Supplementary Appendix (Not for publication): Proofs of Propositions 4–7

Proof of Proposition 4. From the proof of Lemma 3, the derivatives of the LHS−RHS of (18) [(HL) when $c^*=c_a=1$] with respect to a^*, c_m, and $\frac{k_a}{k_m}$ are:

$$a^*: \frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}, \tag{A1}$$

$$c_m \text{ when } \frac{k_a}{k_m}\neq 1: \frac{N_h}{N_l}\frac{1}{(1-\frac{k_a}{k_m})l_m}\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]>0, \tag{A2}$$

$$\frac{k_a}{k_m} \text{ when } \frac{k_a}{k_m}\neq 1: -\frac{c_m}{(1-\frac{k_a}{k_m})^2 l_m}\frac{N_h}{N_l}\left(\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}-\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right)<0, \tag{A3}$$

$$c_m \text{ when } \frac{k_a}{k_m}=1: \frac{N_h}{N_l}\frac{1}{l_a-l_m}\frac{1-c_m}{c_m}>0, \tag{A4}$$

$$\frac{k_a}{k_m} \text{ when } \frac{k_a}{k_m}=1: -\frac{1}{2}\frac{N_h}{N_l}\frac{l_m}{(l_a-l_m)^2}\frac{(1-c_m)^2}{c_m}<0. \tag{A5}$$

When $c^*=c_a=1 \Leftrightarrow c_l(a^*)=c_h(a^*)\geq 1$, (P) can be expressed as

$$\frac{l_m}{k_m}\frac{r}{c_m}\left[\int_0^{c_l^{-1}(1)}\int_0^{c_l(a)}\frac{dcda}{A_l(a)}+\int_{c_l^{-1}(1)}^{a^*}\int_0^1\frac{dcda}{A_l(a)}\right]+\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\frac{r}{c_m}\int_{a^*}^1\frac{da}{A_h(a)}+r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}=1. \tag{A6}$$

The derivative of the LHS of the above equation with respect to a^* and c_m are

$$a^*: \frac{l_m}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{r}{c_m}\int_{a^*}^1\frac{da}{A_h(a)}>0, \tag{A7}$$

$$c_m: -\frac{1}{c_m}\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right]<0. \tag{A8}$$

Noting that $r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}=r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{c[a\frac{k_a}{k_m}+1-a]k_m}$, the derivative with respect to $\frac{k_a}{k_m}$ equals

$$-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{a}{c[a\frac{k_a}{k_m}+1-a]^2k_m}dcda=-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda. \tag{A9}$$

For *given* $\frac{k_a}{k_m}$, the derivative with respect to k_m equals

$$-\frac{1}{k_m}\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right]-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{1}{c[a\frac{k_a}{k_m}+1-a]k_m^2}dcda=-\frac{1}{k_m}. \tag{A10}$$

Hence, the total derivatives of (HL) and (P) when $\frac{k_a}{k_m}\neq 1$ are summarized as follows.

$$\begin{pmatrix} \frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)} & \frac{N_h}{N_l}\frac{1}{(1-\frac{k_a}{k_m})l_m}\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right] \\ \frac{l_m}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{r}{c_m}\int_{a^*}^1\frac{1}{A_h(a)}da & -\frac{1}{c_m}\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right] \end{pmatrix}\begin{pmatrix} da^* \\ dc_m \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ \frac{1}{k_m} \end{pmatrix} dk_m|_{\frac{k_a}{k_m}\text{ fixed}} + \begin{pmatrix} \frac{c_m}{(1-\frac{k_a}{k_m})^2 l_m}\frac{N_h}{N_l}\left(\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}-\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right) \\ r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda \end{pmatrix} d\frac{k_a}{k_m}. \quad \text{(A11)}$$

By solving for da^* and dc_m, the above equation can be expressed as

$$\begin{pmatrix} da^* \\ dc_m \end{pmatrix} = \frac{1}{\widehat{\Delta}}\begin{pmatrix} -\frac{1}{c_m}\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right] & -\frac{N_h}{N_l}\frac{1}{(1-\frac{k_a}{k_m})l_m}\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right] \\ -\frac{l_m}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{r}{c_m}\int_{a^*}^1\frac{1}{A_h(a)}da & \frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)} \end{pmatrix}$$

$$\times\left[\begin{pmatrix} 0 \\ \frac{1}{k_m} \end{pmatrix} dk_m|_{\frac{k_a}{k_m}\text{fixed}} + \begin{pmatrix} \frac{c_m}{(1-\frac{k_a}{k_m})^2 l_m}\frac{N_h}{N_l}\left(\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}-\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right) \\ r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda \end{pmatrix} d\frac{k_a}{k_m}\right], \quad \text{(A12)}$$

where $\widehat{\Delta}\equiv-\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\frac{1}{c_m}\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right]-\frac{\frac{N_h}{N_l}}{(1-\frac{k_a}{k_m})k_m}\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{r}{c_m}\int_{a^*}^1\frac{da}{A_h(a)}<0$.

[Effects on a^***]** From (A12), when $\frac{k_a}{k_m}\neq1$, $\frac{da^*}{dk_a}|_{\frac{k_a}{k_m}\text{ fixed}}=\frac{k_m}{k_a}\frac{da^*}{dk_m}|_{\frac{k_a}{k_m}\text{ fixed}}>0$ and $\frac{da^*}{d\frac{k_a}{k_m}}>0$ ($\lim_{c_m\to1}\frac{da^*}{dk_m}|_{\frac{k_a}{k_m}\text{ fixed}}=\lim_{c_m\to1}\frac{da^*}{d\frac{k_a}{k_m}}=0$). From (A2)–(A5), the same result holds when $\frac{k_a}{k_m}=1$ as well. Hence, $\frac{da^*}{dk_a}=\frac{da^*}{dk_a}|_{\frac{k_a}{k_m}\text{ fixed}}+\frac{1}{k_m}\frac{da^*}{d\frac{k_a}{k_m}}>0$ ($\lim_{c_m\to1}\frac{da^*}{dk_a}=0$. $\lim_{c_m\to1}\frac{da^*}{dk_m}=0$ too is clear from (A12)).

The effect of a change in k_m on a^* when $\frac{k_a}{k_m}\neq1$ is

$$\frac{da^*}{dk_m}=\frac{da^*}{dk_m}|_{\frac{k_a}{k_m}\text{ fixed}}-\frac{k_a}{k_m^2}\frac{da^*}{d\frac{k_a}{k_m}}$$

$$=-\frac{k_a}{k_m^2}\frac{1}{\widehat{\Delta}}\left(\begin{aligned}&-\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right]\frac{1}{(1-\frac{k_a}{k_m})^2 l_m}\frac{N_h}{N_l}\left(\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}-\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right)\\ &+\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda\right]\frac{N_h}{N_l}\frac{\frac{k_m}{k_a}}{(1-\frac{k_a}{k_m})l_m}\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\end{aligned}\right)$$

$$>-\frac{k_a}{k_m^2}\frac{1}{\widehat{\Delta}}\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right]\frac{\frac{N_h}{N_l}}{(1-\frac{k_a}{k_m})l_m}\left(\begin{aligned}&\frac{-1}{1-\frac{k_a}{k_m}}\left(\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}-\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right)\\ &+\frac{k_m}{k_a}\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\end{aligned}\right)$$

$$=-\frac{1}{\widehat{\Delta}}\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right]\frac{\frac{N_h}{N_l}}{(1-\frac{k_a}{k_m})^2 k_m l_m}\left(\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]-\frac{k_a}{k_m}\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right)>0. \quad \text{(A13)}$$

The last expression is positive, because $\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]-\frac{k_a}{k_m}\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}$ equals 0 at $\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}=0\Leftrightarrow c_m=1$ and it decreases with c_m, that is,

$$-\frac{1}{c_m^2}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\frac{1-\frac{k_a}{k_m}\left(1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right)}{1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}}=-\frac{1}{c_m^3}\frac{(1-\frac{k_a}{k_m})^2l_m}{(l_a-l_m\frac{k_a}{k_m})^2}\frac{c_m(l_a-l_m\frac{k_a}{k_m})-\frac{k_a}{k_m}(1-c_m)l_m}{1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}}$$
$$=-\frac{1}{c_m^3}\frac{(1-\frac{k_a}{k_m})^2l_m}{(l_a-l_m\frac{k_a}{k_m})^2}\frac{c_ml_a-\frac{k_a}{k_m}l_m}{1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}}<0, \quad \text{(A14)}$$

where the last inequality is from $c^*=1 \Leftrightarrow c_m \geq \frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}$.

Using (A13), (A4), and (A5), the effect when $\frac{k_a}{k_m}=1$ satisfies

$$\frac{da^*}{dk_m}>-\frac{1}{k_m}\frac{1}{\widehat{\Delta}}\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right]\frac{N_h}{N_l}\frac{1}{l_a-l_m}\frac{1-c_m}{c_m}\left(-\frac{1}{2}\frac{l_m}{l_a-l_m}\frac{1-c_m}{c_m}+1\right)$$
$$\geq-\frac{1}{k_m}\frac{1}{\widehat{\Delta}}\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right]\frac{N_h}{N_l}\frac{1}{l_a-l_m}\frac{1-c_m}{c_m}\left(-\frac{1}{2}a^*+1\right)>0 \ (\because c_m\geq\tfrac{l_m}{A_l(a^*)}). \quad \text{(A15)}$$

[**Effects on** c_m] From (A12), the effect of a change in k_m when $\frac{k_a}{k_m}\neq 1$ equals ($\frac{dc_m}{dk_m}<0$ when $\frac{k_a}{k_m}=1$ can be proved similarly)

$$\frac{dc_m}{dk_m}=\frac{dc_m}{dk_m}\Big|_{\frac{k_a}{k_m}\text{ fixed}}-\frac{k_a}{k_m^2}\frac{dc_m}{d\frac{k_a}{k_m}}$$
$$=\frac{1}{\widehat{\Delta}}\left\{\begin{array}{l}\frac{k_a}{k_m^2}\frac{l_m}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}r\int_{a^*}^1\frac{da}{A_h(a)}\frac{1}{(1-\frac{k_a}{k_m})^2l_m}\frac{N_h}{N_l}\left(\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}-\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right)\\ \qquad+\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\frac{1}{k_m}\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda\right]\end{array}\right\}$$
$$<\frac{1}{\widehat{\Delta}}\left\{\begin{array}{l}\frac{k_a}{k_m^2}\frac{l_m}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}r\int_{a^*}^1\frac{da}{A_h(a)}\frac{1}{(1-\frac{k_a}{k_m})^2l_m}\frac{N_h}{N_l}\left(\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}-\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right)\\ \qquad+\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\frac{1}{k_m}\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{1}{cA_k(a)}dcda\right]\end{array}\right\}<0. \quad \text{(A16)}$$

The effect of k_a when $\frac{k_a}{k_m}\neq 1$ satisfies

$$
\begin{aligned}
\frac{dc_m}{dk_a} &= \frac{k_m}{k_a}\frac{dc_m}{dk_m}\Big|_{\frac{k_a}{k_m}\ \text{fixed}} + \frac{1}{k_m}\frac{dc_m}{d\frac{k_a}{k_m}} \\
&= \frac{1}{\widehat{\Delta}}\left\{\begin{array}{c} -\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*} r\int_{a^*}^1 \frac{1}{A_h(a)}da \frac{1}{(1-\frac{k_a}{k_m})^2 k_m^2}\frac{N_h}{N_l}\left(\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}} - \ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right) \\ +\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\frac{1}{k_a}\left(1+r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1 \frac{ak_a}{c[A_k(a)]^2}dcda\right) \end{array}\right\} \\
&\le \frac{1}{\widehat{\Delta}}\left\{\begin{array}{c} -\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*} r\int_{a^*}^1 \frac{1}{A_h(a)}da \frac{1}{(k_m-k_a)^2}\frac{N_h}{N_l}\left(\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}} - \ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right) \\ +\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\frac{1}{k_a}\frac{l_m}{k_m}\frac{r}{c_m}\left[\begin{array}{c} \frac{1}{l_m-l_a}\ln\left[\frac{l_a k_m - l_m k_a}{(k_m-k_a)l_m-(l_m-l_a)k_m c_m}\frac{l_m}{A_l(a^*)}\right] \\ +\frac{k_m c_m}{(k_m-k_a)l_m}\ln\left[\frac{(k_m-k_a)l_m-(l_m-l_a)k_m c_m}{(l_a k_m - l_m k_a)c_m}\right] + \frac{A_h(a^*)}{A_l(a^*)}\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right) \end{array}\right] \end{array}\right\} \\
&= \frac{1}{\widehat{\Delta}}\frac{r}{c_m}\left\{\begin{array}{c} -\frac{(h-l_a)l_m}{A_l(a^*)^2}\int_{a^*}^1 \frac{1}{A_h(a)}da \frac{1}{(k_m-k_a)^2}\frac{N_h}{N_l}\left((1-c_m)\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}} - c_m\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right) \\ +\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\frac{1}{k_a}\frac{l_m}{k_m}\left[\begin{array}{c} \frac{1}{l_m-l_a}\ln\left[\frac{l_a k_m - l_m k_a}{(k_m-k_a)l_m-(l_m-l_a)k_m c_m}\frac{l_m}{A_l(a^*)}\right] \\ +\frac{k_m c_m}{(k_m-k_a)l_m}\ln\left[\frac{(k_m-k_a)l_m-(l_m-l_a)k_m c_m}{(l_a k_m - l_m k_a)c_m}\right] + \frac{A_h(a^*)}{A_l(a^*)}\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right) \end{array}\right] \end{array}\right\},
\end{aligned} \tag{A17}
$$

where to derive the inequality,

$$
1 - r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1 \frac{1}{cA_k(a)}dcda = \left(\frac{N_l}{N_h}+\frac{A_h(a^*)}{A_l(a^*)}\right)\frac{l_m}{k_m}\frac{r}{c_m}\int_{a^*}^1\frac{1}{A_h(a)}da, \tag{A18}
$$

which is derived from (A6) and (18), is used.

In the last equation, the second term inside the big curly bracket increases with c_m from (54) in the proof of Lemma 3. The first term too increases with c_m, because the derivative of $(1-c_m)\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}} - c_m\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]$ with respect to c_m equals

$$
\begin{aligned}
&-\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}} - \ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right] + \left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]^{-1}\frac{1}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}} \\
&= \left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]^{-1}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\left[\frac{1}{c_m}-\left(1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right)\right] - \ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right] \\
&\quad = -\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]^{-1}\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\frac{l_m-l_a}{l_a-l_m\frac{k_a}{k_m}} - \ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right],
\end{aligned} \tag{A19}
$$

which is negative when $l_m \ge l_a$ since $l_m \ge l_a$ implies $\frac{k_a}{k_m} < 1$ when $\frac{k_a}{k_m} < \frac{l_a}{l_m}$. The expression is negative when $l_m < l_a$ as well, because it equals 0 at $c_m = 1$ and its derivative with respect to c_m equals

$$
\begin{aligned}
&-\frac{1}{c_m^2}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]^{-2}\left\{\frac{l_a-l_m}{l_a-l_m\frac{k_a}{k_m}} - \left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right\} \\
&= -\frac{1}{c_m^2}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]^{-2}\left\{\frac{c_m(l_a-l_m)-\left[c_m(l_a-l_m\frac{k_a}{k_m})+(1-c_m)(1-\frac{k_a}{k_m})l_m\right]}{c_m(l_a-l_m\frac{k_a}{k_m})}\right\} \\
&= \frac{1}{c_m^3}\left(\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right)^2\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]^{-2} > 0.
\end{aligned} \tag{A20}
$$

Thus, using $c_m \geq \frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}$, the expression inside the big curly bracket of (A17) satisfies

$$
\begin{aligned}
& -\frac{(h-l_a)l_m}{A_l(a^*)^2}\int_{a^*}^{1}\frac{da}{A_h(a)}\frac{1}{(k_m-k_a)^2}\frac{N_h}{N_l}\left((1-c_m)\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}-c_m\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right) \\
& +\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\frac{1}{k_a}\frac{l_m}{k_m}\left[\begin{array}{c}\frac{1}{l_m-l_a}\ln\left[\frac{l_ak_m-l_mk_a}{(k_m-k_a)l_m-(l_m-l_a)k_mc_m}\frac{l_m}{A_l(a^*)}\right] \\ +\frac{k_mc_m}{(k_m-k_a)l_m}\ln\left[\frac{(k_m-k_a)l_m-(l_m-l_a)k_mc_m}{(l_ak_m-l_mk_a)c_m}\right]+\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\end{array}\right] \\
& \geq -\frac{(h-l_a)l_m}{A_l(a^*)^2}\int_{a^*}^{1}\frac{da}{A_h(a)}\frac{1}{(k_m-k_a)^2}\frac{N_h}{N_l}\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}\left[\frac{a^*(k_m-k_a)}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right] \\
& +\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\frac{1}{k_a}\frac{l_m}{k_m}\left[\frac{A_k(a^*)}{A_l(a^*)}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\right] \\
& =\frac{l_m}{k_m}\frac{1}{A_l(a^*)}\left\{\begin{array}{c}\frac{(h-l_a)l_m}{A_l(a^*)^2}\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\frac{1}{(k_m-k_a)^2}\frac{N_h}{N_l}A_k(a^*)\left[\begin{array}{c}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\left(\frac{k_m}{A_k(a^*)}\right)^{-1}-1 \\ -\left(\left(\frac{k_m}{A_k(a^*)}\right)^{-1}-1\right)-\left(\frac{k_m}{A_k(a^*)}-1\right)\end{array}\right] \\ +\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\frac{1}{k_a}\left[\frac{A_k(a^*)}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{A_h(a^*)}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\right]\end{array}\right\} \\
& =\frac{l_m}{k_m}\frac{1}{A_l(a^*)}\left\{\begin{array}{c}\frac{(h-l_a)l_m}{A_l(a^*)^2}\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\frac{1}{(k_m-k_a)^2}\frac{N_h}{N_l}A_k(a^*)\left[\ln\left(\frac{k_m}{A_k(a^*)}\right)+\left(\frac{k_m}{A_k(a^*)}\right)^{-1}-1\right] \\ -\frac{(h-l_a)l_m}{A_l(a^*)^2}\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\frac{a^{*2}}{k_m}\frac{N_h}{N_l}+\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\frac{1}{k_a}\left[\frac{A_k(a^*)}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{A_h(a^*)}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\right]\end{array}\right\}
\end{aligned}
$$

$$
=\frac{l_m}{k_m}\frac{1}{A_l(a^*)}\left\{\begin{array}{c}\frac{(h-l_a)l_m}{A_l(a^*)^2}\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\frac{1}{(k_m-k_a)^2}\frac{N_h}{N_l}A_k(a^*)\left[\ln\left(\frac{k_m}{A_k(a^*)}\right)+\left(\frac{k_m}{A_k(a^*)}\right)^{-1}-1\right] \\ +[A_h(a^*)A_l(a^*)k_m-a^{*2}(h-l_a)l_mk_a]\frac{1}{k_ak_mA_l(a^*)^2}\frac{N_h}{N_l}\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right) \\ +\frac{1}{k_a}\left[\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\frac{A_k(a^*)}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\right]\end{array}\right\}. \tag{A21}
$$

In the above equation, $A_h(a^*)A_l(a^*)k_m - a^{*2}(h-l_a)l_mk_a > 0$ because, from $l_ak_m - l_mk_a > 0$,

$$
\begin{aligned}
A_h(a^*)A_l(a^*)k_m-a^{*2}(h-l_a)l_mk_a &> k_m\left[A_h(a^*)A_l(a^*)-a^{*2}(h-l_a)l_a\right] \\
&> k_mA_l(a^*)[A_h(a^*)-a^*(h-l_a)]=k_m[A_l(a^*)]^2>0.
\end{aligned} \tag{A22}
$$

Hence, (A21) is positive and thus $\frac{dc_m}{dk_a} < 0$.

