Appendix D: Proofs of Proposition A1 and Claim 1

Proof of Proposition A1. (i) From (43) in the proof of Lemma 2,

$$\frac{d[w_n(e^+,s_l)h_n(e,s_l)]}{ds_l} = w_n(e^+,s_l)\frac{\partial h_n(e,s_l)}{\partial s_l} + \frac{dw_n(e^+,s_l)}{ds_l}h_n(e,s_l) = h_n(e,s_l)\left[-\frac{w_n(e^+,s_l)}{1-s_l} + \frac{dw_n(e^+,s_l)}{ds_l}\right]$$

$$= w_n(e^+,s_l)\frac{h_n(e,s_l)}{1-s_l}\left\{-1+\frac{(1-\alpha)e^+}{h_l(e^+,s_l)}\frac{\alpha\frac{\underline{h}_l}{e^+}\left[\underline{h}_l F(e^+)+\delta_l\int_0^{e^+}ef(e)de\right]+\left(\frac{\underline{h}_l}{e^+}+\delta_l\right)h_l(e^+,s_l)e^+f(e^+)}{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}}ef(e)de+[1-F(\overline{e})]\overline{e}\right]+h_l(e^+,s_l)e^+f(e^+)}\right\}$$

$$= w_n(e^+,s_l)\frac{h_n(e,s_l)}{1-s_l}\frac{1}{h_l(e^+,s_l)}\frac{\left(\begin{array}{c}-h_l(e^+,s_l)\left\{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}}ef(e)de+[1-F(\overline{e})]\overline{e}\right]+h_l(e^+,s_l)e^+f(e^+)\right\}\\ +(1-\alpha)e^+\left\{\alpha\frac{\underline{h}_l}{e^+}\left[\underline{h}_l F(e^+)+\delta_l\int_0^{e^+}ef(e)de\right]+\left(\frac{\underline{h}_l}{e^+}+\delta_l\right)h_l(e^+,s_l)e^+f(e^+)\right\}\end{array}\right)}{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}}ef(e)de+[1-F(\overline{e})]\overline{e}\right]+h_l(e^+,s_l)e^+f(e^+)}$$

$$= w_n(e^+,s_l)\frac{h_n(e,s_l)}{1-s_l}\frac{1}{h_l(e^+,s_l)}\frac{\left(\begin{array}{c}(1-\alpha)\frac{\underline{h}_l}{e^+}\left\{\alpha e^+\left[\underline{h}_l F(e^+)+\delta_l\int_0^{e^+}ef(e)de\right]-h_l(e^+,s_l)\left[\int_{e^+}^{\overline{e}}ef(e)de+[1-F(\overline{e})]\overline{e}\right]\right\}\\ +h_l(e^+,s_l)e^+f(e^+)\left[(1-\alpha)(\underline{h}_l+\delta_l e^+)-h_l(e^+,s_l)\right]\end{array}\right)}{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}}ef(e)de+[1-F(\overline{e})]\overline{e}\right]+h_l(e^+,s_l)e^+f(e^+)}$$

$$= w_n(e^+,s_l)\frac{h_n(e,s_l)}{1-s_l}\frac{1}{h_l(e^+,s_l)}\frac{\left(\begin{array}{c}(1-\alpha)\underline{h}_l\alpha\left\{\left[\underline{h}_l F(e^+)+\delta_l\int_0^{e^+}ef(e)de\right]-\frac{1}{1-\alpha}\left[\underline{h}_l F(e^+)+\delta_l s_l\int_0^{e^+}ef(e)de\right]\right\}\\ +h_l(e^+,s_l)e^+f(e^+)\left[(1-\alpha)(\underline{h}_l+\delta_l e^+)-h_l(e^+,s_l)\right]\end{array}\right)}{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}}ef(e)de+[1-F(\overline{e})]\overline{e}\right]+h_l(e^+,s_l)e^+f(e^+)}$$

$$\left(\text{because } h_l(e^+,s_l)\left[\int_{e^+}^{\overline{e}}ef(e)de+[1-F(\overline{e})]\overline{e}\right]=\frac{\alpha}{1-\alpha}\left[\delta_l s_l\int_0^{e^+}ef(e)de+\underline{h}_l F(e^+)\right]e^+ \text{from (7)}\right)$$

$$= w_n(e^+,s_l)\frac{h_n(e,s_l)}{1-s_l}\frac{1}{h_l(e^+,s_l)}\frac{\left(\begin{array}{c}\alpha\underline{h}_l\left\{(1-\alpha)\left[\underline{h}_l F(e^+)+\delta_l\int_0^{e^+}ef(e)de\right]-\left[\underline{h}_l F(e^+)+\delta_l s_l\int_0^{e^+}ef(e)de\right]\right\}\\ +h_l(e^+,s_l)e^+f(e^+)\left[-\alpha\underline{h}_l+(1-\alpha-s_l)\delta_l e^+\right]\end{array}\right)}{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}}ef(e)de+[1-F(\overline{e})]\overline{e}\right]+h_l(e^+,s_l)e^+f(e^+)}. \tag{D1}$$

[**Proof for** $\frac{d(w_nh_n)}{ds_l}<0$] When $s_l \geq 1-\alpha \Leftrightarrow 1-\alpha-s_l \leq 0$, $\frac{d(w_nh_n)}{ds_l}<0$ because the expressions inside the big parenthesis of (D1) is negative.

When $s_l < 1-\alpha$, $\frac{d(w_nh_n)}{ds_l}<0$ if $-\alpha\underline{h}_l+(1-\alpha-s_l)\delta_l e^+ \leq 0$ because $-\alpha\underline{h}_l+(1-\alpha-s_l)\delta_l e^+ > \frac{1}{F(e^+)}\left\{(1-\alpha)\left[\underline{h}_l F(e^+)+\delta_l\int_0^{e^+}ef(e)de\right]-\left[\underline{h}_l F(e^+)+\delta_l s_l\int_0^{e^+}ef(e)de\right]\right\}$ for the expressions inside the parenthesis. $-\alpha\underline{h}_l+(1-\alpha-s_l)\delta_l e^+\leq 0$ holds iff $s_l \geq (1-\alpha)-\alpha\frac{\underline{h}_l}{\delta_l e^+(s_l)}$, where $e^+(s_l)$ is, from (7) and (5), a solution for

$$\frac{\alpha}{1-\alpha}\left[\underline{h}_l F(e^+)+\delta_l s_l\int_0^{e^+}ef(e)de\right]e^+ = \left[\int_{e^+}^{\overline{e}}ef(e)de+[1-F(\overline{e})]\,\overline{e}\right]h_l(e^+,s_l) \tag{D2}$$

$$\Leftrightarrow \delta_l s_l e^+\left[\int_0^{\overline{e}}ef(e)de+[1-F(\overline{e})]\overline{e}-\frac{1}{1-\alpha}\int_0^{e^+}ef(e)de\right]=\left\{\frac{\alpha}{1-\alpha}F(e^+)e^+ + \int_0^{e^+}ef(e)de-\left[\int_0^{\overline{e}}ef(e)de+[1-F(\overline{e})]\overline{e}\right]\right\}\underline{h}_l. \tag{D3}$$

$s_l-\left[(1-\alpha)-\alpha\frac{\underline{h}_l}{\delta_l e^+(s_l)}\right]$ increases with s_l because

$$1-\alpha\frac{\underline{h}_l}{\delta_l(e^+)^2}\frac{de^+}{ds_l}$$

$$> 1-\alpha\frac{\underline{h}_l}{\delta_l(e^+)^2}\frac{\delta_l(e^+)^2}{\underline{h}_l} > 0. \text{ (from (37) in the proof of Lemma 1).} \tag{D4}$$

Further, at $s_l = s_l^{\star\star} \equiv (1-\alpha) - \alpha\frac{h_l}{\delta_l \overline{e}}$, $s_l > (1-\alpha) - \alpha\frac{h_l}{\delta_l e^+(s_l)}$. Hence, if $(1-\alpha) - \alpha\frac{h_l}{\delta_l e^+(0)} \leq 0 \Leftrightarrow e^+(0) \leq \frac{\alpha h_l}{(1-\alpha)\delta_l}$, $(1-\alpha)(\underline{h}_l + \delta_l e^+) - h_l(e^+, s_l) \leq 0$ holds for any s_l and thus $\frac{d(w_n h_n)}{ds_l} < 0$ for any s_l. $e^+(0)$ is the solution to $\frac{\alpha}{1-\alpha}F(e^+)e^+ + \int_0^{e^+} ef(e)de = \int_0^{\overline{e}} ef(e)de + [1-F(\overline{e})]\overline{e}$ from (D3), thus this is the case when many individuals have limited wealth.

