Cultural Difference, Social Identity, and Redistribution

Preliminary

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Abstract

Empirical evidence suggests that income redistribution may foster economic development by mitigating inequality and enhancing educational investment among the poor. In particular, addressing inequality among different ethnic groups appears pivotal. However, the scale of redistribution limited in many developing countries. Empirical findings suggest two potential explanations: the presence of culturally diverse ethnic groups and weak national identity.

These findings raise various questions. Under what conditions do inter-ethnic cultural disparities diminish over time? Does cultural convergence always lead to increased redistribution and contribute to development? Under what circumstances do widespread national identities emerge? Does national identity always result in increased redistribution? How do cultural differences and national identity interact?

To address these questions, this paper develops and examines a dynamic model of income redistribution and educational investment augmented with cultural change and social identification.

Keywords: culture, development, ethnic inequality, human capital, redistribution, social identify
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1 Introduction

Cross-country differences in economic growth and development are substantial. Empirical evidence shows that income and asset inequalities are negatively related to these performances (Deininger and Squire, 1998). This suggests that redistributive policies may stimulate growth and development by reducing inequality. Indeed, the empirical study by Berg et al. (2018) indicates that income redistribution, unless very large-scale, increases economic growth by lowering income inequality. They also find that lower inequality is associated with higher levels of human capital. Further, Hanushek and Woessmann (2012) find that an increase in educational achievement, measured by cognitive skills, has a large effect on growth. These findings suggest that redistribution may promote growth by reducing inequality and increasing educational investment of the poor. In particular, addressing inequality among different ethnic groups seems to be crucial: Alesina, Michalopoulos, and Papaioannou (2016) discover that inter-group inequality in economic well-being is strongly and negatively related to development, after controlling for inequality across administrative regions, based on cross-sectional data from 173 countries.

However, Berg et al. (2018) find that the scale of redistribution is considerably smaller in many developing countries compared to developed countries. Why is the scale of redistribution limited in many developing countries? There are two potential explanations.

One is the presence of culturally diverse ethnic groups within these countries. Empirical studies show that ethnic heterogeneity negatively influences redistribution, particularly when groups are culturally different. Desmet, Ortuño-Ortín, and Weber (2008), based on cross-country data for 105 countries, reveal statistically and economically significant negative relationships between measures of linguistic diversity that take into account between-group linguistic distances (interpreted as proxies for ethnolinguistic or cultural heterogeneity) and redistribution.[1] Tabellini (2020), using U.S. city-level data from 1910 to 1930, shows that European immigration led to reduced public spending, particularly on items that primarily benefit poor immigrants, and this effect is more pronounced when cultural differences between immigrants and natives are greater.[2]

The other potential explanations is weak national identity. Empirical evidence suggests national identity has a positive effect on redistribution (Transue, 2007; Chen and Li, 2009; Singh, 2015). Transue (2007), through a survey experiment on American whites, finds that those who feel close to the nation are more supportive of a tax increase to improve educational opportunities for minorities, compared to those who feel close to their racial group. Further, he finds that making American identity salient increases their support for the policy. Chen and Li (2009) conduct lab experiments to examine the impact of induced group identity on social preferences and find that participants are more averse to payoff differences to groups they identify with.

These findings raise various questions. Under what conditions do inter-ethnic cultural disparities diminish over time? Does cultural convergence always lead to increased redistribution and contribute to development? Under what circumstances do many people share national identity? Does national identity always result in increased redistribution? How do cultural differences and national identity interact?

[1] This contrasts with insignificant results of theirs and preceding studies for indexes that do not take into account linguistic distances.

[2] Alesina, Murard, and Rapoport (2021), analyzing regional data for 16 Western European countries, discover that native support for redistribution is lower when the share of immigrants in their region is higher, with a stronger negative relationship observed when immigrants are less skilled than natives and originate from Middle Eastern or Eastern European countries. In contrast, Alesina, Miano, and Stantcheva (2023) conduct large-scale surveys and experiments in six countries (France, Germany, Italy, Sweden, the UK, and the US) and find that making immigration salient to respondents led to decreased support for redistribution, but the perceived cultural distance between respondents and immigrants seems to play a minor role in explaining the result.

To address these questions, this paper develops and examines a dynamic model of income redistribution and educational investment augmented with cultural change and social identification, primarily drawing on the models by Shayo (2009) and Yuki (2023).

The seminal paper by Shayo (2009) presents an extension to the standard political economy model of redistributive taxation (Meltzer and Richard, 1981) by incorporating socio-psychological factors. In the model, there are two classes (the poor and the rich) and the government imposes a proportional tax on their incomes to provide a lump-sum transfer, with the tax rate determined through voting. What sets the Shayo model apart from the standard one is that individual utility depends not only on disposable income (consumption) but also negatively on the *perceived distance* between oneself and the group one identifies with (their class or the nation) and positively on the *group's status*. Specifically, individuals incur a cognitive cost when they differ from other members of the group in income level and non-economic attributes, while deriving high utility from membership when the group's status, determined by an exogenous factor and average income level, is high. These socio-psychological components are major determinants of social identification and intergroup behaviors, as indicated by influential theories in social psychology (Tajfel and Turner, 1986; Turner et al., 1987) and empirical evidence (Manning and Roy, 2010; Hett, Mechtel, and Kröll, 2020; Fouka, Mazumder, and Tabellini, 2022).[3] Importantly, because these components vary depending on which group one identifies with, the social identities of people influence the tax rate and thus their disposable incomes. Meanwhile, social identity is *endogenously determined*: one chooses the identity that yields higher utility. Hence, identity, individual outcomes, and aggregate outcomes interact with one another.

The present model diverges from Shayo (2009) mainly in three respects.[4] First, the focus is on ethnic groups rather than economic classes. In the model, there are two ethnic groups, the majority and the minority. Everyone in the majority is skilled and wealthy, while some minority individuals are unskilled and poor.[5] Thus, redistribution reduces ethnic inequality. Individuals choose to identify either with their ethnic group or with the nation.

Second, a non-economic attribute of individuals, called *culture*, evolves endogenously over time. Parents shape their child's culture taking into account the perceived distance between their own culture and the identity group's culture, as well as the cultural distance between themselves and their child.[6] In other words, parents want their child to develop a cultural affinity with the identity group while also preserving a cultural connection with the parent.

Third, the model is dynamic, and variables such as economic class (skilled or unskilled), social identity, the tax rate, and culture, change endogenously over time. Similarly to Yuki (2023), which is a dynamic extension of Shayo (2009) but does not incorporate endogenous culture,[7] the

[3] Evidence suggests that perceived distance and status influence identity. For example, Hett, Mechtel, and Kröll (2020) find that participants in a lab experiment exhibit a preference for groups with smaller social distances and higher social status, with their social identity preferences aligning with their choices in dictator games.

[4] Another important difference from Shayo (2009) is that the perceived distance depends on the difference in disposable income between oneself and the group one identifies with, rather than the difference in pre-tax pre-transfer income. In contrast, status does depend on disposable income in his model too.

[5] To be precise, as explained just below, whether an individual is skilled or unskilled is determined endogenously by educational investment.

[6] The assumption that parents determine their child's culture is made for analytical tractability. However, as explained later, it can capture a significant portion of cultural influences from environments such as neighborhoods and schools on children in the real society.

[7] Yuki (2023) differs from Shayo (2009) and the present model in the following respects. First, unlike Shayo (2009) and the present study, pre-tax pre-transfer incomes are endogenously determined. Second, unlike the two papers, the tax rate is determined based on a probabilistic voting model rather than majority voting. Third, Yuki (2023) also examines the effect of exogenous skill biased technological change on identity, redistribution, and economic outcomes.

dynamic part of the model largely draws on Galor and Zeira (1993) and Yuki (2007, 2008). In the model, individuals, who vary in wealth inherited from their parent, decide on educational spending that must be self-financed and is needed to become skilled workers. As wealth accumulates, the proportion of those who can afford education and become skilled workers may increase over time, thereby influencing the evolutions of other variables. Conversely, wealth accumulation and the proportion of skilled workers are affected by the rate of redistributive tax and other variables.

Based on such a model, the paper examines the dynamics and long-run outcomes of culture, identity, redistribution, and development. Main results are summarized as follows.

First, given other things equal, the rate of redistributive taxation is higher as the proportion of majority individuals identifying with the nation is higher. This result is consistent with the above-mentioned empirical findings (Transue, 2007; Chen and Li, 2009; Singh, 2015).

Second, social identity also influences cultural investment. Children become culturally closer to children from the other ethnic group when their parent identify with the nation rather than with their ethnic group.

As mentioned earlier, social identity is also endogenously determined. One's identity is affected by variables such as one's culture and disposable income, as well as cultural and economic environments of their ethnic group and the nation. Interactions among identity and other variables shape the dynamics and long-run outcomes.

Third, two exogenous parameters, the exogenous component of *national status* and the prominence of culture in the perceived distance, largely determine the dynamics and long-run outcomes.[8] In the actual society, the exogenous component of national status would be high when people believe that they share a glorious history, rich cultural heritage, or a "right" sense of values, because they feel proud of belonging to such a nation

In particular, when the exogenous component of national status is very high, or when inter-ethnic cultural differences are of little concern, everyone always identifies with the nation, and income redistribution is implemented. Consequently, the disposable income of unskilled workers is relatively high, leading to a rapid increase in the proportion of skilled workers in the minority and a swift decline in ethnic inequality. In the long run, everyone is skilled and inter-group income inequality disappears. Inter-ethnic cultural differences also decrease over time and *cultural integration* occurs eventually, where the integrated culture contains elements from both minority and majority origins in proportion to their population shares.

When the exogenous component of national status is lower and inter-ethnic cultural differences are more prominent in people's minds, the majority identify with their own group and redistribution is not initially implemented. Long-run outcomes differ greatly depending on the levels of these exogenous parameters.

If national status is relatively high due to exogenous reasons, all or skilled minority individuals identify with the nation. Consequently, they become culturally closer to the majority over time. Eventually, the diminished cultural distance between the groups lead the majority to switch to national identity and implement redistributive taxation. Thereafter, the share of skilled workers in the minority increases rapidly, and ethnic inequality declines fast. In the long run, income equality is achieved, and cultural integration occurs, although the integrated culture contains a greater share of the majority-origin element compared to the previous case.

Otherwise, the majority consistently identity with their group, and redistribution is never carried out. Thus, the share of skilled workers in the minority grows only slowly or remains constant, and ethnic inequality diminishes slowly or does not disappear even in the long run.

[8] National (ethnic) status also depends on the average disposable income of the nation (ethnic group) and thus is endogenous.

Long-run cultures depend greatly on the levels of the exogenous parameters. When the exogenous component of national status is low but not extremely so and when inter-group cultural differences are prominent but not very so, only the minority identify with the nation and gradually adopt the majority's culture. The long-run outcome is *cultural assimilation*, in which everyone shares the culture originating solely from the majority. When national status is very low due to exogenous reasons or inter-ethnic cultural disparities are significant concerns, both ethnic groups identity with their group and *cultural segregation* persists, where the ethnic groups remain culturally dissimilar.

The above results show that inter-ethnic cultural disparities diminish over time when either ethnic group identifies with the nation. However, cultural convergence leads to the adoption of income redistribution and fosters economic development *only if* the majority hold national identity, which occurs when the exogenous component of national status is sufficiently high, or when the salience of the inter-group cultural distance in people's minds is sufficiently weak. If these conditions are not met and national identity is held solely by the minority, redistribution is not implemented, and the minority's culture is absorbed into the majority's.

Various empirical studies (Blouin and Mukand, 2019; Cáeres-Delpiano et al., 2021; Chen, Lin, and Yang, 2023) indicate that *nation-building policies*, including school education and government propaganda that emphasize shared history, culture, and values, as well as policies promoting inter-group contact, may effectively strengthen national identity. According to the model, these policies elevate national status or deemphasize inter-group cultural differences, playing a critical role in reducing ethnic inequality and fostering development, if such policies are ethnic-neutral or considerate of the minority

This paper contributes to the theoretical literature on the relationship between social identity and redistribution (Shayo 2009; Lindqvist and Östling, 2013; Holm, 2016; Dhami, Manifold, and al-Nowaihi, 2021; and Ghiglino, Júarez-Lunam, and Müller, 2021; Yuki, 2023). The relations with closely-related papers, Shayo (2009) and Yuki (2023), are elucidated above (see footnotes 4 and 7 for additional details). Diverging from existing works, this paper investigates the interactions among cultural and human capital investment, social identity, and redistribution.

More broadly, this paper contributes to the theoretical literature on the relationship between identity and economic behaviors (Akerlof and Kranton, 2000; Shayo, 2009; Benabou and Tirole, 2011; Bernard, Hett, and Mechtel, 2016; Carvalho and Dippel, 2020; Grossman and Helpman, 2020; Bonomi, Gennaioli, and Tabellini, 2021; Yuki, 2021).[9] Generalizing the pioneering work of Akerlof and Kranton (2000), Shayo (2009) constructs the basic analytical framework and applies it to examine the political economy of income redistribution. Shayo's (2009) framework has been applied to various issues. For instance, Grossman and Helpman (2020) develop a political economy model of trade policy with social identification, motivated by a recent reversal of in trade policies in some Western countries seemingly influenced by the rise of populism and ethnic tensions. They explore how policies are affected by changes in identification patterns triggered by events such as increased ethnic tensions.

This work is also related to the large literature on cultural transmission models, which is initiated by Bisin and Verdier (2000) and surveyed by Bisin and Verdier (2011, 2023a). Recent contributions include Spiro (2020), Bisin and Verdier (2023b), Hiller, Wu, and Zhang (2023), and Carvalho, Koyama, and Williams (2024). Apart from investigating different issues, the current study diverges from existing works by incorporating cultural evolution into the Shayo-type model of social identification.

The rest of the paper is organized as follows. To facilitate understanding, Section 2 presents

[9]In addition to those already mentioned, recent empirical and experimental studies on identity include Dehdari and Gehring (2021) and Assouad (2021).

and Section 3 examines a simpler model without social mobility. Section 4 introduces the full-fledged model with social mobility, and Section 5 analyzes the model and interpret the results. The section also discusses policy implications. Section 6 concludes. Appendix A presents a part of the full-fledged model, Appendix B contains lemmas used for proving propositions in Sections 3 and 5, and Appendix C contains proofs.

2 Model without social mobility

For the sake of clarity, this section presents a simplified model. The full-fledged model is presented in Section 4. Consider a society composed of the majority (group 1), the minority (group 2), and a government. The government levies a proportional tax on earnings to finance lump-sum transfers. The tax-transfer policy is determined through majority voting, effectively under the control of group 1. Utility depends not only on one's disposable income but also on socio-psychological components that are influenced by their social identity. Social identity is determined endogenously: one chooses to identify with their ethnic group or the nation. Markets are competitive.

The model is dynamic: it is a deterministic, discrete-time, and OLG world in which individuals live for two periods, first as a child, then as an adult. Each adult has a single child, so the population is constant over time. Children are born with an ethnicity inherited from their parents and a culture that is shaped by, but may differ from, their parents. In this simplified model, children inherit their parents' class (skilled or unskilled) and do not make any decisions. Adults choose social identities, work and receive earnings, vote on the tax-transfer policy, pay taxes and receive transfers, and mold their children's culture. Although the model is dynamic, a time subscript is *not* used in variables in this section to avoid unnecessary complications.

2.1 Environment

In each generation, the total population is 1, of which N_1 $(1 - N_1)$ belongs to group 1 (group 2), where $N_1 > \frac{1}{2}$. All individuals in group 1 are skilled, while some individuals in group 2 are unskilled and others are skilled. The former assumption is made for analytical tractability, but similar results would be obtained as long as the proportion of skilled workers in the majority is higher than the proportion in the minority. Because children inherit their parents' class, the proportion of skilled workers in group 2, H_2, is constant in this section; H_2 is endogenized in the full-fledge model presented in Section 4. Workers supply 1 unit of labor to receive earnings, pay the proportional tax on earnings, receive the lump-sum transfer, and spend disposable income on consumption. Individual i's disposable income is given by

$$y_i = (1-\tau)w_i + T, \tag{1}$$

where w_i is her earnings, $\tau \in [0, 1)$ is the tax rate, and T is the transfer.

The government uses tax revenue entirely for the lump-sum tranfer, but taxation involves deadweight loss. The deadweight loss is assumed to be quadratic, so the governmental budget constraint can be expressed as

$$T = \left(\tau - \frac{1}{2}\tau^2\right)\overline{w}, \tag{2}$$

where $\overline{w}$ is the average earnings of the population.

2.2 Preferences

As in Shayo (2009), individual utility depends not only on disposable income (consumption) but also negatively on the *perceived distance* to a group with which one identifies (either one's ethnic group or the nation) in cultural and economic dimensions, and positively on the *status* of the group. In other words, one incurs a mental cost when one is different from others in the group culturally or economically, but takes pride in belonging to the group when its status is high. Moreover, the utility function includes components associated with their *child's culture*, which reflect the parents' desire for their child to become culturally similar to them while also being culturally close to the group with which they identify. Perceived distance and status are major determinants of social identification and intergroup behaviors, according to influential theories in social psychology (Tajfel and Turner, 1986; Turner et al., 1987) and empirical evidence in economics (Manning and Roy, 2010; Hett, Mechtel, and Kröll, 2020; Fouka, Mazumder, and Tabellini, 2022).[10]

An individual's perception of the distance or proximity to a social group (her ethnic group or the nation) is based on the difference between her disposable income and the group's average disposable income, and the difference between her culture and the group's average culture. The *perceived distance* of individual i, who belongs to ethnic group J ($J = 1, 2$) and class C ($C = S, U$; S [U] is for skilled [unskilled]), to group G ($G = J, N$; N is for the nation) is represented by

$$\begin{aligned} d^i_{JCG} &\equiv |y_C - \overline{y}_G| + \omega_q \big(q^i_{JC} - \overline{q}_G\big)^2 \\ &= (1-\tau)|w_C - \overline{w}_G| + \omega_q \big(q^i_{JC} - \overline{q}_G\big)^2 \quad (\text{from}(1)), \end{aligned} \tag{3}$$

where y_C and w_C are respectively the disposable income and earnings of the class, and $\overline{y}_G$ and $\overline{w}_G$ are the average income and earnings of the group; $q^i_{JC} \in [0, 1]$ and $\overline{q}_G \in [0, 1]$ represent her culture and the group's average culture, respectively; ω_q is the weight on the cultural component. For analytical tractability, the economic distance is measured in absolute value following Ghiglino, Júarez-Lunam, and Müller (2021), while the cultural distance is measured in squared distance.

The *status of the social group* G ($G = J, N$) one identifies with, S_G, depends on the exogenous component $\widetilde{S_G}$ and the average disposable income of the group:

$$S_G = \delta \widetilde{S_G} + \overline{y}_G, \tag{4}$$

where δ is the weight on the exogenous component.[11]

The exogenous component of *national status*, $\widetilde{S_N}$, would be high when people of the nation believe that they share a glorious history or rich cultural heritage, or when the nation achieves commendable performance in international sports competitions, because this fosters a sense of pride among the people. On the other hand, the exogenous component of *ethnic group J's status*, $\widetilde{S_J}$, would be high when the group excels in these dimensions. It would be reasonable to suppose that it is higher for the majority, i.e., $\widetilde{S_1} > \widetilde{S_2}$.

Finally, individual utility depends negatively on the perceived distance of the child's culture, $(q^i_{JC})'$, to one's own culture q^i_{JC}, and to the average culture of the group with which one identifies, $\overline{q}_G$. The composite distance is given by

[10] For the United Kingdom, Manning and Roy (2010) find that the non-white individuals, whose perceived distance to the "average national" would be greater than that of the whites, are less likely to think of themselves as British. They also find that immigrants from poorer and less democratic (i.e., lower status) countries assimilate faster into a British identity. Hett, Mechtel, and Kröll (2020), based on a lab experiment, find that participants in the experiment prefer groups to which they have a smaller social distance and which have a higher social status and their social identity preferences are related to their choices in dictator games. Fouka, Mazumder, and Tabellini (2022) find that migrations of African Americans from the South to non-southern metropolitan areas stimulated assimilation of European immigrants for the years 1910−30. Further, they provide evidence suggesting that higher integration resulted from decreased perceived distance of native whites to European immigrants.

[11] Similar to works such as Grossman and Helpman (2021), status is an absolute measure. By contrast, in Shayo (2009), status is a relative measure and is defined as the difference from the reference group. The main results remain unchanged under the alternative specification.

$$D^i_{JCG} \equiv \chi\left[(q^i_{JC})' - \overline{q}_G\right]^2 + (1-\chi)\left[(q^i_{JC})' - q^i_{JC}\right]^2, \tag{5}$$

where $\chi \in [0,1]$ is the weight on the cultural distance to the identity group. A high value of χ indicates that the parent wants her child to become culturally close to the identity group rather than to her, who might be quite different from the identity group culturally.

Hence, the utility of an individual who belongs to ethnic group J and class C when she identifies with group G is given by

$$u^i_{JCG} = y_C - \beta d^i_{JCG} + \gamma S_G - \rho D^i_{JCG}, \qquad \beta > 1, \gamma, \rho > 0. \tag{6}$$

$\beta > 1$ is assumed to ensure that the rate of redistributive tax is positive under certain situations.

By substituting (1)$-$(5) into the above equation, the utility for a group J individual of each class-identity type can be expressed as (note that $\overline{w}_N$ and $\overline{q}_N$ are denoted as $\overline{w}$ and $\overline{q}$, respectively)

$$\begin{aligned} u^i_{JSN} &= (1-\tau)w_s + T - \beta\left[(1-\tau)(w_s - \overline{w}) + \omega_q\left(q^i_{JS} - \overline{q}\right)^2\right] + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w} + T\right] - \rho\left\{\chi\left[(q^i_{JS})' - \overline{q}\right]^2 + (1-\chi)\left[(q^i_{JS})' - q^i_{JS}\right]^2\right\} \\ &= (1-\tau)w_s + (1+\gamma)T - \beta\left[(1-\tau)(w_s - \overline{w}) + \omega_q\left(q^i_{JS} - \overline{q}\right)^2\right] + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] - \rho\left\{\chi\left[(q^i_{JS})' - \overline{q}\right]^2 + (1-\chi)\left[(q^i_{JS})' - q^i_{JS}\right]^2\right\}, \end{aligned} \tag{7}$$

$$u^i_{JSJ} = (1-\tau)w_s + (1+\gamma)T - \beta\left[(1-\tau)(w_s - \overline{w}_J) + \omega_q\left(q^i_{JS} - \overline{q}_J\right)^2\right] + \gamma\left[\delta\widetilde{S_J} + (1-\tau)\overline{w}_J\right] - \rho\left\{\chi\left[(q^i_{JS})' - \overline{q}_J\right]^2 + (1-\chi)\left[(q^i_{JS})' - q^i_{JS}\right]^2\right\}, \tag{8}$$

$$u^i_{JUN} = (1-\tau)w_u + (1+\gamma)T - \beta\left[(1-\tau)(\overline{w} - w_u) + \omega_q\left(q^i_{JU} - \overline{q}\right)^2\right] + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] - \rho\left\{\chi\left[(q^i_{JU})' - \overline{q}\right]^2 + (1-\chi)\left[(q^i_{JU})' - q^i_{JU}\right]^2\right\}, \tag{9}$$

$$u^i_{JUJ} = (1-\tau)w_u + (1+\gamma)T - \beta\left[(1-\tau)(\overline{w}_J - w_u) + \omega_q\left(q^i_{JU} - \overline{q}_J\right)^2\right] + \gamma\left[\delta\widetilde{S_J} + (1-\tau)\overline{w}_J\right] - \rho\left\{\chi\left[(q^i_{JU})' - \overline{q}_J\right]^2 + (1-\chi)\left[(q^i_{JU})' - q^i_{JU}\right]^2\right\}. \tag{10}$$

2.3 Timing of decisions

The timing of decisions by adult individuals in each period is as follows. First, they choose their social identity, either their ethnic group or the nation, and work to receive earnings;[12] then they vote on the tax-transfer policy; finally, they choose their child's culture $(q^i_{JC})'$. Similar to Shayo (2009), the policy is determined through majority voting, hence by group 1.[13]

The assumption that parents determine their child's culture is made for analytical tractability, but it can capture a large part of cultural influences from environments such as neighborhoods and schools on children in the real society. For instance, a parent with a national identity may expose her child to an ethnically diverse environment in which the child is culturally influenced by the other ethnic group as well; this influence would be captured by $-\rho\left[(q^i_{JC})' - \overline{q}\right]^2$ in the utility function. In contrast, when she has an ethnic identity, the child would be raised in an ethnically segregated environment, influenced by their own ethnic group alone, which would be captured by $-\rho\left[(q^i_{JC})' - \overline{q}_J\right]^2$.

[12]By assumption, individuals do not identify with the nation and their ethnic group simultaneously. Conversely, in the model of Grossman and Helpman (2021), individuals always identify with their class and also identify with the nation if the additional identity increases their utility, which depends on the sum of the perceived distance to and the status of each group with which they identify. The present paper does not adopt this specification owing to the complexities associated with the additional terms and the difficulties in analyzing the model.

[13]Majority voting is assumed for analytical tractability, while Ghiglino, Júarez-Lunam, and Müller (2021) and Yuki (2023) adopt probabilistic voting.

3 Analysis of the model without social mobility

3.1 Children's culture

Because the model can be solved by backward induction, the determination of children's culture is examined first. For the initial generation, assume that $q^i_{1S}=1$ and $q^i_{2S}=q^i_{2U}=0$ for any i. That is, each group is culturally homogenous, and the inter-ethnic cultural distance is highest in the initial period. In subsequent periods, the child's culture is determined by parental investment $(q^i_{JC})'$, which becomes q^i_{JC} in the next period. From (7)−(10), the parent chooses $(q^i_{JC})'$ to minimize the weighted average of its distance to the average culture of the identity group and the distance to her culture, q^i_{JC},[14] which equals

$$\text{When the parent identifies with her ethnic group: } -\rho\left\{\chi\left[(q^i_{JC})'-\overline{q}_J\right]^2+(1-\chi)\left[(q^i_{JC})'-q^i_{JC}\right]^2\right\}, \quad (11)$$

$$\text{When the parent identifies with the nation: } -\rho\left\{\chi\left[(q^i_{JC})'-\overline{q}\right]^2+(1-\chi)\left[(q^i_{JC})'-q^i_{JC}\right]^2\right\}. \quad (12)$$

Under the ethnic identity, from the first-order condition, $(q^i_{JC})'$ is determined by

$$-2\chi\left[(q^i_{JC})'-\overline{q}_J\right]-2(1-\chi)\left[(q^i_{JC})'-q^i_{JC}\right]=0.$$
$$\Leftrightarrow (q^i_{JC})'=\chi\overline{q}_J+(1-\chi)q^i_{JC}. \quad (13)$$

Under the national identity,

$$(q^i_{JC})'=\chi\overline{q}+(1-\chi)q^i_{JC}. \quad (14)$$

Thus, the child's culture is a weighed average of the average culture of the group with which the parent identifies and the parent's culture, where the weight on the former is χ.

3.2 Tax rate

The tax-transfer policy is determined through majority voting. By substituting (2) and (14) into (7), the utility of group 1 individual i when she identifies with the nation is given by

$$u^i_{1SN}=(1-\tau)w_s+(1+\gamma)\left(\tau-\frac{\tau^2}{2}\right)\overline{w}-\beta(1-\tau)(w_s-\overline{w})+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right]-\left[\rho\chi(1-\chi)+\beta\omega_q\right]\left(q^i_{1S}-\overline{q}\right)^2. \quad (15)$$

Her preferred tax rate is obtained by maximizing the above equation:

$$(1-\tau)(1+\gamma)\overline{w}-w_s+\beta(w_s-\overline{w})-\gamma\overline{w}=0,$$
$$\Leftrightarrow \tau=1-\frac{w_s-\beta(w_s-\overline{w})+\gamma\overline{w}}{(1+\gamma)\overline{w}}=\frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}. \quad (16)$$

By substituting (2) and (13) into (8), her utility under the ethnic identity is given by

$$u^i_{1S1}=(1-\tau)w_s+(1+\gamma)\left(\tau-\frac{\tau^2}{2}\right)\overline{w}+\gamma\left[\delta\widetilde{S_1}+(1-\tau)w_s\right]-\left[\rho\chi(1-\chi)+\beta\omega_q\right]\left(q^i_{1S}-\overline{q}_1\right)^2. \quad (17)$$

In this case, the preferred tax rate is 0 because $(1-\tau)(1+\gamma)\overline{w}-w_s-\gamma w_s<0$.

The preferred tax rate is common among all individuals in group 1 because, as shown below, they share the same identity. Since the tax-transfer policy is determined through majority voting, this implies that the implemented tax rate is equal to their preferred rate. The result is summarized as the following lemma.