[Effect on $c_l(a)$]: Since $c_l(a) = \frac{k_m}{l_m}\frac{A_l(a)}{A_k(a)}c_m$, $c_l(a)$ decreases with k_a from $\frac{dc_m}{dk_a} < 0$.

Since $a \leq c_l^{-1}(1) = \frac{l_m(1-c_m)}{(k_m-k_a)\frac{l_m}{k_m}-(l_m-l_a)c_m}$ (from equation 49),

$$
\begin{aligned}
\frac{dc_l(a)}{dk_m} &= \frac{1}{l_m}\frac{A_l(a)}{A_k(a)}\left(k_m\frac{dc_m}{dk_m}+\frac{ak_a}{A_k(a)}c_m\right)\leq\frac{1}{l_m}\frac{A_l(a)}{A_k(a)}\left(k_m\frac{dc_m}{dk_m}+\frac{c_l^{-1}(1)k_a}{A_k(c_l^{-1}(1))}c_m\right) \\
&= \frac{k_m}{l_m}\frac{A_l(a)}{A_k(a)}\frac{1}{l_ak_m-l_mk_a}\left[(l_ak_m-l_mk_a)\frac{dc_m}{dk_m}+\frac{k_a}{k_m}l_m(1-c_m)\right],
\end{aligned} \tag{A23}
$$

where $\frac{c_l^{-1}(1)k_a}{A_k(c_l^{-1}(1))}c_m = \frac{k_al_m(1-c_m)}{l_ak_m-l_mk_a}$ (from equation 50) is used to derive the last equality.

The expression inside the square bracket of (A23) satisfies

$$
\begin{aligned}
&(l_a k_m - l_m k_a)\frac{dc_m}{dk_m} + \frac{k_a}{k_m} l_m (1-c_m) \\
&= \frac{(l_a k_m - l_m k_a)}{\widehat{\Delta}} \left\{ \begin{array}{c} \frac{k_a}{k_m} \frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*} r \int_{a^*}^1 \frac{da}{A_h(a)} \frac{1}{(k_m-k_a)^2} \frac{N_h}{N_l} \left(\frac{1-c_m}{c_m} \frac{(1-\frac{k_a}{k_m})l_m}{l_a - l_m \frac{k_a}{k_m}} - \ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right) \\ + \left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)} + \frac{1}{A_h(a^*)}\right) \frac{1}{k_m} \left[1 - r\int_0^{c_l^{-1}(1)} \int_{c_l(a)}^1 \frac{a k_a}{c[A_k(a)]^2} dc da \right] \end{array} \right\} \\
&+ \frac{1}{\widehat{\Delta}} \frac{k_a}{k_m} l_m (1-c_m) \left\{ -\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)} + \frac{1}{A_h(a^*)}\right) \frac{1}{c_m} \left[1 - r\int_0^{c_l^{-1}(1)} \int_{c_l(a)}^1 \frac{dc da}{c A_k(a)}\right] - \frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*} \frac{r}{c_m} \int_{a^*}^1 \frac{da}{A_h(a)} \frac{N_h/N_l}{k_m-k_a} \ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right] \right\} \\
&< \frac{1}{\widehat{\Delta}} \frac{r}{c_m} \frac{1}{k_m} \left\{ \begin{array}{c} \int_{a^*}^1 \frac{da}{A_h(a)} \left[\begin{array}{c} (l_a k_m - l_m k_a) \frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*} \frac{k_a c_m}{(k_m-k_a)^2} \frac{N_h}{N_l} \left(\frac{1-c_m}{c_m} \frac{(1-\frac{k_a}{k_m})l_m}{l_a - l_m \frac{k_a}{k_m}} - \ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right) \\ - k_a l_m (1-c_m) \frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*} \frac{1}{k_m - k_a} \frac{N_h}{N_l} \ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right] \end{array} \right] \\ + \left(l_a k_m - \frac{l_m k_a}{c_m}\right) \left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)} + \frac{1}{A_h(a^*)}\right) \frac{l_m}{k_m} \left(\begin{array}{c} \frac{1}{l_m - l_a} \ln\left[\frac{l_a k_m - l_m k_a}{(k_m-k_a)l_m - (l_m-l_a)k_m c_m} \frac{l_m}{A_l(a^*)}\right] \\ + \frac{k_m c_m}{(k_m-k_a) l_m} \ln\left[\frac{(k_m-k_a)l_m - (l_m-l_a)k_m c_m}{(l_a k_m - l_m k_a) c_m}\right] + \frac{A_h(a^*)}{A_l(a^*)} \int_{a^*}^1 \frac{da}{A_h(a)} \end{array} \right) \end{array} \right\} \\
&= \frac{1}{\widehat{\Delta}} \frac{r}{c_m} \frac{1}{k_m} \left\{ \begin{array}{c} \int_{a^*}^1 \frac{da}{A_h(a)} \left[(l_a k_m - l_m k_a) \frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*} \frac{k_a}{(k_m-k_a)^2} \frac{N_h}{N_l} c_m \left(\frac{1-c_m}{c_m} \frac{(1-\frac{k_a}{k_m})l_m}{l_a - l_m \frac{k_a}{k_m}} - \left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right] \ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right) \right] \\ + \left(l_a k_m - \frac{l_m k_a}{c_m}\right) \left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)} + \frac{1}{A_h(a^*)}\right) \frac{l_m}{k_m} \left(\begin{array}{c} \frac{1}{l_m - l_a} \ln\left[\frac{l_a k_m - l_m k_a}{(k_m-k_a)l_m - (l_m-l_a)k_m c_m} \frac{l_m}{A_l(a^*)}\right] \\ + \frac{k_m c_m}{(k_m-k_a) l_m} \ln\left[\frac{(k_m-k_a)l_m - (l_m-l_a)k_m c_m}{(l_a k_m - l_m k_a) c_m}\right] + \frac{A_h(a^*)}{A_l(a^*)} \int_{a^*}^1 \frac{da}{A_h(a)} \end{array} \right) \end{array} \right\},
\end{aligned}
\tag{A24}
$$

where (A6) and (18) are used to derive the last inequality.

The first term inside the big curly bracket of the above equation increases with c_m, since the derivative of $c_m\left(\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}} - \left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right)$ with respect to c_m equals $\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}} - \frac{l_a-l_m}{l_a-l_m\frac{k_a}{k_m}}\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right] > 0$, where the positive derivative is because the expression equals 0 at $\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}} = 0$ and its derivative with respect to $\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}$ equals $\frac{1}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]^{-1}$ ($\gtreqless 0$ when $\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}} \gtreqless 0$). The second term too increases with c_m from (A2) and the fact that $l_a k_m - \frac{l_m k_a}{c_m} \geq l_a k_m - l_m k_a \left(\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}\right)^{-1} = \frac{(1-a^*)k_m}{A_k(a^*)}(l_a k_m - l_m k_a) > 0$.

Hence, from $c_m \geq \frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}$, the expression inside the big curly bracket satisfies

$$
\begin{aligned}
&\int_{a^*}^{1}\frac{da}{A_h(a)}\left\{(l_a k_m - l_m k_a)\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{k_a c_m}{(k_m-k_a)^2}\frac{N_h}{N_l}\left(\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}-\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right)\right\}\\
&+\left(l_a k_m-\frac{l_m k_a}{c_m}\right)\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\frac{l_m}{k_m}\left(\begin{array}{c}\frac{1}{l_m-l_a}\ln\left[\frac{l_a k_m-l_m k_a}{(k_m-k_a)l_m-(l_m-l_a)k_m c_m}\frac{l_m}{A_l(a^*)}\right]\\+\frac{k_m c_m}{(k_m-k_a)l_m}\ln\left[\frac{(k_m-k_a)l_m-(l_m-l_a)k_m c_m}{(l_a k_m-l_m k_a)c_m}\right]+\frac{A_h(a^*)}{A_l(a^*)}\int_{a^*}^{1}\frac{da}{A_h(a)}\end{array}\right)\\
&\geq (l_a k_m-l_m k_a)\frac{l_m}{k_m}\frac{1}{A_l(a^*)}\left\{\begin{array}{l}\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{k_a A_k(a^*)}{(k_m-k_a)^2}\frac{N_h}{N_l}\left[\frac{k_m}{A_k(a^*)}-1-\frac{k_m}{A_k(a^*)}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]\\+\frac{(1-a^*)k_m}{A_k(a^*)}\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\left[\frac{A_k(a^*)}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{A_h(a^*)}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\right]\end{array}\right\}\\
&=(l_a k_m-l_m k_a)\frac{l_m}{k_m}\frac{1}{A_l(a^*)}\left\{\begin{array}{r}\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{k_a A_k(a^*)}{(k_m-k_a)^2}\frac{N_h}{N_l}\left[-\left(\frac{k_m}{A_k(a^*)}-1\right)^2+\frac{k_m}{A_k(a^*)}\left(\frac{k_m}{A_k(a^*)}-1-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\right]\\+\frac{(1-a^*)k_m}{A_k(a^*)}\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\left[\frac{A_k(a^*)}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{A_h(a^*)}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\right]\end{array}\right\}\\
&=(l_a k_m-l_m k_a)\frac{l_m}{k_m}\frac{1}{A_l(a^*)}\left\{\begin{array}{c}\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{k_a k_m}{(k_m-k_a)^2}\frac{N_h}{N_l}\left[\frac{k_m}{A_k(a^*)}-1-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]\\+\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\frac{1}{A_k(a^*)}\left[-\frac{(h-l_a)l_m}{A_l(a^*)^2}\frac{N_h}{N_l}a^{*2}k_a+(1-a^*)k_m\frac{A_h(a^*)}{A_l(a^*)}\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\right]\\+\frac{(1-a^*)k_m}{A_k(a^*)}\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\frac{A_k(a^*)}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\end{array}\right\}.
\end{aligned}
\tag{A25}
$$

In the above equation, the first and third terms are clearly positive. To prove that the second term too is positive, the following inequality is used.

$$
-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)>0. \tag{A26}
$$

The inequality holds because the left-hand side is greater than the corresponding expression when $c_m \leq \frac{l_m}{k_m}\frac{k_a}{h}\frac{A_h(a^*)}{A_l(a^*)} \Leftrightarrow c^*, c_a \leq 1$ (since the expression decreases with a^* and a^* is greater when $c_m \leq \frac{l_m}{k_m}\frac{k_a}{h}\frac{A_h(a^*)}{A_l(a^*)}$) , which equals, from (14) of Lemma 1,

$$
\begin{aligned}
-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*) &= \left[\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]^{-1}\frac{N_h}{N_l}\left[-a^*\ln\left(a^*+(1-a^*)\frac{k_m}{k_a}\right)-(1-a^*)\ln\left(a^*\frac{k_a}{k_m}+1-a^*\right)\right]\\
&=\left[\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]^{-1}\frac{N_h}{N_l}\left[a^*\ln\left(\frac{k_a}{k_m}\right)-\ln\left(a^*\frac{k_a}{k_m}+1-a^*\right)\right].
\end{aligned}
\tag{A27}
$$

The expression inside the large bracket of the above equation is negative, because the expression equals 0 at $\frac{k_a}{k_m}=1$ and its derivative with respect to $\frac{k_a}{k_m}$ equals

$$
a^*\left(\frac{1}{\frac{k_a}{k_m}}-\frac{1}{a^*\frac{k_a}{k_m}+1-a^*}\right)=a^*\frac{(1-a^*)(1-\frac{k_a}{k_m})}{\frac{k_a}{k_m}(a^*\frac{k_a}{k_m}+1-a^*)},
$$

which is negative (positive) for $\frac{k_a}{k_m}>(<)1$. Thus, noting that $\ln\left(\frac{A_k(a^*)}{k_a}\right)<(>)0$ for $\frac{k_a}{k_m}>(<)1$, (A27) is positive.

Using this result, the expression inside the large square bracket of the second term of (A25) satisfies,

$$
\begin{aligned}
&-\frac{(h-l_a)l_m}{A_l(a^*)^2}\frac{N_h}{N_l}a^{*2}k_a+(1-a^*)k_m\frac{A_h(a^*)}{A_l(a^*)}\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\\
&>-\frac{(h-l_a)l_m}{A_l(a^*)^2}\frac{N_h}{N_l}a^{*2}k_a+(1-a^*)k_m\frac{A_h(a^*)}{A_l(a^*)}\left\{\frac{N_h}{N_l}+\frac{1}{1-a^*}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)+\frac{N_h}{N_l}a^*\right]\right\}\\
&>-\frac{(h-l_a)l_m}{A_l(a^*)^2}\frac{N_h}{N_l}a^{*2}k_a+(1-a^*)k_m\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{1-a^*}\frac{N_h}{N_l}\\
&=\frac{N_h}{N_l}\frac{1}{A_l(a^*)^2}\left[A_h(a^*)A_l(a^*)k_m-a^{*2}(h-l_a)l_mk_a\right], \qquad \text{(A28)}
\end{aligned}
$$

which is positive when $l_ak_m-l_mk_a>0$, as shown in the proof of $\frac{dc_m}{dk_a}<0$.

Therefore, when $\frac{k_a}{k_m}\neq 1$, $\frac{dc_l(a)}{dk_m}<0$ holds from (A24) and (A23) (can be proved similarly when $\frac{k_a}{k_m}=1$ as well).

[Effects on earnings] Since $w_l=\frac{l_m}{k_m}\frac{1}{c_m}r$, $\frac{dw_l}{dk_a}>0$ is from $\frac{dc_m}{dk_a}<0$. As for the effect of k_m on $w_h=\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\frac{r}{c_m}$ when $\frac{k_a}{k_m}\neq 1$,

$$
\frac{d\left(\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{k_mc_m}\right)}{dk_m}=\frac{1}{k_mc_m}\left[\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{da^*}{dk_m}-\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{k_mc_m}\left(\frac{dc_m}{dk_m}k_m+c_m\right)\right], \qquad \text{(A29)}
$$

where, using $c_m\widehat{\Delta}\equiv-\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right]-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{r\int_{a^*}^1\frac{da}{A_h(a)}\frac{N_h}{N_l}}{(1-\frac{k_a}{k_m})k_m}\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]$ and (A16),

$$
\begin{aligned}
&\frac{dc_m}{dk_m}k_m+c_m\\
&=\frac{1}{\widehat{\Delta}}\left\{\begin{aligned}&\frac{k_a}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}r\int_{a^*}^1\frac{da}{A_h(a)}\frac{1}{(1-\frac{k_a}{k_m})^2k_m}\frac{N_h}{N_l}\left(\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}-\frac{k_m}{k_a}\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right)\\
&\quad+\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)r\left[\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}-\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda\right]\end{aligned}\right\}\\
&<-\frac{1}{\widehat{\Delta}}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}r\int_{a^*}^1\frac{da}{A_h(a)}\frac{1}{(1-\frac{k_a}{k_m})^2k_m}\frac{N_h}{N_l}\left(\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]-\frac{k_a}{k_m}\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right). \qquad \text{(A30)}
\end{aligned}
$$

Substituting the above equation and (A13) into (A29),

$$
\begin{aligned}
\frac{d\left(\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{k_mc_m}\right)}{dk_m}&>-\frac{1}{\widehat{\Delta}}\frac{1}{k_mc_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{1}{(1-\frac{k_a}{k_m})^2k_m}\frac{N_h}{N_l}\left(\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]-\frac{k_a}{k_m}\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right)\\
&\quad\times\left[\frac{1}{l_m}\left(1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right)-\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{k_mc_m}r\int_{a^*}^1\frac{da}{A_h(a)}\right]\\
=-\frac{1}{\widehat{\Delta}}\frac{1}{k_mc_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}&\frac{1}{(1-\frac{k_a}{k_m})^2k_m}\left(\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]-\frac{k_a}{k_m}\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right)\frac{1}{k_mc_m}r\int_{a^*}^1\frac{da}{A_h(a)}>0, \qquad \text{(A31)}
\end{aligned}
$$

where, the last equality is from (A18), and the last inequality is from $\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]-\frac{k_a}{k_m}\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}>0$ (eq. A13). Hence, $\frac{dw_h}{dk_m}>0$ when $\frac{k_a}{k_m}\neq 1$ (can be proved similarly when $\frac{k_a}{k_m}=1$ as well).