Otherwise, i.e. $e^+(0) > \frac{\alpha h_l}{(1-\alpha)\delta_l}$, there exists unique $s_l^\sharp \in (0, s_l^{\star\star})$ satisfying $s_l^\sharp = (1-\alpha) - \alpha\frac{h_l}{\delta_l e^+(s_l^\sharp)}$ and $-\alpha\underline{h}_l + (1-\alpha-s_l)\delta_l e^+ \leq 0$ for $s_l \geq s_l^\sharp$. Thus, $\frac{d(w_n h_n)}{ds_l} < 0$ for $s_l \geq s_l^\sharp$. (Note that this is a sufficient but not necessary condition. $\frac{d(w_n h_n)}{ds_l} < 0$ could hold for smaller s_l or for any s_l, if the expressions inside the big parenthesis of (D1) is negative.)

[**Proof for** $\frac{d(w_n h_n)}{ds_l} > 0$] As shown above, $\frac{d(w_n h_n)}{ds_l} > 0$ is possible only when $s_l < 1-\alpha$, in which case $-\alpha\underline{h}_l + (1-\alpha-s_l)\delta_l e^+ > \frac{1}{F(e^+)}\left\{(1-\alpha)\left[\underline{h}_l F(e^+) + \delta_l\int_0^{e^+} ef(e)de\right] - \left[\underline{h}_l F(e^+) + \delta_l s_l \int_0^{e^+} ef(e)de\right]\right\}$ holds. Thus, $\frac{d(w_n h_n)}{ds_l} > 0$ when $(1-\alpha)\left[\underline{h}_l F(e^+) + \delta_l\int_0^{e^+} ef(e)de\right] - \left[\underline{h}_l F(e^+) + \delta_l s_l\int_0^{e^+} ef(e)de\right] \geq 0$, which holds iff $s_l \leq 1-\alpha - \alpha\frac{\underline{h}_l F(e^+(s_l))}{\delta_l\int_0^{e^+(s_l)} ef(e)de}$, where $e^+(s_l)$ is a solution to (D3).

At $s_l = s_l^{\star\star} \equiv (1-\alpha) - \alpha\frac{h_l}{\delta_l\overline{e}}$, $s_l > (1-\alpha) - \alpha\frac{\underline{h}_l F(e^+(s_l))}{\delta_l\int_0^{e^+(s_l)} ef(e)de}$. Further, $s_l - \left[(1-\alpha) - \alpha\frac{\underline{h}_l}{\delta_l}\frac{F(e^+(s_l))}{\int_0^{e^+(s_l)} ef(e)de}\right]$ increases with s_l if $\frac{\int_0^{e^+(0)} ef(e)de}{F(e^+(0))} \geq \frac{\alpha}{1+\alpha}e^+(0)$.

This can be proved as follows. The derivative of the expression with respect to s_l equals

$$1 - \alpha\frac{\underline{h}_l}{\delta_l}\frac{F(e^+)e^+ - \int_0^{e^+} ef(e)de}{(\int_0^{e^+} ef(e)de)^2} f(e^+)\frac{de^+}{ds_l}$$

$$= 1 - \alpha\frac{\underline{h}_l}{\delta_l}\frac{F(e^+)e^+ - \int_0^{e^+} ef(e)de}{\left[\int_0^{e^+} ef(e)de\right]^2} f(e^+)\frac{\delta_l e^+\left\{(1-\alpha)\left[\int_{e^+}^{\overline{e}} ef(e)de + [1-F(\overline{e})]\overline{e}\right] - \alpha\int_0^{e^+} ef(e)de\right\}}{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}} ef(e)de + [1-F(\overline{e})]\overline{e}\right] + h_l(e^+, s_l)e^+ f(e^+)} \quad \text{(from (37))}$$

$$= \frac{\left(\begin{array}{c}\left\{\frac{1}{e^+}\left[\int_0^{e^+} ef(e)de\right]^2 - \alpha\left[F(e^+)e^+ - \int_0^{e^+} ef(e)de\right] f(e^+)e^+\right\}(1-\alpha)\underline{h}_l\left[\int_{e^+}^{\overline{e}} ef(e)de + [1-F(\overline{e})]\overline{e}\right] \\ + \left[\int_0^{e^+} ef(e)de\right]^2 h_l(e^+, s_l)e^+ f(e^+) + \alpha^2\underline{h}_l\left[F(e^+)e^+ - \int_0^{e^+} ef(e)de\right] f(e^+)e^+\left[\int_0^{e^+} ef(e)de\right]\end{array}\right)}{\left[\int_0^{e^+} ef(e)de\right]^2\left\{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}} ef(e)de + [1-F(\overline{e})]\overline{e}\right] + h_l(e^+, s_l)e^+ f(e^+)\right\}}$$

$$= \frac{\left(\begin{array}{c}\left\{\frac{1}{e^+}\left[\int_0^{e^+} ef(e)de\right]^2 - \alpha\left[F(e^+)e^+ - \int_0^{e^+} ef(e)de\right] f(e^+)e^+\right\}\alpha\underline{h}_l\left[\underline{h}_l F(e^+) + \delta_l s_l\int_0^{e^+} ef(e)de\right]\frac{e^+}{h_l(e^+, s_l)} \\ + \left[\int_0^{e^+} ef(e)de\right]^2 h_l(e^+, s_l)e^+ f(e^+) + \alpha^2\underline{h}_l\left[F(e^+)e^+ - \int_0^{e^+} ef(e)de\right] f(e^+)e^+\left[\int_0^{e^+} ef(e)de\right]\end{array}\right)}{\left[\int_0^{e^+} ef(e)de\right]^2\left\{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}} ef(e)de + [1-F(\overline{e})]\overline{e}\right] + h_l(e^+, s_l)e^+ f(e^+)\right\}} \quad \text{(from (D2))}$$

$$= \frac{\left(\begin{array}{c}\left\{\frac{1}{e^+}\left[\int_0^{e^+} ef(e)de\right]^2 - \alpha\left[1 - h_l(e^+, s_l)\frac{1}{\delta_l s_l e^+}\right]\left[F(e^+)e^+ - \int_0^{e^+} ef(e)de\right] f(e^+)e^+\right\}\alpha\underline{h}_l\left(\delta_l s_l\int_0^{e^+} ef(e)de\right)\frac{e^+}{h_l(e^+, s_l)} \\ + \left\{\frac{1}{e^+}\left[\int_0^{e^+} ef(e)de\right]^2 - \alpha\left[F(e^+)e^+ - \int_0^{e^+} ef(e)de\right] f(e^+)e^+\right\}\alpha\underline{h}_l\left[\underline{h}_l F(e^+)\right]\frac{e^+}{h_l(e^+, s_l)} + \left[\int_0^{e^+} ef(e)de\right]^2 h_l(e^+, s_l)e^+ f(e^+)\end{array}\right)}{\left[\int_0^{e^+} ef(e)de\right]^2\left\{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}} ef(e)de + [1-F(\overline{e})]\overline{e}\right] + h_l(e^+, s_l)e^+ f(e^+)\right\}}$$

