Social Identity, Redistribution, and Development

Kazuhiro Yuki*

March 2024

Abstract

Empirical studies suggest that income redistribution promotes economic growth and development by reducing inequality and increasing educational investment among the poor. However, the scale of redistribution, to be precise, the inequality-reducing effect of taxes and transfers, is limited in many developing countries. Why is the scale of redistribution small, and how does it influence development? This paper focuses on the role of social identity, whose importance in redistribution and development is supported in existing empirical research. Under what conditions is national identity realized, and how does it influence the economic outcomes?

To answer the questions, this paper develops a dynamic model of income redistribution and educational investment augmented with social identification and explores the interaction among identity, redistribution, and development theoretically.

Keywords: social identity, redistribution, nation-building policies, economic development

JEL classification numbers: D72, O11, O20, I38

---

*Faculty of Economics, Kyoto University, Yoshida-hommachi, Sakyo-ku, Kyoto, 606-8501, Japan; Phone +81-75-753-3532; e-mail yuki@econ.kyoto-u.ac.jp. Valuable comments from participants at the 2022 and 2023 Asian Meeting of Econometric Society in East and South-East Asia are gratefully acknowledged. All remaining errors are the author's own.

1 Introduction

Cross-country differences in economic growth and development are substantial. Empirical evidence shows that income and asset inequalities are negatively related to these performances (Deininger and Squire, 1998; Easterly, 2007). This suggests that redistributive policies may stimulate growth and development by reducing inequality. Indeed, the empirical study by Berg et al. (2018) indicates that income redistribution, unless very large-scale, increases economic growth by lowering income inequality. They also find that lower inequality is associated with higher levels of human capital. Further, Hanushek and Woessmann (2012a) find that an increase in educational achievement, measured by cognitive skills, has a large effect on growth. These findings suggest that redistribution may promote growth by reducing inequality and increasing educational investment of the poor.

However, Berg et al. (2018, Figure 5) find that the scale of redistribution, to be precise, the inequality-reducing effect of taxes and transfers, is much smaller in many developing countries compared to developed countries. Further, while the redistributive effect of the fiscal system is greater in countries with higher market income inequality among developed nations, this tendency is not observed for developing nations. Goni, Lopez, and Serven (2011) find that while market inequality is not very different between Latin American and Western European countries, after-tax after-transfer inequality is much higher in the former group of countries. This holds true even when public expenditures on education and health too are considered.

Why is the scale of redistribution small in many developing countries, and how does it impact economic development? This paper examines the role of social identity in addressing these questions. The lack of a shared national identity, that is, the dominance of subnational identities over national identity, is often blamed for the poor economic performance of socially diverse countries (Collier, 2009; Michalopoulos and Papaioannou, 2015; Fukuyama, 2018). And empirical findings suggest that national identity has a positive effect on redistribution (Chen and Li, 2009; Transue, 2007; Singh, 2015). Under what conditions is national identity realized, and how does it influence redistribution, educational investment, and development?

Technological change, a major driving force of growth and development alongside the accumulation of human and physical capital, is increasingly skill-biased. The technology available to developing countries today is much more skill-biased than that used by developed countries during their periods of modernization and industrialization. What are the implications of skill-biased technological change (SBTC) for identity, redistribution, and development?

This paper theoretically explores the interaction between identity, redistribution, and development. To this end, it develops a dynamic model of income redistribution and educational investment augmented with social identification, primarily drawing on the model by Shayo (2009).

Shayo (2009) develops a model that augments the standard political economy model of redistributive taxation (Meltzer and Richard, 1981) with socio-psychological factors. In the model, there are two classes, the poor and the rich. The government imposes a proportional tax on their incomes to provide a lump-sum transfer, with the tax rate determined through voting. What distinguishes this model from the standard one is that individual utility depends not only on disposable income (consumption) but also negatively on the *perceived distance* between oneself and the group one identifies with (their class or the nation) and positively on the *group's status*. Specifically, one incurs a cognitive cost when they differ from other members of the group in income level and non-economic attributes, while they derive high utility from being a member when the group's status, determined by an exogenous factor and average income level, is high. These socio-psychological components are major determinants of social identification and intergroup behaviors, according to influential theories in social psychology (Tajfel and Turner, 1986; Turner et al., 1987) and empirical evidence (Manning and Roy, 2010; Hett, Mechtel, and Kröll, 2020; Fouka, Mazumder, and Tabellini, 2022).[1] Since these components differ depending on which group one identifies with, the social identities of people influence the tax rate and thus their disposable incomes. Meanwhile, social identity is *endogenously determined*: one chooses the identity that yields higher utility. Hence, identity and individual and aggregate outcomes interact with each other.

The present model differs from Shayo (2009) mainly in two respects.[2] First, pre-tax pre-transfer incomes are endogenously determined. The rich (poor) are skilled (unskilled) workers, and their earnings depend on the proportion of skilled workers. Second, the model is dynamic; variables such as the proportion of skilled workers, earnings, social identity, and tax rate change endogenously over time. The dynamic part of the model is based on Galor and Zeira (1993) and Yuki (2007, 2008), in which individuals with heterogeneous wealth received from their parents decide on educational spending that must be self-financed and is necessary to become skilled workers. In these models, the proportion of those with sufficient wealth for education in the initial period is a critical determinant of the dynamics. If this proportion is high enough, the share of skilled workers and total output increase, the earnings gap between skilled and unskilled workers narrows over time, and in the long run, the welfare level of everyone becomes equal; if this proportion is low, the economy is stuck in a "poverty trap," where the share of skilled workers and total output remain low, and earnings and welfare disparities persist in the long run.

Based on such a model, the paper examines the dynamics and long-run outcomes of the skilled workers' share, identity, redistribution, and development. Main results are summarized as follows.

First, given the skilled workers' share, the rate of redistributive taxation is higher as the proportion of individuals identifying with the nation is higher. This result is opposite to Shayo (2009),[3] but is consistent with empirical findings such as Chen and Li (2009), Transue (2007), Qari, Konrad, and Geys (2012), and Singh (2015).

Second, the dynamics and long-run outcomes are influenced by the exogenous component of *national status* (in comparison to that of class status), as well as inter-class differences in non-economic attributes, which may be interpreted as culture, norms, or values, and their prominence in the perceived distance.[4] In the actual society, the exogenous component of national status would be high when people believe that they share a glorious history, rich culture, or a "right" sense of values, because they feel proud of belonging to such a nation.

In particular, when the exogenous component of national status is greater, or when inter-class cultural differences are smaller or less concern, society is less likely to fall into a "poverty trap", and under a favorable initial condition of wealth distribution that avoids this trap, the share of skilled workers and output increase faster, leading to earlier attainment of welfare equality. This is because these exogenous factors influence redistribution through social identity.[5]

[1] For example, Hett, Mechtel, and Kröll (2020) find that participants in a lab experiment prefer to identify with groups to which they have a smaller social distance and which have a higher social status, and their social identity preferences are related to their choices in dictator games.

[2] Other important differences from Shayo (2009) are the following. First, the perceived distance depends on the difference in disposable income between oneself and the group one identifies with, rather than the difference in pre-tax pre-transfer income. Second, the tax rate is determined based on a probabilistic voting model (Lindbeck and Weibull, 1987), as opposed to majority voting. These settings are common with Ghiglino, Júarez-Lunam, and Müller (2021), but unlike their model, social identities are endogenously determined in the present model.

[3] The different results are due to the different model settings mentioned in footnote 2: unlike Shayo (2009), the perceived distance depends on the difference in disposable income rather than the difference in pre-tax pre-transfer income, and the tax rate is determined based on a probabilistic voting model, not by majority voting. He provides empirical results consistent with his theoretical result, which are discussed in Section 3.1.

[4] National (class) status also depends on the average disposable income of the nation (class) and thus is endogenous.

[5] That is, the proportion of people identifying with the nation and the rate of redistributive tax are higher, when

What is notable is the case in which the exogenous factors are neither very high nor very low, and the initial condition is favorable. In this case, when the share of skilled workers and thus the level of development are low, skilled workers identify with their class, unskilled workers identify with the nation, and the dynamics do not depend on the exogenous factors. When the skilled share becomes sufficiently high, society generally experiences a change in social identity, significantly impacting subsequent dynamics. If national status is relatively high due to exogenous reasons or inter-class cultural differences are small or of little concern, society transitions to *universal national identity*; as a result, the redistributive tax rate increases, and the upward mobility of the poor through education accelerates. Otherwise, society shifts to *universal class identity*, the tax rate falls, and upward mobility *slows down or stops*. That is, under the former (latter) situation, the increased share of skilled workers ultimately has a positive (negative) effect on national identity, redistribution, and the pace of development.

Third, skill-biased technical change (SBTC) negatively impacts upward mobility and long-run outcomes by widening inter-class wage disparity, changing social identity, and influencing redistribution. As SBTC advances, society is more likely to fall into a "poverty trap"; even when starting from a favorable initial condition that avoids this trap, the proportion of skilled workers increases at a slower rate, delaying the attainment of welfare equality. Moreover, when SBTC continues, society generally shifts to an equilibrium in which fewer people identify with the nation. This shift lowers the rate of redistributive tax and exacerbates the adverse effects of SBTC.

The findings indicate that large cross-country disparities in the level and pace of development may stem from differences in the exogenous element of national status, in inter-class distances in culture, norms, and values, or in people's concerns about these distances. In many developing countries, the belief that people share a glorious history, rich culture, or a "right" sense of values is weak, and inter-class differences in culture, norms, and values are large or perceived to be serious. According to the model, such circumstances lower national status or widen the perceived distance to the other class, hindering the formation of a common national identity. As a result, the extent of redistribution is limited, the upward mobility of the poor through education is constrained, and developmental progress is slow. Various empirical studies (Blouin and Mukand, 2019; Cáeres-Delpiano et al., 2021; Chen, Lin, and Yang, 2023) reveal that *nation-building policies*, such as school education and government propaganda emphasizing shared history, culture, and values, as well as policies facilitating inter-group contact, may effectively strengthen national identity. According to the model, such policies elevate national status or decrease (or deemphasize) inter-class differences in culture, norms, and values, potentially playing a critical role in divided societies.

The result on SBTC shows that as technology becomes more skill-biased, inequality rises, making it more difficult to establish national identity and execute large-scale redistribution. As a result, upward mobility and developmental progress slow down. This may be another reason why the pace of development, especially among the poor, is slower in many developing countries compared to the period when developed countries industrialized and modernized their economies. The result can also explain the lack of increased demand for and scale of redistribution in advanced economies over the last several decades (Ashok et al., 2016; Piketty, Saez, and Stantcheva, 2014).

Finally, classic theories of modernization in political science (Deutsch, 1953; Gellner, 1983; Weber, 1979), based on Europe's historical experiences, argue that modernization (including industrialization and universal education) leads to widespread national identity at the expense of subnational identities (Robinson, 2014). However, the results on social identity, particularly the shift in identity, in the present model suggest that these theories are only applicable when national

---

the exogenous component of national status (compared to that of class status) is greater or inter-class cultural differences are smaller or less prominent in people's minds.

status is relatively high or when inter-class distances in culture, norms, and values are relatively small or of little concern. The findings also demonstrate that the identity shift associated with modernization has a positive effect on the level of redistribution and the pace of development if these conditions are met, but a negative effect otherwise.

This paper contributes to the theoretical literature on the relationship between social identity and redistribution (Shayo 2009; Lindqvist and Östling, 2013; Holm, 2016; Dhami, Manifold, and al-Nowaihi, 2021; and Ghiglino, Júarez-Lunam, and Müller, 2021). Besides Shayo (2009), several settings of the model follow Ghiglino, Júarez-Lunam, and Müller (2021) (footnote 2), who consider a society with two ethnicities and three income groups, but unlike their model, identities are endogenous. Neither work explores the relationship among identity, redistribution, and development.

More broadly, the paper adds to the theoretical literature on the relationship between identity and economic behaviors (Akerlof and Kranton, 2000; Shayo, 2009; Benabou and Tirole, 2011; Bisin et al., 2011; Sambanis and Shayo, 2013; Bernard, Hett, and Mechtel, 2016; Carvalho and Dippel, 2020; Grossman and Helpman, 2020; Bonomi, Gennaioli, and Tabellini, 2021; Yuki, 2021).[6] Generalizing the pioneering work of Akerlof and Kranton (2000), the aforementioned Shayo (2009) constructs the basic analytical framework. Shayo's (2009) framework has been applied to various issues, with Yuki (2021) being particularly relevant to the present work. Drawing on the seminal work of Sambanis and Shayo (2013) on the interaction between social identity and ethnic conflict, Yuki (2021) examines how the shift of economic activities from ethnically-segregated traditional sectors to the integrated modern sector, driven by the increased productivity of the latter, affects identity, ethnic conflict, and development. Unlike the present model, his model does not consider educational investment and redistribution, as well as intergenerational linkages. Another notable application is Grossman and Helpman (2020), who, inspired by a recent reversal of trade policies in some Western countries seemingly influenced by the rise of populism and ethnic tensions, develops a political economy model of trade policy with social identification and examine how policies are affected by changes in identification patterns triggered by events such as increased ethnic tensions.

The rest of the paper is organized as follows. Section 2 presents and Section 3 examines the static model. Section 4 introduces and analyzes the dynamic model, while Section 5 presents and explains the main results. Section 6 concludes. Appendix A contains propositions used in Section 3, Appendix B provides supplementary analysis for Section 4, and Appendix C contains proofs.

2 Static model

The society comprises skilled workers, unskilled workers, and a government. The government imposes a proportional tax on earnings to finance a lump-sum transfer, where the tax-transfer policy is determined based on a probabilistic voting model. Utility depends not only on one's disposable income but also on socio-psychological components that depend on her social identity. Identity is determined endogenously: one chooses to identify with their economic class or the nation. Markets are competitive.

2.1 Economic environment

The total population is 1, and the number of skilled workers is H, which is constant in this section but endogenized in Section 4. Workers supply 1 unit of labor to receive earnings, pay a proportional tax on earnings, receive a lump-sum transfer, and spend disposable income on consumption. The disposable income of individual i is denoted by

[6] In addition to those already mentioned, recent empirical and experimental studies on identity include Dehdari and Gehring (2021) and Assouad (2021).

$$y_i = (1 - \tau) w_i + T, \tag{1}$$

where w_i is her earnings, $\tau \in [0, 1)$ is the tax rate, and T is the transfer.

The government uses tax revenue entirely for the lump-sum tranfer, but taxation involves deadweight loss. The deadweight loss is assumed to be quadratic, thus the governmental budget constraint equals

$$T = \left(\tau - \frac{1}{2}\tau^2\right) \overline{w}, \tag{2}$$

where $\overline{w}$ is average earnings of the population.

The final good is produced by using skilled labor and unskilled labor as inputs. The production function takes the following CES form:

$$Y(=\overline{w}) = \left\{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}, \alpha \in (0,1),\ \sigma \in (1,3], \tag{3}$$

where A_s (A_u) is the level of skilled (unskilled) labor augmenting techonology, and σ is the elasticity of substitution between skilled and unskilled workers. $\sigma \in (1, 3]$ is assumed following Autor, Goldin, and Katz (2020) who estimate $\sigma = 1.62$ using U.S. data and state that estimates in the literature typically fall in the 1 to 2.5 range.

From first-order conditions of the profit maximization problem of a representative firm, the wage of skilled workers and that of unskilled workers respectively equal

$$w_s = \left\{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1} \alpha(A_s H)^{\frac{\sigma-1}{\sigma}} \frac{1}{H}, \tag{4}$$

$$w_u = \left\{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1} (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}} \frac{1}{1-H}. \tag{5}$$

2.2 Preferences

As in Shayo (2009), individual utility depends not only on one's disposable income (consumption) but also negatively on the *perceived distance* between oneself and the group one identifies with (their economic class—skilled/unskilled—or the nation) and positively on the *group's status*. In other words, one incurs a mental cost when they are different from others in the group in relevant features, while take pride in belonging to the group when its status is high. These socio-psychological components, based on influential theories of social psychology (Tajfel and Turner, 1986; Turner et al., 1987), are the major factors influencing social identification and intergroup behaviors (Manning and Roy, 2010; Hett, Mechtel, and Kröll, 2020; Fouka, Mazumder, and Tabellini, 2022).[7]

An individual perceives the distance or proximity to a group (her class or the nation) based on the difference between her disposable income and the average disposable income of the group. The *perceived distance* of an individual in class C (where C represents skilled [S] or unskilled [U])

[7]For the United Kingdom, Manning and Roy (2010) find that non-whites, whose perceived distance to the "average national" would be greater than that of whites, are less likely to think of themselves as British. They also find that immigrants from poorer and less democratic (i.e., lower status) countries assimilate faster into a British identity. Hett, Mechtel, and Kröll (2020) find that participants in a lab experiment prefer groups to which they have a smaller social distance and which have a higher social status, and their social identity preferences are related to their choices in dictator games. Fouka, Mazumder, and Tabellini (2022) find that migrations of African Americans from the South to non-southern metropolitan areas stimulated assimilation of European immigrants for the years 1910−30. Further, they provide evidence suggesting that the higher integration resulted from decreased perceived distance of native whites to European immigrants.

to group G (where G represents class C or the nation $[N]$) is represented by[8]

$$\begin{aligned} d_{CG} &= |y_C - y_G| \\ &= (1-\tau)\,|w_C - w_G| \quad (\text{from}(1)), \end{aligned} \tag{6}$$

where y_G and w_G are respectively the average disposable income and earnings of group G, and y_C and w_C are those of the class. For analytical tractability, the distance is measured in absolute value, following Ghiglino, Júarez-Lunam, and Müller (2021). In Shayo's (2009) model, perceived distance also depends on the difference in non-economic attributes that may be interpreted as culture, norms, values etc. For ease of presentation, this dependence is not modeled here, but is considered in Section 5.1.1.

The *status* of group G ($G = C, N$), S_G, depends on the exogenous component $\widetilde{S_G}$ and the average disposable income of the group:

$$S_G = \delta \widetilde{S_G} + y_G, \tag{7}$$

where δ is the weight on the exogenous component.[9] The term y_G may be interpreted as altruism toward the group. To simplify analysis significantly, the exogenous component of *class status* is assumed to be the same for both classes, i.e., $\widetilde{S_S} = \widetilde{S_U} \equiv \widetilde{S_C}$; this would not affect the main results qualitatively.[10]

The level of the exogenous component of *national status* $\widetilde{S_N}$ would be high when people of the nation believe that they share a glorious history, rich culture, or a "right" sense of values, or when the nation records commendable performance in international sports competitions, because people would feel proud of belonging to such a nation.

Finally, the utility of an individual in class C who identifies with group G is given by

$$u_{CG} = y_C - \beta d_{CG} + \gamma S_G, \qquad \beta, \gamma > 0. \tag{8}$$

Thus, she cares not only about her own disposable income but also about the difference in disposable income between herself and the average member of the group, the group's average disposable income, and the exogenous component of the group's status.

By substituting (1), (2), (6), and (7) into the above equation, the utility for each combination of class and identity can be expressed as

$$\begin{aligned} u_{SN} &= (1-\tau)w_s + \left(\tau - \frac{1}{2}\tau^2\right)\overline{w} - \beta(1-\tau)(w_s - \overline{w}) + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w} + \left(\tau - \frac{1}{2}\tau^2\right)\overline{w}\right] \\ &= (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_s - \beta(1-\tau)(w_s - \overline{w}) + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right], \end{aligned} \tag{9}$$

[8] The concept of perceived distance is developed in cognitive psychology to study how a person categorizes information that comes in to her (stimuli) (Nosofsky, 1986). Turner et al. (1987) apply the concept to the categorization of people, including oneself, into social groups, in constructing an influential social psychological theory, self-categorization theory. The theory attempts to explain psychological basis of social identification.

[9] Status is an absolute measure, following works such as Grossman and Helpman (2021), while in Shayo's (2009) model, status is a relative measure defined as the difference from the reference group. The main results remain unchanged under the alternative specification.

[10] In Ghiglino, Júarez-Lunam, and Müller (2021), $\widetilde{S_N} = \widetilde{S_C} = 0$ is assumed. In Shayo (2009), $\widetilde{S_C}$ can be different between the poor and the rich, but the rich's status does not affect the results because, unlike the present paper, only the poor can influence the policy.

$$u_{SS} = (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_s+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_s\right], \quad (10)$$

$$u_{UN} = (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_u-\beta(1-\tau)(\overline{w}-w_u)+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right], \quad (11)$$

$$u_{UU} = (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_u+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_u\right]. \quad (12)$$

In the model, one's social identity, that is, the group with which one identifies, is *not fixed.* Between the nation and their class, one "chooses" the group that brings higher utility because of a shorter perceived distance or higher status.[11] One's identity may change when the levels of the variables influencing utility, either directly or indirectly through the choices of others, change.

2.3 Political environment and Timing of decisions

Similar to Grossman and Helpman (2021) and Ghiglino, Júarez-Lunam, and Müller (2021), the political environment is based on the probabilistic voting model (Lindbeck and Weibull, 1987). Two parties, parties 1 and 2, that differ in the non-policy dimension ("ideology") compete for public office by announcing their electoral platforms on the tax rate, τ_1 and τ_2. They propose platforms to maximize the probability of wining the majority election. Individuals care about both the policy platforms and the "ideologies" of the competing parties and vote sincerely.

Individual i in class C who identifies with group G prefers party 1 if

$$u_{CG}(\tau_1) \geq u_{CG}(\tau_2) + \eta_i + \mu, \quad (13)$$

where $u_{CG}(\tau_j)$ $(j = 1, 2)$ is the "non-ideology" component of the utility given by (8) when party j implements the tax rate τ_j. η_i is an individual-specific parameter that measures the voter's "ideological" bias toward party 2 and has a uniform distribution on $\left[-\frac{1}{2\phi}, \frac{1}{2\phi}\right]$, where $\phi > 0$. A negative value of η_i implies that the individual has an "ideological" bias in favor of party 1. μ measures the average popularity of party 2 relative to party 1 in the population and has a uniform distribution on $\left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right]$, where $\psi > 0$.

The timing of decisions is as follows: first, workers choose their social identities; then, the parties announce their policy platforms independently; finally, workers vote for the party that offers higher utility, and the winning party implements the proposed policy.

3 Analysis of the static model

3.1 Tax rate

Because the model can be solved by backward induction, the determination of the tax rate τ is examined first.

