

Appendix C: Analysis of the dynamics and the proof of Proposition A4

This Appendix analyzes the dynamics of the model and thereby proves Proposition A4, which examines the relationship between initial conditions and steady states. In this Appendix, $P(F_{ht}, F_{mt}, B)$ when $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$ is denoted as $\widehat{P}(F_{ht}+F_{mt}, B)$, because $P(F_{ht}, F_{mt}, B)$ depends on $F_{ht}+F_{mt}$, not F_{ht} and F_{mt} individually (from eq. 16).

The following lemmas are used to examine the dynamics.

Lemma C1 *When $B_0 \leq \overline{B}^*(F_{h0})$, $B_t \leq \overline{B}^*(F_{ht})$ for any $t > 0$.*

Proof. Suppose $B_t \leq \overline{B}^*(F_{ht})$ at $t \geq 0$. Since $F_{ht+1} \geq F_{ht}$, $\overline{B}^*(F_{ht+1}) \geq \overline{B}^*(F_{ht})$. Thus, when B_{t+1} is determined by (41) or (43), $B_{t+1} \leq \overline{B}^*(F_{ht}) \leq \overline{B}^*(F_{ht+1})$. When B_{t+1} is determined by (37) or (39), $B_{t+1} < \overline{B}^*(F_{ht+1})$ from $B^*(F_{ht}, F_{mt}) < \overline{B}^*(F_{ht}) \leq \overline{B}^*(F_{ht+1})$, where the first inequality is from $F_{mt} < \max\{\phi(F_{ht}, B_t), [(\frac{F_h}{F_m})_{ml, \theta}]^{-1}\} F_{ht}$, (38), (40), (42), and (44). When B_{t+1} is determined by (33) or (35), $B_{t+1} < \overline{B}^*(F_{ht+1})$ from $\widehat{B}^*(F_{ht}+F_{mt}) < B^*(F_{ht}, F'_{mt}) < \overline{B}^*(F_{ht}) \leq \overline{B}^*(F_{ht+1})$, where $F'_{mt} \in \left([(\frac{F_h}{F_m})_{hm}]^{-1} F_{ht}, \max\{\phi(F_{ht}, B_t), [(\frac{F_h}{F_m})_{ml, \theta}]^{-1}\} F_{ht} \right)$ and the first inequality is from (34), (36), (38), and (40). ■

Lemma C2 *Suppose $B_0 \leq \overline{B}^*(F_{h0})$. Then,*

- (i) *When $\frac{F_{ht}}{F_{mt}} \in \left[(\frac{F_h}{F_m})_{ml, \theta}, (\frac{F_h}{F_m})_{hm} \right)$ and $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ is possible only if $F_{ht} > F_h^\flat$. Similarly, when $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$ and $\widehat{P}(F_{ht}+F_{mt}, \widehat{B}^*(F_{ht}+F_{mt})) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, $\widehat{P}(F_{ht}+F_{mt}, B) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ is possible only if $F_{ht} > F_h^\flat$.*
- (ii) *When $\frac{F_{ht}}{F_{mt}} \in \left[(\frac{F_h}{F_m})_{ml, \theta}, (\frac{F_h}{F_m})_{hm} \right)$ and $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \leq \theta$, $P(F_{ht}, F_{mt}, B_t) > \theta$ is possible only if $F_{ht} > F_h^\dagger$. Similarly, when $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$ and $\widehat{P}(F_{ht}+F_{mt}, \widehat{B}^*(F_{ht}+F_{mt})) \leq \theta$, $\widehat{P}(F_{ht}+F_{mt}, B) > \theta$ is possible only if $F_{ht} > F_h^\dagger$.*

Proof. When $B_0 \leq \overline{B}^*(F_{h0})$, $B_t \leq \overline{B}^*(F_{ht})$ for any t from Lemma C1.

(i) Suppose $F_{ht} \leq F_h^\flat$. Since $B_t \leq \overline{B}^*(F_{ht})$, $\phi(F_{ht}, \overline{B}^*(F_{ht})) F_{ht} \leq \phi(F_{ht}, B_t) F_{ht}$ and thus, for any $F_h > F_{ht}$, $F_m = \phi(F_h, B_t) F_h$ is located at the right side of $F_m = \phi(F_h, \overline{B}^*(F_h)) F_h$ on the (F_m, F_h) plane (see Figure 1). Hence, $F_h^\flat(B_t) > F_h^\flat$ (see the figure). At given F_h and F_m satisfying $\frac{F_h}{F_m} \in \left[(\frac{F_h}{F_m})_{ml, \theta}, (\frac{F_h}{F_m})_{hm} \right)$, the absolute value of the slope of the equi $P(F_h, F_m, B)$ curve on the (F_m, F_h) plane is strictly greater than that of the equi $P(F_h, F_m, B^*(F_h, F_m))$ curve from (16), (29), and $\widetilde{w}_h > \widetilde{w}_m$. Hence, $P(F_h, F_m, B_t) A_T = \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ ($b^*(w) = e_m$ when $\frac{F_h}{F_m} \in \left[(\frac{F_h}{F_m})_{ml, \theta}, (\frac{F_h}{F_m})_{hm} \right)$) is located above $P(F_h, F_m, B^*(F_h, F_m)) A_T = \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ on the plane. Therefore, when $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ is possible only if $F_{ht} > F_h^\flat$. When $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$, the slopes of both $\widehat{P}(F_{ht}+F_{mt}, B) A_T = \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ and $\widehat{P}(F_{ht}+F_{mt}, \widehat{B}^*(F_{ht}+F_{mt})) A_T = \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ equal -1 from (25) and (30). Hence, when $\widehat{P}(F_{ht}+F_{mt}, \widehat{B}^*(F_{ht}+F_{mt})) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, $\widehat{P}(F_{ht}+F_{mt}, B) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ is possible only if $F_{ht} > F_h^\flat$. (ii) Can be proved in a similar way as (i) (see Figure 2). ■

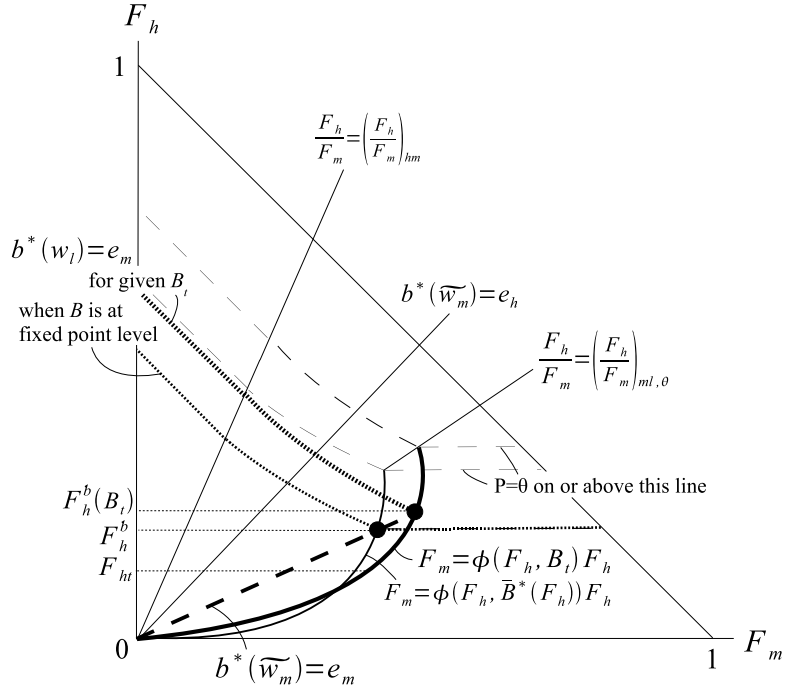


Figure 1: Proof of Lemma C2 (i)