$$= \frac{\left(\begin{array}{c}\left[\int_0^{e^+} ef(e)de\right]^2\left[\delta_l s_l \int_0^{e^+} ef(e)de + \underline{h}_l F(e^+)\right]\alpha \underline{h}_l \\ +\left\{[h_l(e^+,s_l)-\alpha\underline{h}_l]\left[\int_0^{e^+} ef(e)de\right]+\alpha\underline{h}_l F(e^+)e^+\right\}\left\{[h_l(e^+,s_l)+\alpha\underline{h}_l]\left[\int_0^{e^+} ef(e)de\right]-\alpha\underline{h}_l F(e^+)e^+\right\} f(e^+)e^+\end{array}\right)}{h_l(e^+,s_l)\left[\int_0^{e^+} ef(e)de\right]^2\left\{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}} ef(e)de+[1-F(\overline{e})]\overline{e}\right]+h_l(e^+,s_l)e^+f(e^+)\right\}}. \tag{D5}$$

The expression is positive if

$$\begin{aligned}&[h_l(e^+,s_l)+\alpha\underline{h}_l]\left[\int_0^{e^+} ef(e)de\right]-\alpha\underline{h}_l F(e^+)e^+ \geq 0\\ \Leftrightarrow &\left\{2\left[\int_0^{e^+} ef(e)de\right]-(1-\alpha)\left[\int_0^{\overline{e}} ef(e)de+[1-F(\overline{e})]\,\overline{e}\right]\right\}h_l(e^+,s_l)\geq 0,\end{aligned} \tag{D6}$$

where the second equality is from (D2), which can be expressed as $\alpha\underline{h}_l\left[F(e^+)e^+ - \int_0^{e^+} ef(e)de\right] + h_l(e^+,s_l)\int_0^{e^+} ef(e)de = (1-\alpha)\left[\int_0^{\overline{e}} ef(e)de+[1-F(\overline{e})]\,\overline{e}\right]h_l(e^+,s_l)$. The inequality holds for any s_l iff

$$\begin{aligned}&2\left[\int_0^{e^+(0)} ef(e)de\right]-(1-\alpha)\left[\int_0^{\overline{e}} ef(e)de+[1-F(\overline{e})]\,\overline{e}\right]\geq 0\\ \Leftrightarrow &\frac{\int_0^{e^+(0)} ef(e)de}{F(e^+(0))}\geq \frac{\alpha}{1+\alpha}e^+(0) \quad \text{(from (D3))}.\end{aligned} \tag{D7}$$

Hence, if $(1-\alpha)-\alpha\frac{\underline{h}_l F(e^+(0))}{\delta_l\int_0^{e^+(0)} ef(e)de} > 0 \Leftrightarrow \frac{\int_0^{e^+(0)} ef(e)de}{F(e^+(0))} > \frac{\alpha\underline{h}_l}{(1-\alpha)\delta_l}$ and $\frac{\int_0^{e^+(0)} ef(e)de}{F(e^+(0))} \geq \frac{\alpha}{1+\alpha}e^+(0)$, there exists unique $s_l^\flat \in (0, s_l^\sharp)$ satisfying $s_l^\flat = (1-\alpha)-\alpha\frac{\underline{h}_l F(e^+(s_l^\flat))}{\delta_l\int_0^{e^+(s_l^\flat)} ef(e)de}$ and $\frac{d(w_n h_n)}{ds_l} > 0$ for $s_l \leq s_l^\flat$.

(ii) From (43) in the proof of Lemma 2,

$$\begin{aligned}\frac{d[w_l(e^+,s_l)h_l(e,s_l)]}{ds_l} &= w_l(e^+,s_l)\frac{\partial h_l(e,s_l)}{\partial s_l}+\frac{dw_l(e^+,s_l)}{ds_l}h_l(e,s_l)\\ &= w_l(e^+,s_l)\delta_l e - \frac{\alpha}{1-\alpha}\frac{w_l(e^+,s_l)}{w_n(e^+,s_l)}\frac{dw_n(e^+,s_l)}{ds_l}h_l(e,s_l)\end{aligned}$$

$$= w_l(e^+,s_l)\left\{\delta_l e - \alpha\frac{h_l(e,s_l)}{h_l(e^+,s_l)}\frac{e^+}{1-s_l}\frac{\alpha\frac{\underline{h}_l}{e^+}\left[\underline{h}_l F(e^+)+\delta_l\int_0^{e^+} ef(e)de\right]+\left(\frac{\underline{h}_l}{e^+}+\delta_l\right)h_l(e^+,s_l)e^+f(e^+)}{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}} ef(e)de+[1-F(\overline{e})]\overline{e}\right]+h_l(e^+,s_l)e^+f(e^+)}\right\}$$

$$= w_l(e^+,s_l)\frac{\left(\begin{array}{c}\delta_l e h_l(e^+,s_l)(1-s_l)\left\{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}} ef(e)de+[1-F(\overline{e})]\overline{e}\right]+h_l(e^+,s_l)e^+f(e^+)\right\}\\ -\alpha h_l(e,s_l)\left\{\alpha\underline{h}_l\left[\underline{h}_l F(e^+)+\delta_l\int_0^{e^+} ef(e)de\right]+(\underline{h}_l+\delta_l e^+)h_l(e^+,s_l)e^+f(e^+)\right\}\end{array}\right)}{h_l(e^+,s_l)(1-s_l)\left\{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}} ef(e)de+[1-F(\overline{e})]\overline{e}\right]+h_l(e^+,s_l)e^+f(e^+)\right\}}$$

$$= w_l(e^+,s_l)\frac{\left\{\begin{array}{c}\delta_l e h_l(e^+,s_l)(1-s_l)(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}} ef(e)de+[1-F(\overline{e})]\overline{e}\right]\\ -\alpha h_l(e,s_l)\alpha\underline{h}_l\left[\underline{h}_l F(e^+)+\delta_l\int_0^{e^+} ef(e)de\right]+h_l(e^+,s_l)e^+f(e^+)[\delta_l e h_l(e^+,s_l)(1-s_l)-\alpha h_l(e,s_l)(\underline{h}_l+\delta_l e^+)]\end{array}\right\}}{h_l(e^+,s_l)(1-s_l)\left\{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}} ef(e)de+[1-F(\overline{e})]\overline{e}\right]+h_l(e^+,s_l)e^+f(e^+)\right\}}$$

$$= w_l(e^+,s_l)\frac{\left(\begin{array}{c}\alpha\underline{h}_l\left\{\delta_l e(1-s_l)\left[\underline{h}_l F(e^+)+\delta_l s_l\int_0^{e^+} ef(e)de\right]-\alpha h_l(e,s_l)\left[\underline{h}_l F(e^+)+\delta_l\int_0^{e^+} ef(e)de\right]\right\}\\ +h_l(e^+,s_l)e^+f(e^+)\left[\delta_l e(1-s_l)h_l(e^+,s_l)-\alpha h_l(e,s_l)(\underline{h}_l+\delta_l e^+)\right]\end{array}\right)}{h_l(e^+,s_l)(1-s_l)\left\{(1-\alpha)\frac{\underline{h}_l}{e^+}\left[\int_{e^+}^{\overline{e}} ef(e)de+[1-F(\overline{e})]\overline{e}\right]+h_l(e^+,s_l)e^+f(e^+)\right\}} \quad \text{(from (D2))}$$