Lemma 1 *When the majority identify with the nation, $\tau=\frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}>0$, while when they identify with their ethnic group, $\tau=0$.*

[14] For analytical simplicity, the good cost of cultural investment is assumed to be 0.

When the majority identify with the nation, the rate of redistributive tax is positive, whereas when they identify with their ethnic group, the tax rate is 0 and there is no redistribution. This result is consistent with empirical findings that show a positive relationship between national identity and redistribution (Transue, 2007; Chen and Li, 2009; Singh, 2015).[15]

Under national identity, the tax rate increases with β because greater concern for the perceived distance to the "average national" leads the majority to narrow the income gap with the minority more. In contrast, the tax rate decreases with γ because greater concern for national status induces them to mitigate the negative effect of taxation on the national average disposable income and thus national status. The negative effect of the strength of concern for national status, or national pride, on the tax rate aligns with Gustavsson (2019), who finds that national pride is negatively related to support for reducing income inequality after controlling for national identity and political ideology, based on Dutch survey data. It is also consistent with Shayo (2009), who, using survey data, finds that national pride is negatively associated with support for redistribution after controlling for income and education in most of the countries examined. The tax rate is 0 under ethnic identity since the tax reduces the majority's disposable income, does not affect the perceived distance to other majority members, and lowers the group's status.

The following assumption is imposed to ensure that the rate of redistributive tax when the majority identify with the nation is less than $\frac{1}{3}$.[16]

Assumption 1 $\frac{\beta-1}{1+\gamma} \leq \frac{1}{3}$.

This assumption is plausible, considering that the tax revenue is solely used for income redistribution in the model.

3.3 Social identity

Finally, the determination of social identities is examined. The choice of social identity affects one's utility through the perceived distance term, the status term, and the term associated with the child's culture. Thus, one chooses the identity with the higher sum of these terms. (When the level of utility is equal under both identities, it is assumed that one chooses the national identity.) Let p^i_{1S} be the indicator of the national identity for individual i of group 1: $p^i_{1S}=1$ ($p^i_{1S}=0$) if she identifies with the nation (her ethnic group). Similarly, let p^i_{2C} ($C=S,U$) be the indicator of the national identity for individual i who belongs to group 2 and class C.

From (15)$-$(17), $p^i_{1S}=1$ ($p^i_{1S}=0$), i.e., a majority individual identifies with the nation (her ethnic group) iff

$$u^i_{1SN} \geq (<) u^i_{1S1} \Leftrightarrow \gamma\delta\widetilde{S_N} \geq (<) \gamma\delta\widetilde{S_1} + (\beta+\gamma)(1-\tau)(w_s-\overline{w}) + [\rho\chi(1-\chi)+\beta\omega_q](\overline{q}_1-\overline{q})(2q^i_{1S}-\overline{q}_1-\overline{q})$$

$$\Leftrightarrow \widetilde{S_N} \geq (<) \widetilde{S_1} + \frac{1}{\gamma\delta}\left\{(\beta+\gamma)(1-N_1)(1-H_2)(1-\tau)(w_s-w_u) + [\rho\chi(1-\chi)+\beta\omega_q](\overline{q}_1-\overline{q})^2\right\}, \quad \text{(M)}$$

$$\text{where } \tau = \frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}} \quad (\tau=0).$$

[15] Transue (2007), based on a survey experiment on American whites, finds that those who feel close to the nation are more supportive of a tax increase to improve educational opportunities for minorities, compared to those who feel close to their racial group. Further, he finds that making American identity salient increases their support for the policy. Chen and Li (2009) conduct lab experiments to examine the effects of induced group identity on social preferences and find that participants are more averse to payoff differences to groups they identify with. Singh (2015), based on statistical and comparative historical analysis of Indian states, shows that states with a stronger sense of shared identity spend more on education and health.

[16] This is because $\frac{w_s}{\overline{w}} < 2$ when $N_1 > \frac{1}{2}$.

All majority individuals face the same condition and thus make the same identity choice, because $q^i_{1S} = \overline{q}_1$ for any period under the initial condition $q^i_{1S} = 1$.[17] Thereafter, p^i_{1S} and q^i_{1S} are used without superscript i unless necessary.

By substituting (2), (13), and (14) into (7)−(10), the utility for a group 2 individual of each class-identity type is expressed as

$$u^i_{2SN} = (1-\tau)w_s + (1+\gamma)\left(\tau - \frac{\tau^2}{2}\right)\overline{w} - \beta(1-\tau)(w_s - \overline{w}) + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] - [\rho\chi(1-\chi) + \beta\omega_q](\overline{q} - q^i_{2S})^2, \quad (18)$$

$$u^i_{2S2} = (1-\tau)w_s + (1+\gamma)\left(\tau - \frac{\tau^2}{2}\right)\overline{w} - \beta(1-\tau)(w_s - \overline{w}_2) + \gamma\left[\delta\widetilde{S_2} + (1-\tau)\overline{w}_2\right] - [\rho\chi(1-\chi) + \beta\omega_q](\overline{q}_2 - q^i_{2S})^2, \quad (19)$$

$$u^i_{2UN} = (1-\tau)w_u + (1+\gamma)\left(\tau - \frac{\tau^2}{2}\right)\overline{w} - \beta(1-\tau)(\overline{w} - w_u) + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] - [\rho\chi(1-\chi) + \beta\omega_q](\overline{q} - q^i_{2U})^2, \quad (20)$$

$$u^i_{2U2} = (1-\tau)w_u + (1+\gamma)\left(\tau - \frac{\tau^2}{2}\right)\overline{w} - \beta(1-\tau)(\overline{w}_2 - w_u) + \gamma\left[\delta\widetilde{S_2} + (1-\tau)\overline{w}_2\right] - [\rho\chi(1-\chi) + \beta\omega_q](\overline{q}_2 - q^i_{2U})^2. \quad (21)$$

Thus, from (20), (21), and (16), $p^i_{2U} = 1$ ($p^i_{2U} = 0$), i.e., an unskilled minority identifies with the nation (her ethnic group) iff

$$u^i_{2UN} \geq (<) u^i_{2U2} \Leftrightarrow \gamma\delta\widetilde{S_N} \geq (<) \gamma\delta\widetilde{S_2} + (\beta-\gamma)(1-\tau)(\overline{w} - \overline{w}_2) + [\rho\chi(1-\chi) + \beta\omega_q](\overline{q} - \overline{q}_2)[(\overline{q}_2 - q^i_{2U}) + (\overline{q} - q^i_{2U})]$$

$$\Leftrightarrow \widetilde{S_N} \geq (<) \widetilde{S_2} + \frac{1}{\gamma\delta}\{(\beta-\gamma)N_1(1-H_2)(1-\tau)(w_s - w_u) + [\rho\chi(1-\chi) + \beta\omega_q](\overline{q} - \overline{q}_2)[(\overline{q}_2 - q^i_{2U}) + (\overline{q} - q^i_{2U})]\}, \quad \text{(mU)}$$

where $\tau = \frac{\beta-1}{1+\gamma}\frac{w_s - \overline{w}}{\overline{w}}$ when $p_{1S} = 1$, while $\tau = 0$ when $p_{1S} = 0$.

Similary, from (18), (19), and (16), $p^i_{2S} = 1$ ($p^i_{2S} = 0$), i.e., a skilled minority identifies with the nation (her ethnic group) iff

$$u^i_{2SN} > (<) u^i_{2S2}$$

$$\Leftrightarrow \widetilde{S_N} \geq (<) \widetilde{S_2} + \frac{1}{\gamma\delta}\{-(\beta+\gamma)N_1(1-H_2)(1-\tau)(w_s - w_u) + [\rho\chi(1-\chi) + \beta\omega_q](\overline{q} - \overline{q}_2)[-(q^i_{2S} - \overline{q}_2) + (\overline{q} - q^i_{2S})]\}, \quad \text{(mS)}$$

where $\tau = \frac{\beta-1}{1+\gamma}\frac{w_s - \overline{w}}{\overline{w}}$ when $p_{1S} = 1$, while $\tau = 0$ when $p_{1S} = 0$.

In the model with constant H_2, all minority individuals with a given skill level make the same identity choice, because $q^i_{2C} = \overline{q}_{2C}$ ($C = S, U$) for any period under the initial condition $q^i_{2C} = 0$.[18] Thus, thereafter in this section, p^i_{2C} and q^i_{2C} are also used without superscript i unless necessary.

To simplify analysis, the following assumption is imposed on the relationship among several terms in the above equations.

Assumption 2 $\gamma\delta\left(\widetilde{S_1} - \widetilde{S_2}\right) > [\rho\chi(1-\chi) + \beta\omega_q](N_1)^2$

The assumption holds when the exogenous component of the majority's status ($\widetilde{S_1}$) is sufficiently greater than that of the minority ($\widetilde{S_2}$), $N_1 (> \frac{1}{2})$ is relatively small, or the importance of the exogenous component of group status ($\gamma\delta$) is large compared to that of the cultural terms ($\rho\chi(1-\chi) + \beta\omega_q$) in one's utility. The main results remain mostly unchanged qualitatively even when this condition does not hold, although the analysis becomes much more complicated.

[17] To be precise, $q^i_{1S} = \overline{q}_1$ holds in the current period because they make the same identity choices in the previous periods based on (M), starting from the initial period.

[18] As shown in Section 4, when H_2 is endogenous, identity choices and values of the cultural variable can be different among group 2's skilled individuals.

3.3.1 Majority-minority difference in the tendency of national identity

Among the majority, the skilled minority, and the unskilled minority, which group is more likely to identify with the nation? The next proposition examines this question.

Proposition 1 (*i*) *Skilled workers in group 2 are more likely to identify with the nation than unskilled workers.*

(*ii*) *Under Assumption 2,*

(*a*) *Group 1 individuals are less likely to identify with the nation than group 2's skilled workers.*

(*b*) *They are also less likely to identify with the nation than group 2's unskilled workers except when $\beta > \gamma$, $N_1 > \frac{\beta+\gamma}{2\beta}$, and H_2 is small, in which case they can be more likely to have a national identity than the unskilled.*

Proof. See Appendix C. ■

Among the minority, skilled workers are more likely to have a national identity than unskilled workers. This is because the disposable income of skilled workers is at the same level as that of the majority, and thus their perceived distance to the "average national" is smaller compared to the unskilled.

On the other hand, the majority are *less* likely to identify with the nation than the skilled minority and except when $\beta > \gamma$, $N_1 > \frac{\beta+\gamma}{2\beta}$, and H_2 is small, than the unskilled minority (in such a case, the opposite can be true).[19] This is mainly because the majority's status is higher than that of the minority: the majority have higher pride in their cultural accumulation, history, or values than the minority ($\widetilde{S_1} > \widetilde{S_2}$), and their income is higher than the national average. Further, the fact that the income of the skilled minority is closer to the national average than to the minority's average also contributes to the majority's weaker tendency to have a national identity in comparison to the skilled minority. By contrast, the majority's income and cultural distances to the national averages are smaller than those of the unskilled minority (because of the majority's greater population size). Thus, the majority's tendency to have a national identity can be stronger than that of the unskilled minority when the status effect is weak, i.e., $\beta > \gamma$, $N_1 > \frac{\beta+\gamma}{2\beta}$, and H_2 is small.

Note that the majority's weaker tendency to identify with the nation means their lower concerns with the perceived distance to and the welfare of the other ethnic group compared to the minority. In this model, the majority's pride in the nation without considering the minority as part of it is not considered a form of national identity.

Empirical studies examining the majority-minority difference in the tendency of national identity are scarce, and indicators used to measure national identity vary among studies. Masella (2013) employs a measure that is closest to the concept of national identity in the present model: the national identity dummy that equals 1 if, in the case of U.S. data, a respondent chooses "I am an American first and a member of some ethnic group second", as opposed to alternatives such as "Above all I am a White American." in a multiple choice question. Using survey data for 21 countries, he finds that when ethnic diversity is relatively low, minority groups are more likely to identify with the nation than the "majority" (the largest ethnic group), and the opposite is true when ethnic diversity is relatively high. Further, when the sample is divided into the subsample consisting solely of "majority" people and the one comprising only minorities, national identity is negatively related to the size of the respondent's ethnic group for both subsamples. Hence, the theoretical result aligns with his findings, particularly when ethnic diversity is relatively low.[20]

[19] When Assumption 2 is not satisfied, the result may not hold (that is, the majority may have a stronger tendency to have a national identity) especially for large H_2 in the early periods, but it does hold in the longer term.

[20] By contrast, using perceived closeness to one's country as a measure of national identity, Staerklé et al. (2010) find that majority people are more likely to identify with the nation than minorities, although the majority-minority

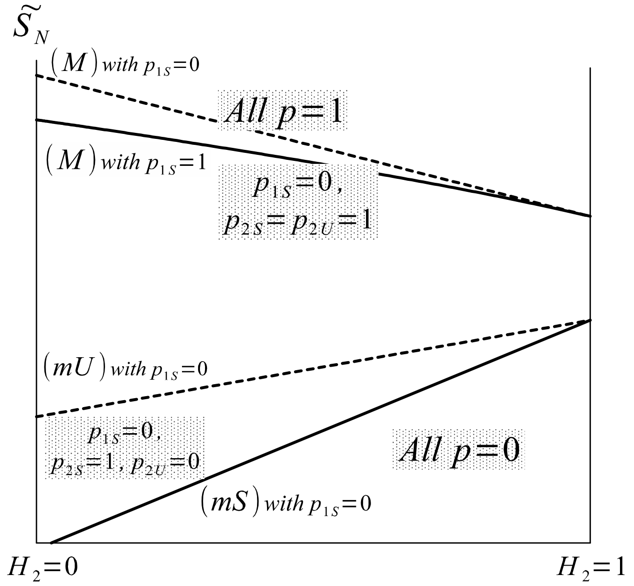


Figure 1: Identity in the initial period when $\beta \leq \gamma$

3.3.2 Dependence of social identity on H_2 and $\widetilde{S_N}$

The equations (M), (mU), and (mS) show that an individual's choice of identity depends on variables such as H_2 and $\widetilde{S_N}$. To analyze how these variables affect identity choices and to prepare for later analysis, it is important to understand the relationship between H_2 and the RHS of each equation. The next lemma shows that the sign of the relationship when $p_{1S}=1$, in which case H_2 affects the RHS through $\tau = \frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}$ as well, is the same as when $p_{1S}=0$ under Assumption 1.

Lemma 2 *Under Assumption 1, the sign of the relationship between H_2 and the RHS of each of (M), (mU), and (mS) when $p_{1S}=1$ is the same as the sign when $p_{1S}=0$.*

Proof. See Appendix C. ■

The assumption, which ensures that the rate of redistributive tax when $p_{1S}=1$ is less than $\frac{1}{3}$, also guarantees that redistribution always increases the disposable income of unskilled workers.

Based on the equations, Proposition 1, and the lemma, it is possible to graphically examine how the identity choices of different types of individuals depend on H_2 and $\widetilde{S_N}$. Figure 1 illustrates identity choices when $\gamma \geq \beta$ on the $(H_2, \widetilde{S_N})$ plane, assuming that society is in the initial period where $q_1^i=1$ and $q_{2S}^i=q_{2U}^i=0$.[21] Note that the same qualitative results hold in subsequent periods. The figure shows that the proportion of individuals having a national identity increases with the level of the exogenous component of national status, $\widetilde{S_N}$. Given H_2, when $\widetilde{S_N}$ is high (the region on and above the upper solid line), everyone identifies with the nation, i.e., $p_{1S}=p_{2S}=p_{2U}=1$. When $\widetilde{S_N}$ is low (the region below the lower solid line), everyone identifies with their ethnic group, i.e., $p_{1S}=p_{2S}=p_{2U}=0$. When $\widetilde{S_N}$ falls within the intermediate range, the majority identify with their ethnic group, while for relatively high $\widetilde{S_N}$ (the region enclosed by the dotted lines), the minority

difference is small.

[21](mU) is located above (mS) on the $(H_2, \widetilde{S_N})$ plane from Proposition 1 (i), and (M) is located above (mU) from (ii)(b) of the proposition. In the initial period, $\overline{q}_1=1$, $\overline{q}_2=0$, and $\overline{q}=N_1$, and the last term of the RHS of (M) equals $\frac{1}{\gamma\delta}\,[\rho\chi\,(1-\chi)+\beta\omega_q](1-N_1)^2$, while that of (mU) and (mS) equals $\frac{1}{\gamma\delta}\,[\rho\chi(1-\chi)+\beta\omega_q](N_1)^2$.

identify with the nation, i.e. $p_{1S}=0$, $p_{2S}=p_{2U}=1$,[22] and for relatively low $\widetilde{S_N}$ (the region between the lower dotted line and the lower solid line), the skilled (unskilled) minority identify with the nation (their ethnic group), i.e., $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=0$. As proven in Proposition 1, the figure shows that the minority are more likely to identify with the nation than the majority, and within the minority, the skilled are more likely to identify with the nation than the unskilled: $p_{1S}=1$ only if $p_{2S}=p_{2U}=1$ and $p_{2U}=1$ only if $p_{2S}=1$.

The figure shows that as the proportion of skilled workers in the minority, H_2, is higher, the regions of mixed national and ethnic identities, i.e., $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=1$ or 0, are smaller, while the region of universal national identity is larger when $\widetilde{S_N}$ is high, and the region of universal *ethnic* identity is larger when $\widetilde{S_N}$ is low. That is, as the average skill level of the minority is higher and thus the inter-ethnic skill gap is smaller, universal national (*ethnic*) identity is more likely to be realized when the exogenous component of national status is high (low). As H_2 is higher, the between-group income disparity is smaller. Thus, the majority's perceived distance to the "average national" is smaller, and national status relative to the majority's status is higher. In contrast, the skilled minority's perceived distance to the "average minority" is smaller, and the relative status of the minority to the nation is higher. When $\widetilde{S_N}$ is high, the effect on the majority dominates, and universal national identity tends to emerge, while when $\widetilde{S_N}$ is low, the effect on the minority dominates, and universal ethnic identity tends to come about.

An increase in the exogenous element of the majority's status $\widetilde{S_1}$ shifts (M) upward on the $(H_2, \widetilde{S_N})$ plane, making universal national identity less likely to be realized. Conversely, an increase in $\widetilde{S_2}$ shifts (mU) and (mS) downward, making universal ethnic identity more likely to be realized.[23]

Taken together with the results on the tax rate in Section 3.2, these findings show that income redistribution is implemented and the disparity in disposable income between ethnic groups is relatively small only when $\widetilde{S_N}$ is sufficiently high. Such a favorable outcome is more likely to occur when H_2 is high and $\widetilde{S_1}$ is low. In other words, when the belief that people share a glorious history or rich cultural heritage is weak and thus national status is low, income redistribution narrowing the inter-ethnic income inequality is not implemented. This is especially true when the inter-ethnic skill gap is large or when the majority has a strong sense of pride in their own culture and history.

Figure 2 presents a similar graph for the case where $\beta > \gamma$ and $N_1 < \frac{\beta+\gamma}{2\beta}$. Unlike the previous figure, (mU) increases with H_2, but the previous results remain unchanged. Figure 3 presents a graph when $\beta > \gamma$, $N_1 > \frac{\beta+\gamma}{2\beta}$, and (mU) is located above (M) for small H_2.[24] Apart from a small difference, the above results remain hold.[25]

These results are summarized in the following proposition.

Proposition 2 (*i*) *When $\widetilde{S_N}$ is high, everyone identifies with the nation, i.e., $p_{1S}=p_{2S}=p_{2U}=1$ (except when $\beta > \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$, in which $p_{1S}=p_{2S}=1$, $p_{2U}=0$ can occur for small H_2); when $\widetilde{S_N}$ is low, everyone identifies with their ethnic group, i.e., $p_{1S}=p_{2S}=p_{2U}=0$; and when $\widetilde{S_N}$*

[22]The figure shows that in the region between the upper dotted line and the upper solid line, both $p_{1S}=p_{2S}=p_{2U}=1$ and $p_{1S}=0$, $p_{2S}=p_{2U}=1$ are equilibria.

[23]An increase in N_1 has multiple effects. It shifts (mS) downward on the $(H_2, \widetilde{S_N})$ plane, making the skilled minority more likely to identify with the nation, thereby reducing the likelihood of universal ethnic identity. An increase in N_1 also shifts (M) downward, increasing the likelihood of universal national identity. However, in the real society, higher N_1 is typically associated with higher $\widetilde{S_1}$, decreasing the likelihood of universal national identity. Hence, the overall effect of N_1 on the realization of universal national identity is ambiguous.

[24]It is possible that (mU) is located above (M) for small H_2 from Proposition 1 (ii)(b). When (M) is located above (mU) for any H_2, a figure similar to Figure 2 can be drawn.

[25]Since the relative position of (mU) to (M) is different for small and large H_2, a new case arises in which the majority and skilled minority identify with the nation, while the unskilled minority identify with their ethnic group, i.e., $p_{1S}=p_{2S}=1$, $p_{2U}=0$, when $\widetilde{S_N}$ is relatively high and H_2 is small.

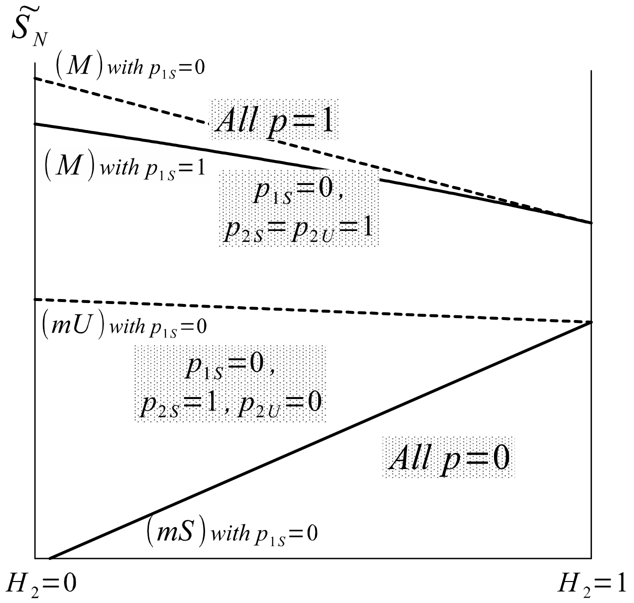


Figure 2: Identity in the initial period when $\beta > \gamma$ and $N_1 < \frac{\beta+\gamma}{2\beta}$

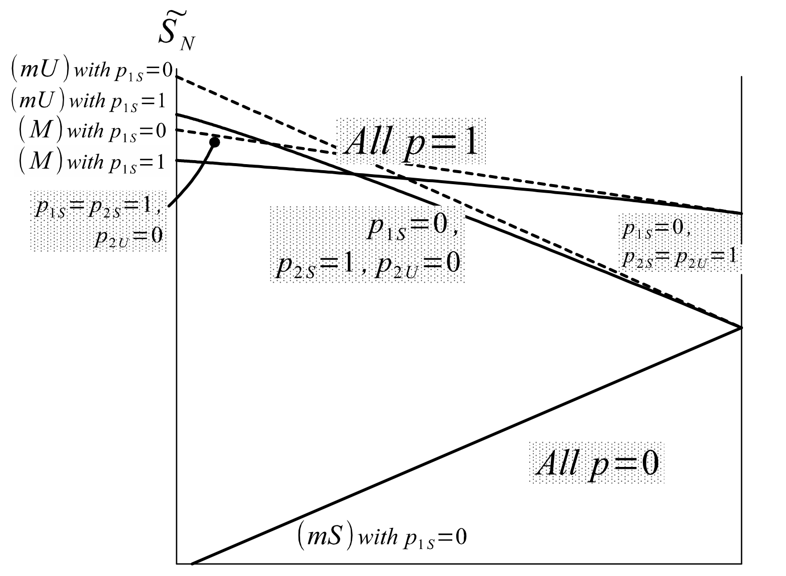


Figure 3: Initial identity when $\beta>\gamma$, $N_1>\frac{\beta+\gamma}{2\beta}$, and (mU) is located above (M) for small H_2

is in the intermediate range, the majority identify with their ethnic group, while all or skilled minority individuals identify with the nation, i.e, $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=1$ *or* 0.

(*ii*) *As* H_2 *is higher, universal national identity is more likely to be realized when* $\widetilde{S_N}$ *is high, while when* $\widetilde{S_N}$ *is low, universal ethnic identity is more likely to be realized.*

(*iii*) *Higher* $\widetilde{S_1}$ *makes universal national identity less likely to occur, while higher* $\widetilde{S_2}$ *makes universal ethnic identity more likely to occur.*

(*iv*) *Income redistribution is implemented only when* $\widetilde{S_N}$ *is sufficiently high. The policy is more likely to be carried out when* H_2 *is high and* $\widetilde{S_1}$ *is low.*

3.4 Dynamics and long-run outcomes

Section 3.3.2 just above has examined how identity choices depend on H_2 and $\widetilde{S_N}$ at a given point in time. However, identity choices evolve over time due to changes in cultural variables. Specifically, as (13) and (14) show, levels of cultural variables, q_{1S}, q_{2S}, and q_{2U}, change over time, and changes in these variables shift graphs of (M), (mU), and (mS) on the $(H_2, \widetilde{S_N})$ plane, influencing identity choices of each type of individuals, p_{1S}, p_{2S}, and p_{2U}. Further, this affects the rate of redistributive tax, then the levels of the cultural variables in the next period. Thus, the dynamics of the cultural variables and identity choices interact with each other. How does this interaction shape social identity and culture in the long run?

Based on Lemmas A1 and A2 in Appendix B and taking into account these interactions, the next proposition presents steady-state levels of the identity and cultural variables for each type of individuals. In the following, superscript $*$ is used to denote steady-state variables.

Proposition 3 (*i*) *When* $\widetilde{S_N}$ *is high,*[26] *everyone identifies with the nation in the long run, i.e.,* $p^*_{1S}=p^*_{2S}=p^*_{2U}=1$, *unless* $\beta>\gamma$, $N_1>\frac{\beta+\gamma}{2\beta}$, *and* H_2 *is low, in which case* $p^*_{1S}=p^*_{2S}=1, p^*_{2U}=0$ *can happen. Cultural integration occurs, i.e., when* $p^*_{1S}=p^*_{2S}=p^*_{2U}=1$, $q^*_{1S}=q^*_{2S}=q^*_{2U}=\overline{q}^{\#}\in(0,1)$, *where* $\overline{q}^{\#}$ *is the value of* $\overline{q}$ *in the first period after which* $p_{1S}=p_{2S}=p_{2U}=1$ *continues to hold.*[27] *As* $\widetilde{S_N}$ *is higher, the proportion of the minority element in the integrated culture is higher, i.e.,* $\overline{q}^{\#}$ *is smaller when* $p^*_{1S}=p^*_{2S}=p^*_{2U}=1$, *with the maximum proportion being* $1-N_1$.

(*ii*) *When* $\widetilde{S_N}$ *is low,*[28] *everyone identifies with their ethnic group in the long run, i.e.,* $p^*_{1S}=p^*_{2S}=p^*_{2U}=0$, *and cultures remain segregated, i.e.,* $q^*_{1S}=1$ *and* $q^*_{2S}=q^*_{2U}=0$.

(*iii*) *When* $\widetilde{S_N}$ *is in the intermediate range,*[29] *the majority identify with their ethnic group, while all or skilled minority individuals identify with the nation in the long run, i.e.,* $p^*_{1S}=0$, $p^*_{2S}=1, p^*_{2U}=0$ *or* 1, *and cultural assimilation occurs, i.e.,* $q^*_{1S}=q^*_{2S}=q^*_{2U}=1$.

(*iv*) *As* H_2 *is higher,* (*i*) [(*ii*)] *is more likely to be realized when* $\widetilde{S_N}$ *is high* (*low*). *Further,* (*i*) *is more likely to occur when* $\widetilde{S_1}$ *is lower, while* (*ii*) *is more likely to occur when* ω_q *and* $\widetilde{S_2}$ *are higher.*

[26] To be accurate, when $(H_2, \widetilde{S_N})$ is located on or above steady-state (M) with $p_{1S}=1$ on the $(H_2, \widetilde{S_N})$ plane.

[27] When $p^*_{1S}=p^*_{2S}=1$, $p^*_{2U}=0$, $q^*_{1S}=q^*_{2S}=q^*_{2U}=\frac{1}{1-(1-H_2)N_1}\left[H_2N_1\overline{q}^{\dagger}_1+(1-N_1)\overline{q}^{\dagger}_2\right]$, where $\overline{q}^{\dagger}_1$ ($\overline{q}^{\dagger}_2$) is the value of $\overline{q}_1$ ($\overline{q}_2$) in the first period after which $p_{1S}=p_{2S}=1$, $p_{2U}=0$ continues to hold.

[28] To be accurate, when $(H_2, \widetilde{S_N})$ is located below initial (mS) with $p_{1S}=0$.