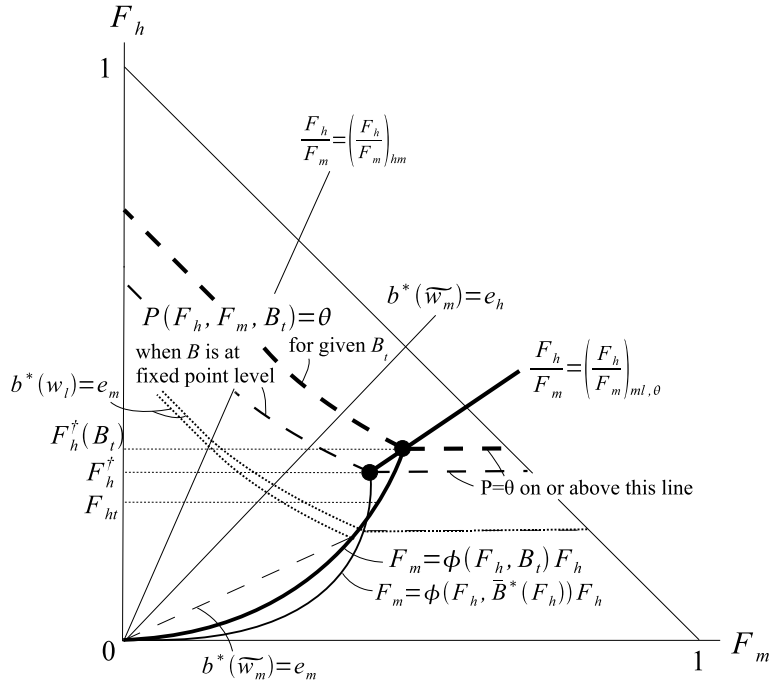


Figure 2: Proof of Lemma C2 (ii)

(I) Dynamics when $\frac{F_{ht}}{F_{mt}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$

From Proposition 1, when $\frac{F_{ht}}{F_{mt}} < (\frac{F_h}{F_m})_{hm}$, $L_{ht} = F_{ht}$, $L_{mt} = \min\{\phi(F_{ht}, B_t)F_{ht}, F_{mt}\}$ for $F_{ht} < F_h^\dagger(B_t)$, and $L_{mt} = \min\{[(\frac{F_h}{F_m})_{ml, \theta}]^{-1} F_{ht}, F_{mt}\}$ for $F_{ht} \geq F_h^\dagger(B_t)$, where $\phi(F_{ht}, B_t) > [(\frac{F_h}{F_m})_{ml, \theta}]^{-1}$ for $F_{ht} < F_h^\dagger(B_t)$. Since $\frac{L_{ht}}{L_{mt}} = \frac{F_{ht}}{L_{mt}} \leq (\frac{F_h}{F_m})_{ml, \theta}$, $\widetilde{w}_m < \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ and thus $L_{ht} = F_{ht}$ is constant.

[1] When $F_{ht} < F_h^b$

Since $B_t \leq \overline{B}^*(F_{ht}) < \overline{B}^*(F_h^b)$ from Lemma C1, $F_{ht} < F_h^b \equiv F_h^b(\overline{B}^*(F_h^b)) < F_h^b(B_t)$ and thus $\widetilde{w}_m(\frac{1}{\phi(F_{ht}, B_t)}) < \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (see Figure 4). Since $B_t < \overline{B}^*(F_h^b) < \overline{B}^*(F_h^\dagger)$, $F_{ht} < F_h^\dagger \equiv F_h^\dagger(\overline{B}^*(F_h^\dagger)) < F_h^\dagger(B_t)$ and thus $\frac{L_{ht}}{L_{mt}} = \frac{F_{ht}}{\min\{\phi(F_{ht}, B_t)F_{ht}, F_{mt}\}}$. Hence, $\widetilde{w}_m < \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ and F_{mt} decreases over time.

(a) When $F_{mt} \leq \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$

Since $F_{mt} \leq \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht} \leq \phi(F_{ht}, B_t)F_{ht}$, $L_{mt} = F_{mt}$ and the dynamics equation of B_t is (37), whose fixed point is $B^*(F_{ht}, F_{mt})$. Since F_{mt} decreases over time, $F_{mt} < \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht} < \phi(F_{ht}, B_t)F_{ht}$ continues to hold, where the second inequality is from $\overline{B}^*(F_{ht}) \equiv B^*(F_{ht}, \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}) > B^*(F_{ht}, F_{mt})$. Eventually, the economy transits to a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$ and $F_{ht} < F_h^b < F_h^b(B_t)$ (case [1](a) i of (II), or case [1](a) i A of (III), (IV), or (V) below).

(b) When $F_{mt} > \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$

i. When $F_{mt} \leq \phi(F_{ht}, B_t)F_{ht}$ and thus $L_{mt} = F_{mt}$

Since $B_t < \overline{B}^*(F_{ht}) \equiv B^*(F_{ht}, \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}) \leq B^*(F_{ht}, F_{mt})$, B_t increases. Since B_t increases and F_{mt} decreases over time, the economy transits to either $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ (case ii), $F_{mt} \leq \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$ (case (a)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$ and $F_{ht} < F_h^b < F_h^b(B_t)$ (case [1](a) i of (II), or case [1](a) i A of (III), (IV), or (V)).

ii. When $F_{mt} \geq \phi(F_{ht}, B_t)F_{ht}$

Since $L_{mt} = \phi(F_{ht}, B_t)F_{ht}$ and $B_t \leq \overline{B}^*(F_{ht})$, B_t non-decreases. Since F_{mt} decreases and B_t non-decreases over time, the economy transits to either $F_{mt} \leq \phi(F_{ht}, B_t)F_{ht}$ (case i), $F_{mt} \leq \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$ (case (a)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$ and $F_{ht} < F_h^b < F_h^b(B_t)$ (case [1](a) i of (II), or case [1](a) i A of (III), (IV), or (V)). The economy does not transit between this case and case i forever, because, with F_{mt} decreasing, $F_{mt} \leq \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$ does hold at some point.

[2] When $F_{ht} \geq F_h^b$ and thus $F_{mt} > \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$

(a) When $F_{ht} < F_h^b(B_t)$

This case arises only when $B_t < \overline{B}^*(F_h^b)$ because $F_h^b \equiv F_h^b(\overline{B}^*(F_h^b)) < F_h^b(B_t)$. As shown in [1], F_{mt} decreases over time.

i. When $F_{mt} \leq \phi(F_{ht}, B_t)F_{ht}$

Since $B_t < \overline{B}^*(F_{ht}) = B^*(F_{ht}, \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}) < B^*(F_{ht}, F_{mt})$, B_t increases. Since F_{mt} decreases and B_t increases, the economy transits to either $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ (case ii), $F_{ht} \geq F_h^b(B_t)$ (case (b)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$ and $F_{ht} \geq F_h^b$ eventually.

ii. When $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$

Since $L_{mt} = \phi(F_{ht}, B_t)F_{ht}$ and $B_t < \bar{B}^*(F_{ht})$, B_t increases. Since F_{mt} decreases and B_t increases, the economy transits to either $F_{mt} \leq \phi(F_{ht}, B_t)F_{ht}$ (case i), $F_{ht} \geq F_h^b(B_t)$ (case (b)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$ and $F_{ht} \geq F_h^b$. Note that the economy does not transit between this case and case i forever, because, with B_t increasing and $F_{ht} \geq F_h^b \equiv F_h^b(\bar{B}^*(F_h^b)) \geq F_h^b(\bar{B}^*(F_{ht}))$, $F_{ht} \geq F_h^b(B_t)$ does hold at some point.