$$
= w_l(e^+, s_l)\frac{\begin{pmatrix}\alpha\underline{h}_l\left\{[(1-s_l-\alpha s_l)\delta_l e-\alpha\underline{h}_l]\,\underline{h}_l F(e^+)+[(1-s_l-\alpha)\delta_l s_l e-\alpha\underline{h}_l]\,\delta_l\int_0^{e^+} ef(e)de\right\}\\ +h_l(e^+,s_l)e^+f(e^+)\{[(1-s_l-\alpha s_l)\delta_l e-\alpha\underline{h}_l]\,\underline{h}_l+[(1-s_l-\alpha)\delta_l s_l e-\alpha\underline{h}_l]\,\delta_l e^+\}\end{pmatrix}}{h_l(e^+,s_l)(1-s_l)\left\{(1-\alpha)\frac{h_l}{e^+}\left[\int_{e^+}^{\overline{e}} ef(e)de+[1-F(\overline{e})]\overline{e}\right]+h_l(e^+,s_l)e^+f(e^+)\right\}}. \quad \text{(D8)}
$$

[**Proof for** $\frac{d(w_l h_l)}{ds_l}>0$] If the expression inside the big parenthesis of (D8) is positive, $(1-s_l-\alpha s_l)\delta_l e-\alpha\underline{h}_l>0$ and thus $[(1-s_l-\alpha s_l)\delta_l e-\alpha\underline{h}_l]\frac{\underline{h}_l F(e^+)}{\int_0^{e^+} ef(e)de}+[(1-s_l-\alpha)\delta_l s_l e-\alpha\underline{h}_l]\delta_l>[(1-s_l-\alpha s_l)\delta_l e-\alpha\underline{h}_l]\frac{\underline{h}_l}{e^+}+[(1-s_l-\alpha)\delta_l s_l e-\alpha\underline{h}_l]\delta_l$ hold. Hence, $\frac{d(w_l h_l)}{ds_l}>0$ if $[(1-s_l-\alpha s_l)\delta_l e-\alpha\underline{h}_l]\frac{\underline{h}_l}{e^+}+[(1-s_l-\alpha)\delta_l s_l e-\alpha\underline{h}_l]\delta_l\geq 0\Leftrightarrow L(s_l)\equiv -ee^+(s_l)s_l^2+\left[(1-\alpha)e^+(s_l)-(1+\alpha)\frac{\underline{h}_l}{\delta_l}\right]es_l+\left[-\alpha\left(\frac{\underline{h}_l}{\delta_l}+e^+(s_l)\right)+e\right]\frac{\underline{h}_l}{\delta_l}\geq 0\Leftrightarrow s_l\in\left[\max(0,s_{l,l}^{\triangle}(e)),s_{l,h}^{\triangle}(e)\right]$, where $s_{l,h}^{\triangle}(e)$ ($s_{l,l}^{\triangle}(e)$) is the solution to

$$
\begin{aligned}
s_l &= \frac{\left[(1-\alpha)e^+(s_l)-(1+\alpha)\frac{\underline{h}_l}{\delta_l}\right]e+(-)\sqrt{\left[(1-\alpha)e^+(s_l)-(1+\alpha)\frac{\underline{h}_l}{\delta_l}\right]^2e^2+4ee^+(s_l)\left[-\alpha\left(\frac{\underline{h}_l}{\delta_l}+e^+(s_l)\right)+e\right]\frac{\underline{h}_l}{\delta_l}}}{2ee^+(s_l)}\\
&= \frac{\left[(1-\alpha)e^+(s_l)-(1+\alpha)\frac{\underline{h}_l}{\delta_l}\right]+(-)\sqrt{\left[(1-\alpha)e^+(s_l)-(1+\alpha)\frac{\underline{h}_l}{\delta_l}\right]^2+4e^+(s_l)\left[-\alpha\left(\frac{\underline{h}_l}{\delta_l}+e^+(s_l)\right)e^{-1}+1\right]\frac{\underline{h}_l}{\delta_l}}}{2e^+(s_l)}\\
&= \frac{\left[(1-\alpha)-(1+\alpha)\frac{\underline{h}_l}{\delta_l e^+(s_l)}\right]+(-)\sqrt{\left[(1-\alpha)-(1+\alpha)\frac{\underline{h}_l}{\delta_l e^+(s_l)}\right]^2+4\left[-\alpha+\frac{e-\alpha\frac{\underline{h}_l}{\delta_l}}{e^+(s_l)}\right]\frac{\underline{h}_l}{\delta_l e}}}{2}. \qquad \text{(D9)}
\end{aligned}
$$

Real solutions to (D9) could exist only if the expression inside the square root is non-negative (from the second equation),

$$
e\geq\Lambda(e^+(s_l))\equiv\frac{4\alpha\left(\frac{\underline{h}_l}{\delta_l}+e^+(s_l)\right)\frac{\underline{h}_l}{\delta_l}}{4\frac{\underline{h}_l}{\delta_l}+\frac{1}{e^+(s_l)}\left[(1-\alpha)e^+(s_l)-(1+\alpha)\frac{\underline{h}_l}{\delta_l}\right]^2}=\frac{\alpha\left(\frac{\underline{h}_l}{\delta_l}+e^+(s_l)\right)}{1+\frac{\delta_l}{4\underline{h}_l e^+(s_l)}\left[(1-\alpha)e^+(s_l)-(1+\alpha)\frac{\underline{h}_l}{\delta_l}\right]^2}, \qquad \text{(D10)}
$$

which holds for any s_l if

$$
e\geq\Lambda(\overline{e})\equiv\frac{\alpha\left(\frac{\underline{h}_l}{\delta_l}+\overline{e}\right)}{1+\frac{\delta_l}{4\underline{h}_l\overline{e}}\left[(1-\alpha)\overline{e}-(1+\alpha)\frac{\underline{h}_l}{\delta_l}\right]^2}, \qquad \text{(D11)}
$$

because the derivative of (D10), which can be expressed as $\frac{4\alpha\left(1+\frac{\underline{h}_l}{\delta_l e^+(s_l)}\right)\frac{\underline{h}_l}{\delta_l}}{4\frac{\underline{h}_l}{\delta_l e^+(s_l)}+\left[(1-\alpha)-(1+\alpha)\frac{\underline{h}_l}{\delta_l e^+(s_l)}\right]^2}$, with respect to s_l is proportional to

