Do Sticky Prices Increase Real Exchange Rate Volatility at the Sector Level?*

Mario J. Crucini†, Mototsugu Shintani‡ and Takayuki Tsuruga§

First draft: June 2010
This version: February 2013

Abstract

We introduce the real exchange rate volatility curve as a useful device to understand the relationship between price stickiness and the fluctuations in Law of One Price deviations. In the presence of both nominal and real shocks, the theory predicts that the real exchange rate volatility curve is a U-shaped function of the degree of price stickiness. Using sector-level US-European real exchange rate data and frequency of price changes, we estimate the volatility curve and find the predominance of real effects over nominal effects. Good-by-good variance decompositions show that the relative contribution of nominal shocks is smaller at the sector level than what previous studies have found at the aggregate level, consistent with significant averaging out of good-specific real microeconomic shocks.

JEL Classification: E31; F31; D40

Keywords: Real exchange rates, Law of One Price, Sticky prices, Nonparametric test for monotonicity.

---

*We thank Yasushi Iwamoto, Oleksiy Kryvtsov and seminar and conference participants at the Bank of Canada, the Board of Governors of the Federal Reserve System, Clemson University, the European Commission, the Federal Reserve Bank of Atlanta, the Federal Reserve Bank of Dallas, Hitotsubashi University, Kobe University, Kyoto University, Nagoya University, Osaka University, Otaru University of Commerce, Econometrics Society World Congress 2010, and Society for Economic Dynamics Annual Meeting 2011 for their helpful discussion and comments. We are grateful to John Taylor for his encouragement at the early stage of the project. We also thank Virgiliu Midrigan for providing the data and Subhashis Ghosal for providing the programing code. Crucini and Shintani gratefully acknowledge the financial support of NSF (SES-1030164). Tsuruga gratefully acknowledges the financial support of Grant-in-aid for Scientific Research (21243027).

†Department of Economics, Vanderbilt University and NBER; e-mail: `mario.crucini@vanderbilt.edu`.

‡Department of Economics, Vanderbilt University; e-mail: `mototsugu.shintani@vanderbilt.edu`.

§Graduate School of Economics, Kyoto University; e-mail: `tsuruga@econ.kyoto-u.ac.jp`.

1 Introduction

Among international macroeconomists, it is widely believed that the variability of real exchange rates is increasing in the degree of price rigidity. A reasoning is found in a prominent textbook by Dornbusch, Fischer, and Startz (2004):

> "Exchange rate overshooting results from the rapid response of exchange rates to monetary policy and the sluggish adjustment of prices. A monetary expansion will lead to an immediate depreciation but only a gradual increase in prices. Exchange rate overshooting implies that real exchange rates are highly volatile (p. 534)."

The basic idea is as follows. The nominal exchange rate is an asset price (since currencies are actively traded in the foreign exchange market) and thus it adjusts instantaneously in response to nominal shocks. However, if prices of many goods and services adjust sluggishly, the real exchange rate will be highly volatile because it comoves with the nominal exchange rate. The expectation, then, is a *positive* correlation between the volatility of real exchange rates and the degree of price stickiness if *nominal* shocks dominate the landscape, as they do in much theorizing on the topic. Quantitative investigations of this prediction have been undertaken, by Chari, Kehoe, and McGrattan (2002) who focus on the aggregate real exchange rates and by Kehoe and Midrigan (2007) who focus on Law of One Price (LOP) deviations.

An early advocate for the role of real shocks in the equilibrium determination of real exchange rates is Stockman (1980). Stockman casts his model in a flexible price setting, so that nominal shocks make no contribution to real exchange rate volatility. Crucini, Shintani, and Tsuruga (2010), on the other hand, neutralize the effect of nominal shocks by focusing on intranational trade and investigate the role of real shocks on good-level real exchange rate volatility across cities in the presence of price rigidity. Unlike models emphasizing the role of the nominal shocks, their model predicts a *negative* correlation between price stickiness and real exchange rate variability because only *real* shocks affect real exchange rates across locations within a country.

The current paper puts these two views of real exchange rate determination on the same playing field by combining the model of Kehoe and Midrigan (2007) which emphasizes nominal shocks with the model of Crucini, Shintani, and Tsuruga (2010) which emphasizes real shocks. These models rely on the time dependent pricing assumption, but allow the frequencies of price changes to vary across goods, as measured in the micro-data. Under the synthesized framework, we theoretically explore the cross-sectional relationship between price stickiness and real exchange rate volatility at the level of individual goods. We refer to this relationship as the real exchange rate volatility curve: the functional relationship between the forecast error variance of the real exchange rate and the infrequency of price changes at the level of a good. When real shocks are absent, the volatility curve is upward-sloping: an increasing function of the price stickiness parameter and the good with the stickiest price should exhibit the greatest amount of real exchange rate variability. When nominal shocks are turned off, the volatility curve is downward-sloping: a decreasing function of the price stickiness parameter and the good with the stickiest price has the least amount of real exchange rate variability. When both real and nominal shocks are present, the real exchange rate volatility curve becomes U-shaped.

We estimate the volatility curve using sector-level real exchanges of Austria, Belgium, France, and Spain vis à vis the US, constructed by Kehoe and Midrigan (2007). We find the estimated U-shaped curve is monotonically decreasing over the majority of the range of price stickiness. Our main findings regarding the shape of the curve are confirmed by both parametric and nonparametric estimation methods. The negative correlation together with the theoretical prediction of our model suggests the predominance of real shocks over nominal shocks in explaining the volatility of real exchange rates at the sector level.

At the aggregate level, the relative contribution of real and nominal shocks to real exchange rate variability has been typically evaluated in terms of forecast error variance decompositions (e.g., Clarida and Galí, 1994, Eichenbaum and Evans, 1995, and Rogers, 1999).[1] Following this literature, we further conduct variance decompositions of real exchange rates at the sector level, and evaluate the relative contribution of nominal and real shocks. For the majority of goods, the contribution of nominal shocks is smaller than that of real shocks, and real shocks rise in dominance as the forecast horizon lengthens. To reconcile our microeconomic evidence with the macroeconomic evidence, it seems necessary to allow for large idiosyncratic real shocks at the sector level such that these microeconomic sources of variation average out in the move to the CPI-based real exchange rate (see Broda and Weinstein, 2008, and Bergin, Glick and Wu, 2012, Crucini and Telmer, 2012).

2 The Model

The theory combines the key features of Kehoe and Midrigan (2007) and Crucini, Shintani and Tsuruga (2010). Both of these models assume heterogeneous price stickiness across goods, but the former relies on nominal exchange rate variations whereas the latter focuses on the labor productivity variations along with trade costs in explaining the volatility of good-level real exchange rates.

In what follows, we present a sketch of our model to discuss its main implications for good-level real exchange rate volatility.[2] The (log) real exchange rate for a bilateral pair of countries is defined as:

[1] Some exceptions, such as Steinsson (2008), focus on the shape of impulse response function to evaluate the relative importance of nominal and real shocks.

[2] The full model is presented in Appendix A.

$$q_{it} = s_t + p_{it}^* - p_{it}, \tag{1}$$

where p_{it} (p_{it}^*) denotes the (log) price index for good i in the home (foreign) country and s_t is the (log) nominal exchange rate, at period t.

To introduce the real exchange rate volatility curve, some simplifying assumptions are made on the sources of real exchange rate variation. The first assumptions concern nominal shocks and exchange rates and we take these assumptions from Kehoe and Midrigan (2007). Let M_t and P_t be the money demand and aggregate price level, respectively. The nominal shocks in the model are the home and foreign money growth rate, $\mu_t = \ln(M_t/M_{t-1})$ and $\mu_t^* = \ln(M_t^*/M_{t-1}^*)$, which are independent and identically distributed (i.i.d.). We also assume that the period utility function is given by $\ln C_t - \chi L_t$, where C_t and L_t denote aggregate consumption and hours worked, respectively. The assumption on household preference, combined with a local-currency cash-in-advance constraint, $M_t = P_t C_t$, leads to the equality of the money growth differential and the nominal exchange rate growth (i.e., $\mu_t - \mu_t^* = \Delta s_t$). Conveniently, the i.i.d. process of the money growth rates gives rise to a nominal exchange rate s_t that follows a random walk, a characteristic similar to the data.[3]

The second assumptions concern real shocks and trade costs. Monopolistically competitive firms set prices of their goods, which are produced using a technology that is linear in labor and subject to productivity shocks: $Y_{it}(v) = A_{it} L_{it}(v)$, where $Y_{it}(v)$ is output and $L_{it}(v)$ is labor demand, for firms producing brand v of good i, and A_{it} is the labor productivity which varies by good i but is common across brands. Productivity shocks in logs in two countries

[3] With the cost of losing computational simplicity of the real exchange rate volatility curve, we can also replace i.i.d. money growth with serially correlated money growth.

($a_{it} = \ln A_{it}$ and $a^*_{it} = \ln A^*_{it}$) are given by:

$$a_{it} = z_t + \eta_t + \varepsilon_{it}, \qquad a^*_{it} = z_t + \eta^*_t + \varepsilon^*_{it}. \tag{2}$$

Due to our microeconomic focus, the productivity shock for each good consists of three components: a possibly nonstationary global trend component (z_t), an i.i.d. nation-specific component (η_t and η^*_t) and an i.i.d. good-specific component (ε_{it} and ε^*_{it}).[4] Finally, firms in each country are required to pay an iceberg transportation cost τ to send a good across the border. This transportation cost leads to home bias in consumption because the home variety of each good is cheaper than the imported variety of the same good. We denote the elasticity of substitution among differentiated products by θ and the discount factor by β; these parameters are assumed to satisfy $\theta > 1$ and $0 < \beta < 1$. Finally, firms reset their prices under Calvo-type pricing with the good-specific degree of price stickiness given by the probability of no price change λ_i.