From (13), a class C $(C = S, U)$ individual identifying with group G $(G = C, N)$ is indifferent between the two parties when η_i equals $\eta_{CG} \equiv u_{CG}(\tau_1) - u_{CG}(\tau_2) - \mu$. Thus, all individuals in this category with $\eta_i < \eta_{CG}$ prefer party 1. Hence, the share of votes party 1 obtains equals

[11] By assumption, one does not identify with the nation and their class simultaneously. Conversely, in the model of Grossman and Helpman (2021), an individual identifies with her class always and with the nation also if the additional identity increases the utility, where the utility depends on the sum of the perceived distance to and the status of each group with which she identifies. The present paper does not adopt this specification owing to the complexities associated with the additional terms and the difficulties when analyzing the model.

$$s_1 = H\phi\left[p\left(\eta_{SN}+\frac{1}{2\phi}\right)+(1-p)\left(\eta_{SS}+\frac{1}{2\phi}\right)\right]+(1-H)\phi\left[q\left(\eta_{UN}+\frac{1}{2\phi}\right)+(1-q)\left(\eta_{UU}+\frac{1}{2\phi}\right)\right]$$
$$=\frac{1}{2}-\phi\mu+\phi\begin{pmatrix} H\{p[u_{SN}(\tau_1)-u_{SN}(\tau_2)]+(1-p)[u_{SS}(\tau_1)-u_{SS}(\tau_2)]\} \\ +(1-H)\{q[u_{UN}(\tau_1)-u_{UN}(\tau_2)]+(1-q)[u_{UU}(\tau_1)-u_{UU}(\tau_2)]\}\end{pmatrix}, \quad (14)$$

where $p \in [0,1]$ ($q \in [0,1]$) is the proportion of skilled (unskilled) workers identifying with the nation. s_1 depends on μ and thus is a random variable.

From the above equation, the probability that party 1 wins the election is

$$\Pr\left[s_1 \geq \frac{1}{2}\right] = \Pr\left[\mu \leq \begin{pmatrix} H\{p[u_{SN}(\tau_1)-u_{SN}(\tau_2)]+(1-p)[u_{SS}(\tau_1)-u_{SS}(\tau_2)]\} \\ +(1-H)\{q[u_{UN}(\tau_1)-u_{UN}(\tau_2)]+(1-q)[u_{UU}(\tau_1)-u_{UU}(\tau_2)]\}\end{pmatrix}\right]$$
$$=\frac{1}{2}+\psi\begin{pmatrix} H\{p[u_{SN}(\tau_1)-u_{SN}(\tau_2)]+(1-p)[u_{SS}(\tau_1)-u_{SS}(\tau_2)]\} \\ +(1-H)\{q[u_{UN}(\tau_1)-u_{UN}(\tau_2)]+(1-q)[u_{UU}(\tau_1)-u_{UU}(\tau_2)]\}\end{pmatrix}. \quad (15)$$

Because the probability that party 2 wins the election equals $1-\Pr\left[s_1 \geq \frac{1}{2}\right]$ and τ_1 and τ_2 enter symmetrically in the above equation, the unique equilibrium is such that the two parties propose the same tax rate, τ, that maximizes the utilitarian social welfare function:

$$H\{pu_{SN}(\tau)+(1-p)u_{SS}(\tau)\} + (1-H)\{qu_{UN}(\tau)+(1-q)u_{UU}(\tau)\}$$
$$= (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)\begin{pmatrix} \overline{w}+\gamma\{H[p\overline{w}+(1-p)w_s]+(1-H)[q\overline{w}+(1-q)w_u]\} \\ -\beta[Hp(w_s-\overline{w})+(1-H)q(\overline{w}-w_u)]\end{pmatrix}+\text{constants}, \quad (16)$$

where (9)−(12) are used to derive the last equation.

From the above equation, the proposed tax rate equals

$$\tau = 1-\frac{1}{(1+\gamma)\overline{w}}\begin{pmatrix} \overline{w}+\gamma\{H[p\overline{w}+(1-p)w_s]+(1-H)[q\overline{w}+(1-q)w_u]\} \\ -\beta\{Hp(w_s-\overline{w})+(1-H)q(\overline{w}-w_u)\}\end{pmatrix}, \quad (17)$$

if the right-hand side of the equation is positive, otherwise, $\tau = 0$. Note that when $\beta = \gamma = 0$, i.e., socio-psychological factors do not affect utility, $\tau = 0$ holds due to the linear utility, the utilitarian social welfare, and the cost of taxation.

The next lemma shows that (p, q) with p or $q \in (0,1)$ cannot be a stable equilibrium, thus only $(p,q) = (0,0),(1,1),(0,1),(1,0)$ can be stable equilibria.

Lemma 1 *Only* $(p,q) = (0,0),(1,1),(0,1),(1,0)$ *can be stable equilibria.*

Proof. See Appendix C. ■

Based on this lemma, the following proposition summarizes how the tax rate depends on identity choices of the two groups and H.

Proposition 1 *(i) (a)* $\tau = 0$ *when* $p = q = 0$*, i.e, everyone identifies with their class.*

(b) $\tau = \frac{2\beta}{1+\gamma}(a(H)-H)$*, where* $a(H) \equiv \frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}} \geq H$ *from* $w_s \geq w_u$*, when* $p = q = 1$*, i.e., everyone identifies with the nation.*

(c) When $p = 0$ *and* $q = 1$*, i.e., the skilled identify with their class and the unskilled identify with the nation,* $\tau = \frac{\beta-\gamma}{1+\gamma}(a(H)-H)$ *if* $\beta > \gamma$ *and* $\tau = 0$ *if* $\beta \leq \gamma$*.*

(d) $\tau = \frac{\beta+\gamma}{1+\gamma}(a(H)-H)$ *when* $p = 1$ *and* $q = 0$*, i.e., the skilled identify with the nation and the unskilled identify with their class.*

(ii) Given H, τ is highest when $p = q = 1$, and if $\beta > (\leq)\gamma$, it is lowest when $p = q = 0$ (when $p = q = 0$ and $p = 0$, $q = 1$). ($p = 1, q = 0$ is not an equilibrium when $\beta \leq \gamma$.)

(iii) Given p and q, there exists an $H^+ \in (0, \overline{H})$, where $\overline{H}$ satisfies $\overline{H} = a(\overline{H}) \Leftrightarrow w_s = w_u$, such that $\frac{d\tau}{dH} > (<)0$ for $H < (>)H^+$.

Proof. See Appendix C. ■

The tax rate is 0 when $p=q=0$, i.e, everyone identifies with their class, and if $\beta \leq \gamma$, when $p=0$ and $q=1$ as well, i.e., the skilled identify with their class and the unskilled identify with the nation.[12] In other cases, τ equals a constant times $a(H) - H$, where $a(H) \equiv \frac{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}$. Given H, the tax rate is highest when $p = q = 1$, i.e, everyone identifies with the nation, and if $\beta > (\leq)\gamma$, it is lowest when $p=q=0$ (when $p=q=0$ and $p=0$, $q=1$).

Hence, given the proportion of skilled workers, the rate of redistributive tax is higher when the proportion of individuals identifying with the nation is higher. This result aligns with Ghiglino, Júarez-Lunam, and Müller (2021) and contrasts with Shayo (2009).[13,14] National identity has a positive effect on the tax rate for the following reasons. First, individuals identifying with the nation are concerned with the perceived distance to the "average national", which decreases with increased redistribution. Second, their concern with national status has a negative effect on the tax rate because tax lowers the average disposable income (due to the taxation cost) and thus national status, but this effect is dominated by the effect through the perceived distance.[15] The result is generally consistent with empirical findings regarding the relationship between national identity and redistribution (Chen and Li, 2009; Transue, 2007; Qari, Konrad and Geys, 2012; Singh, 2015),[16] but it contrasts with the empirical finding of Shayo (2009).[17]

[12] Ghiglino, Júarez-Lunam, and Müller (2021) allow β and γ to differ between the classes. In this case, $\tau > 0$ holds even when $p=q=0$ if γ (the importance of status in the utility) for the unskilled is greater than that for the skilled. In the present paper, β and γ are assumed to be common to both classes to make the subsequent analysis tractable.

[13] The same result holds for the model of Ghiglino, Júarez-Lunam, and Müller (2021), in which β and γ can differ between the classes and social identities are exogenous, except that τ when $p=q=0$ is higher than when $p=0$, $q=1$ iff $\beta \leq \gamma$ for the unskilled and their γ is strictly greater than γ for the skilled.

[14] Shayo (2009) shows that the tax rate preferred by the poor is *lower* under national identity than under class identity. Since the poor determine the tax policy, this is true for the implemented tax rate as well. In contrast, in the present model, τ preferred by the unskilled is higher (lower) under national identity iff $\beta > (<)\gamma$, i.e., perceived distance is more (less) important than status in one's utility, and the implemented τ is *always* (weakly) *higher* when the proportion of those identifying with the nation is higher. The reason τ preferred by the unskilled can be higher under national identity is that perceived distance depends on the difference in disposable income. If perceived distance depends on the difference in *pre-tax pre-transfer* income as in the Shayo model, their preferred τ is lower under national identity. (By contrast, status does depend on *disposable* income in his model too.) The result on the implemented τ holds because not only the unskilled but also the skilled, whose preferred τ is *always* higher under national identity, influence the tax policy.

[15] To be precise, when $\beta < \gamma$ and $p = 0$, i.e., status is more important than perceived distance in one's utility and the skilled identify with their class, this effect dominates the effect through the perceived distance for the unskilled and thus their preferred tax rate is lower when $q = 1$ than when $q = 0$. However, since the implemented τ equals 0 when $p = q = 0$, $\tau = 0$ when $p = 0$, $q = 1$ as well.

[16] Chen and Li (2009) conduct lab experiments to examine the effects of induced group identity on social preferences and find that subjects are more averse to payoff differences to groups they identify with. Transue (2007), based on a survey experiment on American whites, shows that those who feel close to the nation are more supportive of a tax increase to improve educational opportunities for minorities, compared to those who feel close to their racial group. Further, he finds that making American identity salient increases their support for the tax increase. Qari, Konrad and Geys (2012), using data from OECD countries, find that the income tax burden of an above-middle income group is positively related to their strength of national pride (which fosters national identity according to the model), after controlling for the relative income position. Singh (2015), based on statistical and comparative historical analysis of Indian states, shows that states with a stronger sense of shared identity spend more on education and health.

[17] Using survey data from democratic countries, Shayo (2009) finds that the extent of redistribution to the poor is

Unlike Shayo (2009) and Ghiglino, Júarez-Lunam, and Müller (2021), pre-tax pre-transfer incomes are endogenous and depend on H, thus the tax rate changes with H. The relationship between the share of skilled workers and the tax rate is non-monotonic; namely, τ increases with H for relatively small H and decreases with H for relatively large H. This can be explained as follows. The tax rate preferred by an individual with a national identity rises as the perceived distance in pre-tax pre-transfer income to the "average national" increases, because greater redistribution is needed to counteract the larger distance.[18] An increase in H lowers the preferred τ for skilled workers with a national identity because it decreases the difference between their wage and the average national wage, $w_s - \overline{w} = (1-H)(w_s - w_u)$, thus reducing the distance to the "average national". Conversely, it raises (lowers) τ preferred by the unskilled for relatively small (large) H because it increases (decreases) $\overline{w} - w_u = H(w_s - w_u)$ and the distance. When H is small (large), the influence of unskilled (skilled) workers on the tax policy is stronger due to their population share. Hence, the implemented tax rate increases (decreases) with H for small (large) H.[19]

To ensure that a higher tax rate always increases the disposable income of unskilled workers across the possible range of τ, the following assumption is imposed.

Assumption 1 $\frac{2\beta}{1+\gamma} \leq 1$.

The following corollary shows how the disposable incomes of skilled and unskilled workers and the inter-class inequality in disposable income depend on their social identities.

Corollary 1 (*i*) *Given H, the disposable income of skilled workers and the inter-class inequality in disposable income are lowest when $p = q = 1$, and are highest when $p = q = 0$ (when $p = q = 0$ and $p = 0$, $q = 1$), if $\beta > (\leq)\gamma$.*

(*ii*) *Under Assumption 1, given H, the disposable income of unskilled workers is highest when $p = q = 1$, and is lowest when $p = q = 0$ (when $p = q = 0$ and $p = 0$, $q = 1$), if $\beta > (\leq)\gamma$.*

Proof. See Appendix C. ■

Given H, the disposable income of unskilled workers is highest, while that of skilled workers and the inter-class disparity in disposable income are lowest when $p = q = 1$. On the other hand, if $\beta > (\leq)\gamma$, the unskilled income is lowest, while the skilled income and the inter-class inequality are highest when $p = q = 0$ (when $p = q = 0$ and $p = 0$, $q = 1$). That is, as the proportion of individuals identifying with the nation increases, the disposable income of the unskilled (skilled) is higher (lower), and thus the inter-class income inequality is lower.

smaller in countries with a *higher* median level of a national identity measure. His finding differs from other cited works. Several factors may contribute to this difference. First, the national identity measure is constructed from answers to six questions, such as "I would rather be a citizen of (R [respondent]'s country) than of any other country in the world" and "When my country does well in international sports, it makes me proud to be citizen of (R's country)". None of the questions ask respondents to compare their country to subnational groups (e.g., class), and all but one of them involve comparisons to *other countries.* Answers to the questions may differ when the comparison groups are subnational groups. Second, considering the modelling of identity choice, variables measuring the strength of national identity *relative to* class identity might be necessary to examine the theoretical result: it is possible that both the absolute strength of national identity and the relative strength of class identity are large simultaneously.

[18] When one identifies with their class, the perceived distance is 0 and thus does not affect the preferred tax rate.

[19] To be precise, when $p \neq q$, increased H affects τ through the status term as well, unless $p = 0, q = 1$ and $\beta \leq \gamma$, in which case $\tau = 0$. When $p = 0, q = 1$ and $\beta > \gamma$, the effect through the status term and the one through the perceived distance term operate in opposite directions, but the latter dominates, thus the relation between H and τ is as described in the main text. When $p = 1, q = 0$, the effect through the two terms operate in the same direction.

3.2 Social identity

The choice of social identity influences the perceived distance term and the status term of one's utility. Thus, individuals choose the identity with the higher combined value of these two terms. Hence, from (9)−(12) and $H \leq \overline{H}$, i.e., $w_s \geq w_u$, the following conditions are obtained ($\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$). Note that $H \geq \frac{\beta+\gamma}{2\beta}$ holds only when $\beta > \gamma$.

$$p = q = 1 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \geq (1-\tau)(w_s - w_u)\max[(\beta+\gamma)(1-H), (\beta-\gamma)H]$$

$$\Leftrightarrow \gamma\delta\Delta\widetilde{S_N} \geq (1-\tau)(w_s - w_u)(\beta+\gamma)(1-H) \text{ for } H \leq \min\left\{\tfrac{\beta+\gamma}{2\beta}, \overline{H}\right\} \tag{18}$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N} \geq (1-\tau)(w_s - w_u)(\beta-\gamma)H \text{ for } H \in \left[\tfrac{\beta+\gamma}{2\beta}, \overline{H}\right] \text{ when } \overline{H} > \tfrac{\beta+\gamma}{2\beta}, \tag{19}$$

where τ is the tax rate when $p = q = 1$, i.e., $\tau = \frac{2\beta}{1+\gamma}(a(H) - H)$ from Proposition 1 (i)(b).

$$p = q = 0 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \leq (w_s - w_u)\min[(\beta+\gamma)(1-H), (\beta-\gamma)H]$$

$$\Leftrightarrow \gamma\delta\Delta\widetilde{S_N} \leq (w_s - w_u)(\beta-\gamma)H \text{ for } H \leq \min\left\{\tfrac{\beta+\gamma}{2\beta}, \overline{H}\right\} \tag{20}$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N} \leq (w_s - w_u)(\beta+\gamma)(1-H) \text{ for } H \in \left[\tfrac{\beta+\gamma}{2\beta}, \overline{H}\right] \text{ when } \overline{H} > \tfrac{\beta+\gamma}{2\beta}, \tag{21}$$

because $\tau = 0$ from from Proposition 1 (i)(a).

$$p=0, q=1 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \leq (\beta+\gamma)(1-\tau)(1-H)(w_s - w_u) \text{ and } \gamma\delta\Delta\widetilde{S_N} \geq (\beta-\gamma)(1-\tau)H(w_s - w_u), \tag{22}$$

where $\tau = \frac{\beta-\gamma}{1+\gamma}(a(H) - H)$ when $\beta > \gamma$ and $\tau = 0$ when $\beta \leq \gamma$ from Proposition 1 (i)(c). This occurs only for $H \leq \min\left\{\frac{\beta+\gamma}{2\beta}, \overline{H}\right\}$ because the RHS of the first condition must be greater than that of the second condition.

$$p=1, q=0 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \geq (\beta+\gamma)(1-\tau)(1-H)(w_s - w_u) \text{ and } \gamma\delta\Delta\widetilde{S_N} \leq (\beta-\gamma)(1-\tau)H(w_s - w_u), \tag{23}$$

where $\tau = \frac{\beta+\gamma}{1+\gamma}(a(H) - H)$ from Proposition 1 (i)(d). This happens only for $H \in \left[\frac{\beta+\gamma}{2\beta}, \overline{H}\right]$.

Propositions in Appendix A examine combinations of H and $\Delta\widetilde{S_N}$ such that each of these equations hold. To prove the propositions, the following assumption is imposed.

Assumption 2 $\tau = \frac{2\beta}{1+\gamma}(a(H) - H) < \frac{1}{2}$ *at* $H = H^+$, *where* H^+ *satisfies* $a'(H^+) - 1 = 0$.

This assumption states that the maximum possible tax rate is less than $\frac{1}{2}$. It is not restrictive, as a part of the tax revenue allocated to redistribution (rather than non-redistributive governmental expenditure) is much lower than half of aggregate labor income in the real economy. As with Assumption 1, $\frac{2\beta}{1+\gamma} \leq 1$, the assumption holds when $\frac{2\beta}{1+\gamma}$ is sufficiently small.

3.2.1 Result

The following analysis focuses on the more interesting case $\beta > \gamma$; the analysis when $\beta \leq \gamma$ is presented in Appendix A.[20] Based on Proposition A2 (i) in the appendix, Figure 1 shows combinations

[20] The other reason for focusing on the case $\beta > \gamma$ is that when $\beta \leq \gamma$, the figure illustrating combinations of H and $\Delta\widetilde{S_N}$ such that each equilibrium exists (Figure 5 in Appendix A) changes greatly and becomes closer to the corresponding figure for $\beta > \gamma$, when, as assumed in Section 5.1.1 and Shayo (2009), perceived distance also depends on differences in non-economic attributes that represent culture, norms of behavior, and values.

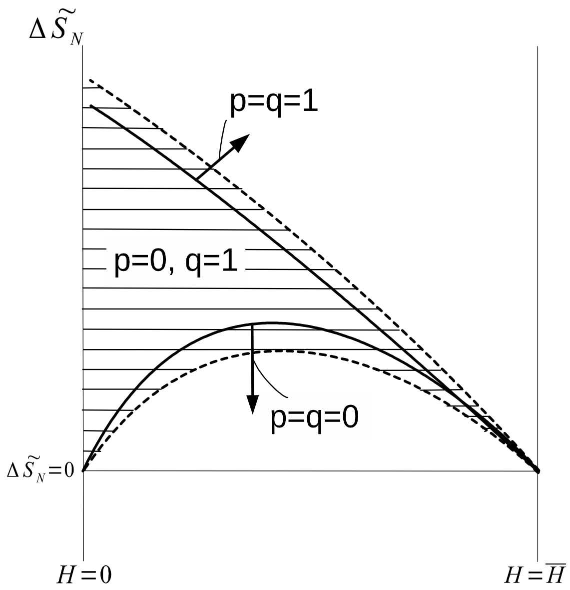

Figure 1: Equilibria when $\beta > \gamma$ and $\frac{A_s}{A_u}$ is relatively low

of H and $\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$ (the difference in the exogenous component of national status and that of class status) each equilibrium exists when $\frac{A_s}{A_u}$ is sufficiently low to satisfy $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$. ($\overline{H}$ represents the H satisfying $H = a(H) \Leftrightarrow w_s = w_u$.) $p = q = 1$, i.e., universal national identity, is realized when $(H, \Delta\widetilde{S_N})$ is in the region on or to the right of the solid downward-sloping curve. $p = q = 0$, i.e., universal class identity, is realized when $(H, \Delta\widetilde{S_N})$ is in the region on or below the solid convex curve. $p=0$, $q=1$, i.e., skilled workers identity with their class and unskilled workers identify with the nation, holds when $(H, \Delta\widetilde{S_N})$ is in the region with horizontal lines.[21]

Given H, everyone identifies with the nation (their class) when $\Delta\widetilde{S_N}$ is high (low), that is, when people are (are not) very proud of the nation relative to their class for non-economic reasons, such as culture and history. When $\Delta\widetilde{S_N}$ is in the intermediate range, skilled (unskilled) workers identity with their class (the nation). This is because skilled worker are more likely to identify with their class than unskilled workers due to the higher status of their class, which aligns with the empirical finding of Shayo (2009).[22]

Importantly, realized equilibria depend on H unless $\Delta\widetilde{S_N}$ is very high or low. When H is small, $p=0$, $q=1$ (the skilled [unskilled] identify with their class [the nation]) is the only equilibrium for a wide range of $\Delta\widetilde{S_N}$. However, as H increases, $p=q=1$, i.e., *universal national identity* ($p=q=0$, i.e., *universal class identity*) becomes more dominant when $\Delta\widetilde{S_N}$ is relatively high (low).

The relationship between realized equilibria and H can be explained mainly by the fact that $w_s - w_u$ decreases with H. When H is low, skilled workers identify with their class because they perceive a large distance between themselves and the "average national", and the status of their class is high relative to that of the nation. Both result from a large wage differential and their small population share. In contrast, unskilled workers identify with the nation when H is low because

[21]The lower dividing line for $p = 0$, $q = 1$ increases (decreases) with H for small (large) H, but when H is intermediate, the relation with H is not analytically clear.

[22]Shayo (2009), based on survey data from developed countries and East European countries, finds that the relationship between national identity and household income is significantly negative in almost all the countries.

