Prudential Capital Controls or Bailouts? The Impact of Different Collateral Constraint Assumptions*

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Abstract

The literature on small open economy models with collateral constraints has provided the theoretical grounds for macroprudential regulations. This paper examines a subsidy on debt during a crisis as a form of bailout in comparison to prudential capital controls. We show that the policy prescription on bailouts differs substantially between the timing assumptions of the collateral constraint of households. If borrowing is constrained by the value of assets that households have purchased before they borrow, the bailout is neutral, suggesting that prudential capital controls are preferable. If, on the other hand, households collateralize their assets that they purchase at the same time as their borrowing, the bailout replicates the unconstrained allocation without collateral constraint and outperforms prudential capital controls. Even in the latter case, however, our numerical experiments suggest that such bailouts restoring the unconstrained allocation may not be implementable in terms of its size and frequency.

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1 Introduction

Should policymakers in emerging market economies rely on macroprudential policies in normal times or bail out borrowers during a financial crisis? Recent studies have highlighted the importance of prudential controls on cross-border capital flows and macroprudential regulations to prevent inefficient boom-bust cycles. Among others, Jeanne and Korinek (2010, hereafter JK), Bianchi (2011), and Bianchi and Mendoza (2012) emphasize pecuniary externalities in the model with an occasionally binding collateral constraint. Based on this framework, these early studies advocate that the government should preemptively impose a Pigouvian tax on debt in normal times to internalize externalities.[1]

This paper investigates the roles of bailouts during crises to explore policy prescriptions for coping with crises. In particular, we consider two assumptions on the collateral constraint and investigate how the difference in the assumptions influences the efficacy of bailouts. The two collateral constraints differ on the timing in which households' assets are collateralized. To emphasize the timing, we call the two collateral constraints a "beginning-of-period" or an "end-of-period" collateral constraint, respectively. The "beginning-of-period" collateral constraint is assumed in JK. Under this constraint, the households collateralize the assets that they have purchased before they borrow. On the other hand, under the "end-of-period" collateral constraints, households collateralize the assets that they purchase at the same time as borrowing. This end-of-period collateral constraint has also been employed by other studies (e.g., Kiyotaki and Moore 1997 and Bianchi and Mendoza 2012).

We find that the choice between the two assumptions is not innocuous. We show that, under the beginning-of-period collateral constraint, subsidizing debt during crises as a form of bailout cannot achieve better allocation than that under the laissez-faire economy. In other words, the bailout is neutral. By contrast, the bailout is highly effective under the "end-of-period" collateral constraint. The subsidy on debt can replicate the unconstrained allocation as if the collateral constraint is not binding.

Our theoretical results deliver important implications for the debate on prudential capital controls versus bailouts. As JK and others suggest, prudential capital controls can achieve the constrained-efficient allocation. If the beginning-of-period collateral constraint is more plausible, a policy prescription is that policymakers should rely on the prudential capital controls via the Pigouvian tax on debt. This is because a bailout through the debt subsidy

[1] More recent papers include Benigno et al. (2012, 2013, 2014), Bianchi (2013), Bianchi and Mendoza (2013), Jeanne and Korinek (2012, 2013), Korinek and Simsek (2014), and Dávila (2015). Farhi and Werning (2012, 2013) and Schmitt-Grohé and Uribe (2012, 2015) argue for prudential capital controls under nominal price or wage rigidities.

is neutral in terms of welfare. Conversely, under the end-of-period collateral constraint, the bailout can restore the unconstrained allocation and thus is more desirable than the prudential capital controls.

If bailouts are theoretically desirable, the next question for policymakers would be whether the bailouts are practically feasible. To answer this question, we perform numerical experiments and assess the size and frequency of policy interventions in credit markets. Our simulation results indicate that unrealistically large-scale bailouts and extremely frequent interventions are required to restore the unconstrained allocation. In fact, on average, a lump-sum tax equivalent to as much as 31.6 percent of annual household income must be imposed to achieve the unconstrained allocation. In terms of the frequency, the government needs to intervene almost every year. These results point to a large gap compared with the actual observation because very few – perhaps no – governments in emerging market economies have embarked on such large bailouts with such a high frequency.

Therefore, we conclude that bailouts should be implemented with caution. The efficacy of the subsidy during a crisis is highly sensitive to timing assumptions of the collateral constraint. Even if the subsidy is theoretically a useful policy option that can restore the unconstrained allocation, such a subsidy would be practically infeasible. Alternatively, our results may imply that the model setup used in the literature may not be the best suited to provide robust policy prescriptions on bailouts.

In the literature, early studies have noted that the government can restore the unconstrained allocation in some models with an occasionally binding collateral constraint. For example, using a model similar to Bianchi (2011), Benigno et al. (2014, hereafter BCORY) show that, if the government can use additional distortionary policy instruments on top of capital controls (e.g., a price support policy in the form of a subsidy on collateral or collateralized nontradable goods), then the collateral constraint can always be removed and the government can achieve the unconstrained allocation.[2] Our finding starkly contrasts with BCORY. In this paper, "bailout" refers to the subsidy on foreign debt. For the sake of comparison between prudential capital controls and bailouts, we assume that the government is given a single policy instrument that affects the cost of borrowing from abroad (i.e., intervention in the credit market). In our model, the government can achieve the unconstrained allocation without relying on additional distortionary policy measures such as

[2] See also Benigno et al. (2012) and Jeanne and Korinek (2013) for the case where multiple policy instruments are available for crisis management. Schmitt-Grohé and Uribe (2015) show that the open economy model with downward nominal wage rigidities can give rise to pecuniary externalities. They argue that bailouts by devaluation of the country's currency could restore the unconstrained allocation. Likewise, using a model of banking, Green (2010) argues that bailouts can lead to a socially efficient outcome.

outright purchase of the collateral and a subsidy on nontradable good consumption.