As for the effect of k_a on $w_h=\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\frac{r}{c_m}$ when $\frac{k_a}{k_m}\neq 1$,

$$
\begin{aligned}
\frac{d\left(\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{c_m}\right)}{dk_a} &= \frac{1}{c_m}\left(\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{da^*}{dk_a} - \frac{A_h(a^*)}{A_l(a^*)}\frac{1}{c_m}\frac{dc_m}{dk_a}\right) \\
&= \frac{1}{c_m}\left[\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{da^*}{dk_a}\Big|_{\frac{k_a}{k_m}\text{ fixed}} + \frac{1}{k_m}\frac{da^*}{d\frac{k_a}{k_m}}\right) - \frac{A_h(a^*)}{A_l(a^*)}\frac{1}{c_m}\left(\frac{dc_m}{dk_a}\Big|_{\frac{k_a}{k_m}\text{ fixed}} + \frac{1}{k_m}\frac{dc_m}{d\frac{k_a}{k_m}}\right)\right],
\end{aligned}
\tag{A32}
$$

where $\frac{da^*}{dk_a}\big|_{\frac{k_a}{k_m}\text{ fixed}} > 0$ and $\frac{dc_m}{dk_a}\big|_{\frac{k_a}{k_m}\text{ fixed}} < 0$ from Proposition 1. Thus, the sign of $\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{da^*}{d\frac{k_a}{k_m}} - \frac{A_h(a^*)}{A_l(a^*)}\frac{1}{c_m}\frac{dc_m}{d\frac{k_a}{k_m}}$ should be examined. From (A12),

$$
\begin{aligned}
&\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{da^*}{d\frac{k_a}{k_m}} - \frac{A_h(a^*)}{A_l(a^*)}\frac{1}{c_m}\frac{dc_m}{d\frac{k_a}{k_m}} \\
&= \frac{1}{\widehat{\Delta}}\left\{\begin{array}{l}
\frac{c_m}{(1-\frac{k_a}{k_m})^2 l_m}\frac{N_h}{N_l}\left(\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}} - \ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right) \\
\quad\times\left[-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{1}{c_m}\left(1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right) + \frac{A_h(a^*)}{A_l(a^*)}\frac{1}{c_m}\frac{l_m}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{r}{c_m}\int_{a^*}^1\frac{1}{A_h(a)}da\right] \\
-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{ak_m dcda}{c[A_k(a)]^2}\left[\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{(1-\frac{k_a}{k_m})l_m}\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right] + \frac{A_h(a^*)}{A_l(a^*)}\frac{1}{c_m}\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\right]
\end{array}\right\} \\
&= \frac{1}{\widehat{\Delta}}\left\{\begin{array}{l}
-\frac{c_m}{(1-\frac{k_a}{k_m})^2 l_m}\frac{N_h}{N_l}\left(\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}} - \ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\right)\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{1}{c_m}\frac{N_l}{N_h}\frac{l_m}{k_m}\frac{r}{c_m}\int_{a^*}^1\frac{1}{A_h(a)}da \\
-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{ak_m dcda}{c[A_k(a)]^2}\left[\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{(1-\frac{k_a}{k_m})l_m}\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right] + \frac{A_h(a^*)}{A_l(a^*)}\frac{1}{c_m}\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\right]
\end{array}\right\} > 0,
\end{aligned}
\tag{A33}
$$

where (A18) is used to derive the last equality. Hence, $\frac{dw_h}{dk_a} > 0$ when $\frac{k_a}{k_m} \neq 1$ (can be proved similarly when $\frac{k_a}{k_m} = 1$ too).

[Effects on Y] Can be proved in the same manner as the constant $\frac{k_a}{k_m}$ case (Proposition 2 (ii)). ■

Proof of Proposition 5. From the proof of Lemma 2, the derivatives of the LHS−RHS of (16) [(HL) when $c^* < c_a = 1$] with respect to a^*, c_m, and $\frac{k_a}{k_m}$ are:

$$a^* \text{ when } \frac{k_a}{k_m} \neq 1: \frac{k_m}{l_m}\frac{c_m}{k_m - k_a}\left\{\left(\frac{N_h}{N_l} + \frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m - k_a}{A_k(a^*)} - \frac{\partial \frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left[1 + \frac{(k_m - k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)} - c_m\right]}{\frac{(hk_m - l_m k_a)c_m}{A_k(a^*)}}\right]\right\} > 0, \tag{A34}$$

$$c_m \text{ when } \frac{k_a}{k_m} \neq 1: \frac{1}{(h - l_m)c_m}\ln\left[1 + \frac{(h - l_m)hk_m\left[c_m - \frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m - l_m k_a)}\right] \geq 0, \tag{A35}$$

$$\frac{k_a}{k_m} \text{ when } \frac{k_a}{k_m} \neq 1: \frac{k_m}{k_m - k_a}\left\{-\left[\frac{N_h}{N_l} + \frac{A_l(a^*)}{A_h(a^*)}\right]\frac{k_m}{l_m}\frac{a^* c_m}{A_k(a^*)} + \frac{k_m\left(1 - c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m - l_m k_a} + \frac{1}{h - l_m}\ln\left[1 + \frac{(h - l_m)hk_m\left[c_m - \frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m - l_m k_a)}\right]\right\}, \tag{A36}$$

$$a^* \text{ when } \frac{k_a}{k_m} = 1: \frac{c_m}{l_m}\left\{\left(\frac{N_h}{N_l} + \frac{A_l(a^*)}{A_h(a^*)}\right) - \frac{\partial \frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\frac{A_h(a^*)\left(\frac{l_m}{A_l(a^*)} - c_m\right)}{(h - l_m)c_m}\right\} > 0, \tag{A37}$$

$$c_m \text{ when } \frac{k_a}{k_m} = 1: \frac{1}{(h - l_m)c_m}\ln\left[1 + \frac{(h - l_m)h\left[c_m - \frac{l_m}{h}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(h - l_m)}\right] \geq 0, \tag{A38}$$

$$\frac{k_a}{k_m} \text{ when } \frac{k_a}{k_m} = 1: -\left\{\left[\frac{N_h}{N_l} + \frac{A_l(a^*)}{A_h(a^*)}\right]\frac{a^{*2}}{l_m}c_m - \frac{l_m\left(1 - c_m\frac{A_l(a^*)}{A_h(a^*)}\right)\left(\frac{1}{c_m}\frac{A_h(a^*)}{A_l(a^*)} - 1\right)}{(h - l_m)^2}\right\}. \tag{A39}$$

When $c_a = 1 \Leftrightarrow c_h(1) \geq 1$, (P) can be expressed as

$$\frac{l_m}{k_m}\frac{r}{c_m}\int_0^{a^*}\int_0^{c_l(a)}\frac{dcda}{A_l(a)} + \frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\frac{r}{c_m}\left[\int_{a^*}^{c_h^{-1}(1)}\int_0^{c_h(a)}\frac{dcda}{A_h(a)} + \int_{c_h^{-1}(1)}^1\int_0^1\frac{dcda}{A_h(a)}\right]$$
$$+ r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)} + \int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{dcda}{cA_k(a)}\right] = 1. \tag{A40}$$

The derivatives of the LHS of the above equation with respect to a^* and c_m are:

$$a^* \text{ when } \frac{k_a}{k_m} \neq 1: \frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{l_m}{k_m}\frac{r}{c_m}\left[\int_{a^*}^{c_h^{-1}(1)}\int_0^{c_h(a)}\frac{dcda}{A_h(a)} + \int_{c_h^{-1}(1)}^1\int_0^1\frac{dcda}{A_h(a)}\right] = \frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m - k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) > 0, \tag{A41}$$

$$c_m \text{ when } \frac{k_a}{k_m} \neq 1: -\frac{1}{c_m}\left\{1 - r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)} + \int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{dcda}{cA_k(a)}\right]\right\} = -\frac{1}{c_m}\left(1 + \frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m - k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) < 0, \tag{A42}$$

$$a^* \text{ when } \frac{k_a}{k_m} = 1: \frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{ra^*}{k_m} > 0, \tag{A43}$$

$$c_m \text{ when } \frac{k_a}{k_m} = 1: -\frac{1}{c_m}\left(1 + \frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{ra^*}{k_m} < 0, \tag{A44}$$

where, to derive (A41), (16) is used; to derive (A42), (16) and (A40) are used; and to derive (A43) and (A44), (17) is used.

Noting that $r\iint\frac{1}{cA_k(a)}dcda = r\iint\frac{1}{c[a\frac{k_a}{k_m} + 1 - a]k_m}dcda$, the derivative of the LHS with respect to $\frac{k_a}{k_m}$ equals

$$-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda + \int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda\right]. \tag{A45}$$

For *given* $\frac{k_a}{k_m}$, the derivative with respect to k_m equals

$$-\frac{1}{k_m}\left\{1-r\left[\int_0^{a^*}\!\!\int_{c_l(a)}^1 \frac{dcda}{cA_k(a)}+\int_{a^*}^{c_h^{-1}(1)}\!\!\int_{c_h(a)}^1 \frac{dcda}{cA_k(a)}\right]\right\}-r\left[\int_0^{a^*}\!\!\int_{c_l(a)}^1 \frac{dcda}{c[a\frac{k_a}{k_m}+1-a]k_m^2}+\int_{a^*}^{c_h^{-1}(1)}\!\!\int_{c_h(a)}^1 \frac{dcda}{c[a\frac{k_a}{k_m}+1-a]k_m^2}\right]=-\frac{1}{k_m}. \tag{A46}$$

Hence, the total derivatives of (HL) and (P) when $\frac{k_a}{k_m}\neq 1$ are summarized as follows.

$$\begin{pmatrix} \frac{k_m c_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right] & \frac{1}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right] \\ \frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) & -\frac{1}{c_m}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) \end{pmatrix}\begin{pmatrix} da^* \\ dc_m \end{pmatrix}$$

$$=\begin{pmatrix} 0 \\ \frac{1}{k_m}\end{pmatrix}dk_m\Big|_{\frac{k_a}{k_m}\text{ fixed}}+\begin{pmatrix} \frac{k_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right] \\ r\left[\int_0^{a^*}\!\!\int_{c_l(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda+\int_{a^*}^{c_h^{-1}(1)}\!\!\int_{c_h(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda\right]\end{pmatrix}d\frac{k_a}{k_m}. \tag{A47}$$

By solving for da^* and dc_m, the above equation can be expressed as

$$\begin{pmatrix} da^* \\ dc_m\end{pmatrix}=\frac{1}{\Delta}\begin{pmatrix} -\frac{1}{c_m}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) & \frac{-1}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right] \\ -\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) & \frac{k_mc_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\end{pmatrix}$$

$$\times\left[\begin{pmatrix} 0 \\ \frac{1}{k_m}\end{pmatrix}dk_m\Big|_{\frac{k_a}{k_m}\text{ fixed}}+\begin{pmatrix} \frac{k_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right] \\ r\left[\int_0^{a^*}\!\!\int_{c_l(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda+\int_{a^*}^{c_h^{-1}(1)}\!\!\int_{c_h(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda\right]\end{pmatrix}d\frac{k_a}{k_m}\right], \tag{A48}$$

$$\text{where } \Delta\equiv-\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left\{\begin{matrix}\frac{k_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right) \\ +\frac{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\frac{N_h}{N_l}\end{matrix}\right\}<0. \tag{A49}$$

[Effects on a^*] Henceforth, results are presented for the case $\frac{k_a}{k_m}\neq 1$, but they can be proved similarly for the case $\frac{k_a}{k_m}=1$ too.

$$
\begin{aligned}
\frac{da^*}{dk_m} &= -\frac{k_a}{k_m^2}\frac{da^*}{d\frac{k_a}{k_m}} - \frac{1}{\Delta}\frac{1}{k_m}\frac{1}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right] \\
&= -\frac{k_a}{k_m^2}\frac{1}{\Delta}\left(\begin{array}{l}
-\frac{k_m}{k_m-k_a}\frac{1}{c_m}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) \\
\times\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)} - \frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a} - \frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right] \\
+\frac{\frac{k_m}{k_a}}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^{1}\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^{1}\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right]\right\}
\end{array}\right) \\
&> -\frac{k_a}{k_m^2}\frac{1}{\Delta}\left(\begin{array}{l}
-\frac{k_m}{k_m-k_a}\frac{1}{c_m}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) \\
\times\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)} - \frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a} - \frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right] \\
+\frac{\frac{k_m}{k_a}}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)
\end{array}\right) \\
&= -\frac{1}{c_m}\frac{1}{\Delta}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) \\
&\times\frac{1}{k_m-k_a}\left(\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right] - \frac{k_a}{k_m}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)} - \frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}\right]\right),
\end{aligned}
\tag{A50}
$$

where (A42) is used to derive the inequality.

Using (16), the expression inside the large parenthesis of the above equation equals

$$
\begin{aligned}
&\left(1-\frac{k_a}{A_k(a^*)}+\frac{k_a}{A_k(a^*)}\right)\frac{1}{k_m-k_a}\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right] - \frac{1}{k_m-k_a}\frac{k_a}{k_m}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)} - \frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}\right] \\
&= \frac{1-a^*}{A_k(a^*)}\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right] + \frac{N_h}{N_l}\frac{k_m}{l_m}\frac{c_m}{(k_m-k_a)^2}\frac{k_a}{A_k(a^*)}\left[\ln\left(\frac{k_m}{A_k(a^*)}\right) - \frac{a^*(k_m-k_a)}{k_m}\right] + \frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{(k_m-k_a)^2}\frac{k_a}{A_k(a^*)} \\
&\quad\times\left\{-\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right] + \left(\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{k_m-k_a}\right)^{-1}\frac{A_k(a^*)}{k_m}\left[\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a} - \frac{A_l(a^*)}{A_h(a^*)}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}\right]\right\} \\
&= \frac{1-a^*}{A_k(a^*)}\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right] + \frac{N_h}{N_l}\frac{k_m}{l_m}\frac{c_m}{(k_m-k_a)^2}\frac{k_a}{A_k(a^*)}\left[\ln\left(\frac{k_m}{A_k(a^*)}\right) + \frac{A_k(a^*)}{k_m} - 1\right] \\
&\quad+\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{(k_m-k_a)^2}\frac{k_a}{A_k(a^*)}\left\{-\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right] + \frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right\},
\end{aligned}
\tag{A51}
$$

where (16) is used to derive the first equality, and the following equation is used to derive the last equality.

$$
\begin{aligned}
\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a} - \frac{A_l(a^*)}{A_h(a^*)}\frac{k_m}{l_m}\frac{a^*}{A_k(a^*)}c_m &= k_m\frac{1-c_m\frac{A_l(a^*)}{A_h(a^*)}\left[1+\frac{hk_m-l_mk_a}{l_m}\frac{a^*}{A_k(a^*)}\right]}{hk_m-l_mk_a} \\
&= k_m\frac{\frac{k_m}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a} > 0.
\end{aligned}
\tag{A52}
$$

Now it is shown that the last two terms of (A51) are positive. As for the first term, the expression inside the bracket is positive since it equals 0 at $\frac{k_m}{A_k(a^*)}=1$ and its derivative with respect to $\frac{k_m}{A_k(a^*)}$ equals

$$\left(\frac{k_m}{A_k(a^*)}\right)^{-2}\left(\frac{k_m}{A_k(a^*)}-1\right)>(<)0 \quad \text{when} \quad \frac{k_m}{A_k(a^*)}>(<)1 \Leftrightarrow \frac{k_a}{k_m}<(>)1. \tag{A53}$$

The second term is positive because the expression inside the big curly bracket equals 0 at $\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}=0$ and its derivative with respect to $\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}$ equals

$$\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]^{-1}\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\geq(\leq)0 \quad \text{when} \quad \frac{k_a}{k_m}<(>)1.$$

Therefore, $\frac{da^*}{dk_m}>0$.

The result that a^* increases when $\frac{k_a}{k_m}$ non-increases can be proved as follows. When both k_m and k_a increase, $da^*=\frac{\partial a^*}{\partial k_m}dk_m+\frac{\partial a^*}{\partial k_a}dk_a=(\frac{da^*}{dk_m}|_{\frac{k_a}{k_m}\text{ fixed}}+\frac{\partial\frac{k_a}{k_m}}{\partial k_m}\frac{\partial a^*}{\partial\frac{k_a}{k_m}})dk_m+(\frac{da^*}{dk_a}|_{\frac{k_a}{k_m}\text{ fixed}}+\frac{\partial\frac{k_a}{k_m}}{\partial k_a}\frac{\partial a^*}{\partial\frac{k_a}{k_m}})dk_a$, where the term with dk_m is positive from $\frac{da^*}{dk_m}>0$. If $\frac{\partial a^*}{\partial\frac{k_a}{k_m}}\geq 0$, the term with dk_a too is positive and thus $da^*>0$ since $\frac{da^*}{dk_a}|_{\frac{k_a}{k_m}\text{ fixed}}>0$ from Proposition 1. If $\frac{\partial a^*}{\partial\frac{k_a}{k_m}}<0$, since the expression equals $\frac{da^*}{dk_m}|_{\frac{k_a}{k_m}\text{ fixed}}dk_m+\frac{da^*}{dk_a}|_{\frac{k_a}{k_m}\text{ fixed}}dk_a+\frac{\partial a^*}{\partial\frac{k_a}{k_m}}d\frac{k_a}{k_m}$, $da^*>0$ from $d\frac{k_a}{k_m}\leq 0$.

[Effects on c_m]

$$\begin{aligned}
\frac{dc_m}{dk_m}&=\frac{dc_m}{dk_m}|_{\frac{k_a}{k_m}\text{ fixed}}-\frac{k_a}{k_m^2}\frac{dc_m}{d\frac{k_a}{k_m}}\\
&=\frac{1}{\Delta}\left(\begin{array}{l}
\frac{k_a}{k_m^2}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
\times\frac{k_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\
+\frac{1}{k_m}\frac{k_mc_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\\
\times\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda\right]\right\}
\end{array}\right)\\
&<\frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
&\quad\times\left(\begin{array}{l}
\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{\frac{k_a}{k_m}}{k_m-k_a}\left\{\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right\}\\
+\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{c_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]
\end{array}\right),
\end{aligned} \tag{A54}$$

where (A42) and (A52) are used to derive the last equation.