The focal equation of the model is the first-order difference equation that nests equations by Kehoe and Midrigan (2007) and Crucini, Shintani and Tsuruga (2010):

$$q_{it} = \lambda_i q_{it-1} + \lambda_i(\mu_t - \mu^*_t) + (1 - \lambda_i)(1 - \lambda_i\beta)\psi\left(a_{it} - a^*_{it}\right), \tag{3}$$

where $\psi = \left[\left(1 - (1+\tau)^{1-\theta}\right) / \left(1 + (1+\tau)^{1-\theta}\right)\right]$. The parameter ψ appears because a productivity shock to good i in one country asymmetrically transmits to the price indexes of the same good in two countries due to the home bias. The derivation of (3) is provided in Appendix A.

[4] Here we impose an i.i.d. assumption on nation- and good-specific component for ease of exposition. As in the case of nominal shocks, however, we can easily relax this assumption and introduce persistence without changing the qualitative implication of real exchange rate volatility curve. For the same reason, we further assume that variance of good-specific component is common across i. In the empirical part of the paper, the possiblity of heterogenous variances of real shocks across goods is considered.

2.1 The real exchange rate volatility curve

The equation (3) together with i.i.d. assumptions of money growth differential $\mu_t - \mu_t^*$ (the nominal shock) and cross-country productivity differential $a_{it} - a_{it}^*$ (the real shocks) implies the k-period-ahead forecast error variance of the sector-level real exchange rate:

$$Var_{t-k}(q_{it}) = \Lambda_{ik}[\lambda_i^2 Var(\mu_t - \mu_t^*) + (1-\lambda_i)^2(1-\lambda_i\beta)^2\psi^2 Var(a_{it} - a_{it}^*)], \tag{4}$$

where $\Lambda_{ik} = \sum_{j=1}^{k} \lambda_i^{2(j-1)}$. Equation (4) attributes the forecast error variance of the good-level real exchange rate to the variance of the nominal shocks and the variance of the real shocks. Recall that price stickiness parameter λ_i is assumed to be common across countries but differs across goods. Viewed as a function of λ_i, this equation is called *the real exchange rate volatility curve.*

Note that the coefficient on the nominal shock $\lambda_i^2\Lambda_{ik}$ is increasing in λ_i, while the coefficient on the real shock $(1-\lambda_i)^2(1-\lambda_i\beta)^2\psi^2\Lambda_{ik}$ is decreasing in λ_i. Therefore, for any forecast horizon k, an increase in λ_i increases the contribution of the nominal effect (measured by $\Lambda_{ik}\lambda_i^2 Var(\mu_t - \mu_t^*)$) and decreases the contribution of the real effect (measured by $\Lambda_{ik}(1-\lambda_i)^2(1-\lambda_i\beta)^2\psi^2 Var(a_{it} - a_{it}^*)$) to the total forecast error variance of the good-level real exchange rate. These two opposing forces give rise to a real exchange rate volatility curve that is U-shaped over the support $\lambda_i \in [0,1]$.[5]

To gain some intuition behind the mechanism, recall that q_{it} is defined as $s_t + p_{it}^* - p_{it}$. To see the impact of nominal shocks, consider a positive money growth rate shock in the home country. The model predicts an immediate depreciation of the nominal exchange rate, that is, an increase in s_t. The responses of p_{it}, however, depend on the good-specific frequencies of price adjustment. For goods with prices that change every period, an increase in p_{it} completely

[5]It can be easily shown that the real exchange rate volatility curve for $k=1$ is convex on $\lambda_i \in [0,1]$ as far as the variances of the two shocks are strictly positive.

offsets an increase in s_t keeping q_{it} unchanged. At the other end of the continuum, goods with prices that are extremely sticky will have q_{it} that basically follows the path of s_t with negligible pass-through of the nominal shock to p_{it}. The nominal effect on real exchange rate variability is amplified by sluggish adjustment of prices. Simply put: conditional on nominal shocks, the correlation between real exchange rate volatility and the degree of price stickiness is positive.

Turning to real shocks, consider a positive shock in home productivity in good i. In our model, this productivity shock has no equilibrium consequences for s_t. What it does is reduce both p_{it} and p_{it}^*, because firms in the home country sell their goods in both countries. However, due to home bias generated by trade costs, p_{it} will decrease more than p_{it}^*. This increases $p_{it}^* - p_{it}$ which results in an increase in $q_{it} = s_t + p_{it}^* - p_{it}$. Because this channel requires prices to actually change and thereby induce asymmetric price changes across countries, it is more quantitatively important when prices are flexible. Conversely, the real effect is mitigated by sticky prices. Simply put: conditional on real shocks, the correlation between real exchange rate volatility and the degree of price stickiness is negative.

We note that household preferences and firm technology are key assumptions, producing a U-shaped real exchange rate volatility curve. The semi-log period utility along with the cash-in-advance constraint ensures that no real shocks affect the nominal exchange rate, generating the equivalence of $Var\left(\Delta s_t\right)$ to $Var\left(\mu_t - \mu_t^*\right)$. The assumption of the firms' linear technology in addition to the semi-log period utility and the cash-in-advance constraint allows us to derive the simple reduced form solution for the real exchange rates by making firms' optimal local-currency pricing depend solely on money supply and productivity and shutting down the quantity effect of the demand for goods on firms' pricing. As long as these assumptions are maintained, the U-shape is robust to the introduction of other real shocks such as preference and markup shocks (see Appendix B).

2.2 Numerical Examples

Let us provide numerical examples to evaluate how different intensities of real and nominal shocks alter the shape of the real exchange rate volatility curve. We focus on the one-period-ahead forecast error variance by setting $k = 1$ in equation (4),

$$Var_{t-1}(q_{it}) = \lambda_i^2 Var(\mu_t - \mu_t^*) + (1 - \lambda_i)^2 (1 - \lambda_i \beta)^2 \psi^2 Var(a_{it} - a_{it}^*). \tag{5}$$

The structural parameters are calibrated as follows: (i) the data is monthly, so the discount factor is set to $\beta = 0.96^{1/12}$; (ii) trade costs, broadly defined at the retail level, are in the neighborhood of $\tau = 0.5$; and (iii) the elasticity of substitution is set at $\theta = 10$.[6] Figure 1 shows the shape of the curve under three distinct stochastic environments: (a) $Std(a_{it} - a_{it}^*)/Std(\mu_t - \mu_t^*) = 5$; (b) $Std(a_{it} - a_{it}^*)/Std(\mu_t - \mu_t^*) = 1$; and (c) $Std(a_{it} - a_{it}^*)/Std(\mu_t - \mu_t^*) = 1/5$.[7] The height of the volatility curve in each panel is the model's prediction for the total forecast error variance of the real exchange rate for a particular good indexed by λ_i. The blue and red segments of a vertical line drawn at each λ_i give the contributions of real and nominal shocks, respectively. For example, real exchange rate fluctuations of goods with fully flexible price (e.g., crude petroleum) would be driven solely by real shocks while goods with completely rigid prices (e.g., postage stamps) would be driven solely by nominal shocks.

Each panel of the figure clearly suggests that the shape of the curve depends crucially on the relative magnitude of the two shocks. The U-shaped curves are minimized at $\lambda_i = 0.75$ in panel (a), 0.40 in panel (b), and 0.06 in panel (c). While shock structures in terms of variance ratios are symmetric between panels (a) and (c), the shapes of their volatility curves

[6] Here we follow Carvalho and Nechio (2011) in setting the substitution across varieties within the same sector as $\theta = 10$. However, the shape of the curve is qualitatively insensitive to the values of θ and τ.

[7] For all three cases, $Std(\mu_t - \mu_t^*)$ is normalized to 1 percent.

are asymmetric because of asymmetric roles played by nominal and real shock volatility.[8] This feature also explains that the curve in panel (b) is minimized at $\lambda_i = 0.40$ rather than 0.50, the midpoint of the unit interval.

When the real shock dominates, the model predicts that the volatility curve is downward sloping over almost its entire range as in panel (a). The middle panel (b), with equal variances of real and nominal shocks, displays an obvious U-shape. When the nominal shock dominates, the curve is upward sloping over almost its entire range as in panel (c). In practice the sign of the correlation of price stickiness and real exchange rate volatility will depend on the distribution of goods in the sample as well as the relative importance of real and nominal shocks. In addition, since the real exchange rate volatility curve is U-shaped in general, introducing a nonlinear functional form in the regression, rather than relying a simple linear regression, is essential for detecting the underlying structure.

When the degree of price stickiness is uniformly distributed across goods, blue and red areas in Figure 1 can be interpreted as the cross-sectional average of the variance decomposition for each individual good. In panel (a), the red area (nominal effects) accounts for only 7 percent and the blue area (real effects) contributes the remaining 93 percent. In contrast, the nominal effects account for 65 percent in panel (b) and 98 percent in panel (c). While the average variance decomposition is a convenient measure under the uniform distribution assumption, it should be noted that variance decompositions can still greatly vary across the individual goods. For example, consider two goods, one with $\lambda_1 = 0.25$ and the other with $\lambda_2 = 0.56$ in panel (b). These two real exchange rates turn out to have the same total forecast error variance. However, the relative contribution of nominal shocks is only 18 percent for

[8] Since β and ψ are close to unity in our setting, the curve can be approximated by $\lambda_i^2 Var(\mu_t - \mu_t^*) + (1 - \lambda_i)^4 Var(a_{it} - a_{it}^*)$. This shows that the relative role of nominal and real shock volatilities is asymmetric since the weight on the former approaches 0 at a slower rate as $\lambda_i \to 0$, compared to the coefficient on the latter which approaches 0 at a faster rate as $\lambda_i \to 1$.

the first good but 90 percent for the second good.

3 Empirical Analysis

3.1 Data

The empirical analysis focuses on (i) examining the relationship between the total variance of real exchange rates and the degree of price stickiness based on the theoretical volatility curve; and (ii) parsing this variance into the real and nominal effects at the disaggregated level. The data is from Kehoe and Midrigan (2007) and consists of 66 sectoral real exchange rates for four European countries (Austria, Belgium, France, and Spain) relative to the US from January 1996 to May 2006. The series are constructed by matching monthly, highly dissaggregated price data from Eurostat and the Bureau of Labor Statistics and converting to a common-currency using spot nominal exchange rates obtained from the FRED database. The goal is to understand the time series properties of the real exchange rate for sector i for country j, vis à vis the United States, $q_{ijt} = s_{jt} + p^*_{it} - p_{ijt}$, for $i = 1, ..., N_j$ and $j = 1, ..., 4$, where N_j is the number of sectors available both in country j and the US, p_{ijt} is the price for sector i in country j expressed in local currency, p^*_{it} is the US price for sector i in US dollar, s_{jt} is the domestic currency value of the US dollar. The Euro was officially introduced part-way through our sample. It should be noted that even before the introduction of the Euro, the volatility of the nominal exchange rate against the US dollar was quite similar across these European countries.[9] Thus, we expect that the role of the nominal shock may be similar across the four bilateral pairs, while that of the shock to productivity differentials is not known. In the analysis, we use a pooled regression of the four country-pairs as a benchmark but also run country-by-country regressions to allow for the different role of the real shocks.