This paper is organized as follows. Section 2 presents two collateral constraints in a simple two-period model with the collateral constraint. Section 3 discusses the debt subsidy as a bailout in the two-period model. Section 4 performs numerical experiments. Section 5 concludes.

2 Two collateral constraints

We begin with a two-period version of the small open economy model discussed in JK. Suppose that the utility of identical atomistic households is given by $u(c_1) + c_2$, where $u(c) = c^{1-\sigma}/(1-\sigma)$, $\sigma > 0$, and c_t represents consumption in period t. Domestic households' budget constraints for each period are given by

$$c_1 + d_1 + p_1\theta_2 = e + d_2 + p_1\theta_1, \tag{1}$$

$$c_2 + d_2 = \theta_2 y, \tag{2}$$

where d_t is the debt to be repaid at the beginning of the period t and θ_t represents the domestic collateral held by the households at the beginning of period t. Here, p_t is the price of collateral traded in a competitive market. The world interest rate is set to zero for simplicity. At the beginning of period 1, the households initially hold θ_1 unit of collateral and d_1 unit of debt. In this period, the households have three sources of inflow: sales of collateral $p_1\theta_1$, new borrowing d_2, and an endowment e that is not pledgeable to foreign lenders. They use them for consumption c_1, repaying d_1, and purchasing collateral θ_2. In period 2, the households must repay d_2 after receiving returns on collateral y. Following JK, we assume that the return on collateral can be acquired only by domestic agents and the value of collateral in period 2 is lost after the households receive y. We also assume that the supply of collateral assets is inelastic and normalized to one. In this model, the linear utility in period 2 implies that, if there is no collateral constraint, consumption in period 1 is unity (i.e., the first-best level of consumption).

The model introduces a collateral constraint for d_2. As JK discuss, a low value of e may result in the binding collateral constraint and precipitate a crisis (e.g., a sudden stop in capital inflows). With the binding collateral constraint in period 1, the desired borrowing is generally impossible and the households must accept a large reduction in c_1. As such, period 1 corresponds to the period of a crisis.

2.1 The beginning-of-period collateral constraint

Each household faces a collateral constraint of the form:

$$d_2 \leq \phi\theta_1 p_1. \tag{3}$$

Note that, following JK, the borrowing capacity is constrained by the market value of collateral at the *beginning* of period 1. The parameter $\phi \in (0, 1]$ represents the ceiling on the leverage in the collateral constraint.[3] Also, θ_1 must be unity because the supply of collateral asset is one. Not surprisingly, pecuniary externalities arise from the feedback loop between the collateral price and borrowing. When a sufficiently low e takes place, the collateral constraint binds. The households try to prevent consumption reduction by decreasing net demand for their collateral. The households' deleveraging results in declines in collateral prices, and the decline in p_1 further tightens their collateral constraints. While each atomistic household takes p_1 as given, the households' decision as a whole has the general equilibrium effect on p_1. As a result, the general equilibrium effect cannot be internalized by price-taking households and the laissez-faire equilibrium is not generally Pareto efficient. Hence, the previous studies widely discuss the "macroprudential policies" in a pre-crisis period to internalize the pecuniary externality.[4] JK show that a Pigouvian tax on the debt repayment can replicate the constrained efficient allocation solved by the social planner.[5]

2.2 The end-of-period collateral constraint

As a variant of the collateral constraint (3), consider

$$d_2 \leq \phi\theta_2 p_1, \tag{4}$$

where the value of collateral is evaluated by the *end*-of-period holding of collateral θ_2, rather than θ_1. This "end-of-period" formulation means that the asset (e.g., house, real estate, etc.) that households are about to purchase can be collateralized against the new borrowing. This is in line with common practice in mortgage contracts, which specify certain collateral property. In a typical mortgage contract, the collateral is the newly purchased property

[3] We consider a slightly more general constraint than JK because they assume $\phi = 1$.

[4] Examples of this research include Bianchi (2011), Korinek (2011), Jeanne and Korinek (2012), Bianchi and Mendoza (2012) and Benigno et al. (2012, 2013, 2014).

[5] JK follow the "constrained efficiency" definition discussed in Stiglitz (1982) and Kehoe and Levine (1993). See also JK for details of the constrained social planner's problem.

financed by the new loan. By contrast, the "beginning-of-period" constraint assumes that the newly purchased assets cannot be collateralized against the new borrowing. JK show that, even with the difference between the two collateral constraints, the key results on prudential taxes remain unaffected:[6] (i) the same collateral constraint in equilibrium (i.e., $d_2 \leq \phi p_1$); (ii) the same feedback loop between the collateral price and borrowing; and (iii) the same form of the Pigouvian tax. These results remain essentially unaltered even under the infinite-horizon setting.