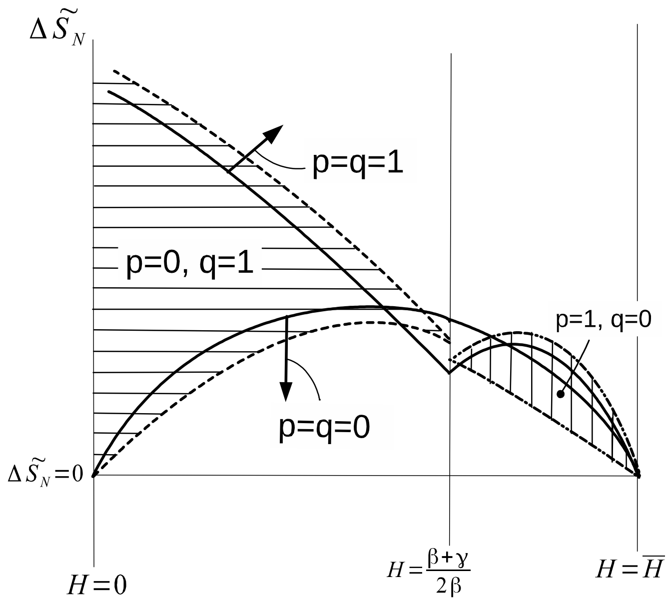

Figure 2: Equilibria when $\beta > \gamma$ and $\frac{A_s}{A_u}$ is relatively high

their large population share makes the perceived distance to the "average national" small, and the large wage disparity makes national status high relative to their class status. As H increases, the wage gap narrows, and the average income becomes closer to the skilled income. This leads to a decrease in the distance of skilled workers to the "average national" and a rise in national status relative to their class status. Thus, when $\Delta\widetilde{S_N}$ is relatively high and thus the utility gain from identifying with the nation rather than with their class is relatively large, skilled workers switch from class identity to national identity when H becomes large enough. As for unskilled workers, their declining population share widens the distance to the "average national", while the falling inequality narrows the distance. It turns out that the former effect dominates unless H is very large.[23] Hence, when $\Delta\widetilde{S_N}$ is relatively low (thus the utility gain from identifying with the nation is relatively small), they switch from national identity to class identity at a large enough H.[24]

The figure shows the presence of regions with multiple equilibria. This arises from two-way positive causations between national identity and tax rate: as the share of those identifying with the nation is higher, so is the tax rate (Proposition 1), while as the tax rate is higher, the inter-class disparity in disposable income is lower, leading to more people identifying with the nation.

Figure 2 depicts equilibria when $\frac{A_s}{A_u}$ is high enough that $\overline{H} > \frac{\beta+\gamma}{2\beta}$, based on Proposition A2 (ii) and (iii) in Appendix A. The figure may appear complicated, but it is similar to Figure 1 for $H \leq \frac{\beta+\gamma}{2\beta}$. For $H > \frac{\beta+\gamma}{2\beta}$, unlike the previous case, $p=0$, $q=1$ is not an equilibrium and $p=1$, $q=0$ is realized when $\Delta\widetilde{S_N}$ is relatively, but not extremely, low, i.e., in the region with vertical lines.

[23] An increase in H also raises (lowers) national status relative to their class status when H is relatively small (large). However, the effect on the perceived distance dominates due to $\beta > \gamma$.

[24] When H is very large, they revert to national identity, although, as will be clear in Section 5, this scenario is not very relevant.

3.3 Summary

Combining the results on the tax rate and social identity, the findings can be summarized as follows. First, given H, the rate of redistributive tax is high (low) when the exogenous status difference $\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$ is relatively high (low), because the proportion of those identifying with the nation is high (low). Second, unless $\Delta\widetilde{S_N}$ is very high or low, identity and the tax rate change with the share of skilled workers and thus the level of development. When H is low, skilled workers identify with their class, unskilled workers identify with the nation, and the tax rate is low. When H is high enough, for relatively high $\Delta\widetilde{S_N}$, everyone identifies with the nation and the tax rate is high, while for relatively low $\Delta\widetilde{S_N}$, everyone identifies with their class and the tax rate is at least as low as when H is low. These results highlight the importance of the levels of $\Delta\widetilde{S_N}$ (which would be high when people believe that they share a glorious history, rich culture, or a "right" sense of values) and H for national identity and redistributive taxation.[25]

The critical question is how H evolves over time. The next section presents a dynamic version of the model in which H is determined endogenously.

4 Dynamics

The rest of the paper examines how social identity, redistribution, earnings, and inter-class inequalities change over time when H changes endogenously and productivities A_s and A_u grow exogenously. This section presents a dynamic version of the model and examines the dynamics of H. The dynamic part of the model is based on Galor and Zeira (1993) and Yuki (2007, 2008), in which individuals who are heterogeneous in wealth received from their parents decide on educational spending needed to become skilled workers.

4.1 Model

For simplicity, A_s and A_u are fixed until the last part of Section 5.1. Consider a deterministic, discrete-time, and small-open OLG economy in which individuals live for two periods, first as a child, then as an adult. Each adult has one child, and thus, the population is constant over time. The population of each generation is normalized to be 1. The lifetime of an individual is as follows.

In childhood, the individual receives a transfer b from her parent and spends it on assets a and educational expenditure e, which is required to become a skilled worker, to maximize the utility given by (24) below. She takes into account the effect of her investment decision not only on future income but also on the socio-psychological components of the utility. The educational investment is discrete, i.e. she takes education or not, costs $\bar{e}$, and yields a gross economic return of $w_s - w_u$. The investment must be self-financed due to the absence of credit markets. Thus, when $b < \bar{e}$, she does not invest in education, i.e., $e = 0$, and becomes an unskilled worker. Investment in assets is a continuous choice, and yields a gross rate of economic return of $1 + r$.

In adulthood, the individual earns income from assets and work, and allocate it to consumption c and a transfer to her child b'. As before, she also chooses a group to identify with and votes for a party that brings her higher utility. When she belongs to class C ($C = S, U$) and identifies with group G ($G = C, N$), she maximizes the following ("non-ideology" component of) utility subject to the budget constraint:

[25] Section 5.1.1 examines a modified model in which perceived distance also depends on differences in non-economic attributes that would represent culture, norms, values, etc., as in Shayo (2009). The analysis there shows that inter-class distances in these attributes and their prominence in perceived distance too are important for the results.

$$\max\ v_{CG} = \frac{1}{(\lambda)^{\lambda}(1-\lambda)^{1-\lambda}}(b')^{\lambda}(c)^{1-\lambda} - \beta d_{CG} + \gamma S_G, \quad \lambda \in (0,1), \tag{24}$$

$$s.t. \quad c + b' = (1-\tau)w_C + T + (1+r)a, \tag{25}$$

where w_C is the wage for class C. By solving the maximization problem, the following consumption and transfer rules are obtained.

$$c = (1-\lambda)[(1-\tau)w_C + T + (1+r)a], \tag{26}$$

$$b' = \lambda[(1-\tau)w_C + T + (1+r)a]. \tag{27}$$

Results on identity choice and the tax rate are the same as before since the indirect utlity function equals the utility function of the original model plus $(1+r)a$.[26]

From the above setting, H is equal to the proportion of individuals who receive $b \geq \overline{e}$ *and* spend $e = \overline{e}$ in childhood. Let F be the proportion of those who receive $b \geq \overline{e}$. Then, if the utility gain from educational investment is non-negative *even when* everyone with $b \geq \overline{e}$ takes education, $H = F$ holds; this is the case when F is not large, as shown in Appendix B. If the utility gain is negative with $H = F$, which is true when F is sufficiently large, H is smaller than F and is determined so that one is indifferent between making the educational investment and not. Denote such H by $H^* \in (0, \overline{H})$, whose determination is explained in Appendix B.

4.2 Dynamics of F and H

For given the distribution of b over the population in the initial period (thus, the initial value of F), the dynamics of H when $F < H^*$, which are equivalent to the dynamics of F, are determined by the dynamics of b of each lineage. (When $F \geq H^*$, $H = H^*$ is time-invariant as long as exogenous variables are constant.)

Suppose $F_t < H^*$ (henceforth, subscript t represents variables for those who become adults in period t), and consider an individual who is born in period $t-1$ and spends her adulthood in period t. Her investment decisions depend on the received transfer:

$$\text{If} \quad b_t < \overline{e}, \quad a_t = b_t,\ e_t = 0, \tag{28}$$

$$\text{If} \quad b_t \geq \overline{e}, \quad a_t = b_t - \overline{e},\ e_t = \overline{e}. \tag{29}$$

By substituting (28) into (27), the dynamic equation linking the received transfer b_t to the transfer to her child b_{t+1} when she received $b_t < \overline{e}$ and thus is an unskilled worker equals

$$b_{t+1} = \lambda\left[(1-\tau_t)w_{ut} + T_t + (1+r)b_t\right]. \tag{30}$$

Similarly, the corresponding equation when she received $b_t \geq \overline{e}$ and thus is a skilled worker is

$$b_{t+1} = \lambda\left[(1-\tau_t)w_{st} + T_t + (1+r)(b_t - \overline{e})\right]. \tag{31}$$

$F_{t+1} \geq F_t \Leftrightarrow H_{t+1} \geq H_t$ holds when all children of skilled workers can afford education, i.e., for any lineage satisfying $b_t \geq \overline{e}$, $b_{t+1} \geq \overline{e}$. From (31), this is the case if

$$\lambda\left[(1-\tau_t)w_{st} + T_t\right] \geq \overline{e}. \tag{32}$$

[26]Asset income is assumed to be non-taxed to keep these results unchanged and to be consistent with the fact that it is less heavily taxed than labor income.

When this condition holds, $H_{t+1} = F_{t+1} > H_t = F_t$ is true when there exist lineages satisfying $b_t < \overline{e}$ and $b_{t+1} \geq \overline{e}$. From (30), such lineages exist only if $\lambda\{(1-\tau_t)w_{ut} + T_t + (1+r)b_t\} \geq \overline{e}$ for some $b_t < \overline{e}$, which is the case when ($\lambda(1+r) < 1$ is assumed)

$$\frac{\lambda}{1-\lambda(1+r)}\left[(1-\tau_t)w_{ut} + T_t\right] > \overline{e}. \tag{33}$$

By contrast, $F_{t+1} = F_t \Leftrightarrow H_{t+1} = H_t$ holds if $b_{t+1} = \lambda\{(1-\tau_t)w_{ut} + T_t + (1+r)b_t\} < \overline{e}$ for any $b_t < \overline{e}$, which is the case when

$$\frac{\lambda}{1-\lambda(1+r)}\left[(1-\tau_t)w_{ut} + T_t\right] \leq \overline{e}. \tag{34}$$

That is, when $F < H^*$ and the disposable income (net of asset income) of skilled workers is sufficiently high, if the net disposable income of unskilled workers is high enough, $H(=F)$ increases over time; otherwise, it is constant. Because the disposable incomes depend on $H = F$, whether H increases or not is determined by the level of F. The next lemma shows that, for *given* values of p and q and under some conditions, if the initial level of F is sufficiently high, H increases over time and reaches H^* eventually; otherwise, it stays low.

Lemma 2 *Suppose that Assumption 1 holds and $\overline{e}$ (λ) is sufficiently but not extremely small (large), with fixed values of p and q. Then,*

(i) There exists $\underline{F} \in (0, H^*)$ such that when $F_0 \in (\underline{F}, H^*)$, H increases over time and converges to H^*, and when $F_0 \leq \underline{F}$, $H_t \leq \underline{F}$ for any t.[27] *When $F_0 \in (\underline{F}, H^*)$, the utility of everyone converges to the same level in the long run.*

(ii) When $\beta > (\leq) \gamma$, $\underline{F}$ is lowest when $p = q = 1$ and is highest when $p = q = 0$ (when $p = q = 0$ and $p = 0$, $q = 1$).[28]

Proof. See Appendix C. ■

When the initial proportion of those who have enough wealth for education is sufficiently high, i.e., $F_0 \in (\underline{F}, H^*)$, the share of skilled workers and thus the level of the unskilled wage are relatively high initially. Thus, unskilled workers with relatively large wealth can make large enough transfers for their children to take education and become skilled workers, as long as the transfer motive λ is sufficiently strong or the cost of education $\overline{e}$ is sufficiently low. This leads to an increase in $H = F$, which in turn raises the unskilled wage and further promotes upward mobility of the children of unskilled workers. In this way, H increases over time and eventually reaches H^* at which one is indifferent between taking education and not.[29,30] Further, since within-class disparities in transfers diminish over time from (30) and (31), the welfare level of everyone becomes equal in the long run.

[27] When $F_0 \leq \underline{F}$, if $p = q = 0$ or if $\beta \leq \gamma$ and $p = 0$, $q = 1$, where $\tau = 0$, $H_t = F_0$ for any t. In other cases, where $\tau > 0$, one cannot rule out the possibility that $H \leq \underline{F}$ *temporarily* increases or decreases. (See the proof of the lemma on this matter.) Further, the possibility that multiple values of H^* exist cannot be ruled out in these cases. If multiple values of H^* exist, H converges to the highest H^* when $F_0 \in (\underline{F}, H^*)$. If $\overline{e}$ (λ) is extremely small (large), from any $F_0 < H^*$, H increases over time and converges to H^*.

[28] Magnitude relations among H^* for different values of p and q are clear when $\frac{A_s}{A_u}$ is small enough that $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$ (as in Figure 3 below), or when $\overline{H} > \frac{\beta+\gamma}{2\beta}$ and H^* for $p = q = 1$ is smaller than $\frac{\beta+\gamma}{2\beta}$. In these cases, H^* for $p = q = 1$ is smallest, H^* for $p = q = 0$ is largest, and H^* for $p = 0$, $q = 1$, which decreases with $\Delta\widetilde{S_N}$, falls in between. When $\overline{H} > \frac{\beta+\gamma}{2\beta}$ and H^* for $p = q = 1$ is greater than $\frac{\beta+\gamma}{2\beta}$ (as in Figure 4 below), the relations are generally ambiguous.

[29] After H reaches H^*, F continues to increase and converges to 1.

[30] When $p = q = 0$ holds at $H = H^*$, the net *economic* return to education is negative; in other words, earnings net of educational expenditure of skilled workers is *lower* than earnings of unskilled workers, i.e., $w_s - (1+r)\overline{e} < w_u$. This happens because education confers an non-economic benefit, i.e., higher status, as well. For other values of p and q, whether the long-run net economic return is positive or not is not clear.

In contrast, when $F_0 < \underline{F}$, generally, the unskilled wage is too low for the children of unskilled workers to receive sufficient wealth for education. Thus, H does not increase continuously and $H < \underline{F} < H^*$ always holds.[31] This implies that skilled workers enjoy higher utility than unskilled workers even in the long run.

Note that the threshold level of F, $\underline{F}$, differs depending on the values of p and q. It is lowest when $p = q = 1$, and if $\beta > (\leq)\gamma$, it is highest when $p = q = 0$ (when $p = q = 0$ and $p = 0$, $q = 1$). In other words, under universal national (class) identity, the minimum level of F_0 required for upward mobility and long-run welfare equalization is lowest (highest). This is because the tax rate is highest (zero) and thus income transfers to the unskilled are largest (not conducted).

5 Main results

Based on Lemma 2 and the results of Section 3 (many of which are illustrated in Figures 1 and 2), this section examines how the endogenous evolution of the skilled workers' share and exogenous skill-biased technical change influence identity, redistribution, and development. Henceforth, the following assumption on identity dynamics is imposed.

Assumption 3 *When society is in equilibrium with specific values of p and q, it stays in the same equilibrium in subsequent periods, as long as the equilibrium continues to exist.*

This means that the values of p and q do not change, as long as they are equilibrium values.

5.1 Effect of H on identity, redistribution, and development

For ease of analysis, first, the effect of H is examined for given levels of productitivies A_s and A_u. As in Section 3.2, the analysis focuses on the case $\beta > \gamma$; however, the following results are mostly unchanged when $\beta \leq \gamma$.[32]

The next proposition analyzes how the dynamics and long-run levels of H, earnings, and earnings inequality, as well as the long-run interclass disparity in welfare depend on F_0 and $\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$. Unlike Lemma 2, p and q are endogernized by taking into account the results in Section 3.

Proposition 2 *Suppose that $\beta > \gamma$, Assumptions 1–3, and the existence conditions for $\underline{F}$ in Lemma 2 hold, and society starts with $F_0 < H^*$.*[33]

(i) When $F_0 \leq \underline{F}$, $H_t \leq \underline{F}$ for any t. Because H is low, Y is low, w_s is high, w_u is low, and inter-class disparities in earnings and welfare are large.

(ii) When $F_0 > \underline{F}$, H increases over time and almost always converges to H^*.[34] *Accordingly, Y increases, w_s decreases, w_u increases, the earnings disparity falls over time, and in the long run, the welfare level of everyone becomes equal.*

(iii) As $\Delta\widetilde{S_N}$ is higher, $\underline{F}$ is lower, and thus $F_0 \geq \underline{F}$ is more likely to hold. In particular, when $\Delta\widetilde{S_N}$ is very high (low), $p = q = 1$ ($p = q = 0$), and thus $\underline{F}$ is lowest (highest).

[31] As mentioned in footnote 27, $H < \underline{F}$ possibly increases or decreases temporarily.

[32] When $\beta \leq \gamma$, two results in (ii) of Proposition 3 below do not hold: since $\tau = 0$ when $p = 0$, $q = 1$, τ and thus the speed of convergence to H^* do not change when the society shifts from $p = 0$, $q = 1$ to $p = q = 0$.

[33] H^* is H^* for values of p and q in the initial period. The same applies to $\underline{F}$ in the equations for F_0 and H_t below.

[34] As explained in footnote 38 attached to the next proposition, when the society starts with $p = 0$, $q = 1$, H could stop increasing after it shifts to $p = q = 0$.

Proof. See Appendix C. ■

When the initial proportion of those who can afford education is sufficiently low, i.e., $F_0 \leq \underline{F}$, the skilled workers' share stays low, i.e., $H_t \leq \underline{F}$ for any t. Hence, output (and thus per capita income) is low, the skilled (unskilled) wage is high (low), and inter-class disparities in earnings and welfare are large. Society is caught in a "poverty trap". In contrast, when the initial proportion of such individuals is sufficiently high, i.e., $F_0 > \underline{F}$, the skilled workers' share increases over time and almost always reaches H^*. As a result, output increases, the skilled (unskilled) wage decreases (increases), the earnings disparity falls over time, and eventually, the welfare level of everyone becomes equal. This result is standard in Galor-Zeira type models that incorporate indivisibilities in human capital investment and credit constraints, and is consistent with empirical findings such as Litschig and Lombardi (2019) and Aiyar and Ebeke (2020).[35]

The latter positive scenario is more likely to occur when $\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$, the difference between the exogenous component of national status and that of class status, is higher: when $\Delta\widetilde{S_N}$ is higher, $\underline{F}$ is lower, and thus $F_0 > \underline{F}$ holds with a smaller F_0, i.e., with a worse initial condition. In particular, when $\Delta\widetilde{S_N}$ is very high (low), everyone identifies with the nation (their class), and the tax rate is highest (zero); thus, $\underline{F}$ is lowest (highest), and the good outcomes are most (least) likely to be realized (see Figure 3 below).

The previous proposition shows that the dynamics and long-run outcomes depend on F_0 and $\Delta\widetilde{S_N}$ since these variables determine whether $F_0 > \underline{F}$ or not. Focusing on the case $F_0 > \underline{F}$, the next proposition shows that $\Delta\widetilde{S_N}$ also impacts the *evolution* of social identity and redistributive taxation, thereby influencing the dynamics and long-run outcomes.

Proposition 3 *Suppose the assumptions and conditions in Proposition 2 hold, and society starts with an* $F_0 \in (\underline{F}, H^*)$.[36]

(i) If $\Delta\widetilde{S_N}$ *is very high (low),* $p = q = 1$ $(p = q = 0)$ *and* τ *is high* $(\tau = 0)$ *all the time; thus, the disposable income of unskilled workers is high (low) for a given* H*, and* H *converges to* H^* *quickly (slowly).*

(ii) Otherwise, when $\Delta\widetilde{S_N}$ *is relatively high (low), society generally shifts from* $p = 0$, $q = 1$ *to* $p = q = 1$ $(p = q = 0)$ *eventually.*[37] *The shift increases* τ *(decreases* τ *to* 0*) and accelerates (slows or halts) the convergence to* H^*.[38]

(iii) When $\Delta\widetilde{S_N}$ *is not very high or low, multiple equilibria may exist for given* $\Delta\widetilde{S_N}$ *and* $H = F$*. The dynamics and long-run outcomes differ depending on which equilibrium is realized initially.*

[35] Litschig and Lombardi (2019) show, based on Brazilian sub-national data for the period 1970−2000, that sub-national units with a higher share of income going to the middle quintile at the expense of the bottom quintile in 1970 grew faster subsequently, while places with a higher initial share of income going to the top at the expense of the middle did not grow faster. Further, they find that the positive effect of a higher share of the middle quintile is observed only in places in which people in the middle quintile are poor. The model can yield a similar result if the initial distribution of wealth is such that many individuals in the middle quintile have b slightly below $\overline{e}$ and those in the top quintile have $b > \overline{e}$. Aiyar and Ebeke (2020) find, using cross-county data, that the negative effect of income inequality on growth is stronger when the degree of the inequality of opportunity, which is measured by father-son correlations of income and education, is higher. In the model, when $F < H^*$, the intergenerational correlations are high, and the effect of increased inequality through lowered τ on H is negative, whereas when $F \geq H^*$, the correlations are low, particularly for those with $b \geq \overline{e}$, and the effect of increased inequality on H is zero or small.

[36] $\underline{F}$ and H^* represent those for the values of p and q in the initial period.

[37] As illustrated in Figure 3 (Figure 4), when $\frac{A_s}{A_u}$ is small (large) enough that $\overline{H} \leq (>) \frac{\beta+\gamma}{2\beta}$ and $\Delta\widetilde{S_N}$ is intermediate for the range of $\Delta\widetilde{S_N}$ considered in (ii), the society could be in $p = 0$, $q = 1$ ($p = 1$, $q = 0$) in the long run.