[9]The standard deviations of the nominal exchange rate growth of the US dollar against the Austrian Schilling, Belgian Franc, French Franc, and Spanish Peseta were 2.36, 2.37, 2.35, and 2.36 percent, respectively.

The country-specific frequencies of price changes for the US are from Bils and Klenow (2004) and those for each of the European countries are taken from the individual country studies by: Baumgartner, Glatzer, Rumler, and Stiglbauer (2005) for Austria; Aucremanne and Dhyne (2004) for Belgium; Baudry, Le Bihan, Sevestre, and Tarrieu (2007) for France; Alvarez and Hernando (2006) for Spain. Since the degree of price stickiness of the same good is assumed to be common between paired countries and constant across time, Kehoe and Midrigan (2007) take the cross-country average monthly infrequencies of price changes within each sector. Thus, observed infrequency of price changes, λ_{ij}, represents the average value between country j and the US.[10]

For the purpose of our analysis, we compute the k-period-ahead forecast error variance, $Var_{t-k}(q_{ijt})$, from the sample variance of the quasi-difference $q_{ijt}-\lambda_{ij}^{k}q_{ijt-k}$, using the observed infrequency of price changes, λ_{ij}. When either λ_{ij} or q_{ijt} is missing or when the sample variance can be computed from only a short time sample, we exclude the sector from the sample. As a result, the number of sectors (N_j) is 57 for Austria, 46 for Belgium, 48 for France, and 31 for Spain. In the pooled case, the number of cross-sectional observations is 182. Figure 2 plots the density estimate of λ_{ij} when all the observation are pooled. It suggests that many observations are clustered around high values of λ_i. The empirical range of λ_{ij} in our dataset is given by $[\lambda_{\min}, \lambda_{\max}] = [0.223, 0.979]$. In what follows, country subscript j is dropped for notational simplicity.

3.2 Estimation of the real exchange rate volatility curve

Let V_i be the one-period-ahead forecast error variance of the real exchange rate. As a preliminary analysis, we first follow Kehoe and Midrigan (2007) and run a simple linear regression of V_i on λ_i pooling the data from all the countries. The estimated coefficient on λ_i is signif-

[10] The details of the data construction are found in the appendix of Kehoe and Midrigan (2007).

icantly negative with the adjusted R^2 of 70 percent.[11] Therefore, linear regression suggests that the real exchange rate volatility curve is downward sloping, which is consistent with the finding of Kehoe and Midrigan (2007) who use the squared residuals from the first-order autoregression instead of our theoretical one-period-ahead forecast error variance based on observed λ_i.

Recall, however, that the theory predicts a nonlinear relationship, rather than the linear relationship, between the frequency of price adjustment and real exchange rate variability. For this reason, we employ Ramsey's (1969) regression specification error test to see whether the null hypothesis of linearity can be rejected. The resulting t-statistic is 5.36, which rejects the null hypothesis at the one percent significance level.[12] In what follows, we conduct two alternative nonlinear regression analyses, one using a parametric approach and the other using a nonparametric approach.

3.2.1 Parametric Approach

Our regression analysis based on a parametric approach relies of the structural form of (5) which suggests both a quadratic term and a quartic term to be included as regressors:

$$V_i = b_1\lambda_i^2 + b_2(1-\lambda_i)^2(1-\lambda_i\beta)^2 + u_i, \tag{6}$$

where u_i is a regression error term. According to (5), regression coefficient b_1 should capture the nominal effects, due to $Var(\mu_t - \mu_t^*)$. The second regressor $(1-\lambda_i)^2(1-\lambda_i\beta)^2$ is constructed by setting $\beta = 0.96^{1/12}$. The regression coefficient, b_2, captures the real effects $\psi^2 Var(a_{it} - a_{it}^*)$, with the restriction that the variance of productivity differentials is common across i. If the

[11] To be specific, the estimated linear model is given by $V_i = 0.013 - 0.014\lambda_i$.

[12] We also tested the linearity for each country. The null hypothesis is rejected at one percent significance level for the Austrian and French cases. Weaker evidence of nonlinearity is obtained for Belgian and Spanish cases possibly because the power of the test is lower in smaller samples.

restriction is not satisfied, it captures the cross-sectional average of variance of real shocks.

Table 1 presents the estimation results of the quartic regression model (6).[13] In all cases, the estimated coefficients, b_1 and b_2, are significantly positive, which is consistent with the theory. The quartic regression is comparable to the linear regression in terms of the goodness of fit. The relative role of nominal and real shocks, as implied by the estimates b_1 and b_2, indicates a much larger role for real shocks.[14] Because the home bias term satisfies $0 \leq \psi^2 \leq 1$, b_2 can be considered as a lower bound on $Var(a_{it} - a_{it}^*)$. The same argument establishes that $\sqrt{b_2/b_1}$ can be used as a lower bound for $Std(a_{it} - a_{it}^*)/Std(\mu_t - \mu_t^*)$. The lower bounds are inferred to be: 6.63, 6.17, 4.42, 5.30 and 9.64, for the pooled case and the Austrian, Belgian, French and Spanish nation-specific cases, respectively. Thus, the quartic regression result is very comparable to the numerical example of case (a) where $Std(a_{it} - a_{it}^*)/Std(\mu_t - \mu_t^*) = 5$.

Figure 3 further elucidates the shape of the volatility curve, showing the fitted curve and its derivative with respect to λ_i of the pooled quartic regression. The figure also includes the 95 percent confidence intervals computed using heteroskedasticity-robust standard errors. The fitted curve shown in the upper panel resembles panel (a) of Figure 1 in terms of the shape of the curve, suggesting the importance of real effects. The fitted curve in the lower panel shows the monotonically increasing first derivative.

As reported in the last two columns of Table 1, estimates of b_1 and b_2 can be also used to locate the degree of price stickiness which minimizes the forecast error variance. For

[13] In the pooled case, we run the regression with and without the cross product of country dummy variables and each regressor. In the table, we only report the result with country dummies D_{ji} for $j = 1, .., 4$, which takes value 1 if the data is from country j. For example, reported b_1 is from $b_1\lambda_i^2 + \sum_{j=1}^4 b_{1j} D_{ji} \lambda_i^2$ with a restriction $\sum_{j=1}^4 b_{1j} = 0$. When country dummies are excluded, estimates of b_1 and b_2 are 0.0016 and 0.0456, respectively.

[14] We also conduct the robustness analysis, using the first-order autocorrelation (ρ_i) of the sector-level real exchange rates, rather than λ_i. In particular, we obtain the sample variance of $q_{it} - \rho_i q_{it-1}$ and run the quartic regression (6) by replacing λ_i with ρ_i. Even when we use ρ_i for the quartic regressions, the estimated coefficients are both significantly positive in the pooled regression and country-by-country regressions except for Spain and again suggest a large role for real shocks as in the case of λ_i.

the pooled case, the variance of the real exchange rate is minimized at $\underline{\lambda} = 0.79$ which is somewhat close to the value of 0.75 from our numerical example with dominant real shocks. This frequency of price changes implies that the duration between price changes is 4.8 months.

3.2.2 Nonparametric Approach

We now turn to the regression analysis based on a nonparametric approach. Note that the quartic regression (6) imposes a strict theoretical shape restriction on the real exchange rate volatility curve. To allow for a more general functional form of the real exchange rate volatility curve, we may consider the following nonparametric regression model,

$$V_i = m(\lambda_i) + u_i,$$

where m is an unknown conditional mean function. We are interested in the shape of $m(x)$ and its first derivative $m'(x)$. Here, we estimate the former using the local linear estimator and the latter using the local quadratic estimator. The p-th order local polynomial estimator at a point $\lambda_i = x$ can be obtained by minimizing the weighted least squares criterion $\sum_{i=1}^{N} \left\{V_i - \sum_{k=0}^{p} \beta_k (\lambda_i - x)^k\right\}^2 K\left(\frac{\lambda_i - x}{h}\right)$, where h is the bandwidth and K is a kernel function such that $\int K(u)du = 1$.[15] The solution is given by $\widehat{\beta} = (\widehat{\beta}_0, \cdots \widehat{\beta}_p)' = (X'WX)^{-1}X'WY$ where

$$X = \begin{bmatrix} 1 & (\lambda_1 - x) & \cdots & (\lambda_1 - x)^p \\ \vdots & \vdots & & \vdots \\ 1 & (\lambda_N - x) & \cdots & (\lambda_N - x)^p \end{bmatrix},$$

$Y = (V_1, \ldots, V_N)'$ and $W = \text{diag}\{K\{(\lambda_1 - x)/h\}, \ldots, K\{(\lambda_N - x)/h\}\}$. Our local linear estimator of the function $m(x)$ is given by $\widehat{m}(x) = \widehat{\beta}_0$ when $p = 1$ while our local quadratic estimator of first derivative is given by $\widehat{m}'(x) = \widehat{\beta}_1$ when $p = 2$. The (point-wise) confidence interval of $\widehat{\beta}_k$ is constructed from the heteroskedasticity-robust standard error defined as a

[15] We employ the standard Gaussian kernel function for K. For the bandwidth h, we use the rule of thumb bandwidth selector designed for the local polynomial regression proposed by Fan and Gijbels (1996, p.111).

square root of the (k, k) element of $s^2(x)(X'WX)^{-1}X'W^2X(X'WX)^{-1}$ where

$$s^2(x) = \frac{1}{\text{tr}\{W - WX(X'WX)^{-1}X'W\}} \sum_{i=1}^{N} \left\{ V_i - \sum_{k=0}^{p} \widehat{\beta}_k(\lambda_i - x)^k \right\}^2 K\left(\frac{\lambda_i - x}{h}\right).$$

The upper panel of Figure 4 shows the estimated curve using the local linear regression estimator with pooled data. The shape of the fitted curve shown as the solid line is very different from the linear regression fit shown as the dashed line. This suggests the plausibility of a nonlinear structure in the real exchange volatility curve. The lower panel of the figure shows the estimated first derivative of real exchange rate volatility curve based on the local polynomial regression, along with the confidence interval. The first derivative tends to be increasing in λ_i, which is consistent with the theoretical prediction. The slope of the curve is negative over the majority of the empirical range of λ_i with the exception of values close to one. The confidence intervals for the derivative include the positive region when λ_i is above 0.9, which could represent the dominating nominal effect.