(b) When $F_{ht} \geq F_h^b(B_t)$

Since $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ and thus $L_{mt} = \phi(F_{ht}, B_t)F_{ht}$, when $F_{ht} > (=) F_h^b(B_t)$, $w_{lt} = \widetilde{w}_m \left(\frac{1}{\phi(F_{ht}, B_t)} \right) > (=) \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ and F_{mt} increases (is constant). Since B_t non-decreases from $B_t \leq \bar{B}^*(F_{ht})$, $F_{ht} \geq F_h^b(B_t)$ continues to hold, and the economy converges to $(F_h, F_m, B) = (F_{ht}, 1 - F_{ht}, \bar{B}^*(F_{ht}))$, unless $B_t = \bar{B}^*(F_{ht})$ and $F_{ht} = F_h^b$, in which case both F_{mt} and B_t remain constant.

Summary of the dynamics when $\frac{F_{ht}}{F_{mt}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$

- When $F_{ht} < F_h^b$, F_{ht} is constant and F_{mt} decreases over time, and eventually the economy transits to a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$ and $F_{ht} < F_h^b < F_h^b(B_t)$.
- When $F_{ht} \geq F_h^b$,
 - If $F_{ht} \geq F_h^b(B_t)$, F_{ht} is constant and F_{mt} increases, and the economy stays in this case and converges to $(F_h, F_m, B) = (F_{ht}, 1 - F_{ht}, \bar{B}^*(F_{ht}))$ (Steady state 3), unless $B_t = \bar{B}^*(F_{ht})$ and $F_{ht} = F_h^b$, in which case both F_{mt} and B_t are constant.
 - If $F_{ht} < F_h^b(B_t)$ (thus $B_t < \bar{B}^*(F_h^b)$), F_{ht} is constant and F_{mt} decreases, and the economy transits to either the case $F_{ht} \geq F_h^b(B_t)$ or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$ and $F_{ht} \geq F_h^b$ eventually.

(II) Dynamics when $\frac{F_{ht}}{F_{mt}} \leq \left[\widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right], \left(\frac{F_h}{F_m} \right)_{ml, \theta} \right]$

As shown in (I), $L_{ht} = F_{ht}$ is constant, and since $L_{mt} \leq F_{mt}$ and $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$, F_{mt} non-decreases over time.

[1] When $F_{ht} < F_h^\dagger$

Since $F_{ht} < F_h^\dagger \equiv F_h^\dagger(\bar{B}^*(F_h^\dagger)) < F_h^\dagger(\bar{B}^*(F_{ht})) \leq F_h^\dagger(B_t)$, $w_{lt} < \theta A_T$.

(a) When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$

Since $F_{mt} \leq \phi(F_{ht}, \bar{B}^*(F_{ht}))F_{ht} \leq \phi(F_{ht}, B_t)F_{ht}$ (as for the first inequality, see Figure 5), $L_{mt} = F_{mt}$.

i. When $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, i.e. $b^*(w_l) \leq e_m$

Since F_{mt} is constant, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ and thus $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ continue to hold. Hence, the economy stays in this case and converges to $(F_h, F_m, B) = (F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))$.

ii. When $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and thus $F_{ht} > F_h^b(B_t)$

This case could arise only when $F_{ht} > F_h^b$ from Lemma C2 (i). Since F_{mt} increases and B_t decreases (because $B_t > B^*(F_{ht}, F_{mt})$) over time, the economy transits to either $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case i), a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (a case in (b) or (c)), or a case satisfying $\frac{F_{ht}}{F_{mt}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m \right]$ and $F_{ht} > F_h^b$ (a case in [2] of (I)).

(b) When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b$) and $F_{mt} \leq \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$

Since $F_{mt} \leq \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht} \leq \phi(F_{ht}, B_t)F_{ht}$, $L_{mt} = F_{mt}$. $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold, since F_{mt} non-decreases.

i. When $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

B_t increases from $B_t < B^*(F_{ht}, F_{mt})$. Since F_{mt} is constant, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $F_{mt} \leq \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$ continue to hold. Thus, the economy transits to $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case ii).

ii. When $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b(B_t)$)

Since F_{mt} increases and $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold. Thus, the economy transits to either a case satisfying $F_{mt} > \phi(F_{ht}, B^*(F_{ht}, F_{mt}))F_{ht}$ and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case (c) ii or iii) or the case satisfying $\frac{F_{ht}}{F_{mt}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m \right]$, $F_{ht} > F_h^b$, and $F_{ht} > F_h^b(B_t)$ (case [2](b) of (I)).

(c) When $F_{mt} > \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$ (thus $F_{ht} > F_h^b$)

Since $B_t \leq \overline{B}^*(F_{ht}) \leq B^*(F_{ht}, F_{mt})$, B_t non-decreases over time. Thus, $F_{mt} > \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$ continues to be satisfied.

i. When $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

$F_{mt} \leq \phi(F_{ht}, B_t)F_{ht}$ holds, because $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ cannot happen from $\phi(F_{ht}, B_t)F_{ht} \geq \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$ (see Figure 4). Thus $L_{mt} = F_{mt}$ is constant, while B_t increases from $B_t < B^*(F_{ht}, F_{mt})$. Hence, the economy transits to a case satisfying $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case ii or iii).

ii. When $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b(B_t)$) and $F_{mt} \leq \phi(F_{ht}, B_t)F_{ht}$

Since F_{mt} and B_t increase (note $B_t < \overline{B}^*(F_{ht}) \leq B^*(F_{ht}, F_{mt})$), $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold and the economy transits to either $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ (case iii) or case [2](b) of (I) $\frac{F_{ht}}{F_{mt}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m \right]$.

iii. When $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ (thus $F_{ht} > F_h^b(B_t)$)

Since $L_{mt} = \phi(F_{ht}, B_t)F_{ht}$, $w_{lt} = \widetilde{w}_m \left(\frac{1}{\phi(F_{ht}, B_t)} \right) > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and F_{mt} increases. Since F_{mt} increases and B_t non-decreases, $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ continues to be true. Hence, the economy converges to $(F_h, F_m, B) = (F_{ht}, 1-F_{ht}, \overline{B}^*(F_{ht}))$ in this case or transits to case [2](b) of (I).

[2] When $F_{ht} \geq F_h^\dagger (> F_h^b)$

(a) When $F_{ht} < F_h^\dagger(B_t)$ (thus $F_{mt} = \phi(F_{ht}, B_t)F_{ht}$ is effective)

Since $B_t < \bar{B}^*(F_{ht}) = B^*(F_{ht}, [\frac{F_h}{F_m}]_{ml, \theta})^{-1} F_{ht}) \leq B^*(F_{ht}, F_{mt})$, B_t increases over time, where $B_t < \bar{B}^*(F_{ht})$ is from $F_h^\dagger(B_t) > F_h^\dagger \equiv F_h^\dagger(\bar{B}^*(F_h^\dagger)) \geq F_h^\dagger(\bar{B}^*(F_{ht}))$.

i. When $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$

Since F_{mt} is constant and B_t increases, the economy transits to a case satisfying $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (case ii or iii) or $F_{ht} \geq F_h^\dagger(B_t)$ (case (b)).

ii. When $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^b(B_t)$) and $F_{mt} \leq \phi(F_{ht}, B_t)F_{ht}$

Since F_{mt} and B_t increase, $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ continues to be satisfied, and the economy transits to either $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ (case iii), $F_{ht} \geq F_h^\dagger(B_t)$ (case (b)), or the case satisfying $\frac{F_{ht}}{F_{mt}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$, $F_{ht} > F_h^\dagger$, and $F_{ht} > F_h^b(B_t)$ (case [2](b) of (I)).

iii. When $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$

F_{mt} increases from $w_{lt} = \widetilde{w}_m \left(\frac{1}{\phi(F_{ht}, B_t)} \right) > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$. Thus, $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ continues to hold, and the economy transits to $F_{ht} \geq F_h^\dagger(B_t)$ (case (b)) or case [2](b) of (I).

