# Appendix C: Analysis of the dynamics and the proof of Proposition A4

This Appendix analyzes the dynamics of the model and thereby proves Proposition A4, which examines the relationship between initial conditions and steady states. In this Appendix, $P(F_{ht},F_{mt},B)$ when $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$ is denoted as $\widehat{P}(F_{ht}+F_{mt},B)$, because $P(F_{ht},F_{mt},B)$ depends on $F_{ht}+F_{mt}$, not $F_{ht}$ and $F_{mt}$ individually (from eq. 15).

The following lemmas are used to examine the dynamics.

**Lemma C1** *When $B_0 \leq \overline{B}^*(F_{h0})$, $B_t \leq \overline{B}^*(F_{ht})$ for any $t > 0$.*

**Proof.** Suppose $B_t \leq \overline{B}^*(F_{ht})$ at $t \geq 0$. Since $F_{ht+1} \geq F_{ht}$, $\overline{B}^*(F_{ht+1}) \geq \overline{B}^*(F_{ht})$. Thus, when $B_{t+1}$ is determined by (37) or (39), $B_{t+1} \leq \overline{B}^*(F_{ht}) \leq \overline{B}^*(F_{ht+1})$. When $B_{t+1}$ is determined by (33) or (35), $B_{t+1} < \overline{B}^*(F_{ht+1})$ from $B^*(F_{ht},F_{mt}) < \overline{B}^*(F_{ht}) \leq \overline{B}^*(F_{ht+1})$, where the first inequality is from $F_{mt} < \max\{\phi(F_{ht},B_t),[(\frac{F_h}{F_m})_{ml,\theta}]^{-1}\}F_{ht}$, (34), (36), (38), and (40). When $B_{t+1}$ is determined by (29) or (31), $B_{t+1} < \overline{B}^*(F_{ht+1})$ from $\widehat{B}^*(F_{ht}+F_{mt}) < B^*(F_{ht},F'_{mt}) < \overline{B}^*(F_{ht}) \leq \overline{B}^*(F_{ht+1})$, where $F'_{mt} \in \left([(\frac{F_h}{F_m})_{hm}]^{-1}F_{ht}, \max\{\phi(F_{ht},B_t),[(\frac{F_h}{F_m})_{ml,\theta}]^{-1}\}F_{ht}\right)$ and the first inequality is from (30), (32), (34), and (36). ■

**Lemma C2** *Suppose $B_0 \leq \overline{B}^*(F_{h0})$. Then,*

*(i) When $\frac{F_{ht}}{F_{mt}} \in \left[(\frac{F_h}{F_m})_{ml,\theta}, (\frac{F_h}{F_m})_{hm}\right)$ and $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ is possible only if $F_{ht} > F_h^\flat$. Similarly, when $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$ and $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, $\widehat{P}(F_{ht}+F_{mt},B)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ is possible only if $F_{ht} > F_h^\flat$.*

*(ii) When $\frac{F_{ht}}{F_{mt}} \in \left[(\frac{F_h}{F_m})_{ml,\theta}, (\frac{F_h}{F_m})_{hm}\right)$ and $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \leq \theta$, $P(F_{ht},F_{mt},B_t) > \theta$ is possible only if $F_{ht} > F_h^\dagger$. Similarly, when $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$ and $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt})) \leq \theta$, $\widehat{P}(F_{ht}+F_{mt},B) > \theta$ is possible only if $F_{ht} > F_h^\dagger$.*

**Proof.** When $B_0 \leq \overline{B}^*(F_{h0})$, $B_t \leq \overline{B}^*(F_{ht})$ for any $t$ from Lemma C1.

(i) Suppose $F_{ht} \leq F_h^\flat$. Since $B_t \leq \overline{B}^*(F_{ht})$, $\phi(F_{ht},\overline{B}^*(F_{ht}))F_{ht} \leq \phi(F_{ht},B_t)F_{ht}$ and thus, for any $F_h > F_{ht}$, $F_m = \phi(F_h,B_t)F_h$ is located at the right side of $F_m = \phi(F_h,\overline{B}^*(F_h))F_h$ on the $(F_m,F_h)$ plane (see Figure 1). Hence, $F_h^\flat(B_t) > F_h^\flat$ (see the figure). At given $F_h$ and $F_m$ satisfying $\frac{F_h}{F_m} \in \left[(\frac{F_h}{F_m})_{ml,\theta}, (\frac{F_h}{F_m})_{hm}\right)$, the absolute value of the slope of the equi $P(F_h,F_m,B)$ curve on the $(F_m,F_h)$ plane is strictly greater than that of the equi $P(F_h,F_m,B^*(F_h,F_m))$ curve from (15), (25), and $\widetilde{w_h} > \widetilde{w_m}$. Hence, $P(F_h,F_m,B_t)A_T = \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ ($b^*(w_l) = e_m$ when $\frac{F_h}{F_m} \in \left[(\frac{F_h}{F_m})_{ml,\theta}, (\frac{F_h}{F_m})_{hm}\right)$) is located above $P(F_h,F_m,B^*(F_h,F_m))A_T = \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ on the plane. Therefore, when $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ is possible only if $F_{ht} > F_h^\flat$. When $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$, the slopes of both $\widehat{P}(F_{ht}+F_{mt},B)A_T = \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T = \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ equal $-1$ from (21) and (26). Hence, when $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, $\widehat{P}(F_{ht}+F_{mt},B)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ is possible only if $F_{ht} > F_h^\flat$. (ii) Can be proved in a similar way as (i) (see Figure 2). ■

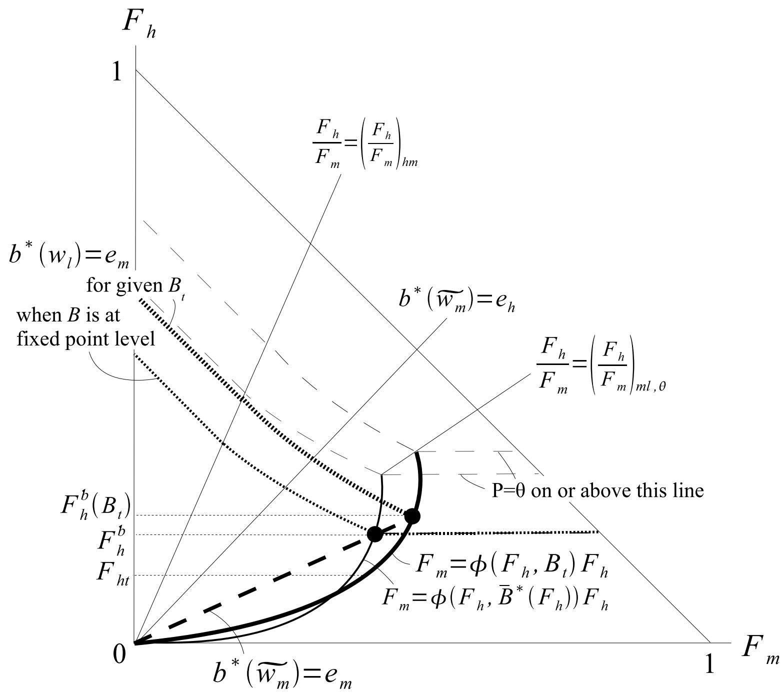


Figure 1: Proof of Lemma C2 (i)