The expression inside the big parenthesis of (A54) equals a positive term plus

$$\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{\frac{k_a}{k_m}}{k_m-k_a}\left\{\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right\}$$

$$+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\frac{c_m}{l_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]$$

$$=\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\left\{\begin{array}{l}\frac{k_a}{k_m}\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]\\ \qquad+\frac{c_m}{k_m-k_a}\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\end{array}\right\}$$

$$=\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{c_m}{(k_m-k_a)^2}\frac{k_m}{l_m}\left\{\begin{array}{l}\frac{k_a}{k_m}\frac{N_h}{N_l}\left[\frac{k_m}{A_k(a^*)}-1-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]\\ +\frac{A_l(a^*)}{A_h(a^*)}\left[-\frac{k_a}{A_k(a^*)}\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}+\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\end{array}\right\},\tag{A55}$$

where (16) is used to derive the last equality. The two terms inside the big curly bracket of the above equation are positive. In particular, the latter term is positive since it equals 0 at $c_m=\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}$ and its derivative with respect to c_m equals

$$-\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}}{\frac{(hk_m-l_mk_a)c_m^2}{A_k(a^*)}}\frac{A_l(a^*)}{A_h(a^*)}\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]^{-1}\left\{1-\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\frac{k_a}{A_k(a^*)}\right\},\tag{A56}$$

which is negative (except zero at $c_m=\frac{k_a}{h}\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}$), because the expression inside the curly bracket of the above equation equals 0 at $c_m=\frac{k_a}{h}\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}$, equals $\frac{(1-a^*)(k_m-k_a)}{A_k(a^*)}$ at $c_m=\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}$, and increases (decreases) with c_m when $\frac{k_a}{k_m}<(>)1$. Therefore, $\frac{dc_m}{dk_m}<0$.

$$\frac{dc_m}{dk_a}=\frac{1}{\Delta}\frac{1}{k_a}\frac{k_mc_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]$$

$$+\frac{1}{\Delta}\frac{1}{k_m}\left\{\begin{array}{l}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{k_m}{k_m-k_a}\\ \qquad\times\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\ +\frac{k_mc_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]r\left(\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_m\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_m\,dc\,da}{c[A_k(a)]^2}\right)\end{array}\right\}$$

$$
=\frac{1}{\Delta}\left\{\begin{array}{l}
-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{1}{k_m-k_a}\\
\quad\times\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\
+\frac{k_mc_m}{k_al_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\left[1+r\left(\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right)\right]
\end{array}\right\}
\tag{A57}
$$

$$
<\frac{1}{\Delta}\left\{\begin{array}{l}
-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{1}{k_m-k_a}\\
\times\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\
\quad+\frac{k_mc_m}{k_al_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\\
\quad\times\left[1-r\left(\int_0^{a^*}\int_{c_l(a)}^1\frac{dc\,da}{cA_k(a)}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{dc\,da}{cA_k(a)}\right)\right]
\end{array}\right\}
$$

$$
=\frac{k_mc_m}{l_m}\frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)
$$

$$
\times\left\{\begin{array}{l}
\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\left[\begin{array}{l}\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{k_m-k_a}\left(\frac{k_m}{A_k(a^*)}\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}-\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right)\\
\qquad+\frac{N_h}{N_l}\frac{1}{k_m-k_a}\left[\ln\left(\frac{k_m}{A_k(a^*)}\right)-\frac{a^*(k_m-k_a)}{A_k(a^*)}\right]\end{array}\right]\\
+\frac{1}{k_a}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)
\end{array}\right\},
\tag{A58}
$$

where (16) and (A42) are used to derive the last equality.

In the above equation, $\frac{k_m}{A_k(a^*)}\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}-\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]$ decreases with c_m because

$$
-\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}}{\frac{(hk_m-l_mk_a)c_m^2}{A_k(a^*)}}\frac{A_l(a^*)}{A_h(a^*)}\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]^{-1}\frac{k_m-k_a}{A_k(a^*)}\left\{\frac{\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}k_m+a^*\right\}<0.
\tag{A59}
$$

Thus, the expression inside the big curly bracket of (A58) is greater than

$$
\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{N_h}{N_l}\right)^2\frac{1}{(k_m-k_a)^2}\left[\ln\left(\frac{k_m}{A_k(a^*)}\right)-\frac{a^*(k_m-k_a)}{A_k(a^*)}\right]+\frac{1}{k_a}\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)
$$

$$
=\frac{N_h}{N_l}\left\{\begin{array}{l}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{(k_m-k_a)^2}\left[\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{A_k(a^*)}{k_m}-1+1-\frac{A_k(a^*)}{k_m}-\frac{a^*(k_m-k_a)}{A_k(a^*)}\right]\\
\qquad+\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\left(\frac{N_l}{N_h}+\frac{A_h(a^*)}{A_l(a^*)}\right)\frac{1}{k_aA_k(a^*)}\end{array}\right\}
$$

$$
=\frac{N_h}{N_l}\left\{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{(k_m-k_a)^2}\left[\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{A_k(a^*)}{k_m}-1-\frac{[a^*(k_m-k_a)]^2}{A_k(a^*)k_m}\right]+\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\left(\frac{N_l}{N_h}+\frac{A_h(a^*)}{A_l(a^*)}\right)\frac{1}{k_aA_k(a^*)}\right\}
$$

$$> \frac{N_h}{N_l}\left\{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{(k_m-k_a)^2}\left[\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{A_k(a^*)}{k_m}-1-\frac{[a^*(k_m-k_a)]^2}{A_k(a^*)k_m}\right]+\frac{1}{1-a^*}\frac{N_h}{N_l}\left(\frac{N_l}{N_h}+\frac{A_h(a^*)}{A_l(a^*)}\right)\frac{1}{k_aA_k(a^*)}\right\},$$

where, to derive the inequality, $-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)>0$, which is shown in the proof of $\frac{dc_l(a)}{dk_m}<0$ of Proposition 4, is used.

The expression inside the large curly bracket of the above equation equals positive terms plus

$$\frac{N_h}{N_l}\frac{1}{A_k(a^*)A_l(a^*)}\left[-\frac{(h-l_a)l_m}{A_l(a^*)}\frac{a^{*2}}{k_m}+\frac{A_h(a^*)}{1-a^*}\frac{1}{k_a}\right]=\frac{N_h}{N_l}\frac{1}{A_k(a^*)A_l(a^*)^2(1-a^*)k_ak_m}\left[A_h(a^*)A_l(a^*)k_m-(1-a^*)a^{*2}(h-l_a)l_mk_a\right]$$
$$> \frac{N_h}{N_l}\frac{1}{A_k(a^*)A_l(a^*)^2(1-a^*)k_ak_m}\left[A_h(a^*)A_l(a^*)k_m-a^{*2}(h-l_a)l_mk_a\right], \qquad \text{(A60)}$$

which is positive when $l_ak_m-l_mk_a\geq 0$, as shown in the proof of $\frac{dc_m}{dk_a}<0$ of Proposition 4. When $l_ak_m-l_mk_a<0$, because $\frac{A_h(a^*)}{A_l(a^*)}\leq\frac{hk_m}{l_mk_a}$ holds from $c_a=\min\left\{\frac{h}{k_a}\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}c_m,1\right\}=1$ and $c_m\leq 1$,

$$A_h(a^*)A_l(a^*)k_m-a^{*2}(h-l_a)l_mk_a\geq\frac{l_mk_a}{h}\left[(A_h(a^*))^2-a^{*2}(h-l_a)h\right]$$
$$>\frac{l_mk_a}{h}A_h(a^*)A_l(a^*)>0. \qquad \text{(A61)}$$

Therefore, $\frac{dc_m}{dk_a}<0$.

[Effects on $c_h(a)$]:

Since $a\leq c_h^{-1}(1)=\frac{l_m\left(\frac{A_h(a^*)}{A_l(a^*)}-c_m\right)}{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}$ (from equation 32),

$$\frac{dc_h(a)}{dk_m}=\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{A_h(a)}{A_k(a)}\left(\frac{dc_m}{dk_m}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_m}+\frac{ak_a}{A_k(a)}\frac{c_m}{k_m}\right)$$
$$\leq\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{A_h(a)}{A_k(a)}\left(\frac{dc_m}{dk_m}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_m}+\frac{c_h^{-1}(1)k_a}{A_k(c_h^{-1}(1))}\frac{c_m}{k_m}\right)$$
$$=\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{A_h(a)}{A_k(a)}\left[\frac{dc_m}{dk_m}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_m}+\frac{\frac{k_a}{k_m}l_m}{hk_m-l_mk_a}\left(\frac{A_h(a^*)}{A_l(a^*)}-c_m\right)\right], \qquad \text{(A62)}$$

where $\frac{c_h^{-1}(1)k_a}{A_k(c_h^{-1}(1))}\frac{c_m}{k_m}=\frac{k_a}{k_m}l_m\frac{\frac{A_h(a^*)}{A_l(a^*)}-c_m}{hk_m-l_mk_a}$ (from equation 33) is used to derive the last equality.

In the above equation, from (A54) and (A51),

$$\frac{dc_m}{dk_m}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_m}$$
$$=\frac{1}{\Delta}\left(\begin{array}{l}\frac{k_a}{k_m^2}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\ \times\frac{k_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\ +\frac{1}{k_m}\frac{k_mc_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\\ \quad\times\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right]\right\}\end{array}\right)$$

$$
\begin{aligned}
&+\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{k_a}{k_m^2}\frac{1}{\Delta}\begin{pmatrix} -\frac{k_m}{k_m-k_a}\frac{1}{c_m}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) \\ \times\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right] \\ +\frac{\frac{k_m}{k_a}}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right]\right\} \end{pmatrix} \\
&=\frac{1}{\Delta}\begin{pmatrix} -\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{k_a}{k_m^2}\frac{k_m}{k_m-k_a}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) \\ \times\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right] \\ +\frac{1}{k_m}\frac{k_mc_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right]\right\} \end{pmatrix} \\
&<\frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\begin{pmatrix} -\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{k_a}{k_m^2}\frac{k_m}{k_m-k_a} \\ \times\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right] \\ +\frac{1}{k_m}\frac{k_mc_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right) \end{pmatrix},
\end{aligned}
\tag{A63}
$$

where, to derive the last equality, (16) is used, and to derive the last inequality, (A42) is used.

From this equation and

$$
\begin{aligned}
\frac{k_a}{k_m}l_m\left(\frac{A_h(a^*)}{A_l(a^*)}-c_m\right) &= -\frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{k_a}{k_m}l_m\left(\frac{A_h(a^*)}{A_l(a^*)}-c_m\right) \\
&\times\begin{Bmatrix} \frac{k_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\partial \frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\Big/(k_m-k_a)\,\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right) \\ +\frac{1}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l} \end{Bmatrix},
\end{aligned}
\tag{A64}
$$

$$
\begin{aligned}
&(hk_m-l_mk_a)\left(\frac{dc_m}{dk_m}-\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_m}\right)+\frac{k_a}{k_m}l_m\left(\frac{A_h(a^*)}{A_l(a^*)}-c_m\right) \\
&<\frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)(hk_m-l_mk_a)\begin{pmatrix} -\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{k_a}{k_m^2}\frac{k_m}{k_m-k_a} \\ \times\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right] \\ +\frac{c_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right) \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{k_a}{k_m}l_m\left(\frac{A_h(a^*)}{A_l(a^*)}-c_m\right)\left\{\frac{k_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\right.\\
&\qquad\left.+\frac{1}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\right\}\\
&=\frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left(\begin{aligned}
&(hk_m-l_mk_a)\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{k_a}{k_m}\frac{1}{k_m-k_a}\\
&\times\left[-\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}+\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}+\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\
&+\left[(hk_m-l_mk_a)\frac{c_m}{l_m}-k_a\left(\frac{A_h(a^*)}{A_l(a^*)}-c_m\right)\right]\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\\
&+k_a\left(\frac{A_h(a^*)}{A_l(a^*)}-c_m\right)\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{l_m}{k_m}\frac{1}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]
\end{aligned}\right)\\
&=\frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left(\begin{aligned}
&\frac{hk_m-l_mk_a}{k_m-k_a}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{k_a}{k_m}\left[\begin{aligned}&-\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}+\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}\\
&+\left(\frac{\frac{A_h(a^*)}{A_l(a^*)}-c_m}{hk_m-l_mk_a}\frac{l_m}{k_m}\frac{(k_m-k_a)k_m}{c_m}+1\right)\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\end{aligned}\right]\\
&+\left[(hk_m-l_mk_a)\frac{c_m}{l_m}-k_a\left(\frac{A_h(a^*)}{A_l(a^*)}-c_m\right)\right]\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)
\end{aligned}\right)\\
&=\frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left(\begin{aligned}
&(hk_m-l_mk_a)\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{k_a}{k_m}\\
&\times\left[\begin{aligned}&\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{c_m}{(k_m-k_a)^2}\left[\frac{k_m}{A_k(a^*)}\ln\left(\frac{k_m}{A_k(a^*)}\right)+1-\frac{k_m}{A_k(a^*)}\right]\\
&+\frac{A_l(a^*)}{A_h(a^*)}\frac{k_m}{l_m}\frac{c_m}{(k_m-k_a)^2}\frac{k_m}{A_k(a^*)}\left[\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}-\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\\
&+\frac{1}{k_m-k_a}\left(\frac{\frac{A_h(a^*)}{A_l(a^*)}-c_m}{hk_m-l_mk_a}\frac{l_m}{k_m}\frac{(k_m-k_a)k_m}{c_m}+1-\frac{k_m}{A_k(a^*)}\right)\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\end{aligned}\right]\\
&+\left[(hk_m-l_mk_a)\frac{c_m}{l_m}-k_a\left(\frac{A_h(a^*)}{A_l(a^*)}-c_m\right)\right]\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)
\end{aligned}\right),
\end{aligned}
\tag{A65}
$$

where, to derive the first equality, (16) and (A52) are used, and to derive the last equality, (16) is used.

In the expression inside the big parenthesis of the above equation, since $c_m\in\left[\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)},\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}\right)$

$$
\frac{1}{k_m-k_a}\left(\frac{\frac{A_h(a^*)}{A_l(a^*)}-c_m}{hk_m-l_mk_a}\frac{l_m}{k_m}\frac{(k_m-k_a)k_m}{c_m}+1-\frac{k_m}{A_k(a^*)}\right)>\frac{1}{k_m-k_a}\left[\frac{\frac{A_h(a^*)}{A_l(a^*)}-\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}}{hk_m-l_mk_a}\frac{l_m}{k_m}\frac{(k_m-k_a)k_m}{\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}}+1-\frac{k_m}{A_k(a^*)}\right]=0,
\tag{A66}
$$

$$
(hk_m-l_mk_a)\frac{c_m}{l_m}-k_a\left(\frac{A_h(a^*)}{A_l(a^*)}-c_m\right)>(hk_m-l_mk_a)\frac{1}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}-k_a\frac{A_h(a^*)}{A_l(a^*)}\left(1-\frac{l_m}{h}\frac{k_a}{k_m}\right)=0.
\tag{A67}
$$

Therefore, (A65) is negative and thus $\frac{dc_h(a)}{dk_m}<0$ from (A62).

Since $a\geq a^*$,

$$
\begin{aligned}
\frac{dc_h(a)}{dk_a} &= \frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{A_h(a)}{A_k(a)}\left(\frac{dc_m}{dk_a}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_a}-\frac{a}{A_k(a)}c_m\right)\\
&\leq \frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{A_h(a)}{A_k(a)}\left(\frac{dc_m}{dk_a}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_a}-\frac{a^*}{A_k(a^*)}c_m\right).
\end{aligned}
\tag{A68}
$$

In the above equation, from (A57) and (A48),

$$
\begin{aligned}
&\frac{dc_m}{dk_a}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_a}\\
&=\frac{1}{\Delta}\left\{\begin{array}{c}
-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{1}{k_m-k_a}\\
\times\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\
+\frac{k_mc_m}{k_al_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}}{k_m-k_a}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\left[\begin{array}{c}1+\\ r\left(\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right)\end{array}\right]
\end{array}\right\}\\
&-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{\Delta}\left(\begin{array}{c}
-\frac{1}{k_m-k_a}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}\right]\\
-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\frac{1}{k_a}\left\{\begin{array}{c}1+r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right]\\ -\frac{k_a}{k_m-k_a}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\end{array}\right\}
\end{array}\right)\\
&=\frac{1}{\Delta}\left\{\begin{array}{c}
\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{k_m-k_a}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
\times\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\
+\frac{k_mc_m}{k_al_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]\left[1+r\left(\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right)\right]
\end{array}\right\}\\
&<\frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
&\times\left\{\begin{array}{c}
\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\left[\frac{1}{k_m-k_a}\left(\begin{array}{c}\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\\ +\frac{k_mc_m}{k_al_m}\frac{N_h}{N_l}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\end{array}\right)\right]\\
+\frac{k_mc_m}{k_al_m}\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)
\end{array}\right\},
\end{aligned}
\tag{A69}
$$

where, to derive the last equality, (16) is used, and to derive the last inequality, (A42) is used. From (16), the expression inside the big square bracket of the above equation equals positive terms plus

$$
\frac{1}{k_m-k_a}\left(\begin{array}{c}\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\\ +\frac{k_mc_m}{k_al_m}\frac{A_l(a^*)}{A_h(a^*)}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\end{array}\right)
$$

$$
=\frac{1}{k_m-k_a}\left(\begin{array}{c}\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[\frac{a^*(k_m-k_a)}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}\\ +\frac{k_m}{k_a}\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\end{array}\right)
$$

$$
=\frac{1}{k_m-k_a}\left(\begin{array}{c}\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[-1+\frac{k_m}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]+\frac{k_m}{k_a}\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\\ -\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}\left(\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right)^{-1}\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\end{array}\right)
$$

$$
=\frac{1}{k_m-k_a}\left(\begin{array}{c}\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[-1+\frac{k_m}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]\\ +\frac{k_m}{k_a}\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{k_m-k_a}\left[\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]-\frac{k_a}{A_k(a^*)}\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\end{array}\right)>0. \tag{A70}
$$

Therefore, $\frac{dc_m}{dk_a}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_a}<0$ from (A69) and thus $\frac{dc_h(a)}{dk_a}<0$ from (A68).