[29] To be accurate, $p^*_{1S}=0, p^*_{2S}=p^*_{2U}=1$ when $(H_2, \widetilde{S_N})$ is located on or above initial (mS) with $p_{1S}=0$ as well as steady-state (mU) with $p_{1S}=0$ and below steady-state (M) with $p_{1S}=0$; $p^*_{1S}=0$, $p^*_{2S}=1, p^*_{2U}=0$ when $(H_2, \widetilde{S_N})$ is located on or above initial (mS) with $p_{1S}=0$ and below steady-state (mU) with $p_{1S}=0$, when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$, also below steady-state (M) with $p_{1S}=0$.

Proof. See Appendix C. ■

The relationships between exogenous variables such as $\widetilde{S_N}$ and H_2 and the long-run social identities are qualitatively similar to the static result of Proposition 2. When $\widetilde{S_N}$ is high, everyone identifies with the nation in the long run (unless $\beta > \gamma$, $N_1 > \frac{\beta+\gamma}{2\beta}$, and H_2 is low, in which case it is possible that the majority and the skilled minority have a national identity and the unskilled minority have an ethnic identity). When $\widetilde{S_N}$ is low, everyone identifies with their ethnic group. When $\widetilde{S_N}$ is in the intermediate range, the majority identify with their ethnic group, while all or skilled minority individuals identify with the nation. As H_2 is higher, universal national (ethnic) identity is more likely to be realized when $\widetilde{S_N}$ is high (low). Further, universal national identity is more likely to occur when $\widetilde{S_1}$ is lower, and universal ethnic identity is more likely to occur when $\widetilde{S_2}$ and ω_q (the weight on the cultural component in perceived distance) are higher.

However, combinations of $\widetilde{S_N}$ and H_2 that lead to specific levels of p_{1S}, p_{2S}, and p_{2U} in the long run do not necessarily coincide with combinations of the variables that yield the same levels of identity variables in the initial or transition periods. This is because changes in the levels of cultural variables shift the positions of graphs (M), (mU), and (mS) over time.

Specifically, when $\widetilde{S_N}$ is high, the region with universal national identity on the $(H_2, \widetilde{S_N})$ plane expands over time as (M) shifts downward (see, for example, Figure 1). Initially, universal national identity is achieved only when society is endowed with a fairly favorable condition, i.e., $\widetilde{S_N}$ is very high or both $\widetilde{S_N}$ and H_2 are high. When society is endowed with a less but relatively favorable condition, i.e., $\widetilde{S_N}$ is relatively but not very high, or $\widetilde{S_N}$ is high but H_2 is low, national identity is held by all or skilled minority individuals only, while the majority identify with their ethnic group initially, i.e., $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=0$ or 1. Because some or all of the minority identify with the nation, they are culturally influenced by the majority, and over time, their culture becomes closer to the majority' culture, i.e., q_{2S} and q_{2U} increase and become closer to $q_{1S}=1$ over time.[30] Due to the decreased inter-ethnic cultural distance, (M) shifts downward, leading the majority to switch from ethnic identity to national identity at some point, resulting in universal national identity.[31]

The proposition also presents the relationship between long-run levels of cultural variables and exogenous variables such as $\widetilde{S_N}$ and H_2. When $\widetilde{S_N}$ is sufficiently high to achieve universal national identity in the long run,[32] *cultural integration* occurs where everyone shares the same culture that contains elements from both minority and majority origins, i.e., $q_{1S}^* = q_{2S}^* = q_{2U}^* \in (0,1)$. The proportion of the minority element in the integrated culture increases with $\widetilde{S_N}$, with the maximum proportion being its population share. When $\widetilde{S_N}$ is low enough that universal ethnic identity is realized in the long run, *cultural segregation* persists, meaning that the ethnic groups remain culturally dissimilar, i.e., $q_{1S}^*=1$ and $q_{2S}^*=q_{2U}^*=0$. When $\widetilde{S_N}$ is in the intermediate range so that the majority identify with their ethnic group and all or skilled minority individuals identify with the nation, *cultural assimilation* occurs, meaning that everyone shares the same culture but the culture *does not* contain the element of the minority origin. As H_2 is higher, cultural integration (segregation) is more likely to occur when $\widetilde{S_N}$ is high (low). Further, cultural integration is more likely to occur when $\widetilde{S_1}$ (the exogenous component of the majority's status) is lower, while cultural segregation is more likely when ω_q and $\widetilde{S_2}$ are higher.

Universal national identity leads to cultural integration because both ethnic groups are culturally influenced by the other group. As $\widetilde{S_N}$ is higher, the integrated culture has a higher share of the

[30]When the unskilled minority identify with their ethnic group, they are culturally influenced by the majority through the skilled minority, who identify with the nation.

[31]When $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=0$, this occurs after switching to $p_{1S}=0$, $p_{2S}=p_{2U}=1$, which is a result of a decrease in the cultural distance among the minority.

[32]Possibly $p_{1S}^*=p_{2S}^*=1, p_{2U}^*=0$ as well when $\beta>\gamma$, $N_1>\frac{\beta+\gamma}{2\beta}$, and H_2 is low.

minority element because the periods during which the majority identify with their ethnic group and thus cultural assimilation proceeds (i.e., q_{2S} and q_{2U} increase with q_{1S} unchanged) are shorter. In contrast, universal ethnic identity leads to cultural segregation because there are no cultural interactions between the ethnic groups. Cultural assimilation occurs when national identity is held by some or all minority individuals but not by the majority, because cultural influence only flows from the majority to the minority.

Finally, the above result has the following implications for the tax rate and disposable incomes of skilled and unskilled workers.

Corollary 1 *When $\widetilde{S_N}$ is high, $p_{1S} = 1$ and $\tau = \frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}$ always or after some point in time, whereas when $\widetilde{S_N}$ is not high, $p_{1S} = 0$ and thus $\tau = 0$ always. The disposable income of unskilled (skilled) workers is higher (lower) and ethnic inequality is lower when $p_{1S} = 1$. $\tau = \frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}$ is more likely to occur when $\widetilde{S_1}$ is low and H_2 is high.*

When $\widetilde{S_N}$ is high, the majority identify with the nation, and thus, the rate of redistributive tax is positive from the beginning or becomes positive after some point in time. On the other hand, when $\widetilde{S_N}$ is not high, they identify with their ethnic group and the tax rate is always 0. Consequently, when $\widetilde{S_N}$ is high, the disposable income of unskilled (skilled) workers is higher (lower) and thus the inter-ethnic inequality in disposable income is smaller.

While social identity has only a static economic effect, i.e., the effect on present disposable incomes and inequalities, in the model with constant H_2, it has important dynamic economic effects in the model with endogenous H_2, as will be shown later.

4 Model with social mobility

So far, for ease of exposition, the model assumed that children inherit their parents' classes and thus H_2 is constant. This section removes this unrealistic assumption and introduces educational investment that can alter H_2 over time. The newly introduced elements of the model are based on Galor and Zeira (1993) and Yuki (2007, 2008), in which individuals with varying levels of wealth inherited from their parents decide on educational spending that must be self-financed and is required to become skilled workers. In this model, H_2 may increase over time and social identity has important dynamic effects on economic outcomes.

4.1 Model

Consider an OLG economy in which individuals live for two periods. The basic structure of the model is similar to the previous model, but several additional settings are introduced to incorporate assets that are transferred intergenerationally and are used for educational investment. Appendix A presents the production part of the model and shows that the skilled wage w_s, the unskilled wage w_u, and the interest rate r are constant.

Lifetime of an individual is described as follows. In childhood, she receives a transfer b from her parent and allocates it to assets a and educational expenditure e, which is required for becoming a skilled worker, to maximize the utility given by (22) below.[33] She considers not just the impact of her investment decision on future income but also on the socio-psychological components of her utility. The educational investment is binary (i.e. taking education or not), costs $\overline{e}$, and yields a gross economic return of $w_s - w_u$. The investment must be self-financed due to the absence of credit markets. Thus, when $b < \overline{e}$, she does not expend on education, i.e., $e = 0$, and becomes an unskilled worker. In adulthood, the individual earns income from assets and work and spends it

[33]For individuals in the initial generation, b is given.

on consumption c and a transfer to her child b'. As in the previous model, she also chooses a group with which she identifies, votes on the tax-transfer policy, and molds her child's culture.

When she belongs to ethnic group J ($J=1,2$) and class C ($J=S,U$), and identifies with group G ($G=J,N$), she maximizes the following utility subject to the budget constraint:

$$\max\ v^i_{JCG} = \frac{1}{(\lambda)^\lambda(1-\lambda)^{1-\lambda}}(b')^\lambda(c)^{1-\lambda} - \beta d^i_{JCG} + \gamma S_G - \rho D^i_{JCG}, \quad \lambda \in (0,1), \tag{22}$$

$$s.t. \quad c + b' = (1-\tau)w_C + T + (1+r)a, \tag{23}$$

where, as before, d^i_{JCG} is her perceived distance, S_G is the status of the identity group, and D^i_{JCG} is the composite distance of the child's culture.

By solving the maximization problem, the following consumption and transfer rules are obtained.

$$c = (1-\lambda)[(1-\tau)w_C + T + (1+r)a], \tag{24}$$

$$b' = \lambda[(1-\tau)w_C + T + (1+r)a]. \tag{25}$$

The results on identity choice and the tax rate remain the same as before, since the indirect utility function equals the utility function of the original model plus $(1+r)a$.[34]

4.2 Determination of H_2

From the above setting, H_2 is equal to the proportion of group 2 individuals who receive $b \geq \bar{e}$ *and* spend $e = \bar{e}$ in childhood. Let F_2 be the proportion of those who receive $b \geq \bar{e}$. Then, if the utility gain from educational investment is non-negative even when everyone with $b \geq \bar{e}$ takes education, $H_2 = F_2$ holds, whereas if the utility gain is negative with $H_2 = F_2$, H_2 is smaller than F_2. In the present model, the proportion of skilled workers in the majority, H_1, is also endogenously determined, under the assumption that $F_1 = 1$ in the initial period.

The following assumption is imposed regarding the economic return to education.

Assumption 3 $\frac{2}{3}(w_s - w_u) - (1+r)\bar{e} > 0$.

The assumption posits that taking education raises disposable income, in other words, the economic return to education is positive, even when the rate of redistributive tax is $\frac{1}{3}$, the highest possible value under Assumption 1.

The next lemma shows how H_1 and H_2 are determined.

Lemma 3 *Suppose $F_1 = 1$. Under Assumption 3:*

(i) $H_1 = 1$.[35]

(ii) (a) When $p_{2S} = p_{2U} = 1$ or when $p_{1S} = 1, p_{2S} = 1, p_{2U} = 0$, $H_2 = F_2$.

(b) If $\widetilde{S_N} - \widetilde{S_2}$ is not large and thus $p_{1S} = 0, p_{2S} = 0$ or $1, p_{2U} = 0$ holds, when $\widetilde{S_N} - \widetilde{S_2}$ is relatively large, $H_2 = F_2$; when $\widetilde{S_N} - \widetilde{S_2}$ is relatively low, $H_2 = 0$ for small F_2 and $H_2 = F_2$ for larger F_2;

[34] Asset income is assumed to be untaxed to keep these results unchanged and to be consistent with the fact that it is less heavily taxed than labor income.

[35] $H_1 = 0$ is also an equilibrium because the utility gain from education becomes negative when everyone with $b \geq \bar{e}$ does not take education. However, this equilibrium appears unrealistic and uninteresting, thus is not considered. The same applies to H_2 as well.

further, when $\beta < \gamma$ *and* $N_1 > \frac{2\beta}{\beta+\gamma}$, *and* $\widetilde{S_N} - \widetilde{S_2}$ *is in the intermediate range,* $H_2 = F_2$ *for small and large* F_2 *and* $H_2 \in (0, F_2)$ *for intermediate* F_2.[36]

Proof. See Appendix C. ■

As for the majority group, $H_1 = 1$ when $F_1 = 1$. Under Assumption 4 (i) below, $F_1 = 1$ for all periods when $F_1 = 1$ initially, thus $H_1 = 1$ always. The result for the minority depends on social identity and $\widetilde{S_N} - \widetilde{S_2}$, the difference between the exogenous component of national status and that of the minority's status. When $p_{2S} = p_{2U} = 1$ or when $p_{1S} = 1, p_{2S} = 1, p_{2U} = 0$, $H_2 = F_2$ for any F_2. In contrast, if $\widetilde{S_N} - \widetilde{S_2}$ is not large and thus $p_{1S} = 0, p_{2S} = 0$ or $1, p_{2U} = 0$ holds (see, for example, Figure 1), when $\widetilde{S_N} - \widetilde{S_2}$ is relatively large, $H_2 = F_2$, whereas when $\widetilde{S_N} - \widetilde{S_2}$ is relatively low, $H_2 = 0$ for small F_2 and $H_2 = F_2$ for larger F_2. (Further, when $\beta < \gamma$ and $N_1 > \frac{2\beta}{\beta+\gamma}$, and $\widetilde{S_N} - \widetilde{S_2}$ is in the intermediate range, $H_2 = F_2$ for small and large F_2 and $H_2 \in (0, F_2)$ for intermediate F_2.) That is, people may not take education despite a *positive* economic return to education. When $p_{1S} = p_{2S} = p_{2U} = 0$, this occurs because the socio-psychological return to education is large negative when the proportion of individuals able to afford education is low, due to the large (small) perceived distance of a skilled (an unskilled) worker to one's ethnic group. When $p_{1S} = 0, p_{2S} = 1, p_{2U} = 0$ and $\widetilde{S_N} - \widetilde{S_2}$ is relatively small, this is the case because the small perceived distance of an unskilled worker to one's ethnic group and the minority's high status make it psychologically attractive to forgo education and identify with their ethnic group.

4.3 Dynamics of F_2 and H_2

Given the distribution of b over the population in the initial period and thus the initial value of F_2, the dynamics of F_2 are determined by the dynamics of b of each lineage. Consider an individual who is born in period $t-1$ and spends her adulthood in period t. Her investment decisions depend on the received transfer and the sign of the utility return to education, which is determined by conditions in Lemma 3 (henceforth, subscript t represents variables for those *born* in period t):

$$\text{If } b_t < \bar{e} \text{ or the utility return to education is negative,} \quad a_t = b_t,\ e_t = 0, \tag{26}$$

$$\text{If } b_t \geq \bar{e} \textit{ and } \text{the utility return to education is positive,} \quad a_t = b_t - \bar{e},\ e_t = \bar{e}. \tag{27}$$

By substituting (26) into (25), the dynamic equation linking the received transfer b_t to the transfer to her child b_{t+1} when she does not take education and thus remains unskilled equals

$$b_{t+1} = \lambda[(1-\tau_t)w_u + T_t + (1+r)b_t]. \tag{28}$$

Similarly, the corresponding equation when she is a skilled worker is

$$b_{t+1} = \lambda[(1-\tau_t)w_s + T_t + (1+r)(b_t - \bar{e})]. \tag{29}$$

When $H_{2,t} = F_{2,t}$, $F_{2,t+1} \geq F_{2,t}$ holds if all the children of skilled workers can afford education, i.e., for any lineage satisfying $b_t \geq \bar{e}$, $b_{t+1} \geq \bar{e}$. From (29), this is the case if

$$\lambda[(1-\tau_t)w_s + T_t] \geq \bar{e}. \tag{30}$$

[36]To be more precise, when $\beta \geq \gamma$ or $N_1 \leq \frac{2\beta}{\beta+\gamma}$, and $\widetilde{S_N} - \widetilde{S_2}$ is relatively low, $H_2 = 0$ for $F_2 < H_2^{\Diamond\Diamond}(\widetilde{S_N} - \widetilde{S_2})$ when $p_{1S} = 0, p_{2S} = 1, p_{2U} = 0$ and for $F_2 < H_2^{\Diamond}$ when $p_{1S} = p_{2S} = p_{2U} = 0$, where $H_2^{\Diamond\Diamond\prime}(\widetilde{S_N} - \widetilde{S_2}) < 0$ and $H_2^{\Diamond\Diamond}(\widetilde{S_N} - \widetilde{S_2}) = H_2^{\Diamond}$ on the dividing line between $p_{2S} = 1$ and $p_{2S} = 0$. When $\beta < \gamma$ and $N_1 > \frac{2\beta}{\beta+\gamma}$, and $\widetilde{S_N} - \widetilde{S_2}$ is in the intermediate range, $H_2 = H_2^{\Diamond\Diamond}(\widetilde{S_N} - \widetilde{S_2})$ for $F_2 \in (H_2^{\Diamond\Diamond}(\widetilde{S_N} - \widetilde{S_2}), H_2^{\Diamond})$ (where $H_2^{\Diamond\Diamond\prime}(\widetilde{S_N} - \widetilde{S_2}) > 0$), whereas when $\widetilde{S_N} - \widetilde{S_2}$ is relatively small, $H_2 = 0$ for $F_2 < H_2^{\Diamond}$. See the proof for the definitions of $H_2^{\Diamond\Diamond}(\widetilde{S_N} - \widetilde{S_2})$ and $H_2^{\Diamond}$ and the critical levels of $\widetilde{S_N} - \widetilde{S_2}$.

As shown later, this is true under Assumption 4 (i) below. (This also implies that $H_1 = F_1 = 1$ always, given $F_1 = 1$ in the initial period.)

$F_{2,t+1} > F_{2,t}$ is true when there exist lineages satisfying $b_t < \overline{e}$ and $b_{t+1} \geq \overline{e}$. From (28), such lineages exist only if $\lambda\{(1-\tau_t)w_u + T_t + (1+r)b_t\} \geq \overline{e}$ holds for some $b_t < \overline{e}$, which is the case when ($\lambda(1+r) < 1$ is assumed)

$$\frac{\lambda}{1-\lambda(1+r)}\left[(1-\tau_t)w_u + T_t\right] > \overline{e}. \tag{31}$$

By contrast, $F_{2,t+1} = F_{2,t}$ if $b_{t+1} = \lambda\{(1-\tau_t)w_u + T_t + (1+r)b_t\} < \overline{e}$ is true for any $b_t < \overline{e}$, which is the case when

$$\frac{\lambda}{1-\lambda(1+r)}\left[(1-\tau_t)w_{ut} + T_t\right] \leq \overline{e}. \tag{32}$$

When $H_2 = F_2$, these conditions determine the dynamics of H_2, while when $H_2 < F_2$, the level of H_2 is determined by Lemma 3 above. To simplify analysis, the following assumption is imposed hereafter.

Assumption 4 *(i) β or w_s is large enough or γ is small enough that $\frac{\lambda}{1-\lambda(1+r)}[(1-\tau)w_u + T]$ at $H_2 = 0$ when $p_{1S} = 1$ is sufficiently larger than $\overline{e}$.*[37]

(ii) A not-small proportion of group 2 individuals do not have wealth in the initial period.

The first part of the assumption implies that the scale of redistribution when $p_{1S} = 1$ is large enough (due to the rate of redistributive tax $\tau = \frac{\beta-1}{1+\gamma}\frac{w_s - \overline{w}}{\overline{w}}$ and/or the tax base $\overline{w}$ being large) that descendants of unskilled workers can gradually accumulate wealth sufficient for education, as long as H_2 is not large. This assumption ensures that H_2 increases over time, at least for not large H_2. The second part states that a not-small proportion of minority individuals in the initial generation do not have wealth. The next lemma shows that the second part of the assumption, together with the first part, ensures that the tax-redistribution policy leads to $H_2 = 1$ in the long run. It also shows that H_2 non-decreases over time, even without redistribution when $H_2 = F_2$.

Lemma 4 *Under Assumption 4, when $p_{1S} = 1$ and thus $\tau = \frac{\beta-1}{1+\gamma}\frac{w_s - \overline{w}}{\overline{w}}$, H_2 increases over time and becomes 1 in the long run, while when $p_{1S} = 0$ and thus $\tau = 0$, H_2 non-decreases over time when $H_2 = F_2$.*

Proof. See Appendix C. ■

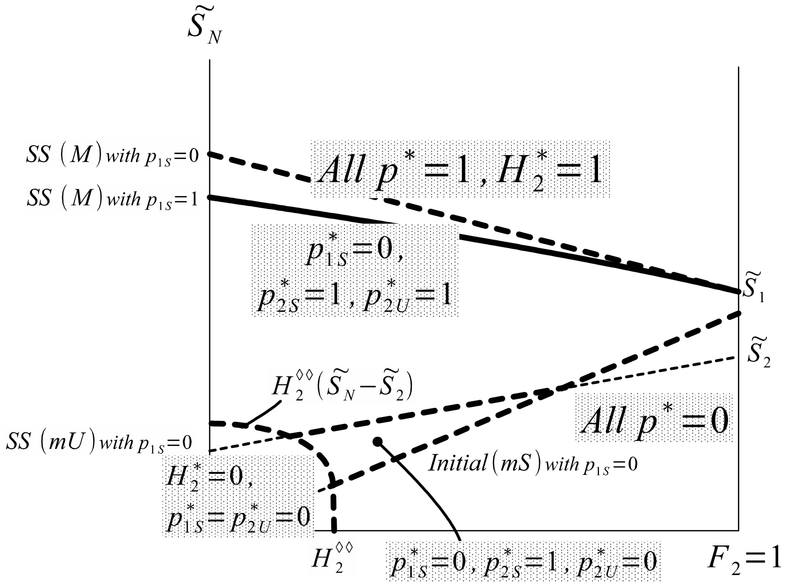
5 Analysis of the model with social mobility

Based on Lemmas 3 and 4, this section analyzes the model with social mobility in which H_2 is an endogenous variable and may change over time. This section imposes the following assumption.

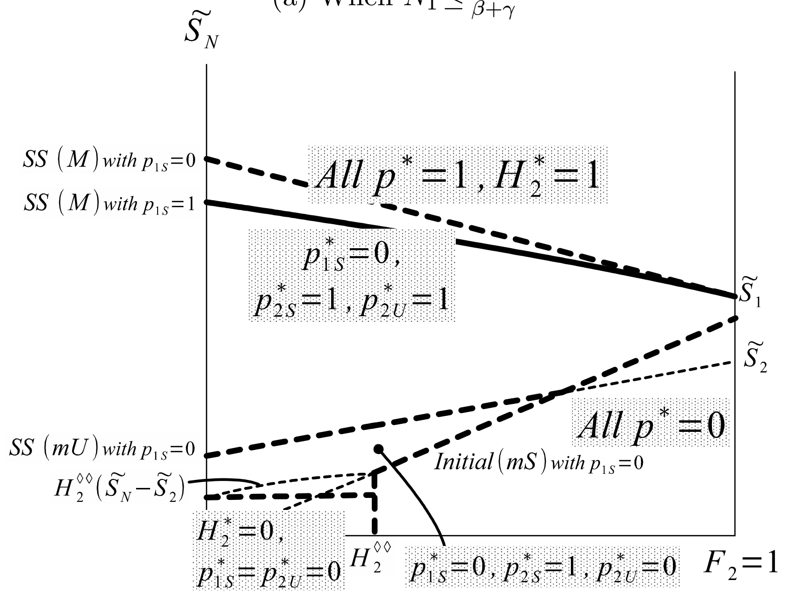
Assumption 5 *χ is sufficiently large, and the distribution of wealth is not very concentrated in a narrow range.*

As shown in Lemma A3 in Appendix B, this assumption ensures that the graphs of (mU) and (mS) shift in the same directions on the $(H_2, \widetilde{S_N})$ plane as in the model with constant H_2.

[37] To be precise, $\frac{\lambda}{1-\lambda(1+r)}\left\{w_u + \frac{\beta-1}{1+\gamma}\frac{(1-N_1)(w_s-w_u)^2}{N_1 w_s + (1-N_1)w_u}\left[N_1 - \frac{1}{2}\frac{\beta-1}{1+\gamma}(1-N_1)\right]\right\}$ is sufficiently greater than $\overline{e}$.



(a) When $N_1 \leq \frac{2\beta}{\beta+\gamma}$



(b) When $N_1 > \frac{2\beta}{\beta+\gamma}$

Figure 4: Relation between initial $(F_2, \widetilde{S_N})$ and steady-state outcomes for the full-fledged model with $\frac{\lambda}{1-\lambda(1+r)}w_u \leq \overline{e}$ when $\beta < \gamma$

5.1 When $\frac{\lambda}{1-\lambda(1+r)}w_u \leq \overline{e}$

First, the case in which the unskilled wage w_u is small enough or the cost of education $\overline{e}$ is large enough that $\frac{\lambda}{1-\lambda(1+r)}w_u \leq \overline{e}$ is analyzed. In this case, when $p_{1S}=0$ and thus $\tau=0$, F_2 and H_2 are time-invariant. H_2 increases over time only when $p_{1S}=1$ and thus $\tau>0$.

The next proposition, which corresponds to Proposition 3 for the model with constant H_2, shows how social identities and cultural variables in the steady state depend on exogenous parameters such as $\widetilde{S_N}$ and ω_q (the weight on the cultural component in the perceived distance), as well as F_2 in the initial period.

Proposition 4 *Suppose that* $\frac{\lambda}{1-\lambda(1+r)}w_u \leq \overline{e}$ *holds.*

(i) When $\widetilde{S_N}$ *is high,*[38] *everyone identifies with the nation in the long run, i.e.,* $p_{1S}^*=p_{2S}^*=p_{2U}^*=1$.

[38]The exact condition is that $(F_2, \widetilde{S_N})$ is located on or above steady-state (M) with $p_{1S}=1$ on the $(F_2, \widetilde{S_N})$ plane.

Cultural integration occurs, i.e., $q_{1S}^*=q_{2S}^*=q_{2U}^*=\overline{q}^*\geq\overline{q}^{\#}\in(0,1)$*, where* $\overline{q}^{\#}$ *is the value of* $\overline{q}$ *in the first period after which* $p_{1S}=p_{2S}=p_{2U}=1$ *continues to hold and* $\overline{q}^*\in(0,1)$*.* $\overline{q}^*=\overline{q}^{\#}$ *when* $p_{2S}=p_{2U}=1$ *initially. As* $\widetilde{S_N}$ *is higher,* $\overline{q}^{\#}$ *is lower, with the minimum value being* N_1*.*

(*ii*) *When* $\widetilde{S_N}$ *is low,*[39] *everyone identifies with their ethnic group in the long run, i.e.,* $p_{1S}^*=p_{2S}^*=p_{2U}^*=0$*, or* $H_2^*=0$ *and* $p_{1S}^*=p_{2U}^*=0$*, and cultures remain segregated, i.e.,* $q_{1S}^*=1, q_{2S}^*=q_{2U}^*=0$*.*

(*iii*) *When* $\widetilde{S_N}$ *is in the intermediate range,*[40] *the majority identify with their ethnic group, while all or skilled minority individuals identify with the nation in the long run, i.e.,* $p_{1S}^*=0$*,* $p_{2S}^*=1$*,* $p_{2U}^*=0$ *or* 1*, and cultural assimilation occurs, i.e.,* $q_{1S}^*=q_{2S}^*=q_{2U}^*=1$*.*

(*iv*) (*i*) *is more likely to be realized when* $\widetilde{S_1}$ *is lower, and* (*ii*) *is more likely to be realized when* ω_q *and* $\widetilde{S_2}$ *are higher.* (*i*) *and* (*ii*) *are more likely to occur when initial* F_2 *is higher.*

Proof. See Appendix C. ■

Similar to the model with constant H_2, when the exogenous component of the national status, $\widetilde{S_N}$, is high, everyone identifies with the nation in the long run, i.e., $p_{1S}^*=p_{2S}^*=p_{2U}^*=1$, and *cultural integration* occurs, i.e. $q_{1S}^*=q_{2S}^*=q_{2U}^*\in(0,1)$; when $\widetilde{S_N}$ is low, everyone identifies with their ethnic group in the long run, i.e., $p_{1S}^*=p_{2S}^*=p_{2U}^*=0$ or $H_2^*=0$ and $p_{1S}^*=p_{2U}^*=0$, and *cultures remain segregated*, i.e., $q_{1S}^*=1, q_{2S}^*=q_{2U}^*=0$; and when $\widetilde{S_N}$ falls within the intermediate range, the majority identify with their ethnic group, while all or skilled minority individuals identify with the nation in the long run, i.e., $p_{1S}^*=0$, $p_{2S}^*=1$, $p_{2U}^*=0$ or 1, and *cultural assimilation* occurs, i.e., $q_{1S}^*=q_{2S}^*=q_{2U}^*=1$.

Universal national identity is more likely to be realized when the exogenous component of the majority's status, $\widetilde{S_1}$, is lower, while universal ethnic identity is more likely to occur when ω_q and $\widetilde{S_2}$ are higher, i.e., people are more concerned about the cultural distance to others and the minority is more proud of their group for exogenous reasons. Further, as initial F_2 is higher, universal national (ethnic) identity is more likely to be realized when $\widetilde{S_N}$ is relatively high (low).