Turning to a comparison between the quartic regression (6) and the nonparametric regression, both similarities and differences are evident. Both estimates imply convexity in the real exchange rate volatility curve. The most notable difference between the parametric and nonparametric estimates is the location of the bottom of the curve. The value of λ_i which minimizes the forecast error variance in the nonparametric regression is close to unity, much larger than the value predicted by the quartic regression.

To formally investigate the shape of the estimated curve, a nonparametric test of monotonicity developed by Ghosal, Sen and van der Vaart (2000) is employed. The testing procedure concerns the null hypothesis that the m function is an increasing function over a certain interval $[a, b]$, or $H_0 : m'(x) \geq 0$ for all $x \in [a, b]$. In the present context, the shape of the curve is examined over the observed range of the data, $[a, b] = [\lambda_{\min}, \lambda_{\max}]$. Their test statistic for

$H_0 : m'(x) \geq 0$ is defined as the supremum of a U-statistic over $x \in [a, b]$.[16] The test for the decreasing function, or $H_0 : m'(x) \leq 0$, can be similarly constructed by replacing λ_i by $1 - \lambda_i$. We also utilize the parametrically estimated value of $\underline{\lambda}$ and apply the test for $H_0 : m'(x) \leq 0$ using the range below $\underline{\lambda}$ and for $H_0 : m'(x) \geq 0$ using the range above $\underline{\lambda}$. Table 2 shows that the hypothesis of an increasing function in λ_i over $[a, b] = [\lambda_{\min}, \lambda_{\max}]$ is rejected, and that of decreasing function is not, based on a conventional significance level. At the same time, both hypotheses of decreasing function over $[a, b] = [\lambda_{\min}, \underline{\lambda}]$ and of increasing function over $[a, b] = [\underline{\lambda}, \lambda_{\max}]$ fail to be rejected. Therefore, the evidence from the nonparametric test on real exchange rate volatility curve does not contradict the U-shape prediction of the theory.

3.3 Variance decomposition

Let us now turn to the relative importance of the real and nominal effects at the sector level by directly using equation (4) at various horizons. Because the nominal shock, $\mu_t - \mu_t^* = \Delta s_t$, is observable according to the theory, the variance due to nominal effect can be estimated by $\lambda_i^2 \Lambda_{ik} \widehat{Var}(\Delta s_t)$, where $\widehat{Var}(\Delta s_t)$ is the sample variance of Δs_t. The relative contribution of nominal shocks to the k-period-ahead forecast error variance of the real exchange rate of

[16] Formally, the test statistic is defined as $\sup_{x \in [a,b]} \sqrt{N} U(x) / \widehat{\sigma}(x)$ with

$$\begin{aligned} U(x) &= \frac{2}{N(N-1)h^2} \sum_{1 \leq i < j \leq N} \operatorname{sign}(V_i - V_j) \operatorname{sign}(\lambda_i - \lambda_j) K\left(\frac{\lambda_i - x}{h}\right) K\left(\frac{\lambda_j - x}{h}\right) \text{ and} \\ \widehat{\sigma}^2(x) &= \frac{4}{3N(N-1)(N-2)h^4} \sum_{1 \leq i < j \leq N, i \neq j \neq k} \operatorname{sign}(\lambda_i - \lambda_j) \operatorname{sign}(\lambda_i - \lambda_k) \\ & \quad \times K^2\left(\frac{\lambda_i - x}{h}\right) K\left(\frac{\lambda_j - x}{h}\right) K\left(\frac{\lambda_k - x}{h}\right), \end{aligned}$$

where $\operatorname{sign}(x)$ denotes the sign function which takes 1 if $x > 0$, -1 if $x < 0$ and 0 otherwise. We follow the example of Ghosal et al.(2000) and use the quadratic kernel $K(x) = 0.75(1 - x^2)$ for $-1 < x < 1$ combined with $h = 0.5N^{-1/5}$.

sector i can be estimated by:

$$FEV_i(k) = \frac{\lambda_i^2 \Lambda_{ik} \widehat{Var}(\Delta s_t)}{\widehat{Var}_{t-k}(q_{it})},$$

where $\widehat{Var}_{t-k}(q_{it})$ is the sample variance of the quasi-difference $q_{it} - \lambda_i^k q_{i,t-k}$. For the limiting case of $k \to \infty$, we utilize the sample variance of q_{it} and $[\lambda_i^2/(1-\lambda_i^2)]\widehat{Var}(\Delta s_t)$ to measure the relative contribution of the nominal shocks to the unconditional variance. For the purpose of evaluating the relative role of shocks, this approach has an advantage over the direct estimation of the volatility curve in the sense that it allows for heterogeneous variance of real shocks across sectors. Furthermore, as shown in Appendix B, this approach is robust in the presence of other types of real shocks, such as preference shocks and markup shocks.

Table 3 reports the summary statistics for the contributions of the nominal shocks to the forecast error variance of sectoral real exchange rates at monthly horizons of $k = 1, 3, 6, 12$ and ∞. Note that, unlike the variance decomposition of aggregate real exchange rates often reported in the literature, the decomposition is calculated for each sector i. The first row of the table shows the average contribution of nominal shocks, with the average taken across all sectors and all four bilateral pairs. The numbers in parentheses in the second row are the standard deviations of these contributions across goods. The remaining rows report corresponding results for each pair of countries.

For the one-period-ahead forecast error decomposition, nominal shocks account for about 40 percent of real exchange rate variation and range from a high of 49 percent for Austria to a low of 35 percent for Spain. The large standard deviations in the table imply that the contributions of nominal shocks differ considerably across goods. This cross-sectional dispersion is similar across countries. At the monthly horizon it seems sensible to conclude that the contribution of real shocks is at least as large as that of nominal shocks for many goods.

The role of nominal shocks becomes smaller as the horizon lengthens. At a horizon of 6 months, the relative contribution is about one half of the 1-month horizon. The long-run contribution of nominal shocks, evaluated at $k = \infty$, is lower than 10 percent for all countries except for Austria, thus leaving 90 percent to be explained by real shocks.

Let us now compare the variance decompositions of sector-level real exchange rates with previous studies involving the aggregate real exchange rate. Using a structural VAR model, Clarida and Galí (1994, Table 3) find that the relative contribution of nominal shocks to one-period-ahead forecast error variance of quarterly real exchange rate is 47 percent for Germany and 36 percent for Japan. In contrast, our three-month (the counterpart to one quarter) ahead variance decomposition indicates nominal shocks account for between 19 to 31 percent, depending on the country (see Table 3). Using over 100 years of annual UK-US real exchange rate data, Rogers (1999) finds that the contribution of nominal shocks to the one-year-ahead forecast error variance ranges from 19 percent to 60 percent, with a median value of 41 percent. Our 12-month ahead forecast error variance decomposition estimates indicate nominal shocks only account for about 14 percent. The benchmark estimates of Eichenbaum and Evans (1995, Table 1a) show a nominal shock contribution at horizons of 31- to 36-months averaging 38 percent for France, while our estimates imply long-run contributions between 9 and 12 percent for France. Thus, largely independent of the horizon or countries examined, nominal shocks play a more important role in accounting for aggregate real exchange rate fluctuations than for sector-level real exchange rate fluctuations.

What accounts for this difference in the micro and macro evidence? Our suspicion is that the real shocks tend to average out across sectors. Recall productivity shocks in (2) are expected to embody idiosyncratic sector-specific shocks ε_{it} and ε_{it}^{*}, at least in part. In our model, the aggregation of goods prices eliminates this idiosyncratic component but has no effect on the nominal shocks, μ_t and μ_t^*, which are common across sectors. In particular, (3) implies that idiosyncratic components of q_{it} is $\sum_{j=0}^{\infty} \lambda_i^j (1-\lambda_i)(1-\lambda_i\beta)\psi\left(\varepsilon_{it-j}-\varepsilon_{it-j}^*\right)$ which is likely to be averaged out when the number of sectors increases. It is therefore not surprising that nominal and real shocks are more on par as contributors to real exchange rate variation at disaggregated level.

To see whether this averaging-out effect is present in our data, we can decompose the total variance of one-period ahead forecast error series, $e_{it} = q_{it} - \lambda_i q_{it-1}$, into the between-group variance and the within-group variance using the basic identity from the analysis of variance:

$$\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(e_{it}-\overline{e}\right)^2 = \frac{1}{T}\sum_{t=1}^{T}\left(e_t-\overline{e}\right)^2 + \frac{1}{T}\sum_{t=1}^{T}\left[\frac{1}{N}\sum_{i=1}^{N}\left(e_{it}-e_t\right)^2\right], \tag{7}$$

where $\overline{e} = (NT)^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T} e_{it}$ and $e_t = N^{-1}\sum_{i=1}^{N} e_{it}$. Substitution of (3) into (7) implies that the between-group variance (the first term) captures the contribution of common shocks $\lambda_i\Delta s_t + (1-\lambda_i)(1-\lambda_i\beta)\psi(\eta_t-\eta_t^*)$ and the within-group variance (the second term) captures the contribution of idiosyncratic shocks $(1-\lambda_i)(1-\lambda_i\beta)\psi(\varepsilon_{it}-\varepsilon_{it}^*)$. Using the analysis of variance, we find that the ratio of within-group variance to total variance is 0.50, 0.57, 0.54, 0.40 and 0.34, for the pooled case and the Austrian, Belgian, French and Spanish nation-specific cases, respectively. Thus, the contribution of idiosyncratic shocks in the total variance seems to be large, which helps to reconcile the differences between the micro and macro findings.