[Effects on $c_l(a)$]:

$\frac{dc_l(a)}{dk_a}=\frac{k_m}{l_m}\frac{A_l(a)}{A_k(a)}\frac{dc_m}{dk_a}-\frac{k_m}{l_m}\frac{aA_l(a)}{A_k(a)^2}c_m<0$ from $\frac{dc_m}{dk_a}<0$.

Since $a\le a^*$,

$$
\frac{dc_l(a)}{dk_m}=\frac{1}{l_m}\frac{A_l(a)}{A_k(a)}\left(k_m\frac{dc_m}{dk_m}+\frac{ak_a}{A_k(a)}c_m\right)\le\frac{k_m}{l_m}\frac{A_l(a)}{A_k(a)}\left(\frac{dc_m}{dk_m}+\frac{k_a}{k_m}\frac{a^*c_m}{A_k(a^*)}\right). \tag{A71}
$$

The expression inside the parenthesis of the above equation satisfies, from (A54) and the definition of Δ,

$$
\frac{dc_m}{dk_m}+\frac{k_a}{k_m}\frac{a^*c_m}{A_k(a^*)}=\frac{1}{\Delta}\left(\begin{array}{l}\frac{k_a}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\ \times\frac{1}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\ +\frac{c_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\\ \quad\times\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right]\right\}\end{array}\right)
$$

$$
-\frac{1}{\Delta}\frac{k_a}{k_m}\frac{a^*c_m}{A_k(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left\{\begin{array}{c}\frac{k_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\\ +\frac{1}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\end{array}\right\}
$$

$$
= \frac{1}{\Delta}\left(\begin{array}{l}
\frac{k_a}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) \\
\times\frac{1}{k_m-k_a}\left\{\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{k_m}{A_k(a^*)}\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right\} \\
+\frac{c_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right] \\
\times\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dcda}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dcda}{c[A_k(a)]^2}\right]-\frac{a^*k_a}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right\}
\end{array}\right)
$$

$$
< \frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left(\begin{array}{l}
\frac{k_a}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\left\{\begin{array}{l}\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a} \\ -\frac{k_m}{A_k(a^*)}\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\end{array}\right\} \\
+\frac{c_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right] \\
\times\frac{(1-a^*)k_m}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)
\end{array}\right), \tag{A72}
$$

where the last inequality is from (A42).

The expression inside the big parenthesis of the above equation equals

$$
\begin{aligned}
&\frac{k_a}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\left\{\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{k_m}{A_k(a^*)}\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right\} \\
&\quad+\frac{c_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\frac{(1-a^*)k_m}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right) \\
&=\frac{k_a}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\left\{\begin{array}{l}\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[\frac{a^*(k_m-k_a)}{A_k(a^*)}-\frac{k_m}{A_k(a^*)}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right] \\ +\frac{A_l(a^*)}{A_h(a^*)}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{k_m}{A_k(a^*)}\left[\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]-\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\end{array}\right\} \\
&\quad+\frac{c_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\frac{(1-a^*)k_m}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right) \\
&=\frac{k_mc_m}{l_m}\left(\begin{array}{l}\frac{k_a}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{(k_m-k_a)^2}\left\{\begin{array}{l}\frac{N_h}{N_l}\left[\frac{k_m}{A_k(a^*)}-1-\frac{k_m}{A_k(a^*)}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]+\frac{A_l(a^*)}{A_h(a^*)}\frac{k_m}{A_k(a^*)} \\ \times\left[\frac{A_k(a^*)}{k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]-\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\end{array}\right\} \\ +\frac{(1-a^*)}{A_k(a^*)}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\end{array}\right),
\end{aligned} \tag{A73}
$$

where (16) is used to derive the first equality.

Since $c_m < \frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}$, the expression inside the big parenthesis of the above equation is strictly greater than

$$\frac{k_a}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{(k_m-k_a)^2}\frac{N_h}{N_l}\left[-\left(\frac{k_m}{A_k(a^*)}-1\right)^2+\frac{k_m}{A_k(a^*)}\left(\frac{k_m}{A_k(a^*)}-1-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\right]+\frac{1-a^*}{A_k(a^*)}\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)$$

$$>\frac{k_a}{k_m}\frac{(h-l_a)l_m}{A_l(a^*)^2}\frac{1}{(k_m-k_a)^2}\left(\frac{N_h}{N_l}\right)^2\left[-\frac{a^{*2}(k_m-k_a)^2}{A_k(a^*)^2}+\frac{k_m}{A_k(a^*)}\left(\frac{k_m}{A_k(a^*)}-1-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\right]+\frac{1}{A_k(a^*)^2}\frac{N_h}{N_l}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right),\quad \text{(A74)}$$

where, to derive the inequality, $-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)>0$, which is shown in the proof of $\frac{dc_l(a)}{dk_m}<0$ of Proposition 4, is used.

The equation equals positive terms plus

$$\frac{\left(\frac{N_h}{N_l}\right)^2}{A_k(a^*)^2A_l(a^*)}\left[-\frac{k_a}{k_m}\frac{(h-l_a)l_m a^{*2}}{A_l(a^*)}+A_h(a^*)\right]=\frac{\left(\frac{N_h}{N_l}\right)^2}{A_k(a^*)^2A_l(a^*)^2k_m}\left[A_h(a^*)A_l(a^*)k_m-a^{*2}(h-l_a)l_mk_a\right]>0,\quad \text{(A75)}$$

where the last inequality is shown in the proof of $\frac{dc_m}{dk_a}<0$. Therefore, $\frac{dc_l(a)}{dk_m}<0$ from (A71) and (A72).

[**Effects on earnings**] Since $w_l=\frac{l_m}{k_m}\frac{r}{c_m}$, $\frac{dw_l}{dk_a}>0$ from $\frac{dc_m}{dk_a}<0$. Since $w_h=\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\frac{r}{c_m}$, the sign of $\frac{dw_h}{dk_m}$ equals that of $\frac{d(\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{k_mc_m})}{dk_m}=\frac{1}{k_mc_m}\left[\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{da^*}{dk_m}-\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{k_mc_m}\left(\frac{dc_m}{dk_m}k_m+c_m\right)\right]$, where, from the second equation of (A54),

$$\frac{dc_m}{dk_m}k_m+c_m$$

$$=\frac{1}{\Delta}\left(\begin{array}{l}\frac{k_a}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\ \times\frac{k_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\ +\frac{k_mc_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\\ \times\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_adcda}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_adcda}{c[A_k(a)]^2}\right]\right\}\end{array}\right)$$

$$+\frac{1}{\Delta}\left(\begin{array}{l}\frac{-k_mc_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\ -\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\end{array}\right)$$

$$=\frac{1}{\Delta}\left(\begin{array}{l}\frac{k_a}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\ \times\frac{k_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{k_m}{k_a}\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\ +\frac{k_mc_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\\ \times\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_adcda}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_adcda}{c[A_k(a)]^2}\right]-\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right\}\end{array}\right).$$

(A76)

From the above equation and the second equation of (A50),

$$
\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{da^*}{dk_m}-\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{k_m c_m}\left(\frac{dc_m}{dk_m}k_m+c_m\right)
$$

$$
=-\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{k_m c_m}\frac{1}{\Delta}\left(\begin{array}{l}
\frac{k_a}{k_m}\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
\times\frac{k_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^* c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_m k_a}-\frac{k_m}{k_a}\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_m k_a)}\right]\right]\\
+\frac{k_m c_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial \frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_m k_a)c_m}{A_k(a^*)}}\right]\right]\\
\times\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right]-\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right\}
\end{array}\right)
$$

$$
-\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{1}{k_m}\frac{1}{\Delta}\left(\begin{array}{l}
-\frac{k_a}{k_m-k_a}\frac{1}{c_m}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
\times\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^* c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_m k_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_m k_a)}\right]\right]\\
+\frac{1}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_m k_a)}\right]\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right]\right\}
\end{array}\right)
$$

$$
=\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{1}{k_m c_m}\frac{1}{\Delta}\left(\frac{k_a}{k_m-k_a}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^* c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_m k_a}\right]\right)
$$

$$
-\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{1}{k_m c_m}\frac{1}{\Delta}\left\{\begin{array}{l}1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right]\\
+\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left[\frac{k_a}{k_m-k_a}-\frac{N_h}{N_l}\frac{A_h(a^*)}{A_l(a^*)}\right]\end{array}\right\}\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_m k_a)}\right]
$$

$$
-\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{\Delta}\frac{1}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial \frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_m k_a)c_m}{A_k(a^*)}}\right]\right]
$$

$$
\times\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right]-\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right\}. \tag{A77}
$$

From (A42), the above equation is strictly greater than

$$
-\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{1}{c_m}\frac{r}{(k_m-k_a)^2}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{1}{\Delta}\left\{\begin{array}{l}\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_m k_a)}\right]\\
-\frac{k_a}{k_m}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^* c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_m k_a}\right]\end{array}\right\}>0 \text{ (from eq. A51).}
$$

Hence, $\frac{dw_h}{dk_m}>0$.

As for the effect of k_a on $w_h=\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\frac{r}{c_m}$,

$$
\frac{d\left(\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{c_m}\right)}{dk_a}=\frac{1}{c_m}\left(\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{da^*}{dk_a}-\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{c_m}\frac{dc_m}{dk_a}\right), \tag{A78}
$$

where

$$\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{da^*}{dk_a}-\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{c_m}\frac{dc_m}{dk_a}$$

$$= \frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{1}{c_m}\frac{1}{\Delta}\left(\begin{array}{c} -\frac{1}{k_m-k_a}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}\right] \\ -\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\frac{1}{k_a}\left\{\begin{array}{c} 1+r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right] \\ -\frac{k_a}{k_m-k_a}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\end{array}\right\}\end{array}\right)$$

$$-\frac{A_h(a^*)}{A_l(a^*)}\frac{1}{c_m}\frac{1}{\Delta}\left\{\begin{array}{l} -\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{1}{k_m-k_a}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) \\ \qquad\times\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right] \\ +\frac{k_mc_m}{k_al_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial \frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\left[1+r\left(\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right)\right]\end{array}\right\}$$

$$= -\frac{1}{c_m}\frac{1}{\Delta}\left(\begin{array}{l} \frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{r}{(k_m-k_a)^2}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right] \\ +\frac{1}{k_a}\left\{1+r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right]\right\} \\ \qquad\times\left\{\begin{array}{c}\frac{A_h(a^*)}{A_l(a^*)}\frac{k_mc_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial \frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right] \\ +\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\end{array}\right\}\end{array}\right). \tag{A79}$$

The expression inside the big parenthesis of the above equation equals a positive term plus

$$\begin{array}{l}\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{r}{(k_m-k_a)^2}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right] \\ +\left\{1+r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right]\right\}\frac{A_h(a^*)}{A_l(a^*)}\frac{k_m}{k_a}\frac{c_m}{l_m}\left[-\frac{\frac{\partial \frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right] \\ > \frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left\{\begin{array}{c}\frac{\partial \frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{1}{k_m-k_a}\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right] \\ +\frac{A_h(a^*)}{A_l(a^*)}\frac{k_m}{k_a}\frac{c_m}{l_m}\left[-\frac{\frac{\partial \frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\end{array}\right\},\end{array} \tag{A80}$$

which is positive from (A55). Hence, $\frac{dw_h}{dk_a}>0$.

[Effect of k_m on Y] The LHS of (HL) when $c^* < c_a = 1$ equals $\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)$ from (16) of Lemma 2, whose derivative with respect to k_m is

$$\frac{N_h}{N_l}\frac{k_m}{l_m}\left[\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left(\frac{dc_m}{dk_m}-\frac{k_a}{k_m}\frac{c_m}{k_m-k_a}\right)+\frac{c_m}{A_k(a^*)}\left(\frac{da^*}{dk_m}+\frac{k_a}{k_m}\frac{a^*}{k_m-k_a}\right)\right], \tag{A81}$$

where, from (A54) and (A50), the terms associated with $\frac{dc_m}{dk_m}$ and $\frac{da^*}{dk_m}$ inside the square bracket satisfy

$$\begin{aligned}
&\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{dc_m}{dk_m}+\frac{c_m}{A_k(a^*)}\frac{da^*}{dk_m}\\
&=\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{1}{\Delta}\left(\begin{array}{l}
\frac{k_a}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
\times\frac{1}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\
+\frac{c_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}-\frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\\
\qquad\times\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right]\right\}
\end{array}\right)\\
&\quad-\frac{1}{k_mA_k(a^*)}\frac{1}{\Delta}\left(\begin{array}{l}
-\frac{k_a}{k_m-k_a}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
\times\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\
+\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right]\right\}
\end{array}\right)\\
&<\frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
&\times\left(\begin{array}{l}
\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\
\qquad\times\frac{k_a}{k_m}\frac{1}{k_m-k_a}\left\{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{1}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\right\}\\
+\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\left\{\begin{array}{l}
\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{c_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\\
\qquad-\frac{1}{k_mA_k(a^*)}\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]
\end{array}\right\}
\end{array}\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
&\times\left(\begin{array}{l}
\frac{k_a}{k_m}\frac{1}{k_m-k_a}\left\{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{1}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\right\}\\
\quad\times\left[\begin{array}{l}
\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left(\frac{a^*(k_m-k_a)}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\\
+\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{k_m-k_a}\left(\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]-\frac{k_m}{A_k(a^*)}\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right)
\end{array}\right]\\
+\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\left\{\begin{array}{l}
\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{c_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\\
-\frac{1}{k_mA_k(a^*)}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left(\frac{N_h}{N_l}\ln\left(\frac{k_m}{A_k(a^*)}\right)-\frac{A_l(a^*)}{A_h(a^*)}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right)
\end{array}\right\}
\end{array}\right)\\
&= \frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
&\times\left\{\begin{array}{l}
\frac{k_a}{k_m}\frac{1}{k_m-k_a}\left\{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{1}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\right\}\\
\times\left[\begin{array}{l}
\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left(\frac{a^*(k_m-k_a)}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\\
+\frac{k_m}{k_a}\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{k_m-k_a}\left(\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]-\frac{k_a}{A_k(a^*)}\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right)
\end{array}\right]\\
+\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{c_m}{l_m}\left\{\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right\}
\end{array}\right\},
\end{aligned}
\tag{A82}
$$

where, to derive the inequality, (A42) and (A52) are used, and, to derive the second last equality, (16) is used.

Using the definition of Δ, the remaining terms inside the square bracket of (A81) equals

$$
\begin{aligned}
&\frac{c_m}{(k_m-k_a)^2}\frac{k_a}{k_m}\left(-\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{a^*(k_m-k_a)}{A_k(a^*)}\right)\\
&= -\frac{1}{\Delta}\frac{c_m}{(k_m-k_a)^2}\frac{k_a}{k_m}\left(\frac{a^*(k_m-k_a)}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
&\times\left(\begin{array}{l}
\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{k_m}{l_m}\left\{\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right\}\\
+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{1}{k_m-k_a}\left(\frac{N_h}{N_l}\ln\left(\frac{k_m}{A_k(a^*)}\right)-\frac{A_l(a^*)}{A_h(a^*)}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right)
\end{array}\right)\\
&= -\frac{1}{\Delta}\frac{c_m}{(k_m-k_a)^2}\frac{k_a}{k_m}\left(\frac{a^*(k_m-k_a)}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
&\times\left(\begin{array}{l}
\frac{k_m}{l_m}\left\{\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right\}\\
+\frac{N_h}{N_l}\frac{k_m}{l_m}\left\{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{1}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\right\}
\end{array}\right),
\end{aligned}
\tag{A83}
$$

where (16) is used to derive the first equality.

From the above equation and (A82),

$$
\begin{aligned}
&\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{dc_m}{dk_m}+\frac{c_m}{A_k(a^*)}\frac{da^*}{dk_m}+\frac{c_m}{(k_m-k_a)^2}\frac{k_a}{k_m}\left(-\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{a^*(k_m-k_a)}{A_k(a^*)}\right)\\
&<\frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
&\times\left\{\begin{array}{l}
\frac{k_a}{k_m}\frac{1}{k_m-k_a}\left\{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{1}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\right\}\\
\times\left[\begin{array}{l}
\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left(\frac{a^*(k_m-k_a)}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\\
+\frac{k_m}{k_a}\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{k_m-k_a}\left(\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]-\frac{k_a}{A_k(a^*)}\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right)
\end{array}\right]\\
+\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{c_m}{l_m}\left\{\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right\}
\end{array}\right\}\\
&\quad-\frac{1}{\Delta}\frac{c_m}{(k_m-k_a)^2}\frac{k_a}{k_m}\left(\frac{a^*(k_m-k_a)}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
&\quad\times\left(\begin{array}{l}
\frac{k_m}{l_m}\left\{\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right\}\\
+\frac{N_h}{N_l}\frac{k_m}{l_m}\left\{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{1}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\right\}
\end{array}\right)\\
&=\frac{1}{\Delta}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
&\times\left\{\begin{array}{l}
\frac{k_a}{k_m}\frac{1}{k_m-k_a}\left\{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{1}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\right\}\\
\times\left[\frac{k_m}{k_a}\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{k_m-k_a}\left(\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]-\frac{k_a}{A_k(a^*)}\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right)\right]\\
+\frac{c_m}{(k_m-k_a)^2}\frac{k_m}{l_m}\left[\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{k_a}{k_m}\left(1-\frac{k_m}{A_k(a^*)}\right)\right]\\
\quad\times\left\{\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right\}
\end{array}\right\}<0.
\end{aligned}
\tag{A84}
$$

Hence, (A81) is negative and thus Y increases with k_m.

