A Note on Recursive Multiple-Priors*

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Abstract

One of the assumptions of recursive multiple-priors utility (Epstein and Schneider, *J. Econ. Theory*, 113(1) (2003), 1-31) is that conditional preferences at every node satisfy an intertemporal version of the multiple-priors axioms (Gilboa and Schmeidler, *J. Math. Econ.*, 18 (1989) 141-153). This note strengthens this assumption as a consequence of a more primitive updating rule: If an agent’s preference *ex-ante* satisfies the multiple-priors axioms (meaning at the beginning of a time horizon), under a dynamically consistent updating rule, conditional preference at each node must also satisfy the same set of axioms.

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1 Introduction

Ambiguity is a notion distinct from risk. Gilboa and Schmeidler’s [3] multiple-priors utility is one of the models that captures this notion. Epstein and Schneider [2] extend this model to a dynamic setting under a fixed information structure (namely, filtration) and show that for a multiple-priors agent to behave consistently over time, each set of priors at every time-event node needs to be rectangular, and each set is updated by Bayes’ Rule applied prior by prior. One of the crucial assumptions of their analysis is that conditional preferences at every node satisfy an intertemporal version of the multiple-priors axioms. Although this coherence is desirable, whether or not it is implied by a more primitive updating rule remains to be shown.\(^1\) The purpose of this note is to provide justification for this coherence assumption as a consequence of a dynamically consistent updating rule.

Formally, we follow Machina [5], who has proposed a dynamically consistent method of updating non-expected utility preferences: A conditional preference relation agrees with an \textit{ex-ante} preference relation on a subset of acts that share the same outcomes in the past as well as on states that were not realized. Given this definition, the main result of this note is that under a fixed filtration, if an \textit{ex-ante} preference relation satisfies an intertemporal version of the multiple-priors axioms and conditional preference relations are updated under a dynamically consistent updating rule, conditional preference relations \textit{must} also satisfy the same set of axioms. This result shows that coherence is not an assumption but a \textit{requirement} for an agent of multiple-priors utility to

\(^1\)In this regard, \textit{consistency} is the usual terminology. However, to avoid confusion between consistency among choices and consistency among sets of axioms, we use \textit{coherence} instead of consistency to refer to the given property.
behave consistently over time; it also relates recursive multiple-priors more clearly to the literature that studies updating rules for multiple-priors utility (for example, Gilboa and Schmeidler [4]). Therefore, our result strengthens the recursive multiple-priors theorem by Epstein and Schneider [2].

In terms of the literature concerning updating rules that ensure both dynamic consistency and coherence, Epstein and Breton [1] show that if preferences based on beliefs are updated to satisfy dynamic consistency and coherence, beliefs can be represented by a unique probability measure and conditional beliefs must be derived by Bayes’ rule. Our argument is similar to the above although it is restricted to a given filtration; however, we obtain a stronger result: If the multiple-priors preference is updated to satisfy dynamic consistency, coherence is implied and a set of priors is updated by Bayes’ rule applied prior by prior.

2 Model and Results

We follow the set-up of Epstein and Schneider [2]. Time is discrete and its horizon is finite; $T = \{0, ..., T\}$. The state space is $\Omega$. The information structure is given and fixed and represented by the filtration $\{\mathcal{F}_t\}_{t=0}^T$, where $\mathcal{F}_0 = \{\Omega\}$ and for each time $t$, $\mathcal{F}_t$ is a finite partition; $\mathcal{F}_t(\omega)$ denotes an event in the partition $\mathcal{F}_t$ containing $\omega$.