Our findings appear consistent with recent micro studies including Crucini and Telmer (2012) who show that only a small fraction of LOP changes are common to all goods, Broda and Weinstein (2008) who find that enormous volatility in the barcode prices is eliminated at the aggregate price level, and Bergin, Glick and Wu (2012) who claim that idiosyncratic industry price shocks account for about 80% of variation in LOP deviations. Thus, explaining the LOP volatility only by nominal exchange rates would leave most of the variation unaccounted for. The decomposition performed here fills this gap with common and sector-specific real shocks.

4 Conclusion

We use a Calvo-type pricing model with real and nominal shocks to develop the concept of a real exchange rate volatility curve. The volatility curve has a U-shape as a function of the degree of price stickiness, implying an ambiguous correlation between the forecast error variance of real exchange rates and the degree of price stickiness. Using US-European real exchange rate data, the correlation was found to be negative over the most of the range of observed degree of price stickiness. The downward-sloping profile suggested that for this micro-sample of goods and countries, real shocks account for most of the volatility of sectoral real exchange rates, though nominal shocks are important as well. The good with minimal real exchange rate volatility was estimated to have a duration between the price changes of about 4.8 months based on the benchmark quartic regression, and longer based on a nonparametric regression. However, to validate the generality of the theory, it is important to explore other samples of goods, cross-sections of countries and historical periods.

Our results also point to the value of examining cross-sectional differences in real exchange rate variability in order to flesh out the rich quantitative predictions of models of micro-price adjustment currently under development. Differences across goods help us to disentangle heterogeneous responses to common shocks due to differences in economic propagation mechanisms such as costs of price adjustment and trade costs from heterogeneity in the underlying shocks themselves. Averaging across goods, as is inevitable in the move to an aggregate real exchange rate, is not innocuous in terms of the weight given to real and nominal shocks. The same averaging may also lead to an under-appreciation of the sources of the risks that individuals and firms face. We hope to explore these possibilities in future work. Much remains to be done.

Appendix A. The model

This appendix derives the real exchange rate volatility curve used in our numerical and empirical analysis. The model is based on a symmetric two-country model with heterogeneous price stickiness. Due to the symmetry of the model, we mostly focus on the equations of the home country.

Households. Households choose consumption and labor supply over an infinite horizon subject to a cash-in-advance (CIA) constraint. Let $C_{it}(\upsilon)$, C_{it}, and C_t denote the consumption of brand υ of good i, the consumption of good i aggregated across brands, and aggregate consumption, respectively. Each country has a continuum of goods $i \in [0,1]$, each of which consists of a continuum of brand $\upsilon \in [0,1]$. Brands of good i in the home country are indexed by $\upsilon \in [0,1/2]$ while those in the foreign country are indexed by $\upsilon \in (1/2,1]$. We define C_{it} and C_t as $C_{it} = \left[\int_0^1 C_{it}(\upsilon)^{(\theta-1)/\theta} d\upsilon\right]^{\theta/(\theta-1)}$ and $C_t = \left[\int_0^1 C_{it}{}^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)}$, respectively. The home consumer maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U\left(C_t\right) - \chi V\left(L_t\right)\right], \tag{A.1}$$

where β is the household's discount factor and $E_0(\cdot)$ is the expectations operator. The functions are: $U\left(C_t\right) = C_t^{1-\sigma}/\left(1-\sigma\right)$ and $V\left(L_t\right) = L_t^{1-\phi}/\left(1-\phi\right)$, where $\sigma \geq 0$ and $\phi \geq 0$.

The households are faced with intertemporal budget and CIA constraints

$$M_t + E_t(\Upsilon_{t,t+1} D_{t+1}) = R_{t-1} W_{t-1} L_{t-1} + D_t + (M_{t-1} - P_{t-1} C_{t-1}) + T_t + \Pi_t \tag{A.2}$$

$$M_t \geq P_t C_t, \tag{A.3}$$

where M_t, D_{t+1}, and $\Upsilon_{t,t+1}$ denote cash holding, state-contingent bond holding denominated in the home currency, and the nominal stochastic discount factor, respectively. Thus, $M_t + E_t(\Upsilon_{t,t+1} D_{t+1})$ on the left hand side represents the nominal total value of wealth brought into the beginning of the period $t+1$. On the right hand side, the household receives labor income $(W_{t-1}L_{t-1})$ at the end of period $t-1$ and earns nominal interest R_{t-1} per unit of labor income within period t in terms of the home currency. The household also carries the nominal bonds in amount of D_t and the cash remaining after consumption in amount of $M_{t-1} - P_{t-1}C_{t-1}$, where P_t is the aggregate price index given by $P_t = \left[\int P_{it}{}^{1-\theta} di\right]^{1/(1-\theta)}$ and P_{it} is the price index for good i given by $P_{it} = \left[\int P_{it}(\upsilon)^{1-\theta} d\upsilon\right]^{1/(1-\theta)}$. Finally, the household receives nominal transfers from the government, T_t and the nominal profits from firms, Π_t.

The foreign consumer has the same preference and the same constraints except that we express the intertemporal budget constraint in the foreign currency with the nominal exchange rate S_t. In particular, the budget constraint for the foreign consumer is given by

$$M_t^* + E_t\left(\frac{\Upsilon_{t,t+1}D_{t+1}^*}{S_t}\right) = \frac{S_{t-1}R_{t-1}}{S_t}W_{t-1}^*L_{t-1}^* + D_t^* + \left(M_{t-1}^* - P_{t-1}^*C_{t-1}^*\right) + T_t^* + \Pi_t^*. \quad \text{(A.4)}$$

The first-order conditions are standard:

$$\frac{W_t}{P_t} = \chi\frac{V_L\left(L_t\right)}{U_C\left(C_t\right)}, \quad \text{(A.5)}$$

$$\Upsilon_{t,t+1} = \beta\left[\frac{U_C\left(C_{t+1}\right)}{U_C\left(C_t\right)}\frac{P_t}{P_{t+1}}\right] = \beta\left[\frac{U_C\left(C_{t+1}^*\right)}{U_C\left(C_t^*\right)}\frac{S_tP_t^*}{S_{t+1}P_{t+1}^*}\right] \quad \text{(A.6)}$$

$$M_t = P_tC_t, \quad \text{(A.7)}$$

where $U_C\left(C_t\right) = C_t^{-\sigma}$ is the marginal utility of consumption and $V_L\left(L_t\right) = L_t^{-\phi}$ is the marginal disutility of labor.

Assumption 1. $\sigma = 1$ *and* $\phi = 0$.

Imposing assumption 1 and combining the first-order conditions (A.5) and (A.7) yields

$$\frac{W_t}{P_t} = \chi\frac{M_t}{P_t}. \quad \text{(A.8)}$$

Using (A.6), the aggregate real exchange rate, Q_t, is proportional to the marginal utility ratio:

$$Q_t = \frac{S_tP_t^*}{P_t} = \kappa\frac{U_C\left(C_t^*\right)}{U_C\left(C_t\right)}, \quad \text{(A.9)}$$

where $\kappa = Q_0U_C\left(C_0\right)/U_C\left(C_0^*\right)$. In what follows, we assume that $\kappa = 1$. Furthermore, using (A.7) and (A.9) gives an expression for the nominal exchange rate:

$$S_t = \frac{M_t}{M_t^*}. \quad \text{(A.10)}$$

Firms. Firms in the home country sell their goods to home and foreign consumers.

Assumption 2. *Firm's technology is linear in labor:* $Y_{it}\left(\upsilon\right) = A_{it}L_{it}\left(\upsilon\right)$, *where* $Y_{it}(\upsilon)$ *is output and* $L_{it}(\upsilon)$ *is labor demand, for firms producing brand* υ *of good* i, *and* A_{it} *is the labor productivity which varies by good* i *but is common across brands.*

Recall that the firm must pay trade cost, τ, to send one unit of goods into another country.

Then, the resource constraint for the home market is

$$C_{it}(\upsilon) + (1+\tau)C^{*}_{it}(\upsilon) = A_{it}L_{it}(\upsilon) \text{ for } \upsilon \in [0, 1/2], \tag{A.11}$$

and the resource constraint for the foreign market is:

$$(1+\tau)C_{it}(\upsilon) + C^{*}_{it}(\upsilon) = A^{*}_{it}L^{*}_{it}(\upsilon) \text{ for } \upsilon \in (1/2, 1]. \tag{A.12}$$

Prices are set in the buyers' currency (local currency pricing). We introduce the Calvo-type nominal price rigidities for firm's pricing. Every month, a fraction λ_i of firms are not allowed to change prices; the remaining fraction of firms reset prices. We allow for heterogeneity of this Calvo parameter across goods whereas we assume that it is common across countries. The optimal price for firms in the home country to sell good i in the home country, $P_{H,it}$, solves

$$\underset{P_{H,it}}{\mathrm{Max}} E_t \sum_{h=0}^{\infty} \lambda_i^h \Upsilon_{t,t+h} \left[P_{H,it} - \frac{W_{t+h}}{A_{it+h}} \right] \left(\frac{P_{H,it}}{P_{it+h}} \right)^{-\theta} C_{it+h},$$

where W_t/A_{it} is the nominal marginal cost of production. Note that all firms face the common nominal marginal cost, implying that the optimal price is independent of υ. The optimal price for firms in the foreign country to sell good i in the home country, $P_{F,it}$, solves

$$\underset{P_{F,it}}{\mathrm{Max}} E_t \sum_{h=0}^{\infty} \lambda_i^h \Upsilon_{t,t+h} \left[P_{F,it} - (1+\tau)\frac{S_{t+h}W^{*}_{t+h}}{A^{*}_{it+h}} \right] \left(\frac{P_{F,it}}{P_{it+h}} \right)^{-\theta} C_{it+h}.$$

Shocks and model identities. We follow Kehoe and Midrigan (2007) and assume that the money growth rates μ_t and μ^{*}_t are i.i.d., such that the model predicts random walk behavior of the nominal exchange rate, consistent with the data. As stated in the main text, real shock is defined by (2).