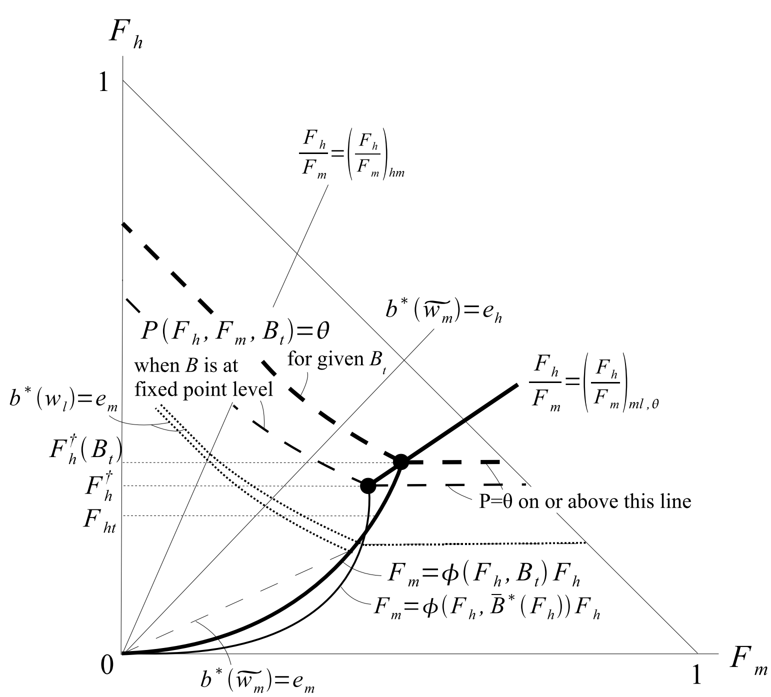


Figure 2: Proof of Lemma C2 (ii)

## (I) Dynamics when $\frac{F_{ht}}{F_{mt}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$

From Proposition 1, when $\frac{F_{ht}}{F_{mt}} < (\frac{F_h}{F_m})_{hm}$, $L_{ht} = F_{ht}$, $L_{mt} = \min\{\phi(F_{ht},B_t)F_{ht},F_{mt}\}$ for $F_{ht} < F_h^{\dagger}(B_t)$, and $L_{mt} = \min\{[(\frac{F_h}{F_m})_{ml,\theta}]^{-1}F_{ht},F_{mt}\}$ for $F_{ht} \geq F_h^{\dagger}(B_t)$, where $\phi(F_{ht},B_t) > [(\frac{F_h}{F_m})_{ml,\theta}]^{-1}$ for $F_{ht} < F_h^{\dagger}(B_t)$. Since $\frac{L_{ht}}{L_{mt}} = \frac{F_{ht}}{L_{mt}} \leq (\frac{F_h}{F_m})_{ml,\theta}$, $\widetilde{w_{mt}} < \frac{1-\gamma_b(1+r)}{\gamma_b}e_h$ and thus $L_{ht} = F_{ht}$ is constant.

[1] When $F_{ht} < F_h^{\flat}$

Since $B_t \leq \overline{B}^*(F_{ht}) < \overline{B}^*(F_h^{\flat})$ from Lemma C1, $F_{ht} < F_h^{\flat} \equiv F_h^{\flat}(\overline{B}^*(F_h^{\flat})) < F_h^{\flat}(B_t)$ and thus $\widetilde{w_m}(\frac{1}{\phi(F_{ht},B_t)}) < \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (see Figure 4). Since $B_t < \overline{B}^*(F_h^{\flat}) < \overline{B}^*(F_h^{\dagger})$, $F_{ht} < F_h^{\dagger} \equiv F_h^{\dagger}(\overline{B}^*(F_h^{\dagger})) < F_h^{\dagger}(B_t)$ and thus $\frac{L_{ht}}{L_{mt}} = \frac{F_{ht}}{\min\{\phi(F_{ht},B_t)F_{ht},F_{mt}\}}$. Hence, $\widetilde{w_{mt}} < \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $F_{mt}$ decreases over time.

(a) When $F_{mt} \leq \phi(F_{ht},\overline{B}^*(F_{ht}))F_{ht}$

Since $F_{mt} \leq \phi(F_{ht},\overline{B}^*(F_{ht}))F_{ht} \leq \phi(F_{ht},B_t)F_{ht}$, $L_{mt} = F_{mt}$ and the dynamics equation of $B_t$ is (33), whose fixed point is $B^*(F_{ht},F_{mt})$. Since $F_{mt}$ decreases over time, $F_{mt} < \phi(F_{ht},\overline{B}^*(F_{ht}))F_{ht} < \phi(F_{ht},B_t)F_{ht}$ continues to hold, where the second inequality is from $\overline{B}^*(F_{ht}) \equiv B^*(F_{ht},\phi(F_{ht},\overline{B}^*(F_{ht}))F_{ht}) > B^*(F_{ht},F_{mt})$. Eventually, the economy transits to a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$ and $F_{ht} < F_h^{\flat} < F_h^{\flat}(B_t)$ (case [1](a) i of (II), or case [1](a) i A of (III), (IV), or (V) below).

(b) When $F_{mt} > \phi(F_{ht},\overline{B}^*(F_{ht}))F_{ht}$

i. When $F_{mt} \leq \phi(F_{ht},B_t)F_{ht}$ and thus $L_{mt} = F_{mt}$

Since $B_t < \overline{B}^*(F_{ht}) \equiv B^*(F_{ht},\phi(F_{ht},\overline{B}^*(F_{ht}))F_{ht}) \leq B^*(F_{ht},F_{mt})$, $B_t$ increases. Since $B_t$ increases and $F_{mt}$ decreases over time, the economy transits to either $F_{mt} > \phi(F_{ht},B_t)F_{ht}$ (case ii), $F_{mt} \leq \phi(F_{ht},\overline{B}^*(F_{ht}))F_{ht}$ (case (a)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$ and $F_{ht} < F_h^{\flat} < F_h^{\flat}(B_t)$ (case [1](a) i of (II), or case [1](a) i A of (III), (IV), or (V)).

ii. When $F_{mt} \geq \phi(F_{ht},B_t)F_{ht}$

Since $L_{mt} = \phi(F_{ht},B_t)F_{ht}$ and $B_t \leq \overline{B}^*(F_{ht})$, $B_t$ non-decreases. Since $F_{mt}$ decreases and $B_t$ non-decreases over time, the economy transits to either $F_{mt} \leq \phi(F_{ht},B_t)F_{ht}$ (case i), $F_{mt} \leq \phi(F_{ht},\overline{B}^*(F_{ht}))F_{ht}$ (case (a)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$ and $F_{ht} < F_h^{\flat} < F_h^{\flat}(B_t)$ (case [1](a) i of (II), or case [1](a) i A of (III), (IV), or (V)). The economy does not transit between this case and case i forever, because, with $F_{mt}$ decreasing, $F_{mt} \leq \phi(F_{ht},\overline{B}^*(F_{ht}))F_{ht}$ does hold at some point.

[2] When $F_{ht} \geq F_h^{\flat}$ and thus $F_{mt} > \phi(F_{ht},\overline{B}^*(F_{ht}))F_{ht}$

(a) When $F_{ht} < F_h^{\flat}(B_t)$

This case arises only when $B_t < \overline{B}^*(F_h^{\flat})$ because $F_h^{\flat} \equiv F_h^{\flat}(\overline{B}^*(F_h^{\flat})) < F_h^{\flat}(B_t)$. As shown in [1], $F_{mt}$ decreases over time.

i. When $F_{mt} \leq \phi(F_{ht},B_t)F_{ht}$

Since $B_t < \overline{B}^*(F_{ht}) = B^*(F_{ht},\phi(F_{ht},\overline{B}^*(F_{ht}))F_{ht}) < B^*(F_{ht},F_{mt})$, $B_t$ increases. Since $F_{mt}$ decreases and $B_t$ increases, the economy transits to either $F_{mt} > \phi(F_{ht},B_t)F_{ht}$ (case ii), $F_{ht} \geq F_h^{\flat}(B_t)$ (case (b)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$ and $F_{ht} \geq F_h^{\flat}$ eventually.

ii. When $F_{mt} > \phi(F_{ht}, B_t) F_{ht}$
Since $L_{mt} = \phi(F_{ht}, B_t) F_{ht}$ and $B_t < \overline{B}^*(F_{ht})$, $B_t$ increases. Since $F_{mt}$ decreases and $B_t$ increases, the economy transits to either $F_{mt} \leq \phi(F_{ht}, B_t) F_{ht}$ (case i), $F_{ht} \geq F_h^\flat(B_t)$ (case (b)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m\right]$ and $F_{ht} \geq F_h^\flat$. Note that the economy does not transit between this case and case i forever, because, with $B_t$ increasing and $F_{ht} \geq F_h^\flat \equiv F_h^\flat(\overline{B}^*(F_h^\flat)) \geq F_h^\flat(\overline{B}^*(F_{ht}))$, $F_{ht} \geq F_h^\flat(B_t)$ does hold at some point.