[38] The increase of H could stop after the shift to $p = q = 0$ when $\Delta\widetilde{S_N}$ falls within the range in which $p_0 = 0$, $q_0 = 1$, and F_0 is greater than $\underline{F}$ for $p = 0, q = 1$ but smaller than $\underline{F}$ for $p = q = 0$. See Figures 3 and 4.

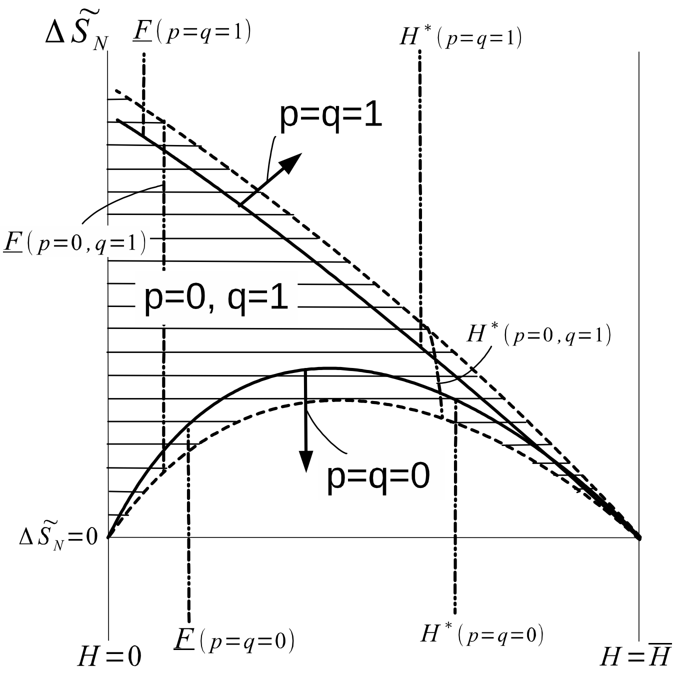

Figure 3: Dynamics when $\beta > \gamma$ and $\frac{A_s}{A_u}$ is relatively low

Proof. See Appendix C. ■

Figure 3, which is based on Figure 1 in Section 3.2, is useful for understanding the dynamics when $\frac{A_s}{A_u}$ is small enough that $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$. The proposition examines the situation in which F_0 is greater than the $\underline{F}$ for the p and q realized in the initial period. $H=F$ moves rightward in the figure as long as the values of p and q are unchanged and $H< H^*$. When the initial values of p and q are no longer equilibrium values, $H=F$ continues to move rightward if H is greater than the $\underline{F}$ and smaller than the H^* for the new values of p and q.

If $\Delta\widetilde{S_N}$ is very high (low), everyone always identifies with the nation (their class), i.e., $p=q=1$ ($p=q=0$), and as a result, redistributive taxation is implemented on a large scale (not implemented). Hence, the disposable income of unskilled workers is high (low) for a given H, and thus the convergence of H to H^* and the equalization of welfare occur quickly (only slowly).

Otherwise, when $\Delta\widetilde{S_N}$ is relatively high (low), society generally shifts from the equilibrium in which the skilled identify with their class and the unskilled identify with the nation, i.e., $p=0$, $q=1$, to the one in which *everyone identifies with the nation* (*their class*) eventually. Before the shift, the tax rate, and thus the dynamics, do not depend on $\Delta\widetilde{S_N}$. When $\Delta\widetilde{S_N}$ is relatively high, the identity shift increases the scale of redistribution and *accelerates* the convergence to H^*, whereas when $\Delta\widetilde{S_N}$ is relatively low, the shift leads to no taxation, *slowing down or halting* the convergence.[39] In the latter case, the increase of H can stop after the shift when $p_0=0$, $q_0=1$, and F_0 is greater than $\underline{F}$ for $p=q=0$ but smaller than $\underline{F}$ for $p=0, q=1$. That is, when $\Delta\widetilde{S_N}$ is relatively high (low), the increased share of skilled workers has a positive (negative) effect on

[39]The result of $\tau=0$ after the shift to $p=q=0$ arises from several assumptions aimed at making the model analytically tractable. Specifically, if ϕ for the unskilled is greater than that for the skilled, i.e., the distribution of σ_i (the parameter capturing a voter's "ideological" bias toward party 2) is narrower for the unskilled, the weight on the unskilled in the social welfare function is greater, resulting in $\tau>0$. However, qualitative results do not change.

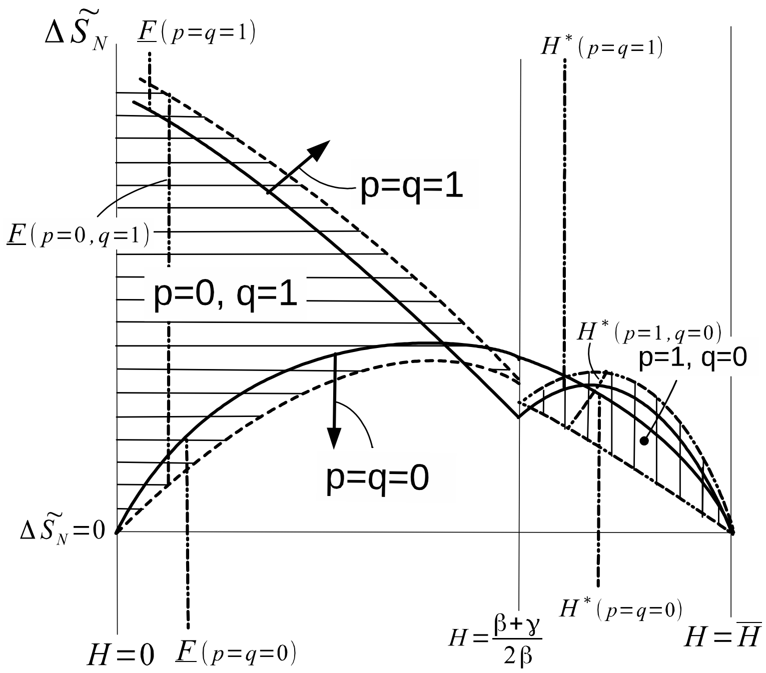

Figure 4: Dynamics when $\beta > \gamma$, $\frac{A_s}{A_u}$ is relatively high, and γ is small

national identity, redistribution, and the pace of development eventually.

Multiple equilibria may exist for given $\Delta\widetilde{S_N}$ and $H = F$ unless $\Delta\widetilde{S_N}$ is very high or low. The figure shows that when $\Delta\widetilde{S_N}$ is particularly high (low) within the range in which $p=0$, $q=1$ is an equilibrium, i.e., in the region near the upper (lower) dividing line for $p=0$, $q=1$, both $p=q=1$ ($p=q=0$) and $p=0$, $q=1$ are equilibria. The dynamics and long-run outcomes differ depending on which equilibrium *happens to* be realized initially. When society is within the upper (lower) region with multiple equilibria and $p_0=0$, $q_0=1$ happens to be the initial equilibrium, convergence to H^* is more (less) likely to occur, i.e., $\underline{F}$ is greater (smaller), and the speed of convergence is slower (faster) than when $p_0=q_0=1$ ($p_0=q_0=0$) is initially realized.

Figure 4, which is based on Figure 2, is a corresponding figure when $\frac{A_s}{A_u}$ is large enough that $\overline{H} > \frac{\beta+\gamma}{2\beta}$ *and* the degree of status concern γ is small enough that $H^* > \frac{\beta+\gamma}{2\beta}$.[40] (When γ is not small, the dynamics are very similar to the previous case.) Although the figure appears more complicated, the above results remain unchanged. The main difference is that $p=1$, $q=0$, rather than $p=0$, $q=1$, could hold in the long run when $\Delta\widetilde{S_N}$ is in the intermediate range.

Both figures demonstrate that society transitions from $p = 0$, $q = 1$ to $p = q = 1$ ($p = q = 0$) when $\Delta\widetilde{S_N}$ is relatively, but not extremely, large (small). This can be explained as follows. An increase in H weakens the existing social identities of *both classes*: the class (national) identity of skilled (unskilled) workers becomes weaker in the sense that the utility gain from identifying with their class (the nation) rather than with the nation (their class) decreases. The class identity of skilled workers weakens because their increasing share in the population and the decreasing inter-class wage disparity reduces the perceived distance of these workers to the "average national,"

[40] For the case illustrated in Figure 4, relations among H^* for different values of p and q are ambiguous. The exception is that, as shown in the figure, H^* for $p = 1$, $q = 0$ at the intersection with the upper (lower) dividing line for $p = 1$, $q = 0$ is greater than H^* for $p = q = 1$ (smaller than H^* for $p = q = 0$).

thereby elevating the status of the nation relative to that of their class. The national identity of unskilled workers weakens because their decreasing population share increases the distance to the "average national" (and dominates other effects operating in the opposite direction). If the exogenous component of national status is high or that of class status is low, i.e., $\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$ is large, the utility gain for skilled workers from identifying with their class is small for a given H; hence, they change identities, and universal national identity is established. Otherwise, the unskilled change identities, and everyone identifies with their class.

5.1.1 Dependence of perceived distance on non-economic attributes

So far, for the sake of clarity, the perceived distance term of the utility function depends solely on the difference in disposable income between oneself and the identity group. However, it would be more realistic to assume that, as in Shayo (2009), the perceived distance also depends on differences in non-economic attributes that would represent culture, norms of behavior, values, etc.. Suppose, for simplicity, that individuals belonging to a particular class share the same non-economic attributes. Then, whether an individual in class C ($C = S, U$; S [U] is for skilled [unskilled]) possesses the non-economic characteristics specific to each class or not can be expressed by the following indicators:

$$q_C^S = 1(=0) \text{ and } q_C^U = 0(=1) \text{ for } C = S(=U). \tag{35}$$

And, the perceived distance between an individual in class C ($C = S, U$) and group G ($G = C, N$; N is for the nation) is given by

$$d_{CG} = \omega_q\left(\left|q_C^S - q_G^S\right| + \left|q_C^U - q_G^U\right|\right) + \left|y_C - y_G\right|, \tag{36}$$

where ω_q is the weight on differences in the indicators, and q_G^S (q_G^U) denotes the average value of the indicator for class S (class U) specific attributes in group G. Specifically, $q_S^S = 1$, $q_S^U = 0$ ($q_U^S = 0$, $q_U^U = 1$), and $q_N^S = H$, $q_N^U = 1 - H$.

A decrease in ω_q implies that people care less about inter-class differences in culture, norms of behavior, values, and so forth. Decreased ω_q may also be interpreted as a decline in class-specific culture, norms, and values, and the *homogenization of these attributes across the classes*, because the indicators do not capture the quantitative importance of the attributes. The next proposition shows that a decrease in ω_q has similar effects to an increase in $\Delta\widetilde{S_N}$ on the dynamics and long-run outcomes.

Proposition 4 *Suppose that the assumptions and conditions of Proposition 2 hold.*

(*i*) *When ω_q is smaller, $\underline{F}$ is lower and thus $F_0 \geq \underline{F}$ is more likely to hold. In particular, if ω_q is very small* (*large*), *$p = q = 1$* (*$p = q = 0$*) *and thus $\underline{F}$ is lowest* (*highest*).

(*ii*) *Suppose that society starts with an $F_0 \in (\underline{F}, H^*)$.*

(*a*) *If ω_q is very small* (*large*), *then $p = q = 1$* (*$p = q = 0$*) *and τ is high* (*$\tau = 0$*) *all the time. Consequently, the disposable income of unskilled workers is high* (*low*) *for a given H, and H converges to H^* quickly* (*slowly*).

(*b*) *Otherwise, when ω_q is relatively small* (*large*), *society generally shifts from $p = 0$, $q = 1$ to $p = q = 1$* (*$p = q = 0$*) *eventually. The shift increases τ* (*decreases τ to* 0) *and speeds up* (*slows or stops*) *the convergence to H^*.*

(*c*) *When ω_q is not very small or large, multiple equilibria may exist for given ω_q and $H = F$. The dynamics and long-run outcomes differ depending on which equilibrium is realized initially.*

Proof. See Appendix C. ■

Graphically, this result holds because all the dividing lines of Figures 3 and 4 shift downward when ω_q declines.

5.1.2 Implications

The results indicate that large cross-country differences in the level and pace of economic development may be attributed to differences in the exogenous element of national status, in inter-class distances in culture, norms, and values, or in people's concerns about these distances, as well as differences in the initial distributions of wealth and productivity. In many developing countries, the belief that people share a glorious history, rich culture, or "right" sense of values is weak, and inter-class differences in culture, norms, and values are large or perceived to be serious. According to the model, such situations lead to a low national status or a large perceived distance to the other class, hindering the formation of a common national identity. Consequently, the scale of redistribution is limited, the upward mobility of the poor through education is constrained, and the pace of development is slow.

This implication of the model aligns with empirical findings. First, empirical research indicates that national identity promotes growth and development by increasing income redistribution and stimulating educational investment. Various studies (Chen and Li, 2009; Transue, 2007; Qari, Konrad and Geys, 2012; Singh, 2015) suggest that national identity has a positive effect on redistribution (see footnote 16 in Section 3.1 for details). Berg et al. (2018), based on cross-country data covering numerous countries, indicates that income redistribution, unless very large-scale, makes growth faster and more sustainable by reducing income inequality. They also find that lower inequality is associated with higher years of education. Further, Hanushek and Woessmann (2012a) find that educational achievement, measured by cognitive skills, has a large effect on growth, using data from 64 countries. Second, there is suggestive evidence for Latin American countries that income redistribution is limited in scale and difficult to expand due to a weak sense of common identity stemming from severe social divisions. Goni, Lopez, and Serven (2011) observe that while market inequality is not very different between Latin American and Western European countries, after-tax after-transfer inequality is much higher in the former group of countries because transfers are small in scale and not well targeted to the poor. This holds true even when public expenditures on education and health are considered.[41] Blofield and Juan Pablo (2011), based on World Values Survey data, find that a significant proportion of the population in Latin American countries actually favors even *higher* income inequalities than those currently prevalent (although, a sizable segment supports lower inequalities), whereas people in European countries generally accept the status quo. Experts on Latin America (O'Donnell, 1998; Vilas, 1997) argue that implementing policies to seriously address poverty and inequality is challenging because people do not have a sense of common belonging or broad solidarity due to sharp social divisions or polarization.

The results also indicate the critical importance of *nation-building policies*, such as school education and government propaganda emphasizing common history, culture, and values, as well as policies promoting inter-group contact, in countries with low national status or large perceived inter-class differences in culture, norms, and values. According to the model, these policies enhance national status or reduce (or make less salient) the inter-class differences, thereby contributing to the formation of a shared identity and fostering economic development. Londoño-Vélez (2022) finds that a Colombian financial aid reform that increased the share of low-income students at an elite university prompted greater interaction between high-income and low-income peers and increased support for progressive redistribution among high-income students. Various studies indicate that nation-building policies can effectively strengthen national identity. Chen, Lin, and Yang

[41] Hanushek and Woessmann (2012b) find that the poor growth and development performance of Latin American nations could be attributed primarily to poor educational achievement. This finding, along with the limited scale of redistribution, suggests that increased redistribution could improve economic outcomes by enabling the poor to access quality education.

(2023) examine a curriculum reform that introduced a large amount of Taiwan-related contents into the history subject for junior high school students and find that students under the new curriculum are much more likely to hold an exclusive Taiwanese identity rather than dual identities of Taiwanese and Chinese. Blouin and Mukand (2019), based on field and lab experiments in post-genocide Rwanda, show that exposure to government radio propaganda weakened ethnic identity and increased interethnic trust and cooperation. By examining data on a lottery that allocates conscripts to various regions in Spain, Cáeres-Delpiano et al. (2021) find that men from regions with weak national identity, when assigned to military service in a different region, significantly and persistently increased their national identity.

Finally, according to the model, when the share of skilled workers and thus the level of development become high, everyone identifies with the nation (their class) when $\Delta\widetilde{S_N}$ is high (low), or ω_q is low (high). In particular, if $\Delta\widetilde{S_N}$ is relatively, but not extremely, high or ω_q is relatively, but not extremely, low, society shifts from the equilibrium in which the skilled identify with their class and the unskilled identify with the nation to *universal national identity*; otherwise, it shifts to *universal class identity*. Classic modernization theories in political science (Deutsch, 1953; Gellner, 1983; Weber, 1979), based on Europe's past experiences, argue that modernization (including industrialization and universal education) leads to widespread national identity at the expense of subnational identities (Robinson, 2014). The result suggests that these theories hold true only when national status is relatively high or perceived inter-class differences in culture, norms, and values are relatively small. The result also shows that the identity shift associated with modernization positively impacts the level of redistribution and the pace of development if these conditions are met, but *negatively* impacts the outcomes otherwise.

5.1.3 Time-varying exogenous variables

So far, productitivies A_s and A_u, and the cost of education $\overline{e}$ have been time-invariant. The results remain unchanged qualitatively when these variables grow over time at the same constant rate, as long as the model is adjusted so that the exogenous components of status, $\widetilde{S_N}$ and $\widetilde{S_C}$, are multiplied by a variable that also grows at the same rate in the utility function.[42] It would be reasonable to suppose that $\overline{e}$ grows at the same rate as the productivities and thus earnings, considering that the main cost of education is the cost of hiring teachers and other staff. The assumption on the status variables would also be plausible because the importance of exogenous factors influencing status, such as culture, history, and values, in one's welfare does not seem to diminish with economic growth in the real society.

5.2 Effect of SBTC on identity, redistribution, and development

Skill-biased technical change (SBTC) is another primary driver of economic growth and development, along with human capital accumulation. This section examines the effect of increasing $\frac{A_s}{A_u}$ for a given level of H. To simplify the analysis, exogenous variables such as A_u and $\Delta\widetilde{S_N}$ are

[42]Suppose that these variables grow at rate g, and let $A_{st} = g^t A_s$, $A_{ut} = g^t A_u$, and $\overline{e_t} = g^{t-1}\overline{e}$ (note that $\overline{e_t}$ is the cost in period $t-1$). Then, given H_t, w_{st}, w_{ut}, and T_t also grow at g. Denote detrended endogenous variables with a tilde, e.g., $\widetilde{w_{st}} \equiv \frac{w_{st}}{g^t}$ and $\widetilde{b_t} \equiv \frac{b_t}{g^{t-1}}$. By dividing both sides of the equations by g^t, (28) and (29) can be respectively expressed as $\widetilde{b_{t+1}} = \lambda\{(1-\tau_t)\widetilde{w_{ut}} + \widetilde{T_t} + (1+r)\frac{\widetilde{b_t}}{g}\}$ and $\widetilde{b_{t+1}} = \lambda\{(1-\tau_t)\widetilde{w_{st}} + \widetilde{T_t} + (1+r)\frac{1}{g}(\widetilde{b_t} - \overline{e})\}$. Then, the conditions for $H_{t+1} = F_{t+1} > H_t = F_t$ become $\lambda[(1-\tau_t)\widetilde{w_{st}} + \widetilde{T_t}] \geq \overline{e}$ and $\frac{\lambda}{1-\frac{\lambda}{g}(1+r)}\left[(1-\tau_t)\widetilde{w_{ut}} + \widetilde{T_t}\right] > \overline{e}$, which are very similar to (32) and (33), respectively. Results on identity choice and the tax rate do not change because given H, all the terms in the utility function grow at g. Results on earnings and welfare disparities do not change when relative measures are used. Results on output, earnings, disposable incomes, and the long-run welfare level hold when they are detrended.

fixed: as mentioned in Section 5.1.3, the qualitative results remain unchanged when A_u grows over time at a constant rate and the exogenous components of status, $\widetilde{S_N}$ and $\widetilde{S_C}$, are multiplied by a variable that grows at the same rate. Unlike the original setting, $\overline{e}$ is assumed to be proportional to w_s, since the main cost of education in the real world is the cost of hiring teachers (thus skilled workers), and w_s now grows over time. The following proposition summarizes the result.

Proposition 5 *Suppose that the assumptions and conditions of Proposition 2 hold. Then, an increase in* $\frac{A_s}{A_u}$ *has the following effects:*

(*i*) *The inter-class disparity in earnings increases.*

(*ii*) *All the dividing lines for identity choice shift upward on the* $(H, \Delta\widetilde{S_N})$ *plane. Hence, when the rise of* $\frac{A_s}{A_u}$ *continues, society eventually shifts to an equilibrium with weaker national identity, unless* $\Delta\widetilde{S_N}$ *is small or very large:*

(*a*) *If* $H < \frac{\beta+\gamma}{2\beta}$ *or* $\frac{A_s}{A_u}$ *is small enough that* $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$, *society shifts from* $p=q=1$ *to* $p=0$, $q=1$ (*from* $p=0, q=1$ *to* $p=q=0$) *when* $\Delta\widetilde{S_N}$ *is relatively large* (*small*).[43]

(*b*) *Otherwise, it shifts from* $p=q=1$ *to* $p=q=0$ *when* $\Delta\widetilde{S_N}$ *is relatively small.*[44]

(*iii*) *Given* p *and* q, τ *rises, but the inter-class disparity in welfare increases. When the identity shift occurs,* τ *falls, exacerbating the welfare disparity.*

(*iv*) *When the cost of education is sufficiently high,*[45] $\underline{F}_t$ *increases and for* $F_0 > \underline{F}_0$, *the speed of the convergence to* H^*, *which increases with* $\frac{A_s}{A_u}$, *slows down. The latter is particularly so when the identity shift occurs.*

Proof. See Appendix C. ■

SBTC widens the wage gap between skilled and unskilled workers. To counteract this growing disparity, the tax rate is raised for given values of p and q (unless $p = q = 0$), but the expansion of redistribution is not sufficient to fully offset the increased wage inequality. Hence, the perceived distances to the other class and thus to the "average national" increase. SBTC also raises (lowers) the status of the skilled (unskilled) class relative to that of the nation, due to increased inter-class disparity in disposable income. Consequently, the national identity of skilled workers becomes weaker in the sense that the utility gain from identifying with the nation rather than with their class decreases. In contrast, for unskilled workers, the lowered relative status of their class partially offsets the increased perceived distance to the "average national", but the latter effect dominates (due to $\beta > \gamma$), resulting in a weaker national identity for them as well. As a result, all the dividing lines for identity choice in Figures 3 and 4 shift upward. Therefore, if $\frac{A_s}{A_u}$ continues to increase, society eventually shifts to an equilibrium with weaker national identity and a *lower* redistributive tax, unless $\Delta\widetilde{S_N}$ is small or very large.