Total transfers from the governments satisfy $T_t = M_t - M_{t-1} - (R_{t-1} - 1)W_{t-1}L_{t-1}$ and $T^{*}_t = M^{*}_t - M^{*}_{t-1} - (R_{t-1}S_{t-1}/S_t - 1)W^{*}_{t-1}L^{*}_{t-1}$. Hence, the total transfers in each country equal domestic money injections and interest payment of the government to correct distortion in the intra-temporal first-order condition for the households. The profits of firms accrue exclusively to consumers in the same country: $\Pi_t = \int_{i=0}^{1}\int_{\upsilon=0}^{1/2} \Pi_{it}(\upsilon) d\upsilon di$ and $\Pi^{*}_t = \int_{i=0}^{1}\int_{\upsilon=1/2}^{1} \Pi^{*}_{it}(\upsilon) d\upsilon di$. The labor market clearing conditions are $L_t = \int_{i=0}^{1}\int_{\upsilon=0}^{1/2} L_{it}(\upsilon) d\upsilon di$ and $L^{*}_t = \int_{i=0}^{1}\int_{\upsilon=1/2}^{1} L^{*}_{it}(\upsilon) d\upsilon di$. The state-contingent bond market clearing condition is $D_t + D^{*}_t = 0$. Finally, good market clearing conditions are given by (A.11) and (A.12).

Equilibrium dynamics. To derive the real exchange rate volatility curve, we approximate the first-order conditions and resource constraints by the log deviation from the steady state and derive the reduced form solution for the real exchange rates. In what follows, unless otherwise indicated, the log deviation of a generic variable X_t from the steady state is expressed as $x_t = \ln(X_t/X)$, where X is the steady state value of X_t. Also, we normalize all nominal prices by dividing by the nominal money stock in the place of consumption and by multiplying the labor productivity in the place of production to ensure the stationarity. For example, the normalized price of the nominal home price index for good i is given by $\bar{P}_{it} = P_{it}A_{it}/M_t$. Likewise, the normalized optimal reset prices for the home market are $\bar{P}_{H,it} = P_{H,it}A_{it}/M_t$ and $\bar{P}_{F,it} = P_{F,it}A^*_{it}/M_t$.

The log-linearized first-order conditions are:

$$\bar{p}_{H,it} = \sum_{h=1}^{\infty}(\lambda_i\beta)^h E_t\left[\mu_{t+h} - g^{A_i}_{t+h}\right] = \lambda_i\beta\left(\eta_t + \varepsilon_{it}\right) \tag{A.13}$$

$$\bar{p}_{F,it} = \sum_{h=1}^{\infty}(\lambda_i\beta)^h E_t\left[\mu_{t+h} - g^{A^*_i}_{t+h}\right] = \lambda_i\beta\left(\eta^*_t + \varepsilon^*_{it}\right), \tag{A.14}$$

where $g^{A_i}_{t+1} = a_{it+1} - a_{it}$. Equations (A.13) and (A.14) indicate that the optimal reset prices are determined only by exogenous shocks. The second equality in each equation arises from our assumptions about the stochastic processes of the shocks.

We next turn to the price index for good i. Due to Calvo pricing, $P^{1-\theta}_{it} = \lambda_i P^{1-\theta}_{it-1} + (1-\lambda_i)\left(P^{opt}_{it}\right)^{1-\theta}$, where P^{opt}_{it} denotes the composite of domestic and foreign reset prices in the home country, defined as $P^{opt}_{it} = \left[(1/2)\,P^{1-\theta}_{H,it} + (1/2)\,P^{1-\theta}_{F,it}\right]^{1/(1-\theta)}$. Log-linearizing the above indexes yields

$$\bar{p}_{it} = \lambda_i\bar{p}_{it-1} - \lambda_i\mu_t + \lambda_i g^{A_i}_t + (1-\lambda_i)\,\bar{p}^{opt}_{it} \tag{A.15}$$

$$\bar{p}^{opt}_{it} = \omega\bar{p}_{H,it} + (1-\omega)\left(\bar{p}_{F,it} + \eta_t + \varepsilon_{it} - \eta^*_t - \varepsilon^*_{it}\right), \tag{A.16}$$

where ω is the expenditure share of the household in the home country given by $1/[1+(1+\tau)^{1-\theta}]$.

Let the sectoral real exchange rate be $Q_{it} = S_t P^*_{it}/P_{it}$. Note that the log real exchange rate at the sector level can be expressed by $q_{it} = \bar{p}^*_{it} - \bar{p}_{it} + a_{it} - a^*_{it}$. Then, (A.13) - (A.15) yields

$$q_{it} = \lambda_i q_{it-1} + \lambda_i(\mu_t - \mu^*_t) + (1-\lambda_i)(1-\lambda_i\beta)\psi\left(a_{it} - a^*_{it}\right), \tag{A.17}$$

where $\psi = 2\omega - 1 = \left[\left(1-(1+\tau)^{1-\theta}\right)/\left(1+(1+\tau)^{1-\theta}\right)\right]$. To derive (A.17), we used the fact that $\eta_t + \varepsilon_{it} - \eta_t^* - \varepsilon_{it}^*$ can be replaced with $a_{it} - a_{it}^*$ because a common stochastic trend z_t cancels out of the productivity differential.

Equation (A.17) describes equilibrium dynamics of real exchange rates at the sector level. The real exchange rate follows the AR(1) process and it immediately follows that the k-period-ahead forecast error variance is given by (4):

Appendix B. Model robustness

We present the robustness of the U-shape in the real exchange rate volatility curve by introducing two additional real shocks into the model: a preference shock in marginal labor disutility χ and a markup shock arising from variations in θ. We replace χ and θ by χ_t and θ_t, respectively. These shocks are country-specific so we use χ_t^* and θ_t^* for the foreign country counterparts while the steady state values are common across countries. We assume that these shocks follow i.i.d. processes.

Given the above assumptions, the normalized log optimal prices can be written as

$$\begin{aligned} \bar{p}_{H,it} &= (1-\lambda_i\beta)\left[\hat{\chi}_t - \frac{\hat{\theta}_t}{\theta-1}\right] + \lambda_i\beta\left(\eta_t + \varepsilon_{it}\right) && \text{(B.1)} \\ \bar{p}_{F,it} &= (1-\lambda_i\beta)\left[\hat{\chi}_t^* - \frac{\hat{\theta}_t}{\theta-1}\right] + \lambda_i\beta\left(\eta_t^* + \varepsilon_{it}^*\right), && \text{(B.2)} \end{aligned}$$

where $\hat{\theta}_t = \ln(\theta_t/\theta)$, $\hat{\chi}_t = \ln(\chi_t/\chi)$, and $\hat{\chi}_t^* = \ln(\chi_t^*/\chi)$. Note that $\bar{p}_{F,it}$ has $\hat{\chi}_t^*$ rather than $\hat{\chi}_t$ because the labor in the foreign country is used for the production of the imported goods. In contrast, $\bar{p}_{F,it}$ remains affected by $\hat{\theta}_t$, because the markup is determined by the elasticity of substitution at the place of consumption.

Turning to the log-linearized price indexes, (A.15) continues to hold but (A.16) must be replaced by

$$\bar{p}_{it}^{opt} = \omega\bar{p}_{H,it} + (1-\omega)\left(\bar{p}_{F,it} + \eta_t + \varepsilon_{it} - \eta_t^* - \varepsilon_{it}^*\right) - \Psi\hat{\theta}_t, \qquad \text{(B.3)}$$

where the coefficient Ψ is defined as:

$$\begin{aligned} \Psi &= \frac{\theta}{\theta-1}\left\{\ln\bar{P}_{ii}^{opt} - \left[\omega\ln\bar{P}_{Hi} + (1-\omega)\ln\bar{P}_{Fi}\right]\right\} \\ &= \frac{\theta}{(\theta-1)^2}\left[(1-\omega)\ln(1-\omega) + \omega\ln\omega + \ln(2)\right] > 0, \end{aligned}$$

The second equality obtained from the definitions of ω and Ψ can be proved to be positive for any $\tau > 0$ and $\theta > 1$.[17] Combining (A.15) and (B.1) - (B.3), we have the equation for q_{it}:

$$
\begin{aligned}
q_{it} &= \lambda_i q_{it-1} + \lambda_i(\mu_t - \mu_t^*) + (1-\lambda_i)(1-\lambda_i\beta)\psi\left(a_{it} - a_{it}^*\right) \\
&\quad - (1-\lambda_i)(1-\lambda_i\beta)\psi\left(\hat{\chi}_t - \hat{\chi}_t^*\right) + (1-\lambda_i)\left(\frac{1-\lambda_i\beta}{\theta-1} + \Psi\right)\left(\hat{\theta}_t - \hat{\theta}_t^*\right). \qquad \text{(B.4)}
\end{aligned}
$$

Consequently, the extended version of the real exchange rate volatility curve is:

$$
\begin{aligned}
& Var_{t-k}(q_{it}) \\
= \; & \left(\sum_{j=1}^{k} \lambda_i^{2(j-1)}\right)\left[\lambda_i^2 Var\left(\mu_t - \mu_t^*\right) + (1-\lambda_i)^2(1-\lambda_i\beta)^2\psi^2\left[Var\left(a_{it} - a_{it}^*\right) + Var\left(\hat{\chi}_t - \hat{\chi}_t^*\right)\right]\right. \\
& \left. +(1-\lambda_i)^2\left(\frac{1-\lambda_i\beta}{\theta-1} + \Psi\right)^2 Var\left(\hat{\theta}_t - \hat{\theta}_t^*\right)\right]. \qquad \text{(B.5)}
\end{aligned}
$$

In (B.5), all the coefficients on the variance of real shocks are decreasing in λ_i. By contrast, the coefficient on the variance of nominal shock is increasing in λ_i. Therefore, for any forecast horizon, an increase in λ_i decreases the contribution of real shocks and increases the contribution of nominal shock.

[17] The detail of the derivation is available upon request.

References

[1] Alvarez, L. J., and I. Hernando. 2006. "Price setting behaviour in Spain: Stylised facts using consumer price micro data," *Economic Modelling*, 24 (4), 699-716.