At each $t$ and $\omega$, the outcome space is defined as a space of simple lotteries over $C \subseteq \mathbb{R}_+$, denoted by $\Delta_s(C)$. Let $\mathcal{H}$ be a set of all $\Delta_s(C)$-valued adapted processes $h = (h_t)$ (called an act), where each $h_t : \Omega \rightarrow \Delta_s(C)$ is $\mathcal{F}_t$-measurable. We use the notation $h_t(\omega)$ to refer to a specific lottery assigned at $(t, \omega)$. Also, define a lottery act $l = (l_t)$ as a $\Delta_s(C)$-valued adapted process
such that for each $t$, $l_t(\omega) = l_t(\omega')$ for all $\omega, \omega' \in \Omega$; lottery acts involve risk but not ambiguity. In addition, $(l_{-\tau,-(\tau+k)}, q, q')$ denotes the lottery act $l'$ in which $l'_t = l_t$ for $t \neq \tau, \tau + k$, $l'_{e}(\omega) = q$ and $l'_{e+k}(\omega) = q'$ for all $\omega \in \Omega$, where $q, q' \in \Delta_s(C)$. Furthermore, for any given act $h$ and any given $\omega$, $h(\omega)$ is the lottery act $l$ such that $l_{e}(\omega') = h_{e}(\omega)$ in every period $\tau$ and in every state $\omega'$. We also define the following operation: $(\alpha h + (1 - \alpha)h')_t(\omega) = \alpha h_t(\omega) + (1 - \alpha)h'_t(\omega)$ for any $\alpha \in [0,1]$.

The decision maker has a preference ordering on $H$ at any $(t, \omega) \in T \times \Omega$, denoted by $\succeq_{t,\omega}$. Each preference ordering in the collection of $\{\succeq_{t,\omega}\} \equiv \{\succeq_{t,\omega}(t, \omega) \in T \times \Omega\}$ satisfies the following axioms:

**Axiom 1 (Conditional Preference - CP).** For each $t$ and $\omega$: (i) $\succeq_{t,\omega} = \succeq_{t,\omega}^*$ if $F_t(\omega) = F_t(\omega^*)$. (ii) If $h'_t(\omega') = h_t(\omega')$ for all $\tau \geq t$ and $\omega' \in F_t(\omega)$, then $h' \sim_{t,\omega} h$.

**Axiom 2 (Multiple-Priors - MP).** For each $t$ and $\omega$: (i) $\succeq_{t,\omega}$ is complete and transitive. (ii) For all $h, h'$ and lottery acts $l$, and for all $\alpha \in (0,1)$, $h' \succ_{t,\omega} h$ if and only if $\alpha h' + (1 - \alpha)l \succ_{t,\omega} \alpha h + (1 - \alpha)l$. (iii) If $h'' \succ_{t,\omega} h', h' \succ_{t,\omega} h$, then $\alpha h'' + (1 - \alpha)h \succ_{t,\omega} h' \succ_{t,\omega} \beta h'' + (1 - \beta)h$ for some $\alpha$ and $\beta \in (0,1)$. (iv) If $h'(\omega') \succeq_{t,\omega} h(\omega')$ for all $\omega'$, then $h' \succeq_{t,\omega} h$. (v) If $h' \sim_{t,\omega} h$, then $\alpha h' + (1 - \alpha)h \succeq_{t,\omega} h$ for all $\alpha \in (0,1)$. (vi) $h' \succ_{t,\omega} h$ for some $h'$ and $h$.

**Axiom 3 (Risk Preference - RP).** For any lottery act $l$, for all $p, p', q$ and $q' \in \Delta_s(C)$, if

$$(l_{-\tau,-(\tau+1)}, p, p') \succeq_{t,\omega} (l_{-\tau,-(\tau+1)}, q, q')$$

for some $\omega, t$ and $\tau \geq t$, then it is true for every $\omega, t$ and $\tau \geq t$.

For the following axioms, we define the usual notion of nullity: for any $\tau > t$, the event $A$ in $F_\tau$ is $\succeq_{t,\omega}$-null if $h'(\cdot) = h(\cdot)$ on $A^c$ implies that $h' \sim_{t,\omega} h$. 

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Axiom 4 (Dynamic Consistency - DC). For every $t$ and $\omega$ and for all acts $h'$ and $h$, if $h'_\tau(.) = h_\tau(.)$ for all $\tau \leq t$ and if $h' \succeq_{t+1,\omega'} h$ for all $\omega'$, then $h' \succeq_{t,\omega} h$; the latter ranking is strict if the former ranking is strict at every $\omega'$ in a $\succeq_{t,\omega}$-nonnull event.

Axiom 5 (Full Support - FS). Each nonempty event in $\bigcup_{t=0}^T F_t$ is $\succeq_{0,\omega}$-nonnull.