(b) When $F_{ht} \geq F_h^\dagger(B_t)$

Since $w_{lt} = \theta A_T$, F_{mt} increases, and since B_t non-decreases, $F_{ht} \geq F_h^\dagger(B_t)$ continues to hold. Thus, the economy converges to $(F_h, F_m, B) = (F_{ht}, 1 - F_{ht}, \bar{B}^*(F_{ht}))$ or transits to case [2](b) of (I).

Summary of the dynamics when $\frac{F_{ht}}{F_{mt}} \leq \left[\widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right], \left(\frac{F_h}{F_m} \right)_{ml, \theta} \right]$

• When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$

– When $F_{ht} \leq F_h^b$ or $B_t \leq B^*(F_{ht}, F_{mt})$, both F_{ht} and F_{mt} are constant and the economy converges to $(F_h, F_m, B) = (F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))$ in this case (Steady state 4).

– When $F_{ht} > F_h^b$ and $B_t > B^*(F_{ht}, F_{mt})$, if $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, the result is same as the previous case, otherwise (thus $F_{ht} > F_h^b(B_t)$), F_{ht} is constant and F_{mt} increases and the economy transits to either the case $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, the case $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, or a case satisfying $\frac{F_{ht}}{F_{mt}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$ and $F_{ht} > F_h^b$.

• When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^b$), F_{ht} is constant and F_{mt} increases (could be constant temporarily) over time, and eventually the economy either converges to $(F_h, F_m, B) = (F_{ht}, 1 - F_{ht}, \bar{B}^*(F_{ht}))$ in this case (Steady state 3) or transits to the case satisfying $\frac{F_{ht}}{F_{mt}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$, $F_{ht} > F_h^b$, and $F_{ht} > F_h^b(B_t)$.

(III) Dynamics when $\frac{F_{ht}}{F_{mt}} \in \left(\left(\frac{F_h}{F_m} \right)_{ml, \theta}, \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right] \right)$

As shown in (I), $L_{ht} = F_{ht}$ is constant. Since $\frac{F_{ht}}{F_{mt}} \in \left(\left(\frac{F_h}{F_m} \right)_{ml, \theta}, \left(\frac{F_h}{F_m} \right)_{hm} \right)$, $L_{mt} = F_{mt}$ from Proposition 1 (ii)(a). $L_{mt} = F_{mt}$ non-decreases since $\widetilde{w}_m \left(\frac{F_{ht}}{F_{mt}} \right) > \widetilde{w}_m \left(\left(\frac{F_h}{F_m} \right)_{ml, \theta} \right) > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$.

[1] When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) < \theta$

(a) When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$

i. When $P(F_{ht}, F_{mt}, B_t) < \theta$

A. When $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$

Since F_{mt} is constant and $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ continues to hold. The economy stays in this case and converges to $(F_h, F_m, B) = (F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))$.

B. When $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^b(B_t)$)

This case could arise only when $F_{ht} > F_h^b$ from Lemma C2 (i). Since F_{mt} increases and B_t decreases (note $B_t > B^*(F_{ht}, F_{mt})$), the economy transits to either $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (case A), $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case ii), a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (a case in (b)), a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ (a case in [2]), or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml, \theta}$ and $F_{ht} > F_h^b$ (a case of (II), or a case in [2] of (I)).

ii. When $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (thus $F_{ht} > F_h^\dagger(B_t)$)

This case could arise only when $F_{ht} > F_h^\dagger$ from Lemma C2 (ii). Since $w_{lt} = \theta A_T$, F_{mt} increases, and since $B_t > B^*(F_{ht}, F_{mt})$, B_t decreases. The economy transits to either a case satisfying $P(F_{ht}, F_{mt}, B_t) < \theta$ (case i A or B), a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (a case in (b)), a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ (a case in [2]), or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml, \theta}$ and $F_{ht} > F_h^\dagger$ (a case in [2] of (II) or (I)). Note that it does not transit between this case and case i B forever, because, with $B^*(F_{ht}, F_{mt})$ increasing, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ eventually.

(b) When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^b$)

$P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ continues to hold, since F_{ht} is constant and F_{mt} non-decreases.

i. When $P(F_{ht}, F_{mt}, B_t) < \theta$

A. When $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$

B_t increases from $B_t < B^*(F_{ht}, F_{mt})$. With constant F_{mt} , $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) < \theta$ and thus $P(F_{ht}, F_{mt}, B_t) < \theta$ continue to hold, and the economy transits to the case satisfying $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ and $B_t \leq B^*(F_{ht}, F_{mt})$ (case B).

B. When $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^b(B_t)$)

$P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ continues to hold, since $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ and F_{mt} rises. Thus, the economy transits to $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case (ii)), a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (case [2](a) ii or (b)), a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml, \theta}$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^b$), and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^b(B_t)$) [case [1](b) ii, case [1](c) ii or iii, or a case in [2], except (a) i and (b) ii, of (II);

case [2](b) of (I)]. A transition to $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case (ii)) occurs only when $B_t > B^*(F_{ht}, F_{mt})$.

ii. When $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (thus $F_{ht} > F_h^\dagger(B_t)$)

This case arises only when $F_{ht} > F_h^\dagger$ from Lemma C2 (ii). F_{mt} increases from $w_{lt} = \theta A_T$ and B_t decreases from $B_t > B^*(F_{ht}, F_{mt})$. Since $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ continues to hold, so as $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$. Hence, the economy transits to a case satisfying $P(F_{ht}, F_{mt}, B_t) < \theta$ and $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (case iB), a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ and $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (case [2](a) ii or (b)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml, \theta}$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, and $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (see iB for details). Note that it does not transit between this case and case iB forever, because, with B_t decreasing and $B^*(F_{ht}, F_{mt})$ increasing, $B_t \leq B^*(F_{ht}, F_{mt})$ holds eventually.

[2] When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ and thus $F_{ht} > F_h^\dagger (> F_h^\flat)$

$P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ continues to hold, since F_{ht} is constant and F_{mt} non-decreases.

(a) When $P(F_{ht}, F_{mt}, B_t) < \theta$.

Since $B_t < B^*(F_{ht}, F_{mt})$, B_t increases over time.

i. When $P(F_{ht}, F_{mt}, B_t) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$

Since F_{mt} is constant and B_t increases, it transits to either $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (case ii) or $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case (b)).

ii. When $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^\flat(B_t)$)

$P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ continues to hold, since F_{mt} increases and $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$. Thus, the economy transits to $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case (b)), a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml, \theta}$, $F_{ht} > F_h^\dagger (> F_h^\flat)$, and $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^\flat(B_t)$) [case [2](a) ii or iii, or [2](b) of (II); case [2](b) of (I)].

(b) When $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (thus $F_{ht} > F_h^\dagger(B_t)$)

$P(F_{ht}, F_{mt}, B_t) \geq \theta$ continues to hold, since F_{mt} increases and $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$. Thus, the economy stays in this case and converges to $(F_{ht}, 1 - F_{ht}, B^*(F_{ht}, 1 - F_{ht}))$, or transits to a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml, \theta}$, $F_{ht} > F_h^\dagger (> F_h^\flat)$, and $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case [2](b) of (II) or case [2](b) of (I)).