(b) When $F_{ht} \geq F_h^\flat(B_t)$
Since $F_{mt} > \phi(F_{ht}, B_t) F_{ht}$ and thus $L_{mt} = \phi(F_{ht}, B_t) F_{ht}$, when $F_{ht} > (=) F_h^\flat(B_t)$, $w_{lt} = \widetilde{w_m}\left(\frac{1}{\phi(F_{ht}, B_t)}\right) > (=) \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ and $F_{mt}$ increases (is constant). Since $B_t$ non-decreases from $B_t \leq \overline{B}^*(F_{ht})$, $F_{ht} \geq F_h^\flat(B_t)$ continues to hold, and the economy converges to $(F_h, F_m, B) = (F_{ht}, 1-F_{ht}, \overline{B}^*(F_{ht}))$, unless $B_t = \overline{B}^*(F_{ht})$ and $F_{ht} = F_h^\flat$, in which case both $F_{mt}$ and $B_t$ remain constant.

**Summary of the dynamics when $\frac{F_{ht}}{F_{mt}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m\right]$**

- When $F_{ht} < F_h^\flat$, $F_{ht}$ is constant and $F_{mt}$ decreases over time, and eventually the economy transits to a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m\right]$ and $F_{ht} < F_h^\flat < F_h^\flat(B_t)$.
- When $F_{ht} \geq F_h^\flat$,
  - If $F_{ht} \geq F_h^\flat(B_t)$, $F_{ht}$ is constant and $F_{mt}$ increases, and the economy stays in this case and converges to $(F_h, F_m, B) = (F_{ht}, 1-F_{ht}, \overline{B}^*(F_{ht}))$ (SS 3), unless $B_t = \overline{B}^*(F_{ht})$ and $F_{ht} = F_h^\flat$, in which case both $F_{mt}$ and $B_t$ are constant.
  - If $F_{ht} < F_h^\flat(B_t)$ (thus $B_t < \overline{B}^*(F_h^\flat)$), $F_{ht}$ is constant and $F_{mt}$ decreases, and the economy transits to either the case $F_{ht} \geq F_h^\flat(B_t)$ or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m\right]$ and $F_{ht} \geq F_h^\flat$ eventually.

## (II) Dynamics when $\frac{F_{ht}}{F_{mt}} \leq \left[\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m\right], \left(\frac{F_h}{F_m}\right)_{ml,\theta}\right]$

As shown in (I), $L_{ht} = F_{ht}$ is constant, and since $L_{mt} \leq F_{mt}$ and $\frac{F_{ht}}{F_{mt}} \geq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m\right]$, $F_{mt}$ non-decreases over time.

[1] When $F_{ht} < F_h^\dagger$
Since $F_{ht} < F_h^\dagger \equiv F_h^\dagger(\overline{B}^*(F_h^\dagger)) < F_h^\dagger(\overline{B}^*(F_{ht})) \leq F_h^\dagger(B_t)$, $w_{lt} < \theta A_T$.

(a) When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$
Since $F_{mt} \leq \phi(F_{ht}, \overline{B}^*(F_{ht})) F_{ht} \leq \phi(F_{ht}, B_t) F_{ht}$ (as for the first inequality, see Figure 5), $L_{mt} = F_{mt}$.

i. When $P(F_{ht}, F_{mt}, B_t) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, i.e. $b^*(w_l) \leq e_m$
Since $F_{mt}$ is constant, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ and thus $P(F_{ht}, F_{mt}, B_t) A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ continue to hold. Hence, the economy stays in this case and converges to $(F_h, F_m, B) = (F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))$.

ii. When $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and thus $F_{ht} > F_h^\flat(B_t)$

This case could arise only when $F_{ht} > F_h^\flat$ from Lemma C2 (i). Since $F_{mt}$ increases and $B_t$ decreases (because $B_t > B^*(F_{ht},F_{mt})$) over time, the economy transits to either $P(F_{ht},F_{mt},B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case i), a case satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (a case in (b) or (c)), or a case satisfying $\frac{F_{ht}}{F_{mt}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$ and $F_{ht} > F_h^\flat$ (a case in [2] of (I)).

(b) When $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat$) and $F_{mt} \leq \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$

Since $F_{mt} \leq \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht} \leq \phi(F_{ht},B_t)F_{ht}$, $L_{mt} = F_{mt}$. $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold, since $F_{mt}$ non-decreases.

i. When $P(F_{ht},F_{mt},B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

$B_t$ increases from $B_t < B^*(F_{ht},F_{mt})$. Since $F_{mt}$ is constant, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $F_{mt} \leq \phi(F_{ht},\overline{B}^*(F_{ht}))F_{ht}$ continue to hold. Thus, the economy transits to $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case ii).

ii. When $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat(B_t)$)

Since $F_{mt}$ increases and $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold. Thus, the economy transits to either a case satisfying $F_{mt} > \phi(F_{ht},B^*(F_{ht},F_{mt}))F_{ht}$ and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case (c) ii or iii) or the case satisfying $\frac{F_{ht}}{F_{mt}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$, $F_{ht} > F_h^\flat$, and $F_{ht} > F_h^\flat(B_t)$ (case [2](b) of (I)).

(c) When $F_{mt} > \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$ (thus $F_{ht} > F_h^\flat$)

Since $B_t \leq \overline{B}^*(F_{ht}) \leq B^*(F_{ht},F_{mt})$, $B_t$ non-decreases over time. Thus, $F_{mt} > \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$ continues to be satisfied.

i. When $P(F_{ht},F_{mt},B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

$F_{mt} \leq \phi(F_{ht}, B_t)F_{ht}$ holds, because $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ cannot happen from $\phi(F_{ht}, B_t)F_{ht} \geq \phi(F_{ht}, \overline{B}^*(F_{ht}))F_{ht}$ (see Figure 4). Thus $L_{mt} = F_{mt}$ is constant, while $B_t$ increases from $B_t < B^*(F_{ht},F_{mt})$. Hence, the economy transits to a case satisfying $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case ii or iii).

ii. When $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat(B_t)$) and $F_{mt} \leq \phi(F_{ht}, B_t)F_{ht}$

Since $F_{mt}$ and $B_t$ increase (note $B_t < \overline{B}^*(F_{ht}) \leq B^*(F_{ht},F_{mt})$), $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold and the economy transits to either $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ (case iii) or case [2](b) of (I) $\frac{F_{ht}}{F_{mt}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$.

iii. When $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ (thus $F_{ht} > F_h^\flat(B_t)$)

Since $L_{mt} = \phi(F_{ht}, B_t)F_{ht}$, $w_{lt} = \widetilde{w_m}\left(\frac{1}{\phi(F_{ht},B_t)}\right) > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $F_{mt}$ increases. Since $F_{mt}$ increases and $B_t$ non-decreases, $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ continues to be true. Hence, the economy converges to $(F_h,F_m,B) = (F_{ht},1-F_{ht},\overline{B}^*(F_{ht}))$ in this case or transits to case [2](b) of (I).

[2] When $F_{ht} \geq F_h^\dagger (> F_h^\flat)$

(a) When $F_{ht} < F_h^\dagger(B_t)$ (thus $F_{mt} = \phi(F_{ht}, B_t)F_{ht}$ is effective)

Since $B_t < \overline{B}^*(F_{ht}) = B^*(F_{ht}, [(\frac{F_h}{F_m})_{ml,\theta}]^{-1}F_{ht}) \leq B^*(F_{ht}, F_{mt})$, $B_t$ increases over time, where $B_t < \overline{B}^*(F_{ht})$ is from $F_h^\dagger(B_t) > F_h^\dagger \equiv F_h^\dagger(\overline{B}^*(F_h^\dagger)) \geq F_h^\dagger(\overline{B}^*(F_{ht}))$.

i. When $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$

Since $F_{mt}$ is constant and $B_t$ increases, the economy transits to a case satisfying $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (case ii or iii) or $F_{ht} \geq F_h^\dagger(B_t)$ (case (b)).

ii. When $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^\flat(B_t)$) and $F_{mt} \leq \phi(F_{ht}, B_t)F_{ht}$

Since $F_{mt}$ and $B_t$ increase, $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ continues to be satisfied, and the economy transits to either $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ (case iii), $F_{ht} \geq F_h^\dagger(B_t)$ (case (b)), or the case satisfying $\frac{F_{ht}}{F_{mt}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m\right]$, $F_{ht} > F_h^\dagger$, and $F_{ht} > F_h^\flat(B_t)$ (case [2](b) of (I)).

iii. When $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$

$F_{mt}$ increases from $w_{lt} = \widetilde{w_m}\left(\frac{1}{\phi(F_{ht}, B_t)}\right) > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$. Thus, $F_{mt} > \phi(F_{ht}, B_t)F_{ht}$ continues to hold, and the economy transits to $F_{ht} \geq F_h^\dagger(B_t)$ (case (b)) or case [2](b) of (I).

