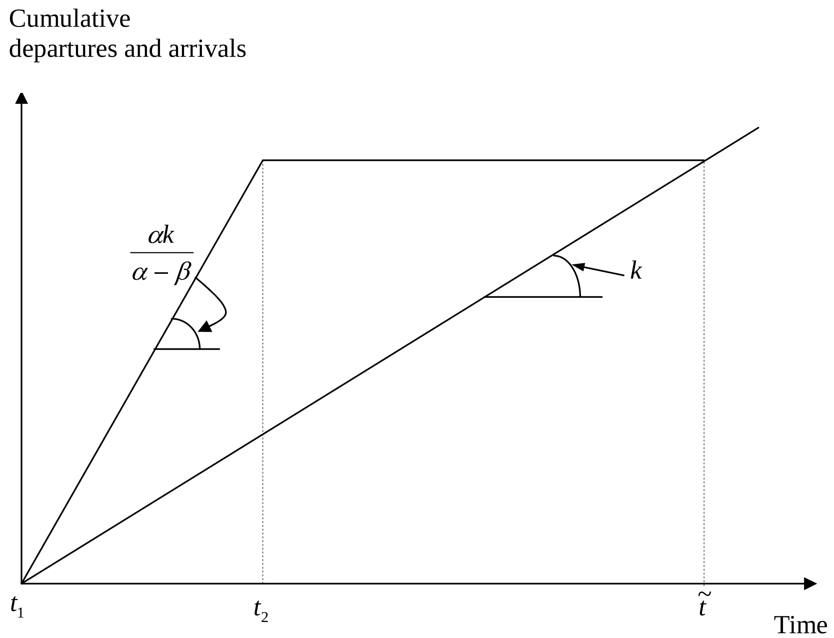


Figure 1   Cumulative departures and arrivals in Case A