The increased inequality in disposable income enlarges the disparity in welfare. The effect is particularly large when the identity shift occurs because it reduces the scale of redistribution.

SBTC also impacts the dynamics and long-run outcomes. It increases $\underline{F}_t$ by raising the cost of education, which is proportional to the skilled wage, at a higher rate than the unskilled wage. Consequently, escaping the "poverty trap" becomes more difficult for societies starting with a relatively small share of individuals able to afford education. Further, the pace of convergence

[43]To be precise, when $\overline{H} > \frac{\beta+\gamma}{2\beta}$ and $H(< \frac{\beta+\gamma}{2\beta})$ is close to $\frac{\beta+\gamma}{2\beta}$, shifts from $p = q = 1$ to $p = q = 0$ and from $p = 0$, $q = 1$ to $p = q = 1$, then to $p = q = 0$ are also possible (see Figure 4).

[44]To be precise, the shift from $p = q = 1$ to $p = 1$, $q = 0$, then to $p = q = 0$ can also occur (see Figure 4).

[45]To be accurate, this is true when the constant s of $\overline{e}_t = sw_{st-1}$ is sufficiently large.

to H^* slows down when $F_0 > \underline{F}_0$, especially when the identity shift occurs and thus the scale of redistribution shrinks.

The result suggests that as technology becomes more skill-biased, establishing national identity becomes more challenging, and redistribution becomes less effective. Consequently, achieving upward mobility for the poor and equalizing welfare becomes more difficult or slower. The higher skill bias of current technology, along with a weaker sense of shared history, culture, and values, as well as greater inter-class differences in culture, norms, and values, may contribute to the slower pace of upward mobility and development in many developing countries compared to to the rate experienced by developed countries during their modernization period.

In advanced economies, income inequality has increased greatly in recent decades, but the demand for and scale of income redistribution have not increased; in some measures, they have even decreased (Ashok et al., 2016; Piketty, Saez, and Stantcheva, 2014). To explain this phenomenon, Windsteiger (2022) develops a model in which individuals are segregated according to income and mainly interact with those at similar income levels, resulting in biased information on income inequality. Consequently, under certain conditions, increased inequality leads to a fall in perceived inequality and thus a decrease in support for and the level of redistribution. With a slight modification, the present model can also explain this phenomenon. Suppose that, unlike the original setting, individuals are heterogeneous in the weight on the exogenous component of group status in utility, δ, with a continuous distribution. Then, unless $\Delta\widetilde{S_N}$ is small or very large, as $\frac{A_s}{A_u}$ increases, p and q, and thus τ, could decrease continuously.

6 Conclusion

This paper developed a dynamic model of income redistribution and educational investment incorporating social identification and explored the interaction among identity, redistribution, and development theoretically. Analysis showed that, given the skilled workers' share, the rate of redistributive taxation is higher as the proportion of people identifying with the nation is higher. When the exogenous component of national status is higher (which would be the case when, for example, people have stronger pride in the nation), or when inter-class differences in culture, norms, and values are smaller or less of a concern, the proportion of people having a national identity and thus the redistributive tax rate are higher. Consequently, society is less likely to fall into a "poverty trap", and under a favorable initial condition that avoids this trap, the share of skilled workers and output increase faster. As skill-biased technical change (SBTC) proceeds, society becomes more prone to the "poverty trap", and under favorable initial conditions, the upward mobility of the poor slows down. Further, when SBTC continues, society generally transitions to an equilibrium in which fewer people identify with the nation, leading to reduced tax rate and redistribution, thereby intensifying the negative effects of SBTC.

The result suggests that large cross-country differences in the level and speed of development may be partly due to differences in the exogenous component of national status and in inter-class distances in culture, norms, and values or people's concerns about these distances. In many developing countries, the belief that people share a glorious history, rich culture, or a "right" sense of values is weak and inter-class differences in culture, norms, and values are large or perceived to be serious. According to the model, these factors make the formation of a common national identity difficult, and as a result, the scale of redistribution limited, the upward mobility of the poor and the pace of development slow. For these countries, nation-building policies, such as school education and government propaganda emphasizing common history, culture, and values and policies promoting between-group contact, would be crucial for good outcomes. Technology available for present developing countries is much more skill-biased than what developed countries used when they underwent the modernization and industrialization of their economies. The result on SBTC suggests that this may be another reason for the slower pace of development, particularly of the poor, in many developing countries.

References

[1] Aiyar, Shekhar and Christian Ebeke (2020), "Inequality of Opportunity, Inequality of Income, and Economic Growth," *World Development* 136,105115.

[2] Akerlof, G. A. and R. E. Kranton (2000), "Economics and identity, " *Quarterly Journal of Economics* 115 (3): 715-53.

[3] Ashok, Vivekinan, Ilyana Kuziemko, and Ebonya Washington (2016), "Support for Redistribution in an Age of Rising Inequality: New Stylized Facts and Some Tentative Explanations," *Brookings Papers on Economic Activity* 2015 (1), 367−433.

[4] Assouad, Lydia (2021), "Charismatic Leaders and Nation-Building", mimeo, Paris School of Economics.

[5] Atkin, David, Eve Colson-Sihra, and Moses Shayo (2021), "How do we Choose our Indentity? A Revealed Preference Approach Using Food Consumption," *Journal of Political Economy* 129(4), 1193–1251.

[6] Autor, David, Claudia Goldin, and Lawrence F. Katz (2020), "Extending the Race between Education and Technology," NBER Working Paper No. 26705.

[7] Benabou, R. and J. Tirole (2011), "Identity, morals, and taboos: Beliefs as assets," *Quarterly Journal of Economics* 126 (2): 805−855.

[8] Berg, Andrew, Jonathan D. Ostry, Charalambos G. Tsangarides, and Yorbol Yakhshilikov (2018), "Redistribution, inequality, and growth: new evidence," *Journal of Economic Growth* 23, 259−305.

[9] Bernard, Mark, Florian Hett, and Mario Mechtel (2016), "Social identity and social free-riding," *European Economic Review* 90, 4–17.

[10] Bisin, A., E. Patacchini, T. Verdier, and Y. Zenou (2011), "Formation and persistence of oppositional identities," *European Economic Review* 55 (8), 1046–1071.

[11] Blofield, Merike, and Luna, Juan Pablo (2011) "Public Opinion on Income Inequalities in Latin America." In Blofield, Merike, ed., *The Great Gap: Inequality and the Politics of Redistribution in Latin America*. University Park, Pa.: Pennsylvania State University Press.

[12] Blouin, Arthur and Sharun W. Mukand (2019), "Erasing ethnicity? Propaganda, nation building, and identity in Rwanda," *Journal of Political Economy* 127(3), 1008−1062.

[13] Bonomi, Giampaolo, Nicola Gennaioli, and Guido Tabellini (2021), "Identity, Beliefs, and Political Conflict", *Quarterly Journal of Economics* 136(4), 2371–2411.

[14] Cáeres-Delpiano, Julio, Antoni-Italo De Moragas, Gabriel Facchini, and Ignacio Gonzárez (2021), "Intergroup contact and nation building: Evidence from military service in Spain," *Journal of Public Economics* 201(C), 104477.

[15] Carvalho, Jean-Paul and Christian Dippel (2020), "Elite Identity and Political Accountability: A Tale of Ten Islands," *Economic Journal* 130(631), 1995–2029.

[16] Chen, Wei-Lin, Ming-Jen Lin, and Tzu-Ting Yang (2023), "Curriculum and National Identity: Evidence from the 1997 Curriculum Reform in Taiwan," *Journal of Development Economics* 163, 103078.

[17] Chen, Y. and S. X. Li (2009), "Group identity and social preferences," *American Economic Review* 99 (1), 431–457.

[18] Collier, P. (2009), *Wars, Guns, and Votes*. New York, NY: Harper Perennial.

[19] Dehdari, Sirus H. and Kai Gehring (2021), "The Origins of Common Identity: Evidence from Alsace-Lorraine, " forthcoming in *American Economic Journal: Applied Economics.*

[20] Deininger, Klaus and Lyn Squire (1998), "New ways of looking at old issues: Inequality and growth," *Journal of Development Economics* 57 (2), 259−287.

[21] Deutsch, K. W. (1953), *Nationalism and Social Communication: An Inquiry into the Foundations of Nationality*. Cambridge, MA: MIT Press.

[22] Dhami, Sanjit, Emma Manifold, and Ali al-Nowaihi (2021), "Identity and Redistribution: Theory and Evidence," *Economica* 88 (350), 499−531.

[23] Easterly, William (2007), "Inequality does cause underdevelopment: Insights from a new instrument," *Journal of Development Economics* 84 (2), 755−776.

[24] Fouka, Vasiliki, Soumyajit Mazumder, and Marco Tabellini (2022), "From Immigrants to Americans: Race and Assimilation during the Great Migration," *Review of Economic Studies* 89 (2), 811−842.

[25] Fukuyama, Francis (2018), "Why National Identity Matters," *Journal of Democracy* 29 (4), 5−15.

[26] Galor, Oded and Joseph Zeira (1993), "Income distribution and macroeconomics," *Review of Economic Studies* 60, 35−52.

[27] Gellner, E. (1983), *Nations and Nationalism*. Ithaca, NY: Cornell University Press.

[28] Ghiglino, Christian, David Júarez-Lunam, and Andreas Müller (2021), "Class Altruism and Redistribution," *Economic Journal* 131(64), 3274−3295.

[29] Goni, Edwin, J. Humberto Lopez, and Luis Serven (2011), "Fiscal Redistribution and Income Inequality in Latin America," *World Development* 39 (9), 1558−1569.

[30] Grossman, G. and E. Helpman (2021) "Identity politics and trade policy," *Review of Economic Studies* 88 (3), 1101–1126.

[31] Hanushek, Eric A. and Ludger Woessmann (2012a), "Do better schools lead to more growth? Cognitive skills, economic outcomes, and causation," *Journal of Economic Growth* 17 267−321

[32] Hanushek, Eric A. and Ludger Woessmann (2012b), "Schooling, educational achievement, and the Latin American growth puzzle," *Journal of Development Economics* 99, 497−512.

[33] Hett, Florian, Mario Mechtel, and Markus Kröll (2020),"The Structure and Behavioral Effects of Revealed Social Identity Preferences," *Economic Journal* 130 (632), 2569–2595.

[34] Holm, Joshua (2016), "A model of redistribution under social identification in heterogeneous federations," *Journal of Public Economics* 143, 39−48.

[35] Lindbeck, Assar and Weibull, Jörgen (1987), "Balanced-Budget Redistribution as the Outcome of Political Competition", *Public Choice* 52, 273–297.

[36] Lindqvist, Erik and Robert Östling (2013), "Identity and redistribution," *Public Choice* 155, 469−491.

[37] Litschig, Stephan and María Lombardi (2019), "Which tail matters? Inequality and growth in Brazil," *Journal of Economic Growth* 24 (2), 155−187.

[38] Londoño-Vélez, Juliana (2022), "The impact of diversity on perceptions of income distribution and preferences for redistribution," *Journal of Public Economics* 214, 104732.

[39] Manning, A., and S. Roy (2010), “Culture clash or culture club? National identity in Britain,” *Economic Journal* 120: F72–F100.

[40] Meltzer, Allan H. and Scott F. Richard (1981), ”A Rational Theory of the Size of Government,” *Journal of Political Economy* 89 (5), 914−927.

[41] Michalopoulos, S. and E. Papaioannou (2015), “On the ethnic origins of African development: Chiefs and precolonial political centralization”, *Academy of Management Perspectives* 29: 32−71.

[42] Nosofsky, R. M. (1986), “Attention, similarity and the identification-categorization relationship,” *Journal of Experimental Psychology: General* 115 (1): 39–57.

[43] O'Donnell, Guillermo (1998) ”Poverty and Inequality in Latin America: Some Political Reflections,” in V. T. Tokman and G. O'Donnell (eds), *Poverty and Inequality in Latin America. Issues and New Challenges*, Notre Dame: Notre Dame University Press.

[44] Piketty, Thomas, Emmanuel Saez, and Stefanie Stantcheva (2014), ”Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities,” *American Economic Journal: Economic Policy* 6 (1), 230−71.

[45] Qari, Salmai, Kai A. Konrad and Benny Geys (2012), ”Patriotism, taxation and international mobility,” *Public Choice* 151 (3/4), 695−717.

[46] Robinson, A. L. (2014), ”National versus ethnic identification in Africa: Modernization, colonial legacy, and the origins of territorial nationalism,” *World Politics* 66, 709–746.

[47] Sambanis, N. and M. Shayo (2013), ”Social identification and ethnic conflict,” *American Political Science Review* 107 (2): 294–325.

[48] Shayo, M. (2009), “A Model of social identity with an application to political economy: Nation, class and redistribution,” *American Political Science Review* 103 (2): 147–74.

[49] Singh, P. (2015), ”Subnationalism and Social Development: A Comparative Analysis of Indian States,” *World Politics* 67 (3), 506−62.

[50] Tajfel, H. and J. C. Turner (1986), ”The social identity theory of intergroup behavior.” In S. Worchel and W. Austin eds., *Psychology of Intergroup Relations*, Chicago: Nelson Hall, 7–24.

[51] Transue, J. E. (2007), ”Identity salience, identity acceptance, and racial policy attitudes: American national identity as a uniting force,” *American Journal of Political Science* 51, 78−91.

[52] Turner, J. C., M. Hogg, P. Oakes, S. Reicher, and M. Wetherell (1987), *Rediscovering the Social Group: A Self-Categorization Theory*. Oxford: Blackwell.

[53] Vilas, Carlos M. (1997), ”Participation, Inequality, and the Whereabouts of Democracy”, in Douglas A. Chalmers and others (eds), *The New Politics of Inequality in Latin America: Rethinking Participation and Representation*, Oxford: Oxford University Press.

[54] Weber, E. (1979), *Peasants into Frenchmen : The Modernization of Rural France, 1870-1914*. Stanford, CA: Stanford University Press.

[55] Windsteiger, Lisa (2022), ”The redistributive consequences of segregation and misperceptions,” *European Economic Review* 144, 104073.

[56] Yuki, K. (2007), ”Urbanization, informal sector, and development,” *Journal of Development Economics* 84 (1), 76–103.

[57] Yuki, K. (2008), ”Sectoral shift, wealth distribution, and development,” *Macroeconomic Dynamics* 12 (4), 527–559.

[58] Yuki, K. (2021), ”Modernization, social identity, and ethnic conflict,” *European Economic Review* 140, 103919.

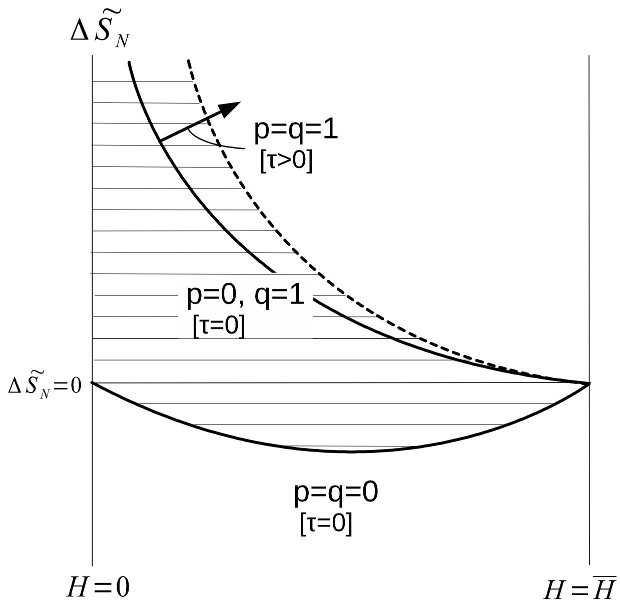

Figure 5: Equilibria when $\beta \leq \gamma$

Appendix A Propositions A1 and A2

This Appendix presents results on shapes of dividing lines for quilibria, which are the basis for Figures 1 and 2 in Section 3.2. The next proposition presents the result when $\beta \leq \gamma$.

Proposition A1 *Suppose that $\beta \leq \gamma$ and Assumption 2 holds.*

(i) $p = q = 1$, $p = 0$, $q = 1$, and $p = q = 0$ can be stable equilibria.

(ii) The dividing line for $p = q = 1$ and the upper dividing line for $p = 0$ and $q = 1$ decrease with H, go to $+\infty$ as $H \to 0$, and go to 0 as $H \to \overline{H}$, where $\overline{H}$ is H satisfying $H = a(H) \Leftrightarrow w_s = w_u$, on the $(H, \Delta\widetilde{S_N})$ plane.

(iii) There exists an $H^\sharp \in (0, \overline{H})$ such that the dividing line for $p = q = 0$ and the lower dividing line for $p = 0$ and $q = 1$ decrease (increase) with H for $H < (>) H^\sharp$. They go to 0 as $H \to 0$ and $H \to \overline{H}$.

(iv) On the $(H, \Delta\widetilde{S_N})$ plane, the dividing line for $p = q = 0$ and the lower dividing line for $p = 0$ and $q = 1$ are the same; they are located below the dividing line for $p = q = 1$ and the upper dividing line for $p = 0$ and $q = 1$; the dividing line for $p = q = 1$ is located below the upper dividing line for $p = 0$ and $q = 1$.

Proof. See Appendix C. ■

Based on the proposition, Figure 5 illustrates combinations of H and $\Delta\widetilde{S_N}$ each equilibrium exists when $\beta \leq \gamma$. There are two differences from the figures when $\beta > \gamma$ (Figures 1 and 2 in Section 3.2). First, the dividing line for $p = q = 0$ decreases (increases) with H for relatively small (large) H. That is, the relation with H is opposite to the case $\beta > \gamma$. This is because the effect of H on status dominates the effect on perceived distance when $\beta \leq \gamma$, where the former effect is that, when H is relatively small (large), an increase in H increases (decreases) national status relative to class status and thus the utility from identifying with the nation. Second, the lower dividing line for $p = 0$, $q = 1$ coincides with the dividing line for $p = q = 0$ because τ when $p = 0$, $q = 1$ is 0 when $\beta \leq \gamma$. Except these points, the figure is similar to the one when $\beta > \gamma$ and A_s is relatively low (Figure 1).

The next proposition presents the result when $\beta > \gamma$. Based on (i) [(ii) and (iii)] of the proposition, Figure 1 [Figure 2] in Section 3.2 illustrates combinations of H and $\Delta\widetilde{S_N}$ each equilibrium exists when $\frac{A_s}{A_u}$ is relatively low [high].

Proposition A2 *Suppose that $\beta > \gamma$ and Assumption 2 holds.*

(*i*) *When $\frac{A_s}{A_u}$ is small enough that $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$, Proposition A1 applies except the following.*

(*a*) *The dividing line for $p = q = 0$ and the lower dividing line for $p = 0$ and $q = 1$ are different. The former increases (decreases) with H for $H < (>) H^\sharp$. The latter increases (decreases) with H for small (large) enough H.*[46]

(*b*) *On the $(H, \Delta\widetilde{S_N})$ plane, the dividing line for $p = q = 0$ is located above the lower dividing line for $p = 0$ and $q = 1$; when H is relatively high, the dividing line for $p = q = 0$ could be located above the dividing line for $p = q = 1$ and the upper dividing line for $p = 0$ and $q = 1$.*

(*ii*) *When $\frac{A_s}{A_u}$ is large enough that $\overline{H} > \frac{\beta+\gamma}{2\beta}$ and $H \leq \frac{\beta+\gamma}{2\beta}$, the results are same as (i) except the following.*

(*a*) *When A_s is large enough that $H^\sharp \geq \frac{\beta+\gamma}{2\beta}$, the dividing line for $p = q = 0$ and the lower dividing line for $p = 0$ and $q = 1$ increase with H.*

(*b*) *When H is relatively high, the dividing line for $p = q = 0$ is located above the dividing line for $p = q = 1$ and the upper dividing line for $p = 0$ and $q = 1$. The lower and upper dividing lines for $p = 0$ and $q = 1$ intersect at $H = \frac{\beta+\gamma}{2\beta}$.*

(*iii*) *When $\overline{H} > \frac{\beta+\gamma}{2\beta}$ and $H > \frac{\beta+\gamma}{2\beta}$,*

(*a*) *$p = q = 1$, $p = 1$ and $q = 0$, and $p = q = 0$ can be stable equilibria.*

(*b*) *The dividing line for $p = q = 0$ and the lower dividing line for $p = 1$ and $q = 0$ decrease with H and go to 0 as $H \to \overline{H}$ on the $(H, \Delta\widetilde{S_N})$ plane.*

(*c*) *The dividing line for $p = q = 1$ and the upper dividing line for $p = 1$ and $q = 0$ decrease with H for sufficiently large H and go to 0 as $H \to \overline{H}$. They could increase with H for sufficiently small H.*[47]

(*d*) *The dividing line for $p = q = 0$ is located above the lower dividing line for $p = 1$ and $q = 0$; the dividing line for $p = q = 1$ is located below the upper dividing line for $p = 1$ and $q = 0$; when H is relatively low (high), the dividing line for $p = q = 0$ is located above (below) the dividing line for $p = q = 1$ and the upper dividing line for $p = 1$ and $q = 0$; the lower and upper dividing lines for $p = 1$ and $q = 0$ intersect at $H = \frac{\beta+\gamma}{2\beta}$.*[48]

Proof. See Appendix C. ■

[46] When H is intermediate, the relationship with H is not analytically clear.
[47] When H is intermediate, the relationship with H is not analytically clear.
[48] When H is intermediate, the relative positions of these dividing lines are not analytically clear.