(b) When $F_{ht} \geq F_h^\dagger(B_t)$

Since $w_{lt} = \theta A_T$, $F_{mt}$ increases, and since $B_t$ non-decreases, $F_{ht} \geq F_h^\dagger(B_t)$ continues to hold. Thus, the economy converges to $(F_h, F_m, B) = (F_{ht}, 1-F_{ht}, \overline{B}^*(F_{ht}))$ or transits to case [2](b) of (I).

**Summary of the dynamics when** $\frac{F_{ht}}{F_{mt}} \leq \left[\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m\right], (\frac{F_h}{F_m})_{ml,\theta}\right]$

- When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$
  - When $F_{ht} \leq F_h^\flat$ or $B_t \leq B^*(F_{ht}, F_{mt})$, both $F_{ht}$ and $F_{mt}$ are constant and the economy converges to $(F_h, F_m, B) = (F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))$ in this case (SS 4).
  - When $F_{ht} > F_h^\flat$ and $B_t > B^*(F_{ht}, F_{mt})$, if $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, the result is same as the previous case, otherwise (thus $F_{ht} > F_h^\flat(B_t)$), $F_{ht}$ is constant and $F_{mt}$ increases and the economy transits to either the case $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, the case $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$, or a case satisfying $\frac{F_{ht}}{F_{mt}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m\right]$ and $F_{ht} > F_h^\flat$.
- When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$ (thus $F_{ht} > F_h^\flat$), $F_{ht}$ is constant and $F_{mt}$ increases (could be constant temporarily) over time, and eventually the economy either converges to $(F_h, F_m, B) = (F_{ht}, 1-F_{ht}, \overline{B}^*(F_{ht}))$ in this case (SS 3) or transits to the case satisfying $\frac{F_{ht}}{F_{mt}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_m\right]$, $F_{ht} > F_h^\flat$, and $F_{ht} > F_h^\flat(B_t)$.

## (III) Dynamics when $\frac{F_{ht}}{F_{mt}} \in \left((\frac{F_h}{F_m})_{ml,\theta}, \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b} e_h\right]\right]$

As shown in (I), $L_{ht} = F_{ht}$ is constant. Since $\frac{F_{ht}}{F_{mt}} \in ((\frac{F_h}{F_m})_{ml,\theta}, (\frac{F_h}{F_m})_{hm})$, $L_{mt} = F_{mt}$ from Proposition 1 (ii)(a). $L_{mt} = F_{mt}$ non-decreases since $\widetilde{w_m}\left(\frac{F_{ht}}{F_{mt}}\right) > \widetilde{w_m}\left((\frac{F_h}{F_m})_{ml,\theta}\right) > \frac{1-\gamma_b(1+r)}{\gamma_b} e_m$.

[1] When $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))<\theta$

(a) When $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T\le\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

i. When $P(F_{ht},F_{mt},B_t)<\theta$

A. When $P(F_{ht},F_{mt},B_t)A_T\le\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since $F_{mt}$ is constant and $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T\le\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, $P(F_{ht},F_{mt},B_t)A_T\le\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold. The economy stays in this case and converges to $(F_h,F_m,B)=(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))$.

B. When $P(F_{ht},F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht}>F_h^\flat(B_t)$)

This case could arise only when $F_{ht}>F_h^\flat$ from Lemma C2 (i). Since $F_{mt}$ increases and $B_t$ decreases (note $B_t>B^*(F_{ht},F_{mt})$), the economy transits to either $P(F_{ht},F_{mt},B_t)A_T\le\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case A), $P(F_{ht},F_{mt},B_t)\ge\theta$ (case ii), a case satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (a case in (b)), a case satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))\ge\theta$ (a case in [2]), or a case satisfying $\frac{F_{ht}}{F_{mt}}\le(\frac{F_h}{F_m})_{ml,\theta}$ and $F_{ht}>F_h^\flat$ (a case of (II), or a case in [2] of (I)).

ii. When $P(F_{ht},F_{mt},B_t)\ge\theta$ (thus $F_{ht}>F_h^\dagger(B_t)$)

This case could arise only when $F_{ht}>F_h^\dagger$ from Lemma C2 (ii). Since $w_{lt}=\theta A_T$, $F_{mt}$ increases, and since $B_t>B^*(F_{ht},F_{mt})$, $B_t$ decreases. The economy transits to either a case satisfying $P(F_{ht},F_{mt},B_t)<\theta$ (case i A or B), a case satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (a case in (b)), a case satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))\ge\theta$ (a case in [2]), or a case satisfying $\frac{F_{ht}}{F_{mt}}\le(\frac{F_h}{F_m})_{ml,\theta}$ and $F_{ht}>F_h^\dagger$ (a case in [2] of (II) or (I)). Note that it does not transit between this case and case i B forever, because, with $B^*(F_{ht},F_{mt})$ increasing, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ eventually.

(b) When $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht}>F_h^\flat$)

$P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold, since $F_{ht}$ is constant and $F_{mt}$ non-decreases.

i. When $P(F_{ht},F_{mt},B_t)<\theta$

A. When $P(F_{ht},F_{mt},B_t)A_T\le\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

$B_t$ increases from $B_t<B^*(F_{ht},F_{mt})$. With constant $F_{mt}$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))<\theta$ and thus $P(F_{ht},F_{mt},B_t)<\theta$ continue to hold, and the economy transits to the case satisfying $P(F_{ht},F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $B_t\le B^*(F_{ht},F_{mt})$ (case B).

B. When $P(F_{ht},F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht}>F_h^\flat(B_t)$)

$P(F_{ht},F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold, since $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $F_{mt}$ rises. Thus, the economy transits to $P(F_{ht},F_{mt},B_t)\ge\theta$ (case (ii)), a case satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))\ge\theta$ and $P(F_{ht},F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [2](a) ii or (b)), a case satisfying $\frac{F_{ht}}{F_{mt}}\le(\frac{F_h}{F_m})_{ml,\theta}$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht}>F_h^\flat$), and $P(F_{ht},F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht}>F_h^\flat(B_t)$) [case [1](b) ii, case [1](c) ii or iii, or a case in [2], except (a) i and (b) ii, of (II);

case [2](b) of (I)]. A transition to $P(F_{ht},F_{mt},B_t) \geq \theta$ (case (ii)) occurs only when $B_t > B^*(F_{ht},F_{mt})$.

ii. When $P(F_{ht},F_{mt},B_t) \geq \theta$ (thus $F_{ht} > F_h^{\dagger}(B_t)$)

This case arises only when $F_{ht} > F_h^{\dagger}$ from Lemma C2 (ii). $F_{mt}$ increases from $w_{lt} = \theta A_T$ and $B_t$ decreases from $B_t > B^*(F_{ht},F_{mt})$. Since $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold, so as $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$. Hence, the economy transits to a case satisfying $P(F_{ht},F_{mt},B_t) < \theta$ and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case i B), a case satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \geq \theta$ and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [2](a) ii or (b)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml,\theta}$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (see i B for details). Note that it does not transit between this case and case i B forever, because, with $B_t$ decreasing and $B^*(F_{ht},F_{mt})$ increasing, $B_t \leq B^*(F_{ht},F_{mt})$ holds eventually.

[2] When $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \geq \theta$ and thus $F_{ht} > F_h^{\dagger} (> F_h^{\flat})$

$P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \geq \theta$ continues to hold, since $F_{ht}$ is constant and $F_{mt}$ non-decreases.