$$
\begin{aligned}
&-\left\{\frac{4\underline{h}_l}{\delta_l e^+(s_l)}+\left[(1-\alpha)-(1+\alpha)\frac{\underline{h}_l}{\delta_l e^+(s_l)}\right]^2\right\}\frac{\underline{h}_l}{\delta_l[e^+(s_l)]^2}e^{+\prime}(s_l)-\left(1+\frac{\underline{h}_l}{\delta_l e^+(s_l)}\right)\left\{\left[(1-\alpha)-(1+\alpha)\frac{\underline{h}_l}{\delta_l e^+(s_l)}\right](1+\alpha)-2\right\}\frac{2\underline{h}_l}{\delta_l[e^+(s_l)]^2}e^{+\prime}(s_l)\\
&=-\left\{(1-\alpha)^2+\left[(1+\alpha)\frac{\underline{h}_l}{\delta_l e^+(s_l)}\right]^2+2(1+\alpha^2)\frac{\underline{h}_l}{\delta_l e^+(s_l)}-2\left(1+\frac{\underline{h}_l}{\delta_l e^+(s_l)}\right)\left[(1+\alpha)^2\frac{\underline{h}_l}{\delta_l e^+(s_l)}+(1+\alpha^2)\right]\right\}\frac{\underline{h}_l}{\delta_l[e^+(s_l)]^2}e^{+\prime}(s_l)\\
&=-\left\{(1-\alpha)^2-\left[(1+\alpha)\frac{\underline{h}_l}{\delta_l e^+(s_l)}\right]^2-2(1+\alpha^2)-2(1+\alpha)^2\frac{\underline{h}_l}{\delta_l e^+(s_l)}\right\}\frac{\underline{h}_l}{\delta_l[e^+(s_l)]^2}e^{+\prime}(s_l)\\
&=(1+\alpha)^2\left(1+\frac{\underline{h}_l}{\delta_l e^+(s_l)}\right)^2\frac{\underline{h}_l}{\delta_l[e^+(s_l)]^2}e^{+\prime}(s_l)>0. \qquad \text{(D12)}
\end{aligned}
$$

$s^{\triangle}_{l,h}(e) \in (0,1)$ satisfying (D9) exists and is unique when $e > \alpha\left(\frac{h_l}{\delta_l} + e^+(0)\right)$ and $e \geq \Lambda(\overline{e})$ or when $e^+(0) > \frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$ and $e \geq \Lambda(\overline{e})$, because [1] the RHS of (D9) decreases with $e^+(s_l)$ and thus s_l, [2] the RHS at $s_l = 0$ is greater than 0, which holds clearly for the above ranges of e from (D9), and [3] the RHS at $s_l = 1$ is smaller than 1.

[1] can be proved as follows. The derivative of the numerator of the RHS of (D9) with respect to e^+ equals

$$\frac{(1+\alpha)h_l}{\delta_l[e^+(s_l)]^2} + (-)\left\{\left[(1-\alpha) - \frac{(1+\alpha)h_l}{\delta_l e^+(s_l)}\right]^2 + 4\left(-\alpha + \frac{e-\alpha\frac{h_l}{\delta_l}}{e^+(s_l)}\right)\frac{h_l}{\delta_l e}\right\}^{-\frac{1}{2}}\left\{\left[(1-\alpha) - \frac{(1+\alpha)h_l}{\delta_l e^+(s_l)}\right]\frac{(1+\alpha)h_l}{\delta_l[e^+(s_l)]^2} - 2\frac{e-\alpha\frac{h_l}{\delta_l}}{[e^+(s_l)]^2}\frac{h_l}{\delta_l e}\right\}$$

$$= \left\{\left[(1-\alpha) - \frac{(1+\alpha)h_l}{\delta_l e^+(s_l)}\right]^2 + 4\left(-\alpha + \frac{e-\alpha\frac{h_l}{\delta_l}}{e^+(s_l)}\right)\frac{h_l}{\delta_l e}\right\}^{-\frac{1}{2}}\frac{h_l}{\delta_l[e^+(s_l)]^2}\left(\begin{array}{l}\left\{\left[(1-\alpha) - \frac{(1+\alpha)h_l}{\delta_l e^+(s_l)}\right]^2 + 4\left(-\alpha + \frac{e-\alpha\frac{h_l}{\delta_l}}{e^+(s_l)}\right)\frac{h_l}{\delta_l e}\right\}^{\frac{1}{2}}(1+\alpha) \\ \quad + (-)\left\{\left[(1-\alpha) - \frac{(1+\alpha)h_l}{\delta_l e^+(s_l)}\right](1+\alpha) - 2\frac{e-\alpha\frac{h_l}{\delta_l}}{e}\right\}\end{array}\right). \tag{D13}$$

The second term inside the big parenthesis of the above equation is negative because

$$\begin{aligned}
&\left[(1-\alpha) - \frac{(1+\alpha)h_l}{\delta_l e^+(s_l)}\right](1+\alpha) - 2\frac{e-\alpha\frac{h_l}{\delta_l}}{e} \\
\leq\ &\left[(1-\alpha) - \frac{(1+\alpha)h_l}{\delta_l e^+(s_l)}\right](1+\alpha) - 2 + 2\alpha\frac{h_l}{\delta_l}\left\{\frac{4\alpha\left(1+\frac{h_l}{\delta_l\overline{e}}\right)\frac{h_l}{\delta_l}}{\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l\overline{e}}\right]^2 + 4\frac{h_l}{\delta_l\overline{e}}}\right\}^{-1} \quad \text{(from (D11))} \\
<\ &\frac{-\left[\frac{(1+\alpha)^2h_l}{\delta_l\overline{e}} + (1+\alpha^2)\right]2\left(1+\frac{h_l}{\delta_l\overline{e}}\right) + \left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l\overline{e}}\right]^2 + 4\frac{h_l}{\delta_l\overline{e}}}{2\left(1+\frac{h_l}{\delta_l\overline{e}}\right)} \\
=\ &\frac{-\left[(1+\alpha)\frac{h_l}{\delta_l\overline{e}}\right]^2 - 2\frac{(1+\alpha)^2h_l}{\delta_l\overline{e}} - (1+\alpha^2)2\left(1+\frac{h_l}{\delta_l\overline{e}}\right) + \left[(1-\alpha)^2 - 2(1-\alpha^2)\frac{h_l}{\delta_l\overline{e}}\right] + 4\frac{h_l}{\delta_l\overline{e}}}{2\left(1+\frac{h_l}{\delta_l\overline{e}}\right)} \\
=\ &-\frac{(1+\alpha)^2}{2}\left(1+\frac{h_l}{\delta_l\overline{e}}\right) < 0.
\end{aligned} \tag{D14}$$

Then, the RHS of (D9) decreases with e^+ when the solution is $s^{\triangle}_{l,h}(e)$ because

$$\begin{aligned}
&\left\{\left[(1-\alpha) - \frac{(1+\alpha)h_l}{\delta_l e^+(s_l)}\right]^2 + 4\left(-\alpha + \frac{e-\alpha\frac{h_l}{\delta_l}}{e^+(s_l)}\right)\frac{h_l}{\delta_l e}\right\}(1+\alpha)^2 < \left\{-\left[(1-\alpha) - \frac{(1+\alpha)h_l}{\delta_l e^+(s_l)}\right](1+\alpha) + 2\frac{e-\alpha\frac{h_l}{\delta_l}}{e}\right\}^2 \\
\Leftrightarrow\ &\left(-\alpha + \frac{e-\alpha\frac{h_l}{\delta_l}}{e^+(s_l)}\right)\frac{h_l}{\delta_l e}(1+\alpha)^2 < \frac{e-\alpha\frac{h_l}{\delta_l}}{e}\left\{-\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(s_l)}\right](1+\alpha) + \frac{e-\alpha\frac{h_l}{\delta_l}}{e}\right\} \\
\Leftrightarrow\ &-\frac{h_l}{\delta_l}(1+\alpha)^2 < (e-\alpha\frac{h_l}{\delta_l})(\alpha - \frac{h_l}{\delta_l e}) \\
\Leftrightarrow\ &-2\frac{h_l}{\delta_l} < e + \frac{h_l}{\delta_l}\frac{h_l}{\delta_l e}.
\end{aligned} \tag{D15}$$