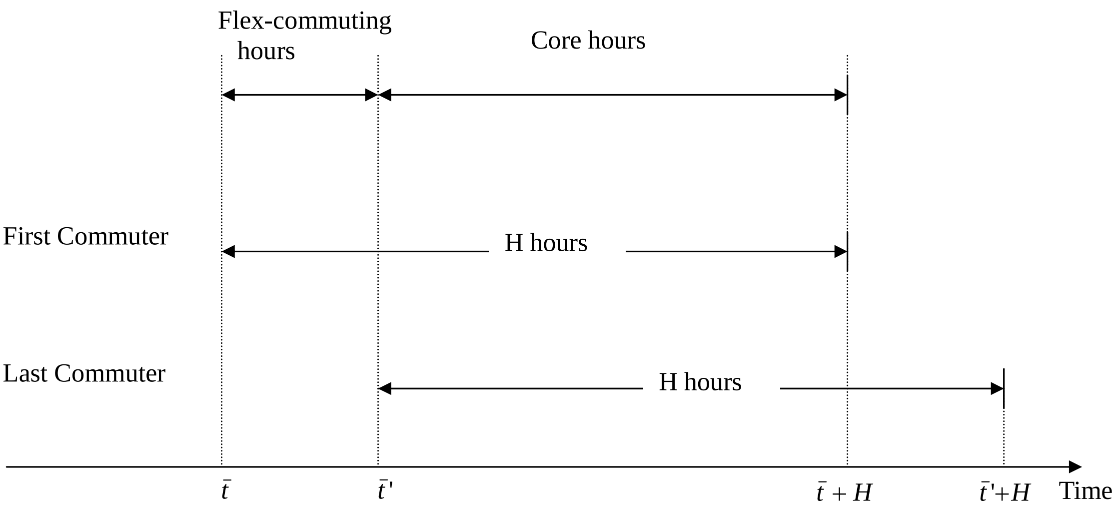
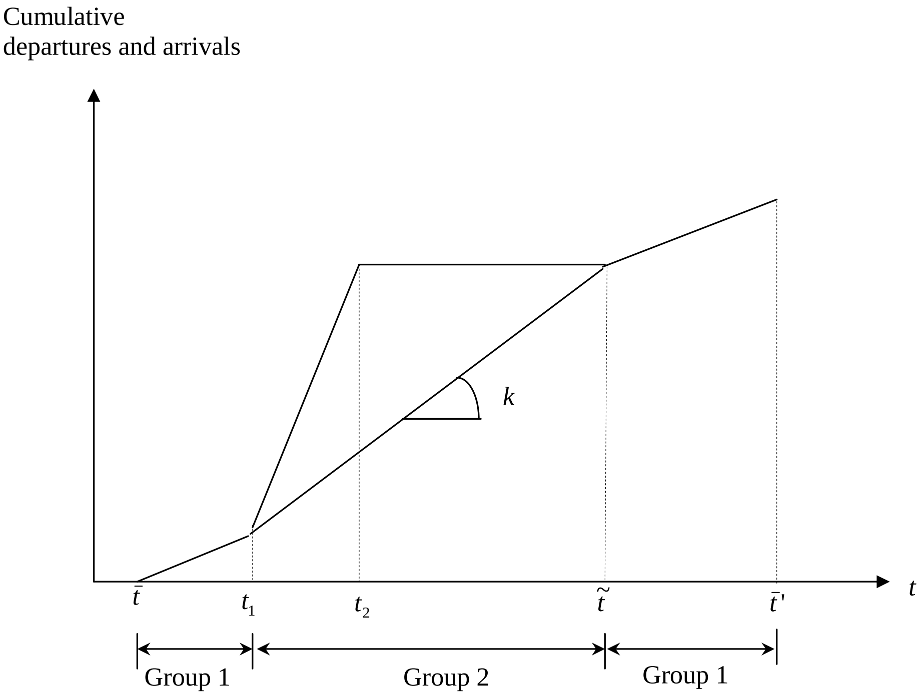
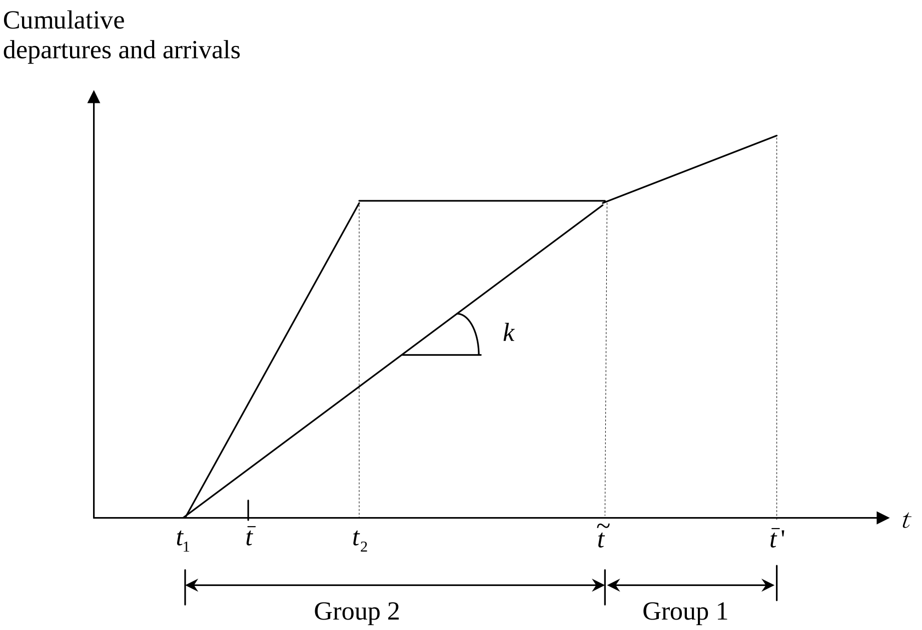