We say that a measure $p \in \Delta(\Omega, F_T)$ has full support if $p(A) > 0$ for every non-empty $A \in F_T$. In addition, for any measure $p \in \Delta(\Omega, F_T)$, $p_t(\omega) = p(.|F_t)$ is its $F_t$-conditional and $p_{t+1}^+(\omega)$ is the restriction of $p_t(\omega)$ to $F_{t+1}$. Define the following two sets, $P_t(\omega)$ and $P_{t+1}^+(\omega)$, as

$$P_t(\omega) = \{ p_t(\omega) | p \in P \} \quad \text{and} \quad P_{t+1}^+(\omega) = \{ p_{t+1}^+(\omega) | p \in P \},$$

where $P$ is a subset of $\Delta(\Omega, F_T)$. Then, $P$ is said to be $\{F_t\}$-rectangular if for all $t$ and $\omega$, $P_t(\omega) = \int P_{t+1}dP_{t+1}^+(\omega)$. Furthermore, $u : \Delta_s(C) \to \mathbb{R}$ is called mixture linear provided that $u(\alpha p + (1 - \alpha)q) = \alpha u(p) + (1 - \alpha)u(q)$ for all $p, q \in \Delta_s(C)$ and $\alpha \in [0, 1]$.

Given the above terminology, Epstein and Schneider [2] show that the above set of axioms is necessary and sufficient for the representation of preferences to satisfy recursive multiple-priors utility.

Theorem (Epstein and Schneider [2]): Each preference ordering in $\{\succeq_{t,\omega}\}$ satisfies CP, MP, RP and DC, and each preference ordering in $\{\succeq_{0,\omega}\}$ also satisfies FS if and only if there exists $P \subset \Delta(\Omega, F_T)$, closed, convex and $\{F_t\}$-rectangular, with all measures in $P$ having full support, $\beta > 0$ and a mixture linear and nonconstant $u : \Delta_s(C) \to \mathbb{R}$ such that: for every $t$ and $\omega$, $\succeq_{t,\omega}$ is represented by $V_t(., \omega)$, where

$$V_t(h, \omega) = \min_{m \in P_t(\omega)} \int \sum_{\tau \geq t} \beta^{\tau-t} u(h_\tau(\omega')) dm. \tag{1}$$
Moreover, $\beta$ and $\mathcal{P}$ are unique and $u$ is unique up to a positive affine transformation.

In Epstein and Schneider [2], MP and RP directly specify the structure of a conditional preference ordering $\succeq_{t,\omega}$. Then, at each $t$ and $\omega$, $\succeq_{t,\omega}$ is represented by an intertemporal version of multiple-priors utility without rectangularity, where $\beta_{t,\omega}$ and $u_{t,\omega}$ replace $\beta$ and $u$ in (1). For defining consistency among preference orderings in $\{\succeq_{t,\omega}\}$, RP establishes equivalence among $\beta_{t,\omega}$ and among $u_{t,\omega}$ (up to a positive affine transformation); DC derives rectangularity. Clearly, DC is not sufficient to relate each $\beta_{t,\omega}$ and each $u_{t,\omega}$.

On the other hand, in this note we do not impose MP and RP on each conditional preference ordering $\succeq_{t,\omega}$ for $t > 0$. Rather, we consider that an agent updates her preference ordering under some rules.\(^2\) In particular, we want updated preference orderings to satisfy dynamic consistency. A commonly used method for non-expected utility theory is that as proposed by Machina [5], where a conditional preference relation agrees with an \textit{ex-ante} preference relation on a subset of acts that share the same outcomes in the past as well as on states that were not realized. The following axioms are in line with this approach.

\textbf{Axiom 2b (Ex-ante Multiple-Priors - EAMP).} At $t = 0$, for each $\omega$: (i) $\succeq_{0,\omega}$ is complete and transitive. (ii) For all $h, h'$ and lottery acts $l$, and for all $\alpha \in (0, 1)$, $h' \succ_{0,\omega} h$ if and only if $\alpha h' + (1 - \alpha)l \succ_{0,\omega} \alpha h + (1 - \alpha)l$. (iii) If $h'' \succ_{0,\omega} h' \succ_{0,\omega} h$, then $\alpha h'' + (1 - \alpha)h \succ_{0,\omega} h' \succ_{0,\omega}$ $\beta h'' + (1 - \beta)h$ for some $\alpha$ and $\beta \in (0, 1)$. (iv) If $h'(\omega') \succeq_{0,\omega} h(\omega')$ for all $\omega'$, then $h' \succeq_{0,\omega} h$. (v) If $h' \sim_{0,\omega} h$, then $\alpha h' + (1 - \alpha)h \succeq_{0,\omega} h$ for all $\alpha \in (0, 1)$. (vi) $h' \succ_{0,\omega} h$ for some $h'$ and $h$.