(a) When $P(F_{ht},F_{mt},B_t) < \theta$.

Since $B_t < B^*(F_{ht},F_{mt})$, $B_t$ increases over time.

i. When $P(F_{ht},F_{mt},B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since $F_{mt}$ is constant and $B_t$ increases, it transits to either $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case ii) or $P(F_{ht},F_{mt},B_t) \geq \theta$ (case (b)).

ii. When $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^{\flat}(B_t)$)

$P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold, since $F_{mt}$ increases and $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \geq \theta$. Thus, the economy transits to $P(F_{ht},F_{mt},B_t) \geq \theta$ (case (b)), a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml,\theta}$, $F_{ht} > F_h^{\dagger} (> F_h^{\flat})$, and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^{\flat}(B_t)$) [case [2](a) ii or iii, or [2](b) of (II); case [2](b) of (I)].

(b) When $P(F_{ht},F_{mt},B_t) \geq \theta$ (thus $F_{ht} > F_h^{\dagger}(B_t)$)

$P(F_{ht},F_{mt},B_t) \geq \theta$ continues to hold, since $F_{mt}$ increases and $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \geq \theta$. Thus, the economy stays in this case and converges to $(F_{ht}, 1-F_{ht}, B^*(F_{ht},1-F_{ht}))$, or transits to a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml,\theta}$, $F_{ht} > F_h^{\dagger} (> F_h^{\flat})$, and $P(F_{ht},F_{mt},B_t) \geq \theta$ (case [2](b) of (II) or case [2](b) of (I)).

**Summary of the dynamics when $\frac{F_{ht}}{F_{mt}} \in \left((\frac{F_h}{F_m})_{ml,\theta}, \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]\right]$**

- When $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$,
  - When $F_{ht} \leq F_h^{\flat}$ or $B_t \leq B^*(F_{ht},F_{mt})$, both $F_{ht}$ and $F_{mt}$ are constant and the economy converges to $(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))$ in this case (SS 4).
  - When $F_{ht} > F_h^{\flat}$ and $B_t > B^*(F_{ht},F_{mt})$, $F_{ht}$ is constant and $F_{mt}$ non-decreases, and the economy converges to $(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))$ (SS 4), or it transits to the case $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \geq \theta$, the case satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) < \theta$ and $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml,\theta}$ and $F_{ht} > F_h^{\flat}$.

- When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) < \theta$ and $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat$), $F_{ht}$ is constant and $F_{mt}$ increases (could be constant temporarily), and the economy converges to $(F_{ht}, 1-F_{ht}, B^*(F_{ht}, 1-F_{ht}))$ of the next case (SS 3), or transits to a case satisfying a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml,\theta}$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat$), and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat(B_t)$)
- When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ (thus $F_{ht} > F_h^\dagger$), $F_{ht}$ is constant and $F_{mt}$ increases (could be constant temporarily), and eventually the economy converges to $(F_{ht}, 1-F_{ht}, B^*(F_{ht}, 1-F_{ht}))$ of this case (SS 3), or transits to a case satisfying $\frac{F_{ht}}{F_{mt}} \leq (\frac{F_h}{F_m})_{ml,\theta}$, $F_{ht} > F_h^\dagger$(thus $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $F_{ht} > F_h^\flat$), and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat(B_t)$).

## (IV) Dynamics when $\frac{F_{ht}}{F_{mt}} \in \left(\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right], (\frac{F_h}{F_m})_{hm}\right)$

Since $\widetilde{w_m}\left(\frac{F_{ht}}{F_{mt}}\right) > \frac{1-\gamma_b(1+r)}{\gamma_b}e_h$, $L_{ht} = F_{ht}$ (shown in (I)), and $L_{mt} = F_{mt}$ (shown in (III)), $F_{ht}$ increases and $F_{ht}+F_{mt}$ non-decreases over time.

[1] When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) < \theta$

(a) When $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

i. When $P(F_{ht}, F_{mt}, B_t) < \theta$

A. When $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since $F_{ht}$ increases and $F_{mt}$ decreases with $F_{ht}+F_{mt}$ constant, the economy transits to either $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case B), $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case ii), a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (a case in (b)), one satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ (a case in [2]), or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$ (a case in (V)). (Transition to a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ is possible, since the slope of $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) = \theta$ is strictly less than $-1$.) Note that a transition to a case satisfying either $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ or $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) < \theta$ and $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case B, case ii, and case (b) ii below; case [1](a) i B or ii, or [1](b) ii of (V)) is possible only when $B_t > B^*(F_{ht}, F_{mt})$, since $B_{t+1} \leq B^*(F_{ht+1}, F_{mt+1})$ holds when $B_t \leq B^*(F_{ht}, F_{mt})$.

B. When $P(F_{ht}, F_{mt}, B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

This case could arise only when $F_{ht} > F_h^\flat$ from Lemma C2 (i). $F_{ht}$ and $F_{ht}+F_{mt}$ increase, and $B_t$ decreases (note $B_t > B^*(F_{ht}, F_{mt})$) over time. Eventually, the economy transits to either $P(F_{ht}, F_{mt}, B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case A), $P(F_{ht}, F_{mt}, B_t) \geq \theta$ (case ii), a case satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (a case in (b)), one satisfying $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt})) \geq \theta$ (a case in [2]), one satisfying $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$ and $F_{ht} > F_h^\flat$ (a case in (V)), or one satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]$ and $F_{ht} > F_h^\flat$ (a case in (II) or (III), or [2] of (I)).

ii. When $P(F_{ht},F_{mt},B_t)\geq\theta$

This case could arise only when $F_{ht}>F_h^\flat$ from Lemma C2 (i). Since $w_{lt}=\theta A_T$, $F_{ht}$ and $F_{ht}+F_{mt}$ increase, and since $B_t>B^*(F_{ht},F_{mt})$, $B_t$ decreases over time. Eventually, the economy transits to either $P(F_{ht},F_{mt},B_t)<\theta$ (a case in i), a case satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (a case in (b)), one satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))\geq\theta$ (a case in [2]), one satisfying $\frac{F_{ht}}{F_{mt}}\geq(\frac{F_h}{F_m})_{hm}$ and $F_{ht}>F_h^\flat$ (a case in (V)), or one satisfying $\frac{F_{ht}}{F_{mt}}\leq\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]$ and $F_{ht}>F_h^\flat$ (a case in (II) or (III), or [2] of (I)). It does not transit between the cases satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T\leq\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ forever, because, with $F_{ht}$ increasing and $F_{ht}+F_{mt}$ non-decreasing, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ does hold eventually.

(b) When $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht}>F_h^\flat$)

$P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold, since $F_{ht}$ increases and $F_{ht}+F_{mt}$ non-decreases over time.

i. When $P(F_{ht},F_{mt},B_t)<\theta$

A. When $P(F_{ht},F_{mt},B_t)A_T\leq\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

$F_{ht}$ increases and $F_{mt}$ decreases with $F_{ht}+F_{mt}$ constant and $B_t$ increases (note $B_t<B^*(F_{ht},F_{mt})$). The economy transits to either $P(F_{ht},F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case B), a case satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))\geq\theta$ (a case in [2]), or a case satisfying $\frac{F_{ht}}{F_{mt}}\geq(\frac{F_h}{F_m})_{hm}$ and $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (a case in [1](b) or [2] of (V)). It does not transit to $P(F_{ht},F_{mt},B_t)\geq\theta$ (case (ii)), because $P(F_{ht},F_{mt},B_t)<P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))$ always holds. A shift to a case satisfying $\widehat{P}(F_{ht}+F_{mt},B_t)\geq\theta>\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))$ (case [1](b) ii of (V)) occurs only when $B_t>B^*(F_{ht},F_{mt})$.