Figure 4 illustrates the relationship between $(F_2,\widetilde{S_N})$ in the initial period and social identities in the steady state for the full-fledge model with $\frac{\lambda}{1-\lambda(1+r)}w_u\leq\overline{e}$ when $\beta<\gamma$. Panels (a) and (b) depict the relationship for the case $N_1\leq\frac{2\beta}{\beta+\gamma}$ and $N_1>\frac{2\beta}{\beta+\gamma}$, respectively. (The relationship for the case $\beta\geq\gamma$ is shown in Figures 9 and 10 in the proof of the proposition.)

As shown in the figure, there are several differences from the the model without social mobility. First, when $\widetilde{S_N}$ is high, $p_{1S}^*=p_{2S}^*=1, p_{2U}^*=0$ is not the steady state. This occurs as H_2 increases over time and becomes 1 in the long run due to $p_{1S}=1$ and thus $\tau>0$. Second, when $\widetilde{S_N}$ is low, $H_2^*=0$ and $p_{1S}^*=p_{2U}^*=0$ is the steady state for low F_2. This is because the socio-psychological return to education is large negative when $\widetilde{S_N}$ and F_2 are low, as explained after Lemma 3. Third, when cultural integration occurs, $q_{1S}^*=q_{2S}^*=q_{2U}^*\geq\overline{q}^{\#}$, where $\overline{q}^{\#}$ is the value of $\overline{q}$ in the first period after which $p_{1S}=p_{2S}=p_{2U}=1$ continues to hold, except when $p_{2S}=p_{2U}=1$ initially, in which case $q_{1S}^*=q_{2S}^*=q_{2U}^*=\overline{q}^{\#}$ as before. That is, the share of the majority-origin element in the integrated culture tends to be higher than the previous model.

[39] To be precise, when $\beta\leq\gamma$ or $N_1\leq\frac{\beta+\gamma}{2\beta}$, this is the case when $(F_2,\widetilde{S_N})$ is below initial (mS) with $p_{1S}=0$ or when F_2 and $\widetilde{S_N}$ are small enough that $F_2\leq H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$, where $H_2^{\Diamond\Diamond\prime}(\widetilde{S_N}-\widetilde{S_2})\leq 0$. When $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$, the condition is that $(F_2,\widetilde{S_N})$ is below initial (mS) with $p_{1S}=0$ for $F_2\geq H_2^{\Diamond}$ and $\widetilde{S_N}-\widetilde{S_2}$ is smaller than the level satisfying $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})=0$ for $F_2<H_2^{\Diamond}$, where $H_2^{\Diamond\Diamond\prime}(\widetilde{S_N}-\widetilde{S_2})>0$. See the proof for the definitions of $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$ and $H_2^{\Diamond}$.

[40] $p_{1S}^*=0$, $p_{2S}^*=1$, $p_{2U}^*=0$ or 1 when $(F_2,\widetilde{S_N})$ is above the region for $p_{1S}^*=p_{2S}^*=p_{2U}^*=0$ or $H_2^*=0$ and $p_{1S}^*=p_{2U}^*=0$, and below steady-state (M) with $p_{1S}=0$, when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$, also below steady-state (mU) with $p_{1S}=0$.

The next proposition examines how the dynamics and long-run level of H_2 and ethnic inequality depend on exogenous parameters such as $\widetilde{S_N}$ and ω_q.

Proposition 5 *Suppose that* $\frac{\lambda}{1-\lambda(1+r)} w_u \leq \bar{e}$ *holds.*

(*i*) *When* $\widetilde{S_N}$ *is very high so that* $p_{1S} = p_{2S} = 1, p_{2U} = 0$ *or* 1 *in the initial period.*[41]

(*a*) *The majority identify with the nation, i.e.,* $p_{1S} = 1$*, and* $\tau > 0$ *always.* H_2 *increases and ethnic inequality declines over time, and everyone becomes skilled workers, i.e.,* $H_2^* = 1$*, and inter-group inequality disappears in the long run.*

(*b*) *This case is more likely to be realized when* ω_q *and* $\widetilde{S_1}$ *are lower and initial* F_2 *is higher.*

(*ii*) *When* $\widetilde{S_N}$ *is not very high, the majority identify with their ethnic group initially.*

(*a*) *If* $\widetilde{S_N}$ *is relatively high,*[42] *the majority switch to a national identity,* τ *becomes positive,* H_2 *starts increasing, and ethnic inequality begins to decline at some point. In the long run,* $H_2^* = 1$ *and ethnic inequality disappears.*

(*b*) *Otherwise,* $\tau = 0$*,* F_2 *is constant* (H_2 *is constant in most occasions*)*,* $H_2^* < 1$*, and ethnic inequality persists.*

(*c*) (*b*) *is more likely to hold as* $\widetilde{S_1}$ *is higher* (*when* $\beta > \gamma$ *and* $N_1 > \frac{\beta+\gamma}{2\beta}$*, also* $\widetilde{S_2}$ *is higher*) *and initial* F_2 *is lower.*

Proof. See Appendix C. ■

When $\widetilde{S_N}$ is very high, the majority consistently identify with the nation and the rate of redistributive taxation is positive. Thus, the disposable income of unskilled workers is high enough for their descendants to gradually accumulate wealth sufficient for education. Consequently, the proportion of the skilled minority increases and inter-ethnic income inequality declines over time. In the long run, everyone is skilled, i.e., $H_2^* = 1$, and ethnic inequality disappears. This case is more likely to occur when ω_q and $\widetilde{S_1}$ are lower and initial F_2 is higher, in other words, when people are less concerned about cultural differences with others, the majority's pride in their group is weaker for exogenous reasons, and the initial share of the minority accessible to education is higher.

If $\widetilde{S_N}$ is not very high, but relatively so, the majority identify with their ethnic group and thus income redistribution is not initially implemented. Hence, none of the children of unskilled workers can afford education, and H_2 stays constant. In contrast, since skilled or all minority individuals identify with the nation, they are culturally influenced by the majority, leading to a decrease in the cultural distance between the groups over time. When the cultural distance diminishes sufficiently, due to the relatively high $\widetilde{S_N}$, the majority begins to adopt a national identity, and income redistribution starts to be implemented. Thereafter, the proportion of the skilled minority increases and ethnic inequality decreases over time, and $H_2^* = 1$ and inter-group equality are attained eventually.

Otherwise, the majority never identity with the nation, redistribution is never implemented, and the share of the minority accessible to education remains unchanged. Hence, some or all of the minority remain unskilled and ethnic inequality persists even in the long run. This case is more likely to hold as $\widetilde{S_1}$ is higher (when $\beta > \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$, also $\widetilde{S_2}$ is higher) and initial F_2 is lower.

[41] To be precise, when $(F_2, \widetilde{S_N})$ in the initial period is on or above initial (M) with $p_{1S} = 1$ on the $(F_2, \widetilde{S_N})$ plane.

[42] To be precise, when initial $(F_2, \widetilde{S_N})$ is on or above steady-state (M) with $p_{1S} = 1$ and below initial (M) with $p_{1S} = 0$.

5.2 When $\frac{\lambda}{1-\lambda(1+r)}w_u > \overline{e}$

When $\frac{\lambda}{1-\lambda(1+r)}w_u > \overline{e}$, F_2 always increases over time and $H_2^* = 1$. The next proposition analyzes social identities and cultural compositions in the steady state.

Proposition 6 *Suppose that* $\frac{\lambda}{1-\lambda(1+r)}w_u > \overline{e}$ *holds.*

(*i*) *When* $\widetilde{S_N}$ *is high,*[43] $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$. $q_{1S}^* = q_{2S}^* = q_{2U}^* = \overline{q}^* \geq \overline{q}^{\#} \in (0,1)$, *where* $\overline{q}^{\#}$ *is the value of* $\overline{q}$ *in the first period after which* $p_{1S} = p_{2S} = p_{2U} = 1$ *continues to hold and* $\overline{q}^* \in (0,1)$. $\overline{q}^* = \overline{q}^{\#}$ *when* $p_{2S} = p_{2U} = 1$ *initially. As* $\widetilde{S_N}$ *is higher,* $\overline{q}^{\#}$ *is lower, with the minimum value being* N_1.

(*ii*) *When* $\widetilde{S_N}$ *is low,*[44] $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$. $q_{1S}^* = 1$, $q_{2S}^* = q_{2U}^* \geq \overline{q}_2^{\flat}$, *where* $\overline{q}_2^{\flat}$ *is the value of* $\overline{q}_2$ *in the first period after which* $p_{2S} = p_{2U} = 0$ *continues to hold.*

(*iii*) *When* $\widetilde{S_N}$ *is in the intermediate range,*[45] $p_{1S}^* = 0$, $p_{2S}^* = p_{2U}^* = 1$ *and* $q_{1S}^* = q_{2S}^* = q_{2U}^* = 1$.

(*iv*) (*i*) *is more likely to be realized when* $\widetilde{S_1}$ *is lower, and* (*ii*) *is more likely to occur when* ω_q *and* $\widetilde{S_2}$ *are higher.*

Proof. See Appendix C. ■

Similar to the case $\frac{\lambda}{1-\lambda(1+r)}w_u \leq \overline{e}$, when $\widetilde{S_N}$ is high (low), everyone identifies with the nation (their ethnic group) and cultural integration occurs (cultures remain segregated) in the long run; and when $\widetilde{S_N}$ falls within the intermediate range, the majority (minority) identify with their ethnic group (the nation) and cultural assimilation occurs in the long run. However, since F_2 increases over time and $H_2^* = 1$, unlike the previous case, only three identity equilibria are possible in the steady state: $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$ when $\widetilde{S_N}$ is high; $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$ when $\widetilde{S_N}$ is low; and $p_{1S}^* = 0, p_{2S}^* = 1$, $p_{2U}^* = 1$ when $\widetilde{S_N}$ is in the intermediate range. Further, compared to the previous case, the ranges of initial $(F_2, \widetilde{S_N})$ leading to $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$ and $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$ are greater, because F_2 increases over time.

While F_2 consistently increases over time, leading to $H_2^* = 1$, the speed of convergence to $H_2^* = 1$ varies depending on $\widetilde{S_N}$, $\widetilde{S_1}$, ω_q, and initial F_2, as demonstrated in the next proposition.

Proposition 7 *Suppose that* $\frac{\lambda}{1-\lambda(1+r)}w_u > \overline{e}$ *holds.*

(*i*) *When* $\widetilde{S_N}$ *is very high so that* $p_{1S} = p_{2S} = 1, p_{2U} = 0$ *or* 1 *in the initial period,*[46] $p_{1S} = 1$ *and* $\tau > 0$ *always, and convergence to* $H_2^* = 1$ *occurs fastest. This case is more likely to be realized when* ω_q *and* $\widetilde{S_1}$ *are lower and initial* F_2 *is higher.*

(*ii*) *When* $\widetilde{S_N}$ *is not very high:*

  (*a*) *If* $\widetilde{S_N}$ *is relatively high,*[47] *the shift from* $p_{1S} = 0$ *and* $\tau = 0$ *to* $p_{1S} = 1$ *and* $\tau > 0$ *occurs at some point, and convergence to* $H_2^* = 1$ *accelerates. As* $\widetilde{S_N}$ *is higher, the speed of convergence is higher.*

  (*b*) *Otherwise,* $p_{1S} = 0$ *and* $\tau = 0$ *always, and convergence to* $H_2^* = 1$ *occurs slowest.*

[43] The exact condition is $\widetilde{S_N} > \widetilde{S_1}$.

[44] $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$ if $(F_2, \widetilde{S_N})$ is below initial (mS) with $p_{1S} = 0$. Otherwise, $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$ occurs for certain when $\widetilde{S_N} < \widetilde{S_2}$ and is possible when $\widetilde{S_N} < \widetilde{S_2} + \frac{1}{\gamma\delta}[\rho\chi(1-\chi) + \beta\omega_q](N_1)^2$ (the level of $\widetilde{S_N}$ at which initial (mS) with $p_{1S} = 0$ intersects with $F_2 = 1$).

[45] $p_{1S}^* = 0$, $p_{2S}^* = p_{2U}^* = 1$ for certain when $\widetilde{S_N} \in \left[\widetilde{S_2} + [\frac{1}{\gamma\delta}[\rho\chi(1-\chi) + \beta\omega_q](N_1)^2, \widetilde{S_1}\right)$ and is possible when $\widetilde{S_N} \in \left[\widetilde{S_2}, \widetilde{S_2} + \frac{1}{\gamma\delta}[\rho\chi(1-\chi) + \beta\omega_q](N_1)^2\right)$.

[46] To be precise, when $(F_2, \widetilde{S_N})$ in the initial period is on or above initial (M) with $p_{1S} = 1$ on the $(F_2, \widetilde{S_N})$ plane.

[47] To be precise, when initial $(F_2, \widetilde{S_N})$ is below initial (M) with $p_{1S} = 0$ and $\widetilde{S_N} > \widetilde{S_1}$.

(*c*) *The speed of convergence to $H_2^* = 1$ tends to be faster when $\widetilde{S_1}$ and ω_q are lower.*

Proof. See Appendix C. ■

When $\widetilde{S_N}$ is very high, the majority consistently identify with the nation, and income redistribution is always implemented. Thus, the proportion of the skilled minority increases fast, and $H_2^* = 1$ and inter-ethnic equality are achieved most rapidly. When $\widetilde{S_N}$ is not very high but relatively high, the majority identify with their ethnic group and the tax rate is zero initially. However, as the proportion of skilled workers in the minority increases and the minority's culture becomes closer to the majority's, the perceived distance of the majority to the minority decreases, eventually leading the majority to adopt a national identity. Consequently, income redistribution begins to take place, and an increase in the proportion of the skilled minority accelerates. Higher $\widetilde{S_N}$ and lower $\widetilde{S_1}$ and ω_q (the weight on the cultural component in the perceived distance) lead to an earlier transition to national identity, thereby achieving $H_2^* = 1$ and inter-ethnic equality at a faster rate. In contrast, when $\widetilde{S_N}$ is lower, the majority always identify with their group and redistribution is not carried out, resulting in a slower convergence to $H_2^* = 1$.

5.3 Summary

To summarize, when the exogenous component of national status, $\widetilde{S_N}$, is very high, or when inter-ethnic cultural differences are of little concern, i.e., ω_q is very small, in all periods, everyone identifies with the nation, and income redistribution is implemented. Consequently, the proportion of skilled workers in the minority increases rapidly and ethnic inequality declines swiftly. In the long run, everyone is skilled and inter-group income inequality disappears. Inter-ethnic cultural differences also decrease over time and cultural integration occurs eventually, where the integrated culture contains elements from both minority and majority origins in proportion to their population shares.

When $\widetilde{S_N}$ is lower and ω_q is greater, the majority identify with their own group and redistribution is not initially implemented. Long-run outcomes differ greatly depending on the levels of these exogenous parameters.

If $\widetilde{S_N}$ is relatively high, all or skilled minority individuals identify with the nation. Consequently, they become culturally closer to the majority over time. Eventually, the diminished cultural distance between the groups lead the majority to switch to a national identity and implement redistributive taxation. Thereafter, H_2 increases rapidly, and ethnic inequality declines fast. In the long run, income equality is achieved, and cultural integration occurs, although the integrated culture contains a greater share of the majority-origin element compared to the previous case.

Otherwise, the majority consistently identity with their group, and redistribution is never carried out. Thus, H_2 grows only slowly (when $\frac{\lambda}{1-\lambda(1+r)} w_u > \bar{e}$) or remains constant (when $\frac{\lambda}{1-\lambda(1+r)} w_u \leq \bar{e}$), and ethnic inequality diminishes slowly or does not disappear even in the long run. Long-run cultures depend greatly on the levels of the exogenous parameters. When $\widetilde{S_N}$ is low but not extremely so and when ω_q is large but not very so, only the minority identify with the nation and gradually adopt the majority's culture. The long-run outcome is cultural assimilation, in which everyone shares the culture originating solely from the majority. When $\widetilde{S_N}$ is very low or ω_q is very large, both ethnic groups identity with their group and cultural segregation persists.

These results show that inter-ethnic cultural disparities diminish over time when either ethnic group identifies with the nation. However, cultural convergence leads to the adoption of income redistribution and fosters economic development *only if* the majority hold national identity, which occurs when the exogenous component of national status is sufficiently high, or when the salience of the inter-group cultural distance in people's minds is sufficiently weak. If these conditions are not met and national identity is held solely by the minority, redistribution is not implemented, and the minority's culture is absorbed into the majority's.

5.4 Policy Implications

[To be added]

6 Conclusion

[To be added]

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Appendix A Production part of the model with social mobility

This Appendix presents the production part of the model with social mobility and shows that wages and the interest rate are constant. The model economy is small open and, following Galor and Zeira (1993), has two production technologies to produce final goods that can be used for consumption or investment. One technology uses skilled labor and capital to produce the good, which is expressed as

$$Y_S = F(K, L_S), \tag{33}$$

where Y_S is output, K is physical capital, $L_S(= N_1 + (1-N_1)H_2$ in equilibrium) is labor input, and the function F is concave and constant returns to scale (CRS).

The other technology uses unskilled labor as the sole input, which is given by

$$Y_U = w_u L_U, \tag{34}$$

where Y_U is output, L_U is labor input $(= (1-N_1)(1-H_2)$ in equilibrium), and w_u is constant productivity. Thus, the unskilled wage is constant and equals w_u.

Capital is freely mobile internationally. Let r be the time-invariant world interest rate. From the profit maximization condition of a representative firm with the CRS technology (33),

$$r = F_K(K, L_S). \tag{35}$$

Thus, $\frac{K}{L_S}$ is constant. Consequently, the skilled wage w_s is also time-invariant.

Appendix B Lemmas A1–A4

This section presents lemmas that are used to prove Proposition 3 that examine steady-state levels of the identity and cultural variables for each type of individuals for the model with constant H_2 and a similar proposition (Proposition 4) for the model with endogenous H_2. The next lemma examines the directions of shifts of graphs of (M), (mU), and (mS) on the $(H_2, \widetilde{S_N})$ plane for every possible combination of p_{1S}, p_{2S}, and p_{2U} for the model with constant H_2.[48]

Lemma A1 *In the model with constant H_2, under the initial condition $q_1^i = 1$ and $q_{2S}^i = q_{2U}^i = 0$ for any i, the following holds as levels of the cultural variables change over time.*

(i) (M) and (mS) in subsequent periods are located at lower positions than or the same positions as those in the initial period on the $(H_2, \widetilde{S_N})$ plane.

(ii) When $p_{1S} = p_{2S} = p_{2U} = 1$, (M) and (mU) shift downward over time on the $(H_2, \widetilde{S_N})$ plane, while (mS) shifts downward (upward) when $\overline{q}_2 + \overline{q} - 2q_{2S}^i > (<) 0$.

(iii) When $p_{1S} = 1$, $p_{2S} = 1$, $p_{2U} = 0$ or when $p_{1S} = 0$, $p_{2S} = p_{2U} = 1$, (M) and (mU) shift downward. The direction of the shift of (mS) is ambiguous in the short run, but in the long run, it shifts downward.

(iv) When $p_{1S} = 0$, $p_{2S} = 1$, $p_{2U} = 0$, (M) shifts downward. (mU) and (mS) shift downward in the long run.

(v) When $p_{1S} = p_{2S} = p_{2U} = 0$, (M) does not shift, while (mU) shifts downward and (mS) shifts upward unless $q_{2S}^i = q_{2U}^i$ for any i, in which case they do not shift.

Proof. See Appendix B. ■

The next lemma examines steady-state levels of the cultural variables for every possible steady-state combination of p_{1S}, p_{2S}, and p_{2U} when H_2 is constant.

Lemma A2 *Suppose that H_2 is constant.*

(i) When $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$, $q_{1S}^* = q_{2S}^* = q_{2U}^* = \overline{q}^{\#}$, where $\overline{q}^{\#}$ is the value of $\overline{q}$ in the first period after which $p_{1S} = p_{2S} = p_{2U} = 1$ continues to hold.

(ii) When $p_{1S}^* = p_{2S}^* = 1$, $p_{2U}^* = 0$, $q_{1S}^* = q_{2S}^* = q_{2U}^* = \frac{1}{1-(1-H_2)N_1}\left[H_2 N_1 \overline{q}_1^{\dagger} + (1-N_1)\overline{q}_2^{\dagger}\right]$, where $\overline{q}_1^{\dagger}$ ($\overline{q}_2^{\dagger}$) is the value of $\overline{q}_1$ ($\overline{q}_2$) in the first period after which $p_{1S} = p_{2S} = 1$, $p_{2U} = 0$ continues to hold.

(iii) When $p_{1S}^* = 0$ and at least one of p_{2S}^* and p_{2U}^* equals 1, $q_{1S}^* = q_{2S}^* = q_{2U}^* = \overline{q}_1^{\flat}$.

(iv) When $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$, $q_{1S}^* = \overline{q}_1^{\flat}$ and $q_{2S}^* = q_{2U}^* = \overline{q}_2^{\flat}$, where $\overline{q}_1^{\flat}$ ($\overline{q}_2^{\flat}$) is the value of $\overline{q}_1$ ($\overline{q}_2$) in the first period after which $p_{1S} = 0$ ($p_{2S} = p_{2U} = 0$) continues to hold.

Proof. See Appendix C. ■

The next lemma presents similar results as Lemma A1 for the model with endogenous H_2.

[48] When there exist multiple equilibria for a given combination of H_2 and $\widetilde{S_N}$, it is assumed that the equilibrium chosen in the previous period is selected in the present period as well.

Lemma A3 *In the model with endogenous H_2, under the initial condition $q_1^i = 1$ and $q_{2S}^i = q_{2U}^i = 0$ for any i, the following holds as levels of the cultural variables change over time.*

(i) (M) and (mS) in subsequent periods are located at lower positions than or the same positions as those in the initial period on the $(H_2, \widetilde{S_N})$ plane.

(ii) When $p_{1S} = p_{2S} = p_{2U} = 1$, (M) shifts downward over time. (mU), as well as (mS) when $\overline{q}_2 + \overline{q} - 2q_{2S}^i > 0$, shift downward under Assumption 5. (mS) shifts downward in the long run.

(iii) When $p_{1S} = 1$, $p_{2S} = 1$, $p_{2U} = 0$ or when $p_{1S} = 0$, $p_{2S} = p_{2U} = 1$, (M) shifts downward, so does (mU) under Assumption 5. (mS) shifts downward in the long run.

(iv) When $p_{1S} = 0$, $p_{2S} = 1$, $p_{2U} = 0$, (M) shifts downward. (mU) and (mS) shift downward in the long run.

(v) When $p_{1S} = p_{2S} = p_{2U} = 0$, (M) shifts upward unless $q_{2S}^i = q_{2U}^i$ for any i, in which case it does not shift. When $q_{2S}^i = q_{2U}^i$ for any i, (mU) and (mS) too do not shift. When $H_2 = 0, p_{1S} = p_{2U} = 0$, (M), (mU), and (mS) all do not shift.

Proof. See Appendix C. ■

With the additional Assumption 5, all results important for proving Proposition 4 are same as the constant H_2 case.

Finally, the next lemma presents similar results as Lemma A2 for the model with endogenous H_2 under the initial condition $q_1^i = 1$ and $q_{2S}^i = q_{2U}^i = 0$ for any i.

Lemma A4 *Consider the model with endogenous H_2 under the initial condition $q_1^i = 1$ and $q_{2S}^i = q_{2U}^i = 0$ for any i.*

(i) When $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$, $q_{1S}^* = q_{2S}^* = q_{2U}^* = \overline{q}^* \geq \overline{q}^{\#} \in (0,1)$, where $\overline{q}^{\#}$ is the value of $\overline{q}$ in the first period after which $p_{1S} = p_{2S} = p_{2U} = 1$ continues to hold and $\overline{q}^* \in (0,1)$. $\overline{q}^* = \overline{q}^{\#}$ when $p_{2S} = p_{2U} = 1$ initially.

(ii) When $p_{1S}^* = 0$ and at least one of p_{2S}^* and p_{2U}^* equals 1, $q_{1S}^* = q_{2S}^* = q_{2U}^* = \overline{q}_1^\flat$.

(iii) When $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$, $q_{1S}^* = \overline{q}_1^\flat$ and $q_{2S}^* = q_{2U}^* \geq \overline{q}_2^\flat$, where $\overline{q}_1^\flat$ $(\overline{q}_2^\flat)$ is the value of $\overline{q}_1$ $(\overline{q}_2)$ in the first period after which $p_{1S} = 0$ $(p_{2S} = p_{2U} = 0)$ continues to hold. When $H_2^* = 0$ and $p_{1S}^* = p_{2U}^* = 0$, $q_{1S}^* = \overline{q}_1^\flat$ and $q_{2U}^* = \overline{q}_2^\flat$

Proof. See Appendix C. ■

The differences from the constant H_2 case are: $p_{1S}^* = p_{2S}^* = 1$, $p_{2U}^* = 0$ does not occur; when $p_{1S}^* = p_{2S}^* = p_{2U}^* = 1$, $q_{1S}^* = q_{2S}^* = q_{2U}^* = \overline{q}^* \geq \overline{q}^{\#}$, where $\overline{q}^{\#}$ is the value of $\overline{q}$ in the first period after which $p_{1S} = p_{2S} = p_{2U} = 1$ continues to hold, unless $p_{2S} = p_{2U} = 1$ initially, in which case $\overline{q}^* = \overline{q}^{\#}$ as before; and when $p_{1S}^* = p_{2S}^* = p_{2U}^* = 0$, $q_{2S}^* = q_{2U}^* \geq \overline{q}_2^\flat$, where $\overline{q}_2^\flat$ is the value of $\overline{q}_2$ in the first period after which $p_{2S} = p_{2U} = 0$ continues to hold.

Appendix C Proofs of Lemmas and Propositions

Proof of Proposition 1. (i) The statement is true iff the RHS of (mU) is greater than that of (mS). From these equations (multiplied by $\gamma\delta$),

$$\begin{aligned}
&\gamma\delta\widetilde{S_2} + (\beta-\gamma)N_1(1-H_2)(1-\tau)(w_s - w_u) + [\rho\chi(1-\chi) + \beta\omega_q](\overline{q} - \overline{q}_2)\left[(\overline{q}_2 - q_{2U}^i) + (\overline{q} - q_{2U}^i)\right] \\
> \;& \gamma\delta\widetilde{S_2} - (\beta+\gamma)N_1(1-H_2)(1-\tau)(w_s - w_u) + [\rho\chi(1-\chi) + \beta\omega_q](\overline{q} - \overline{q}_2)\left[-(q_{2S}^i - \overline{q}_2) + (\overline{q} - q_{2S}^i)\right] \\
\Leftrightarrow \;& 2\beta N_1(1-H_2)(1-\tau)(w_s - w_u) + [\rho\chi(1-\chi) + \beta\omega_q](\overline{q} - \overline{q}_2)2\left(q_{2S}^i - q_{2U}^i\right) > 0. \qquad (36)
\end{aligned}$$

Given the initial condition $q^i_{2S}=q^i_{2U}=0$, (36) is true for the initial period and thus $q^i_{2S}\geq q^i_{2U}$ for the next period from (13), (14), and the initial condition $q^i_{1S}=1$. Then, (36) is true for the next period and $q^i_{2S}\geq q^i_{2U}$ for the next next period. Continuing in this way, one can prove (36) for all periods.

(ii) (a) The statement is true iff the RHS of (M) is greater than that of (mS). From these equations (multiplied by $\gamma\delta$),

$$\begin{aligned}&\gamma\delta\widetilde{S_1}+(\beta+\gamma)(1-N_1)(1-H_2)(1-\tau)(w_s-w_u)+[\rho\chi(1-\chi)+\beta\omega_q](\overline{q}_1-\overline{q})^2\\ &>\gamma\delta\widetilde{S_2}-(\beta+\gamma)N_1(1-H_2)(1-\tau)(w_s-w_u)+[\rho\chi(1-\chi)+\beta\omega_q](\overline{q}-\overline{q}_2)[-(q^i_{2S}-\overline{q}_2)+(\overline{q}-q^i_{2S})]\\ &\Leftrightarrow\gamma\delta\left(\widetilde{S_1}-\widetilde{S_2}\right)+(\beta+\gamma)(1-H_2)(1-\tau)(w_s-w_u)>-[\rho\chi(1-\chi)+\beta\omega_q]\left[(\overline{q}_1-\overline{q})^2-(\overline{q}-\overline{q}_2)[-(q^i_{2S}-\overline{q}_2)+(\overline{q}-q^i_{2S})]\right],\end{aligned}\tag{37}$$

The RHS of (37) equals $[\rho\chi(1-\chi)+\beta\omega_q]$ times

$$\begin{aligned}&-(\overline{q}_1-\overline{q}_2)\left\{(1-N_1)^2(\overline{q}_1-\overline{q}_2)-N_1\left[-2\left(q^i_{2S}-\overline{q}_2\right)+N_1(\overline{q}_1-\overline{q}_2)\right]\right\}\\ &=(\overline{q}_1-\overline{q}_2)\left[(2N_1-1)(\overline{q}_1-\overline{q}_2)-2N_1\left(q^i_{2S}-\overline{q}_2\right)\right]\leq 2N_1-1<(N_1)^2,\end{aligned}\tag{38}$$

where $q^i_{2S}\geq\overline{q}_2$ is used to prove the second last inequality. Hence, (37) is true under Assumption 2.