3 The efficacy of the debt subsidy

We consider how changes in the assumption on the collateral constraint affect the efficacy of bailouts. For simplicity, we assume in this section that e is deterministic rather than stochastic. This simplification allows us to obtain an explicit solution for the optimal debt subsidy, but has no effect on our argument.

To introduce bailouts, we replace (1) by

$$c_1 + d_1 + p_1\theta_2 = e + (1+s)\,d_2 - S + p_1\theta_1, \tag{5}$$

where $s \geq 0$ is a subsidy on debt and S is the lump-sum tax, satisfying $S = sd_2$.[7] A balanced government budget ensures that household resources are kept unchanged both intra- and inter-temporally. The only distinction between the prudential capital controls and bailouts is whether to raise the cost of debt before a crisis or to reduce it during one.

In the following two subsections, we will present propositions on the optimal subsidy under the two differing collateral constraints, (3) and (4). Then, we will interpret the two propositions in the context of policy implications.

[6] See footnote 4 in JK.

[7] Following Jeanne and Korinek (2013), our policy analysis rules out the possibility that the government uses non-distortionary inter-temporal lump-sum taxes and transfers to fully relax the collateral constraint.

3.1 The beginning-of-period collateral constraint

The households maximize their utility $u(c_1) + c_2$, subject to the budget constraints (5), (2) and the beginning-of-period collateral constraint (3). The first-order conditions are

$$(1+s)\,u'(c_1) = 1 + \lambda_m \tag{6}$$

$$p_1 = \frac{y}{u'(c_1)}. \tag{7}$$

Here λ_m represents the Lagrange multiplier for (3). In (6), the households choose d_2 by comparing the marginal cost $1+\lambda_m$ on the right-hand side with the marginal benefit $(1+s)\,u'(c_1)$ on the left-hand side. Other things being equal, a higher subsidy on debt encourages households to hold more debt during a crisis. The asset pricing equation is given by (7).

The first proposition establishes that, under the beginning-of-period collateral constraint, subsidizing debt during a crisis does not improve the welfare, compared to the laissez-faire economy.

Proposition 1 *Suppose that the households maximize the utility of $u(c_1)+c_2$ subject to (5), (2) and the beginning-of-period collateral constraint (3). Then, the optimal subsidy on debt s^* is zero if $(1-e+d_1)/y \leq \phi \leq 1$. If $0 < \phi < (1-e+d_1)/y$, on the other hand, the allocation is fully independent of s and is equivalent to the allocation under the laissez-faire economy.*

Proof. Let a variable with a subscript UA be the variables under the unconstrained allocation. It is straightforward to obtain the unconstrained (first-best) allocation: $c_{UA,1} = 1$, $c_{UA,2} = y-(1-e+d_1)$, and $d_{UA,2} = 1-e+d_1$. Likewise, it can be easily shown that the price of collateral under the unconstrained allocation is $p_{UA,1} = y$. For $(1-e+d_1)/y \leq \phi \leq 1$, the collateral constraint does not bind under the laissez-faire economy ($d_{UA,2} < \phi p_{UA,1}$). Hence, the optimal subsidy s^* is trivially zero. Next, consider the case of $0 < \phi < (1-e+d_1)/y$, which means that the collateral constraint binds under the laissez-faire economy. In this case, together with (7), (3) implies (i) $d_2 = \phi y / u'(c_1)$. Because of the market clearing conditions for collateral (i.e., $\theta_1 = \theta_2 = 1$) and the government budget constraint, (5) and (2) become (ii) $c_1 + d_1 = e + d_2$, and (iii) $c_2 + d_2 = y$, respectively. The allocation in this decentralized economy, $\{c_1, c_2, d_2\}$, can be fully determined by (i)–(iii) if the unique equilibrium exists. Because (i)–(iii) do not include s, the resulting allocation is fully independent of s. Therefore, the allocation must be equivalent to that under the laissez-faire economy. ■

3.2 The end-of-period collateral constraint

We next consider the same bailout under the end-of-period collateral constraint (4). While the first-order condition (6) remains the same as before, (7) must be replaced by

$$p_1 = \frac{y}{u'(c_1) - \lambda_m \phi}. \tag{8}$$

In contrast to (7), the extra benefit of holding the collateral ($\lambda_m \phi p_1$) affects p_1 under the end-of-period collateral constraint.

The next proposition states that the optimal s can replicate the unconstrained allocation.

Proposition 2 *Suppose that the households maximize the utility of $u(c_1) + c_2$ subject to (5), (2), and the end-of-period collateral constraint (4). Then, there exists s^* with which the decentralized equilibrium replicates the unconstrained allocation without the collateral constraint, and s^* is equal to the equilibrium shadow price of holding debt in the decentralized economy:*

$$s^* = \lambda_m(s^*) \geq 0. \tag{9}$$

If $0 < \phi < (1 - e + d_1)/y$, *$s^*$ is given by*

$$s^* = \frac{1}{\phi} - \frac{y}{1 - e + d_1} > 0. \tag{10}$$

Otherwise, $s^* = 0$.