B. When $P(F_{ht},F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

$P(F_{ht},F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continue to be satisfied, because $F_{ht}$ and $F_{ht}+F_{mt}$ increase over time and $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$. The economy transits to $P(F_{ht},F_{mt},B_t)\geq\theta$ (case (ii)), a case satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))\geq\theta$ and $P(F_{ht},F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [2](a) ii or (b)), a case satisfying $\frac{F_{ht}}{F_{mt}}\geq(\frac{F_h}{F_m})_{hm}$, $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, and $\widehat{P}(F_{ht}+F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [1](b) i B or ii, or case [2](a) ii or (b) of (V)), or a case satisfying $\frac{F_{ht}}{F_{mt}}\leq\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht}>F_h^\flat$), and $P(F_{ht},F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht}>F_h^\flat(B_t)$) [a case in [1](b), except i A, or a case in [2](a) ii or (b), of (III); case [1](b) ii, (c) ii, (c) iii, or a case in [2], except (a) i, of (II); or case [2](b) of (I)]. A shift to a case satisfying $P(F_{ht},F_{mt},B_t)\geq\theta>P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))$ (case (ii) just below; case [1](b) ii of (V); case [1](b) ii of (III)) occurs only when $B_t>B^*(F_{ht},F_{mt})$.

ii. When $P(F_{ht},F_{mt},B_t)\geq\theta$

This case could arise only when $F_{ht}>F_h^\dagger$ from Lemma C2 (ii). Since $w_{lt}=\theta A_T$, $F_{ht}$ and $F_{ht}+F_{mt}$ increase, and since $B_t>B^*(F_{ht},F_{mt})$, $B_t$ decreases. Thus, $P(F_{ht},F_{mt},B_t)A_T>$

$\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to be satisfied from $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$. The economy shifts to the case satisfying $P(F_{ht},F_{mt},B_t) < \theta$ and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case i B), one satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \geq \theta$ and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [2](a) ii or (b)), one satisfying $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$, $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, and $\widehat{P}(F_{ht}+F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [1](b) i B or ii, or case [2](a) ii or (b) of (V)), or one satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]$, $F_{ht} > F_h^{\dagger}$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ [a case in [1](b), except i A, or in [2], except (a) i, of (III); a case in [2], except (a) i, of (II); or case [2](b) of (I)]. The economy does not transit between this case and case i B forever, because, with both $F_{ht}$ and $F_{ht}+F_{mt}$ increasing, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \geq \theta$ does hold eventually.

[2] When $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \geq \theta$ (thus $F_{ht} > F_h^{\dagger}$)

$P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \geq \theta$ continues to hold, since $F_{ht}$ increases and $F_{ht}+F_{mt}$ nondecreases.

(a) When $P(F_{ht},F_{mt},B_t) < \theta$

Since $B_t < B^*(F_{ht},F_{mt})$, $B_t$ increases over time.

i. When $P(F_{ht},F_{mt},B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since $F_{ht}$ increases and $F_{mt}$ decreases with $F_{ht}+F_{mt}$ constant and $B_t$ increases, the economy transits to either $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case (ii)), $P(F_{ht},F_{mt},B_t) \geq \theta$ (case (b)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$ and $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt})) \geq \theta$ (a case in [2] of (V)).

ii. When $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since $F_{ht}$ and $F_{ht}+F_{mt}$ increase, $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold. The economy transits to $P(F_{ht},F_{mt},B_t) \geq \theta$ (case (b)), a case satisfying $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$, $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt})) \geq \theta$, and $\widehat{P}(F_{ht}+F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [2](a) ii or (b) of (V)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) > \theta$ (thus $F_{ht} > F_h^{\dagger}$), and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^{\flat}(B_t)$) [a case in [2], except (a) i, of (III); a case in [2], except (a) i, of (II); or case [2](b) of (I)).

(b) When $P(F_{ht},F_{mt},B_t) \geq \theta$

Since $w_{lt} = \theta A_T$, $F_{ht}$ and $F_{ht}+F_{mt}$ increase, and since $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \geq \theta$, $P(F_{ht},F_{mt},B_t) \geq \theta$ continues to hold. Hence, the economy transits to either a case satisfying $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$, $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt})) \geq \theta$, and $\widehat{P}(F_{ht}+F_{mt},B_t) > \theta$ (case [2](b) of (V)), or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) > \theta$ (thus $F_{ht} > F_h^{\dagger}$), and $P(F_{ht},F_{mt},B_t) > \theta$ (thus $F_{ht} > F_h^{\dagger}(B_t)$) [case [2](b) of (III), (II), or (I)].

**Summary of the dynamics when $\frac{F_{ht}}{F_{mt}} \in \left(\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right], (\frac{F_h}{F_m})_{hm}\right)$**

- $F_{ht}$ increases and $F_{ht}+F_{mt}$ non-decreases. When $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $B_t \leq B^*(F_{ht},F_{mt})$, $F_{ht}+F_{mt}$ is constant.
- When $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$
  - Eventually, the economy shifts to the case satisfying $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) < \theta$ and $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, the case $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \geq \theta$, a case satisfying $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$, or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]$ and $F_{ht} > F_h^{\flat}$.
  - A shift to a case satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]$ and $F_{ht} > F_h^{\flat}$, one satisfying $P(F_{ht},F_{mt},B_t) \geq \theta > P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))$, and one $\widehat{P}(F_{ht}+F_{mt},B_t) \geq \theta > \widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))$ occurs only when $B_t > B^*(F_{ht},F_{mt})$.
- When $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) < \theta$
  - Eventually, the economy shifts to the case $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \geq \theta$, a case satisfying $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$ and $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^{\flat}$), and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^{\flat}(B_t)$).
  - A shift to a case satisfying $P(F_{ht},F_{mt},B_t) \geq \theta > P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))$ or $\widehat{P}(F_{ht}+F_{mt},B_t) \geq \theta > \widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))$ occurs only when $B_t > B^*(F_{ht},F_{mt})$.
- When $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) \geq \theta$
  - The economy shifts to a case satisfying $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$ and $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt})) \geq \theta$, or a case satisfying $\frac{F_{ht}}{F_{mt}} \leq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) > \theta$ (thus $F_{ht} > F_h^{\dagger}$), and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^{\flat}(B_t)$) eventually.

## (V) Dynamics when $\frac{F_{ht}}{F_{mt}} \geq (\frac{F_h}{F_m})_{hm}$

Since $\widetilde{w_{ht}} = \widetilde{w_{mt}} > \frac{1-\gamma_b(1+r)}{\gamma_b}e_h$ and $L_{ht}+L_{mt} = F_{ht}+F_{mt}$ with $\frac{L_{ht}}{L_{mt}} = (\frac{F_h}{F_m})_{hm}$, $F_{ht}$ increases and $F_{ht}+F_{mt}$ non-decreases over time.

[1] When $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt})) < \theta$

(a) When $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

i. When $\widehat{P}(F_{ht}+F_{mt},B_t) < \theta$

A. When $\widehat{P}(F_{ht}+F_{mt},B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since $F_{ht}+F_{mt}$ is constant (with $F_{ht}$ increasing and $F_{mt}$ decreasing) and $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, $\widehat{P}(F_{ht}+F_{mt},B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to be satisfied, and the economy converges to $(F_h,F_m,B) = (F_{ht}+F_{mt},0,\widehat{B}^*(F_{ht}+F_{mt}))$.

B. When $\widehat{P}(F_{ht}+F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

This case could arise only when $F_{ht} > F_h^\flat$ from Lemma C2 (i). $F_{ht}$ and $F_{ht}+F_{mt}$ increase (the direction of motion of $F_{mt}$ is ambiguous), and $B_t$ decreases from $B_t > \widehat{B}^*(F_{ht}+F_{mt})$. The economy transits to $\widehat{P}(F_{ht}+F_{mt},B_t)A_T \le \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case A), $\widehat{P}(F_{ht}+F_{mt},B_t) \ge \theta$ (case ii), a case satisfying $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (a case in (b)), one satisfying $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt})) \ge \theta$ (a case in [2]), or one satisfying $\frac{F_{ht}}{F_{mt}} < (\frac{F_h}{F_m})_{hm}$ and $F_{ht} > F_h^\flat$ (a case of (II)-(VI) or a case in [2] of (I)).

ii. When $\widehat{P}(F_{ht}+F_{mt},B_t) \ge \theta$

This case could arise only when $F_{ht} > F_h^\flat$ from Lemma C2 (ii). $F_{ht}$ and $F_{ht}+F_{mt}$ increase from $w_{lt} = \theta A_T$, and $B_t$ decreases from $B_t > \widehat{B}^*(F_{ht}+F_{mt})$. The economy transits to a case satisfying $\widehat{P}(F_{ht}+F_{mt},B_t) < \theta$ (a case in i), one satisfying $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (a case in (b)), one satisfying $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt})) \ge \theta$ (a case in [2]), or a case satisfying $\frac{F_{ht}}{F_{mt}} < (\frac{F_h}{F_m})_{hm}$ and $F_{ht} > F_h^\flat$ (a case of (II)-(VI) or a case in [2] of (I)). It does not transit between this case and case i B forever, because, with $F_{ht}+F_{mt}$ growing, $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ holds eventually.