Figure 2 Working schedule in a firm adopting flextime



Case C1



Case C2

Figure 3 Cumulative departures and arrivals in Case C

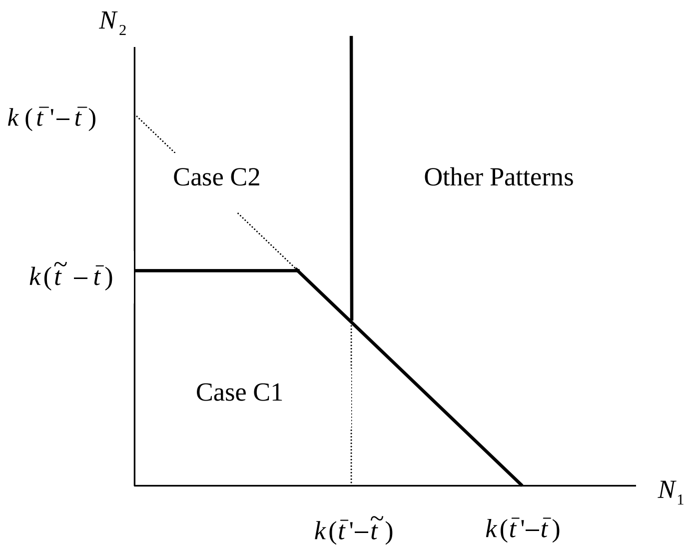
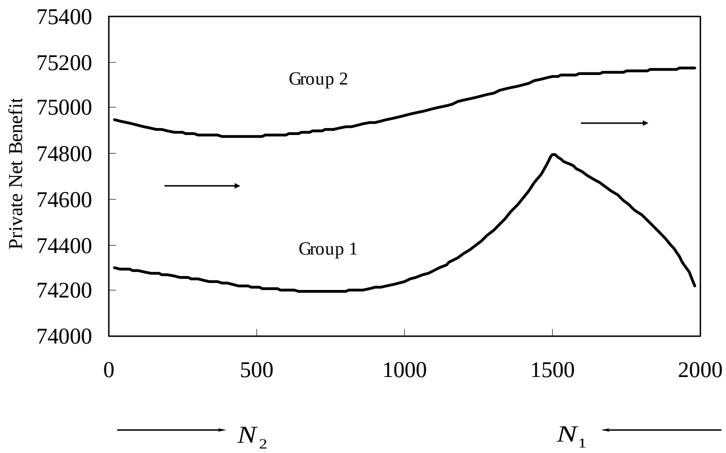
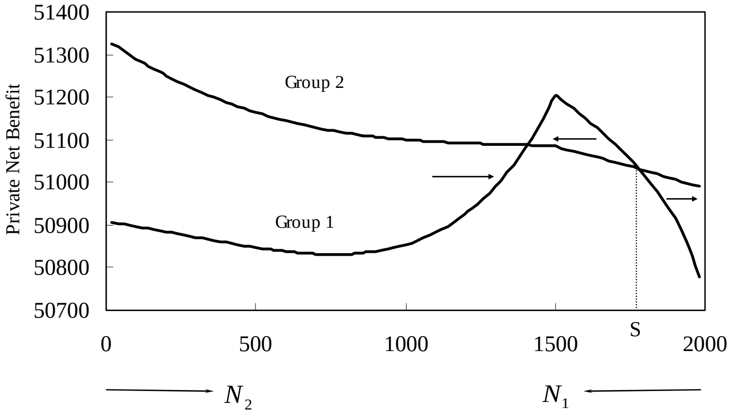


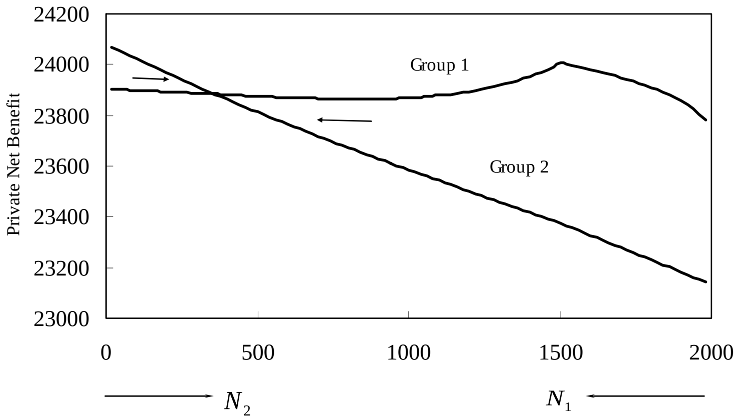
Figure 4 Parameters and departure patterns