Proof. As in the proof of Proposition 1, the optimal subsidy for $(1 - e + d_1)/y \leq \phi \leq 1$ is trivial: $s^* = \lambda_m = 0$ and the collateral constraint does not bind under s^*. For $0 < \phi < (1 - e + d_1)/y$, $d_{UA,2} > \phi p_{UA,1}$. If a positive s^* exists, it must be the case that $\lambda_m(s^*) > 0$ and $d_{UA,2} = \phi p_1$. Here, from (6), we note that λ_m depends on s. Substituting the unconstrained allocation into (6) and (8) yields $s = \lambda_m(s)$ and $p_1 = y/(1 - s\phi)$, respectively. Therefore, s^* in (10) is obtained by eliminating p_1 from $p_1 = y/(1 - s\phi)$ and $d_{UA,2} = \phi p_1$. Because $d_{UA,2}$ is feasible under p_1 satisfying $p_1 = y/(1 - s^*\phi)$, s^* achieves the unconstrained allocation $\{c_{UA,1}, c_{UA,2}, d_{UA,2}\}$.[8] ■

3.3 Interpretation

Propositions 1 and 2 suggest that the efficacy of subsidizing debt during crises is highly sensitive to the timing assumption of the collateral constraint. If the collateral constraint is

[8] In Appendix A.1, we extend Proposition 2 to the continuous-time version of the model.

given by (3), the bailout does not improve the welfare. Hence, the prudential capital controls that can achieve the constrained-efficient allocation are strictly preferred to the bailout. By contrast, if the collateral constraint is replaced by (4), the optimal subsidy s^* can prevent fire sales of collateral and achieve the unconstrained (first-best) allocation. In this case, the bailout outperforms the prudential capital controls.

The key to understanding our result is the price of collateral p_1 at a time of crisis. Suppose that the collateral constraint binds in period 1 (i.e., $0 < \phi < (1 - e + d_1)/y$). In this case, the government may wish to intervene in the credit market with $s > 0$ if it can inflate the price of collateral. Moreover, the households can enjoy even the first-best level of consumption if the asset price is inflated to ensure that $d_{UA,2} = \phi p_1$. Therefore, $d_{UA,2}$ uniquely determines the target level of the asset price for the government:

$$p_1 = \frac{d_{UA,2}}{\phi} = \frac{1 - e + d_1}{\phi} > y, \tag{11}$$

where the strict inequality is ensured by the assumption of $\phi < (1 - e + d_1)/y$.

The question is whether the government can inflate asset prices. Under (3), the subsidy on borrowing has no effect on households' decisions, because their borrowing capacity is predetermined by $\phi\theta_1 p_1$. Households' demand for collateral (θ_2) is not stimulated by the subsidy. As a result, p_1 remains unaffected by the subsidy and the only option for the households is to demand less collateral (θ_2) to compensate for consumption. After all, the allocation turns out to be the same as that under the laissez-faire economy with a low price of collateral. By contrast, under (4), households know that if they buy more collateral (θ_2), then they can borrow more, because θ_2 affects their borrowing capacity. The lower cost of debt owing to the subsidy provides them with financing for the purchase of new collateral (θ_2). This financing for the new collateral purchase stimulates the demand, which results in higher p_1.

From the viewpoint of the government, p_1 under (8) can be seen as a function of the policy instrument s. In particular, along with (6), the asset pricing equation (8) becomes

$$p_1 = \frac{y}{[1 - (1 + s)\phi] u'(c_1) + \phi}.$$

This equation indicates that the government can control the price of collateral by choosing s. If the government sets $s = s^* = \lambda_m(s^*)$,

$$p_1 = \frac{y}{1 - s^*\phi}. \tag{12}$$

This s^* is consistent with the target price given by (11), while satisfying all the first-order conditions and the constraints. Hence, under the end-of-period collateral constraint, the government can achieve the unconstrained allocations.

Our results in the propositions are related to what Fostel and Geanakoplos (2008) call the "collateral value." They show that the price of assets can always be decomposed into the payoff value and the collateral value. The collateral value of an asset represents the additional benefit of holding the asset that is used as a collateral. In our context, the price of the asset is p_1 and the payoff value is its discounted future returns $y/u'(c_1)$, because the discount factor of households and the marginal utility of c_2 are both unity. Equation (8) can also be written as $p_1 = y/u'(c_1) + \lambda_m \phi p_1/u'(c_1) \geq y/u'(c_1)$, which indicates the presence of the collateral value under the end-of-period collateral constraint. However, (7) shows that p_1 reflects only the payoff value. This is because households cannot change the amount of the collateral under the beginning-of-period collateral constraint. Thus, our results shown in the propositions rely on the presence of the collateral value.

Proposition 2 in our paper is also related to BCORY, who discover that the government can restore the unconstrained allocation using the additional policy instrument. The optimal subsidy in our model, however, starkly differs from BCORY's policy prescription in two respects. First, the policy instrument in this paper differs from that of BCORY. The government in BCORY subsidizes the household collateral purchase, while that in our model subsidizes household borrowing. Intervening in collateral markets may require additional capacity (i.e., additional policy instruments) of the government.[9] Our argument is applicable to any government that has the capacity to implement capital controls (i.e., the capacity to affect the cost of borrowing). The second difference is slightly technical. Under BCORY's prescription, the collateral constraint never binds. In the context of our model, this means that $d_{UA,2} < \phi p_1$ and $\lambda_m = 0$. To ensure the efficacy of the optimal subsidy in our model, λ_m needs to take strictly positive values because, as indicated by (8), the non-zero Lagrange multiplier enables asset price inflation.