[Effect of k_a on Y]

$$
\begin{aligned}
\frac{da^*}{dk_a}=&-\frac{1}{\Delta}\left(\frac{1}{k_a}\frac{1}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right)\\
&+\frac{1}{\Delta}\left(\begin{array}{l}
-\frac{1}{k_m-k_a}\frac{1}{c_m}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
\times\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{k_m\left(1-c_m\frac{A_l(a^*)}{A_h(a^*)}\right)}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\
-\frac{1}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]r\left[\int_0^{a^*}\int_{c_l(a)}^{1}\frac{adcda}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^{1}\frac{adcda}{c[A_k(a)]^2}\right]
\end{array}\right)
\end{aligned}
$$

$$
=\frac{1}{c_m}\frac{1}{\Delta}\left(\begin{array}{l}
-\frac{1}{k_m-k_a}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
\times\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\
-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\frac{1}{k_a}\left[1+r\left(\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right)\right]
\end{array}\right). \quad \text{(A85)}
$$

The LHS of (HL) when $c^*<c_a=1$ equals $\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)$ from (16) of Lemma 2, whose derivative with respect to k_a is

$$
\frac{N_h}{N_l}\frac{k_m}{l_m}\left[\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left(\frac{dc_m}{dk_a}+\frac{c_m}{k_m-k_a}\right)+\frac{c_m}{A_k(a^*)}\left(\frac{da^*}{dk_a}-\frac{a^*}{k_m-k_a}\right)\right], \quad \text{(A86)}
$$

where, from (A57) and (A48), the terms associated with $\frac{dc_m}{dk_a}$ and $\frac{da^*}{dk_a}$ inside the square bracket equal

$$
\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{dc_m}{dk_a}+\frac{c_m}{A_k(a^*)}\frac{da^*}{dk_a}
$$

$$
=\frac{1}{\Delta}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left\{\begin{array}{l}
-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{1}{k_m-k_a}\\
\times\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\
+\frac{k_mc_m}{k_al_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\\
\quad\times\left[1+r\left(\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right)\right]
\end{array}\right\}
$$

$$
+\frac{1}{\Delta}\frac{1}{A_k(a^*)}\left(\begin{array}{l}
-\frac{1}{k_m-k_a}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
\times\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\
-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\frac{1}{k_a}\left[1+r\left(\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right)\right]
\end{array}\right)
$$

$$
=-\frac{1}{\Delta}\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\frac{r}{(k_m-k_a)^2}\ln\left(\frac{k_m}{A_k(a^*)}\right)
$$

$$
\times\left\{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{1}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\right\}
$$

$$
+\frac{1}{\Delta}\left[1+r\left(\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}+\int_{a^*}^{c_h^{-1}(1)}\int_{c_h(a)}^1\frac{ak_a\,dc\,da}{c[A_k(a)]^2}\right)\right]
$$

$$
\times\left\{\begin{array}{c}
\frac{1}{k_a}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left[\frac{\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\\
+\frac{1}{k_aA_k(a^*)}\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]
\end{array}\right\}, \quad \text{(A87)}
$$

where (16) is used to derive the last equality.

Using the definition of Δ, the remaining terms inside the square bracket of (A86) equals

$$
\begin{aligned}
&\frac{c_m}{(k_m-k_a)^2}\left(\ln\left(\frac{k_m}{A_k(a^*)}\right)-\frac{a^*(k_m-k_a)}{A_k(a^*)}\right)\\
&=\frac{1}{\Delta}\frac{c_m}{(k_m-k_a)^2}\left(\frac{a^*(k_m-k_a)}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\\
&\quad\times\left\{\begin{array}{l}
\frac{k_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
\qquad+\frac{1}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)
\end{array}\right\}\\
&=\frac{1}{\Delta}\frac{c_m}{(k_m-k_a)^2}\left(\frac{a^*(k_m-k_a)}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\\
&\quad\times\left\{\begin{array}{l}
\frac{k_m}{l_m}\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
\quad+\frac{k_m}{l_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\
+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left[\begin{array}{l}
\frac{1}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\\
+\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]
\end{array}\right]
\end{array}\right\}\\
&=\frac{1}{\Delta}\frac{c_m}{(k_m-k_a)^2}\left(\frac{a^*(k_m-k_a)}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\frac{r}{k_m-k_a}\frac{k_m}{l_m}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left\{\begin{array}{l}
\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{N_h}{N_l}\right)^2\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\\
+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]
\end{array}\right\},
\end{aligned}
\tag{A88}
$$

where (16) is used to derive the last equality.

The sum of the above equation and the first term of (A87) equals $\frac{1}{\Delta}\frac{r}{(k_m-k_a)^2}\ln\left(\frac{k_m}{A_k(a^*)}\right)$ times (the second term of (A87) is clearly negative)

$$
\left(\begin{array}{l}
-\left[\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{a^*c_m}{A_k(a^*)}-\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a}-\frac{1}{h-l_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m-\frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}\right]\right]\\
\qquad\times\left\{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{1}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\right\}\\
+\frac{c_m}{k_m-k_a}\left(\frac{a^*(k_m-k_a)}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\frac{k_m}{l_m}\left\{\begin{array}{l}
\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{N_h}{N_l}\right)^2\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\\
+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]
\end{array}\right\}
\end{array}\right)
$$

$$
= \left(\begin{array}{c} \left[\frac{\frac{k_m^2}{l_m}\frac{A_l(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{hk_m-l_mk_a} - \frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right] \\ \times\left\{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{1}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\right\} \\ +\frac{c_m}{k_m-k_a}\left(\frac{a^*(k_m-k_a)}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\frac{k_m}{l_m}\left\{ \begin{array}{c} \frac{1}{A_k(a^*)}\left(\frac{A_l(a^*)}{A_h(a^*)}+\frac{N_h}{N_l}\right) \\ +\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right] \end{array} \right\} \end{array} \right)
$$

$$
= \left(\begin{array}{c} \frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{k_m-k_a}\left(\frac{k_m}{A_k(a^*)}\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}-\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right) \\ \times\left\{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)+\frac{1}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\right\} \\ +\frac{c_m}{k_m-k_a}\left(\frac{k_m}{A_k(a^*)}-1-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right)\frac{k_m}{l_m}\left\{ \begin{array}{c} \frac{1}{A_k(a^*)}\left(\frac{A_l(a^*)}{A_h(a^*)}+\frac{N_h}{N_l}\right) \\ +\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{1}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right] \end{array} \right\} \end{array} \right) > 0,
$$

where (16) is used to derive the first equality. The expression can be proved to be positive since $\frac{1}{k_m-k_a}\left(\frac{k_m}{A_k(a^*)}\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}-\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right) > 0$, which is true because it equals 0 at $\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}} = 0$ and the derivative with respect to $\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}$ equals

$$
\frac{1}{k_m-k_a}\left(1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right)^{-1}\left[\frac{k_m}{A_k(a^*)}\left(1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right)-1\right] > 0, \tag{A89}
$$

where the expression when $\frac{k_a}{k_m} > 1$ is positive from

$$
\frac{k_m}{A_k(a^*)}\left(1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)}-c_m\right]}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right)-1 < \frac{k_m}{A_k(a^*)}-1 < 0. \tag{A90}
$$

Hence, (A86) is negative and thus Y increases with k_a. ■

Proof of Proposition 6. From the proof of Lemma 1, when $c^*, c_a < 1$, derivatives of the LHS−RHS of (HL) with respect to a^* and $\frac{k_a}{k_m}$ are

$$
a^* \text{ when } \frac{k_a}{k_m} \neq 1 : \frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right] > 0, \tag{A91}
$$

$$
\frac{k_a}{k_m} \text{ when } \frac{k_a}{k_m} \neq 1 : \frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{k_m}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right] > 0, \tag{A92}
$$

$$
a^* \text{ when } \frac{k_a}{k_m} = 1 : \frac{c_m}{l_m}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}(1-a^*)\right] > 0, \tag{A93}
$$

$$
\frac{k_a}{k_m} \text{ when } \frac{k_a}{k_m} = 1 : \frac{c_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}(1-a^*) > 0. \tag{A94}
$$

When $c_a < 1 \Leftrightarrow c_h(1) < 1$, (P) can be expressed as

$$\frac{l_m}{k_m}\frac{r}{c_m}\int_0^{a^*}\int_0^{c_l(a)}\frac{dcda}{A_l(a)}+\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\frac{r}{c_m}\int_{a^*}^1\int_0^{c_h(a)}\frac{dcda}{A_h(a)}+r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}+\int_{a^*}^1\int_{c_h(a)}^1\frac{dcda}{cA_k(a)}\right]=1. \tag{A95}$$

The derivatives of the LHS of the above equation with respect to c_m and a^* are

$$c_m \text{ when } \frac{k_a}{k_m}\neq 1: -\frac{1}{c_m}\left\{\frac{l_m}{k_m}\frac{r}{c_m}\int_0^{a^*}\int_0^{c_l(a)}\frac{dcda}{A_l(a)}+\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\frac{r}{c_m}\int_{a^*}^1\int_0^{c_h(a)}\frac{dcda}{A_h(a)}\right\}$$

$$=-\frac{r}{c_m(k_m-k_a)}\left\{\ln\left(\frac{k_m}{A_k(a^*)}\right)+\ln\left(\frac{A_k(a^*)}{k_a}\right)\right\}=-\frac{r}{c_m(k_m-k_a)}\ln\left(\frac{k_m}{k_a}\right)<0, \tag{A96}$$

$$a^* \text{ when } \frac{k_a}{k_m}\neq 1: \frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)>0, \tag{A97}$$

$$c_m \text{ when } \frac{k_a}{k_m}=1: -\frac{1}{c_m}\frac{r}{k_m}<0, \tag{A98}$$

$$a^* \text{ when } \frac{k_a}{k_m}=1: \frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m}(1-a^*)>0, \tag{A99}$$

where (21), (24), (22), and (25) in the proof of Lemma 1 are used to derive the equations.

Noting that $r\iint\frac{1}{cA_k(a)}dcda=r\iint\frac{1}{c[a\frac{k_a}{k_m}+1-a]k_m}dcda$, the derivative of the LHS of (A95) with respect to $\frac{k_a}{k_m}$ equals

$$-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda+\int_{a^*}^1\int_{c_h(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda\right]<0. \tag{A100}$$

For *given* $\frac{k_a}{k_m}$, the derivative with respect to k_m equals

$$-\frac{1}{k_m}\left\{1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}+\int_{a^*}^1\int_{c_h(a)}^1\frac{dcda}{cA_k(a)}\right]\right\}-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{dcda}{c[a\frac{k_a}{k_m}+1-a]k_m^2}+\int_{a^*}^1\int_{c_h(a)}^1\frac{dcda}{c[a\frac{k_a}{k_m}+1-a]k_m^2}\right]=-\frac{1}{k_m}. \tag{A101}$$

Hence, the total derivative of (HL) and (P) when $\frac{k_a}{k_m}\neq 1$ are summarized as follows.

$$\begin{pmatrix}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right] & 0\\ \frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right) & -\frac{r}{c_m(k_m-k_a)}\ln\left(\frac{k_m}{k_a}\right)\end{pmatrix}\begin{pmatrix}da^*\\ dc_m\end{pmatrix}$$

$$=\begin{pmatrix}0\\ \frac{1}{k_m}\end{pmatrix}dk_m\Big|_{\frac{k_a}{k_m}\text{ fixed}}+\begin{pmatrix}-\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{k_m}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]\\ r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda+\int_{a^*}^1\int_{c_h(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda\right]\end{pmatrix}d\frac{k_a}{k_m}. \tag{A102}$$

By solving for da^* and dc_m, the above equation can be expressed as:

$$\begin{pmatrix}da^*\\ dc_m\end{pmatrix}=\frac{1}{\widetilde{\Delta}}\begin{pmatrix}-\frac{r}{c_m(k_m-k_a)}\ln\left(\frac{k_m}{k_a}\right) & 0\\ -\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right) & \frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]\end{pmatrix}$$

$$\times\left[\begin{pmatrix}0\\ \frac{1}{k_m}\end{pmatrix}dk_m\Big|_{\frac{k_a}{k_m}\text{ fixed}}+\begin{pmatrix}-\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{k_m}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]\\ r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda+\int_{a^*}^1\int_{c_h(a)}^1\frac{ak_m}{c[A_k(a)]^2}dcda\right]\end{pmatrix}d\frac{k_a}{k_m}\right], \tag{A103}$$

where $\widetilde{\Delta} \equiv -\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right)\frac{k_m}{l_m}\frac{1}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)} - \frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right] < 0.$

[Effects on a^* and c_m] When $\frac{k_a}{k_m}\neq 1$, $\frac{da^*}{dk_m}|_{\frac{k_a}{k_m}\text{ fixed}} = 0$, $\frac{da^*}{d\frac{k_a}{k_m}} < 0$, $\frac{dc_m}{dk_m}|_{\frac{k_a}{k_m}\text{ fixed}} < 0$, and $\frac{dc_m}{d\frac{k_a}{k_m}} < 0$ are straightforward from (A103). The same result holds when $\frac{k_a}{k_m}=1$ as well from (A93), (A94), (A98), and (A99). Hence, $\frac{da^*}{dk_a} = \frac{k_m}{k_a}\frac{da^*}{dk_m}|_{\frac{k_a}{k_m}\text{ fixed}} + \frac{1}{k_m}\frac{da^*}{d\frac{k_a}{k_m}} = \frac{1}{k_m}\frac{da^*}{d\frac{k_a}{k_m}} < 0$ (and $\frac{da^*}{dk_m} = -\frac{k_a}{k_m^2}\frac{da^*}{d\frac{k_a}{k_m}} > 0$) and $\frac{dc_m}{dk_a} = \frac{k_m}{k_a}\frac{dc_m}{dk_m}|_{\frac{k_a}{k_m}\text{ fixed}} + \frac{1}{k_m}\frac{dc_m}{d\frac{k_a}{k_m}} < 0$. As for the effect of k_m on c_m when $\frac{k_a}{k_m}\neq 1$,

$$
\begin{aligned}
\frac{dc_m}{dk_m} &= \frac{1}{\widetilde{\Delta}}\left\{\begin{array}{c}
-\frac{k_a}{k_m^2}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{k_m}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right] \\
+\frac{1}{k_m}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)} - \frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right] \\
\times\left(1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda+\int_{a^*}^1\int_{c_h(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda\right]\right)
\end{array}\right\} \\
&< \frac{1}{\widetilde{\Delta}}\frac{1}{l_m}\left\{\begin{array}{c}
-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{c_m}{k_m-k_a}\frac{k_a}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right] \\
+\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)} - \frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right] \\
\times\left(1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}+\int_{a^*}^1\int_{c_h(a)}^1\frac{dcda}{cA_k(a)}\right]\right)
\end{array}\right\} \\
&= \frac{1}{\widetilde{\Delta}}\frac{1}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right) \\
&\quad\times\left\{\begin{array}{c}
-\frac{c_m}{k_m-k_a}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{k_a}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right] \\
+\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)} - \frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]\left(\frac{N_l}{N_h}+\frac{A_h(a^*)}{A_l(a^*)}\right)
\end{array}\right\},
\end{aligned}
\tag{A104}
$$

where, to derive the last equation, the following equation, which is obtained from (A95), (21), and (24), is used.

$$
1-r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}+\int_{a^*}^1\int_{c_h(a)}^1\frac{dcda}{cA_k(a)}\right] = \left(\frac{N_l}{N_h}+\frac{A_h(a^*)}{A_l(a^*)}\right)\frac{r}{k_m-k_a}\frac{A_l(a^*)}{A_h(a^*)}\ln\left(\frac{A_k(a^*)}{k_a}\right)\left(=\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right)\right). \tag{A105}
$$

The expression inside the large curly bracket of the last equation of (A104) equals a positive term plus

$$
\begin{aligned}
&-\frac{c_m}{k_m-k_a}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{k_a}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right] - \frac{c_m}{k_m-k_a}\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\left(\frac{N_l}{N_h}+\frac{A_h(a^*)}{A_l(a^*)}\right) \\
&= \frac{c_m}{k_m-k_a}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left\{-\frac{k_a}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]+\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\ln\left(\frac{A_k(a^*)}{k_a}\right)\left(\frac{N_l}{N_h}+\frac{A_h(a^*)}{A_l(a^*)}\right)\right\} \\
&= \frac{c_m}{k_m-k_a}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left\{-\frac{k_a}{A_k(a^*)}\left[-\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)a^*+\frac{A_l(a^*)}{A_h(a^*)}\frac{A_k(a^*)}{k_a}\right]+\frac{A_l(a^*)}{A_h(a^*)}\frac{N_h}{N_l}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left(\frac{N_l}{N_h}+\frac{A_h(a^*)}{A_l(a^*)}\right)\right\} \\
&= \frac{c_m}{k_m-k_a}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left\{-\frac{A_l(a^*)}{A_h(a^*)}+\left[\frac{a^*k_a}{A_k(a^*)}+\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\right\} \\
&= \frac{c_m}{k_m-k_a}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\left[\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]^{-1}\left\{-\ln\left(\frac{k_m}{A_k(a^*)}\right)+\left[\frac{a^*k_a}{A_k(a^*)}+\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]\ln\left(\frac{k_m}{k_a}\right)\right\},
\end{aligned}
\tag{A106}
$$

where (14) is used to derive the third equation and the last equation. The expression inside the curly bracket of the last equation is positive because it equals 0 at $a^*=0$ and the derivative of the expression with respect to a^* is

$$\frac{1}{A_k(a^*)}\left[\left(\ln\left(\frac{k_m}{k_a}\right)-1\right)(k_m-k_a)+\frac{k_a k_m}{A_k(a^*)}\ln\left(\frac{k_m}{k_a}\right)\right]>0. \tag{A107}$$

The expression inside the square bracket of the above equation is positive because it increases with a^* and the value at $a^*=0$, $k_m\left[-\ln\left(\frac{k_a}{k_m}\right)-1+\frac{k_a}{k_m}\right]$, is positive. Therefore, $\frac{dc_m}{dk_m}<0$ when $\frac{k_a}{k_m}\neq 1$. $\frac{dc_m}{dk_m}<0$ when $\frac{k_a}{k_m}=1$ can be proved similarly. (Henceforth, results are proved only for the case $\frac{k_a}{k_m}\neq 1$, but they can be proved similarly when $\frac{k_a}{k_m}=1$ as well.)