[2] Aucremanne, L., and E. Dhyne. 2004. "How frequently do prices change? Evidence based on the micro data underlying the Belgian CPI," National Bank of Belgium Working Paper, No. 44.

[3] Baudry, L., H. Le Bihan, P. Sevestre, and S. Tarrieu. 2007. "What do thirteen million price records have to say about consumer price rigidity?" *Oxford Bulletin of Economics and Statistics*, 69 (2), 139-183.

[4] Baumgartner, J., E. Glatzer, F. Rumler, and A. Stiglbauer. 2005. "How frequently do consumer prices change in Austria?" ECB Working Paper Series, No. 523.

[5] Bergin, P. R., R. Glick, and J.-L. Wu. 2012. "The micro-macro disconnect of purchasing power parity," *Review of Economics and Statistics*, forthcoming.

[6] Bils, M., and P. J. Klenow. 2004. "Some evidence on the importance of sticky prices," *Journal of Political Economy*, 112 (5), 947-985.

[7] Broda, C., and D. E. Weinstein. 2008. "Understanding international price differences using barcode data," NBER Working Paper, No. 14017.

[8] Carvalho, C., and F. Nechio. 2011. "Aggregation and the PPP puzzle in a sticky price model," *American Economic Review*, 101 (6), 2391-2424.

[9] Chari, V. V., P. J. Kehoe, and E. R. McGrattan. 2002. "Can sticky price models generate volatile and persistent real exchange rates?" *Review of Economics Studies*, 69 (3), 533-563.

[10] Clarida, R., and J. Galí. 1994. "Sources of real exchange-rate fluctuations: How important are nominal shocks?" *Carnegie-Rochester Conference Series on Public Policy*, 41 (1), 1-56.

[11] Crucini, M. J., M. Shintani, and T. Tsuruga. 2010. "The law of one price without the border: the role of distance versus sticky prices," *Economic Journal*, 120, 462-480.

[12] Crucini, M. J., and C. Telmer. 2012. "Microeconomic sources of real exchange rate variability," NBER Working Paper, No. 17978.

[13] Dornbusch, R., S. Fischer, and R. Startz. 2004. *Macroeconomics*. McGraw-Hill, New York.

[14] Eichenbaum, M., and C. Evans. 1995. "Some empirical evidence on the effects of monetary policy shocks on exchange rates," *Quarterly Journal of Economics*, 110, 975–1010.

[15] Fan, J., and I Gijbels. 1996. *Local Polynomial Modelling and Its Applications*. Chapman & Hall. London.

[16] Ghosal, S., A. Sen, and A. W. van der Vaart. 2000. "Testing monotonicity of regression," *The Annals of Statistics*, 28 (4), 1054-1082.

[17] Kehoe, P. J., and V. Midrigan. 2007. "Sticky prices and sectoral real exchange rates," Federal Reserve Bank of Minneapolis Working Paper, No. 656.

[18] Ramsey, J. B., 1969. "Tests for specification errors in classical linear least-squares regression analysis," *Journal of the Royal Statistical Society B,* 31, 350–371.

[19] Rogers, J. H., 1999. "Monetary shocks and real exchange rates," *Journal of International Economics*, 49 (2), 269-288.

[20] Steinsson, J., 2008. "The dynamic behavior of the real exchange rate in sticky price models," *American Economic Review*, 98 (1), 519-533.

[21] Stockman, A. C., 1980. "A theory of exchange rate determination," *Journal of Political Economy*, 88 (4), 673-698.

Table 1: Structural regressions

	λ_i^2	$(1-\lambda_i)^2(1-\lambda_i\beta)^2$	Adj. R^2_{uc}	$\underline{\lambda}$
Pooled	0.0016 (0.0001)	0.0718 (0.0076)	0.789	0.793 (0.008)
Austria	0.0012 (0.0001)	0.0475 (0.0066)	0.895	0.784 (0.011)
Belgium	0.0021 (0.0004)	0.0419 (0.0056)	0.623	0.735 (0.017)
France	0.0015 (0.0001)	0.0411 (0.0040)	0.905	0.763 (0.009)
Spain	0.0017 (0.0002)	0.1567 (0.0292)	0.716	0.836 (0.011)

Notes: The heteroskedasticity-consistent standard errors are in parentheses. The "Adj. R^2_{uc}" denotes the adjusted uncentered R^2. $\underline{\lambda}$ is the estimate of λ which minimizes the total variance.

Table 2: Tests of monotonicity

H_0 for range	$m'(x) \leq 0$ $[\lambda_{\min}, \lambda_{\max}]$	$m'(x) \geq 0$ $[\lambda_{\min}, \lambda_{\max}]$	$m'(x) \leq 0$ $[\lambda_{\min}, \underline{\lambda}]$	$m'(x) \geq 0$ $[\underline{\lambda}, \lambda_{\max}]$
Pooled	-1.831 (1.000)	10.329 (0.001)	-1.822 (1.000)	10.329 (0.182)
Austria	-1.546 (1.000)	4.927 (0.026)	-1.546 (1.000)	4.783 (0.587)
Belgium	-0.647 (1.000)	4.608 (0.051)	-0.643 (1.000)	4.535 (0.708)
France	-1.212 (1.000)	6.369 (0.004)	-1.212 (1.000)	6.243 (0.578)
Spain	-1.281 (1.000)	5.098 (0.067)	-1.281 (1.000)	3.906 (0.767)

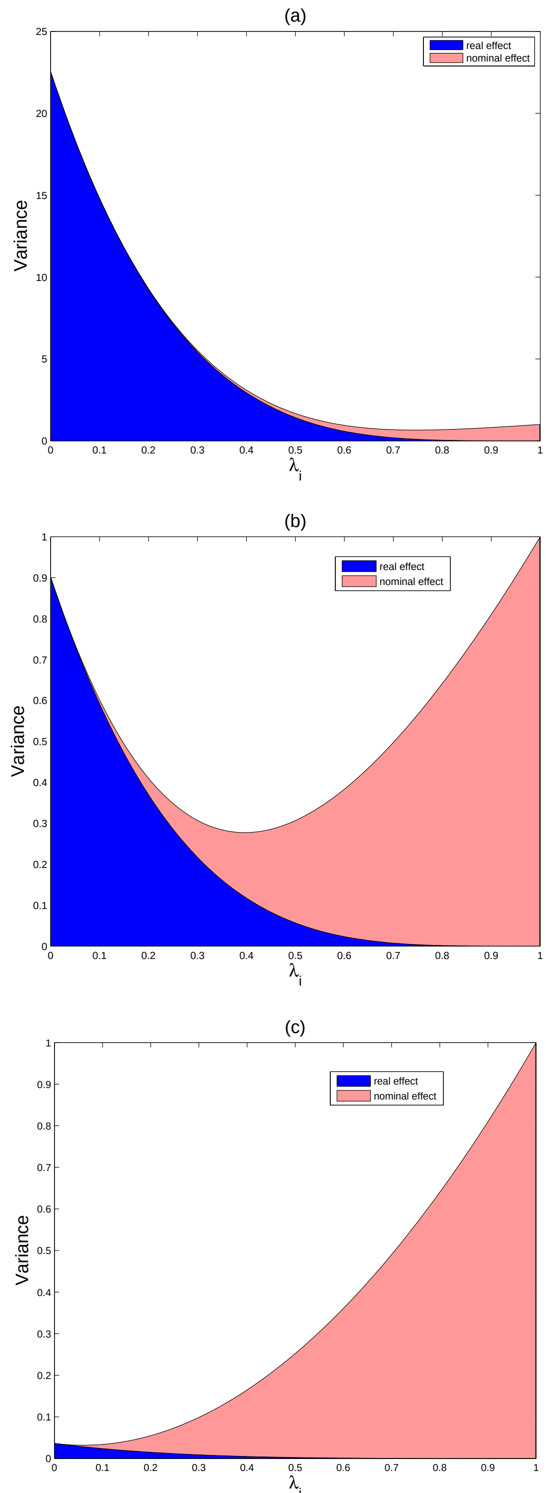
Notes: Numbers in parentheses are P-values computed using the procedure explained in Ghosal et al.(2000).

Table 3: Percentage of forecast error variance accounted for by nominal shocks

k	1	3	6	12	∞
Pooled	40.6	23.6	18.7	14.2	11.4
	(24.1)	(16.5)	(15.7)	(13.5)	(11.8)
Austria	48.6	30.5	25.7	20.3	17.1
	(24.4)	(16.9)	(17.1)	(15.7)	(16.3)
Belgium	34.9	19.9	15.3	11.4	8.9
	(23.0)	(14.9)	(13.6)	(11.2)	(8.2)
France	40.2	21.8	16.2	11.7	9.2
	(22.7)	(15.2)	(13.5)	(10.2)	(7.6)
Spain	35.2	18.9	14.6	11.1	7.9
	(24.5)	(16.4)	(15.6)	(13.3)	(8.0)

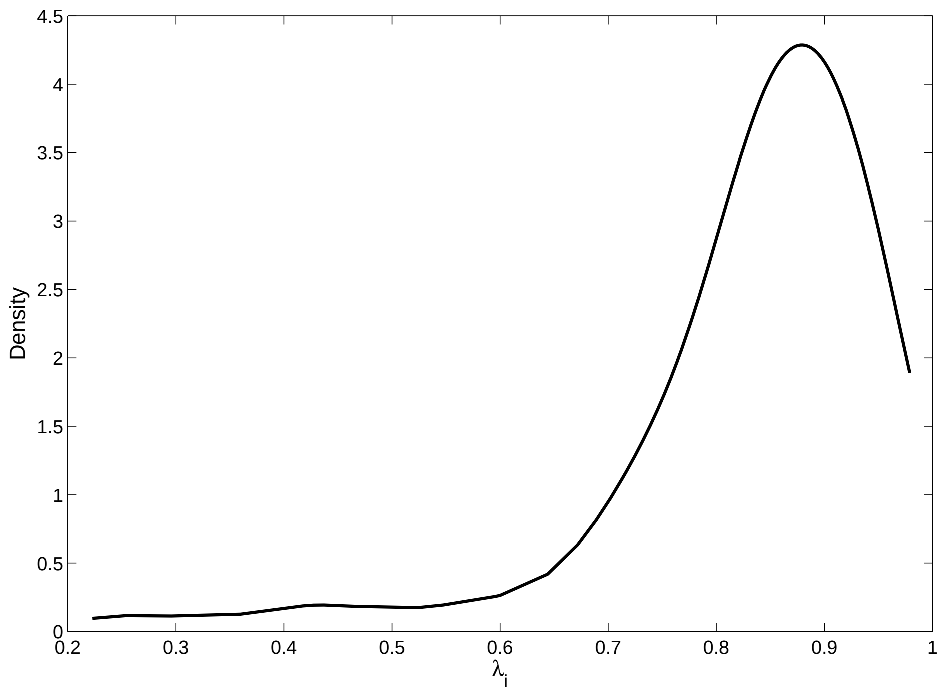
Notes: Numbers are in percent. Each column corresponds to the cross-sectional average of the k-period-ahead forecast error variance of sector-level real exchange rates accounted for by nominal shocks. Numbers in parentheses are standard deviations.