(b) When $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat$)

$\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold, since $F_{ht}+F_{mt}$ non-decreases.

i. When $\widehat{P}(F_{ht}+F_{mt},B_t) < \theta$

A. When $\widehat{P}(F_{ht}+F_{mt},B_t)A_T \le \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

$\widehat{P}(F_{ht}+F_{mt},B_t) < \theta$ continue to hold, since $F_{ht}+F_{mt}$ is constant and $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt})) < \theta$. Hence, since $B_t < \widehat{B}^*(F_{ht}+F_{mt})$ and thus $B_t$ increases, the economy transits to $\widehat{P}(F_{ht}+F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case B).

B. When $\widehat{P}(F_{ht}+F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since $F_{ht}+F_{mt}$ increases and $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, $\widehat{P}(F_{ht}+F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold. The economy transits to either $\widehat{P}(F_{ht}+F_{mt},B_t) \ge \theta$ (case ii), a case satisfying $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt})) \ge \theta$ and $\widehat{P}(F_{ht}+F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [2](a) ii or (b)), or a case satisfying $\frac{F_{ht}}{F_{mt}} < (\frac{F_h}{F_m})_{hm}$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat$), and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat(B_t)$) [case [1](b) i B or ii, or [2](a) ii or (b) of (VI); case [1](b) i B or ii, or a case in [2], except (a) i, of (III); case [1](b) ii, (c) ii, (c) iii, or a case in [2], except (a) i, of (II); or case [2](b) of (I)]. A transition to a case satisfying $P(F_{ht},F_{mt},B_t) \ge \theta > P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))$ (case ii below; case [1](b) ii of (III) or (V)) does not occur unless $B_t > \widehat{B}^*(F_{ht}+F_{mt})$.

ii. When $\widehat{P}(F_{ht}+F_{mt},B_t) \ge \theta$

Since $B_t > \widehat{B}^*(F_{ht}+F_{mt})$, $B_t$ decreases over time. $\widehat{P}(F_{ht}+F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold, since $F_{ht}+F_{mt}$ increases and $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$. Thus, the economy transits to either case i B, a case satisfying $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+$

$F_{mt})) \geq \theta$ and $\widehat{P}(F_{ht}+F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [2](a) ii or (b)), or a case satisfying $\frac{F_{ht}}{F_{mt}} < (\frac{F_h}{F_m})_{hm}$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat$), and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat(B_t)$) (see case i B for details). Clearly, the economy does not transit between this case and case i B forever.

[2] When $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))\geq\theta$ (thus $F_{ht}>F_h^\dagger$)

$\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))\geq\theta$ continues to be satisfied, since $F_{ht}+F_{mt}$ non-decreases.

(a) When $\widehat{P}(F_{ht}+F_{mt},B_t)<\theta$

Since $B_t<\widehat{B}^*(F_{ht}+F_{mt})$, $B_t$ increases over time.

i. When $\widehat{P}(F_{ht}+F_{mt},B_t)A_T\leq\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since $F_{ht}+F_{mt}$ is constant and $B_t$ increases, the economy transits to either $\widehat{P}(F_{ht}+F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case ii) or $\widehat{P}(F_{ht}+F_{mt},B_t)\geq\theta$ (case (b)).

ii. When $\widehat{P}(F_{ht}+F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$

Since $F_{ht}+F_{mt}$ and $B_t$ increase, $\widehat{P}(F_{ht}+F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ continues to hold and the economy transits to $\widehat{P}(F_{ht}+F_{mt},B_t)\geq\theta$ (case (b)), or a case satisfying $\frac{F_{ht}}{F_{mt}}\leq(\frac{F_h}{F_m})_{hm}$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt})) > \theta$ (thus $F_{ht} > F_h^\dagger$), and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht}>F_h^\flat(B_t)$) [case [2](a) ii or (b) of (VI) or (III); a case in [2], except (a) i, of (II); or case [2](b) of (I)].

(b) When $\widehat{P}(F_{ht}+F_{mt},B_t)\geq\theta$

Since $F_{ht}+F_{mt}$ increases and $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))\geq\theta$, $\widehat{P}(F_{ht}+F_{mt},B_t)\geq\theta$ continues to hold and the economy converges to $(F_h,F_m,B) = (1,0,\widehat{B}^*(1))$, or transits to a case satisfying $\frac{F_{ht}}{F_{mt}}\leq(\frac{F_h}{F_m})_{hm}$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))>\theta$ (thus $F_{ht}>F_h^\dagger$), and $P(F_{ht}+F_{mt},B_t)>\theta$ (thus $F_{ht}>F_h^\dagger(B_t)$) [case [2](b) of (VI), (III), (II), or (I)].

**Summary of the dynamics when $\frac{F_{ht}}{F_{mt}}\geq(\frac{F_h}{F_m})_{hm}$**

- When $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T\leq\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$
  - When $F_{ht} \leq F_h^\flat$ or $B_t \leq \widehat{B}^*(F_{ht}+F_{mt})$, $F_{ht}$ increases and $F_{ht}+F_{mt}$ is constant, and the economy converges to $(F_h,F_m,B) = (F_{ht}+F_{mt},0,\widehat{B}^*(F_{ht}+F_{mt}))$ in this case (SS 2).
  - When $F_{ht}>F_h^\flat$ and $B_t>\widehat{B}^*(F_{ht}+F_{mt})$, $F_{ht}$ increases and $F_{ht}+F_{mt}$ non-decreases, and the economy converges to $(F_{ht}+F_{mt},0,\widehat{B}^*(F_{ht}+F_{mt}))$, or it transits to the two cases below, or a case satisfying $\frac{F_{ht}}{F_{mt}}<(\frac{F_h}{F_m})_{hm}$ and $F_{ht}>F_h^\flat$.
- When $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ and $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt})) < \theta$, $F_{ht}$ and $F_{ht}+F_{mt}$ increase over time ($F_{ht}+F_{mt}$ could be constant temporarily), and the economy transits to the case $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt})) \geq \theta$, or a case satisfying $\frac{F_{ht}}{F_{mt}} < (\frac{F_h}{F_m})_{hm}$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat$), and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht} > F_h^\flat(B_t)$). A transition to a case satisfying $P(F_{ht},F_{mt},B_t) \geq \theta > P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))$ does not occur unless $B_t>\widehat{B}^*(F_{ht}+F_{mt})$.

- When $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))\geq\theta$, $F_{ht}$ and $F_{ht}+F_{mt}$ increase over time ($F_{ht}+F_{mt}$ could be constant temporarily), and the economy converges to $(F_h,F_m,B)=(1,0,\widehat{B}^*(1))$ of this case (SS 1), or transits to a case satisfying $\frac{F_{ht}}{F_{mt}}\leq(\frac{F_h}{F_m})_{hm}$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))>\theta$ (thus $F_{ht}>F_h^{\dagger}$), and $P(F_{ht},F_{mt},B_t)A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (thus $F_{ht}>F_h^{\flat}(B_t)$).