(b) The majority are less likely to have a national identity than the minority's unskilled iff the RHS of (M) is greater than that of (mU). From these equations,

$$\begin{aligned}&\gamma\delta\widetilde{S_1}+(\beta+\gamma)(1-N_1)(1-H_2)(1-\tau)(w_s-w_u)+[\rho\chi(1-\chi)+\beta\omega_q](\overline{q}_1-\overline{q})^2\\ &>\gamma\delta\widetilde{S_2}+(\beta-\gamma)N_1(1-H_2)(1-\tau)(w_s-w_u)+[\rho\chi(1-\chi)+\beta\omega_q](\overline{q}-\overline{q}_2)[(\overline{q}_2-q^i_{2U})+(\overline{q}-q^i_{2U})]\\ &\Leftrightarrow\gamma\delta\left(\widetilde{S_1}-\widetilde{S_2}\right)+[\gamma-\beta(2N_1-1)](1-H_2)(1-\tau)(w_s-w_u)>-[\rho\chi(1-\chi)+\beta\omega_q]\left\{(\overline{q}_1-\overline{q})^2-(\overline{q}-\overline{q}_2)[(\overline{q}_2-q^i_{2U})+(\overline{q}-q^i_{2U})]\right\},\end{aligned}\tag{39}$$

where the RHS equals $[\rho\chi(1-\chi)+\beta\omega_q]$ times

$$\begin{aligned}&(\overline{q}_1-\overline{q}_2)\left[(2N_1-1)(\overline{q}_1-\overline{q}_2)+2N_1\left(\overline{q}_2-q^i_{2U}\right)\right]\\ \leq\ &(1-\overline{q}_2)[(2N_1-1)(1-\overline{q}_2)+2N_1\overline{q}_2]=(1-\overline{q}_2)[(2N_1-1)+\overline{q}_2]\leq(N_1)^2,\end{aligned}\tag{40}$$

where the last inequality holds because the derivative of the second last expression with respect to $\overline{q}_2$ equals $-[(2N_1-1)+\overline{q}_2]+(1-\overline{q}_2)$ and thus the expression is highest at $\overline{q}_2=1-N_1$.

Hence, when $\gamma-\beta(2N_1-1)\geq 0\Leftrightarrow N_1\leq\frac{\beta+\gamma}{2\beta}$, (39) is true under Assumption 2. When $N_1>\frac{\beta+\gamma}{2\beta}$, (39) holds for large H_2, but it may not hold for small H_2. ■

Proof of Lemma 2. From (M), (mU), and (mS), the statement of the lemma holds iff $(1-\tau)(1-H_2)$ decreases with H_2 when $p_{1S}=1$. From (16), $(1-\tau)(1-H_2)=\frac{1}{1+\gamma}\left[\beta+\gamma-\frac{(\beta-1)w_s}{\overline{w}}\right](1-H_2)$, thus its derivative with respect to H_2 equals $\frac{1}{1+\gamma}$ times $-\left[\beta+\gamma-\frac{(\beta-1)w_s}{\overline{w}}\right]+(1-H_2)\frac{(\beta-1)w_s}{(\overline{w})^2}(1-N_1)(w_s-w_u)=-\left[\beta+\gamma-(\beta-1)\left(\frac{w_s}{\overline{w}}\right)^2\right]$. Hence,

$$\frac{d[(1-\tau)(1-H_2)]}{dH_2}<0\Leftrightarrow\frac{1+\gamma}{\beta-1}>\left(\frac{w_s}{\overline{w}}\right)^2-1.\tag{41}$$

From Assumption 1, the above condition holds if $3>\left(\frac{w_s}{\overline{w}}\right)^2-1\Leftrightarrow\frac{w_s}{\overline{w}}<2$, which is always true because $\frac{w_s}{\overline{w}}\leq\frac{w_s}{N_1w_s+(1-N_1)w_u}<2$ from $N_1>\frac{1}{2}$. ■

Proof of Proposition 3.

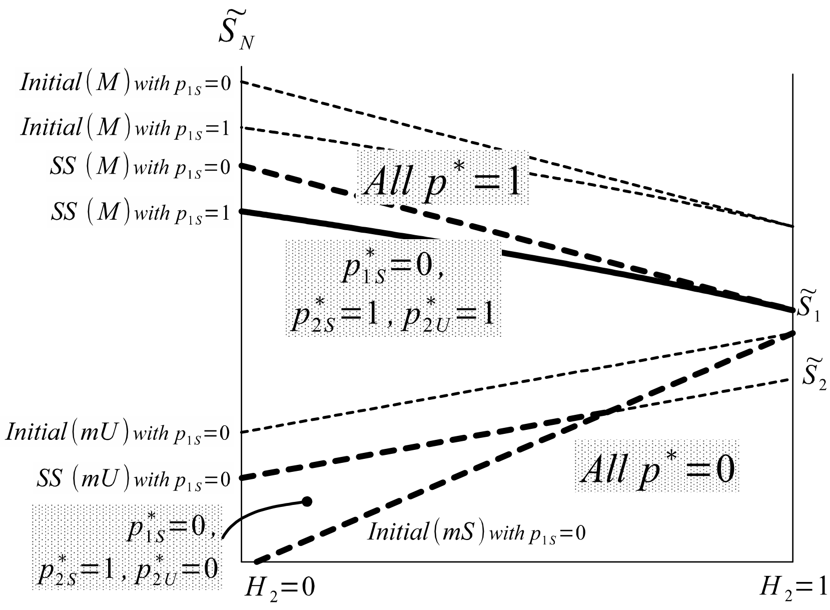

Figure 5: Relation between initial $(H_2, \widetilde{S_N})$ and steady-state identity when $\gamma \geq \beta$ and H_2 is constant

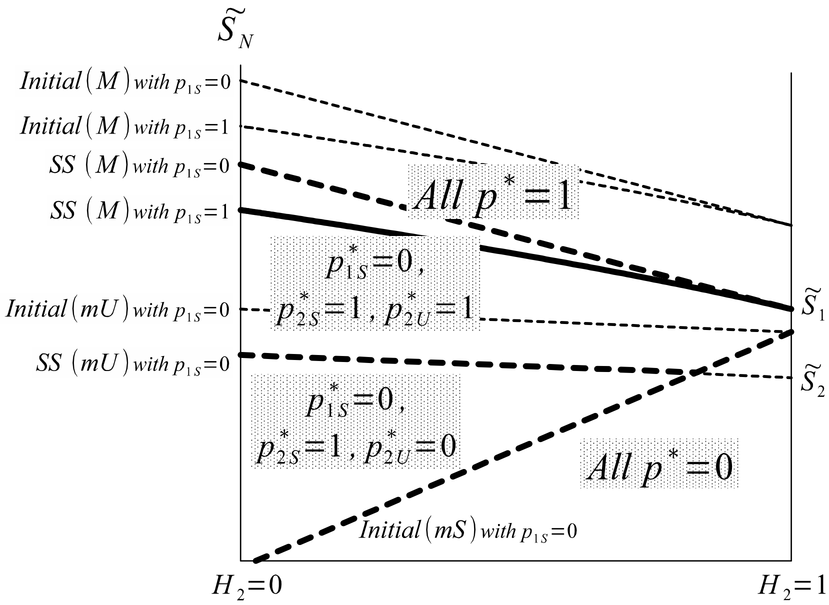

Figure 6: Relation between initial $(H_2, \widetilde{S_N})$ and steady-state identity when $\beta > \gamma$, $N_1 \leq \frac{\beta+\gamma}{2\beta}$, and H_2 is constant

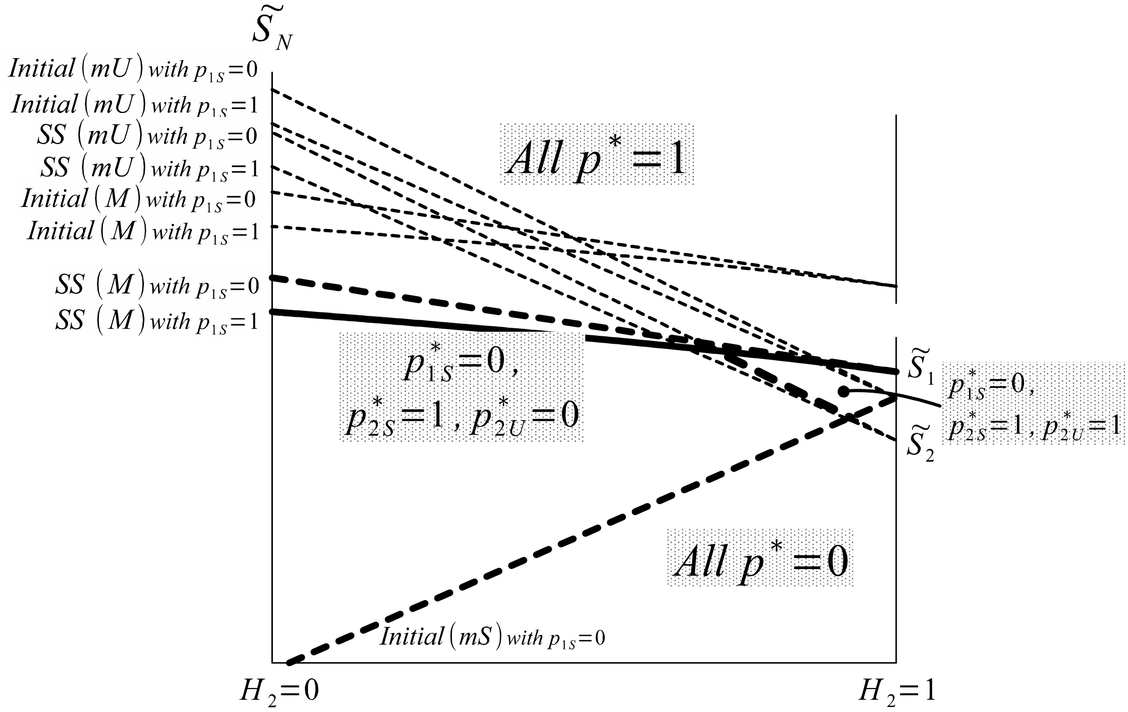

Figure 7: Relation between initial $(H_2, \widetilde{S_N})$ and steady-state identity when $\beta > \gamma$, $N_1 > \frac{\beta+\gamma}{2\beta}$, and H_2 is constant

To prove the results, we use Figures 5−7 that show the positions of (M) and (mU) (with $p_{1S}=0$ and with $p_{1S}=1$) in the initial period and in the steady state, and of (mS) in the initial period on the $(H_2, \widetilde{S_N})$ plane. The steady-state dividing lines are for when $p^*_{1S}=p^*_{2S}=p^*_{2U}=0$ does not hold in the long run, in which case $q^*_{1S}=q^*_{2S}=q^*_{2U}$ from Lemma A2.

Relative positions of the dividing lines for given period and p_{1S} are based on the following theoretical results. (mU) is located above (mS) from Proposition 1 (i), and unless $\beta > \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$, (M) is above (mU) from (ii)(b) of the proposition. When $\beta > \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$, (mU) may be above (M) for small H_2 from Proposition 1 (ii)(b), and Figure 7 illustrates such a case. (M) and (mU) with $p_{1S}=0$ are located above (M) and (mU) with $p_{1S}=1$ respectively, because $\tau > (=)0$ when $p_{1S}=1(=0)$ from Section 3.2. The dividing lines in the steady state are below the corresponding ones in the initial period from Lemma A1 (ii)−(iv). At $H_2=1$, the vertical level of initial (M) on the $(H_2, \widetilde{S_N})$ plane equals $\widetilde{S_1}+\frac{1}{\gamma\delta}[\rho\chi(1-\chi)+\beta\omega_q](1-N_1)^2$, that of initial (mU) and (mS) equals $\widetilde{S_2}+\frac{1}{\gamma\delta}[\rho\chi(1-\chi)+\beta\omega_q](N_1)^2$, while the vertical level of steady-state (M) is $\widetilde{S_1}$ and that of steady-state (mU) and (mS) is $\widetilde{S_2}$. Relative levels of these values are from Assumption 2.

(i) Given H_2, when $\widetilde{S_N}$ is very high so that $p_{1S}=p_{2S}=p_{2U}=1$ in the initial period (i.e., $(H_2, \widetilde{S_N})$ is on or above initial (M) with $p_{1S}=1$ and when $\beta > \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$ [Figure 7], also on or above initial (mU) with $p_{1S}=1$ on the $(H_2, \widetilde{S_N})$ plane), $p_{1S}=p_{2S}=p_{2U}=1$ holds in subsequent periods because (M) and (mU) shift downward over time on the $(H_2, \widetilde{S_N})$ plane from Lemma A1 (ii).

When $p_{1S}=0$, $p_{2S}=p_{2U}=1$ initially (i.e., $(H_2, \widetilde{S_N})$ is on or above initial (mU) with $p_{1S}=0$ and below initial (M) with $p_{1S}=0$) and $\widetilde{S_N}$ is relatively high for given H_2 (i.e., $(H_2, \widetilde{S_N})$ is on or above steady-state (M) with $p_{1S}=1$), society shifts to $p_{1S}=p_{2S}=p_{2U}=1$ eventually (i.e., $(H_2, \widetilde{S_N})$ is on or above (M) with $p_{1S}=1$, when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$ [Figure 7], also on or above (mU) with $p_{1S}=1$) and stays in this state, because (M) and (mU) shift downward over time from Lemma A1 (iii).

Similarly, when $p_{1S}=0$, $p_{2S}=1, p_{2U}=0$ initially (i.e., $(H_2,\widetilde{S_N})$ is on or above initial (mS) with $p_{1S}=0$ and below initial (M) and (mU) with $p_{1S}=0$) and $\widetilde{S_N}$ is relatively high (i.e., $(H_2,\widetilde{S_N})$ is on or above steady-state (M) and (mU) with $p_{1S}=1$), which occur only when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$, society shifts to $p_{1S}=p_{2S}=p_{2U}=1$ eventually (typically, after shifting to $p_{1S}=0$, $p_{2S}=p_{2U}=1$) because (M) shifts downward over time and (mU) and (mS) shift downward in the long term from Lemma A1 (iv).

When $p_{1S}=1$, $p_{2S}=1$, $p_{2U}=0$ initially (i.e., $(H_2,\widetilde{S_N})$ is on or above initial (M) with $p_{1S}=1$ and below initial (mU) with $p_{1S}=1$), which may occur only when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$ (Figure 7), and $\widetilde{S_N}$ is relatively high (i.e., $(H_2,\widetilde{S_N})$ is on or above steady-state (mU) with $p_{1S}=1$), society shifts to $p_{1S}=p_{2S}=p_{2U}=1$ because (M) and (mU) shift downward over time from Lemma A1 (iii).

To summarize, $p^*_{1S}=p^*_{2S}=p^*_{2U}=1$ when $(H_2,\widetilde{S_N})$ is located on or above steady-state (M) with $p_{1S}=1$, and when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$ (Figure 7), also on or above steady-state (mU) with $p_{1S}=1$.

When $p_{1S}=1$, $p_{2S}=1$, $p_{2U}=0$ initially (i.e., $(H_2,\widetilde{S_N})$ is on or above initial (M) with $p_{1S}=1$ and below initial (mU) with $p_{1S}=1$), which occurs only when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$ (Figure 7), and $\widetilde{S_N}$ is not very high (i.e., $(H_2,\widetilde{S_N})$ is below steady-state (mU) with $p_{1S}=1$), society stays in $p_{1S}=1$, $p_{2S}=1$, $p_{2U}=0$ because (M) and (mU) shift downward from Lemma A1 (iii).

When $\beta>\gamma$, $N_1>\frac{\beta+\gamma}{2\beta}$ (Figure 7), $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=0$ initially (i.e., $(H_2,\widetilde{S_N})$ is on or above initial (mS) with $p_{1S}=0$ and below initial (M) and (mU) with $p_{1S}=0$), and $\widetilde{S_N}$ is relatively, but not very, high (i.e., $(H_2,\widetilde{S_N})$ is on or above steady-state (M) with $p_{1S}=1$ and below steady-state (mU) with $p_{1S}=1$), society shifts to $p_{1S}=1$, $p_{2S}=1$, $p_{2U}=0$ because (M) and (mU) shift downward in the long term from Lemma A1 (iv).

To summarize, $p^*_{1S}=1, p^*_{2S}=1, p^*_{2U}=0$ when $\beta>\gamma$, $N_1>\frac{\beta+\gamma}{2\beta}$ (Figure 7), and $(H_2,\widetilde{S_N})$ is located on or above steady-state (M) with $p_{1S}=1$ and below steady-state (mU) with $p_{1S}=1$.

The result on the steady state level of the cultural variable is from Lemma A2 (i) and (ii). The negative relation between $\widetilde{S_N}$ and $\overline{q}^{\#}$ holds because, as $\widetilde{S_N}$ is lower, the period during which cultural assimilation proceeds, i.e., $p_{1S}=0$, $p_{2S}=1, p_{2U}=0$ or 1, is longer. The maximum proportion of the minority element in the integrated culture equals $1-N_1$ because $\overline{q}=N_1$ in the initial period.

(ii) Given H_2, when $\widetilde{S_N}$ is low enough that $p_{1S}=p_{2S}=p_{2U}=0$ initially (i.e., $(H_2,\widetilde{S_N})$ is below initial (mS) with $p_{1S}=0$), $p_{1S}=p_{2S}=p_{2U}=0$ holds in subsequent periods, because $q_{2S}=q_{2U}=0$ continues to hold and thus (mS) does not shift from Lemma A1 (v) and (13).

The equilibrium with $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=0$ does not shift to $p_{1S}=p_{2S}=p_{2U}=0$ because (mS) with $p_{1S}=0$ in the initial period is at a higher position than or the same position as those in subsequent periods on the $(H_2,\widetilde{S_N})$ plane from Lemma A1 (i). $q^*_{1S}=1$ and $q^*_{2S}=q^*_{2U}=0$ is from Lemma A2 (iv) and the result that only the society starting with $p_{1S}=p_{2S}=p_{2U}=0$ ends up with $p^*_{1S}=p^*_{2S}=p^*_{2U}=0$.

(iii) When $p_{1S}=0$, $p_{2S}=p_{2U}=1$ initially (i.e., $(H_2,\widetilde{S_N})$ is on or above initial (mU) with $p_{1S}=0$ and below initial (M) with $p_{1S}=0$) and $\widetilde{S_N}$ is relatively low for given H_2 (i.e., $(H_2,\widetilde{S_N})$ is below steady-state (M) with $p_{1S}=0$), $p_{1S}=0$, $p_{2S}=p_{2U}=1$ holds in subsequent periods because (M) and (mU) shift downward over time on the $(H_2,\widetilde{S_N})$ plane from Lemma A1 (iii).

When $p_{1S}=0$, $p_{2S}=1, p_{2U}=0$ initially (i.e., $(H_2,\widetilde{S_N})$ is on or above initial (mS) with $p_{1S}=0$ and below initial (mU) with $p_{1S}=0$; when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$ [Figure 7], also below initial (M) with $p_{1S}=0$) and $\widetilde{S_N}$ is relatively high (i.e., $(H_2,\widetilde{S_N})$ is on or above steady-state (mU) with $p_{1S}=0$;

when $\beta > \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$, also below steady-state (M) with $p_{1S}=0$), society shifts to $p_{1S}=0$, $p_{2S}=p_{2U}=1$ eventually and stays in this state, because (M) shifts downward over time, so does (mU) in the long run, from Lemma A1 (iv), and (mS)s in subsequent periods are not located above the one in the initial period from Lemma A1 (i).

When $p_{1S}=0$, $p_{2S}=1, p_{2U}=0$ initially and $\widetilde{S_N}$ is relatively low (i.e., $(H_2, \widetilde{S_N})$ is below steady-state (mU) with $p_{1S}=0$; when $\beta > \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$, also below steady-state (M) with $p_{1S}=0$), society stays in this state because because (M) shifts downward over time, so does (mU) in the long run, from Lemma A1 (iv), and (mS)s in subsequent periods are not located above the initial one from Lemma A1 (i).

To summarize, $p^*_{1S}=0, p^*_{2S}=p^*_{2U}=1$ when $(H_2, \widetilde{S_N})$ is on or above initial (mS) with $p_{1S}=0$, as well as steady-state (mU) with $p_{1S}=0$, and below steady-state (M) with $p_{1S}=0$; $p^*_{1S}=0$, $p^*_{2S}=1, p^*_{2U}=0$ when $(H_2, \widetilde{S_N})$ is on or above initial (mS) with $p_{1S}=0$ and below steady-state (mU) with $p_{1S}=0$, when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$ (Figure 7), also below steady-state (M) with $p_{1S}=0$.

$q^*_{1S}=q^*_{2S}=q^*_{2U}=1$ is from Lemma A2 (iii) and the result that only the society starting with $p_{1S}=1$ and never satisfying $p_{1S}=0$ ends up with $p^*_{1S}=0$, $p^*_{2S}=1, p^*_{2U}=0$ or 1.

(iv) The result on $\widetilde{S_1}$ ($\widetilde{S_2}$) holds because as $\widetilde{S_1}$ is lower ($\widetilde{S_2}$ is higher), (M) [(mS)] is located at a lower (higher) position on the $(H_2, \widetilde{S_N})$ plane. The result on ω_q holds because as ω_q is higher, (mS) in the initial period, whose last term equals $[\rho\chi(1-\chi)+\beta\omega_q](N_1)^2$, is located at a higher position. The level of ω_q does not affect the likelihood of universal national identity because steady-state (M) does not depend on ω_q when $q^*_{1S}=q^*_{2S}=q^*_{2U}$. The result on H_2 is from the figures. ■

Proof of Lemma 3. (i) The claim is proved if the difference in utility between when a group 1 individual takes education and when she does not is positive at $H_1=1$. To compute the utility when not taking education, the value of p_{1U} needs to be specified. It is reasonable to suppose $p_{1U} \geq p_{1S}$ since for a group 1 individual with cultural variable q^i_{1S}, from (9), (10), (13), and (14),

$$\begin{aligned} & p_{1U}=1(=0) \Leftrightarrow u^i_{1UN} \geq (<) u^i_{1U1} \\ \Leftrightarrow\; & \gamma\delta\widetilde{S_N} \geq (<) \gamma\delta\widetilde{S_1}-(\beta-\gamma)(1-\tau)(\overline{w}_1-\overline{w})+[\rho\chi(1-\chi)+\beta\omega_q](\overline{q}_1-\overline{q})\left(2q^i_{1S}-\overline{q}_1-\overline{q}\right) \end{aligned} \tag{42}$$

and thus the RHS of the equation is smaller than that of (M). Hence, the cases to be examined are $p_{1S}=p_{1U}=1$, $p_{1S}=p_{1U}=0$, and $p_{1S}=0, p_{1U}=1$.

When $p_{1S}=p_{1U}=1$, for a group 1 individual with q^i_{1S}, from (7), (9), (14), and the fact $v^i_{JCG}=u^i_{JCG}+(1+r)a$, the difference in utility between when taking education and not at $H_1=1$ equals (note $\tau=\frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}$)

$$\begin{aligned} v^i_{1SN}-v^i_{1UN} &= (1-\tau)\{(w_s-w_u)-\beta[(w_s-\overline{w})-(\overline{w}-w_u)]\}-(1+r)\overline{e} \\ &= (1-\tau)(w_s-w_u)(1-\beta\{(1-N_1)(1-H_2)-[N_1+(1-N_1)H_2]\})-(1+r)\overline{e} \\ &= (1-\tau)(w_s-w_u)\{1+\beta\,[(2N_1-1)+2(1-N_1)H_2]\}-(1+r)\overline{e} \;>\; 0 \text{ under Assumption 3.} \end{aligned} \tag{43}$$

When $p_{1S}=p_{1U}=0$, from (8), (10), (13), $\tau=0$, $\overline{w}_1=w_s$, and $v^i_{JCG}=u^i_{JCG}+(1+r)a$, the difference in utility between when taking education and when not at $H_1=1$ equals

$$\begin{aligned} v^i_{1S1}-v^i_{1U1} &= (w_s-w_u)-\beta[(w_s-\overline{w}_1)-(\overline{w}_1-w_u)]-(1+r)\overline{e} \\ &= (1+\beta)(w_s-w_u)-(1+r)\overline{e}>0 \text{ under Assumption 3.} \end{aligned} \tag{44}$$

When $p_{1S}=0, p_{1U}=1$, from (8), (9), (13), (14), $\tau=0$, and $v^i_{JCG}=u^i_{JCG}+(1+r)a$, the difference in utility at $H_1=1$ equals

$$v^i_{1S1}-v^i_{1UN}=(w_s-w_u)-\beta[(w_s-\overline{w}_1)-(\overline{w}-w_u)]+\gamma\Big[-\delta\left(\widetilde{S_N}-\widetilde{S_1}\right)+(\overline{w}_1-\overline{w})\Big]$$
$$-[\rho\chi(1-\chi)+\beta\omega_q]\Big[\left(q^i_{1S}-\overline{q}_1\right)^2-\left(q^i_{1S}-\overline{q}\right)^2\Big]-(1+r)\overline{e}$$
$$> (w_s-w_u)-\beta[(w_s-\overline{w}_1)-(\overline{w}-w_u)]-(1+r)\overline{e}-\beta(\overline{w}_1-\overline{w}) \text{ (from (M) with " < ")}$$
$$= (w_s-w_u)-\beta[(w_s-\overline{w})-(\overline{w}-w_u)]-(1+r)\overline{e} > 0 \text{ from the first equation of (43).} \quad (45)$$

The differences in utility are all positive and thus $H_1=1$.

(ii) From Proposition 2, the cases to be examined are $p_{2S}=p_{2U}=1$, $p_{2S}=1, p_{2U}=0$, and $p_{1S}=p_{2S}=p_{2U}=0$.

(a) When $p_{2S}=p_{2U}=1$, for a group 2 individual with q^i_{2S}, from (18), (20), and $v^i_{JCG}=u^i_{JCG}+(1+r)a$, the difference in utility between when she takes education and when she does not equals

$$v^i_{2SN}-v^i_{2UN} = (1-\tau)\{(w_s-w_u)-\beta[(w_s-\overline{w})-(\overline{w}-w_u)]\}-(1+r)\overline{e}, \quad (46)$$
$$\text{where } \tau = \frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}} \text{ when } p_{1S}=1 \text{ and } \tau=0 \text{ when } p_{1S}=0,$$

which is positive under Assumption 3 from the first equation of (43).

When $p_{2S}=1, p_{2U}=0$, from (18), (21), and $v^i_{JCG}=u^i_{JCG}+(1+r)a$, the difference in utility between when taking education and when not equals

$$v^i_{2SN}-v^i_{2U2} = (1-\tau)\{(w_s-w_u)-\beta[(w_s-\overline{w})-(\overline{w}_2-w_u)]+\gamma(\overline{w}-\overline{w}_2)\}+\gamma\delta\left(\widetilde{S_N}-\widetilde{S_2}\right)$$
$$-[\rho\chi(1-\chi)+\beta\omega_q]\Big[\left(\overline{q}-q^i_{2S}\right)^2-\left(\overline{q}_2-q^i_{2S}\right)^2\Big]-(1+r)\overline{e}$$
$$= (1-\tau)\{1-\beta[(1-N_1)(1-H_2)-H_2]+\gamma N_1(1-H_2)\}(w_s-w_u)+\gamma\delta\left(\widetilde{S_N}-\widetilde{S_2}\right)$$
$$-[\rho\chi(1-\chi)+\beta\omega_q]\Big[\left(\overline{q}-q^i_{2S}\right)^2-\left(\overline{q}_2-q^i_{2S}\right)^2\Big]-(1+r)\overline{e}, \quad (47)$$
$$\text{where } \tau = \tfrac{\beta-1}{1+\gamma}\tfrac{w_s-\overline{w}}{\overline{w}} \text{ when } p_{1S}=1 \text{ and } \tau=0 \text{ when } p_{1S}=0.$$

When $p_{1S}=1$, from (M),

$$v^i_{2SN}-v^i_{2U2}\geq(1-\tau)[1+\beta H_2+\gamma(1-H_2)](w_s-w_u)+\gamma\delta\left(\widetilde{S_1}-\widetilde{S_2}\right)$$
$$+[\rho\chi(1-\chi)+\beta\omega_q]\Big\{(\overline{q}_1-\overline{q})^2-\Big[\left(\overline{q}-q^i_{2S}\right)^2-\left(\overline{q}_2-q^i_{2S}\right)^2\Big]\Big\}-(1+r)\overline{e}$$
$$>(1-\tau)[1+\beta H_2+\gamma(1-H_2)](w_s-w_u)$$
$$+[\rho\chi(1-\chi)+\beta\omega_q]\Big\{(N_1)^2+(\overline{q}_1-\overline{q})^2-\Big[\left(\overline{q}-q^i_{2S}\right)^2-\left(\overline{q}_2-q^i_{2S}\right)^2\Big]\Big\}-(1+r)\overline{e} \quad \text{(from Assumption 2)}, \quad (48)$$

which is positive under Assumption 3 because

$$(N_1)^2+(\overline{q}_1-\overline{q})^2-\Big[\left(\overline{q}-q^i_{2S}\right)^2-\left(\overline{q}_2-q^i_{2S}\right)^2\Big] = (N_1)^2+(\overline{q}_1-\overline{q})^2-(\overline{q}-\overline{q}_2)\left(\overline{q}-q^i_{2S}+\overline{q}_2-q^i_{2S}\right)$$
$$\geq (N_1)^2+(\overline{q}_1-\overline{q})^2-\left[(\overline{q})^2-(\overline{q}_2)^2\right]$$
$$= (N_1)^2+(\overline{q}_1)^2+(\overline{q}_2)^2-2\overline{q}_1\overline{q}$$
$$= (N_1)^2+(\overline{q}_1-\overline{q}_2)^2-2N_1\overline{q}_1(\overline{q}_1-\overline{q}_2)$$
$$= [(N_1)-(\overline{q}_1-\overline{q}_2)]^2+2N_1(1-\overline{q}_1)(\overline{q}_1-\overline{q}_2)>0.$$

Hence, when $p_{2S}=p_{2U}=1$ ($p_{1S}=0$ or 1) and when $p_{1S}=1, p_{2S}=1, p_{2U}=0$, the utility return to education is positive at $H_2=F_2$ and thus $H_2=F_2$ is an equilibrium for any F_2.