Appendix B Determination of H^* and Proof on the value of H

This Appendix formally explains how H^* is determined and proves that $H = F$ ($H = H^*$) holds when F is small (large). By substituting (26) and (27) into (24), the indirect utility function equals

$$v_{CG} = (1-\tau)w_C + T + (1+r)a - \beta d_{CG} + \gamma S_G. \tag{37}$$

From this equation, (9)−(12), (28), and (29),

$$\begin{aligned} v_{SN} &= (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_s - \beta(1-\tau)(w_s - \overline{w}) + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] + (1+r)a, \\ &= (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_s - \beta(1-\tau)(w_s - \overline{w}) + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] + (1+r)(b-\overline{e}), \quad (38) \\ v_{SS} &= (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_s + \gamma\left[\delta\widetilde{S_C} + (1-\tau)w_s\right] + (1+r)(b-\overline{e}), \quad (39) \\ v_{UN} &= (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_u - \beta(1-\tau)(\overline{w} - w_u) + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] + (1+r)b, \quad (40) \\ v_{UU} &= (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_u + \gamma\left[\delta\widetilde{S_C} + (1-\tau)w_u\right] + (1+r)b. \quad (41) \end{aligned}$$

H^* when $p = q = 1$ is H satisfying $v_{SN} = v_{UN}$, thus from (38) and (40), H satisfying

$$\begin{gathered} (1-\tau)w_s - \beta(1-\tau)(w_s - \overline{w}) - (1+r)\overline{e} = (1-\tau)w_u - \beta(1-\tau)(\overline{w} - w_u) \\ \Leftrightarrow [1-\beta(1-2H)](1-\tau)(w_s - w_u) - (1+r)\overline{e} = 0, \quad (42) \\ \text{where } \tau = \tfrac{2\beta}{1+\gamma}(a(H) - H). \end{gathered}$$

H^* when $p = q = 0$ is H satisfying $v_{SS} = v_{UU}$, thus from (39) and (41), H satisfying

$$(1+\gamma)(w_s - w_u) - (1+r)\overline{e} = 0. \tag{43}$$

H^* when $p = 0$, $q = 1$ is H satisfying $v_{SS} = v_{UN}$, thus from (39) and (40), H satisfying

$$\begin{gathered} (1-\tau)w_s + \gamma\left[\delta\widetilde{S_C} + (1-\tau)w_s\right] - (1+r)\overline{e} = (1-\tau)w_u - \beta(1-\tau)(\overline{w} - w_u) + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] \\ \Leftrightarrow [1+\beta H + \gamma(1-H)](1-\tau)(w_s - w_u) - (1+r)\overline{e} - \gamma\delta\Delta\widetilde{S_N} = 0, \quad (44) \\ \text{where } \tau = \tfrac{\beta-\gamma}{1+\gamma}(a(H) - H) \text{ when } \beta > \gamma \text{ and } \tau = 0 \text{ when } \beta \leq \gamma. \end{gathered}$$

H^* when $p = 1$, $q = 0$ is H satisfying $v_{SN} = v_{UU}$, thus from (38) and (41), H satisfying

$$\begin{gathered} (1-\tau)w_s - \beta(1-\tau)(w_s - \overline{w}) + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] - (1+r)\overline{e} = (1-\tau)w_u + \gamma\left[\delta\widetilde{S_C} + (1-\tau)w_u\right] \\ \Leftrightarrow [1-\beta(1-H) + \gamma H](1-\tau)(w_s - w_u) - (1+r)\overline{e} + \gamma\delta\Delta\widetilde{S_N} = 0, \quad (45) \\ \text{where } \tau = \tfrac{\beta+\gamma}{1+\gamma}(a(H) - H). \end{gathered}$$

H^* exists because the LHSs of (42)−(45) go to $+\infty$ as $H \to 0$ from $\lim_{H\to 0}(w_s - w_u) = +\infty$, and the LHSs become negative as $H \to \overline{H}$ from $\lim_{H\to\overline{H}}(w_s - w_u) = 0$. To be more precise, the latter is true when $p = 0$, $q = 1$, which is an equlibrium only when $\beta > \gamma$ from Propositions A1 and A2, because $\Delta\widetilde{S_N}$ in (44) is positive from (22). It is true when $p = 1$, $q = 0$ because the LHS of (45) is smaller than $[1-\beta(1-H)+\gamma H](1-\tau)(w_s - w_u) - (1+r)\overline{e} + (\beta-\gamma)(1-\tau)H(w_s - w_u) = [1-\beta(1-2H)](1-\tau)(w_s - w_u) - (1+r)\overline{e}$ from (23).

Proof that $H = F$ ($H = H^*$) when F is small (large)

When $p = q = 0$, $H = F$ ($H = H^*$) holds when $v_{SS} \geq (<) v_{UU}$ is satisfied with $H = F$, i.e., $(1+\gamma)(w_s - w_u) - (1+r)\overline{e} \geq (<) 0$ with $H = F$ from (43). Because $w_s - w_u$ decreases with H, this is the case when $F \leq (>) H^*$. Similarly, when $p = 0$, $q = 1$ and $\beta \leq \gamma$ (thus $\tau = 0$), $H = F$ ($H = H^*$) holds when $F \leq (>) H^*$ from (44).

For other cases, the relation between H and the LHSs of the above equations determining H^* is generally unclear. Whether $H = F$ or $H = H^*$ is clear only for sufficiently small or large F. As for the range of F satisfying $H = F$, the following is true. When $p = 0$, $q = 1$ and $\beta > \gamma$, from (22), the LHS of (44) is greater than $[1 - \beta(1-2H)](1-\tau)(w_s - w_u) - (1+r)\overline{e} > (1-\beta)(1-\tau)(w_s - w_u) - (1+r)\overline{e}$. Hence, $H = F$ holds at least for F weakly smaller than the smallest H satisfying $(1-\beta)(1-\tau)(w_s - w_u) - (1+r)\overline{e} = 0$, which is smaller than H^* for $p = q = 0$. It is easy to see that a similar result holds when $p = q = 1$ from (42). When $p = 1$, $q = 0$, from (23), the LHS of (45) is greater than $(1+\gamma)(1-\tau)(w_s - w_u) - (1+r)\overline{e}$. Hence, $H = F$ holds at least for F weakly smaller than the smallest H satisfying $(1+\gamma)(1-\tau)(w_s - w_u) - (1+r)\overline{e} = 0$, which is smaller than H^* for $p = q = 0$.

As for the range of F satisfying $H = H^*$, the following is true. When $p = 0$, $q = 1$ and $\beta > \gamma$, $H = H^*$ at least for F weakly greater than H^* for $p = q = 0$. This is because, from (22), the LHS of (44) is smaller than $(1+\gamma)(1-\tau)(w_s - w_u) - (1+r)\overline{e}$, which is smaller than the LHS of (43). Similarly, when $p = q = 1$ and $\beta \leq \gamma$, $H = H^*$ at least for F weakly greater than H^* for $p = q = 0$ because the LHS of (42) is smaller than that of (43) from $[1 - \beta(1-2H)](1-\tau)(w_s - w_u) < (1+\gamma)(w_s - w_u)$. When $p = q = 1$ and $\beta > \gamma$, the LHS of (42) is smaller than $(1+\beta)(w_s - w_u) - (1+r)$, thus $H = H^*$ at least for F weakly greater than H satisfying $(1+\beta)(w_s - w_u) - (1+r) = 0$, which is greater than H^* for $p = q = 0$. A similar result holds when $p = 1$, $q = 0$ also because the LHS of (45) is smaller than $[1 - \beta(1-2H)](1-\tau)(w_s - w_u) - (1+r)\overline{e}$, which is smaller than $(1+\beta)(w_s - w_u)(w_s - w_u) - (1+r)\overline{e}$.

Appendix C Proofs of lemmas, propositions, and a corollary

Proof of Lemma 1. Suppose that $q \in (0,1)$ is an equilibrium, which implies that unskilled workers are indifferent between the two identities. Then, from (11) and (12),

$$-\beta(1-\tau)(\overline{w} - w_u) + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w} + T\right] = \gamma\left[\delta\widetilde{S_C} + (1-\tau)w_u + T\right]$$
$$\Leftrightarrow \gamma\delta\Delta\widetilde{S_N} = (\beta - \gamma)(1-\tau)(\overline{w} - w_u), \tag{46}$$

where from (17),

$$\tau = 1 - \frac{1}{(1+\gamma)\overline{w}}\begin{pmatrix} [Hw_s + (1-H)w_u] + \gamma\{H[p\overline{w} + (1-p)w_s] + (1-H)[q\overline{w} + (1-q)w_u]\} \\ -\beta\{Hp(w_s - \overline{w}) + (1-H)q(\overline{w} - w_u)\} \end{pmatrix}.$$

From the above equation, τ increases (decreases) with q when $\beta > (<) \gamma$. Thus, when q increases, the RHS of (46) decreases and thus identifying with the nation becomes more attractive than identifying with their class. This means that the equilibrium is unstable. (Further, when $\beta = \gamma$, (46) holds only when $\Delta\widetilde{S_N} = 0$.) $p \in (0,1)$ is not a stable equilibrium can be proved similarly. ■

Proof of Proposition 1. (i) Values of τ are obtained from (17). $a(H) \equiv \frac{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}} \geq H \Leftrightarrow w_s \geq w_u$ from (4) and (5). (ii) Straightforward from (i) except that $p = 1, q = 0$ is not an equilibrium when $\beta \leq \gamma$, which is shown in the proof of Proposition A1 (i). (iii) From (i)(b), $a(H) - H = 0$ at $H = 0, a(H)$, and

$$a'(H)=\frac{\sigma-1}{\sigma}\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}\frac{1}{H}-\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}}{\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^2}$$
$$=\frac{\sigma-1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)}$$
$$=\frac{\sigma-1}{\sigma}\frac{a(H)[1-a(H)]}{H(1-H)}>0, \tag{47}$$

which implies that

$$\lim_{H\to 0}[a'(H)-1] = \frac{\sigma-1}{\sigma}\lim_{H\to 0}\left[\frac{a(H)}{H}\right]-1$$
$$= \frac{\sigma-1}{\sigma}\lim_{H\to 0}\left\{\frac{1}{H}\frac{1}{1+\frac{1-\alpha}{\alpha}\left[\frac{A_u(1-H)}{A_sH}\right]^{\frac{\sigma-1}{\sigma}}}\right\}-1=+\infty, \tag{48}$$
$$\lim_{H\to\overline{H}}[a'(H)-1] = -\frac{1}{\sigma}<0. \tag{49}$$

Further,

$$a''(H) = \frac{\sigma-1}{\sigma}\frac{H(1-H)a'(H)[1-2a(H)]-a(H)[1-a(H)](1-2H)}{[H(1-H)]^2}$$
$$= \frac{\sigma-1}{\sigma}a(H)[1-a(H)]\frac{\frac{\sigma-1}{\sigma}[1-2a(H)]-(1-2H)}{[H(1-H)]^2}$$
$$= \frac{\sigma-1}{\sigma}a(H)[1-a(H)]\frac{-\frac{1}{\sigma}[1-2a(H)]-2(a(H)-H)}{[H(1-H)]^2}<0. \tag{50}$$

Hence, there exists unique $H^+\in(0,\overline{H})$ satisfying $a'(H)-1=0$, and $a(H)-H$ increases (decreases) with H for $H<(>)H^+$. This implies that $\frac{d\tau}{dH}>(<)0$ for $H<(>)H^+$. ■

Proof of Corollary 1 . (i) The result on inequality is from $(1-\tau)w_s+T-[(1-\tau)w_u+T]=(1-\tau)(w_s-w_u)$ and Proposition 1 (ii). The result on the disposable income of skilled workers holds because $(1-\tau)w_s+T=(1-\tau)w_s+\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}$ decreases with τ from $-w_s+(1-\tau)\overline{w}<0$. (ii) The derivative of $(1-\tau)w_u+T=(1-\tau)w_u+\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}$ with respect to τ equals $-w_u+(1-\tau)\overline{w}$, which is positive under Assumption 1 because

$$-w_u+(1-\tau)\,\overline{w}>0\Leftrightarrow\tau<\frac{\overline{w}-w_u}{\overline{w}}$$
$$\Leftrightarrow\tau<\frac{H\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\text{ (from (3) and (5))}$$
$$\Leftarrow\frac{2\beta}{1+\gamma}(a(H)-H)<\frac{1}{1-H}(a(H)-H)\text{ (from Proposition 1)}, \tag{51}$$

where the last inequality is true under Assumption 1. ■

Proof of Lemma 2. (i) When $p=q=0$ or when $\beta\le\gamma$ and $p=0$, $q=1$, $\tau=0$ from Proposition 1 and thus $(1-\tau)w_s+T=w_s$, which equals $w_u+\frac{(1+r)\overline{e}}{1+\gamma}$ $(w_u+\frac{(1+r)\overline{e}+\gamma\delta\Delta\widetilde{S_N}}{1+\beta H+\gamma(1-H)})$ at $H=H^*$ when $p=q=0$ from (43) (when $\beta\le\gamma$ and $p=0$, $q=1$ from (44)). Because w_s decreases with H from (4),

(32) is satisfied for any $H \le H^*$ if it holds at $H = H^*$, where the condtion can be expressed as $\lambda w_u \ge \left[1 - \frac{\lambda(1+r)}{1+\gamma}\right]\bar{e}$ when $p = q = 0$ and as $\lambda\left[w_u + \frac{\gamma\delta\Delta\widetilde{S_N}}{1+\beta H+\gamma(1-H)}\right] \ge \left[1 - \frac{\lambda(1+r)}{1+\beta H+\gamma(1-H)}\right]\bar{e}$ when $p = 0$, $q = 1$. Because $w_s - w_u$ decreases with H and w_u increases with H from (4) and (5), H^* and w_u at $H = H^*$ decrease with $\bar{e}$. Hence, (32) holds when $\bar{e}$ is sufficiently small or λ is sufficiently large. Because $w_s < w_u + (1+r)\bar{e}$ at $H = H^*$, (33) too holds at $H = H^*$. For $H < H^*$, the LHS of (33) increases with H. Thus, if (34) holds at $H = 0$, there exists $\underline{F} \in (0, H^*)$ such that $\frac{\lambda}{1-\lambda(1+r)}w_u = \bar{e}$ at $H = \underline{F}$; $\frac{\lambda}{1-\lambda(1+r)}w_u > (\le)\bar{e}$ and thus $H = F < H^*$ increases over time (is time-invariant) for $H > (\le)\underline{F}$. Because $w_u = (1-\alpha)^{\frac{\sigma}{\sigma-1}}A_u$ at $H = 0$, the condition holds if $\bar{e}$ is not too small or λ is not too large so that $\frac{\lambda}{1-\lambda(1+r)}(1-\alpha)^{\frac{\sigma}{\sigma-1}}A_u < \bar{e}$ is true.

When values of p and q are such that $\tau > 0$, whether $(1-\tau)w_s \ge (1-\tau)w_u + (1+r)\bar{e}$ at $H = H^*$ or not is unclear from (42), (44), and (45). Consider the case in which $(1-\tau)w_s \ge (1-\tau)w_u + (1+r)\bar{e}$ for any $H \le H^*$ first. In this case, $\underline{F}$ smaller than the one for $\tau = 0$ exists if the above conditions hold. This is because (34) holds at $H = 0$ from $(1-\tau)w_u + T = w_u$ at $H = 0$, $\frac{\lambda}{1-\lambda(1+r)}[(1-\tau)w_u + T] > \frac{\lambda}{1-\lambda(1+r)}w_u \ge \bar{e}$, i.e., (33), holds for H weakly greater than $\underline{F}$ for $\tau = 0$, where $(1-\tau)w_u + T > w_u$ under Assumption 1 from the proof of Corollary 1 (ii), and (32) holds from $(1-\tau)w_s \ge (1-\tau)w_u + (1+r)\bar{e}$. (Since the relation between $(1-\tau)w_u + T$ and H is unclear, the possibility that there exist multiple levels of H satisfying $\frac{\lambda}{1-\lambda(1+r)}[(1-\tau)w_u + T] = \bar{e}$ cannot be excluded; by definition, $\underline{F}$ is the highest H satisfying the equation. Because the relation between $(1-\tau)w_s + T$ and H too is ambiguous, the possible existence of $H < \underline{F}$ satisfying $\lambda[(1-\tau)w_s + T] = \bar{e}$ too cannot be ruled out, although this is unlikely given that w_s is large for small H. Thus, the possibility that $H < \underline{F}$ increases or decreases temporarily cannot be ruled out.)

Next, consider the case in which $(1-\tau)w_s < (1-\tau)w_u + (1+r)\bar{e}$ for some $H \le H^*$. As in the previous case, if the conditions for values of p and q such that $\tau = 0$ are satisfied, (34) holds at $H = 0$ and (33) holds for H weakly greater than $\underline{F}$ for $\tau = 0$. By contrast, these conditions do not assure that (32) is true. Because $(1-\tau)w_s + T = (1-\tau)w_s + \left(\tau - \frac{1}{2}\tau^2\right)\overline{w} > \frac{1}{2}\left(w_s + \frac{3}{4}\overline{w}\right) > \frac{7}{8}\overline{w}$ from Assumption 2 and $\overline{w}$ increases with H from (3), (4), and (5), (32) holds if $\lambda\frac{7}{8}\overline{w} \ge \bar{e}$ holds for the minimum H satisfying $(1-\tau)w_s = (1-\tau)w_u + (1+r)\bar{e}$, which is denoted by $H^\dagger$. Because $(1-\tau)(w_s - w_u)$ decreases with H at $H = H^\dagger$, $H^\dagger$ decreases with $\bar{e}$. Hence, (32) is true if $\bar{e}$ is sufficiently small or λ is sufficiently large. The result on the utility is true because within-class disparities in transfers diminish over time from (30) and (31).

(ii) The result for $\underline{F}$ is straightforward from the proof of (i) and Corollary 1 (ii), and the result for H^* is straightforward from the definition of H^* and Proposition 1. ■

Proof of Proposition 2. (i) The result on H is straightforward from Lemma 2 (i) when values of p and q are time-invariant, which is the case when $p = q = 0$ in the initial period from footnote 27 attached to the lemma. For other values of p_0 and q_0, as mentioned in the footnote, the possibility that $H \le \underline{F}$ increases or decreases temporarily cannot be ruled out. Then, from Figures 1 and 2 based on Proposition A2, values of p and q may change. When the shift from $p = 0, q = 1$ to $p = q = 0$ occurs, H remains smaller than $\underline{F}$ for $p = 0, q = 1$, because H is time-invariant after the shift to $p = q = 0$ from footnote 27. The same is true for the shift from $p = q = 1$ to $p = q = 0$. When the shift from $p = q = 1$ to $p = 0, q = 1$ occurs, H remains smaller than $\underline{F}$ for $p = q = 1$, because $\underline{F}$ for $p = 0, q = 1$ is greater than $\underline{F}$ for $p = q = 1$ from Lemma 2 (ii). By contrast, when $p_0 = 0, q_0 = 1$ and F_0 is smaller than $\underline{F}$ for $p = 0, q = 1$ and greater than $\underline{F}$ for $p = q = 1$, the possibility that the shift from $p = 0, q = 1$ to $p = q = 1$ occurs and H converges to H^* cannot be ruled out. (The shift from or to $p = 1, q = 0$ is not considered, because $p = 1, q = 0$ is realized only for large H when $\beta > \gamma$ and $\frac{A_s}{A_u}$ is relatively high,) The results on Y, w_s, w_u, and the earnings disparity are straightforward from (3)−(5). The welfare disparity is greater than the long-run level for $F_0 > \underline{F}$, which is 0 from (ii). Thus, the disparity is large in the sense that it is greater than the one for $F_0 > \underline{F}$ when H is sufficiently large.

(ii) The result on H is from Figures 1 and 2 based on Proposition A2, Lemma 2 (i), and Assumption 3. The result on the long-run welfare is from Lemma 2 (i), and the results on w_s, w_u, and the earnings disparity are from (4) and (5). The result on Y is from $\frac{dY}{dH} > 0$, which is proved as follows. $\frac{dY}{dH} > 0 \Leftrightarrow w_s > w_u$ because, from (3),

$$\begin{aligned}\frac{dY}{dH} &= \left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\} \\ &= w_s-w_u \text{ (from (4) and (5))}, \qquad (52)\end{aligned}$$

where $w_s > w_u$ from $H < \overline{H}$.

(iii) The results are from Lemma 2 (ii) and Proposition A2 (Figures 1 and 2). ■

Proof of Proposition 3. (i) and (ii) The results on identities and τ are from Figures 1 and 2 that are based on Proposition A2, Proposition 1, and Assumption 3. Convergence to H^* is from Proposition 2 (ii), except the stop of the convergence when $\Delta\widetilde{S_N}$ is relatively low, which is explained in footnote 38 attached to the proposition. (iii) Existence of multiple equilibria are from Figures 1 and 2 (Proposition A2). Assumption 3 implies the persistent effect of the initial equilibrium on the subsequent dynamics and long-run outcomes. ■

Proof of Proposition 4. First, it is proved that shapes of the dividing lines for each combination of p and q are similar to those under the original setting. From (35) and (36), the utility of an individual in class C ($C = S, U$) identifying with social group G ($G = C, N$), u_{CG}, which is given by (9)−(12) under the original setting, is expressed as:

$$u_{SN} = (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_s-\beta\left[2\omega_q(1-H)+(1-\tau)(w_s-\overline{w})\right]+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right], \quad (53)$$

$$u_{SS} = (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_s+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_s\right], \quad (54)$$

$$u_{UN} = (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_u-\beta\left[2\omega_qH+(1-\tau)(\overline{w}-w_u)\right]+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right], \quad (55)$$

$$u_{UU} = (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_u+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_u\right]. \quad (56)$$

From these equations, the condition for $p = q = 0$ equals ($\tau = 0$ from Proposition 1 (i)(a))

$$\begin{aligned} p = q = 0 \text{ iff } &\gamma\delta\Delta\widetilde{S_N} \le \min\{[(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H), [(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H\} \\ &\Leftrightarrow \gamma\delta\Delta\widetilde{S_N} \le [(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H \quad \text{for } H \le \min\left\{H^\flat_{00}, \overline{H}\right\} \qquad (57) \\ &\text{and } \gamma\delta\Delta\widetilde{S_N} \le [(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H) \quad \text{for } H \in \left[H^\flat_{00}, \overline{H}\right] \text{when } \overline{H} > H^\flat_{00}, \qquad (58) \\ &\text{where } H^\flat_{00} \text{ is } H \text{ satisfying } H = \frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{w_s-w_u}{(w_s-w_u)+2\omega_q}\right\}. \end{aligned}$$

$H^\flat_{00}$ is unique because $\frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{w_s-w_u}{(w_s-w_u)+2\omega_q}\right\}$ equals $\frac{\beta+\gamma}{2\beta}$ at $H = 0$ and decreases with H. Note that $\overline{H} \le (>) H^\flat_{00} \Leftrightarrow \overline{H} \le (>) \frac{1}{2}$ and $\frac{\beta+\gamma}{2\beta}$ of the original model is replaced by $H^\flat_{00}$.