\(^2\)In this respect, (ii) of CP can also be regarded as an updating rule.
Axiom 3b (Ex-ante Risk Preference - EARP). For each \( \omega \) and for any lottery act \( l \), for all \( p, p', q \) and \( q' \in \Delta_s(C) \), if

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(l_{-\tau,-(\tau+1)}, p, p') \succeq_{0,\omega} (l_{-\tau,-(\tau+1)}, q, q')
\]

for some \( \tau \geq 0 \), then it is true for every \( \tau \geq 0 \).

Axiom 4b (Dynamically Consistent Updating - DCU). For every \( t \) and \( \omega \) and for all acts \( h' \) and \( h \), if \( h'_t(.) = h_t(.) \) for all \( \tau \leq t \) and \( h'_t(\omega') = h_t(\omega') \) for all \( \tau > t \) and for all \( \omega' \notin \mathcal{F}_{t+1}(\omega) \subseteq \mathcal{F}_t(\omega) \), \( h' \succeq_{t,\omega} h \) if and only if \( h' \succeq_{t+1,\omega'} h \) at every \( \omega' \in \mathcal{F}_{t+1}(\omega) \); this equivalence also holds on a strict ranking if \( \mathcal{F}_{t+1}(\omega) \) is a \( \succeq_{t,\omega} \)-nonnull event.\(^3\,^4\)

We now state the conclusion.\(^5\)

Proposition: Suppose that each preference ordering in \( \{\succeq_{t,\omega} \} \) satisfies CP and DCU, and each preference ordering in \( \{\succeq_{0,\omega} \} \) also satisfies EAMP, EARP and FS. Then, for each \( t \) and \( \omega \), \( \succeq_{t,\omega} \) satisfies MP, RP and DC, i.e., all conditional preference orderings follow recursive multiple-priors

\(^3\)We can change the statement as follows: \( h' \succeq_{0,\omega} h \) if and only if \( h' \succeq_{t+1,\omega'} h \) at every \( \omega' \in \mathcal{F}_{t+1}(\omega) \).

\(^4\)DCU and CP together imply that DCU holds for any event \( A \in \mathcal{F}_{t+1} \) where \( A \subseteq \mathcal{F}_t(\omega) \) (i.e., for any \( \succeq_{t,\omega} \)-nonnull event in \( \mathcal{F}_{t+1} \)).

\(^5\)As long as we assume Axioms 7 and 8 of Epstein and Schneider [2], the same conclusion holds for an infinite horizon case (i.e., \( T \) is countable) because the proof of the proposition is based on induction on \( T \). In fact, we only need to assume Axioms 7 and 8 for \textit{ex-ante} preference orderings.
Here, the proposition shows that coherence (i.e., MP and RP) is a consequence of a dynamically consistent updating rule; it is not an assumption but a requirement for an agent of multiple-priors utility to behave consistently over time. Furthermore, given CP, EAMP, EARP and FS, DCU establishes equivalence among $\beta_{t,\omega}$ and among $u_{t,\omega}$ (up to a positive affine transformation) as well as rectangularity of the set of priors. Hence, DCU alone derives all required consistency among preference orderings in $\{\succeq_{t,\omega}\}$.

Proof. Clearly, CP and the repeated application of DCU on all events $A \in F_{t+1}$ with $A \subseteq F_t(\omega)$ imply DC. For RP, we first show that the following holds at each $(t, \omega) \in T \times \Omega$:

**Axiom 3c (RP)].** For each $\omega$ and for any lottery act $l$, for all $p, p', q$ and $q' \in \Delta_s(C)$, if

$$(l_{-(\tau,-(\tau+1)}, p, p') \succeq_{t,\omega} (l_{-(\tau,-(\tau+1)}, q, q')$$

for some $\tau \geq t$, then it is true for every $\tau \geq t$.