[Effects on c_a] $\frac{dc_a}{dk_m}<0$ ($\frac{dc_a}{dk_a}<0$) from $\frac{dc_h(a)}{dk_m}<0$ ($\frac{dc_h(a)}{dk_a}<0$), which is proved below.

[Effects on $c_l(a)$]: Since $c_l(a)=\frac{k_m}{l_m}\frac{A_l(a)}{A_k(a)}c_m$, $c_l(a)$ decreases with k_a from $\frac{dc_m}{dk_a}<0$. Since $a\leq a^*$,

$$\frac{dc_l(a)}{dk_m}=\frac{1}{l_m}\frac{A_l(a)}{A_k(a)}\left(k_m\frac{dc_m}{dk_m}+\frac{ak_a}{A_k(a)}c_m\right)\leq\frac{1}{l_m}\frac{A_l(a)}{A_k(a)}\left(k_m\frac{dc_m}{dk_m}+\frac{a^*k_a}{A_k(a^*)}c_m\right). \tag{A108}$$

The expression inside the parenthesis of the above equation satisfies, from (A104) and the definition of $\widetilde{\Delta}$,

$$\begin{aligned}
k_m\frac{dc_m}{dk_m}+\frac{a^*k_a}{A_k(a^*)}c_m &< \frac{1}{\widetilde{\Delta}}\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)\\
&\quad\times\left\{\begin{array}{l}
-\frac{c_m}{k_m-k_a}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{k_a}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]\\
+\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]\left(\frac{N_l}{N_h}+\frac{A_h(a^*)}{A_l(a^*)}\right)
\end{array}\right\}\\
&\quad-\frac{1}{\widetilde{\Delta}}\frac{k_m}{l_m}\frac{a^*k_a}{A_k(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right)\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]\\
&=\frac{1}{\widetilde{\Delta}}\frac{r}{k_m-k_a}\frac{k_m^2}{l_m}\frac{c_m}{k_m-k_a}\left\{\begin{array}{l}
-\frac{k_a}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{1}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]\\
+\frac{(1-a^*)}{A_k(a^*)}\ln\left(\frac{k_m}{k_a}\right)\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]
\end{array}\right\},
\end{aligned} \tag{A109}$$

where the last inequality is from (A105).

In the above equation, the expression inside the big curly bracket equals a positive term plus

$$\begin{aligned}
&\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{A_k(a^*)}\left\{-\frac{k_a}{k_m}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]+(1-a^*)\ln\left(\frac{k_m}{k_a}\right)\frac{A_l(a^*)}{A_h(a^*)}\right\}\\
=&\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{A_k(a^*)}\left\{-\frac{k_a}{k_m}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]+(1-a^*)\left[\ln\left(\frac{k_m}{k_a}\right)-1+\frac{k_a}{k_m}+1-\frac{k_a}{k_m}\right]\frac{A_l(a^*)}{A_h(a^*)}\right\}\\
=&\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{A_k(a^*)}\left\{\frac{k_a}{k_m}\left[\frac{N_h}{N_l}a^*-\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\right]+(1-a^*)\left[\ln\left(\frac{k_m}{k_a}\right)-1+\frac{k_a}{k_m}\right]\frac{A_l(a^*)}{A_h(a^*)}\right\},
\end{aligned} \tag{A110}$$

where, using (14),

$$\frac{N_h}{N_l}a^* - \frac{A_l(a^*)}{A_h(a^*)}(1-a^*)$$
$$= \frac{N_h}{N_l}\left[\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]^{-1}\left[a^*\ln\left(\frac{A_k(a^*)}{k_a}\right) - (1-a^*)\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]. \tag{A111}$$

The expression $a^*\ln\left(\frac{A_k(a^*)}{k_a}\right) - (1-a^*)\ln\left(\frac{k_m}{A_k(a^*)}\right)$ is positive since it equals 0 at $a^* = 0, 1$, and its first derivative with respect to a^* equals $\ln\left(\frac{k_m}{k_a}\right) - \frac{k_m - k_a}{A_k(a^*)}$ and thus the second derivative is negative. Hence $k_m\frac{dc_m}{dk_m} + \frac{a^* k_a}{A_k(a^*)}c_m < 0$ and thus $\frac{dc_l(a)}{dk_m} < 0$

[Effects on $c_h(a)$]: Since $a \le 1$,

$$\begin{aligned}\frac{dc_h(a)}{dk_m} &= \frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{A_h(a)}{A_k(a)}\left(\frac{dc_m}{dk_m} - \frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_m} + \frac{ak_a}{A_k(a)}\frac{c_m}{k_m}\right)\\ &\le \frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{A_h(a)}{A_k(a)}\left(\frac{dc_m}{dk_m} - \frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_m} + \frac{c_m}{k_m}\right).\end{aligned} \tag{A112}$$

In the above equation,

$$\begin{aligned}&\frac{dc_m}{dk_m} - \frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_m}\\
&= \frac{1}{\widetilde{\Delta}}\left\{\begin{array}{l} -\frac{k_a}{k_m^2}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{k_m}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^* + \frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]\\ +\frac{1}{k_m}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l} + \frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)} - \frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]\\ \qquad\times\left(1 - r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda + \int_{a^*}^1\int_{c_h(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda\right]\right)\end{array}\right\}\\
&+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{\widetilde{\Delta}}\frac{k_a}{k_m^2}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right)\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{k_m}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^* + \frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]\\
&= \frac{1}{\widetilde{\Delta}}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left\{\begin{array}{l}\frac{k_a}{k_m^2}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{k_m}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^* + \frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]\\ +\frac{1}{k_m}\left[\left(\frac{N_h}{N_l} + \frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)} - \frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]\\ \qquad\times\left(1 - r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda + \int_{a^*}^1\int_{c_h(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda\right]\right)\end{array}\right\}\\
&< \frac{1}{\widetilde{\Delta}}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{r}{k_m-k_a}\frac{1}{k_m}\left\{\begin{array}{l}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{k_a}{A_k(a^*)}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left[-\frac{N_h}{N_l}a^* + \frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]\\ +\ln\left(\frac{k_m}{k_a}\right)\left[\left(\frac{N_h}{N_l} + \frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)} - \frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]\end{array}\right\},\end{aligned} \tag{A113}$$

and

$$\frac{c_m}{k_m} = -\frac{1}{\widetilde{\Delta}}\frac{1}{k_m}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right)\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l} + \frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)} - \frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]. \tag{A114}$$

Hence,

$$\frac{dc_m}{dk_m}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_m}+\frac{c_m}{k_m}$$
$$<\frac{1}{\widetilde{\Delta}}\frac{1}{l_m}\frac{c_m}{k_m-k_a}\frac{r}{k_m-k_a}\left\{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{k_a}{A_k(a^*)}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]\right\}<0. \quad (A115)$$

Therefore, $\frac{dc_h(a)}{dk_m}<0$.

Since $a\geq a^*$,

$$\frac{dc_h(a)}{dk_a} = \frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{A_h(a)}{A_k(a)}\left(\frac{dc_m}{dk_a}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_a}-\frac{a}{A_k(a)}c_m\right)$$
$$\leq \frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{A_h(a)}{A_k(a)}\left(\frac{dc_m}{dk_a}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_a}-\frac{a^*}{A_k(a^*)}c_m\right). \quad (A116)$$

In the above equation,

$$\frac{dc_m}{dk_a}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_a}$$
$$=\frac{1}{\widetilde{\Delta}}\left\{\begin{array}{l}\frac{1}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{k_m}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]\\ +\frac{1}{k_a}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]\\ \qquad\times\left(1+r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda+\int_{a^*}^1\int_{c_h(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda\right]\right)\end{array}\right\}$$
$$-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{\widetilde{\Delta}}\frac{1}{k_m}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right)\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{k_m}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]$$
$$=\frac{1}{\widetilde{\Delta}}\frac{1}{k_a}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left\{\begin{array}{l}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{k_a}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]\\ +\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]\\ \qquad\times\left(1+r\left[\int_0^{a^*}\int_{c_l(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda+\int_{a^*}^1\int_{c_h(a)}^1\frac{ak_a}{c[A_k(a)]^2}dcda\right]\right)\end{array}\right\}$$
$$<\frac{1}{\widetilde{\Delta}}\frac{1}{k_a}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{r}{k_m-k_a}\left\{\begin{array}{l}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{k_a}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]\\ +\ln\left(\frac{k_m}{k_a}\right)\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]\end{array}\right\}, \quad (A117)$$

The expression inside the big curly bracket of the above equation equals a positive term plus

$$-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{k_a}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]+\ln\left(\frac{k_m}{k_a}\right)\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\ln\left(\frac{A_k(a^*)}{k_a}\right)$$
$$=\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left\{-\frac{k_a}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right]+\ln\left(\frac{k_m}{k_a}\right)\frac{N_h}{N_l}\right\}$$

$$=\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left[\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]^{-1}\frac{N_h}{N_l}\left[\ln\left(\frac{A_k(a^*)}{k_a}\right)\ln\left(\frac{k_m}{k_a}\right)+\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{a^*k_a}{A_k(a^*)}-\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{(1-a^*)k_m}{A_k(a^*)}\right]$$
$$=\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left[\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]^{-1}\frac{N_h}{N_l}\left\{\ln\left(\frac{k_m}{k_a}\right)\left(\ln\left(\frac{A_k(a^*)}{k_a}\right)-\frac{(1-a^*)k_m}{A_k(a^*)}\right)+\ln\left(\frac{A_k(a^*)}{k_a}\right)\right\} \quad (A118)$$

where, to derive the first and the second equalities, (14) is used.

The expression inside the large curly bracket of the above equation can be expressed as

$$\ln\left(\frac{k_m}{k_a}\right)\left[\ln\left((1-a^*)\frac{k_m}{k_a}+a^*\right)-(1-a^*)\left((1-a^*)+a^*\left(\frac{k_m}{k_a}\right)^{-1}\right)^{-1}\right]+\ln\left((1-a^*)\frac{k_m}{k_a}+a^*\right), \quad \text{(A119)}$$

whose value at $\frac{k_m}{k_a}=1$ equals 0 and its derivative with respect to $\frac{k_m}{k_a}$ equals

$$\left(\frac{k_m}{k_a}\right)^{-1}\left[\ln\left((1-a^*)\frac{k_m}{k_a}+a^*\right)-(1-a^*)\left((1-a^*)+a^*\left(\frac{k_m}{k_a}\right)^{-1}\right)^{-1}\right]+(1-a^*)\left((1-a^*)\frac{k_m}{k_a}+a^*\right)^{-1}$$
$$+\ln\left(\frac{k_m}{k_a}\right)\left[(1-a^*)\left((1-a^*)\frac{k_m}{k_a}+a^*\right)^{-1}-(1-a^*)\left((1-a^*)+a^*\left(\frac{k_m}{k_a}\right)^{-1}\right)^{-2}a^*\left(\frac{k_m}{k_a}\right)^{-2}\right]$$
$$=\left(\frac{k_m}{k_a}\right)^{-1}\ln\left((1-a^*)\frac{k_m}{k_a}+a^*\right)+\ln\left(\frac{k_m}{k_a}\right)(1-a^*)^2\frac{k_m}{k_a}\left((1-a^*)\frac{k_m}{k_a}+a^*\right)^{-2}>(<)0 \text{ when } \frac{k_m}{k_a}>(<)1. \quad \text{(A120)}$$

Hence, (A119) is positive and thus (A117) is negative. Therefore, $\frac{dc_h(a)}{dk_a}<0$ from (A116).

[Effects on earnings] Since $w_h=\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\frac{r}{c_m}=\frac{h}{k_a}\frac{r}{c_a}$, $\frac{dw_h}{dk_m}>0$ from $\frac{dc_a}{dk_m}<0$. The sign of $\frac{dw_h}{dk_a}$ is opposite to that of $\frac{d(c_m\frac{A_l(a^*)}{A_h(a^*)})}{dk_a}=\frac{A_l(a^*)}{A_h(a^*)}(\frac{dc_m}{dk_a}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{dk_a})$, which is negative from the proof of $\frac{dc_h(a)}{dk_a}<0$ above. Hence, $\frac{dw_h}{dk_a}>0$.

Since $w_l=\frac{l_m}{k_m}\frac{r}{c_m}$, $\frac{dw_l}{dk_a}>0$ from $\frac{dc_m}{dk_a}<0$. As for the effect of a proportional change in k_m and k_a on w_l,

$$\frac{d(k_mc_m)}{dk_m}|_{\frac{k_a}{k_m}\text{ fixed}}=k_m\frac{dc_m}{dk_m}|_{\frac{k_a}{k_m}\text{ fixed}}+c_m$$
$$=\frac{1}{\tilde{\Delta}}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]\left(1-\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right)\right)<0, \quad \text{(A121)}$$

from (A103) and (A105). Thus, $\frac{dw_l}{dk_a}|_{\frac{k_a}{k_m}\text{ fixed}}=\frac{k_m}{k_a}\frac{dw_l}{dk_m}|_{\frac{k_a}{k_m}\text{ fixed}}>0$. Then, the result that w_l increases when $\frac{k_a}{k_m}$ non-decreases can be proved in a similar way as the result for c^* when $hk_m-l_mk_a<0$.

[Effects on Y] Y decreases with the RHS and the LHS of (HL) from (8). The RHS of (HL) when $c^*<c_a<1$ equals $\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)$ from (24) in the proof of Lemma 1, whose derivative with respect to k_m is

$$\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{dc_m}{dk_m}+\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)-\frac{A_l(a^*)}{A_h(a^*)}\frac{k_m-k_a}{A_k(a^*)}\right]\frac{da^*}{dk_m}$$
$$-\frac{1}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_mk_a}{(k_m-k_a)^2}\ln\left(\frac{A_k(a^*)}{k_a}\right)+\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{c_m}{k_m-k_a}\frac{1-a^*}{A_k(a^*)}$$

$$
\begin{aligned}
&< \frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{1}{\tilde{\Delta}}\left\{\begin{array}{l} -\frac{k_a}{k_m^2}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{k_m}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right] \\ +\frac{1}{k_m}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right)\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]\end{array}\right\} \\
&-\frac{k_a}{k_m^2}\frac{k_m}{l_m}\frac{1}{k_m-k_a}\frac{1}{\tilde{\Delta}}\left[\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)-\frac{A_l(a^*)}{A_h(a^*)}\frac{k_m-k_a}{A_k(a^*)}\right]\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right)\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{k_m}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right] \\
&-\frac{1}{\tilde{\Delta}}\frac{A_l(a^*)}{A_h(a^*)}\frac{k_a}{(k_m-k_a)^2}\frac{1}{l_m}\left[-\ln\left(\frac{A_k(a^*)}{k_a}\right)+\frac{k_m}{k_a}\frac{(1-a^*)(k_m-k_a)}{A_k(a^*)}\right]\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right)\frac{k_m}{l_m}\frac{c_m}{k_m-k_a} \\
&\times\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right],
\end{aligned}
\tag{A122}
$$

where (A103), (A104), and (A105) are used to derive the last equation, which can be expressed as

$$
\begin{aligned}
&\frac{1}{\tilde{\Delta}}\left\{\begin{array}{c}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right) \\ -\left[\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)-\frac{A_l(a^*)}{A_h(a^*)}\frac{k_m-k_a}{A_k(a^*)}\right]\ln\left(\frac{k_m}{k_a}\right)\end{array}\right\}\frac{r}{(k_m-k_a)^2}\frac{k_a}{k_m^2}\left(\frac{k_m}{l_m}\right)^2\frac{c_m}{k_m-k_a}\frac{k_m}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right] \\
&+\frac{1}{\tilde{\Delta}}\frac{k_m}{(k_m-k_a)^2}\left[\ln\left(\frac{A_k(a^*)}{k_a}\right)-\left(\frac{A_k(a^*)}{k_a}\right)^{-1}-1\right]\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right)\frac{k_m}{l_m^2}\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right] \\
&=\frac{1}{\tilde{\Delta}}\left\{-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{k_m}{A_k(a^*)}\right)\ln\left(\frac{A_k(a^*)}{k_a}\right)+\frac{A_l(a^*)}{A_h(a^*)}\frac{k_m-k_a}{A_k(a^*)}\ln\left(\frac{k_m}{k_a}\right)\right\}\frac{k_a}{k_m^2}\left(\frac{k_m}{l_m}\right)^2\frac{rc_m}{(k_m-k_a)^3}\frac{k_m}{A_k(a^*)}\left[-\frac{N_h}{N_l}a^*+\frac{A_l(a^*)}{A_h(a^*)}(1-a^*)\frac{k_m}{k_a}\right] \\
&+\frac{1}{\tilde{\Delta}}\frac{k_m}{(k_m-k_a)^2}\left[\ln\left(\frac{A_k(a^*)}{k_a}\right)-\left(\frac{A_k(a^*)}{k_a}\right)^{-1}-1\right]\frac{A_l(a^*)}{A_h(a^*)}\frac{rc_m}{(k_m-k_a)^2}\ln\left(\frac{k_m}{k_a}\right)\frac{k_m}{l_m^2}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]<0.
\end{aligned}
\tag{A123}
$$

Hence, the RHS of (HL) decreases with k_m and thus Y increases with k_m.

The LHS of (HL) when $c^* < c_a < 1$ equals $\frac{N_h}{N_l}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)$ from (21) in the proof of Lemma 1, whose derivative with respect to k_a is

$$
\frac{N_h}{N_l}\frac{k_m}{l_m}\left[\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left(\frac{dc_m}{dk_a}+\frac{c_m}{k_m-k_a}\right)+\frac{c_m}{A_k(a^*)}\left(\frac{da^*}{dk_a}-\frac{a^*}{k_m-k_a}\right)\right],
\tag{A124}
$$

where $\frac{dc_m}{dk_a}<0$ and $\frac{da^*}{dk_a}<0$ from (i), and

$$
\frac{c_m}{(k_m-k_a)^2}\left(\ln\left(\frac{k_m}{A_k(a^*)}\right)-\frac{a^*(k_m-k_a)}{A_k(a^*)}\right)=\frac{c_m}{(k_m-k_a)^2}\left(\ln\left(\frac{k_m}{A_k(a^*)}\right)+1-\frac{k_m}{A_k(a^*)}\right)<0.
\tag{A125}
$$

Hence, the LHS of (HL) decreases with k_a and thus Y increases with k_a. ■

Proof of Proposition 7. : The results for c_m, a^*, w_l, and $\frac{w_h}{w_l}$ are proved in Proposition 3. $c_l(a)$ decreases with $\frac{N_h}{N_l}$ from $\frac{dc_m}{d\frac{N_h}{N_l}}<0$. Henceforce, remaining results are proved when $\frac{k_a}{k_m}\neq 1$, but they can be proved similarly when $\frac{k_a}{k_m}=1$ too.