(a) Pattern 1

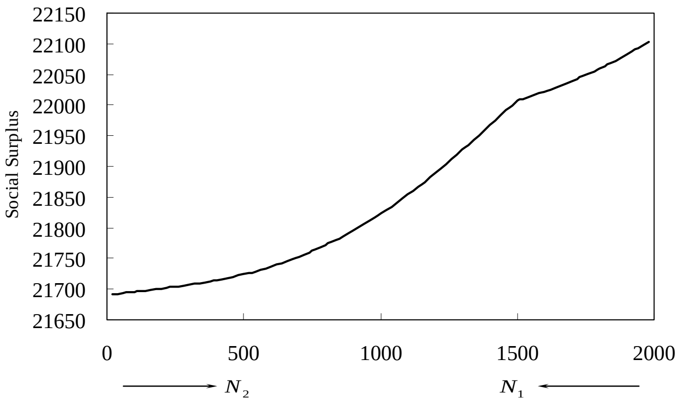


(b) Pattern 2

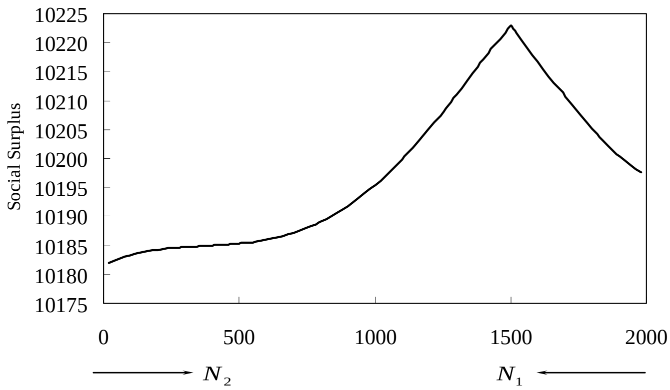


(c) Pattern 3

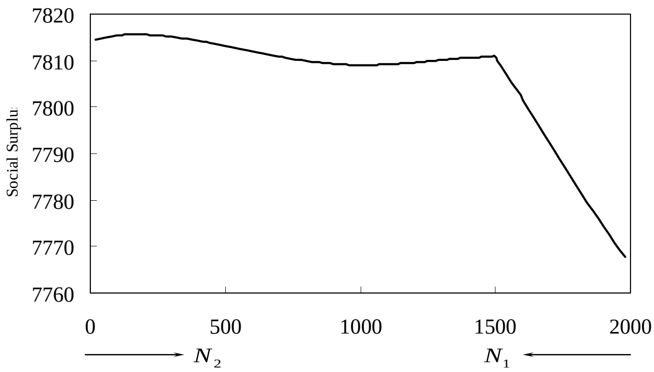
Figure 5 Patterns of equilibrium solutions