Summary of the dynamics when $\frac{F_{ht}}{F_{mt}} \in \left((\frac{F_h}{F_m})_{ml, \theta}, \widetilde{\omega}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right] \right)$

• When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$,

– When $F_{ht} \leq F_h^\flat$ or $B_t \leq B^*(F_{ht}, F_{mt})$, both F_{ht} and F_{mt} are constant and the economy converges to $(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))$ in this case (Steady state 4).

– When $F_{ht} > F_h^\flat$ and $B_t > B^*(F_{ht}, F_{mt})$, F_{ht} is constant and F_{mt} non-decreases, and the economy converges to $(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))$ (Steady state 4), or it transits to the case $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$, the case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) < \theta$ and $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml, \theta}$ and $F_{ht} > F_h^\flat$.

- When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) < \theta$ and $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b$), F_{ht} is constant and F_{mt} increases (could be constant temporarily), and the economy converges to $(F_{ht}, 1-F_{ht}, B^*(F_{ht}, 1-F_{ht}))$ of the next case (Steady state 3), or transits to a case satisfying a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml, \theta}$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b$), and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b(B_t)$)
- When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ (thus $F_{ht} > F_h^\dagger$), F_{ht} is constant and F_{mt} increases (could be constant temporarily), and eventually the economy converges to $(F_{ht}, 1-F_{ht}, B^*(F_{ht}, 1-F_{ht}))$ of this case (Steady state 3), or transits to a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml, \theta}$, $F_{ht} > F_h^\dagger$ (thus $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $F_{ht} > F_h^b$), and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b(B_t)$).

(IV) Dynamics when $\frac{F_{ht}}{F_{mt}} \in \left(\widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right], \left(\frac{F_h}{F_m} \right)_{hm} \right)$

Since $\widetilde{w}_m \left(\frac{F_{ht}}{F_{mt}} \right) > \frac{1-\gamma_b(1+r)}{\gamma_b} e_h$, $L_{ht} = F_{ht}$ (shown in (I)), and $L_{mt} = F_{mt}$ (shown in (III)), F_{ht} increases and $F_{ht} + F_{mt}$ non-decreases over time.

[1] When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) < \theta$

(a) When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

i. When $P(F_{ht}, F_{mt}, B_t) < \theta$

A. When $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since F_{ht} increases and F_{mt} decreases with $F_{ht} + F_{mt}$ constant, the economy transits to either $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case B), $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case ii), a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (a case in (b)), one satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ (a case in [2]), or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \left(\frac{F_h}{F_m} \right)_{hm}$ (a case in (V)). (Transition to a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ is possible, since the slope of $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) = \theta$ is strictly less than -1 .) Note that a transition to a case satisfying either $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ or $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) < \theta$ and $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case B, case ii, and case (b) ii below; case [1](a) iB or ii, or [1](b) ii of (V)) is possible only when $B_t > B^*(F_{ht}, F_{mt})$, since $B_{t+1} \leq B^*(F_{ht+1}, F_{mt+1})$ holds when $B_t \leq B^*(F_{ht}, F_{mt})$.

B. When $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

This case could arise only when $F_{ht} > F_h^b$ from Lemma C2 (i). F_{ht} and $F_{ht} + F_{mt}$ increase, and B_t decreases (note $B_t > B^*(F_{ht}, F_{mt})$) over time. Eventually, the economy transits to either $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case A), $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case ii), a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (a case in (b)), one satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ (a case in [2]), one satisfying $\frac{F_{ht}}{F_{mt}} \geq \left(\frac{F_h}{F_m} \right)_{hm}$ and $F_{ht} > F_h^b$ (a case in (V)), or one satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right]$ and $F_{ht} > F_h^b$ (a case in (II) or (III), or [2] of (I)).

ii. When $P(F_{ht}, F_{mt}, B_t) \geq \theta$

This case could arise only when $F_{ht} > F_h^b$ from Lemma C2 (i). Since $w_{lt} = \theta A_T$, F_{ht} and $F_{ht} + F_{mt}$ increase, and since $B_t > B^*(F_{ht}, F_{mt})$, B_t decreases over time. Eventually, the economy transits to either $P(F_{ht}, F_{mt}, B_t) < \theta$ (a case in i), a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (a case in (b)), one satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ (a case in [2]), one satisfying $\frac{F_{ht}}{F_{mt}} \geq \left(\frac{F_h}{F_m}\right)_{hm}$ and $F_{ht} > F_h^b$ (a case in (V)), or one satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right]$ and $F_{ht} > F_h^b$ (a case in (II) or (III), or [2] of (I)). It does not transit between the cases satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ forever, because, with F_{ht} increasing and $F_{ht} + F_{mt}$ non-decreasing, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ does hold eventually.

(b) When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^b$)

$P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ continues to hold, since F_{ht} increases and $F_{ht} + F_{mt}$ non-decreases over time.

i. When $P(F_{ht}, F_{mt}, B_t) < \theta$

A. When $P(F_{ht}, F_{mt}, B_t) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$

F_{ht} increases and F_{mt} decreases with $F_{ht} + F_{mt}$ constant and B_t increases (note $B_t < B^*(F_{ht}, F_{mt})$). The economy transits to either $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (case B), a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ (a case in [2]), or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \left(\frac{F_h}{F_m}\right)_{hm}$ and $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (a case in [1](b) or [2] of (V)). It does not transit to $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case (ii)), because $P(F_{ht}, F_{mt}, B_t) < P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))$ always holds. A shift to a case satisfying $\widehat{P}(F_{ht} + F_{mt}, B_t) \geq \theta > \widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt}))$ (case [1](b) ii of (V)) occurs only when $B_t > B^*(F_{ht}, F_{mt})$.

B. When $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$

$P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ continue to be satisfied, because F_{ht} and $F_{ht} + F_{mt}$ increase over time and $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$. The economy transits to $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case (ii)), a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ and $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (case [2](a) ii or (b)), a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \left(\frac{F_h}{F_m}\right)_{hm}$, $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, and $\widehat{P}(F_{ht} + F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (case [1](b) iB or ii, or case [2](a) ii or (b) of (V)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right]$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^b$), and $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^b(B_t)$) [a case in [1](b), except iA, or a case in [2](a) ii or (b), of (III); case [1](b) ii, (c) ii, (c) iii, or a case in [2], except (a) i, of (II); or case [2](b) of (I)]. A shift to a case satisfying $P(F_{ht}, F_{mt}, B_t) \geq \theta > P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))$ (case (ii) just below; case [1](b) ii of (V); case [1](b) ii of (III)) occurs only when $B_t > B^*(F_{ht}, F_{mt})$.

ii. When $P(F_{ht}, F_{mt}, B_t) \geq \theta$

This case could arise only when $F_{ht} > F_h^\dagger$ from Lemma C2 (ii). Since $w_{lt} = \theta A_T$, F_{ht} and $F_{ht} + F_{mt}$ increase, and since $B_t > B^*(F_{ht}, F_{mt})$, B_t decreases. Thus, $P(F_{ht}, F_{mt}, B_t) A_T >$

$\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to be satisfied from $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$. The economy shifts to the case satisfying $P(F_{ht}, F_{mt}, B_t) < \theta$ and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case i B), one satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [2](a) ii or (b)), one satisfying $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$, $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, and $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [1](b) i B or ii, or case [2](a) ii or (b) of (V)), or one satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right]$, $F_{ht} > F_h^\dagger$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ [a case in [1](b), except i A, or in [2], except (a) i, of (III); a case in [2], except (a) i, of (II); or case [2](b) of (I)]. The economy does not transit between this case and case i B forever, because, with both F_{ht} and $F_{ht} + F_{mt}$ increasing, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ does hold eventually.