When $c^*=c_a=1$ and $\frac{k_a}{k_m}\neq 1$, from (A12) in the proof of Proposition 4 and (18),

$$\begin{pmatrix} da^* \\ dc_m \end{pmatrix} = \frac{1}{\widehat{\Delta}} \begin{pmatrix} -\frac{1}{c_m}\left[1 - r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1 \frac{1}{cA_k(a)} dcda\right] & -\frac{N_h}{N_l}\frac{1}{(1-\frac{k_a}{k_m})l_m}\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a - l_m\frac{k_a}{k_m}}\right] \\ -\frac{l_m}{k_m}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{r}{c_m}\int_{a^*}^1 \frac{1}{A_h(a)}da & \frac{N_h}{N_l}\frac{1}{A_l(a^*)} + \frac{1}{A_h(a^*)} \end{pmatrix}$$

$$\times \begin{pmatrix} -\frac{N_l}{N_h}\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right) \\ 0 \end{pmatrix} d\frac{N_h}{N_l}, \tag{A126}$$

where $\widehat{\Delta} < 0$, as shown in the proof of the proposition.

When $c^* < c_a = 1$ and $\frac{k_a}{k_m} \neq 1$, which arises only when $\frac{k_a}{k_m} < \frac{h}{l_m}$, from (A48) in the proof of Proposition 5 and (16),

$$\begin{pmatrix} da^* \\ dc_m \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} -\frac{1}{c_m}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) & \frac{-1}{(h-l_m)c_m}\ln\left[1+\frac{(h-l_m)hk_m\left[c_m - \frac{l_m}{h}\frac{k_a}{k_m}\frac{A_h(a^*)}{A_l(a^*)}\right]}{l_m\frac{A_h(a^*)}{A_l(a^*)}(hk_m - l_m k_a)}\right] \\ -\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) & \frac{k_m c_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)} - \frac{\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[1+\frac{(k_m-k_a)\frac{A_h(a^*)}{A_k(a^*)}\left[\frac{l_m}{k_m}\frac{A_k(a^*)}{A_l(a^*)} - c_m\right]}{\frac{(hk_m - l_m k_a)c_m}{A_k(a^*)}}\right]\right] \end{pmatrix}$$

$$\times \begin{pmatrix} -\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) \\ 0 \end{pmatrix} d\frac{N_h}{N_l}, \tag{A127}$$

where $\Delta < 0$, as shown in the proof of the proposition.

When $c^*, c_a < 1$ and $\frac{k_a}{k_m} \neq 1$, from (A103) in the proof of Proposition 6 and (14),

$$\begin{pmatrix} da^* \\ dc_m \end{pmatrix} = \frac{1}{\widetilde{\Delta}} \begin{pmatrix} -\frac{r}{c_m(k_m-k_a)}\ln\left(\frac{k_m}{k_a}\right) & 0 \\ -\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right) & \frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)} - \frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right] \end{pmatrix}$$

$$\times \begin{pmatrix} -\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) \\ 0 \end{pmatrix} d\frac{N_h}{N_l}, \tag{A128}$$

where $\widetilde{\Delta} < 0$, as shown in the proof of the proposition.

[**Effects on** w_h and c_a] As shown below, $\frac{dc_h(a)}{d\frac{N_h}{N_l}} > 0$. Thus, $\frac{dc_a}{d\frac{N_h}{N_l}} > 0$, and since $c_a = \frac{h}{k_a}\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}c_m = \frac{h}{k_a}\frac{r}{w_h}$ from (12) and (13), $\frac{dw_h}{d\frac{N_h}{N_l}} < 0$.

[**Effect on** c^*] When $\frac{k_a}{k_m} \leq \frac{l_a}{l_m}$, c^* decreases from the proof of Proposition 3.

When $\frac{k_a}{k_m} \geq \frac{h}{l_m}$ and thus $c_a < c^* < 1$, if $\frac{k_a}{k_m} \neq 1$, from (A128),

$$\frac{dc^*}{d\frac{N_h}{N_l}} \propto \frac{d\frac{A_l(a^*)}{A_k(a^*)}c_m}{d\frac{N_h}{N_l}} = \frac{\partial\frac{A_l(a^*)}{A_k(a^*)}}{\partial a^*}c_m\frac{da^*}{d\frac{N_h}{N_l}} + \frac{A_l(a^*)}{A_k(a^*)}\frac{dc_m}{d\frac{N_h}{N_l}}$$

$$= \frac{1}{\widetilde{\Delta}}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left[\frac{\partial\frac{A_l(a^*)}{A_k(a^*)}}{\partial a^*}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right) + \frac{A_l(a^*)}{A_k(a^*)}\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]$$

$$= -\frac{1}{\widetilde{\Delta}}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{1}{(A_k(a^*))^2}\left[(l_m k_a - l_a k_m)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right) + \frac{A_l(a^*)(l_m k_a - hk_m)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right] > 0. \tag{A129}$$

[Effect on $c_h(a)$]

$$\frac{dc_h(a)}{d\frac{N_h}{N_l}}=\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{A_h(a)}{A_k(a)}\left(\frac{dc_m}{d\frac{N_h}{N_l}}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{d\frac{N_h}{N_l}}\right). \tag{A130}$$

When $c^* < c_a = 1$, from (A127), the expression inside the big parenthesis of the above equation equals

$$\frac{dc_m}{d\frac{N_h}{N_l}}-\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}c_m\frac{da^*}{d\frac{N_h}{N_l}}=-\frac{1}{\Delta}\frac{k_m}{l_m}\frac{c_m r}{(k_m-k_a)^2}\left[\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]^2\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{A_l(a^*)}{A_h(a^*)}>0. \tag{A131}$$

When $c^*, c_a < 1$, the same equation holds from (A128). Hence, $\frac{dc_h(a)}{d\frac{N_h}{N_l}} > 0$.

[Effect on Y] $Y = \frac{N_h}{\iint_{n_h(a,c)>0}\frac{1}{A_h(a)}dadc} = \frac{N_l}{\iint_{n_l(a,c)>0}\frac{1}{A_l(a)}dadc}$ from (8). When $c^* = c_a = 1$ and $\frac{k_a}{k_m} \neq 1$, $\frac{N_h}{\iint_{n_h(a,c)>0}\frac{1}{A_h(a)}dadc} = \frac{N_h}{\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)}$ from Lemma 3, whose derivative with respect to $\frac{N_h}{N_l}$ with constant $N_h + N_l$ equals $\left[\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\right]^{-2}$ times

$$\begin{aligned}
&\frac{N_l^2}{N}\frac{1}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)+\frac{N_h}{A_h(a^*)}\frac{da^*}{d\frac{N_h}{N_l}}\\
&=\frac{N_l}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\frac{1}{\widehat{\Delta}}\left\{\begin{array}{c}-\frac{N_l}{N}\left[\begin{array}{c}\left(\frac{N_h}{N_l}\frac{1}{A_l(a^*)}+\frac{1}{A_h(a^*)}\right)\frac{1}{c_m}\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right]\\ +\frac{\frac{N_h}{N_l}}{(1-\frac{k_a}{k_m})k_m}\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{r}{c_m}\int_{a^*}^1\frac{da}{A_h(a)}\end{array}\right]\\ +\frac{1}{A_h(a^*)}\frac{1}{c_m}\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right]\end{array}\right\}\\
&=-\frac{N_l}{h-l_m}\ln\left(\frac{h}{A_h(a^*)}\right)\frac{1}{\widehat{\Delta}}\frac{N_h}{N}\left\{\begin{array}{c}\left(\frac{1}{A_l(a^*)}-\frac{1}{A_h(a^*)}\right)\frac{1}{c_m}\left[1-r\int_0^{c_l^{-1}(1)}\int_{c_l(a)}^1\frac{dcda}{cA_k(a)}\right]\\ +\frac{1}{(1-\frac{k_a}{k_m})k_m}\ln\left[1+\frac{1-c_m}{c_m}\frac{(1-\frac{k_a}{k_m})l_m}{l_a-l_m\frac{k_a}{k_m}}\right]\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{r}{c_m}\int_{a^*}^1\frac{da}{A_h(a)}\end{array}\right\}>0,
\end{aligned} \tag{A132}$$

where (A126) and the definition of $\widehat{\Delta}$, respectively, in the proof of this proposition and Proposition 4 are used to derive the first equality. Thus, Y increases.

When $c^* < c_a = 1$ and $\frac{k_a}{k_m} \neq 1$, $\frac{N_h}{\iint_{n_h(a,c)>0}\frac{1}{A_h(a)}dadc} = \frac{N_h}{\text{RHS of (16)}}$ from Lemma 2, whose derivative with respect to $\frac{N_h}{N_l}$ with constant $N_h + N_l$ equals $[\text{RHS of (16)}]^{-2}$ times

$$\begin{aligned}
&\frac{N_l^2}{N}RHS-N_h\left(\frac{\partial RHS}{\partial a^*}\frac{da^*}{d\frac{N_h}{N_l}}+\frac{\partial RHS}{\partial c_m}\frac{dc_m}{d\frac{N_h}{N_l}}\right)\\
&=\frac{N_l^2}{N}RHS-N_h\left(\begin{array}{c}-\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left\{\frac{A_l(a^*)}{A_h(a^*)}\frac{k_m-k_a}{A_k(a^*)}-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left[\frac{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right\}\frac{da^*}{d\frac{N_h}{N_l}}\\ +\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{k_m-k_a}\ln\left[\frac{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\frac{dc_m}{d\frac{N_h}{N_l}}\end{array}\right),
\end{aligned} \tag{A133}$$

where (37) and (39) in the proof of Lemma 2 and (16) are used to derive the last equality.

The first term of the above equation equals, using the definition of Δ in the proof of Proposition 5,

$$\frac{N_l^2}{N}RHS=-\frac{1}{\Delta}\frac{N_l}{N}N_h\frac{k_m}{l_m}\frac{c_m r}{(k_m-k_a)^2}\left[\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]^2\left\{\begin{array}{l}\frac{k_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[\frac{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\\ \times\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)+\frac{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}}{(h-l_m)c_m}\ln\left[\frac{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}{\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}h\right]\frac{N_h}{N_l}\end{array}\right\}, \tag{A134}$$

and the second term equals, using (A127),

$$-N_h\left(\begin{array}{l}-\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\left\{\frac{A_l(a^*)}{A_h(a^*)}\frac{k_m-k_a}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\ln\left[\frac{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right\}\frac{da^*}{d\frac{N_h}{N_l}}\\ +\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{k_m-k_a}\ln\left[\frac{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\frac{dc_m}{d\frac{N_h}{N_l}}\end{array}\right)$$

$$=\frac{1}{\Delta}N_h\frac{c_m r}{(k_m-k_a)^2}\left[\frac{k_m}{l_m}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]^2\left(\begin{array}{l}\frac{1}{k_m-k_a}\left\{\frac{A_l(a^*)}{A_h(a^*)}\frac{k_m-k_a}{A_k(a^*)}+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\ln\left[\frac{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right\}\\ \times\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)-\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{k_m-k_a}\ln\left[\frac{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{N_h}{N_l}\end{array}\right)$$

$$=\frac{1}{\Delta}N_h\frac{c_m r}{(k_m-k_a)^2}\left[\frac{k_m}{l_m}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]^2\frac{A_l(a^*)}{A_h(a^*)}\left(\begin{array}{c}\frac{1}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\\ +\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{\frac{A_l(a^*)}{A_h(a^*)}}{k_m-k_a}\ln\left[\frac{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\end{array}\right). \tag{A135}$$

Thus, (A133) equals $-\frac{1}{\Delta}N_h\frac{k_m}{l_m}\frac{c_m r}{(k_m-k_a)^2}\left[\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]^2$ times

$$\frac{N_l}{N}\left\{\begin{array}{c}\frac{k_m}{l_m}\left[\frac{\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}}{A_k(a^*)}+\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[\frac{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\\ +\frac{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}}{(h-l_m)c_m}\ln\left[\frac{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}{\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}h\right]\frac{N_h}{N_l}\end{array}\right\}$$

$$-\frac{k_m}{l_m}\frac{A_l(a^*)}{A_h(a^*)}\left[\frac{1}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)+\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}\frac{\frac{A_l(a^*)}{A_h(a^*)}}{k_m-k_a}\ln\left[\frac{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\right]$$

$$=\frac{N_h}{N}\frac{k_m}{l_m}\frac{1}{A_k(a^*)}\left(1+\frac{A_h(a^*)}{A_l(a^*)}\frac{N_h}{N_l}\right)\left(1-\frac{A_l(a^*)}{A_h(a^*)}\right)+\frac{N_h}{N}\left\{\begin{array}{c}\frac{k_m}{l_m}\left(\frac{A_l(a^*)}{A_h(a^*)}\right)^2\frac{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}}{k_m-k_a}\ln\left[\frac{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}{\frac{(hk_m-l_mk_a)c_m}{A_k(a^*)}}\right]\left(\frac{A_h(a^*)}{A_l(a^*)}-1\right)\\ +\frac{\frac{\partial\frac{A_h(a^*)}{A_l(a^*)}}{\partial a^*}}{(h-l_m)c_m}\ln\left[\frac{(k_m-k_a)\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}+(h-l_m)c_m}{\frac{l_m}{k_m}\frac{A_h(a^*)}{A_l(a^*)}(hk_m-l_mk_a)}h\right]\end{array}\right\}>0. \tag{A136}$$

Hence, (A133) is positive.

When $c^*, c_a<1$ and $\frac{k_a}{k_m}\neq 1$, $\frac{N_h}{\iint_{n_h(a,c)>0}\frac{1}{A_h(a)}dadc}=\frac{N_h}{\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{A_l(a^*)}{A_h(a^*)}\ln\left(\frac{A_k(a^*)}{k_a}\right)}$ from Lemma 1, whose derivative with respect to $\frac{N_h}{N_l}$ with constant N_h+N_l equals (using eq. A128 and the definition of $\widetilde{\Delta}$ in the proof of this proposition and Proposition 6 respectively) $\left[\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{A_l(a^*)}{A_h(a^*)}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]^{-2}$ times

$$\frac{N_l^2}{N}\frac{k_m}{l_m}\frac{c_m}{k_m-k_a}\frac{A_l(a^*)}{A_h(a^*)}\ln\left(\frac{A_k(a^*)}{k_a}\right)$$

$$-N_h\frac{k_m}{l_m}\left[\frac{1}{k_m-k_a}\frac{A_l(a^*)}{A_h(a^*)}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{dc_m}{d\frac{N_h}{N_l}}+c_m\left(\frac{1}{k_m-k_a}\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)-\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{A_k(a^*)}\right)\frac{da^*}{d\frac{N_h}{N_l}}\right]$$

$$=-\frac{1}{\widetilde{\Delta}}\frac{N_l^2}{N}\left(\frac{k_m}{l_m}\right)^2\frac{c_m}{k_m-k_a}\frac{A_l(a^*)}{A_h(a^*)}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{r}{(k_m-k_a)^2}\ln\left(\frac{k_m}{k_a}\right)\left[\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)\frac{k_m-k_a}{A_k(a^*)}-\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\ln\left(\frac{A_k(a^*)}{k_a}\right)\right]$$

$$-\frac{1}{\widetilde{\Delta}}N_h\left(\frac{k_m}{l_m}\right)^2\frac{c_m}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\left[\begin{array}{c}\frac{1}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\\ -\frac{A_l(a^*)}{A_h(a^*)}\frac{1}{A_k(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right)\end{array}\right], \qquad \text{(A137)}$$

where the terms with $\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}$ equal $\frac{1}{\widetilde{\Delta}}\left(\frac{k_m}{l_m}\right)^2\frac{c_m}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}$ times

$$\frac{N_l^2}{N}\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right)\frac{1}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)-N_h\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)$$

$$=N_h\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{r}{k_m-k_a}\left[\frac{N_l}{N}\ln\left(\frac{k_m}{k_a}\right)-\ln\left(\frac{k_m}{A_k(a^*)}\right)\right] \text{ (from Lemma 1)}$$

$$=N_h\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{r}{k_m-k_a}\left[\frac{N_l}{N}\ln\left(\frac{A_k(a^*)}{k_a}\right)-\frac{N_h}{N}\ln\left(\frac{k_m}{A_k(a^*)}\right)\right]$$

$$=N_h\frac{1}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)\frac{r}{k_m-k_a}\frac{N_l}{N}\ln\left(\frac{A_k(a^*)}{k_a}\right)\left(1-\frac{A_l(a^*)}{A_h(a^*)}\right)>0, \text{ (from Lemma 1)} \qquad \text{(A138)}$$

and the terms without $\frac{\partial\frac{A_l(a^*)}{A_h(a^*)}}{\partial a^*}$ equal $-\frac{1}{\widetilde{\Delta}}\left(\frac{k_m}{l_m}\right)^2\frac{A_l(a^*)}{A_h(a^*)}\frac{r}{k_m-k_a}\ln\left(\frac{k_m}{k_a}\right)\frac{1}{A_k(a^*)}$ times

$$\frac{N_l^2}{N}\frac{c_m}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)\left(\frac{N_h}{N_l}+\frac{A_l(a^*)}{A_h(a^*)}\right)-N_h\frac{c_m}{k_m-k_a}\ln\left(\frac{k_m}{A_k(a^*)}\right)$$

$$=\frac{c_m}{k_m-k_a}\ln\left(\frac{A_k(a^*)}{k_a}\right)N_l\frac{N_h}{N}\left(1-\frac{A_l(a^*)}{A_h(a^*)}\right)>0. \qquad \text{(A139)}$$

Hence, the derivative is positive. ■