(b) When $p_{1S}=p_{2S}=p_{2U}=0$, from (19), (21), and $v^i_{JCG}=u^i_{JCG}+(1+r)a$, the difference in utility equals

$$v^i_{2S2}-v^i_{2U2}=[1-\beta(1-2H_2)](w_s-w_u)-(1+r)\bar{e}. \tag{49}$$

Thus, $v^i_{2S2}-v^i_{2U2}<0$ when H_2 is close to 0 from $\beta>1$, whereas $v^i_{2S2}-v^i_{2U2}>0$ when $1-\beta(1-2H_2)\geq \frac{2}{3} \Leftrightarrow H_2\geq \frac{1}{2}(1-\frac{1}{3\beta})$ from Assumption 3. Hence, there exists unique $H_2^{\diamondsuit}\in(0,\frac{1}{2}(1-\frac{1}{3\beta}))$ such that $H_2=0$, or H_2 is smaller than the lowest H_2 satisfying $p_{1S}=p_{2S}=p_{2U}=0$, for $F_2<H_2^{\diamondsuit}$ and $H_2=F_2$ for greater F_2.

When $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=0$, if $N_1\leq\frac{2\beta}{\beta+\gamma}\Leftrightarrow\beta(2-N_1)-\gamma N_1\geq 0$, which is always true when $\beta\geq\gamma$, (47) and thus $v^i_{2SN}-v^i_{2U2}$ increase with H_2. Because $\widetilde{S_N}-\widetilde{S_2}$ is greater than when $p_{1S}=p_{2S}=p_{2U}=0$ for given H_2 (see, for example, Figure 7), the critical H_2 satisfying $v^i_{2SN}-v^i_{2U2}=0$ is smaller than $H_2^{\diamondsuit}$ and decreases with $\widetilde{S_N}-\widetilde{S_2}$. Denote the critical H_2 by $H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2})$, where $H_2^{\diamondsuit\diamondsuit\prime}(\widetilde{S_N}-\widetilde{S_2})<0$. Then, $H_2=0$ for $F_2<H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2})$ and $H_2=F_2$ for greater F_2. Because $H_2^{\diamondsuit\diamondsuit\prime}(\widetilde{S_N}-\widetilde{S_2})<0$, there exists the unique $\widetilde{S_N}-\widetilde{S_2}$ such that $H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2})=0$. When $\widetilde{S_N}-\widetilde{S_2}$ is greater than this level, $H_2=F_2$ always.

If $N_1>\frac{2\beta}{\beta+\gamma}$, which happens only when $\beta<\gamma$, (47) decreases with H_2. $H_2=F_2$ for any $F_2\geq H_2^{\diamondsuit}$ because $v^i_{2SN}-v^i_{2U2}\geq 0$ for $F_2\geq H_2^{\diamondsuit}$ on the dividing line between $p_{2S}=1$ and $p_{2S}=0$, (mS), and thus $v^i_{2SN}-v^i_{2U2}>0$ for greater $\widetilde{S_N}-\widetilde{S_2}$ (see Figure 5). For $F_2<H_2^{\diamondsuit}$, when $\widetilde{S_N}-\widetilde{S_2}$ is greater than the level at which the dividing line and $H_2=H_2^{\diamondsuit}$ intersect, where $v^i_{2SN}-v^i_{2U2}=0$, $H_2=F_2$ because $v^i_{2SN}-v^i_{2U2}$ decreases with H_2. When $\widetilde{S_N}-\widetilde{S_2}$ is smaller than this level, $H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2})$ satisfying $H_2^{\diamondsuit\diamondsuit\prime}(\widetilde{S_N}-\widetilde{S_2})>0$ exists and $H_2=F_2$ for $F_2\leq H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2})$, $H_2=H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2})$ for greater F_2 (satisfying $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=0$), because $v^i_{2SN}-v^i_{2U2}$ decrease with H_2. Because $H_2^{\diamondsuit\diamondsuit\prime}[\widetilde{S_N}-\widetilde{S_2}]>0$, there exists the unique $\widetilde{S_N}-\widetilde{S_2}$ such that $H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2})=0$. When $\widetilde{S_N}-\widetilde{S_2}$ is smaller than this level, $H_2=0$.

To summarize the results when $\beta\geq\gamma$ or $N_1\leq\frac{2\beta}{\beta+\gamma}$, when $\widetilde{S_N}-\widetilde{S_2}$ is greater than the level such that $H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2})=0$ (where $H_2^{\diamondsuit\diamondsuit\prime}(\widetilde{S_N}-\widetilde{S_2})\leq 0$), $H_2=F_2$; when $\widetilde{S_N}-\widetilde{S_2}$ is smaller than the level satisfying $H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2})=H_2^{\diamondsuit}$, $H_2=0$ for $F_2<H_2^{\diamondsuit}$ and $H_2=F_2$ for larger F_2; when $\widetilde{S_N}-\widetilde{S_2}$ is in the intermediate range, $H_2=0$ for $F_2<H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2})$ and $H_2=F_2$ for larger F_2.

The results when $\beta<\gamma$ and $N_1>\frac{2\beta}{\beta+\gamma}$ are summarized as follows: when $\widetilde{S_N}-\widetilde{S_2}$ is greater than the level such that $H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2})=H_2^{\diamondsuit}$ (where $H_2^{\diamondsuit\diamondsuit\prime}(\widetilde{S_N}-\widetilde{S_2})>0$), $H_2=F_2$; when $\widetilde{S_N}-\widetilde{S_2}$ is smaller than the level satisfying $H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2})=0$, $H_2=0$ for $F_2<H_2^{\diamondsuit}$ and $H_2=F_2$ for larger F_2; when $\widetilde{S_N}-\widetilde{S_2}$ is in the intermediate range, $H_2=F_2$ for $F_2\leq H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2})$, $H_2=H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2})$ for $F_2\in(H_2^{\diamondsuit\diamondsuit}(\widetilde{S_N}-\widetilde{S_2}),H_2^{\diamondsuit})$, and $H_2=F_2$ for larger F_2. ■

Proof of Lemma 4. [Proof that H_2 non-decreases over time when $H_2=F_2$] This is the case when $\lambda[(1-\tau)w_s+T]>\bar{e}$ for any H_2. It can be shown that $(1-\tau)w_s+T$ increases with H_2 and thus is lowest at $H_2=0$ from the equations similar to (50) and (51) below. Then, because $(1-\tau)w_s+T-(1+r)\bar{e}>(1-\tau)w_u+T$ for any H_2 from Assumption 3, $\lambda[(1-\tau)w_s+T]>\bar{e}$ always from Assumption 4 (i).

[Proof that H_2 increases over time when $p_{1S}=1$] The result is obvious when $\frac{\lambda}{1-\lambda(1+r)}w_u\geq\bar{e}$, thus the proof focuses on the case $\frac{\lambda}{1-\lambda(1+r)}w_u<\bar{e}$. When $p_{1S}=1$, from (2) and (16), the disposable labor income of unskilled workers is expressed as,

$$(1-\tau)w_u+\left(\tau-\frac{\tau^2}{2}\right)\overline{w}=w_u+\frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}\left[-w_u+\left(1-\frac{1}{2}\frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}\right)\overline{w}\right]. \tag{50}$$

The derivative of this equation with respect to H_2 equals

$$\frac{\beta-1}{1+\gamma}\left\{-\frac{w_s}{(\overline{w})^2}(1-N_1)(w_s-w_u)\left[-w_u+\left(1-\frac{1}{2}\frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}\right)\overline{w}\right]+\frac{w_s-\overline{w}}{\overline{w}}\left(1+\frac{1}{2}\frac{\beta-1}{1+\gamma}\right)(1-N_1)(w_s-w_u)\right\}$$
$$= (1-N_1)(w_s-w_u)\frac{\beta-1}{1+\gamma}\left\{-\frac{w_s}{(\overline{w})^2}\left[\left(1+\frac{1}{2}\frac{\beta-1}{1+\gamma}\right)\overline{w}-\left(w_u+\frac{1}{2}\frac{\beta-1}{1+\gamma}w_s\right)\right]+\frac{w_s-\overline{w}}{\overline{w}}\left(1+\frac{1}{2}\frac{\beta-1}{1+\gamma}\right)\right\}$$
$$= (1-N_1)(w_s-w_u)\frac{\beta-1}{1+\gamma}\left[\frac{w_s}{(\overline{w})^2}\left(w_u+\frac{1}{2}\frac{\beta-1}{1+\gamma}w_s\right)-\left(1+\frac{1}{2}\frac{\beta-1}{1+\gamma}\right)\right]. \tag{51}$$

The second derivative is negative because $\overline{w}$ increases with H_2.

At $H_2=0$, (50) equals

$$(1-\tau)w_u+\left(\tau-\frac{\tau^2}{2}\right)\overline{w}=w_u+\frac{\beta-1}{1+\gamma}\frac{(1-N_1)(w_s-w_u)^2}{N_1w_s+(1-N_1)w_u}\left[N_1-\frac{1}{2}\frac{\beta-1}{1+\gamma}(1-N_1)\right]. \tag{52}$$

Thus, $\frac{\lambda}{1-\lambda(1+r)}[(1-\tau)w_u+T]>\overline{e}$ holds at $H_2=0$ from footnote 37 of Assumption 4 (i). Then, because (50) equals w_u at $H_2=1$ and the second derivative of (50) is negative, there exists $\widetilde{H}_2\in(0,1)$ such that $\frac{\lambda}{1-\lambda(1+r)}[(1-\tau)w_u+T]>(<)\overline{e}$ for $H_2<(>)\widetilde{H}_2$.

From Assumption 4 (ii), a not-small proportion of group 2 individuals do not have wealth initially. If the proportion of such individuals is greater than $1-\widetilde{H}_2$, their descendants can accumulate wealth greater than $\overline{e}$ and thus H_2 jumps from a value less than $\widetilde{H}_2$ to 1 at some point in time because $\frac{\lambda}{1-\lambda(1+r)}[(1-\tau)w_u+T]>\overline{e}$ always holds for their lineages. If $\frac{\lambda}{1-\lambda(1+r)}[(1-\tau)w_u+T]$ at $H_2=0$ is sufficiently greater than $\overline{e}$ (Assumption 4 (i)), $\widetilde{H}_2$ is large enough that the initial proportion of those without wealth is greater than $1-\widetilde{H}_2$. This is the case if β or w_s is sufficiently large or γ is sufficiently small because $(1-\tau)w_u+T$ increases with β and w_s and decreases with γ, as shown next.

The derivative of the disposable income (50) with respect to $\frac{\beta-1}{1+\gamma}$ equals $\frac{w_s-\overline{w}}{\overline{w}}$ times

$$-w_u+\left(1-\frac{1}{2}\frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}\right)\overline{w}-\frac{1}{2}\frac{\beta-1}{1+\gamma}(w_s-\overline{w})=(\overline{w}-w_u)-\frac{\beta-1}{1+\gamma}(w_s-\overline{w})$$
$$=\left\{[N_1+(1-N_1)H_2]-\frac{\beta-1}{1+\gamma}(1-N_1)(1-H_2)\right\}(w_s-w_u)$$
$$\geq\left[N_1-\frac{\beta-1}{1+\gamma}(1-N_1)\right](w_s-w_u)\geq\left[N_1-\frac{1}{3}(1-N_1)\right](w_s-w_u)>0,$$

where the second last inequality is from Assumption 1.

The derivative of the disposable income with respect to w_s equals $\frac{\beta-1}{1+\gamma}$ times

$$\frac{1}{\overline{w}}\left[-w_u+\left(1-\frac{1}{2}\frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}\right)\overline{w}\right]-\frac{w_s-\overline{w}}{\overline{w}}\frac{1}{2}\frac{\beta-1}{1+\gamma}$$
$$+[N_1+(1-N_1)H_2]\left\{-\frac{w_s}{(\overline{w})^2}\left[-w_u+\left(1-\frac{1}{2}\frac{\beta-1}{1+\gamma}\frac{w_s-\overline{w}}{\overline{w}}\right)\overline{w}\right]+\frac{w_s-\overline{w}}{\overline{w}}\left(1+\frac{1}{2}\frac{\beta-1}{1+\gamma}\right)\right\}$$
$$= \frac{\overline{w}-w_u}{\overline{w}}-\frac{w_s-\overline{w}}{\overline{w}}\frac{\beta-1}{1+\gamma}+\frac{N_1+(1-N_1)H_2}{\overline{w}}\left\{\frac{w_s}{\overline{w}}\left[-(\overline{w}-w_u)+\frac{1}{2}\frac{\beta-1}{1+\gamma}(w_s-\overline{w})\right]+(w_s-\overline{w})\left(1+\frac{1}{2}\frac{\beta-1}{1+\gamma}\right)\right\}$$
$$= \frac{1}{\overline{w}}\left\{\overline{w}-w_u+[N_1+(1-N_1)H_2]\left(\frac{w_s w_u}{\overline{w}}-\overline{w}\right)\right\}+\frac{w_s-\overline{w}}{\overline{w}}\frac{\beta-1}{1+\gamma}\left\{-1+\frac{1}{2}\frac{[N_1+(1-N_1)H_2]}{\overline{w}}(w_s+\overline{w})\right\}$$
$$= \frac{1}{\overline{w}}\left[(1-N_1)(1-H_2)\overline{w}-(1-N_1)(1-H_2)\frac{(w_u)^2}{\overline{w}}\right]-\frac{w_s-\overline{w}}{\overline{w}}\frac{\beta-1}{1+\gamma}\frac{(1-N_1)(1-H_2)(w_u+\overline{w})}{2\overline{w}}$$
$$= \frac{(1-N_1)(1-H_2)(w_u+\overline{w})}{(\overline{w})^2}(w_s-w_u)\left\{[N_1+(1-N_1)H_2]-\frac{\beta-1}{1+\gamma}\frac{(1-N_1)(1-H_2)}{2}\right\}$$
$$\geq \frac{(1-N_1)(1-H_2)(w_u+\overline{w})}{(\overline{w})^2}(w_s-w_u)\left(N_1-\frac{\beta-1}{1+\gamma}\frac{1-N_1}{2}\right)>0,$$

where the last inequality is from Assumption 1. ■

Proof of Proposition 4. Figures 8−11 would be helpful to understand the proof.

(i) The proof of Proposition 3 (i) applies for most results on steady-state social identity, because $H_2=F_2$ from Lemma 3 (ii)(a) and, under Assumption 5, the relevant results of Lemma A3 (ii) are same as those of Lemma A1 (ii). The exception is that, unlike the constant H_2 case, $p_{1S}=1$, $p_{2S}=1$, $p_{2U}=0$, which may be realized for low H_2 when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$ (see Figure 3), shifts to $p_{1S}=p_{2S}=p_{2U}=1$ eventually because H_2 increases over time from Lemma 4.

The result on the cultural variable is from Lemma A4 (i) and Proposition 3 (i).

(ii) F_2 is constant under $p_{2S}=0$ from $\tau=0$. When $(F_2,\widetilde{S_N})$ is located below initial (mS) with $p_{1S}=0$ and $F_2\geq H_2^{\Diamond}$, $H_2=F_2$ from the proof of Lemma 3 (ii)(b) and thus $p_{1S}=p_{2S}=p_{2U}=0$ initially. $p_{1S}=p_{2S}=p_{2U}=0$ holds in subsequent periods, because $q_{2S}=q_{2U}=0$ continues to hold and thus (mS) does not shift from Lemma A3 (v) and (13).

When $\beta\leq\gamma$ or $N_1\leq\frac{\beta+\gamma}{2\beta}$ (Figures 8−10), if $(F_2,\widetilde{S_N})$ is below initial (mS) with $p_{1S}=0$ and $F_2\leq H_2^{\Diamond}$ or if $(F_2,\widetilde{S_N})$ is on or above initial (mS) with $p_{1S}=0$ and below initial (mU) with $p_{1S}=0$ and $F_2\leq H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$ (where $H_2^{\Diamond\Diamond\prime}(\widetilde{S_N}-\widetilde{S_2})\leq 0$), $H_2=0$ and $p_{1S}=p_{2U}=0$ initially from the proof of Lemma 3 (ii)(b). Similarly, when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$ (Figure 11), if $F_2\leq H_2^{\Diamond}$ and $\widetilde{S_N}-\widetilde{S_2}$ is smaller than the level satisfying $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})=0$ (where $H_2^{\Diamond\Diamond\prime}(\widetilde{S_N}-\widetilde{S_2})>0$), $H_2=0$ and $p_{1S}=p_{2U}=0$ initially from the proof. In these cases, $p_{1S}=p_{2U}=0$ holds in subsequent periods. This is because (mS) and (mU) do not shift from Lemma A3 (v), $H_2^{\Diamond}$ is constant from (49), and $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$ does not change from (47) and the fact that $q_{1S}=1, q_{2U}=0$ continues to hold from (13).

The equilibrium with $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=0$ does not shift to $p_{1S}=p_{2S}=p_{2U}=0$ because (mS) with $p_{1S}=0$ in the initial period is located at a higher position than or the same position as those in subsequent periods on the $(F_2,\widetilde{S_N})$ plane from Lemma A3 (i).

$q^*_{1S}=1$ and $q^*_{2S}=q^*_{2U}=0$ ($q^*_{2U}=0$ when $H^*_2=0$) is from Lemma A4 (iii) and the result that only the society starting with $p_{1S}=p_{2S}=p_{2U}=0$ ($H_2=0$ and $p_{1S}=p_{2U}=0$) ends up with $p^*_{1S}=p^*_{2S}=p^*_{2U}=0$ ($H^*_2=0$ and $p^*_{1S}=p^*_{2U}=0$).

The result on the cultural variable is from Lemma A4 (iii). Unlike the constant H_2 case, the initial equilibrium $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=0$ or 1 can converge to $p^*_{1S}=p^*_{2S}=p^*_{2U}=0$, thus $q^*_{2S}=q^*_{2U}$ can be greater than 0.

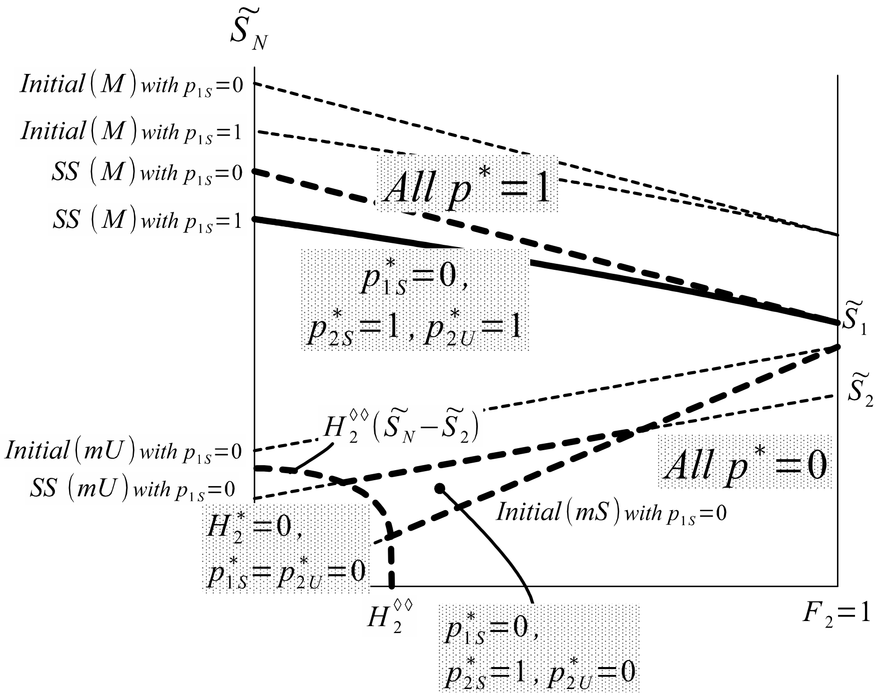

Figure 8: Relation between initial $(F_2, \widetilde{S_N})$ and steady-state identity for the full-fledged model with $\frac{\lambda}{1-\lambda(1+r)}w_u \leq \bar{e}$ when $\beta \leq \gamma$ and $N_1 \leq \frac{2\beta}{\beta+\gamma}$

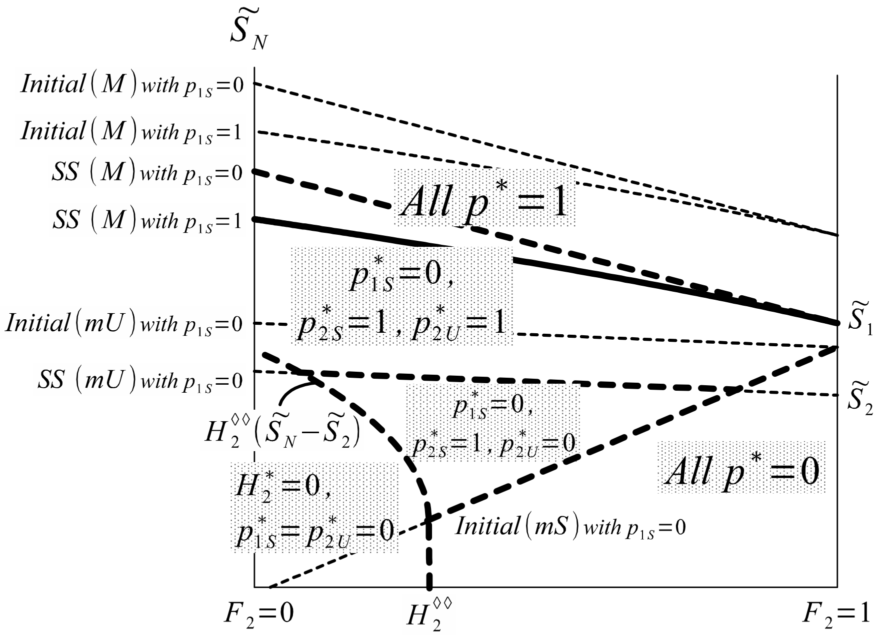

Figure 9: Relation between initial $(F_2, \widetilde{S_N})$ and steady-state identity for the full-fledged model with $\frac{\lambda}{1-\lambda(1+r)}w_u \leq \bar{e}$ when $\beta > \gamma$ and $N_1 \leq \frac{\beta+\gamma}{2\beta}$

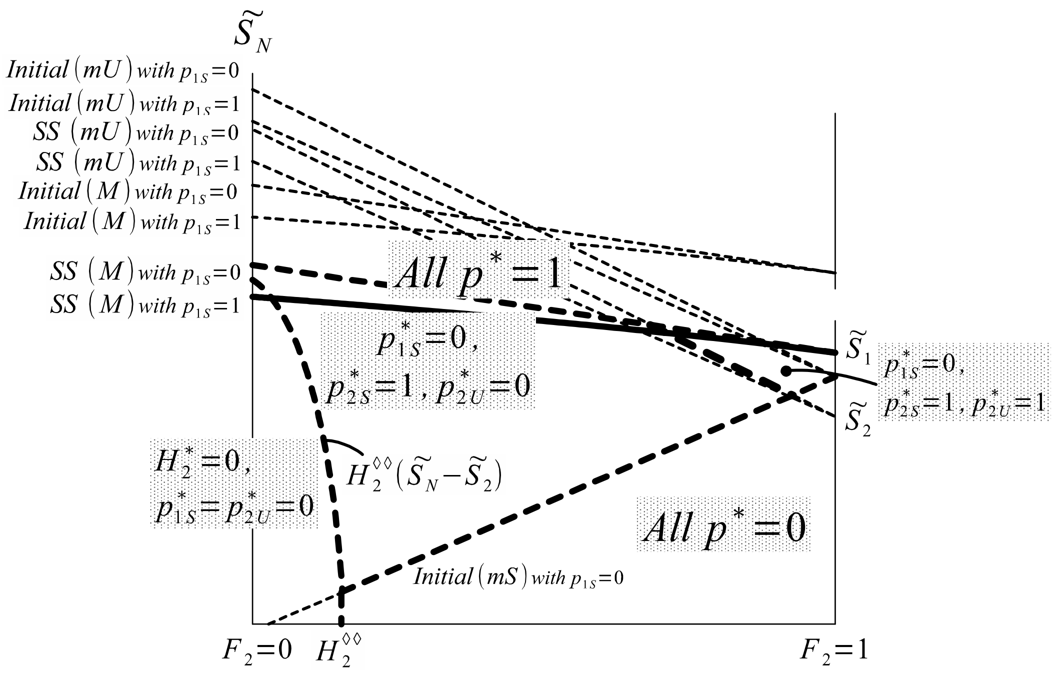

Figure 10: Relation between initial $(F_2, \widetilde{S_N})$ and steady-state identity for the full-fledged model with $\frac{\lambda}{1-\lambda(1+r)} w_u \leq \overline{e}$ when $\beta > \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$

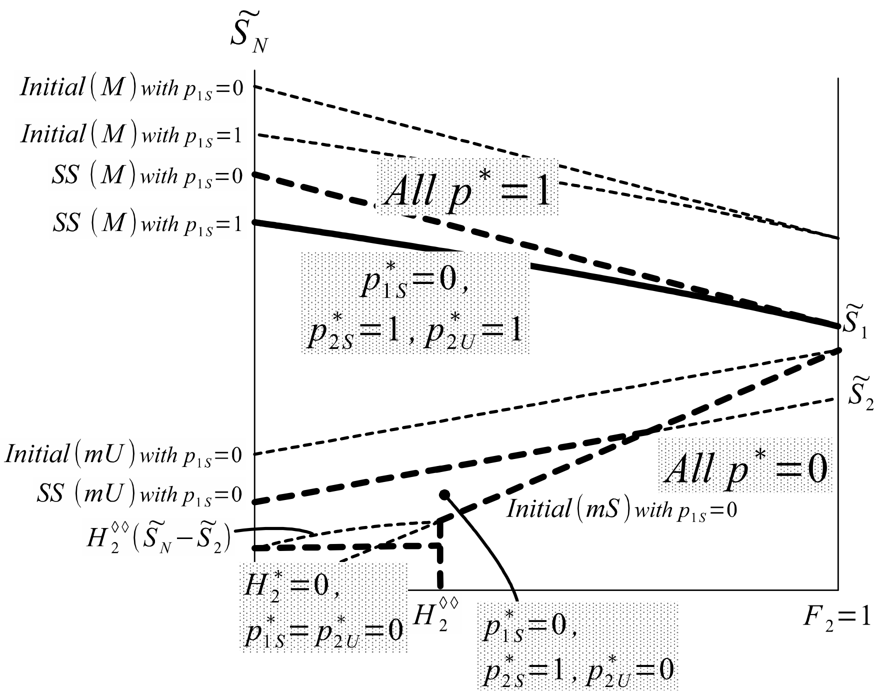

Figure 11: Relation between initial $(F_2, \widetilde{S_N})$ and steady-state identity for the full-fledged model with $\frac{\lambda}{1-\lambda(1+r)} w_u \leq \overline{e}$ when $\beta < \gamma$ and $N_1 > \frac{2\beta}{\beta+\gamma}$

(iii) F_2 is constant under $p_{2S}=0$ from $\tau=0$. When $p_{1S}=0$, $p_{2S}=p_{2U}=1$ initially (i.e., $(F_2, \widetilde{S_N})$ is on or above initial (mU) with $p_{1S}=0$ and below initial (M) with $p_{1S}=0$; $H_2=F_2$ from Lemma 3 (ii)(a)) and $\widetilde{S_N}$ is relatively low for given $H_2=F_2$ (i.e., $(F_2, \widetilde{S_N})$ is below steady-state (M) with $p_{1S}=0$), $p_{1S}=0$, $p_{2S}=p_{2U}=1$ holds in subsequent periods because (M) and (mU) shift downward over time on the $(F_2, \widetilde{S_N})$ plane from Lemma A3 (iii).