(a) Pattern R



(b) Pattern M



(c) Pattern L

Figure 6 Patterns of social surplus variation

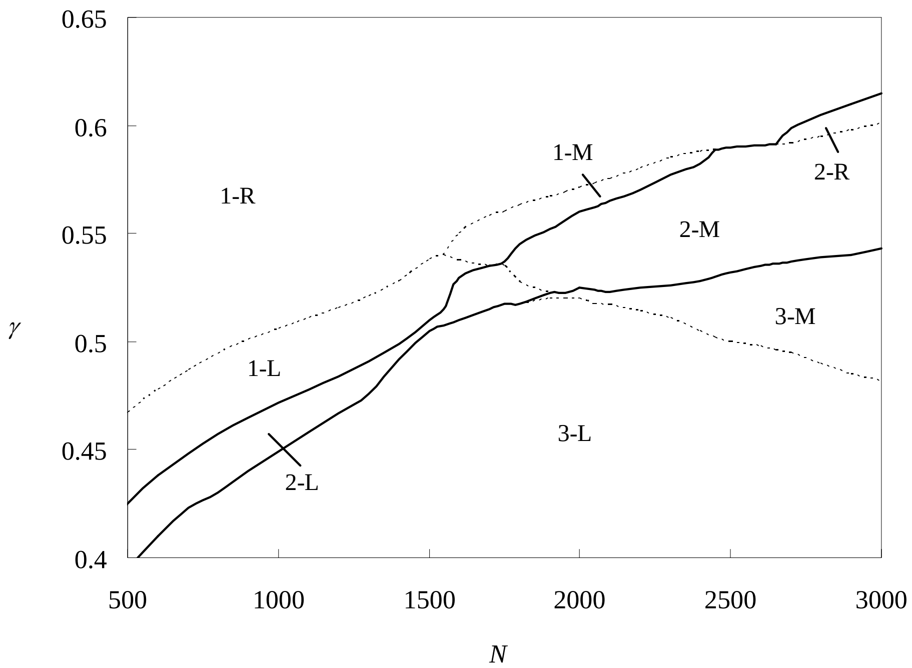
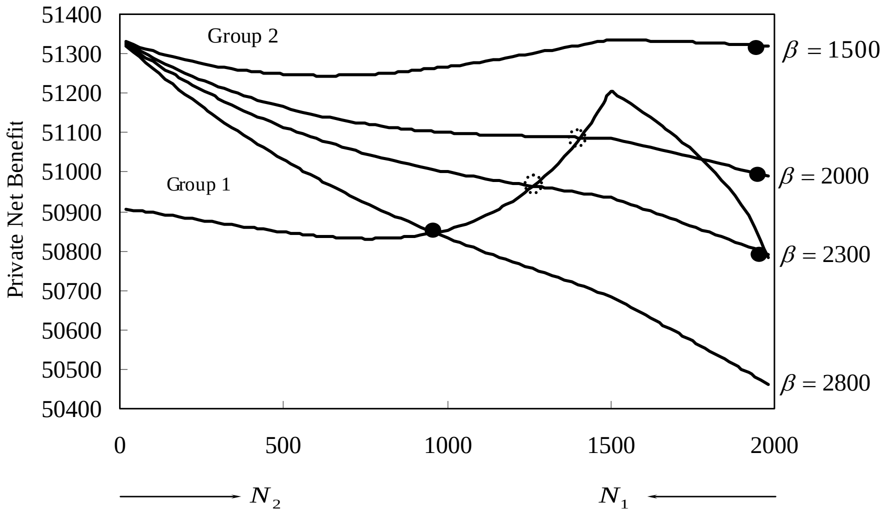
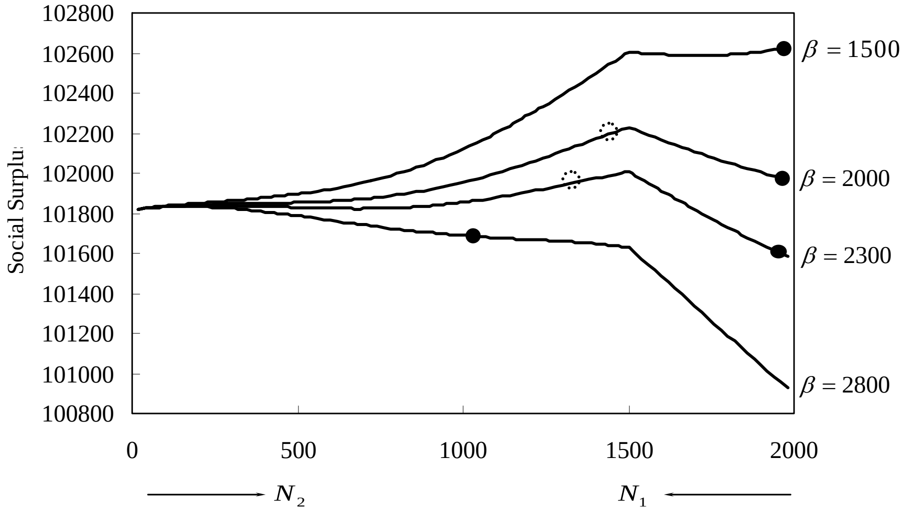