[3] is true since

$$\frac{\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(1)}\right]+\sqrt{\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(1)}\right]^2+4\left(-\alpha+\frac{e-\alpha\frac{h_l}{\delta_l}}{e^+(1)}\right)\frac{h_l}{\delta_l e}}}{2}<1$$

$$\Leftrightarrow \left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(1)}\right]^2+4\left(-\alpha+\frac{e-\alpha\frac{h_l}{\delta_l}}{e^+(1)}\right)\frac{h_l}{\delta_l e}<(1+\alpha)^2\left(1+\frac{h_l}{\delta_l e^+(1)}\right)^2$$

$$\Leftrightarrow 2(1+\alpha^2)\frac{h_l}{\delta_l e^+(1)}+(1-\alpha)^2-4\alpha\left(1+\frac{h_l}{\delta_l e^+(1)}\right)\frac{h_l}{\delta_l e}<(1+\alpha)^2\left(1+2\frac{h_l}{\delta_l e^+(1)}\right). \tag{D16}$$

$s^{\triangle}_{l,l}(e)\in(0,s^{\triangle}_{l,h}(e))$ satisfying (D9) exists when $e^+(0)>\frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$, $e<\alpha\left(\frac{h_l}{\delta_l}+e^+(0)\right)$, and $e\geq\Lambda(\overline{e})$, because [1] the RHS of (D9) increases with $e^+(s_l)$ and thus s_l from (D13) and (D14), [2] the RHS at $s_l=0$ is greater than 0, and [3] the RHS at $s_l=1$ is smaller than 1. [2] and [3] hold clearly for the above range of e and $e^+(0)$ from (D9).

$s^{\triangle}_{l,l}(e)>0$ does not exist when $\overline{e}\leq\frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$ or when $e\geq\alpha\left(\frac{h_l}{\delta_l}+\overline{e}\right)$, which is clear from (D9).

Finally, it is formally proved that $L(s_l)\geq 0$ and thus $\frac{d(w_lh_l)}{ds_l}>0$ when $s_l\in\left[\max(0,s^{\triangle}_{l,l}(e)),s^{\triangle}_{l,h}(e)\right]$. This is true because for s_l greater than $s^{\triangle}_{l,l}(e)$, $s_l<s^{\triangle}_{l,h}(e)$ implies from (D9)

$$s_l<\frac{\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(s_l)}\right]+\sqrt{\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(s_l)}\right]^2+4\left(-\alpha+\frac{e-\alpha\frac{h_l}{\delta_l}}{e^+(s_l)}\right)\frac{h_l}{\delta_l e}}}{2}$$

($\because$ the RHS decreases with s_l)

$$\Leftrightarrow 2s_l-\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(s_l)}\right]<\sqrt{\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(s_l)}\right]^2+4\left(-\alpha+\frac{e-\alpha\frac{h_l}{\delta_l}}{e^+(s_l)}\right)\frac{h_l}{\delta_l e}}$$

$$\Leftrightarrow \left\{2s_l-\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(s_l)}\right]\right\}^2<\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(s_l)}\right]^2+4\left(-\alpha+\frac{e-\alpha\frac{h_l}{\delta_l}}{e^+(s_l)}\right)\frac{h_l}{\delta_l e}$$

$$\left(\begin{array}{c}\because s_l>s^{\triangle}_{l,l}(e)\Leftrightarrow\ 2s_l>\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(s_l)}\right]-\sqrt{\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(s_l)}\right]^2+4\left(-\alpha+\frac{e-\alpha\frac{h_l}{\delta_l}}{e^+(s_l)}\right)\frac{h_l}{\delta_l e}}\\ [\because \text{the RHS increases with } s_l]\\ \Leftrightarrow\ 2s_l-\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(s_l)}\right]>-\sqrt{\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(s_l)}\right]^2+4\left(-\alpha+\frac{e-\alpha\frac{h_l}{\delta_l}}{e^+(s_l)}\right)\frac{h_l}{\delta_l e}}\end{array}\right)$$

$$\Leftrightarrow -s_l^2+\left[(1-\alpha)-(1+\alpha)\frac{h_l}{\delta_l e^+(s_l)}\right]s_l-\left(-\alpha+\frac{e-\alpha\frac{h_l}{\delta_l}}{e^+(s_l)}\right)\frac{h_l}{\delta_l e}>0$$

$$\Leftrightarrow L(s_l)\equiv -ee^+(s_l)s_l^2+\left[(1-\alpha)e^+(s_l)-(1+\alpha)\frac{h_l}{\delta_l}\right]es_l+\left[-\alpha\left(\frac{h_l}{\delta_l}+e^+(s_l)\right)+e\right]\frac{h_l}{\delta_l}>0. \tag{D17}$$

To summarize the result, (a) $\frac{d(w_lh_l)}{ds_l}>0$ for $s_l\leq s^{\triangle}_{l,h}(e)$ when $e\geq\alpha\left(\frac{h_l}{\delta_l}+\overline{e}\right)$ and when $\overline{e}\leq\frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$, $e\in(\alpha\left(\frac{h_l}{\delta_l}+e^+(0)\right),\alpha\left(\frac{h_l}{\delta_l}+\overline{e}\right))$, and $e\geq\Lambda(\overline{e})\equiv\frac{\alpha\left(\frac{h_l}{\delta_l}+\overline{e}\right)}{1+\frac{\delta_l}{4h_l\overline{e}}\left[(1-\alpha)\overline{e}-(1+\alpha)\frac{h_l}{\delta_l}\right]^2}$;[D1] (b) $\frac{d(w_lh_l)}{ds_l}>0$

[D1] $\alpha(\frac{h_l}{\delta_l}+\overline{e})>\Lambda(\overline{e})$ holds in the former case.

for $s_l \in \left[s_{l,l}^{\triangle}(e), s_{l,h}^{\triangle}(e)\right]$ when $e^+(0) > \frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$ and $e \in [\Lambda(\overline{e}), \alpha\left(\frac{h_l}{\delta_l}+e^+(0)\right))$; (c) $\dfrac{d(w_l h_l)}{ds_l} > 0$ for $s_l \in \left[\max\left(0, s_{l,l}^{\triangle}(e)\right), s_{l,h}^{\triangle}(e)\right]$ when $\overline{e} > \frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}, e^+(0) \leq \frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$, and $e \in [\Lambda(\overline{e}), \alpha\left(\frac{h_l}{\delta_l}+\overline{e}\right))$ and when $e^+(0) > \frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$, $e \in [\alpha\left(\frac{h_l}{\delta_l}+e^+(0)\right), \alpha\left(\frac{h_l}{\delta_l}+\overline{e}\right))$, and $e \geq \Lambda(\overline{e})$.[D2]

From (a), the statement on $\dfrac{d(w_l h_l)}{ds_l} > 0$ of (ii)(a) of the proposition is obtained, and from (a)−(c), the statement of (ii)(b) of the proposition is obtained.