Before closing this section, two important remarks on Proposition 2 should be made. First, it should be noted that the subsidy on debt must be financed by non-distortionary tax for Proposition 2 to hold. In our model, we assumed that the government relies on lump-sum tax for financing the subsidy at a time of a crisis. In a related vein, another important

[9] See Propositions 2 and 3 in BCORY. They employ the collateral constraint in which the income from tradable and nontradable endowments can be pledged as collateral as in Bianchi (2011). In this setup of the model, the government supports the relative price of nontradables to tradables during crises by either a subsidy on nontradable good consumption or a tax on tradable good consumption.

condition for Proposition 2 to hold is that working capital does not exist in the economy. With working capital as introduced in Bianchi and Mendoza (2012), the subsidy on borrowing cannot replicate the unconstrained allocation because an additional distortion is generated by the subsidy, which makes inputs subject to working capital more costly. Second, following the literature, we assumed that the ceiling on the leverage in the collateral constraint was unaffected by policy interventions. However, if we drop this assumption, the bailouts may not achieve the unconstrained allocation. For example, if the households can default and divert their borrowing together with the subsidy from the government, the subsidy would strongly incentivize the households to divert borrowing. This may lead to a tighter collateral constraint imposed by foreign lenders. If such a moral hazard to divert borrowing leads to a tighter collateral constraint, the subsidy would not fully relax the collateral constraint. We thus note that whether the government can achieve the unconstrained allocation may also depend on how we model the collateral constraint including how the government subsidizes the debt during crises.

4 The practical feasibility of the debt subsidy

Proposition 2 in the previous section indicates that the subsidy on debt during a crisis can theoretically achieve the unconstrained allocation as if there were no collateral constraint. However, a natural question for policymakers would be whether such a subsidy is *practically* feasible. To answer this question, we extend the two-period model to a stochastic infinite-horizon model that can be calibrated to the data. Using the extended model, we examine (i) whether the policy intervention to restore the unconstrained allocation is realistic in terms of the size; and (ii) how frequently the policy intervention needs to be made to restore the unconstrained allocation. As a preparatory step, we first show that the main result in Proposition 2 continues to hold even in the case of a stochastic infinite-horizon model. We then explore how s^* fluctuates over time to assess the size and frequency of bailouts in a calibrated model.

4.1 The infinite-horizon model

We consider the stochastic infinite-horizon model, similar to Jeanne and Korinek (2012) and Bianchi and Mendoza (2012). The households choose d_{t+1} and θ_{t+1} to maximize $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$, where β is the discount factor satisfying $\beta \in (0, 1)$. Each household faces the period-by-period budget constraint:

$$c_t + d_t + p_t \theta_{t+1} = \theta_t e_t + (1 + s_t) \frac{d_{t+1}}{R} - S_t + p_t \bar{\theta}, \tag{13}$$

and the end-of-period occasionally binding collateral constraint:

$$\frac{d_{t+1}}{R} \leq \phi p_t \theta_{t+1}. \tag{14}$$

In this maximization problem, d_{t+1} is non-state-contingent one-period debt. The real interest rate on the non-state-contingent debt is $R > 1$ rather than unity. Every period, each household receives the exogenous endowment of collateral $\bar{\theta}$, which is normalized to one. It receives a stochastic income e_t based on the predetermined share of collateral assets θ_t (i.e., dividends). As before, the value of collateral is lost after receiving the return on collateral. The budget constraint here is basically the same as (5), but is also similar to (2) in terms of the returns on collateral. Finally, the collateral constraint is the same as (4) except for R.

The first-order conditions are standard:

$$(1 + s_t) u'(c_t) = \beta R \mathbb{E}_t u'(c_{t+1}) + \lambda_{m,t}, \tag{15}$$

$$p_t = \beta \frac{\mathbb{E}_t [u'(c_{t+1}) e_{t+1}]}{u'(c_t) - \lambda_{m,t} \phi}, \tag{16}$$

$$0 = \left[\phi p_t \theta_{t+1} - \frac{d_{t+1}}{R} \right] \lambda_{m,t}, \ \lambda_{m,t} \geq 0, \text{ and } \phi p_t \theta_{t+1} - \frac{d_{t+1}}{R} \geq 0. \tag{17}$$

In equilibrium, the markets for collateral and consumption goods clear: $\theta_{t+1} = \bar{\theta}$ and $c_t + d_t = e_t + d_{t+1}/R$ for all t, respectively. As before, we assume a balanced budget of the government: $S_t = s_t d_{t+1}/R$.

The unconstrained allocation without the collateral constraint must satisfy the following first-order conditions:

$$u'(c_{UA,t}) = \beta R \mathbb{E}_t [u'(c_{UA,t+1})], \tag{18}$$

$$p_{UA,t} = \beta \frac{\mathbb{E}_t [u'(c_{UA,t+1}) e_{t+1}]}{u'(c_{UA,t})}, \tag{19}$$

which yield the policy functions $c_{UA,t} = c_{UA}(d_t, e_t)$ and $d_{UA,t+1} = d_{UA}(d_t, e_t)$ as functions of state variables. Likewise, the asset pricing function is obtained from the model without the collateral constraint: $p_{UA,t} = p_{UA}(d_t, e_t)$. The following proposition shows that there exists a state-contingent subsidy $s_t = s(d_t, e_t)$ consistent with the unconstrained allocation in the stochastic infinite-horizon model.