Now, we prove MP and RP$_t$ by induction. At any $(0, \omega)$, MP and RP$_0$ hold under EAMP and EARP. Suppose that MP and RP$_t$ hold at any $(t, \omega) \in \{t\} \times \Omega$ for some $t < T$. Let $\mathcal{H}(t, \omega; h)$ be a collection of all acts $h' \in \mathcal{H}$ such that $h'_t(.) = h_t(.)$ for all $\tau \leq t$ and $h'_t(\omega') = h_t(\omega')$ for all $\tau > t$ and for all $\omega' \notin F_{t+1}(\omega)$. Also, let $\mathcal{H} \mid F_{t+1}(\omega)$ be a restriction of $\mathcal{H}$ on $(\tau, \omega) \in [t+1, T] \times F_{t+1}(\omega)$.

By CP, to prove MP and RP$_{t+1}$ for $(t+1, \omega)$, it suffices to show that (i) to (vi) of MP and RP$_{t+1}$

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6 More precisely, we require (i) of CP only for ex-ante preference orderings. Then, for all other conditional preference orderings, DCU and (ii) of CP imply (i) of CP.

7 This proposition is an application of the results in Section 3.2 of Chapter III by Wakai [6].
hold on $\mathcal{H} \upharpoonright \mathcal{F}_{t+1}(\omega)$.

For MP, under the induction hypothesis, (i), (iii) and (iv) hold on $\mathcal{H}(t, \omega; h)$; by CP and DCU, (i), (iii) and (iv) hold on $\mathcal{H} \upharpoonright \mathcal{F}_{t+1}(\omega)$. In terms of (ii), let $f, g \in \mathcal{H}(t, \omega; h)$. By (ii) of MP, for any lottery act $l$ and $\alpha \in (0, 1)$, $f \succeq_{t, \omega} g$ if and only if $\alpha f + (1 - \alpha) l \succeq_{t, \omega} \alpha g + (1 - \alpha) l$. Then $\alpha f + (1 - \alpha) l$ and $\alpha g + (1 - \alpha) l$ belong to $\mathcal{H}(t, \omega; h')$, where $h'_{\tau}(\cdot) = \alpha h_{\tau}(\cdot) + (1 - \alpha) l_{\tau}(\cdot)$ for all $\tau \leq t$ and $h'_{\tau}(\omega') = \alpha h_{\tau}(\omega') + (1 - \alpha) l_{\tau}(\omega')$ for all $\tau > t$ and for all $\omega' \notin \mathcal{F}_{t+1}(\omega)$. By CP and DCU, this implies that (ii) holds on $\mathcal{H} \upharpoonright \mathcal{F}_{t+1}(\omega)$. As for (v), for all $f, g \in \mathcal{H}(t, \omega; h)$ and $\alpha \in (0, 1)$, if $f \succeq_{t, \omega} g$, then $\alpha f + (1 - \alpha) g \succeq_{t, \omega} f$. Clearly, $f, g$, and $\alpha f + (1 - \alpha) g$ are in $\mathcal{H}(t, \omega; h)$. Again, by CD and DCU, (v) holds on $\mathcal{H} \upharpoonright \mathcal{F}_{t+1}(\omega)$. For (vi), since MP and RP$_t$ hold at $(t, \omega)$, by Lemma A.1 of Epstein and Schneider [2], (1) holds without rectangularity by replacing $\beta$ and $u$ with $\beta_{t, \omega}$ and $u_{t, \omega}$. Then, there exist $p, q \in \Delta_s(C)$ such that $p \succ_{t, \omega} q$. Since FS and DCU imply that $\mathcal{F}_{t+1}(\omega)$ is $\succeq_{t, \omega}$-nonnull, there exist $f, g \in \mathcal{H}(t, \omega; h)$ such that $f \succ_{t, \omega} g$, where $f_{\tau}(\omega') = p$ and $g_{\tau}(\omega') = q$ for all $\tau > t$ and for all $\omega' \in \mathcal{F}_{t+1}(\omega)$. Then, by CP and DCU, (vi) holds on $\mathcal{H} \upharpoonright \mathcal{F}_{t+1}(\omega)$. Hence, under the induction hypothesis, (i)-(vi) of MP hold for $\succeq_{t+1, \omega'}$ for all $(t + 1, \omega') \in \{t + 1\} \times \Omega$.