[Proof for $\frac{d(w_l h_l)}{ds_l} < 0$] If the expression inside the big parenthesis of (D8) is negative, $(1-s_l-\alpha)s_l e - \alpha\frac{h_l}{\delta_l} < 0$ must hold and thus $\left[(1-s_l-\alpha s_l)e - \alpha\frac{h_l}{\delta_l}\right]\underline{h}_l + \left[(1-s_l-\alpha)s_l e - \alpha\frac{h_l}{\delta_l}\right]\delta_l \frac{\int_0^{e^+} ef(e)de}{F(e^+)} > \left[(1-s_l-\alpha s_l)e - \alpha\frac{h_l}{\delta_l}\right]\underline{h}_l + \left[(1-s_l-\alpha)s_l e - \alpha\frac{h_l}{\delta_l}\right]\delta_l e^+$. Hence, $\frac{d(w_l h_l)}{ds_l} < 0$ if $\left[(1-s_l-\alpha s_l)e - \alpha\frac{h_l}{\delta_l}\right]\underline{h}_l F(e^+)$ $+\left[(1-s_l-\alpha)s_l e - \alpha\frac{h_l}{\delta_l}\right]\delta_l \int_0^{e^+} ef(e)de \leq 0 \Leftrightarrow M(s_l) \equiv -e\frac{\int_0^{e^+(s_l)} ef(e)de}{F(e^+(s_l))}s_l^2 + \left[(1-\alpha)\frac{\int_0^{e^+(s_l)} ef(e)de}{F(e^+(s_l))} - (1+\alpha)\frac{h_l}{\delta_l}\right]es_l + \left[-\alpha\left(\frac{h_l}{\delta_l} + \frac{\int_0^{e^+(s_l)} ef(e)de}{F(e^+(s_l))}\right) + e\right]\frac{h_l}{\delta_l} \leq 0$, $\Leftrightarrow s_l \leq \max\{0, s_{l,l}^{\triangledown}(e)\}$ and $s_l \geq s_{l,h}^{\triangledown}(e)$, where $s_{l,h}^{\triangledown}(e)$ and $s_{l,l}^{\triangledown}(e)$ are the solutions to (D9) with $e^+(s_l)$ replaced by $E(e|e<e^+(s_l)) \equiv \frac{\int_0^{e^+(s_l)} ef(e)de}{F(e^+(s_l))}$.

Hence, following the same steps as the proof of $\frac{d(w_l h_l)}{ds_l} < 0$, it is shown that $s_{l,h}^{\triangledown}(e) \in (0,1)$ satisfying (D9) (with $e^+(s_l)$ replaced by $E(e|e<e^+(s_l))$) exists and is unique when $e > \alpha\left[\frac{h_l}{\delta_l} + E(e|e<e^+(0))\right]$ and $e \geq \Omega(\overline{e}) \equiv \dfrac{\alpha\left(\frac{h_l}{\delta_l} + E(e|e<\overline{e})\right)}{1 + \frac{\delta_l}{4\underline{h}_l E(e|e<\overline{e})}\left[(1-\alpha)E(e|e<\overline{e}) - (1+\alpha)\frac{h_l}{\delta_l}\right]^2}$ and when $E(e|e < e^+(0)) > \frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$ and $e \geq \Omega(\overline{e})$; $s_{l,l}^{\triangledown}(e) \in (0, s_{l,h}^{\triangledown}(e))$ satisfying (D9) exists when $E(e|e < e^+(0)) > \frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$, $e < \alpha\left[\frac{h_l}{\delta_l} + E(e|e<e^+(0))\right]$, and $e \geq \Omega(\overline{e})$; $s_{l,l}^{\triangledown}(e) > 0$ does not exist when $e \geq \alpha\left[\frac{h_l}{\delta_l} + E(e|e<\overline{e})\right]$ and when $E(e|e<\overline{e}) \leq \frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$.

Summarizing these results, (a) $\dfrac{d(w_l h_l)}{ds_l} < 0$ for $s_l \geq s_{l,h}^{\triangledown}(e)$ when $e \geq \alpha\left[\frac{h_l}{\delta_l} + E(e|e<\overline{e})\right]$ and when $E(e|e<\overline{e}) \leq \frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$, $e \in \left(\alpha\left[\frac{h_l}{\delta_l} + E(e|e<e^+(0))\right], \alpha\left[\frac{h_l}{\delta_l} + E(e|e<\overline{e})\right]\right)$, and $e \geq \Omega(\overline{e})$; (b) $\dfrac{d(w_l h_l)}{ds_l} < 0$ for $s_l \leq s_{l,l}^{\triangledown}(e)$ and $s_l \geq s_{l,h}^{\triangledown}(e)$ when $E(e|e<e^+(0)) > \frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$ and $e \in \left[\Omega(\overline{e}), \alpha\left[\frac{h_l}{\delta_l} + E(e|e<e^+(0))\right]\right)$; (c) $\dfrac{d(w_l h_l)}{ds_l} < 0$ for $s_l \leq \max\{0, s_{l,l}^{\triangledown}(e)\}$ and $s_l \geq s_{l,h}^{\triangledown}(e)$ when $E(e|e<\overline{e}) > \frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$, $E(e|e<e^+(0)) \leq \frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$, and $e \in \left[\Omega(\overline{e}), \alpha\left[\frac{h_l}{\delta_l} + E(e|e<\overline{e})\right]\right)$ or when $E(e|e<e^+(0)) > \frac{1+\alpha}{1-\alpha}\frac{h_l}{\delta_l}$, $e \in \left[\alpha\left[\frac{h_l}{\delta_l} + E(e|e<e^+(0))\right], \alpha\left[\frac{h_l}{\delta_l} + E(e|e<\overline{e})\right]\right)$ and $e \geq \Omega(\overline{e})$.

From (a), the statement on $\dfrac{d(w_l h_l)}{ds_l} < 0$ of (ii)(a) of the proposition is obtained, and from (a)−(c), the statement of (ii)(b) of the proposition is obtained.

Further, from the equation for $M(s_l)$ and (D9), it can be seen that $M(s_l) \leq 0$ and thus

[D2] $\Lambda(\overline{e}) > \alpha(\frac{h_l}{\delta_l} + e^+(0))$ holds in the former case.

$\frac{d(w_l h_l)}{ds_l} < 0$ for any s_l when $e \leq \Omega(e^+(0)) \equiv \dfrac{\alpha\left(\frac{\underline{h}_l}{\delta_l}+E(e|e<e^+(0))\right)}{1+\frac{\delta_l}{4\underline{h}_l E(e|e<e^+(0))}\left[(1-\alpha)E(e|e<e^+(0))-(1+\alpha)\frac{\underline{h}_l}{\delta_l}\right]^2}$ (real solutions to $M(s_l)=0$ do not exist) when $E(e|e<e^+(0)) \leq \frac{1+\alpha}{1-\alpha}\frac{\underline{h}_l}{\delta_l}$ and $e \leq \alpha\left[\frac{\underline{h}_l}{\delta_l}+E(e|e<e^+(0))\right]$ (the RHS of (D9) at $s_l=0$ is negative and thus $s_{l,h}^{\bigtriangledown}(e)<0$).

[Proof for $s_{l,h}^{\bigtriangledown}(e) > s_{l,h}^{\triangle}(e) > s_l^{\star}(e)$ and $s_{l,l}^{\bigtriangledown}(e) < s_{l,l}^{\triangle}(e) < s_l^{\circ}(e)$] $s_{l,h}^{\bigtriangledown}(e) > s_{l,h}^{\triangle}(e) > s_l^{\star}(e)$ ($s_l^{\star}(e)$ is s_l maximizing $w_l(e^+,s_l)h_l(e,s_l)$ when the return to educational investment for local jobs is positive and $e^+=\overline{e}$ and is given by (52) in the proof of Proposition 3) is true, because, for given e^+, the RHS of the equation determining $s_{l,h}^{\triangle}(e)$, (D9), is smaller (greater) than the RHS of the equation determining $s_{l,h}^{\bigtriangledown}(e)$ ($s_l^{\star}(e)$) (this can been seen from the fact that, as shown above, the RHS decreases with e^+ when the solution is $s_{l,h}^{\triangle}(e)$), and the RHSs decrease with e^+ and thus s_l. Similarly, $s_{l,l}^{\bigtriangledown}(e) < s_{l,l}^{\triangle}(e) < s_l^{\circ}(e)$, where $s_l^{\circ}(e)$ is s_l minimizing $w_l(e^+,s_l)h_l(e,s_l)$ when the return is positive and $e^+=\overline{e}$, holds, because the RHS of the equation determining $s_{l,l}^{\triangle}(e)$, (D9), is greater (smaller) than the RHS of the equation determining $s_{l,h}^{\bigtriangledown}(e)$ ($s_l^{\circ}(e)$) (this can been seen from the fact that, as shown above, the RHS increases with e^+ when the solution is $s_{l,l}^{\triangle}(e)$), and the RHSs increase with e^+ and thus s_l. ■