When $\beta \geq \gamma$ or $N_1 \leq \frac{2\beta}{\beta+\gamma}$ (Figures 8–10) and $p_{1S}=0$, $p_{2S}=1, p_{2U}=0$ initially (i.e., $(F_2, \widetilde{S_N})$ is on or above initial (mS) with $p_{1S}=0$ and below initial (mU) with $p_{1S}=0$; when $\beta > \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$ [Figure 10], also below initial (M) with $p_{1S}=0$; *and* $F_2 \geq H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$, where $H_2=F_2$ from the proof of Lemma 3 (ii)(b)) and $\widetilde{S_N}$ is relatively high (i.e., $(F_2, \widetilde{S_N})$ is on or above steady-state (mU) with $p_{1S}=0$; when $\beta > \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$ [Figure 10], also below steady-state (M) with $p_{1S}=0$), society shifts to $p_{1S}=0$, $p_{2S}=p_{2U}=1$ eventually and stays in this state, unless F_2 is close to $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$, in which case the shift to $H_2=0$ and $p_{1S}=p_{2U}=0$ cannot be ruled out. This is because (M) shifts downward over time, so does (mU) in the long run, from Lemma A3 (iv) and (mS)s in subsequent periods are not located above initial (mS) from Lemma A3 (i). When F_2 is close to $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$, such shift might occur because $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$ might decrease with changes in the cultural variables.

When $\beta < \gamma$, $N_1 > \frac{2\beta}{\beta+\gamma}$ (Figure 11), $p_{1S}=0$, $p_{2S}=1, p_{2U}=0$ initially and $\widetilde{S_N}$ is relatively high (i.e., $(F_2, \widetilde{S_N})$ is on or above initial (mS) and steady-state (mU) with $p_{1S}=0$, below initial (mU) with $p_{1S}=0$), where $H_2=F_2$ from the proof of Lemma 3 (ii)(b), society shifts to $p_{1S}=0$, $p_{2S}=p_{2U}=1$ and stays in this state due to the reasons explained for the previous case.

To summarize, when $\beta \geq \gamma$ or $N_1 \leq \frac{2\beta}{\beta+\gamma}$ (Figures 8–10), $p_{1S}^*=0, p_{2S}^*=p_{2U}^*=1$ if $(F_2, \widetilde{S_N})$ is located on or above initial (mS) with $p_{1S}=0$, as well as steady-state (mU) with $p_{1S}=0$, and below steady-state (M) with $p_{1S}=0$, and $F_2 \geq H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$, except when F_2 is close to $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$, in which case $H_2^*=0$ and $p_{1S}^*=p_{2U}^*=0$ might occur; when $\beta < \gamma$ and $N_1 > \frac{2\beta}{\beta+\gamma}$ (Figure 11), $p_{1S}^*=0, p_{2S}^*=p_{2U}^*=1$ if $(F_2, \widetilde{S_N})$ is located on or above initial (mS) with $p_{1S}=0$, as well as steady-state (mU) with $p_{1S}=0$, and below steady-state (M) with $p_{1S}=0$.

When $\beta \geq \gamma$ or $N_1 \leq \frac{2\beta}{\beta+\gamma}$ (Figures 8–10), $p_{1S}=0$, $p_{2S}=1, p_{2U}=0$ initially, and $\widetilde{S_N}$ is relatively low (i.e., $(F_2, \widetilde{S_N})$ is on or above initial (mS) with $p_{1S}=0$, below steady-state (mU) with $p_{1S}=0$; when $\beta > \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$ [Figure 10], also below steady-state (M) with $p_{1S}=0$; and $F_2 \geq H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$), society stays in this state, unless F_2 is close to $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$, in which case the shift to $H_2=0$ and $p_{1S}=p_{2U}=0$ might occur due to the reasons explained for the case before the previous case.

When $\beta < \gamma$, $N_1 > \frac{2\beta}{\beta+\gamma}$ (Figure 11), $p_{1S}=0$, $p_{2S}=1, p_{2U}=0$ initially, and $\widetilde{S_N}$ is relatively low (i.e., $(F_2, \widetilde{S_N})$ is on or above initial (mS) with $p_{1S}=0$ for $F_2 \geq H_2^{\Diamond\Diamond}$ and $\widetilde{S_N}-\widetilde{S_2}$ is greater than the level satisfying $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})=0$ for $F_2 < H_2^{\Diamond\Diamond}$ and $(F_2, \widetilde{S_N})$ is below steady-state (mU) with $p_{1S}=0$, where $H_2=F_2$ when $\widetilde{S_N}-\widetilde{S_2}$ is greater than the level satisfying $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})=F_2$ and $H_2=H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$ when $\widetilde{S_N}-\widetilde{S_2}$ is smaller than such level from the proof of Lemma 3 (ii)(b)), society stays in this state, unless $\widetilde{S_N}-\widetilde{S_2}$ is close to the level satisfying $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})=0$, in which case the shift to $H_2=0$ and $p_{1S}=p_{2U}=0$ cannot be ruled out, because of the reasons explained for two cases before the previous case.

To summarize, $p_{1S}^*=0$, $p_{2S}^*=1, p_{2U}^*=0$ if $(F_2, \widetilde{S_N})$ is located above the region for $p_{1S}^*=p_{2S}^*=p_{2U}^*=0$ and the one for $H_2^*=0$ and $p_{1S}^*=p_{2U}^*=0$ and below steady-state (M) with $p_{1S}=0$, when $\beta > \gamma$ and $N_1 > \frac{\beta+\gamma}{2\beta}$ (Figure 10), also below steady-state (mU) with $p_{1S}=0$.

$q_{1S}^*=q_{2S}^*=q_{2U}^*=1$ is from Lemma A4 (ii) and the result that only the society starting with

$p_{1S}=1$ and never satisfying $p_{1S}=0$ ends up with $p^*_{1S}=0$, $p^*_{2S}=1, p^*_{2U}=0$ or 1.

(iv) The result can be proved similarly to Proposition 3 (iv). ■

Proof of Proposition 5. (i) (a) When $\widetilde{S_N}$ is very high so that $p_{1S}=p_{2S}=p_{2U}=1$ in the initial period (i.e., initial $(F_2, \widetilde{S_N})$ is on or above initial (M) with $p_{1S}=1$, when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$, also on or above initial (mU) with $p_{1S}=1$ on the $(F_2, \widetilde{S_N})$ plane), $p_{1S}=p_{2S}=p_{2U}=1$ always because, as noted in the proof of Proposition 4 (i), the proof of Proposition 3 (i) applies under Assumption 5. When $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$, and $p_{1S}=1$, $p_{2S}=1$, $p_{2U}=0$ initially (i.e., initial $(F_2, \widetilde{S_N})$ is on or above initial (M) with $p_{1S}=1$ and below initial (mU) with $p_{1S}=1$), the equilibrium shifts to $p_{1S}=p_{2S}=p_{2U}=1$ eventually from the proof of Proposition 4 (i). Hence, $p_{1S}=1$ always holds, thus H_2 increases over time and $H^*_2=1$ from Lemma 4.

(b) The result holds because initial (M) with $p_{1S}=1$ is downward sloping, and as $\widetilde{S_1}$ and ω_q are lower, it is located at a lower position on the $(F_2, \widetilde{S_N})$ plane.

(ii)(a) When $\widetilde{S_N}$ is high enough that $p_{1S}=0$, $p_{2S}=p_{2U}=1$ initially (i.e., initial $(F_2, \widetilde{S_N})$ is on or above initial (mU) with $p_{1S}=0$ and below initial (M) with $p_{1S}=0$) or $p_{1S}=0$, $p_{2S}=1, p_{2U}=0$ initially (i.e., initial $(F_2, \widetilde{S_N})$ is on or above initial (mS) with $p_{1S}=0$ and below initial (M) and (mU) with $p_{1S}=0$; occurs only when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$) *and* $\widetilde{S_N}$ is relatively high (i.e., initial $(F_2, \widetilde{S_N})$ is on or above steady-state (M) with $p_{1S}=1$, when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$, also on or above (mU) with $p_{1S}=1$), society shifts to $p_{1S}=p_{2S}=p_{2U}=1$ eventually because the proof of Proposition 3 (i) applies under Assumption 5. Hence, H_2 increases after the shift (when $p_{1S}=0$, $p_{2S}=1, p_{2U}=0$ initially, the equilibrium may shift to $p_{1S}=1$, $p_{2S}=1$, $p_{2U}=0$ first, in which case the increase of H_2 starts earlier) and $H^*_2=1$ from Lemma 4.

Similarly, when $\beta>\gamma$, $N_1>\frac{\beta+\gamma}{2\beta}$, $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=0$ initially (i.e., initial $(F_2, \widetilde{S_N})$ is on or above initial (mS) with $p_{1S}=0$ and below initial (M) and (mU) with $p_{1S}=0$), *and* $\widetilde{S_N}$ is relatively, but not very, high (i.e., initial $(F_2, \widetilde{S_N})$ is on or above steady-state (M) with $p_{1S}=1$ and below steady-state (mU) with $p_{1S}=1$), society shifts to $p_{1S}=1$, $p_{2S}=1$, $p_{2U}=0$ eventually since the proof of Proposition 3 (i) applies, thus H_2 starts increasing after the shift and $H^*_2=1$ from Lemma 4.

(b) From the proof of Proposition 4 (ii) and (iii), when initial $(F_2, \widetilde{S_N})$ is located below steady-state (M) with $p_{1S}=0$, when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$, also below steady-state (mU) with $p_{1S}=0$, $p_{1S}=0$ and thus $\tau=0$ always. Hence, F_2 is constant over time and thus $H^*_2<1$ from $\frac{\lambda}{1-\lambda(1+r)}w_u\leq\bar{e}$. H_2 too is constant in most cases, but may change in the following occasions. When $\beta\geq\gamma$ or $N_1\leq\frac{2\beta}{\beta+\gamma}$, H_2 may increase (decrease) when F_2 is smaller (greater) than but close to $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$ and $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$ decreases (increases) with changes in the cultural variables from the proof of Proposition 4 (iii). When $\beta<\gamma$ and $N_1>\frac{2\beta}{\beta+\gamma}$, H_2 may increase when $F_2\in\left(H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2}), H_2^{\Diamond\Diamond}\right)$ and $H_2^{\Diamond\Diamond}(\widetilde{S_N}-\widetilde{S_2})$ increases with changes in the cultural variables from the proof of Proposition 4 (iii).

(c) The result holds because steady-state (M) and (mU) (when $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$) are downward sloping, and as $\widetilde{S_1}$ [$\widetilde{S_2}$] is higher, steady-state (M) [(mU)] is located at a higher position on the $(F_2, \widetilde{S_N})$ plane. ■

Proof of Proposition 6. Figures 8−11 in the proof of Proposition 4 would be helpful to understand the proof.

(i) Since F_2 increases over time even under $p_{2S}=0$ and $H_2=F_2$ from Lemma 3 (ii)(a), $p^*_{1S}=p^*_{2S}=p^*_{2U}=1$ when $\widetilde{S_N}$ is greater than the level at which steady-state (M) with $p_{1S}=1$ intersects with $H_2=1$, i.e., when $\widetilde{S_N}>\widetilde{S_1}$. The result on the cultural variable is from Lemma A4 (i).

(ii) When $(F_2, \widetilde{S_N})$ is located below initial (mS) with $p_{1S}=0$ and $F_2\geq H_2^{\Diamond}$ in the initial period, i.e., $p_{1S}=p_{2S}=p_{2U}=0$ initially, the proof of Proposition 3 (ii) applies. This is because the relevant result of Lemma A3 (v) is same as that of Lemma A1 (v).

Unlike the constant H_2 case, when $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=0$ or 1 initially or when $H_2=0, p_{1S}=p_{2U}=0$ initially,and $\widetilde{S_N}$ is low, the shift to $p_{1S}=p_{2S}=p_{2U}=0$ and $p_{1S}^*=p_{2S}^*=p_{2U}^*=0$ can occur since H_2 increases over time. Such shift is possible when $\widetilde{S_N}$ is smaller than the level at which initial (mS) intersects with $H_2=1$, $\widetilde{S_2}+\frac{1}{\gamma\delta}[\rho\chi(1-\chi)+\beta\omega_q](N_1)^2$, because $p_{1S}=p_{2S}=p_{2U}=0$ holds for sufficiently large H_2. When $\widetilde{S_N}<\widetilde{S_2}$, the shift to $p_{1S}=p_{2S}=p_{2U}=0$ occurs for certain because the level of $\widetilde{S_N}$ on steady-state (mS) with $p_{1S}=0$ at $H_2=1$ is weakly greater than $\widetilde{S_2}$, the corresponding level when $q_{1S}^*=q_{2S}^*=q_{2U}^*$.

The result on the cultural variable is from Lemma A4 (iii). Unlike the constant H_2 case, $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=0$ or 1 can converge to $p_{1S}^*=p_{2S}^*=p_{2U}^*=0$, thus $q_{2S}^*=q_{2U}^*$ can be greater than 0.

(iii) When $\widetilde{S_N}\geq\widetilde{S_2}+[\frac{1}{\gamma\delta}[\rho\chi(1-\chi)+\beta\omega_q](N_1)^2$ and $\widetilde{S_N}\leq\widetilde{S_1}$, the proofs of (i) and (ii) do not apply, thus $p_{1S}^*=0$, $p_{2S}^*=p_{2U}^*=1$ holds, because H_2 increases over time and thus the society starting with $p_{1S}=0$, $p_{2S}=1, p_{2U}=0$ or $H_2=0, p_{1S}=p_{2U}=0$ transits to $p_{1S}=0$, $p_{2S}=p_{2U}=1$ eventually (Figure 10). When $\widetilde{S_N}\geq\widetilde{S_2}$ and $\widetilde{S_N}<\widetilde{S_2}+\frac{1}{\gamma\delta}[\rho\chi(1-\chi)+\beta\omega_q](N_1)^2$ and either $H_2=0, p_{1S}=p_{2U}=0$ or $p_{1S}=0$, $p_{2S}=1$, $p_{2U}=0$ or 1 initially, $p_{1S}^*=0$, $p_{2S}^*=p_{2U}^*=1$ holds if initial $(F_2,\widetilde{S_N})$ is located far above initial (mS) with $p_{1S}=0$ or an increase in H_2 is slow compared to the (long term) downward shift of (mS) with $p_{1S}=0$. Otherwise, as shown in the proof of (ii), $p_{1S}^*=p_{2S}^*=p_{2U}^*=0$.

The result on the cultural variable is from Lemma A4 (ii). $q_{1S}^*=q_{2S}^*=q_{2U}^*=1$ because the states with $p_{2S}=1$ do not transit to $p_{1S}=0$, $p_{2S}=p_{2U}=1$.

(iv) The result can be proved similarly to Proposition 3 (iv). ■

Proof of Proposition 7. (i) When initial $(F_2,\widetilde{S_N})$ is located on or above initial (M) with $p_{1S}=1$ on the $(F_2,\widetilde{S_N})$ plane, $p_{1S}=1$ and thus $\tau>0$ always hold from Lemma A3 (ii). Hence, the speed of convergence to $H_2^*=1$ is highest.

(ii) (a) When initial $(F_2,\widetilde{S_N})$ is located below initial (M) with $p_{1S}=0$ and $\widetilde{S_N}>\widetilde{S_1}$, $p_{2S}=1$ always when $H_2>0$ from Figures 8−11 in the proof of Proposition 4 and Lemma A3 (i). In this case, $p_{1S}=1$ and $\tau>0$ hold (thus the convergence to $H_2^*=1$ accelerates) eventually from Proposition 6 (i). Further, given F_2, $p_{1S}=1$ holds earlier and thus the speed of convergence to $H_2^*=1$ is higher as $\widetilde{S_N}$ is higher. This is because (M) shifts downward over time or does not shift (when $H_2=0$, $p_{1S}=p_{2U}=0$) from Lemma A3 (iii)−(v) and (M) is downward sloping on the $(F_2,\widetilde{S_N})$ plane.

(b) When $\widetilde{S_N}\leq\widetilde{S_1}$, $p_{1S}=0$ and thus $\tau=0$ always from the proof of Proposition 6 (ii) and (iii).

(c) The result holds because as $\widetilde{S_1}$ or ω_q is lower, (M) is located at a lower position on the $(F_2,\widetilde{S_N})$ plane and thus the case of (i) is more likely to hold and the case considered in (ii) (b) is less likely to be realized. ■

Proof of Lemma A1. Because $q_{2C}^i=\overline{q}_{2C}$ $(C=S,U)$ holds in any period under the initial condition $q_{2C}^i=0$ in the model with constant H_2, the notation q_{2C}, not q_{2C}^i, is used. In the following proofs, $q_{2S}\geq\overline{q}_2\geq q_{2U}$ is used, which is from Proposition 1 (i), (13), and (14).

(i) The last term of the RHS of (mS) in the initial period equals $\frac{1}{\gamma\delta}[\rho\chi(1-\chi)+\beta\omega_q](N_1)^2$ from $q_{1S}=1$ and $q_{2S}=q_{2U}=0$. In subsequent periods, $(\overline{q}-\overline{q}_2)[(\overline{q}_2-q_{2S})+(\overline{q}-q_{2S})]\leq(N_1)^2$, because when $(\overline{q}_2-q_{2S})+(\overline{q}-q_{2S})>0$ (the result is straightforward when $(\overline{q}_2-q_{2S})+(\overline{q}-q_{2S})\leq0$), $(\overline{q}-\overline{q}_2)[(\overline{q}_2-q_{2S})+(\overline{q}-q_{2S})]\leq N_1(1-\overline{q}_2)[N_1(1-\overline{q}_2)-2(q_{2S}-\overline{q}_2)]\leq(N_1)^2$ from $\overline{q}_1\leq1$ and thus $\overline{q}\leq N_1+(1-N_1)\overline{q}_2$. The last term of the RHS of (M) in the initial period equals $\frac{1}{\gamma\delta}[\rho\chi(1-\chi)+\beta\omega_q](1-N_1)^2$. The expression in subsequent periods is smaller because $\overline{q}_1-\overline{q}=(1-N_1)(\overline{q}_1-\overline{q}_2)\leq1-N_1$.

(ii) When $p_{1S}=p_{2S}=p_{2U}=1$, $(\overline{q}_J)'=\overline{q}_J+\chi(\overline{q}-\overline{q}_J)$ from (14) and thus $(\overline{q})'=N_1[\overline{q}_1+\chi(\overline{q}-\overline{q}_1)]+$

$(1-N_1)[\overline{q}_2+\chi(\overline{q}-\overline{q}_2)]=\overline{q}$. Hence, $(\overline{q})'-(\overline{q}_J)'=(1-\chi)(\overline{q}-\overline{q}_J)$ and thus (M) shifts downward over time. $(\overline{q}_2)'+(\overline{q})'-2(q^i_{2C})'=\overline{q}_2+\overline{q}+\chi(\overline{q}-\overline{q}_2)-2\left[\chi\overline{q}+(1-\chi)q^i_{2C}\right]=(1-\chi)(\overline{q}_2+\overline{q}-2q^i_{2C})$ $(C=S,U)$. Since $\overline{q}_2+\overline{q}-2q_{2U}>0$ from $q_{2S}\geq q_{2U}$, (mU) shifts downward, while (mS) shifts downward (upward) when $\overline{q}_2+\overline{q}-2q^i_{2S}>(<)0$.

(iii) [When $p_{1S}=0$, $p_{2S}=p_{2U}=1$] $(\overline{q}_1)'=\overline{q}_1$ from (13) and $(\overline{q}_2)'=\overline{q}_2+\chi(\overline{q}-\overline{q}_2)$ from (14). Thus, $(\overline{q})'=N_1\overline{q}_1+(1-N_1)[\overline{q}_2+\chi(\overline{q}-\overline{q}_2)]=\overline{q}+(1-N_1)\chi(\overline{q}-\overline{q}_2)$. Hence, $(\overline{q}_1)'-(\overline{q})'=\overline{q}_1-\overline{q}-(1-N_1)\chi(\overline{q}-\overline{q}_2)$ and thus (M) shifts downward. (mU) shifts downward because $(\overline{q})'-(\overline{q}_2)'=\overline{q}+(1-N_1)\chi(\overline{q}-\overline{q}_2)-[\overline{q}_2+\chi(\overline{q}-\overline{q}_2)]=(1-\chi N_1)(\overline{q}-\overline{q}_2)$ and

$$\begin{aligned}(\overline{q}_2)'+(\overline{q})'-2(q_{2U})' &= \overline{q}_2+\chi(\overline{q}-\overline{q}_2)+\overline{q}+(1-N_1)\chi(\overline{q}-\overline{q}_2)-2[\chi\overline{q}+(1-\chi)q_{2U}]\\ &= \overline{q}_2+\overline{q}-2q_{2U}+\chi[(2-N_1)(\overline{q}-\overline{q}_2)-2(\overline{q}-q_{2U})]\\ &= \overline{q}_2+\overline{q}-2q_{2U}-\chi[N_1(\overline{q}-\overline{q}_2)+2(\overline{q}_2-q_{2U})]\leq\overline{q}+\overline{q}_2-2q_{2U}.\end{aligned}$$

As long as $p_{1S}=0$ and $p_{2S}=p_{2U}=1$ holds, the cultural distance between individuals becomes 0 in the long run from Lemma A2 (iii), thus (mS) shifts downward in the long run.

[When $p_{1S}=1$, $p_{2S}=1$, $p_{2U}=0$] $(\overline{q}_1)'=\chi\overline{q}+(1-\chi)\overline{q}_1$ from (14), $(\overline{q}_2)'=H_2[\chi\overline{q}+(1-\chi)\overline{q}_{2S}]+(1-H_2)[\chi\overline{q}_2+(1-\chi)\overline{q}_{2U}]=\chi H_2\overline{q}+(1-\chi H_2)\,\overline{q}_2$ from (14) and (13), thus $(\overline{q})'=N_1\,[\chi\overline{q}+(1-\chi)\overline{q}_1]+(1-N_1)\,[\chi H_2\overline{q}+(1-\chi H_2)\,\overline{q}_2]$. Hence,

$$\begin{aligned}(\overline{q}_1)'-(\overline{q})' &= (1-N_1)\,\{[\chi\overline{q}+(1-\chi)\overline{q}_1]-[\chi H_2\overline{q}+(1-\chi H_2)\overline{q}_2]\}\\ &= \overline{q}_1-\overline{q}-[1-(1-\chi)(1-N_1)]\,(\overline{q}_1-\overline{q})+(1-N_1)(1-\chi H_2)(\overline{q}-\overline{q}_2)\\ &= \{(1-\chi)(1-N_1)+(1-\chi H_2)\,N_1\}\,(\overline{q}_1-\overline{q})\\ &= \{1-\chi[1-N_1(1-H_2)]\}(\overline{q}_1-\overline{q})<\overline{q}_1-\overline{q}.\end{aligned}$$

Thus, (M) shifts downward over time.

$$\begin{aligned}(\overline{q})'-(\overline{q}_2)' &= N_1\{[\chi\overline{q}+(1-\chi)\overline{q}_1]-[\chi H_2\overline{q}+(1-\chi H_2)\overline{q}_2]\}\\ &= \overline{q}-\overline{q}_2-(1-N_1+\chi N_1H_2)(\overline{q}-\overline{q}_2)+N_1(1-\chi)(\overline{q}_1-\overline{q})\\ &= \overline{q}-\overline{q}_2-[1-N_1\,(1-H_2)]\,\chi N_1(\overline{q}_1-\overline{q}_2)\,.\\ (\overline{q})'+(\overline{q}_2)'-2(q_{2U})' &= N_1[\chi\overline{q}+(1-\chi)\overline{q}_1]+(2-N_1)[\chi H_2\overline{q}+(1-\chi H_2)\,\overline{q}_2]-2[\chi\overline{q}_2+(1-\chi)q_{2U}]\\ &= (\overline{q}+\overline{q}_2-2q_{2U})+N_1(1-\chi)(\overline{q}_1-\overline{q})-(1-N_1)(\overline{q}-\overline{q}_2)+(2-N_1)\chi H_2(\overline{q}-\overline{q}_2)-2\chi(\overline{q}_2-q_{2U})\\ &= (\overline{q}+\overline{q}_2-2q_{2U})+\{(1-\chi)(1-N_1)-[(1-N_1)-(2-N_1)\chi H_2]\}N_1(\overline{q}_1-\overline{q}_2)-2\chi(\overline{q}_2-q_{2U})\\ &= (\overline{q}+\overline{q}_2-2q_{2U})-\chi\{[(1-N_1)-(2-N_1)H_2]N_1(\overline{q}_1-\overline{q}_2)+2(\overline{q}_2-q_{2U})\}\,.\end{aligned}$$

From these equations,

$$\begin{aligned}&\left[(\overline{q})'-(\overline{q}_2)'\right]\left[(\overline{q})'+(\overline{q}_2)'-2(q_{2U})'\right]\\ &=(\overline{q}-\overline{q}_2)(\overline{q}+\overline{q}_2-2q_{2U})-\chi N_1(\overline{q}_1-\overline{q}_2)\begin{bmatrix}\{[(1-N_1)-(2-N_1)H_2]N_1(\overline{q}_1-\overline{q}_2)+2(\overline{q}_2-q_{2U})\}\\ +[1-N_1(1-H_2)]\begin{pmatrix}(\overline{q}+\overline{q}_2-2q_{2U})\\ -\chi\{[(1-N_1)-(2-N_1)H_2]N_1(\overline{q}_1-\overline{q}_2)+2(\overline{q}_2-q_{2U})\}\end{pmatrix}\end{bmatrix}\\ &=(\overline{q}-\overline{q}_2)(\overline{q}+\overline{q}_2-2q_{2U})-\chi N_1(\overline{q}_1-\overline{q}_2)\begin{pmatrix}\{1-\chi[1-N_1(1-H_2)]\}\{[(1-N_1)-(2-N_1)H_2]N_1(\overline{q}_1-\overline{q}_2)+2(\overline{q}_2-q_{2U})\}\\ +[1-N_1\,(1-H_2)](\overline{q}+\overline{q}_2-2q_{2U})\end{pmatrix}\\ &=(\overline{q}-\overline{q}_2)(\overline{q}+\overline{q}_2-2q_{2U})-\chi N_1(\overline{q}_1-\overline{q}_2)\begin{pmatrix}\{1-\chi[1-N_1(1-H_2)]\}\{(1-N_1)(1-H_2)N_1(\overline{q}_1-\overline{q}_2)+2(\overline{q}_2-q_{2U})\}\\ -\{1-\chi\,[1-N_1(1-H_2)]\}H_2N_1(\overline{q}_1-\overline{q}_2)\\ +[1-N_1(1-H_2)][N_1(\overline{q}_1-\overline{q}_2)+2(\overline{q}_2-q_{2U})]\end{pmatrix}\\ &=(\overline{q}-\overline{q}_2)(\overline{q}+\overline{q}_2-2q_{2U})-\chi N_1(\overline{q}_1-\overline{q}_2)\begin{pmatrix}\{1-\chi[1-N_1(1-H_2)]\}\{(1-N_1)(1-H_2)N_1(\overline{q}_1-\overline{q}_2)+2(\overline{q}_2-q_{2U})\}\\ +\{(1-N_1)(1-H_2)+\chi H_2[1-N_1(1-H_2)]\}N_1(\overline{q}_1-\overline{q}_2)\\ +[1-N_1(1-H_2)]2(\overline{q}_2-q_{2U})\end{pmatrix}.\end{aligned}$$

Thus, (mU) shifts downward over time.

The result on (mS) can be proved similarly to the result when $p_{1S}=0$ and $p_{2S}=p_{2U}=1$.

(iv) Because $(\overline{q}_1)'=\overline{q}_1$ from (13) and $(\overline{q}_2)'=\chi H_2\overline{q}+(1-\chi H_2)\,\overline{q}_2$ from the proof of (iii) when $p_{1S}=1$, $p_{2S}=1$, $p_{2U}=0$, $(\overline{q})'=\overline{q}+\chi(1-N_1)H_2(\overline{q}-\overline{q}_2)$. Hence, $(\overline{q}_1)'-(\overline{q})'=\overline{q}_1-\overline{q}-\chi(1-N_1)H_2(\overline{q}-\overline{q}_2)$ and thus (M) shifts downward. The result on (mU) and (mS) can be proved similarly to the one on (mS) of (iii).