For RP$_{t+1}$, we only need to prove the case where $t + 1 < T - 1$; otherwise, RP$_{t+1}$ is trivially true (when $t + 1 = T - 1$) or vacuously true (when $t + 1 = T$). Let $f, g \in \mathcal{H}(t, \omega; h)$ such that $f_{\tau}(\omega') = p$, $f_{\tau+1}(\omega') = p'$, $g_{\tau}(\omega') = q$, $g_{\tau+1}(\omega') = q'$ for some $\tau \geq t + 1$ and for all $\omega' \in \mathcal{F}_{t+1}(\omega)$, and $f_{\tau'}(\omega') = g_{\tau'}(\omega') = l_{\tau'}(\omega')$ for all $\tau' \geq t + 1$ with $\tau' \neq \tau, \tau + 1$ and for all $\omega' \in \mathcal{F}_{t+1}(\omega)$, where $l$ is a lottery act. Then, by CP and DCU, $(l_{\tau, -(\tau+1)}, p, p') \succeq_{t+1, \omega} (l_{\tau, -(\tau+1)}, q, q')$ on $\mathcal{H} \upharpoonright \mathcal{F}_{t+1}(\omega)$ if and only if $f \succeq_{t, \omega} g$. Again, since MP and RP$_t$ hold at $(t, \omega)$, by Lemma A.1 of Epstein and Schneider [2], (1) holds without rectangularity by replacing $\beta$ and $u$ with $\beta_{t, \omega}$ and $u_{t, \omega}$. Since FS
and DCU imply that \( \mathcal{F}_{t+1}(\omega) \) is \( \succeq_{t,\omega} \)-nonnull, the form of (1) clearly implies that \( f \succeq_{t,\omega} g \) if and only if \( (l_{\tau, -(\tau+1)}, p, p') \succeq_{t,\omega} (l_{\tau, -(\tau+1)}, q, q') \) (i.e., (iv) of MP). Then, under CP and DCU, if \( \text{RP}_t \) holds at \((t, \omega)\), \( \text{RP}_{t+1} \) holds at \((t + 1, \omega)\). Hence, under the induction hypothesis, \( \text{RP}_{t+1} \) holds for \( \succeq_{t+1,\omega'} \) for all \((t + 1, \omega') \in \{t + 1\} \times \Omega\).

By the induction principle, the above two paragraphs show that MP and \( \text{RP}_t \) hold for all \((t, \omega) \in T \times \Omega\).

Finally, by Lemma A.1 of Epstein and Schneider [2], at each \( t \) and \( \omega \), \( \succeq_{t,\omega} \) is represented by an intertemporal version of multiple-priors utility without rectangularity, where \( \beta_{t,\omega} \) and \( u_{t,\omega} \) replace \( \beta \) and \( u \) in (1). Then, under CP, \( \beta_{0,\omega} = \beta_{0,\omega'} \) for all \( \omega, \omega' \in \Omega \), and \( u_{0,\omega} \) and \( u_{0,\omega'} \) represent the same ordering on \( \Delta_s(C) \) for all \( \omega, \omega' \in \Omega \). Let \( \beta_{0,\omega} \equiv \beta \) and \( u_{0,\omega} \equiv u \). Then, DCU implies that \( \beta_{t,\omega} = \beta \) for all \((t, \omega) \in T \times \Omega\), and \( u_{t,\omega} \) and \( u \) represent the same ordering on \( \Delta_s(C) \) for all \((t, \omega) \in T \times \Omega\) (refer to the proof of (vi) of MP and the proof of \( \text{RP}_{t+1} \)). Hence, DCU alone establishes all required consistency among preference orderings in \( \{\succeq_{t,\omega}\} \). The above result also implies that \( \text{RP} \) holds for all \((t, \omega) \in T \times \Omega\). In other words, \( \text{RP} \) is required only because DC is not sufficient to relate each \( \beta_{t,\omega} \) and each \( u_{t,\omega} \). ■
References