Figure 1: Simulated real exchange rate volatility curves



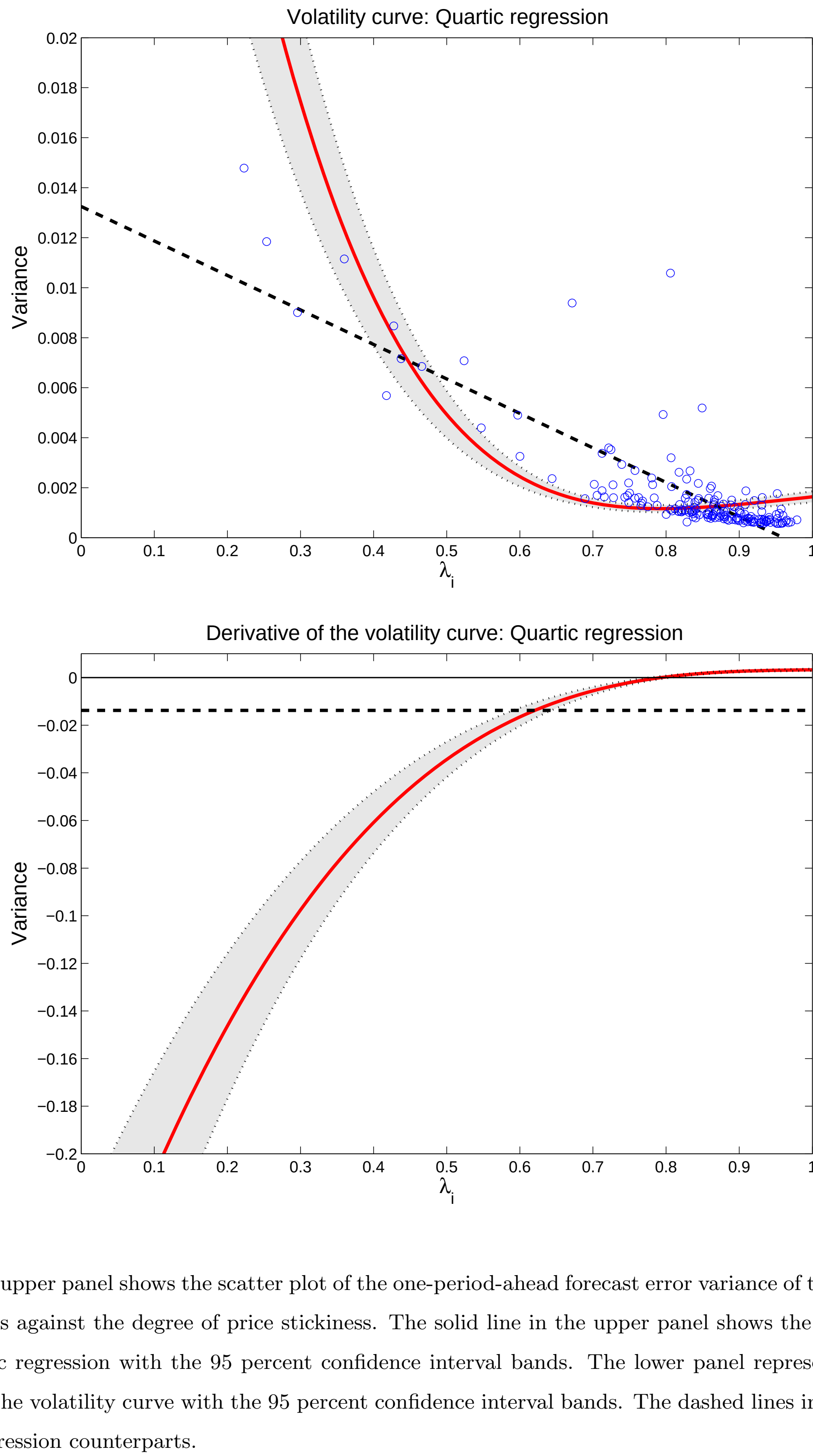
NOTES: Each panel of the figure shows contributions of the nominal and real shocks on the sector-level real exchange rate volatility over the range of the degree of price stickiness. The volatility is measured by the one-period-ahead forecast error variance. For each good, the variances due to real and nominal shocks are expressed by the heights of blue and red areas. The combined height of the two areas corresponds to the total variance. Panels (a), (b) and (c) show the cases of $Std(a_{it} - a_{it}^{*})/Std(\mu_t - \mu_t^{*}) = 5$, 1, and 1/5, respectively. The standard deviation of the nominal shocks is set to one.

Figure 2: Empirical distribution of price stickiness



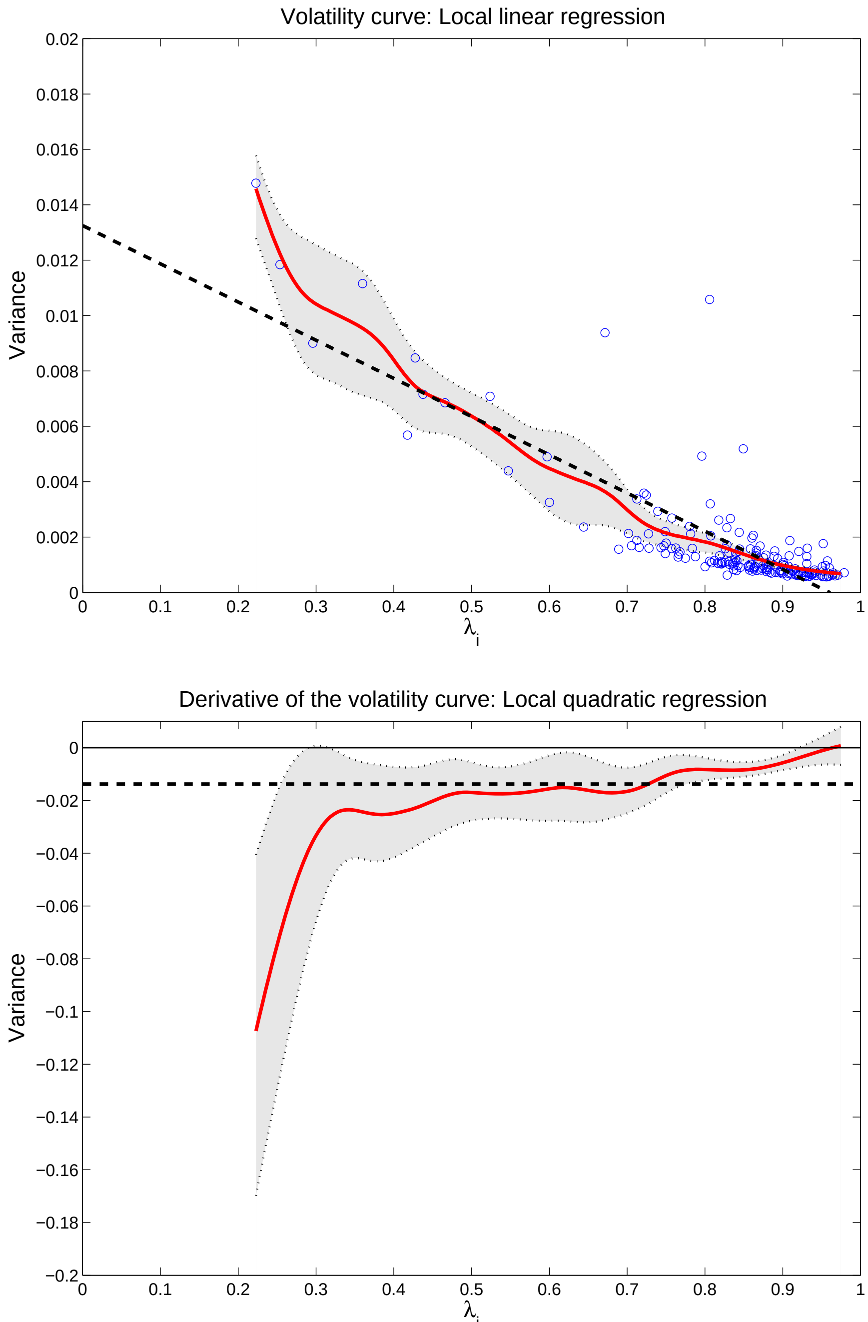
NOTES: The kernel density estimate of pooled λ_i's using standard Gaussian kernel combined with Silverman's rule of thumb for the bandwidth.

Figure 3: Estimated real exchange rate volatility curves: Parametric regression



NOTES: The upper panel shows the scatter plot of the one-period-ahead forecast error variance of the sector-level real exchange rates against the degree of price stickiness. The solid line in the upper panel shows the fitted curve based on the quartic regression with the 95 percent confidence interval bands. The lower panel represents the estimated derivative of the volatility curve with the 95 percent confidence interval bands. The dashed lines in both panels show the linear regression counterparts.

Figure 4: Estimated real exchange rate volatility curves: Nonparametric regression



NOTES: The upper panel shows the scatter plot of the one-period-ahead forecast error variance of the sector-level real exchange rates against the degree of price stickiness. The solid line in the upper panel shows the fitted curve based on the nonparametric regression with the 95 percent confidence interval bands. The lower panel represents the estimated derivative of the volatility curve with with the 95 percent confidence interval bands. The dashed lines in both panels show the linear regression counterparts.