[2] When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ (thus $F_{ht} > F_h^\dagger$)

$P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ continues to hold, since F_{ht} increases and $F_{ht} + F_{mt}$ non-decreases.

(a) When $P(F_{ht}, F_{mt}, B_t) < \theta$

Since $B_t < B^*(F_{ht}, F_{mt})$, B_t increases over time.

i. When $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since F_{ht} increases and F_{mt} decreases with $F_{ht} + F_{mt}$ constant and B_t increases, the economy transits to either $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case (ii)), $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case (b)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$ and $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) \geq \theta$ (a case in [2] of (V)).

ii. When $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since F_{ht} and $F_{ht} + F_{mt}$ increase, $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold. The economy transits to $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case (b)), a case satisfying $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$, $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) \geq \theta$, and $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [2](a) ii or (b) of (V)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right]$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) > \theta$ (thus $F_{ht} > F_h^\dagger$), and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b(B_t)$) [a case in [2], except (a) i, of (III); a case in [2], except (a) i, of (II); or case [2](b) of (I)].

(b) When $P(F_{ht}, F_{mt}, B_t) \geq \theta$

Since $w_{lt} = \theta A_T$, F_{ht} and $F_{ht} + F_{mt}$ increase, and since $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$, $P(F_{ht}, F_{mt}, B_t) \geq \theta$ continues to hold. Hence, the economy transits to either a case satisfying $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$, $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) \geq \theta$, and $\widehat{P}(F_{ht} + F_{mt}, B_t) > \theta$ (case [2](b) of (V)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right]$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) > \theta$ (thus $F_{ht} > F_h^\dagger$), and $P(F_{ht}, F_{mt}, B_t) > \theta$ (thus $F_{ht} > F_h^\dagger(B_t)$) [case [2](b) of (III), (II), or (I)].

Summary of the dynamics when $\frac{F_{ht}}{F_{mt}} \in \left(\widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right], \left(\frac{F_h}{F_m} \right)_{hm} \right)$

- F_{ht} increases and $F_{ht} + F_{mt}$ non-decreases. When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ and $B_t \leq B^*(F_{ht}, F_{mt})$, $F_{ht} + F_{mt}$ is constant.
- When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$
 - Eventually, the economy shifts to the case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) < \theta$ and $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, the case $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$, a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \left(\frac{F_h}{F_m} \right)_{hm}$, or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right]$ and $F_{ht} > F_h^b$.
 - A shift to a case satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right]$ and $F_{ht} > F_h^b$, one satisfying $P(F_{ht}, F_{mt}, B_t) \geq \theta > P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))$, and one $\widehat{P}(F_{ht} + F_{mt}, B_t) \geq \theta > \widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt}))$ occurs only when $B_t > B^*(F_{ht}, F_{mt})$.
- When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ and $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) < \theta$
 - Eventually, the economy shifts to the case $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$, a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \left(\frac{F_h}{F_m} \right)_{hm}$ and $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right]$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^b$), and $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^b(B_t)$).
 - A shift to a case satisfying $P(F_{ht}, F_{mt}, B_t) \geq \theta > P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))$ or $\widehat{P}(F_{ht} + F_{mt}, B_t) \geq \theta > \widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt}))$ occurs only when $B_t > B^*(F_{ht}, F_{mt})$.
- When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$
 - The economy shifts to a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \left(\frac{F_h}{F_m} \right)_{hm}$ and $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) \geq \theta$, or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right]$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) > \theta$ (thus $F_{ht} > F_h^b$), and $P(F_{ht}, F_{mt}, B_t) A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^b(B_t)$) eventually.

(V) Dynamics when $\frac{F_{ht}}{F_{mt}} \geq \left(\frac{F_h}{F_m} \right)_{hm}$

Since $\widetilde{w}_{ht} = \widetilde{w}_{mt} > \frac{1-\gamma_b(1+r)}{\gamma_b} e_h$ and $L_{ht} + L_{mt} = F_{ht} + F_{mt}$ with $\frac{L_{ht}}{L_{mt}} = \left(\frac{F_h}{F_m} \right)_{hm}$, F_{ht} increases and $F_{ht} + F_{mt}$ non-decreases over time.

[1] When $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) < \theta$

(a) When $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$

i. When $\widehat{P}(F_{ht} + F_{mt}, B_t) < \theta$

A. When $\widehat{P}(F_{ht} + F_{mt}, B_t) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$

Since $F_{ht} + F_{mt}$ is constant (with F_{ht} increasing and F_{mt} decreasing) and $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, $\widehat{P}(F_{ht} + F_{mt}, B_t) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ continues to be satisfied, and the economy converges to $(F_h, F_m, B) = (F_{ht} + F_{mt}, 0, \widehat{B}^*(F_{ht} + F_{mt}))$.

B. When $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

This case could arise only when $F_{ht} > F_h^b$ from Lemma C2 (i). F_{ht} and $F_{ht} + F_{mt}$ increase (the direction of motion of F_{mt} is ambiguous), and B_t decreases from $B_t > \widehat{B}^*(F_{ht} + F_{mt})$. The economy transits to $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case A), $\widehat{P}(F_{ht} + F_{mt}, B_t) \geq \theta$ (case ii), a case satisfying $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (a case in (b)), one satisfying $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) \geq \theta$ (a case in [2]), or one satisfying $\frac{F_{ht}}{F_{mt}} < (\frac{F_h}{F_m})_{hm}$ and $F_{ht} > F_h^b$ (a case of (II)-(VI) or a case in [2] of (I)).

ii. When $\widehat{P}(F_{ht} + F_{mt}, B_t) \geq \theta$

This case could arise only when $F_{ht} > F_h^b$ from Lemma C2 (ii). F_{ht} and $F_{ht} + F_{mt}$ increase from $w_{lt} = \theta A_T$, and B_t decreases from $B_t > \widehat{B}^*(F_{ht} + F_{mt})$. The economy transits to a case satisfying $\widehat{P}(F_{ht} + F_{mt}, B_t) < \theta$ (a case in i), one satisfying $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (a case in (b)), one satisfying $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) \geq \theta$ (a case in [2]), or a case satisfying $\frac{F_{ht}}{F_{mt}} < (\frac{F_h}{F_m})_{hm}$ and $F_{ht} > F_h^b$ (a case of (II)-(VI) or a case in [2] of (I)). It does not transit between this case and case i B forever, because, with $F_{ht} + F_{mt}$ growing, $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ holds eventually.

(b) When $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b$)

$\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold, since $F_{ht} + F_{mt}$ non-decreases.

i. When $\widehat{P}(F_{ht} + F_{mt}, B_t) < \theta$

A. When $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

$\widehat{P}(F_{ht} + F_{mt}, B_t) < \theta$ continue to hold, since $F_{ht} + F_{mt}$ is constant and $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) < \theta$. Hence, since $B_t < \widehat{B}^*(F_{ht} + F_{mt})$ and thus B_t increases, the economy transits to $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case B).