As with the corresponding equations of the original model, the RHS of (57) when $\beta>\gamma$ increases (decreases) with H for small (large) H and the RHS of (58) decreases with H. The former can be shown as follows. The proof of (iii) of Proposition A1 shows that there exists $H^{\sharp}\in(0,\overline{H})$ such that $\frac{d[(w_s-w_u)H]}{dH}>(<)0$ for $H<(>)H^{\sharp}$. Further, $\frac{d^2[(w_s-w_u)H]}{dH^2}<0$ for $H\geq H^{\sharp}$ can be easily proven from the equations in the proof. Hence, unless $\beta\omega_q$ is very large, there exists $H^{\sharp\sharp}\in(H^{\sharp},\overline{H})$ such that the RHS of (57) when $\beta>\gamma$ increases (decreases) with H for $H<(>)H^{\sharp\sharp}$.

The condition for $p=q=1$ equals

$$p=q=1 \text{ iff } \gamma\delta\Delta\widetilde{S_N}\geq\max\{[(1-\tau)(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H),[(1-\tau)(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H\}$$

$$\Leftrightarrow \gamma\delta\Delta\widetilde{S_N}\geq[(1-\tau)(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H) \text{ for } H\leq\min\left\{H_{11}^{\flat},\overline{H}\right\} \tag{59}$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N}\geq[(1-\tau)(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H \text{ for } H\in\left[H_{11}^{\flat},\overline{H}\right] \text{when } \overline{H}>H_{11}^{\flat}, \tag{60}$$

where $\tau=\frac{2\beta}{1+\gamma}(a(H)-H)$ and $H_{11}^{\flat}$ is $H\in(0,H_{00}^{\flat})$ satisfying $H=\frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{(1-\tau)(w_s-w_u)}{(1-\tau)(w_s-w_u)+2\omega_q}\right\}$.

$H_{11}^{\flat}<H_{00}^{\flat}$ because $\frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{(1-\tau)(w_s-w_u)}{(1-\tau)(w_s-w_u)+2\omega_q}\right\}<\frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{w_s-w_u}{(w_s-w_u)+2\omega_q}\right\}$. $\overline{H}\leq(>)H_{11}^{\flat}\Leftrightarrow\overline{H}\leq(>)\frac{1}{2}$.

As with the corresponding equations of the original model, the RHS of (59) decreases with H because it equals the original equation plus $2\beta\omega_q(1-H)$ and the RHS of (60) increases with H for small H and decreases with H for H close to $\overline{H}$. To be precise, the RHS of (60) increases with H at least for $H\leq H^{\sharp\sharp}\in(H^{\sharp},\overline{H})$ from $\frac{d[(1-\tau)(w_s-w_u)H]}{dH}>0$ at least for $H\leq H^{\sharp}$ (the proof of Proposition A2 (i)(a)) and the above proof on the RHS of (57), and it decreases with H for H close to $\overline{H}$ from $\frac{d[(1-\tau)(w_s-w_u)H]}{dH}=\frac{d[(w_s-w_u)H]}{dH}$ at $H=\overline{H}$ and the proof on the RHS of (57).

The condition for $p=0,q=1$ equals

$$p=0,q=1 \text{ iff } \gamma\delta\Delta\widetilde{S_N}\leq[(1-\tau)(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H) \tag{61}$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N}\geq[(1-\tau)(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H, \text{ where } \tau=\frac{\beta-\gamma}{1+\gamma}(a(H)-H). \tag{62}$$

This occurs only for $H\leq\min\left\{H_{01}^{\flat},\overline{H}\right\}$, where $H_{01}^{\flat}$ is $H\in(H_{11}^{\flat},H_{00}^{\flat})$ satisfying $H=\frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{(1-\tau)(w_s-w_u)}{(1-\tau)(w_s-w_u)+2\omega_q}\right\}$ with $\tau=\frac{\beta-\gamma}{1+\gamma}(a(H)-H)$. From the above proofs on the equations for $p=q=0$ and $p=q=1$, the relations between the RHSs of (61) and (62) and H are similar to those under the original setting.

Finally, the condition for $p=1,q=0$ equals

$$p=1,q=0 \text{ iff } \gamma\delta\Delta\widetilde{S_N}\geq[(1-\tau)(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H) \tag{63}$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N}\leq[(1-\tau)(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H, \text{ where } \tau=\frac{\beta+\gamma}{1+\gamma}(a(H)-H). \tag{64}$$

This happens only for $H\in\left[H_{10}^{\flat},\overline{H}\right]$, where $H_{10}^{\flat}$ is $H\in(H_{11}^{\flat},H_{01}^{\flat})$ satisfying $H=\frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{(1-\tau)(w_s-w_u)}{(1-\tau)(w_s-w_u)+2\omega_q}\right\}$ with $\tau=\frac{\beta+\gamma}{1+\gamma}(a(H)-H)$, thus only when $\overline{H}>H_{10}^{\flat}\Leftrightarrow\overline{H}>\frac{1}{2}$. The relations between the RHSs of these equations and H are similar to those under the original setting from the above proofs on the equations for $p=q=0$ and $p=q=1$.

From these results, the figure that illustrates combinations of H and $\Delta\widetilde{S_N}$ such that each equilibrium exists when $\overline{H}\leq\frac{1}{2}$ is very similar to Figures 1 and 3. (The difference is that the highest value of H such that the dividing line for $p=q=0$ is upward sloping and the corresponding H for the lower dividing line for $p=0,q=1$ are greater.) Shapes of the dividing lines of the figure when $\overline{H}>\frac{1}{2}$ too are similar to those of Figures 2 and 4, although, unlike these figures, the critical value of H at which the equation for the dividing line for $p=q=1$ changes, the one above which $p=1, q=0$ holds, the one below which $p=0, q=1$ holds, and the one at which the equation for the dividing line for $p=q=1$ changes are all different, i.e., $H^{\flat}_{11} < H^{\flat}_{10} < H^{\flat}_{01} < H^{\flat}_{00}$.

A decrease in ω_q decreases the RHSs of (57)−(64). Hence, given H, $\Delta\widetilde{S_N}$ satisfying the equations decrease, i.e., all the dividing lines shift downward on the $(H, \Delta\widetilde{S_N})$ plane. ■

Proof of Proposition 5. (i) From (4) and (5),

$$\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} - \frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} = A_u\alpha(H)^{\frac{\sigma-1}{\sigma}}(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\left\{\begin{array}{c}\frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}} + \frac{1}{\sigma-1}\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\right] \\ -\frac{1}{\sigma-1}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\end{array}\right\}$$

$$= A_u\alpha(H)^{\frac{\sigma-1}{\sigma}}(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\left\{\begin{array}{c}\frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right] \\ +\frac{1}{\sigma-1}\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right]\end{array}\right\} > 0, \quad (65)$$

where the last inequality sign is from $w_s > w_u$.

(ii) The RHSs of the conditions for identity choice, (18)−(23), are expressed as $(1-\tau)(w_s-w_u)$ times an expression that does not depend on A_s and A_u. Consider the case $p = q = 1$, in which $\tau = \frac{2\beta}{1+\gamma}(a(H)-H)$ from Proposition 1 (i). (Other cases can be proved similarly.) From (65) and the proposition $\left(\Gamma \equiv \alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right)$,

$$\begin{aligned}\frac{d[(1-\tau)(w_s-w_u)]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} &= -\frac{2\beta}{1+\gamma}\frac{\alpha(H)^{\frac{\sigma-1}{\sigma}}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}{\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right]^2}A_u(\Gamma)^{\frac{\sigma}{\sigma-1}-1}\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right] \\ &\quad + \left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]A_u\alpha(H)^{\frac{\sigma-1}{\sigma}}(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\left\{\begin{array}{c}\frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right] \\ +\frac{1}{\sigma-1}\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right]\end{array}\right\} \\ &= A_u(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\alpha(H)^{\frac{\sigma-1}{\sigma}}\left\{\begin{array}{c}-\frac{2\beta}{1+\gamma}(1-a(H))\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right] \\ +\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right] \\ +\frac{1}{\sigma-1}\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right]\end{array}\right\}\end{aligned}$$

$$= A_u(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\alpha(H)^{\frac{\sigma-1}{\sigma}}\left\{\begin{array}{c}\left[1-\frac{2\beta}{1+\gamma}(1-H)\right]\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} + \frac{2\beta}{1+\gamma}(1-a(H))(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H} \\ +\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\frac{1}{H}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}} \\ +\frac{1}{\sigma-1}\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right]\end{array}\right\} > 0, \quad (66)$$

where $1-\frac{2\beta}{1+\gamma}(1-H) > 0$ from Assumption 1. Hence, the dividing lines for identity choice shift upward. The result on the identity shift is from Proposition A2 in Appendix A or Figures 3 and 4.

(iii) [Result on τ] From Proposition 1, when $p = q = 0$ is not true, τ equals a constant times $a(H)-H$. The derivatives of $a(H)-H$ with respect to $\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}$ equal

$$\frac{d(a(H)-H)}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}=\frac{1}{\left\{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right\}^2}$$

$$\times\left\{\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right]\alpha(H)^{\frac{\sigma-1}{\sigma}}(1-H)-\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}(1-H)-(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}H\right]\alpha(H)^{\frac{\sigma-1}{\sigma}}\right\}$$

$$=\frac{\alpha(H)^{\frac{\sigma-1}{\sigma}}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}{\left\{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right\}^2}>0. \tag{67}$$

When the identity shift occurs, the result is straightforward from (ii) and Proposition 1 (ii).

[Result on the welfare disparity] The difference in welfare between skilled and unskilled workers equal to the LHSs of (42)−(45) in Appendix B. Given values of p and q, an increase in $\frac{A_s}{A_u}$ raises $(1-\tau)(w_s-w_u)$ of the LHSs from (66) in the proof of (ii) and thus the inter-class welfare disparity. (Increased $\frac{A_s}{A_u}$ does not affect $\overline{e}$ because it is proportional to w_s in the previous period.) The proof of the result when the identity shift occurs is as follows.

Shift from $p=q=1$ to $p=q=0$ (case mentioned in footnote 43): When $H\leq\frac{\beta+\gamma}{2\beta}$, the LHS of (43) is greater than that of (42), thus, together with the fact that w_s-w_u increases with $\frac{A_s}{A_u}$ from (i), the identity shift increases the inter-class welfare disparity. When $H>\frac{\beta+\gamma}{2\beta}$, because the condition for $p=q=1$ is (19) and the one for $p=q=0$ is (21),

$$(w_{s,00}-w_{u,00})(\beta+\gamma)(1-H)\geq(1-\tau_{11})(w_{s,11}-w_{u,11})(\beta-\gamma)H, \tag{68}$$

where subscript 00 is for $p=q=0$ and subscript 11 is for $p=q=1$. The difference between the LHS of (43) and that of (42) equals

$$\begin{aligned}
& (1+\gamma)(w_{s,00}-w_{u,00})-[1-\beta(1-2H)](1-\tau_{11})(w_{s,11}-w_{u,11}) \\
\geq\ & (1+\gamma)(w_{s,00}-w_{u,00})-[1-\beta(1-2H)](w_{s,00}-w_{u,00})\frac{(\beta+\gamma)(1-H)}{(\beta-\gamma)H}\ \text{(from (68))} \\
=\ & \frac{w_{s,00}-w_{u,00}}{(\beta-\gamma)H}\{(1+\gamma)(\beta-\gamma)H-[1-\beta(1-2H)](\beta+\gamma)(1-H)\} \\
>\ & \frac{w_{s,00}-w_{u,00}}{H}\frac{\beta+\gamma}{2\beta}\{(1+\gamma)-(1-\beta+\beta+\gamma)\}=0,\ \left(\text{from } H>\frac{\beta+\gamma}{2\beta}\right)
\end{aligned} \tag{69}$$

where $[1-\beta(1-2H)](1-H)$ decreases with H from $H>\frac{\beta+\gamma}{2\beta}$.

Shift from $p=q=1$ to $p=0$, $q=1$: From Propositions A1 and A2, $p=0$, $q=1$ is realized only for $H\leq\frac{\beta+\gamma}{2\beta}$. Given H, the LHS of (44) is lowest when $\gamma\delta\Delta\widetilde{S_N}=(\beta+\gamma)(1-\tau)(1-H)(w_s-w_u)$ from (22), in which case the LHS equals

$$\begin{aligned}
& [1+\beta H+\gamma(1-H)]\,(1-\tau)(w_s-w_u)-(1+r)\overline{e}-(\beta+\gamma)(1-\tau)(1-H)(w_s-w_u) \\
& =[1-\beta(1-2H)](1-\tau)(w_s-w_u)-(1+r)\overline{e},
\end{aligned} \tag{70}$$

which is greater than the LHS of (42) because τ is lower when $p=0$, $q=1$ from Proposition 1 (ii) and w_s-w_u being increasing in $\frac{A_s}{A_u}$.

Shift from $p=0$, $q=1$ to $p=q=0$: Given H, the LHS of (44) is highest when $\gamma\delta\Delta\widetilde{S_N}=(\beta-\gamma)(1-\tau)H(w_s-w_u)$ from (22), in which case the LHS equals

$$\begin{aligned}
& [1+\beta H+\gamma(1-H)](1-\tau)(w_s-w_u)-(1+r)\overline{e}-(\beta-\gamma)(1-\tau)H(w_s-w_u) \\
& =(1+\gamma)(1-\tau)(w_s-w_u)-(1+r)\overline{e},
\end{aligned} \tag{71}$$

which is smaller than the LHS of (43) from $\tau > 0$ and $w_s - w_u$ being increasing in $\frac{A_s}{A_u}$.

Shift from $p=q=1$ to $p=1$, $q=0$ (case noted in footnote 44): From Propositions A1 and A2, $p=1$, $q=0$ is realized only for $H > \frac{\beta+\gamma}{2\beta}$. The difference between the LHS of (45) and that of (42) is

$$\begin{aligned}
& [1-\beta(1-H)+\gamma H](1-\tau_{10})(w_{s,10}-w_{u,10})+\gamma\delta\Delta\widetilde{S_N}-[1-\beta(1-2H)](1-\tau_{11})(w_{s,11}-w_{u,11}) \\
\geq\ & [1-\beta(1-H)+\gamma H](1-\tau_{10})(w_{s,10}-w_{u,10}) \\
& +(1-\tau_{11})(w_{s,11}-w_{u,11})(\beta-\gamma)H-[1-\beta(1-2H)](1-\tau_{11})(w_{s,11}-w_{u,11}) \text{ (from (19))} \\
=\ & [1-\beta(1-H)+\gamma H][(1-\tau_{10})(w_{s,10}-w_{u,10})-(1-\tau_{11})(w_{s,11}-w_{u,11})] > 0, \qquad (72)
\end{aligned}$$

where the last inequality holds because $\tau_{11} > \tau_{10}$ from Proposition 1 (ii) and $w_{s,10} - w_{u,10} > w_{s,11}-w_{u,11}$ from $w_s - w_u$ being increasing in $\frac{A_s}{A_u}$.

Shift from $p = 1$, $q = 0$ to $p = q = 0$ (case mentioned in footnote 44): The difference in the LHS of (43) and that of (45) equals

$$\begin{aligned}
& (1+\gamma)(w_{s,00}-w_{u,00})-\left\{[1-\beta(1-H)+\gamma H](1-\tau_{10})(w_{s,10}-w_{u,10})+\gamma\delta\Delta\widetilde{S_N}\right\} \\
& \geq (1+\gamma)(w_{s,00}-w_{u,00})-(w_{s,00}-w_{u,00})(\beta+\gamma)(1-H)-[1-\beta(1-H)+\gamma H](1-\tau_{10})(w_{s,10}-w_{u,10}) \text{ (from (21))} \\
& = [1-\beta(1-H)+\gamma H][(w_{s,00}-w_{u,00})-(1-\tau_{10})(w_{s,10}-w_{u,10})] > 0, \qquad (73)
\end{aligned}$$

where the last inequality holds from $w_s - w_u$ being increasing in $\frac{A_s}{A_u}$.

(iv) [Result on the speed of convergence] From (30) and $\overline{e}_{t+1} = sw_{st}$, where s is a constant, in order for the child of an unskilled worker to be financially accessible to education, the following must hold for b_t the worker receives.

$$\begin{aligned}
& \lambda\{(1-\tau_t)w_{ut}+T_t+(1+r)b_t\} \geq sw_{st} \\
\Leftrightarrow\ & \lambda(1+r)b_t \geq sw_{st} - \lambda\left[(1-\tau_t)w_{ut}+T_t\right]. \qquad (74)
\end{aligned}$$

If the RHS of the above equation increases with $\frac{A_s}{A_u}$, increased $\frac{A_s}{A_u}$ slows down the upward mobility of children of unskilled workers. When $p_t = q_t = 0$ and thus $\tau_t = 0$, the condition for the slowed mobility is (henceforth, time subscripts are omitted unless necessary) $s\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)} - \lambda\frac{dw_u}{d\left(\frac{A_s}{A_u}\right)} > 0 \Leftrightarrow \frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} / \frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} > \frac{\lambda}{s}$, where the LHS of the last equation equals, from (4), (5), and (65),

$$\begin{aligned}
\frac{\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}}{\frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}} &= \frac{\frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}+\frac{1}{\sigma-1}\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\right]}{\frac{1}{\sigma-1}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}} \\
&= \frac{1-H}{H}\left[\sigma - 1+\sigma\frac{\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}}}{(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}\right]. \qquad (75)
\end{aligned}$$

From (43) in Appendix B,

$$\begin{aligned}
H_t &< H^* \Leftrightarrow (1+\gamma)(w_{st}-w_{ut}) > (1+r)sw_{st-1} \\
&\Rightarrow (1+\gamma)(w_{st}-w_{ut}) > (1+r)sw_{st} \text{ (since } H_t \geq H_{t-1}) \\
&\Leftrightarrow \frac{1-H}{H}\frac{\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}}}{(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}} > \frac{1+\gamma}{(1+\gamma)-(1+r)s} \text{ (from (4) and (5)).} \qquad (76)
\end{aligned}$$

Thus,

$$\frac{\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}}{\frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}} > \frac{1-H}{H}(\sigma-1)+\sigma\frac{1+\gamma}{(1+\gamma)-(1+r)s}$$

$$> \frac{1+\gamma}{(1+\gamma)-(1+r)s} \text{ (from } \sigma\in(1,3]). \tag{77}$$

Hence, the condition for the slowed upward mobility holds if

$$\frac{1+\gamma}{(1+\gamma)-(1+r)s} \geq \frac{\lambda}{s} \Leftrightarrow (1+\gamma)s-[(1+\gamma)-(1+r)s]\lambda \geq 0, \tag{78}$$

where $s\leq\lambda$ must be true because from (31) and $\tau_t=0$, $b_{t+1}=\lambda\{w_{st}+(1+r)(b_t-\overline{e}_t)\}\geq\overline{e}_{t+1}=sw_{st}$ must hold for children of skilled workers to be accessible to education, which is necessary for H_t to non-decrease over time. The above inequality holds (does not hold) at $s=\lambda$ $(s=0)$ and the LHS of the second inequality increases with s. Hence, if s is sufficiently high, increased $\frac{A_s}{A_u}$ slows down the upward mobility of children of unskilled workers when $p=q=0$.

When $p=q=0$ does not hold and thus $\tau>0$, the condition for the decreased mobility is $\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}/\frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>\frac{\lambda}{s}$ from (74). This condition is less likely to hold than the condition when $p=q=0$ because $\frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>\frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$ from $\frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}=\frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}+\frac{\partial(T-\tau w_u)}{\partial\tau}\frac{d\tau}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}+\frac{\partial(T-\tau w_u)}{\partial\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$, where $\frac{d(T-\tau w_u)}{d\tau}>0$ from the proof of Corollary 1 (ii), $\frac{d\tau}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>0$ from (iii), and $\frac{\partial(T-\tau w_u)}{\partial\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}=A_u\alpha(H)^{\frac{\sigma-1}{\sigma}}(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\left\{\left(1-\frac{1}{2}\tau\frac{\sigma}{\sigma-1}\right)\Gamma+\frac{1}{\sigma-1}\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}-(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{H}{1-H}\right]\right\}>0$ from $\sigma\leq3$ and $\tau<\frac{1}{2}$ (Assumption 2). But the condition does hold when s is sufficiently high because $\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>\frac{d[(1-\tau)w_s+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>\frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$, where $\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>\frac{d[(1-\tau)w_s+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$ from $\frac{d(-\tau w_s+T)}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}=-\frac{d\left[\tau(w_s-w_u)(1-H)+\frac{1}{2}\tau^2(w_sH+w_u(1-H))\right]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}<0$ ($\frac{d(w_s-w_s)}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>0$ from (i)) and $\frac{d[(1-\tau)w_s+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>\frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$ from $\frac{d[(1-\tau)(w_s-w_s)]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>0$, (66) in the proof of (ii).

When the identity shift occurs, the slowed upward mobility is more likely, i.e., it happens with smaller s, because τ falls and thus the change in $(1-\tau)w_u+T$, which could be positive or negative, is smaller than the change when τ is constant from $\frac{d(T-\tau w_u)}{d\tau}>0$.

[Result on H^*] H^* for different values of p and q are solutions to (42)−(45) in Appendix B. An increase in $\frac{A_s}{A_u}$ raises $(1-\tau)(w_s-w_u)$ in the LHSs of the equations from (66) in the proof of (ii). (Increased $\frac{A_s}{A_u}$ does not affect $\overline{e}$, which is proportional to w_s in the previous period.) While the relation between $(1-\tau)(w_s-w_u)$ and H is generally unclear, the fact that the LHSs of these equations equal $+\infty$ at $H=0$ and $-(1+r)\overline{e}<0$ at $H=\overline{H}$ implies that the LHSs decrease with H at $H=H^*$ (or when multiple levels of H^* exist, at the highest H^*, to which H converges). Hence, increased $\frac{A_s}{A_u}$ raises H^*.