(v) When $p_{1S}=p_{2S}=p_{2U}=0$, $(\overline{q}_J)'=\overline{q}_J$ $(J=1,2)$ and $(\overline{q})'=\overline{q}$ from (13). Hence, $(\overline{q})'-(\overline{q}_J)'=\overline{q}-\overline{q}_J$ and thus (M) does not shift over time. From (13) and $q_{2S}\geq q_{2U}$,

$$\begin{aligned}(\overline{q})'+(\overline{q}_2)'-2(q_{2U})' &= \overline{q}_2+\overline{q}-2q_{2U}-2\chi(\overline{q}_2-q_{2U})\leq\overline{q}_2+\overline{q}-2q_{2U},\\ (\overline{q})'+(\overline{q}_2)'-2(q_{2S})' &= \overline{q}_2+\overline{q}-2q^i_{2S}+2\chi(q_{2S}-\overline{q}_2)\geq\overline{q}_2+\overline{q}-2q^i_{2S},\end{aligned}$$

where the first (second) inequality holds with "$<$" ("$>$") unless $q_{2S}=q_{2U}$ for any i. Thus, the results on (mU) and (mS) hold. ■

Proof of Lemma A2. (i) When $p^*_{1S}=p^*_{2S}=p^*_{2U}=1$, $q^*_{1S}=q^*_{2S}=q^*_{2U}=\overline{q}^*$ from (14). $\overline{q}^*=\overline{q}^\#$ because $(\overline{q})'=\overline{q}$ when $p_{1S}=p_{2S}=p_{2U}=1$ from (14).

(ii) When $p^*_{1S}=p^*_{2S}=1$ and $p^*_{2U}=0$, $q^*_{1S}=q^*_{2S}=q^*_{2U}=\overline{q}^*_1=\overline{q}^*_2$ from (13) and (14). From (14), when $p_{1S}=p_{2S}=1$, $p_{2U}=0$,

$$\begin{aligned}(\overline{q}_1)'&=\chi\,[N_1\overline{q}_1+(1-N_1)\overline{q}_2]+(1-\chi)\overline{q}_1\\ &=[\chi N_1+(1-\chi)]\overline{q}_1+\chi(1-N_1)\overline{q}_2,\end{aligned}\tag{53}$$

and from (14) and (13),

$$\begin{aligned}(\overline{q}_2)'&=H_2[\chi\overline{q}+(1-\chi)\,\overline{q}_{2S}]+(1-H_2)[\chi\overline{q}_2+(1-\chi)\overline{q}_{2U}]\\ &=\chi H_2[N_1\overline{q}_1+(1-N_1)\overline{q}_2]+(1-H_2)\overline{q}_2\\ &=\chi H_2N_1\overline{q}_1+(1-\chi H_2N_1)\,\overline{q}_2.\end{aligned}\tag{54}$$

These equations can be expressed as

$$\begin{pmatrix}(\overline{q}_1)'\\(\overline{q}_2)'\end{pmatrix}=\begin{pmatrix}a & 1-a\\ b & 1-b\end{pmatrix}\begin{pmatrix}\overline{q}_1\\ \overline{q}_2\end{pmatrix},\tag{55}$$

where $a\equiv\chi N_1+(1-\chi)$ and $b\equiv\chi H_2N_1$ in this proof, in which $a>b$.

Thus,

$$\begin{pmatrix}\overline{q}^*_1\\ \overline{q}^*_2\end{pmatrix}=\lim_{n\to\infty}\begin{pmatrix}a & 1-a\\ b & 1-b\end{pmatrix}^n\begin{pmatrix}\overline{q}^\dagger_1\\ \overline{q}^\dagger_2\end{pmatrix},\tag{56}$$

where

$$\lim_{n\to\infty}\begin{pmatrix} a & 1-a \\ b & 1-b \end{pmatrix}^n = \lim_{n\to\infty}\left[\begin{pmatrix} a & 1-a \\ b & 1-b \end{pmatrix}^2\begin{pmatrix} a & 1-a \\ b & 1-b \end{pmatrix}^{n-2}\right]$$

$$= \lim_{n\to\infty}\left[\begin{pmatrix} a^2+(1-a)b & (1+a-b)(1-a) \\ (1+a-b)b & (1-b)^2+(1-a)b \end{pmatrix}\begin{pmatrix} a & 1-a \\ b & 1-b \end{pmatrix}^{n-2}\right]$$

$$= \lim_{n\to\infty}\left[\begin{pmatrix} a^3+[1+(a-b)+a](1-a)b & \left[1+(a-b)+(a-b)^2\right](1-a) \\ \left[1+(a-b)+(a-b)^2\right]b & (1-b)^3+[1+(a-b)+(1-b)](1-a)b \end{pmatrix}\begin{pmatrix} a & 1-a \\ b & 1-b \end{pmatrix}^{n-3}\right]$$

$$= \lim_{n\to\infty}\left[\begin{pmatrix} a^4+\left\{1+(a-b)+(a-b)^2+[1+(a-b)+a]a\right\}(1-a)b & \left[1+(a-b)+(a-b)^2+(a-b)^3\right](1-a) \\ \left[1+(a-b)+(a-b)^2+(a-b)^3\right]b & (1-b)^4+\left\{1+(a-b)+(a-b)^2+[1+(a-b)+(1-b)](1-b)\right\}(1-a)b \end{pmatrix} \times \begin{pmatrix} a & 1-a \\ b & 1-b \end{pmatrix}^{n-4}\right]$$

$$= \lim_{n\to\infty}\begin{pmatrix} \sum_{t=0}^{n-1}(a-b)^t b & \sum_{t=0}^{n-1}(a-b)^t(1-a) \\ \sum_{t=0}^{n-1}(a-b)^t b & \sum_{t=0}^{n-1}(a-b)^t(1-a) \end{pmatrix} \text{ (because } \overline{q}_1^* = \overline{q}_2^*)$$

$$= \frac{1}{1-a+b}\begin{pmatrix} b & 1-a \\ b & 1-a \end{pmatrix}. \tag{57}$$

Hence, $q_{1S}^*=q_{2S}^*=q_{2U}^*=\frac{1}{1-(1-H_2)N_1}\left[H_2N_1\overline{q}_1^\dagger+(1-N_1)\overline{q}_2^\dagger\right]$.

(iii) When at least one of p_{2S}^* and p_{2U}^* equals 1, $q_{2C}^*=\overline{q}^*$ must hold for C such that $p_{2C}^*=1$ from (14) and $q_{2C'}^*=\overline{q}_2^*$ must hold for C' such that $p_{2C'}^*=0$ from (13). Thus, $q_{2C}^*=q_{2C'}^*=\overline{q}_2^*=\overline{q}_1^*$, which equals $q_{1S}^*=\overline{q}_1^\flat$ from (iv). (iv) When $p_{1S}^*=0$, $q_{1S}^*=\overline{q}_1^*$ from (13). $\overline{q}_1^*=\overline{q}_1^\flat$ because $(\overline{q}_1)'=\overline{q}_1$ when $p_{1S}=0$ from (13). The result for q_{2S}^* and q_{2U}^* can be proved similarly. ■

Proof of Lemma A3. (i) The proof of the constant H_2 case applies. (ii) When $p_{1S}=p_{2S}=p_{2U}=1$, $(\overline{q}_1)'=\chi\overline{q}+(1-\chi)\overline{q}_1$ and $(\overline{q}_2)'=(H_2)'[\chi\overline{q}+(1-\chi)\overline{q}_{2S}]+[1-(H_2)'][\chi\overline{q}+(1-\chi)\overline{q}_{2U}]=\chi\overline{q}+(1-\chi)\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\,\overline{q}_{2U}\}$ from (14). Thus,

$$\begin{aligned}(\overline{q}_1)'-(\overline{q})' &= \chi\overline{q}+(1-\chi)\overline{q}_1-\left[N_1[\chi\overline{q}+(1-\chi)\overline{q}_1]+(1-N_1)\left(\chi\overline{q}+(1-\chi)\left\{(H_2)'\overline{q}_{2S}+\left[1-(H_2)'\right]\overline{q}_{2U}\right\}\right)\right] \\ &\leq (1-N_1)\{(1-\chi)\overline{q}_1-(1-\chi)[H_2\overline{q}_{2S}+(1-H_2)\overline{q}_{2U}]\} \\ &= (1-\chi)(1-N_1)(\overline{q}_1-\overline{q}_2)=(1-\chi)(\overline{q}_1-\overline{q}). \end{aligned}\tag{58}$$

Hence, (M) shifts downward over time.

$(\overline{q})'-(\overline{q}_2)'=N_1[(\overline{q}_1)'-(\overline{q}_2)']\leq(1-\chi)N_1(\overline{q}_1-\overline{q}_2)=(1-\chi)(\overline{q}-\overline{q}_2)$ from the above equation. Thus,

$$\begin{aligned}&\left[(\overline{q})'-(\overline{q}_2)'\right]\left[(\overline{q}_2)'+(\overline{q})'-2(q_{2C}^i)'\right] \\ \leq\; & (1-\chi)(\overline{q}-\overline{q}_2)\begin{bmatrix} \chi\overline{q}+(1-\chi)\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\,\overline{q}_{2U}\}-2\left[\chi\overline{q}+(1-\chi)q_{2C}^i\right] \\ +N_1[\chi\overline{q}+(1-\chi)\overline{q}_1]+(1-N_1)(\chi\overline{q}+(1-\chi)\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\,\overline{q}_{2U}\}) \end{bmatrix} \\ =\; & (\overline{q}-\overline{q}_2)(1-\chi)^2\begin{pmatrix} (H_2)'\overline{q}_{2S}+[1-(H_2)']\,\overline{q}_{2U}-2q_{2C}^i \\ +N_1\overline{q}_1+(1-N_1)\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\,\overline{q}_{2U}\} \end{pmatrix}.\end{aligned}$$

This equation is smaller than $(\overline{q}-\overline{q}_2)\begin{Bmatrix} H_2\overline{q}_{2S}+(1-H_2)\overline{q}_{2U}-2q_{2C}^i \\ +N_1\overline{q}_1+(1-N_1)[H_2\overline{q}_{2S}+(1-H_2)\overline{q}_{2U}] \end{Bmatrix}=(\overline{q}-\overline{q}_2)\left(\overline{q}_2+\overline{q}-2q_{2C}^i\right)$ when $\overline{q}_2+\overline{q}-2q_{2C}^i>0$, χ is sufficiently large, and $(H_2)'$ is not very large relative to H_2. $\overline{q}_2+\overline{q}-2q_{2U}^i>0$ holds from $q_{2S}^i\geq q_{2U}^i$, while $\overline{q}_2+\overline{q}-2q_{2S}^i\leq 0$ is possible. The increase of H_2 is not very large when not many unskilled individuals do not have similar levels of wealth, which is the case when the initial distribution of wealth is not concentrated in a narrow range. Hence, (mU) shifts downward, while (mS) shifts downward when $\overline{q}_2+\overline{q}-2q^i_{2S}>0$ under Assumption 5. As long as $p_{1S}=p_{2S}=p_{2U}=1$ holds, the cultural distance between individuals becomes 0 in the long run from **Lemma A2 (iii)**, thus (mS) shifts downward in the long run.

(iii) [When $p_{1S}=0$, $p_{2S}=p_{2U}=1$] $(\overline{q}_1)'=\overline{q}_1$ from (13) and $(\overline{q}_2)'=\chi\overline{q}+(1-\chi)\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\,\overline{q}_{2U}\}$ from the proof of (ii). Thus,

$$\begin{aligned}(\overline{q}_1)'-(\overline{q})' &= \overline{q}_1-\left[N_1\overline{q}_1+(1-N_1)\left(\chi\overline{q}+(1-\chi)\left\{(H_2)'\overline{q}_{2S}+\left[1-(H_2)'\right]\overline{q}_{2U}\right\}\right)\right]\\ &\le \overline{q}_1-(N_1\overline{q}_1+(1-N_1)\{\chi\overline{q}+(1-\chi)[H_2\overline{q}_{2S}+(1-H_2)\overline{q}_{2U}]\})\\ &= (1-\chi N_1)(1-N_1)(\overline{q}_1-\overline{q}_2)=(1-\chi N_1)(\overline{q}_1-\overline{q}).\end{aligned} \tag{59}$$

Hence, (M) shifts downward.

$(\overline{q})'-(\overline{q}_2)'=N_1[(\overline{q}_1)'-(\overline{q}_2)']\le(1-\chi N_1)N_1(\overline{q}_1-\overline{q}_2)=(1-\chi N_1)(\overline{q}-\overline{q}_2)$ from the above equation. Thus, similar to the equation in the proof of (ii),

$$\begin{aligned}&\left[(\overline{q})'-(\overline{q}_2)'\right]\left[(\overline{q}_2)'+(\overline{q})'-2(q^i_{2U})'\right]\\ \le\;& (1-\chi N_1)(\overline{q}-\overline{q}_2)\begin{bmatrix}\chi\overline{q}+(1-\chi)\left\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\,\overline{q}_{2U}\right\}-2\left[\chi\overline{q}+(1-\chi)q^i_{2U}\right]\\ +N_1\overline{q}_1+(1-N_1)(\chi\overline{q}+(1-\chi)\left\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\,\overline{q}_{2U}\right\})\end{bmatrix}\\ =\;& (1-\chi N_1)(\overline{q}-\overline{q}_2)\begin{pmatrix}(1-\chi)\left\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\,\overline{q}_{2U}-2q^i_{2U}\right\}\\ N_1\overline{q}_1+(1-\chi)(1-N_1)\left\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\,\overline{q}_{2U}\right\}-N_1\chi\overline{q}\end{pmatrix}.\end{aligned}$$

This equation is smaller than $(\overline{q}-\overline{q}_2)\begin{Bmatrix}H_2\overline{q}_{2S}+(1-H_2)\overline{q}_{2U}-2q^i_{2U}\\ +N_1\overline{q}_1+(1-N_1)[H_2\overline{q}_{2S}+(1-H_2)\overline{q}_{2U}]\end{Bmatrix}=(\overline{q}-\overline{q}_2)\left(\overline{q}_2+\overline{q}-2q^i_{2U}\right)$ under Assumption 5. Hence, (mU) shifts downward. The result on (mS) can be proved similarly to that in (ii).

[When $p_{1S}=1$, $p_{2S}=1$, $p_{2U}=0$] $(\overline{q}_1)'=\chi\overline{q}+(1-\chi)\overline{q}_1$ from (14), and $(\overline{q}_2)'=(H_2)'[\chi\overline{q}+(1-\chi)\overline{q}_{2S}]+[1-(H_2)'][\chi\overline{q}_2+(1-\chi)\overline{q}_{2U}]=\chi(H_2)'\overline{q}+(1-\chi)\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\overline{q}_{2U}\}+\chi[1-(H_2)']\overline{q}_2$ from (14) and (13). Thus,

$$\begin{aligned}(\overline{q})' &= N_1[\chi\overline{q}+(1-\chi)\overline{q}_1]+(1-N_1)\left(\chi(H_2)'\overline{q}+(1-\chi)\left\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\overline{q}_{2U}\right\}+\chi[1-(H_2)']\overline{q}_2\right)\\ &\ge N_1[\chi\overline{q}+(1-\chi)\overline{q}_1]+(1-N_1)(\chi H_2\overline{q}+(1-\chi)\{(H_2\overline{q}_{2S}+(1-H_2)\overline{q}_{2U}\}+\chi\,(1-H_2)\overline{q}_2)\\ &\ge N_1[\chi\overline{q}+(1-\chi)\overline{q}_1]+(1-N_1)[\chi H_2\overline{q}+(1-\chi H_2)\overline{q}_2]\,.\end{aligned}$$

Hence,

$$\begin{aligned}(\overline{q}_1)'-(\overline{q})' &\le (1-N_1)\{[\chi\overline{q}+(1-\chi)\overline{q}_1]-[\chi H_2\overline{q}+(1-\chi H_2)\overline{q}_2]\}\\ &= \overline{q}_1-\overline{q}-[1-(1-\chi)(1-N_1)](\overline{q}_1-\overline{q})+(1-N_1)(1-\chi H_2)(\overline{q}-\overline{q}_2)\\ &= [(1-\chi)(1-N_1)+(1-\chi H_2)N_1](\overline{q}_1-\overline{q})\\ &= \{1-\chi[1-N_1(1-H_2)]\}(\overline{q}_1-\overline{q})<\overline{q}_1-\overline{q}.\end{aligned}$$

Thus, (M) shifts downward over time.

$(\overline{q})'-(\overline{q}_2)'=N_1[(\overline{q}_1)'-(\overline{q}_2)']\le\{1-\chi[1-N_1(1-H_2)]\}N_1(\overline{q}_1-\overline{q}_2)=\{1-\chi[1-N_1(1-H_2)]\}(\overline{q}-\overline{q}_2)$ from the above equation. Further,

$$\begin{aligned}(\overline{q})'+(\overline{q}_2)'-2(q_{2U})' &= N_1[\chi\overline{q}+(1-\chi)\overline{q}_1]+(1-N_1)\left(\chi(H_2)'\overline{q}+(1-\chi)\left\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\overline{q}_{2U}\right\}+\chi[1-(H_2)']\overline{q}_2\right)\\ &\quad+\chi(H_2)'\overline{q}+(1-\chi)\left\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\overline{q}_{2U}\right\}+\chi[1-(H_2)']\overline{q}_2-2[\chi\overline{q}_2+(1-\chi)q_{2U}]\\ &= (1-\chi)\left(N_1\overline{q}_1+(1-N_1)\left\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\overline{q}_{2U}\right\}+(H_2)'\overline{q}_{2S}+[1-(H_2)']\overline{q}_{2U}-2q^i_{2U}\right)\\ &\quad+\left[N_1+(2-N_1)(H_2)'\right]\chi\overline{q}+\left\{(2-N_1)[1-(H_2)']-2\right\}\chi\overline{q}_2\\ &= (1-\chi)\left((1-N_1)\left\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\overline{q}_{2U}\right\}+(H_2)'\overline{q}_{2S}+[1-(H_2)']\overline{q}_{2U}-2q^i_{2U}\right)\\ &\quad+\left\{1-\chi+\chi\left[N_1+(2-N_1)(H_2)'\right]\right\}N_1\overline{q}_1\\ &\quad+\left[(1-N_1)\left[N_1+(2-N_1)(H_2)'\right]+(2-N_1)[1-(H_2)']-2\right]\chi\overline{q}_2.\end{aligned}$$

From these equations,

$$\left[(\overline{q})'-(\overline{q}_2)'\right]\left[(\overline{q})'+(\overline{q}_2)'-2(q_{2U})'\right] \leq \{1-\chi[1-N_1(1-H_2)]\}(\overline{q}-\overline{q}_2)$$

$$\times\begin{pmatrix}(1-\chi)\left((1-N_1)\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\overline{q}_{2U}\}+(H_2)'\overline{q}_{2S}+[1-(H_2)']\overline{q}_{2U}-2q^i_{2U}\right)\\ +\{1-\chi+\chi[N_1+(2-N_1)(H_2)']\}N_1\overline{q}_1\\ +\begin{Bmatrix}(1-N_1)[N_1+(1-N_1)(H_2)']+(1-N_1)[1-(H_2)']-2\\ (1-N_1)(H_2)'+[1-(H_2)']\end{Bmatrix}\chi\overline{q}_2\end{pmatrix}$$

$$= \{1-\chi[1-N_1(1-H_2)]\}(\overline{q}-\overline{q}_2)$$

$$\times\begin{pmatrix}(1-\chi)\left((1-N_1)\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\overline{q}_{2U}\}+(H_2)'\overline{q}_{2S}+[1-(H_2)']\overline{q}_{2U}-2q^i_{2U}\right)\\ +\{1-\chi+\chi[N_1+(2-N_1)(H_2)']\}N_1\overline{q}_1\\ -\left\{(N_1)^2+\left[1-(1-N_1)^2\right](H_2)'\right\}\chi\overline{q}_2\end{pmatrix}$$

$$< (\overline{q}-\overline{q}_2)\{1-\chi[1-N_1(1-H_2)]\}\begin{bmatrix}(1-\chi)\begin{pmatrix}(1-N_1)\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\overline{q}_{2U}\}\\ +(H_2)'\overline{q}_{2S}+[1-(H_2)']\overline{q}_{2U}-2q^i_{2U}\end{pmatrix}\\ +\{1-\chi[1-N_1-(2-N_1)(H_2)']\}N_1\overline{q}_1\end{bmatrix}.$$

This equation is smaller than $(\overline{q}-\overline{q}_2)\begin{Bmatrix}N_1\overline{q}_1+(1-N_1)[H_2\overline{q}_{2S}+(1-H_2)\overline{q}_{2U}]\\ +H_2\overline{q}_{2S}+(1-H_2)\overline{q}_{2U}-2q^i_{2U}\end{Bmatrix} = (\overline{q}-\overline{q}_2)(\overline{q}+\overline{q}_2-2q^i_{2U})$ under Assumption 5. In particular, $\{1-\chi[1-N_1(1-H_2)]\}\{1-\chi[1-N_1-(2-N_1)(H_2)']\} \leq 1$ when χ is sufficiently large or $(H_2)'$ is not very large relative to H_2, which is show as follows.

$$\{1-\chi[1-N_1(1-H_2)]\}\left\{1-\chi\left[1-N_1-(2-N_1)(H_2)'\right]\right\} \leq 1$$

$$\Leftrightarrow -[1-N_1(1-H_2)]-\left[1-N_1-(2-N_1)(H_2)'\right]+[1-N_1(1-H_2)]\chi\left[1-N_1-(2-N_1)(H_2)'\right] \leq 0$$

$$\Leftrightarrow -[1-N_1(1-H_2)]-(1-N_1)\left[1-(H_2)'\right]+(H_2)'+[1-N_1(1-H_2)]\chi(1-N_1)\left[1-(H_2)'\right]-[1-N_1(1-H_2)]\chi(H_2)' \leq 0$$

$$\Leftrightarrow -(1-N_1)\left[1-(H_2)'\right]\{1-\chi[1-N_1(1-H_2)]\}-[1-N_1(1-H_2)]\left[1+\chi(H_2)'\right]+(H_2)' \leq 0,$$

where the sum of the last two terms is negative when $(H_2)' \to H_2$ from $-(1+\chi H_2)+N_1(1-H_2)(1+\chi H_2)+H_2 = -(1-H_2)(1-N_1)-\chi H_2[1-N_1(1-H_2)] < 0$, and it is negative when $\chi \to 1$ and $(H_2)'$ is not very large relative to H_2 from $-[1-N_1(1-H_2)][1+(H_2)']+(H_2)' = -1+N_1(1-H_2)[1+H_2+(H_2)'-H_2]$. Hence, (mU) shifts downward over time. The result on (mS) can be proved similarly to the result when $p_{1S}=0$ and $p_{2S}=p_{2U}=1$.

(iv) $(\overline{q}_1)'=\overline{q}_1$ from (13) and $(\overline{q})' \geq N_1[\chi\overline{q}+(1-\chi)\overline{q}_1]+(1-N_1)[\chi H_2\overline{q}+(1-\chi H_2)\overline{q}_2]$ from the proof of (iii) when $p_{1S}=1$, $p_{2S}=1$, $p_{2U}=0$. Hence,

$$(\overline{q}_1)'-(\overline{q})' \leq \overline{q}_1-N_1[\chi\overline{q}+(1-\chi)\overline{q}_1]-(1-N_1)[\chi H_2\overline{q}+(1-\chi H_2)\overline{q}_2]$$

$$=\overline{q}_1-N_1[\chi\overline{q}+(1-\chi)\overline{q}_1]-(1-N_1)[\chi\overline{q}+(1-\chi)\overline{q}_2]+(1-N_1)[\chi\overline{q}+(1-\chi)\overline{q}_2]-(1-N_1)[\chi H_2\overline{q}+(1-\chi H_2)\overline{q}_2]$$

$$= \overline{q}_1-\overline{q}-\chi(1-N_1)(1-H_2)(\overline{q}-\overline{q}_2).$$

and thus (M) shifts downward. The result on (mU) and (mS) can be proved similarly to that on (mS) of (iii).

(v) When $p_{1S}=p_{2S}=p_{2U}=0$, $(\overline{q}_1)'=\overline{q}_1$, $(\overline{q}_2)'=(H_2)'[\chi\overline{q}_2+(1-\chi)\overline{q}_{2S}]+[1-(H_2)'][\chi\overline{q}_2+(1-\chi)\overline{q}_{2U}] \geq H_2[\chi\overline{q}_2+(1-\chi)\overline{q}_{2S}]+(1-H_2)[\chi\overline{q}_2+(1-\chi)\overline{q}_{2U}]=\overline{q}_2$ from (13). Thus, $(\overline{q})' \geq \overline{q}$. Hence, $(\overline{q})'-(\overline{q}_1)' \geq \overline{q}-\overline{q}_1$ and (M) shifts upward over time unless $\overline{q}_{2S}=\overline{q}_{2U}$, in which case it does not shift. When $q^i_{2S}=q^i_{2U}$, $(\overline{q}_2)'=\overline{q}_2$ and $(q^i_{2C})'=q^i_{2C}$ $(C=S,U)$ hold. Thus, (mU) and (mS) do not shift.

When $H_2=0, p_{1S}=p_{2U}=0$, $(\overline{q}_1)'=\overline{q}_1$ and $(\overline{q}_2)'=\overline{q}_2$ from (13). Thus, (M), (mU), and (mS) all do not shift. ■

Proof of Lemma A4. Proofs are provided only for results different from Lemma A2. (i) When $\frac{\lambda}{1-\lambda(1+r)}w_u \leq \overline{e}$, $p^*_{1S}=p^*_{2S}=p^*_{2U}=1$ when $p_{1S}=p_{2S}=p_{2U}=1$ or $p_{1S}=p_{2S}=1, p_{2U}=0$ initially, and

$p^*_{1S}=p^*_{2S}=p^*_{2U}=1$ may hold when $p_{1S}=0, p_{2S}=p_{2U}=1$ or $p_{1S}=0, p_{2S}=1, p_{2U}=0$ initially from the proof of Proposition 4 (i). When $p_{2S}=p_{2U}=1$ initially, $p_{2S}=p_{2U}=1$ always from the proof. Hence, $(\overline{q})'=\overline{q}$ under the initial condition, $q_{1S}=1, q_{2S}=q_{2U}=0$, and $q^*_{1S}=q^*_{2S}=q^*_{2U}=\overline{q}^{\#}\in(0,1)$ holds. When $p_{2S}=1, p_{2U}=0$ initially, $q_{2S}\neq q_{2U}$ except the initial period. Hence, after society shifts to $p_{1S}=p_{2S}=p_{2U}=1$, $(\overline{q})'=N_1[\chi\overline{q}+(1-\chi)\overline{q}_1]+(1-N_1)(\chi\overline{q}+(1-\chi)\{(H_2)'\overline{q}_{2S}+[1-(H_2)']\,\overline{q}_{2U}\})\geq\overline{q}$. Thus, $q^*_{1S}=q^*_{2S}=q^*_{2U}=\overline{q}^*\geq\overline{q}^{\#}$. Because society shifts to $p_{1S}=p_{2S}=p_{2U}=1$ in finite periods, $\overline{q}_{2S}, \overline{q}_{2U}\in(0,1)$ in the initial period in which $p_{1S}=p_{2S}=p_{2U}=1$ holds. Hence, $0<(\overline{q})'\leq N_1[\chi\overline{q}+(1-\chi)\overline{q}_1]+(1-N_1)[\chi\overline{q}+(1-\chi)\overline{q}_{2S}]<1$ and thus $\overline{q}^*\in(0,1)$.

When $\frac{\lambda}{1-\lambda(1+r)}w_u>\overline{e}$, $p^*_{1S}=p^*_{2S}=p^*_{2U}=1$ when $\widetilde{S_N}>\widetilde{S_1}$ from Proposition 6 (i). Thus, from Figures 8−11, unless $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$, $p_{2S}=p_{2U}=1$ always. Hence, $(\overline{q})'=\overline{q}$ and $q^*_{1S}=q^*_{2S}=q^*_{2U}=\overline{q}^{\#}\in(0,1)$. When $\beta>\gamma$ and $N_1>\frac{\beta+\gamma}{2\beta}$, $p_{1S}=0$ or $1, p_{2S}=1, p_{2U}=0$ may converge to $p^*_{1S}=p^*_{2S}=p^*_{2U}=1$ from Figure 10. Thus, $q^*_{1S}=q^*_{2S}=q^*_{2U}=\overline{q}^*\geq\overline{q}^{\#}$, where $\overline{q}^*\in(0,1)$.

(iii) When $p^*_{2C}=0$ $(C=S,U)$, $q^*_{2C}=\overline{q}^*_2$ from (13). $\overline{q}^*_2\geq\overline{q}^{\flat}_2$ because $(\overline{q}_2)'\geq\overline{q}_2$ from the proof of Lemma A3 (v). ■