Proposition 3 *Suppose that the households maximize* $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$ *subject to (13) and the end-of-period collateral constraint (14). Then, there exists a price function* $p_s(d_t, e_t)$ *and a time consistent subsidy* $s^*(d_t, e_t)$*, with which the decentralized equilibrium characterized by (13)–(17) replicates the unconstrained allocation without the collateral constraint* $\{c_{UA,t}, d_{UA,t+1}\}_{t=0}^{\infty}$*. Also, the subsidy* $s_t^* = s^*(d_t, e_t)$ *is proportional to the Lagrange multiplier for (14):*

$$s^*(d_t, e_t) = \frac{\lambda_m(d_t, e_t; s_t^*)}{u'[c_{UA}(d_t, e_t)]} \geq 0, \tag{20}$$

where $\lambda_m(d_t, e_t; s_t)$ *represents the Lagrange multiplier for the collateral constraint, given* s_t*. Furthermore, if* $d_{UA}(d_t, e_t)/R > \phi p_{UA}(d_t, e_t)$*,* $s^*(d_t, e_t)$ *is given by*

$$s^*(d_t, e_t) = \frac{1}{\phi} - \frac{p_{UA}(d_t, e_t)}{d_{UA}(d_t, e_t)/R} > 0. \tag{21}$$

Otherwise, $s^*(d_t, e_t) = 0$*.*

Proof. We consider two cases for the states of the economy (d_t, e_t): (i) $d_{UA}(d_t, e_t)/R \leq \phi p_{UA}(d_t, e_t)$ and (ii) $d_{UA}(d_t, e_t)/R > \phi p_{UA}(d_t, e_t)$. Here, the conditions distinguish whether or not the level of debt under the unconstrained allocation is feasible in the laissez-faire economy with the collateral constraint. For each case, we will confirm that the first-order conditions (15)–(17) are satisfied under $s^*(d_t, e_t)$ when they are evaluated at the unconstrained allocation $c_{UA}(d_t, e_t)$ and $d_{UA}(d_t, e_t)$.

Consider the states satisfying (i). If $s^*(d_t, e_t) = 0$, $\lambda_m[d_t, e_t, s^*(d_t, e_t)] = 0$. Then, the first-order conditions of (15) and (16) are the same as (18) and (19), and the allocation is equivalent to the unconstrained allocation without the collateral constraint. Therefore, $p_s(d_t, e_t) = p_{UA}(d_t, e_t)$ characterized by (19). We thus confirm that, for the states satisfying (i), $s^*(d_t, e_t) = 0$ is consistent with the unconstrained allocation.

Next, for the states satisfying (ii), consider the price of collateral that achieves $d_{UA}(d_t, e_t)$ with the binding collateral constraint. If

$$p_s(d_t, e_t) = \frac{d_{UA}(d_t, e_t)}{\phi R}, \tag{22}$$

then (17) is satisfied together with $\lambda_m(d_t, e_t; s^*(d_t, e_t)) \geq 0$. Combining (16), (20), and (22) yields

$$\frac{d_{UA}(d_t, e_t)}{\phi R} = \beta \frac{\mathbb{E}_t\left(u'\{c_{UA}[d_{UA}(d_t, e_t), e_{t+1}], e_{t+1}\} e_{t+1}\right)}{u'[c_{UA}(d_t, e_t)](1 - s_t^* \phi)}, \tag{23}$$

for all states (d_t, e_t) satisfying (ii). Using the above equation and (19), we can solve for $s_t^* = s^*(d_t, e_t)$ and the solution turns out to be (21). Because s_t^* is chosen to satisfy (23), (16) and (17) are obviously satisfied at the unconstrained allocation. Finally, using (20), (18) can be rewritten as

$$[1 + s^*(d_t, e_t)]\, u'[c_{UA}(d_t, e_t)] = \beta R \mathbb{E}_t \left(u' \left\{c_{UA}\left[d_{UA}(d_t, e_t), e_{t+1}\right]\right\}\right) + \lambda_m [d_t, e_t; s^*(d_t, e_t)],$$

which is exactly (15) under $s_t = s^*(d_t, e_t)$. Therefore, all the first-order conditions (15)–(17) are satisfied under $s^*(d_t, e_t)$. This s_t^* is completely determined by the current state of the economy and hence is time-consistent. ■

The proposition confirms that, as in the two-period model, $s^*(d_t, e_t)$ can replicate the unconstrained allocation. Note, however, that the allocation replicated by the bailout is no longer the first-best allocation, because of the unavailability of the state-contingent debt that fully insures against endowment risks between domestic borrowers and foreign lenders. Nevertheless, the mechanism behind this proposition is the same as that behind Proposition 2. Namely, the government can fully control the asset price to avoid fire sales.