B. When $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since $F_{ht} + F_{mt}$ increases and $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold. The economy transits to either $\widehat{P}(F_{ht} + F_{mt}, B_t) \geq \theta$ (case ii), a case satisfying $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) \geq \theta$ and $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [2](a) ii or (b)), or a case satisfying $\frac{F_{ht}}{F_{mt}} < (\frac{F_h}{F_m})_{hm}$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b$), and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b(B_t)$) [case [1](b) i B or ii, or [2](a) ii or (b) of (VI); case [1](b) i B or ii, or a case in [2], except (a) i, of (III); case [1](b) ii, (c) ii, (c) iii, or a case in [2], except (a) i, of (II); or case [2](b) of (I)]. A transition to a case satisfying $P(F_{ht}, F_{mt}, B_t) \geq \theta > P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))$ (case ii below; case [1](b) ii of (III) or (V)) does not occur unless $B_t > \widehat{B}^*(F_{ht} + F_{mt})$.

ii. When $\widehat{P}(F_{ht} + F_{mt}, B_t) \geq \theta$

Since $B_t > \widehat{B}^*(F_{ht} + F_{mt})$, B_t decreases over time. $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold, since $F_{ht} + F_{mt}$ increases and $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$. Thus, the economy transits to either case i B, a case satisfying $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} +$

$F_{mt}) \geq \theta$ and $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [2](a) ii or (b)), or a case satisfying $\frac{F_{ht}}{F_{mt}} < (\frac{F_h}{F_m})_{hm}$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b$), and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b(B_t)$) (see case iB for details). Clearly, the economy does not transit between this case and case iB forever.

[2] When $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) \geq \theta$ (thus $F_{ht} > F_h^\dagger$)

$\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) \geq \theta$ continues to be satisfied, since $F_{ht} + F_{mt}$ non-decreases.

(a) When $\widehat{P}(F_{ht} + F_{mt}, B_t) < \theta$

Since $B_t < \widehat{B}^*(F_{ht} + F_{mt})$, B_t increases over time.

i. When $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since $F_{ht} + F_{mt}$ is constant and B_t increases, the economy transits to either $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case ii) or $\widehat{P}(F_{ht} + F_{mt}, B_t) \geq \theta$ (case (b)).

ii. When $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since $F_{ht} + F_{mt}$ and B_t increase, $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold and the economy transits to $\widehat{P}(F_{ht} + F_{mt}, B_t) \geq \theta$ (case (b)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{hm}$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) > \theta$ (thus $F_{ht} > F_h^\dagger$), and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b(B_t)$) [case [2](a) ii or (b) of (VI) or (III); a case in [2], except (a) i, of (II); or case [2](b) of (I)].

(b) When $\widehat{P}(F_{ht} + F_{mt}, B_t) \geq \theta$

Since $F_{ht} + F_{mt}$ increases and $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) \geq \theta$, $\widehat{P}(F_{ht} + F_{mt}, B_t) \geq \theta$ continues to hold and the economy converges to $(F_h, F_m, B) = (1, 0, \widehat{B}^*(1))$, or transits to a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{hm}$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) > \theta$ (thus $F_{ht} > F_h^\dagger$), and $P(F_{ht} + F_{mt}, B_t) > \theta$ (thus $F_{ht} > F_h^\dagger(B_t)$) [case [2](b) of (VI), (III), (II), or (I)].

Summary of the dynamics when $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$

• When $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

– When $F_{ht} \leq F_h^b$ or $B_t \leq \widehat{B}^*(F_{ht} + F_{mt})$, F_{ht} increases and $F_{ht} + F_{mt}$ is constant, and the economy converges to $(F_h, F_m, B) = (F_{ht} + F_{mt}, 0, \widehat{B}^*(F_{ht} + F_{mt}))$ in this case (Steady state 2).

– When $F_{ht} > F_h^b$ and $B_t > \widehat{B}^*(F_{ht} + F_{mt})$, F_{ht} increases and $F_{ht} + F_{mt}$ non-decreases, and the economy converges to $(F_{ht} + F_{mt}, 0, \widehat{B}^*(F_{ht} + F_{mt}))$, or it transits to the two cases below, or a case satisfying $\frac{F_{ht}}{F_{mt}} < (\frac{F_h}{F_m})_{hm}$ and $F_{ht} > F_h^b$.

• When $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) < \theta$, F_{ht} and $F_{ht} + F_{mt}$ increase over time ($F_{ht} + F_{mt}$ could be constant temporarily), and the economy transits to the case $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt})) \geq \theta$, or a case satisfying $\frac{F_{ht}}{F_{mt}} < (\frac{F_h}{F_m})_{hm}$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b$), and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^b(B_t)$). A transition to a case satisfying $P(F_{ht}, F_{mt}, B_t) \geq \theta > P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))$ does not occur unless $B_t > \widehat{B}^*(F_{ht} + F_{mt})$.

- When $\widehat{P}(F_{ht}+F_{mt}, \widehat{B}^*(F_{ht}+F_{mt})) \geq \theta$, F_{ht} and $F_{ht}+F_{mt}$ increase over time ($F_{ht}+F_{mt}$ could be constant temporarily), and the economy converges to $(F_h, F_m, B) = (1, 0, \widehat{B}^*(1))$ of this case (Steady state 1), or transits to a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{hm}$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) > \theta$ (thus $F_{ht} > F_h^\dagger$), and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat(B_t)$).

0.1 Proof of Proposition A4

From the summaries of the dynamics of (I)–(V), F_{ht} non-decreases and, except when $\frac{F_{ht}}{F_{mt}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$ and either $F_{ht} < F_h^\flat$ or $F_h^\flat(B_t) > F_{ht} \geq F_h^\flat$ hold, $F_{ht}+F_{mt}$ too non-decreases over time. Thus, except the two cases, when $F_{mt} \leq \left[\left(\frac{F_h}{F_m} \right)_{hm} \right]^{-1} F_{ht}$, if $B_t \leq \widehat{B}^*(F_{ht}+F_{mt})$, ($B_t \leq$) $B_{t+1} \leq \widehat{B}^*(F_{ht}+F_{mt}) \leq \widehat{B}^*(F_{ht+1}+F_{mt+1})$; when $F_{mt} \in \left(\left[\left(\frac{F_h}{F_m} \right)_{hm} \right]^{-1} F_{ht}, \max\{\phi(F_{ht}, B_t), \left[\left(\frac{F_h}{F_m} \right)_{ml, \theta} \right]^{-1} F_{ht} \} \right)$, if $B_t \leq B^*(F_{ht}, F_{mt})$, $B_{t+1} \leq B^*(F_{ht}, F_{mt}) \leq B^*(F_{ht+1}, F_{mt+1})$; when $F_{mt} \geq \max\{\phi(F_{ht}, B_t), \left[\left(\frac{F_h}{F_m} \right)_{ml, \theta} \right]^{-1} F_{ht} \}$, if $B_t \leq \overline{B}^*(F_{ht})$, $B_{t+1} \leq \overline{B}^*(F_{ht}) \leq \overline{B}^*(F_{ht+1})$. Hence, as long as Assumption 4 (B_0 is smaller than the fixed point level at $F_h = F_{h0}, F_m = F_{m0}$) is imposed and unless $\frac{F_{h0}}{F_{m0}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$ and either $F_{h0} < F_h^\flat$ or $F_h^\flat(B_0) > F_{h0} \geq F_h^\flat$ hold, B_t is smaller than the fixed point level at $(F_h, F_m) = (F_{ht}, F_{mt})$ for any t . Therefore, from the summaries of the dynamics and Figure 5 of the paper, the relationship between initial conditions and steady states is summarized as follows, which is essentially the content of the proposition. (The results for the two exceptions are explained after the following summary.)