[Result on $\underline{F}$] From (30), there exist lineages satisfying $b_t<\overline{e}_t=sw_{st-1}$ and $b_{t+1}\geq\overline{e}_{t+1}=sw_{st}$ only if $\lambda\{(1-\tau_t)w_{ut}+T_t+(1+r)b_t\}\geq sw_{st}$ for some $b_t<sw_{st-1}$, which is the case when

$$\lambda\{(1-\tau_t)w_{ut}+T_t+(1+r)sw_{st-1}\}-sw_{st}>0$$

$$\Leftrightarrow \lambda(1+r)sw_{st-1}>sw_{st}-\lambda[(1-\tau_t)w_{ut}+T_t]. \tag{79}$$

The equation corresponds to (33) when $\overline{e}$ is time-invariant, thus H_t satisfying it with equality is $\underline{F}_t$, though unlike before, it depends on H_{t-1}. Because the relation between the RHS of (79) and H_t is unclear, multiple values of H_t satisfying the equation with equality could exist; by definition, $\underline{F}_t$ is the highest value of such H_t, whose existence can be proved in a similar way as the proof of Lemma 2 (i) for the constant $\overline{e}$ case. Since the RHS equals $(s-\lambda)w_{st} \leq 0$ at $H_t = \overline{H}$ from $\tau_t = 0$ (Proposition 1) and $\lambda \geq s$ (see the proof on the speed of convergence), the RHS decreases with H_t at $H_t = \underline{F}_t$. From the proof on the speed of convergence, the RHS of (79) increases with $\frac{A_{st}}{A_{ut}}$ when s is sufficiently high. Hence, $\underline{F}_t$ increases with $\frac{A_{st}}{A_{ut}}$. ■

Proof of Proposition A1. (i) $p=1$, $q=0$ cannot hold because the two conditions of (23) do not hold simultaneously when $\beta \leq \gamma$. (ii) Since $\beta \leq \gamma$, $\frac{\beta+\gamma}{2\beta} \geq 1$ and thus $H < \frac{\beta+\gamma}{2\beta}$ always holds. Hence, the RHS of the condition for $p=q=1$, (18), and that of the first condition for $p=0$, $q=1$, (22), equal $(\beta+\gamma)(1-\tau)(1-H)(w_s-w_u)$, where τ equals a constant times $a(H)-H$ from Proposition 1. In the following, the proof for the condition for $p=q=1$, where $\tau = \frac{2\beta}{1+\gamma}(a(H)-H)$, is provided.

From (47) in the proof of Proposition 1,

$$a'(H) = \frac{\sigma-1}{\sigma}\frac{a(H)[1-a(H)]}{H(1-H)} > 0. \tag{80}$$

From (4) and (5) ($\Omega \equiv \alpha(A_sH)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}$),

$$\begin{aligned}
\frac{dw_s}{dH} &= \frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}(H)^{-\frac{1}{\sigma}}\begin{pmatrix}\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H} \\ -\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}\frac{1}{H}\end{pmatrix} \\
&= -\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}(H)^{-\frac{1}{\sigma}}\frac{1}{H(1-H)} < 0. \qquad (81) \\
\frac{dw_u}{dH} &= \frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}(1-H)^{-\frac{1}{\sigma}}\begin{pmatrix}\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H} \\ +\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}\frac{1}{1-H}\end{pmatrix} \\
&= \frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}(1-H)^{-\frac{1}{\sigma}}\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H(1-H)} > 0. \qquad (82)
\end{aligned}$$

From the above two equations,

$$\begin{aligned}
\frac{d(w_s-w_u)}{dH} &= -\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}(H)^{-\frac{1}{\sigma}}(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}(1-H)^{-\frac{1}{\sigma}}\frac{1}{H(1-H)} \\
&= -\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}}\frac{a(H)(1-a(H))}{H^2(1-H)^2}.
\end{aligned} \tag{83}$$

Hence, from the above equation and (80),

$$\begin{aligned}
\frac{d[(1-\tau)(1-H)(w_s-w_u)]}{dH} &= -\frac{2\beta}{1+\gamma}\left\{\frac{\sigma-1}{\sigma}\frac{a(H)[1-a(H)]}{H(1-H)} - 1\right\} \\
&\times(1-H)\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\} \\
&- \left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}\frac{1}{H^2(1-H)}\left[\frac{1}{\sigma}a(H)(1-a(H)) + H(a(H)-H)\right]
\end{aligned}$$

$$= -\frac{\Omega^{\frac{\sigma}{\sigma-1}}}{H^2(1-H)}\begin{pmatrix} \frac{2\beta}{1+\gamma}(a(H)-H)\left\{\frac{\sigma-1}{\sigma}a(H)[1-a(H)]-H(1-H)\right\} \\ +\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\left[\frac{1}{\sigma}a(H)(1-a(H))+H(a(H)-H)\right] \end{pmatrix}$$

$$= -\frac{\Omega^{\frac{\sigma}{\sigma-1}}}{H^2(1-H)}\begin{pmatrix} \frac{2\beta}{1+\gamma}(a(H)-H)\left\{\begin{matrix} a(H)[1-a(H)]-H(1-H)+H(a(H)-H) \\ -\left[\frac{1}{\sigma}a(H)(1-a(H))+H(a(H)-H)\right] \end{matrix}\right\} \\ +\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\left[\frac{1}{\sigma}a(H)(1-a(H))+H(a(H)-H)\right] \end{pmatrix}$$

$$= -\frac{\Omega^{\frac{\sigma}{\sigma-1}}}{H^2(1-H)}\left\{\begin{matrix} \frac{2\beta}{1+\gamma}(a(H)-H)^2[1-a(H)] \\ +\left[1-\frac{4\beta}{1+\gamma}(a(H)-H)\right]\left[\frac{1}{\sigma}a(H)(1-a(H))+H(a(H)-H)\right] \end{matrix}\right\}, \tag{84}$$

which is negative since $1-\frac{4\beta}{1+\gamma}(a(H)-H)\geq 0$ from Assumption 2.

Further, from the first and second lines of (84) ($\overline{H}$ is H satisfying $H = a(H)$),

$$\lim_{H\to 0}[(1-\tau)(1-H)(w_s-w_u)] = +\infty, \quad \lim_{H\to\overline{H}}[(1-\tau)(1-H)H(w_s-w_u)] = 0. \tag{85}$$

(iii) The RHS of the condition for $p = q = 0$, (20), and that of the second condition for $p = 0$, $q = 1$, (22), equal $(\beta-\gamma)H(w_s-w_u) < 0$ since $\tau = 0$ in both cases when $\beta \leq \gamma$. From (83) in the proof of (ii) ($\Omega \equiv \alpha(A_sH)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}$),

$$\frac{d[H(w_s-w_u)]}{dH} = \Omega^{\frac{\sigma}{\sigma-1}-1}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}$$
$$-\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}(H)^{-\frac{1}{\sigma}}(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}(1-H)^{-\frac{1}{\sigma}}\frac{1}{1-H}$$
$$= \Omega^{\frac{\sigma}{\sigma-1}-1}\left\{\begin{matrix} \alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H} \\ -\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2} \end{matrix}\right\}, \tag{86}$$

where ($\overline{H}$ is H satisfying $H = a(H)$)

$$\lim_{H\to 0}H(w_s-w_u) = 0, \ \lim_{H\to 0}\frac{d[H(w_s-w_u)]}{dH} = +\infty, \tag{87}$$

$$\lim_{H\to\overline{H}}H(w_s-w_u) = 0, \ \lim_{H\to\overline{H}}\frac{d[H(w_s-w_u)]}{dH} = -\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}}\frac{1}{1-H} < 0. \tag{88}$$

In the following, it is proved that there exists $H^\sharp \in (0, \overline{H})$ such that the first term of inside the big parenthesis of (86) is greater (smaller) than the second term for $H < (>) H^\sharp$. This implies that $\frac{d[H(w_s-w_u)]}{dH} > (<) 0$ for $H < (>) H^\sharp$ and thus, when $\beta \leq \gamma$, the RHS of the condition for $p = q = 0$, (20), and that of the second condition for $p = 0$, $q = 1$ decrease (increase) with H for $H < (>) H^\sharp$.

The derivative of the first term with respect to H equals

$$-\frac{1}{\sigma}\frac{1}{H^2(1-H)^2}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-H)^2+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}H^2\right\} < 0. \tag{89}$$

The derivative of the second term with respect to H equals

$$\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{3H-1}{H^2(1-H)^3}$$

$$+\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{H(1-H)^2}\frac{\sigma-1}{\sigma}\frac{\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}\left(\frac{1}{H}-\frac{1}{1-H}\right)-\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}}{\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^2}$$

$$=\frac{1}{\sigma}\frac{1}{H(1-H)^2}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\left\{\frac{\sigma-1}{\sigma}\frac{-\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{H}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}+\frac{3H-1}{H(1-H)}\right\}, \tag{90}$$

which is negative (positive) for small (large) H.

The difference between the derivative of the first term and that of the second term is proportional to

$$-(1-H)\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-H)^2+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}H^2\right\}$$
$$-\frac{\alpha(1-\alpha)(A_sH)^{\frac{\sigma-1}{\sigma}}[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\left\{\frac{\sigma-1}{\sigma}\frac{-\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}H+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}(1-H)}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}-(1-3H)\right\}. \tag{91}$$

In the following, it is proved that the difference is negative. This fact, together with the fact that the first term inside the big parenthesis of (86) is greater than the second term when $H\to 0$,

$$\lim_{H\to 0}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}=\alpha(A_s)^{\frac{\sigma-1}{\sigma}}\lim_{H\to 0}\left(\frac{1}{H}\right)^{\frac{1}{\sigma}}-(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}$$
$$>\lim_{H\to 0}\left\{\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}=\frac{1}{\sigma}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}\lim_{H\to 0}\left(\frac{1}{H}\right)^{\frac{1}{\sigma}}, \tag{92}$$

implies that the first term is greater than the second term for $H<(>)H^\sharp$.

Let $J\equiv(A_sH)^{\frac{\sigma-1}{\sigma}}$ and $K\equiv[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}$. If $-\alpha JH+(1-\alpha)K(1-H)\geq 0$, (91) is smaller than

$$\frac{1}{\alpha J+(1-\alpha)K}\left\{-(1-H)\left[\alpha J(1-H)^2+(1-\alpha)KH^2\right][\alpha J+(1-\alpha)K]+\alpha(1-\alpha)JK(1-3H)\right\}, \tag{93}$$

which is negative when $1-3H\leq 0$. When $1-3H>0$, if $J\geq K$, (93) is weakly smaller than

$$\frac{1}{\alpha J+(1-\alpha)K}\left\{-(1-H)\left[\alpha J(1-H)^2+(1-\alpha)KH^2\right]K+\alpha(1-\alpha)JK(1-3H)\right\}$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\left\{\alpha JK\left[-(1-H)^3+(1-\alpha)(1-3H)\right]-(1-H)(1-\alpha)K^2H^2\right\}$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\left\{\alpha JK\left[-\alpha(1-3H)-2H^2-(1-H)H^2\right]-(1-H)(1-\alpha)K^2H^2\right\}<0. \tag{94}$$

If $J<K$, (93) equals

$$\frac{1}{\alpha J+(1-\alpha)K}\left(-(1-H)\left\{\left[\alpha J(1-H)^2+(1-\alpha)KH^2\right]\alpha J+[(1-\alpha)KH]^2\right\}-\alpha(1-\alpha)JK\left[(1-H)^3-(1-3H)\right]\right)$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\left(-\alpha(1-\alpha)JK\left[2H^2+(1-H)H^2\right]-(1-H)\left\{\left[\alpha J(1-H)^2+(1-\alpha)KH^2\right]\alpha J+[(1-\alpha)KH]^2\right\}\right)<0. \tag{95}$$

If $-\alpha JH+(1-\alpha)K(1-H)<0$, (91) is smaller than

$$\frac{1}{\alpha J+(1-\alpha)K}\left\{-(1-H)\left[\alpha J(1-H)^2+(1-\alpha)KH^2\right][\alpha J+(1-\alpha)K]-\alpha(1-\alpha)JK\left[\frac{-\alpha JH+(1-\alpha)K(1-H)}{\alpha J+(1-\alpha)K}-(1-3H)\right]\right\}$$
$$<\frac{1}{\alpha J+(1-\alpha)K}\left\{-(1-H)\left[\alpha J(1-H)^2+(1-\alpha)KH^2\right]\frac{(1-\alpha)K}{H}+\alpha(1-\alpha)JK\left[H+(1-3H)\right]\right\}$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\frac{1}{H}\left\{\alpha(1-\alpha)JK\left[H(1-2H)-(1-H)^3\right]-(1-H)[(1-\alpha)HK]^2\right\}$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\frac{1}{H}\left(\alpha(1-\alpha)JK\left\{H(1-2H)-\left[1-3H+2H^2+(1-H)H^2\right]\right\}-(1-H)[(1-\alpha)HK]^2\right)$$
$$=-\frac{1}{\alpha J+(1-\alpha)K}\frac{1}{H}\left\{\alpha(1-\alpha)JKH\left[(2H-1)^2+(1-H)H^2\right]+(1-\alpha)^2H^2(1-H)K^2\right\}<0. \tag{96}$$

(iv) The RHS of the condition for $p = q = 0$ and that of the second condition for $p = 0$, $q = 1$ are the same from (20) and (22) because $\tau = 0$ in both cases when $\beta \leq \gamma$. Hence, the dividing line for $p = q = 0$ and the lower dividing line for $p = 0$, $q = 1$ are the same. Because the RHS of the condition for $p = q = 0$ (and of the second condition for $p = 0$ and $q = 1$) is non-positive from $\beta \leq \gamma$, it is always smaller than the RHS of the condition for $p = q = 1$ and that of the first condition for $p = 0$, $q = 1$. Thus, the dividing line for $p = q = 0$ is located below the dividing line for $p = q = 1$ and the upper dividing line for $p = 0$, $q = 1$ on the $(H, \Delta\widetilde{S_N})$ plane. From (18) and (22), the RHS of the condition for $p = q = 1$ and that of the first condition for $p = 0$, $q = 1$ are the same except the value of τ, which is higher when $p = q = 1$ from Proposition 1. Hence, the RHS of the former condition is smaller than that of the latter condition, that is, the dividing line for $p = q = 1$ is located below the upper dividing line for $p = 0$, $q = 1$. ■

Proof of Proposition A2. (i) If H satisfying $H = a(H)$ is smaller than $\frac{\beta+\gamma}{2\beta}$, the equations for the dividing lines are the same as when $\beta \leq \gamma$. Hence, Proposition A1 applies except the following. (a) Because the RHS of the condition for $p = q = 0$, (20), is positive from $\beta > \gamma$, the dividing line for $p = q = 0$ increases (decreases) with H for $H < (>) H^\sharp$. Since $\beta > \gamma$, the RHS of the second condition for $p = 0$, $q = 1$, (22), equals $(\beta-\gamma)(1-\tau)H(w_s-w_u)$, where $\tau = \frac{\beta-\gamma}{1+\gamma}(a(H)-H)$ from Proposition 1.

From (80) and (83) in the proof of Proposition A1 (ii),

$$\frac{d[(1-\tau)H(w_s-w_u)]}{dH} = \left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1}$$
$$\times\left(\begin{array}{c} -\frac{\beta-\gamma}{1+\gamma}\left\{\frac{\sigma-1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)} - 1\right\} \\ \times H\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\} \\ +\left[1-\frac{\beta-\gamma}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H} - \frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}\end{array}\right). \tag{97}$$

where the expression inside the big parenthesis equals

$$\left[1-\frac{2(\beta-\gamma)}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H} - \frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}$$
$$-\frac{\beta-\gamma}{1+\gamma}a(H)\left\{\frac{\sigma-1}{\sigma}\frac{(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{1-H} - 1\right\}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}$$
$$-\frac{\beta-\gamma}{1+\gamma}(a(H)-H)\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}$$
$$=\left[1-\frac{2(\beta-\gamma)}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H} - \frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}$$
$$-\frac{\beta-\gamma}{1+\gamma}a(H)\left[\frac{1-a(H)}{1-H} - 1\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}$$
$$+\frac{1}{\sigma}\frac{\beta-\gamma}{1+\gamma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{1-H}\left\{-\frac{a(H)-H}{H(1-H)} + \frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\right\}$$

$$= \left[1-\frac{2(\beta-\gamma)}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}-\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}$$
$$+\frac{\beta-\gamma}{1+\gamma}a(H)\frac{a(H)-H}{1-H}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}, \qquad (98)$$

where $1-\frac{2(\beta-\gamma)}{1+\gamma}(a(H)-H)\geq 0$ from Assumption 2.

Hence, the above expression and thus $\frac{d[(1-\tau)H(w_s-w_u)]}{dH}$ are positive at least when $H\leq H^\sharp\in(0,\overline{H})$ in which the first term of (98) is non-negative from the proof of Proposition A1 (iii), and they are negative when H is close to $\overline{H}$.

(b) Because $\tau>0$ when $p=0$, $q=1$ from $\beta>\gamma$, the RHS of the second condition for $p=0$, $q=1$, (22), is smaller than that of the condition for $p=q=0$, (20). The RHS of the condition for $p=q=0$ equals $(\beta-\gamma)(1-\tau)H(w_s-w_u)$, while the RHS of the condition for $p=q=1$, (18), and that of the first condition for $p=0$, $q=1$, (22), equal $(\beta+\gamma)(1-\tau)(1-H)(w_s-w_u)$. Because $(\beta-\gamma)H=(\beta+\gamma)(1-H)$ at $H=\frac{\beta+\gamma}{2\beta}$ and $\tau=0$ when $p=q=0$, for relatively high H, the RHS of the condition for $p=q=0$ could be greater than the RHSs of the other two conditions. By contrast, as Proposition A1, when H is relatively low, the RHS of the condition for $p=q=0$ is smaller than the RHSs of the other two conditions because from (ii) and (iii) of the proposition, the RHS of the former goes to 0 as $H\to 0$, while the RHSs of the other conditions go to $+\infty$ as $H\to 0$.

(ii) When $\overline{H}>\frac{\beta+\gamma}{2\beta}$ and $H\leq\frac{\beta+\gamma}{2\beta}$, similar to (i), the equations for the dividing lines are the same as when $\beta\leq\gamma$ and thus the results of (i) hold except the two points. (a) From (86) in the proof of Proposition A1 (iii),

$$H=H^\sharp\Leftrightarrow\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}-\frac{1}{\sigma}\frac{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}=0, \quad (99)$$

where the LHS of the equation decreases with H from the proof. Further, the derivative of the LHS with respect to $\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}$ equals

$$\frac{1}{H}-\frac{1}{\sigma}\frac{\left\{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right\}-\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}}{\left\{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right\}^2}\frac{(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}{H(1-H)^2}$$
$$=\frac{1}{H}\left[1-\frac{1}{\sigma}\frac{(1-a(H))^2}{(1-H)^2}\right]>0. \qquad (100)$$

Thus, $H^\sharp$ increases with $\frac{A_s}{A_u}$. From (98) in the proof of (i)(a) and (99), $\frac{d[(1-\tau)H(w_s-w_u)]}{dH}>0$ when $H\leq H^\sharp$. Hence, when $\frac{A_s}{A_u}$ is large enough that $H^\sharp\geq\frac{\beta+\gamma}{2\beta}$, the dividing line for $p=q=0$ and the lower dividing line for $p=0$, $q=1$ increase with H for $H\leq\frac{\beta+\gamma}{2\beta}$.

(b) When H is relatively high, the dividing line for $p=q=0$ is definitely located above the other two dividing lines because given τ, the RHS of the condition for $p=q=0$ is the same as the RHS of the condition for $p=q=1$ and that of the first condition for $p=0$, $q=1$ at $H=\frac{\beta+\gamma}{2\beta}$ from (20) and (22) and $\tau=0$ when $p=q=0$. The last result is straightfowrard from (22).

(iii) (a) $p=0$, $q=1$ cannot hold because the two conditions of (22) do not hold simultaneously when $\beta>\gamma$. (b) When $H>\frac{\beta+\gamma}{2\beta}$, the dividing line for $p=q=0$ and the lower dividing line for $p=1$, $q=0$ equal $(\beta+\gamma)(1-\tau)(1-H)(w_s-w_u)$ from (21) and (23). Hence, the proof of Proposition A2 (ii) applies for them. (c) When $H > \frac{\beta+\gamma}{2\beta}$, the dividing line for $p=q=1$ and the upper dividing line for $p=1$, $q=0$ equal $(\beta-\gamma)(1-\tau)H(w_s-w_u)$ from (19) and (23). Hence, the proof of (i)(b) and (ii)(b) applies for them. When $\frac{A_s}{A_u}$ is small enough that $\overline{H}$ is close to $\frac{\beta+\gamma}{2\beta}$, the dividing lines decrease with H from the proof of (i)(a); hence, the term "could" is used in the last sentence of (c).

(d) From (21) and (23), the RHS of the condition for $p = q = 0$ and that of the second condition for $p = 1$, $q = 0$ are the same except the value of τ, which is 0 when $p = q = 0$. Hence, the RHS of the former condition is greater than that of the latter condition, that is, the dividing line for $p = q = 0$ is located above the lower dividinding line for $p = 1$, $q = 0$. From (19) and (23), the RHS of the condition for $p = q = 1$ and that of the first condition for $p = 1$, $q = 0$ are the same except the value of τ, which is higher when $p = q = 1$ from $\beta > \gamma$. Hence, the RHS of the former condition is smaller than that of the latter condition. From (21), (19), and (23), given τ, the RHS of the condition for $p = q = 0$ is smaller than the RHS of the condition for $p = q = 1$ and that of the first condition for $p = 1$, $q = 0$ when $H > \frac{\beta+\gamma}{2\beta}$ and they are equal at $H = \frac{\beta+\gamma}{2\beta}$, while $\tau = 0$ when $p = q = 0$. Hence, when H is relatively low, the dividing line for $p = q = 0$ is located above the dividling line for $p = q = 1$ and the upper dividinding line for $p = 1$, $q = 0$. From (4) and (5), the RHSs of all the conditions go to 0 as $H \to \overline{H}$. Further, because $\overline{H} > \frac{\beta+\gamma}{2\beta} > \frac{1}{2}$, $\left|\lim_{H\to\overline{H}} \frac{d[(1-H)(w_s-w_u)]}{dH}\right| < \left|\lim_{H\to\overline{H}} \frac{d[(1-\tau)H(w_s-w_u)]}{dH}\right|$ from (84) and (88). Hence, when H is relatively high, the dividing line for $p = q = 0$ is located below the other two dividling lines. The last result is straightfowrard from (23). ■