4.2 The size and frequency of bailouts

4.2.1 Calibration

The parameters are mainly taken from Bianchi and Mendoza (2012). For the household's preference, $\sigma = 2$. The discount factor β is set to 0.96, calibrating the model to the annual frequency. The ceiling on the household borrowing per collateral asset ϕ is set at 0.36. We assume that the total factor productivity in Bianchi and Mendoza (2012) can be translated into the stochastic process of the dividend e_t in our model. The stochastic process of the total factor productivity in Bianchi and Mendoza (2012) follows a log-normal AR(1) process. We thus employ the same stochastic process for e_t as theirs: $\log(e_{t+1}) = \rho \log(e_t) + \eta_{t+1}$, where $\eta_t \sim i.i.d.N(0, \sigma_\varepsilon)$ for all t. Here ρ and σ_ε are set to 0.53 and 0.014, respectively. To ensure stationarity of $d_{UA}(d_t, e_t)$, we assume that households face a small risk premium on their foreign debt.[10] Specifically, we replace the budget constraint (13) with a slightly different form:

$$c_t + d_t + p_t \theta_{t+1} = \theta_t e_t + (1 + s_t) \frac{d_{t+1}}{R_{t+1}} - S_t + p_t \bar{\theta},$$

[10] See Schmitt-Grohé and Uribe (2003). We also note that Proposition 3 can be easily extended to the case with a risk premium on foreign debt.

where $R_{t+1} = R + \psi[\exp(d_{t+1}) - 1]$, R =1.028 and ψ =0.01.

To confirm that parameters are reasonably calibrated, we simulate the laissez-faire economy with the occasionally binding collateral constraint. We interpret GDP in our model as $e_t\bar{\theta}$. In the simulation, the mean of the debt-to-GDP ratio is 33.5 percent. This ratio is in close proximity to the calibration target in Benigno et al. (2013), who calibrate their model to the Mexican economy based on the updated dataset of Lane and Milesi-Ferretti (2007). We also compute the probability of binding collateral constraints in the laissez-faire economy as a proxy of the crisis probability. The probability is 6.57 percent, which is broadly in line with the literature.[11]

4.2.2 Simulation results

Table 1 compares the moments generated by the model without intervention (i.e., the laissez-faire economy) with those generated under $s_t^* = s^*(d_t, e_t)$. Overall, the subsidy on debt takes extremely large values, meaning that massive bailouts are required to restore the unconstrained allocation without the collateral constraint. On average, s_t^* is 67.0 percent. To finance this subsidy, the government needs to impose a large lump-sum tax equivalent to as much as 31.6 percent of annual household income ($e\bar{\theta}$). Figure 1 plots the simulated path of s_t^* over 300 periods. The figure indicates that the government activates bailouts very frequently. In the figure, s_t^* takes a value of zero only in period 13. This means that, over 300 periods, bailouts are activated 299 times.

The reason for the massive and frequent bailouts is straightforward. As long as the debt in the unconstrained allocation is sufficiently large compared to that in the laissez-faire economy, the collateral constraint is likely to bind. Whenever the collateral constraint binds under s_t^*, the government bails out households by encouraging new borrowing that is used to repay the large amount of existing debt. Because the bailouts improve the welfare with no side effects, it is optimal for households to roll over the large amount of new borrowing. This new borrowing substantially increases the probability that the collateral constraint will continue to bind in the next period. As a result, the government almost always bails out households under s_t^*.

We argue that there is a gap between theory and data. For comparison, we take an

[11] For reasonable values of the crisis probability, Benigno et al. (2013) target the crisis probability at 8 percent per year. In the empirical studies, Basel Committee on Banking Supervision (2010) reports two empirical crisis probabilities for 25 countries over 1985–2009, based on the datasets of Reinhart and Rogoff (2013) and Laeven and Valencia (2013). The estimated crisis probabilities are 5.2 percent in Reinhart and Rogoff (2013) and 3.6 percent in Laeven and Valencia (2013).

empirical estimate reported by Frydl (1999). Based on Frydl (1999), an empirically realistic bailout would amount to somewhere between 1 to 3 percent of GDP for a single year in the aftermath of a crisis.[12] Comparisons of the sizes of intervention between the model and the practice suggest that the subsidy consistent with the unconstrained allocation would be difficult to implement, perhaps because of political conflicts, which are not taken into account in our model.

4.2.3 Sensitivity of simulation results

Although unrealistically large and frequent bailouts seem difficult to implement, the size and frequency depend on parameters in the model. Therefore, one could argue that it is possible to attain a realistic size and frequency of bailouts by changing parameters in the model. To consider this argument, this subsection discusses whether the difficulty of the bailout is robust to changes in the model parameters.

Proposition 3 suggests that s_t^* can take lower values under a looser collateral constraint (i.e., a larger ϕ). Under a larger ϕ, the government may activate bailouts less often, because the collateral constraint is less likely to bind. Hence, in the economy with a large ϕ, the subsidy consistent with the unconstrained allocation can be practically feasible. In Figure 2, we plot s_t^* in the economy with $\phi = 0.60$. With this parameterization, the maximum value of s_t^* over 300 periods is 2.90 percent and the size of policy intervention measured by the lump-sum tax relative to GDP is 1.71 percent. In terms of the frequency, the incidence of the policy activation is two out of 300 periods. Thus, bailouts appear to be easier to implement in an economy with $\phi = 0.60$. However, the model fails to explain crisis probabilities under the laissez-faire economy: in our simulations, the probability of binding collateral constraints in the laissez-faire economy is only 0.25 percent, much lower than that reported in the previous studies. Therefore, it is sensible to conclude that, although the bailout which replicates the unconstrained allocation may be implementable in an economy resilient to negative shocks, it would not be feasible in most economies that are fragile to shocks.