Proof of Claim 1. From (27) of the proof of Proposition 1, the dividing line between $e^+<\overline{e}$ and $e^+=\overline{e}$ when the return to educational investment for local jobs is positive is

$$F(\overline{e}) = 1-\alpha\frac{\underline{h}_l + s_l\delta_l\int_0^{\overline{e}} ef(e)de}{\underline{h}_l + s_l\delta_l(1-\alpha)\overline{e}}. \tag{D18}$$

For given s_l, $F(\overline{e})$ satisfying the equation is highest when $\int_0^{\overline{e}} ef(e)de=0$ and thus the equation becomes $F(\overline{e})=1-\alpha\frac{\underline{h}_l}{\underline{h}_l+s_l\delta_l(1-\alpha)\overline{e}}$, and $F(\overline{e})$ satisfying the equation is lowest when $\int_0^{\overline{e}} ef(e)de = \overline{e}F(\overline{e})$ and thus the equation becomes $F(\overline{e})=1-\alpha$.

From (29) of the proof of Proposition 1, the dividing line when the return is negative is

$$\frac{F(\overline{e})-(1-\alpha)}{[F(\overline{e})]^{\alpha}[1-F(\overline{e})]^{1-\alpha}}A[\delta_n(1-s_l)]^{\alpha}\left(\frac{\underline{h}_l}{\overline{e}}\right)^{1-\alpha}=1, \tag{D19}$$

where the LHS increases with $F(\overline{e})$ from the proof of the proposition.

By substituting (D18) into (D19),

$$\frac{\alpha s_l\delta_l\left[(1-\alpha)\overline{e}-\int_0^{\overline{e}} ef(e)de\right]}{\left\{(1-\alpha)\underline{h}_l+s_l\delta_l\left[(1-\alpha)\overline{e}-\alpha\int_0^{\overline{e}} ef(e)de\right]\right\}^{\alpha}\left[\alpha\left(\underline{h}_l+s_l\delta_l\int_0^{\overline{e}} ef(e)de\right)\right]^{1-\alpha}}A[\delta_n(1-s_l)]^{\alpha}\left(\frac{\underline{h}_l}{\overline{e}}\right)^{1-\alpha}=1$$

$$\Leftrightarrow A[\alpha\delta_n(1-s_l)]^{\alpha}s_l\delta_l\frac{(\overline{e})^{\alpha}\left[(1-\alpha)-\frac{\int_0^{\overline{e}} ef(e)de}{\overline{e}}\right]}{\left\{(1-\alpha)\underline{h}_l+s_l\delta_l\left[(1-\alpha)\overline{e}-\alpha\int_0^{\overline{e}} ef(e)de\right]\right\}^{\alpha}\left(1+\frac{s_l\delta_l\int_0^{\overline{e}} ef(e)de}{\underline{h}_l}\right)^{1-\alpha}}=1. \tag{D20}$$

The two dividing lines intersect if there exist s_l and $F(\overline{e})$ satisfying the above equation. They do intersect when $F(\overline{e})$ satisfying (D18) for given s_l is highest, i.e. $F(\overline{e})=1-\alpha\frac{\underline{h}_l}{\underline{h}_l+s_l\delta_l(1-\alpha)\overline{e}}$, because (D20) when $\int_0^{\overline{e}} ef(e)de=0$ is same as the equation for the return being zero when $e^+=\overline{e}$

((23) of the proof of Proposition 1), which implies that the dividing lines intersect on the loci for zero return. By contrast, they do not intersect when $F(\overline{e})$ satisfying (D18) for given s_l is lowest, i.e. $F(\overline{e}) = 1-\alpha$, because the LHS of (D20) when $\int_0^{\overline{e}} ef(e)de = \overline{e}F(\overline{e}) = (1-\alpha)\overline{e}$ is zero.

From (23), the return when $e^+ = \overline{e}$ equals

$$(1-\alpha)A\left[\frac{\alpha\delta_n(1-s_l)\overline{e}(s_l)^{\frac{1}{\alpha}}}{(1-\alpha)(\underline{h}_l+\delta_l s_l\overline{e})}\right]^\alpha \delta_l - 1 = A\left[\alpha\delta_n(1-s_l)\right]^\alpha s_l\delta_l(1-\alpha)\left[\frac{\overline{e}}{(1-\alpha)(\underline{h}_l+\delta_l s_l\overline{e})}\right]^\alpha - 1. \tag{D21}$$

The LHS of (D20) is smaller than the first term of (D21) when $\int_0^{\overline{e}} ef(e)de > 0$, because

$$\frac{\left[(1-\alpha)-\frac{\int_0^{\overline{e}} ef(e)de}{\overline{e}}\right]}{\left\{(1-\alpha)\underline{h}_l+s_l\delta_l\left[(1-\alpha)\overline{e}-\alpha\int_0^{\overline{e}} ef(e)de\right]\right\}^\alpha\left(1+\frac{s_l\delta_l\int_0^{\overline{e}} ef(e)de}{\underline{h}_l}\right)^{1-\alpha}} < (1-\alpha)\left[\frac{1}{(1-\alpha)(\underline{h}_l+\delta_l s_l\overline{e})}\right]^\alpha$$

$$\Leftrightarrow \frac{1-\frac{\int_0^{\overline{e}} ef(e)de}{(1-\alpha)\overline{e}}}{1+\frac{s_l\delta_l\int_0^{\overline{e}} ef(e)de}{\underline{h}_l}} < \left[\frac{1-\frac{s_l\delta_l\alpha\int_0^{\overline{e}} ef(e)de}{(1-\alpha)(\underline{h}_l+\delta_l s_l\overline{e})}}{1+\frac{s_l\delta_l\int_0^{\overline{e}} ef(e)de}{\underline{h}_l}}\right]^\alpha = \left[\frac{1-\frac{\alpha\int_0^{\overline{e}} ef(e)de}{(1-\alpha)(\frac{\underline{h}_l}{s_l\delta_l}+\overline{e})}}{1+\frac{s_l\delta_l\int_0^{\overline{e}} ef(e)de}{\underline{h}_l}}\right]^\alpha. \tag{D22}$$

Hence, when the dividing lines intersect, the return is positive at the intersection. Further, the dividing line when the return is positive (D18) intersects with $s_l = 0$ at $F(\overline{e}) = 1-\alpha$, while the line when the return is negative (D19) intersects with $s_l = 0$ at $F(\overline{e}) \in (1-\alpha, 1)$; the former intersects with $s_l = 1$ at $F(\overline{e}) \in [1-\alpha, 1)$, while the latter approaches $F(\overline{e}) = 1$ as $s_l \to 1$. Therefore, irrespective of whether the dividing lines intersect or not, the line between $e^+ < \overline{e}$ and $e^+ = \overline{e}$ when the positive return is located below the line when the negative return on the loci for zero return on the $(s_l, F(\overline{e}))$ plane. ■