- (i) When $\frac{F_{h0}}{F_{m0}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$
- If $F_{h0} < F_h^\flat$, F_{ht} is constant and F_{mt} decreases, and the economy is most likely to converge to F_h and F_m satisfying $F_h = F_{h0}$ and $\frac{F_h}{F_m} \in \left[\widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right], \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right] \right]$, and $B = B^*(F_{h0}, F_m)$ (Steady state 4). If F_{mt} "jumps over" the region $\frac{F_h}{F_m} \in \left[\widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right], \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right] \right]$, the economy converges to the other types of steady states, particularly Steady state 3.
 - If $F_{h0} \geq F_h^\flat$, when $F_{h0} \geq F_h^\flat(B_0)$, F_{ht} is constant and F_{mt} increases, and the economy converges to $(F_h, F_m, B) = (F_{h0}, 1 - F_{h0}, \overline{B}^*(F_{h0}))$ (Steady state 2), unless $F_{h0} = F_h^\flat$ and $B_0 = \overline{B}^*(F_{h0})$, in which case both F_{mt} and B_t are constant. When $F_{h0} < F_h^\flat(B_0)$, F_{ht} is constant and F_{mt} decreases at first, and the economy could converge to any of the four types of steady states or could cycle between this case and the region $\frac{F_h}{F_m} \in \left[\widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right], \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right] \right]$.
- (ii) When $\frac{F_{h0}}{F_{m0}} \in \left[\widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right], \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right] \right]$
- If $P(F_{h0}, F_{m0}, B^*(F_{h0}, F_{m0}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, both F_{ht} and F_{mt} are constant and the economy converges to $(F_h, F_m, B) = (F_{h0}, F_{m0}, B^*(F_{h0}, F_{m0}))$ (Steady state 4).
 - If $P(F_{h0}, F_{m0}, B^*(F_{h0}, F_{m0}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, F_{ht} is constant and F_{mt} increases (could be constant temporarily), and the economy converges to $(F_{h0}, 1 - F_{h0}, B^*(F_{h0}, 1 - F_{h0}))$ if $\frac{F_{h0}}{1-F_{h0}} \geq \left(\frac{F_h}{F_m} \right)_{ml, \theta}$, otherwise, to $(F_{h0}, 1 - F_{h0}, \overline{B}^*(F_{h0}))$ (Steady state 2).

(iii) When $\frac{F_{h0}}{F_{m0}} > \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right]$, F_{ht} increases and $F_{ht} + F_{mt}$ non-decreases at first.

a. If $\frac{F_{h0}}{F_{m0}} \geq \left(\frac{F_h}{F_m}\right)_{hm}$ and $\widehat{P}(F_{h0} + F_{m0}, \widehat{B}^*(F_{h0} + F_{m0}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, $F_{ht} + F_{mt}$ is constant and the economy converges to $(F_h, F_m, B) = (F_{h0} + F_{m0}, 0, \widehat{B}^*(F_{h0} + F_{m0}))$ (Steady state 3).

b. If $\frac{F_{h0}}{F_{m0}} < \left(\frac{F_h}{F_m}\right)_{hm}$ and $P(F_{h0}, F_{m0}, B^*(F_{h0}, F_{m0}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, the following three scenarios are possible.

1. The more likely is the same scenario as a.

2. $F_{ht} + F_{mt}$ rises from the start or after some period and the final state is $(1, 0, \widehat{B}^*(1))$ (Steady state 1).

3. After $F_{ht} + F_{mt}$ increases for a while, F_{ht} becomes constant and F_{mt} increases, and the economy converges to (F_h, F_m, B) described in (ii)(b) (Steady state 2).

The first scenario is more likely to be realized as F_{h0} and F_{m0} are lower, and the second one is more likely than the third one as $\frac{F_{h0}}{F_{m0}}$ is higher.

c. Otherwise, the same scenarios as 2. and 3. of b. are possible. The first scenario is definitely realized when $\frac{F_{h0}}{1-F_{h0}} \geq \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right]$.

When $\frac{F_{h0}}{F_{m0}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$ and either $F_{h0} < F_h^b$ or $F_h^b(B_0) > F_{h0} \geq F_h^b$ are satisfied, $F_{ht} + F_{mt}$ decreases at least temporarily and B_t could exceed the fixed point level at $(F_h, F_m) = (F_{ht}, F_{mt})$. Thus, the result is complicated.

When $\frac{F_{h0}}{F_{m0}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$ and $F_{h0} < F_h^b$ ((i)(a) of the above result), if F_{mt} "jumps" to the region $\frac{F_h}{F_m} \in \left(\widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right], \left(\frac{F_h}{F_m}\right)_{hm} \right)$, F_{ht} increases and $F_{ht} + F_{mt}$ non-decreases and the economy could transit back to the region $\frac{F_{ht}}{F_{mt}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$. However, it does not transit to the case satisfying $\frac{F_{ht}}{F_{mt}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$, $F_{ht} \geq F_h^b$, and $F_{ht} < F_h^b(B_t)$ (a case in [2](a) of (I)), where F_{mt} decreases, hence the economy does converge to a steady state (does not cycle). The proof is as follows. From Figure 1, as long as $F_{ht} < F_h^b$ and thus $B_t \leq \overline{B}^*(F_{ht}) < \overline{B}^*(F_h^b)$, $P(F_{ht}, F_{mt}, B_t)A_T < \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ must hold when $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$. Thus, when F_{mt} "jumps" to the region $\frac{F_{ht}}{F_{mt}} \in \left(\widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right], \left(\frac{F_h}{F_m}\right)_{hm} \right)$, $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (case [1](a) i A of (IV)) must hold, where F_{ht} increases and F_{mt} decreases with $F_{ht} + F_{mt}$ constant. Hence, in order for the economy to transit back to the region $\frac{F_{ht}}{F_{mt}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$, it must first transit to a case satisfying either $\frac{F_h}{F_m} \in \left(\widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right], \left(\frac{F_h}{F_m}\right)_{hm} \right)$ and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ or $\frac{F_h}{F_m} \geq \left(\frac{F_h}{F_m}\right)_{hm}$ and $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, where $F_{ht} + F_{mt}$ increases. From the summaries of the dynamics, eventually the economy could transit to the case satisfying $\frac{F_{ht}}{F_{mt}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$, $F_{ht} \geq F_h^b$, and $F_{ht} < F_h^b(B_t)$ only when the economy first transit to either a case satisfying $\frac{F_{ht}}{F_{mt}} \in \left(\widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right], \left(\frac{F_h}{F_m}\right)_{hm} \right)$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (case [I](a) i B or ii of (VI)) or one satisfying $\frac{F_h}{F_m} \geq \left(\frac{F_h}{F_m}\right)_{hm}$, $\widehat{P}(F_{ht} + F_{mt}, \widehat{B}^*(F_{ht} + F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, and $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$

(case [I](a) iB or ii of (V)), where $B_t > B^*(F_{ht}, F_{mt})$ must hold. Since $B^*(F_{ht}, F_{mt})$ increases over time in the region $\frac{F_{ht}}{F_{mt}} \in \left(\widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right], \left(\frac{F_h}{F_m} \right)_{hm} \right)$, $B_t > B^*(F_{ht}, F_{mt})$ must continue to hold and thus B_t must continue to decrease after the economy "jumps" to this region. The decrease in B_t , however, implies rightward shifts of $F_{mt} = \phi(F_{ht}, B_t)F_{ht}$, $P(F_{ht}, F_{mt}, B_t)A_T = \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, and $\widehat{P}(F_{ht} + F_{mt}, B_t)A_T = \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (see Figure 1), hence the economy cannot transit to these cases.

By contrast, when $\frac{F_{h0}}{F_{m0}} < \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right]$, $F_{h0} \geq F_h^b$, and $F_{h0} < F_h^b(B_t)$ ((i)(b) of the result), a scenario in which the economy cycles among different cases cannot be ruled out. From the summaries of the dynamics, the economy could cycle between this case and a case satisfying $\frac{F_{ht}}{F_{mt}} \in \left[\widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m \right], \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right] \right]$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, $F_{ht} > F_h^b$, and $B_t > B^*(F_{ht}, F_{mt})$. The economy cannot cycle between this case and a case in the region $\frac{F_{ht}}{F_{mt}} > \widetilde{w}_m^{-1} \left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h \right]$, since F_{ht} increases in this region.