The subsidy s_t^* can also be affected by the risk-premium parameter ψ. In our benchmark simulation, ψ is set to a somewhat large value of 0.01. In the literature, this parameter is

[12] Frydl (1999) discusses two empirical works estimating (i) fiscal costs (resolution cost) and (ii) the length of financial crises as his baseline estimates (Caprio and Klingebiel 1996 and Lindgren, Garcia, and Saal 1996). In the two empirical papers, the average fiscal costs are 13.6 percent of GDP in the former and 7.2 percent in the latter, whereas the average duration of crises is 4.5 years and 6.2 years, respectively. We compute the average fiscal cost for a single year by dividing average total fiscal costs by the average length of financial crises, suggesting that the average fiscal cost for a single year ranges from 1.16 (7.2/6.2) to 3.02 (13.6/4.5) percent of GDP.

typically set to a very small number to calibrate the model without the collateral constraint to match the volatility of the debt-to-GDP ratio. For example, Schmitt-Grohé and Uribe (2003) set this parameter to 0.0007. We could have used their parameterization, but the level of the debt in this case is much higher than what we obtained in the benchmark simulation. Because (21) indicates that a larger amount of foreign debt *increases* the size of bailouts, this low ψ makes bailouts more unrealistic.

5 Concluding remarks

This paper analyzed the roles of bailouts in managing financial crises. Using a simple deterministic model with a collateral constraint, we discussed that the policy prescription on bailouts is sensitive to the timing assumption of the collateral constraint. If we employ the timing assumption of the collateral constraint employed by JK, the policy that subsidizes debt during crises is neutral and hence the bailout fails to outperform the prudential capital controls. If households can collateralize their assets that they purchase at the same time as their borrowing, the same policy can improve the welfare much more than the prudential capital controls. To assess the practical feasibility of the bailout in the latter assumption, we performed numerical simulations using the stochastic infinite-horizon version of the model. We found that the "optimal bailout" would be difficult to implement, because it requires an unrealistically large size and extremely frequent interventions.

Our findings suggest that policymakers should implement bailouts with caution. The model suggests that policymakers should know which type of the collateral constraint is more common. Even when the end-of-period collateral constraint is more plausible, the present setup of the model provides unrealistic policy prescriptions. To provide more robust policy prescriptions on bailouts, the model would need to be extended into a more general framework. This would be an important step for future research.

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Table 1: Average and standard deviations of the model variables

	Laissez-faire economy	Bailouts
Debt	0.335	0.471
	(0.016)	(0.045)
Asset price	0.962	1.273
	(0.018)	(0.121)
Consumption	0.990	0.985
	(0.015)	(0.009)
Subsidy (percent)	-	67.0
		(22.4)
Fiscal cost (percentage of GDP)	-	31.6
		(13.2)

Note: The numbers in each entry report the average and the standard deviations of the debt position, asset price, and consumption in the laissez-faire economy and the economy with bailouts. The numbers without parentheses refer to the average of variables, and the numbers in parentheses below the average are the standard deviations of the corresponding variables. These moments are from the numerically approximated ergodic distributions with 1,000 periods and 1,000 simulations. The second column also reports the average and standard deviation of the optimal subsidy and the fiscal cost of bailouts measured by the lump-sum tax relative to the GDP.

Figure 1: Simulated path of s_t^*: Benchmark calibration

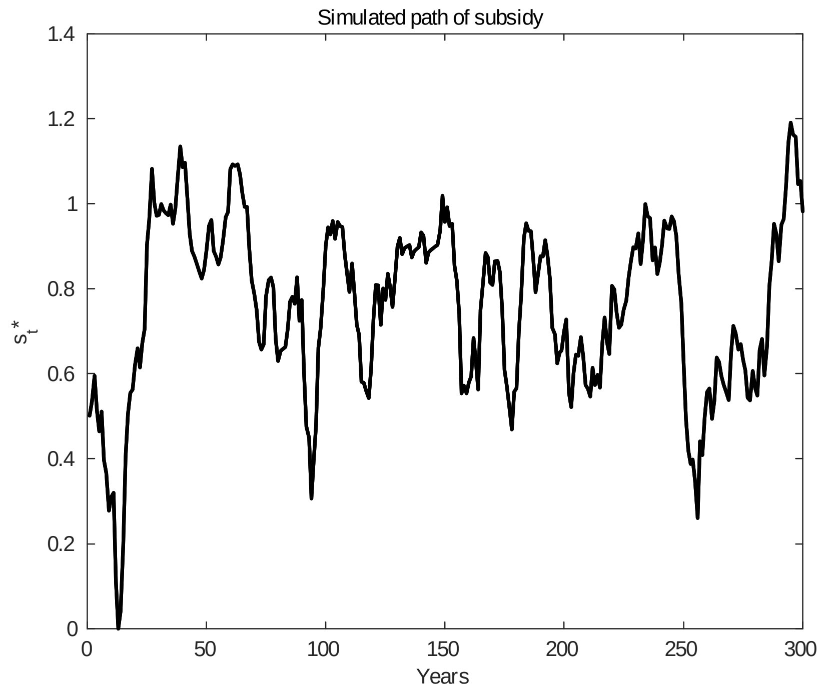

Figure 2: Simulated path of s_t^*: Economy with $\phi = 0.60$