Figure 7 Parameters and patterns of solutions

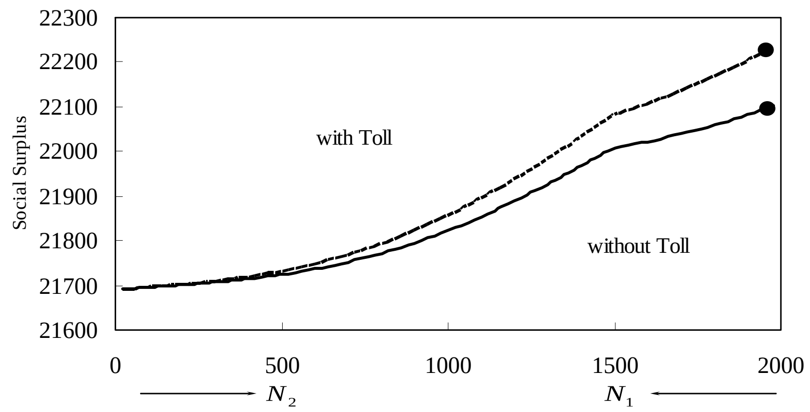


(a) Private net benefit and positions of equilibrium solutions

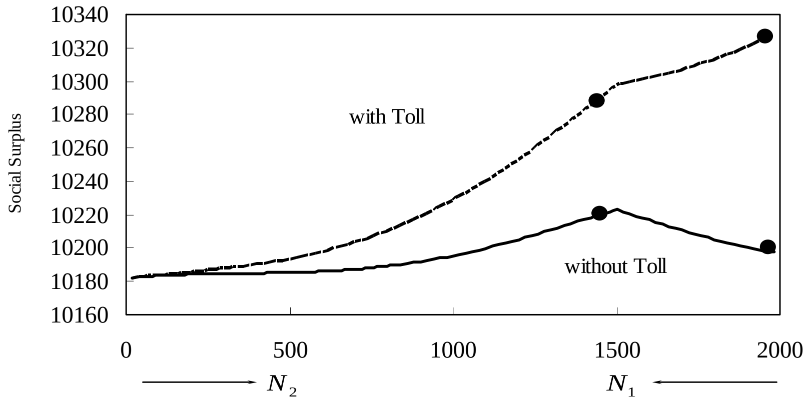


(b)Positions of equilibrium solutions and social surplus

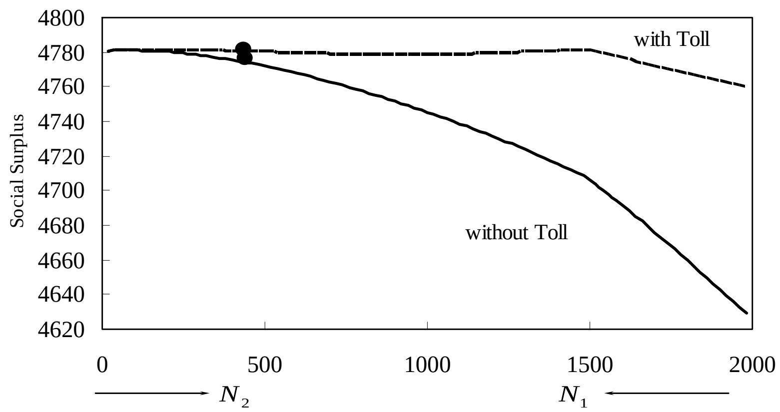
Figure 8 Changes in values of scheduling cost and patterns of solutions



(a) $N = 2000, \gamma = 0.65$



(b) $N = 2000, \gamma = 0.55$



(c) $N = 2000, \gamma = 0.45$

Figure 9 Effects of peak-load toll