## 0.1 Proof of Proposition A4

From the summaries of the dynamics of (I)$-$(V), $F_{ht}$ non-decreases and, except when $\frac{F_{ht}}{F_{mt}}<\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$ and either $F_{ht}<F_h^{\flat}$ or $F_h^{\flat}(B_t)>F_{ht}\geq F_h^{\flat}$ hold, $F_{ht}+F_{mt}$ too non-decreases over time. Thus, except the two cases, when $F_{mt}\leq[(\frac{F_h}{F_m})_{hm}]^{-1}F_{ht}$, if $B_t\leq\widehat{B}^*(F_{ht}+F_{mt})$, $(B_t\leq)B_{t+1}\leq\widehat{B}^*(F_{ht}+F_{mt})\leq\widehat{B}^*(F_{ht+1}+F_{mt+1})$; when $F_{mt}\in\left([(\frac{F_h}{F_m})_{hm}]^{-1}F_{ht},\max\{\phi(F_{ht},B_t),[(\frac{F_h}{F_m})_{ml,\theta}]^{-1}\}F_{ht}\right)$, if $B_t\leq B^*(F_{ht},F_{mt})$, $B_{t+1}\leq B^*(F_{ht},F_{mt})\leq B^*(F_{ht+1},F_{mt+1})$; when $F_{mt}\geq\max\{\phi(F_{ht},B_t),[(\frac{F_h}{F_m})_{ml,\theta}]^{-1}\}F_{ht}$, if $B_t\leq\overline{B}^*(F_{ht})$, $B_{t+1}\leq\overline{B}^*(F_{ht})\leq\overline{B}^*(F_{ht+1})$. Hence, as long as Assumption 4 ($B_0$ is smaller than the fixed point level at $F_h=F_{h0},F_m=F_{m0}$) is imposed and unless $\frac{F_{h0}}{F_{m0}}<\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$ and either $F_{h0}<F_h^{\flat}$ or $F_h^{\flat}(B_0)>F_{h0}\geq F_h^{\flat}$ hold, $B_t$ is smaller than the fixed point level at $(F_h,F_m)=(F_{ht},F_{mt})$ for any $t$. Therefore, from the summaries of the dynamics and Figure 5 of the paper, the relationship between initial conditions and steady states is summarized as follows, which is essentially the content of the proposition. (The results for the two exceptions are explained after the following summary.)

$(i)$ When $\frac{F_{h0}}{F_{m0}}<\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$

*a.* If $F_{h0}<F_h^{\flat}$, $F_{ht}$ is constant and $F_{mt}$ decreases, and the economy is most likely to converge to $F_h$ and $F_m$ satisfying $F_h=F_{h0}$ and $\frac{F_h}{F_m}\in\left[\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right],\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]\right]$, and $B=B^*(F_{h0},F_m)$ (SS 4). If $F_{mt}$ "jumps over" the region $\frac{F_h}{F_m}\in\left[\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right],\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]\right]$, the economy converges to the other types of steady states, particularly SS 3.

*b.* If $F_{h0}\geq F_h^{\flat}$, when $F_{h0}\geq F_h^{\flat}(B_0)$, $F_{ht}$ is constant and $F_{mt}$ increases, and the economy converges to $(F_h,F_m,B)=(F_{h0},1-F_{h0},\overline{B}^*(F_{h0}))$ (SS 2), unless $F_{h0}=F_h^{\flat}$ and $B_0=\overline{B}^*(F_{h0})$, in which case both $F_{mt}$ and $B_t$ are constant. When $F_{h0}<F_h^{\flat}(B_0)$, $F_{ht}$ is constant and $F_{mt}$ decreases at first, and the economy could converge to any of the four types of steady states or could cycle between this case and the region $\frac{F_h}{F_m}\in\left[\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right],\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]\right]$.

$(ii)$ When $\frac{F_{h0}}{F_{m0}}\in\left[\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right],\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]\right]$

*a.* If $P(F_{h0},F_{m0},B^*(F_{h0},F_{m0}))A_T\leq\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, both $F_{ht}$ and $F_{mt}$ are constant and the economy converges to $(F_h,F_m,B)=(F_{h0},F_{m0},B^*(F_{h0},F_{m0}))$ (SS 4).

*b.* If $P(F_{h0},F_{m0},B^*(F_{h0},F_{m0}))A_T>\frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, $F_{ht}$ is constant and $F_{mt}$ increases (could be constant temporarily), and the economy converges to $(F_{h0},1-F_{h0},B^*(F_{h0},1-F_{h0}))$ if $\frac{F_{h0}}{1-F_{h0}}\geq(\frac{F_h}{F_m})_{ml,\theta}$, otherwise, to $(F_{h0},1-F_{h0},\overline{B}^*(F_{h0}))$ (Steady state 2).

$(iii)$ When $\frac{F_{h0}}{F_{m0}}>\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]$, $F_{ht}$ increases and $F_{ht}+F_{mt}$ non-decreases at first.

*a*. If $\frac{F_{h0}}{F_{m0}} \geq (\frac{F_h}{F_m})_{hm}$ and $\widehat{P}(F_{h0}+F_{m0},\widehat{B}^*(F_{h0}+F_{m0}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, $F_{ht}+F_{mt}$ is constant and the economy converges to $(F_h,F_m,B) = (F_{h0}+F_{m0},0,\widehat{B}^*(F_{h0}+F_{m0}))$ (SS 3).

*b*. If $\frac{F_{h0}}{F_{m0}} < (\frac{F_h}{F_m})_{hm}$ and $P(F_{h0},F_{m0},B^*(F_{h0},F_{m0}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, the following three scenarios are possible.

1. The more likely is the same scenario as *a*.
2. $F_{ht}+F_{mt}$ rises from the start or after some period and the final state is $(1,0,\widehat{B}^*(1))$ (SS 1).
3. After $F_{ht}+F_{mt}$ increases for a while, $F_{ht}$ becomes constant and $F_{mt}$ increases, and the economy converges to $(F_h,F_m,B)$ described in $(ii)(b)$ (SS 2).

The first scenario is more likely to be realized as $F_{h0}$ and $F_{m0}$ are lower, and the second one is more likely than the third one as $\frac{F_{h0}}{F_{m0}}$ is higher.

*c*. Otherwise, the same scenarios as 2. and 3. of *b*. are possible. The first scenario is definitely realized when $\frac{F_{h0}}{1-F_{h0}} \geq \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]$.

When $\frac{F_{h0}}{F_{m0}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$ and either $F_{h0} < F_h^\flat$ or $F_h^\flat(B_0) > F_{h0} \geq F_h^\flat$ are satisfied, $F_{ht}+F_{mt}$ decreases at least temporarily and $B_t$ could exceed the fixed point level at $(F_h,F_m)=(F_{ht},F_{mt})$. Thus, the result is complicated.

When $\frac{F_{h0}}{F_{m0}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$ and $F_{h0} < F_h^\flat$ ($(i)(a)$ of the above result), if $F_{mt}$ "jumps" to the region $\frac{F_h}{F_m} \in \left(\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right],(\frac{F_h}{F_m})_{hm}\right)$, $F_{ht}$ increases and $F_{ht}+F_{mt}$ non-decreases and the economy could transit back to the region $\frac{F_{ht}}{F_{mt}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$. However, it does not transit to the case satisfying $\frac{F_{ht}}{F_{mt}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$, $F_{ht} \geq F_h^\flat$, and $F_{ht} < F_h^\flat(B_t)$ (a case in [2](a) of (I)), where $F_{mt}$ decreases, hence the economy does converge to a steady state (does not cycle). The proof is as follows. From Figure 1, as long as $F_{ht} < F_h^\flat$ and thus $B_t \leq \overline{B}^*(F_{ht}) < \overline{B}^*(F_h^\flat)$, $P(F_{ht},F_{mt},B_t)A_T < \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ must hold when $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$. Thus, when $F_{mt}$ "jumps" to the region $\frac{F_{ht}}{F_{mt}} \in \left(\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right],(\frac{F_h}{F_m})_{hm}\right)$, $P(F_{ht},F_{mt},B_t)A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [1](a) i A of (IV)) must hold, where $F_{ht}$ increases and $F_{mt}$ decreases with $F_{ht}+F_{mt}$ constant. Hence, in order for the economy to transit back to the region $\frac{F_{ht}}{F_{mt}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$, it must first transit to a case satisfying either $\frac{F_h}{F_m} \in \left(\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right],(\frac{F_h}{F_m})_{hm}\right)$ and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ or $\frac{F_h}{F_m} \geq (\frac{F_h}{F_m})_{hm}$ and $\widehat{P}(F_{ht}+F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, where $F_{ht}+F_{mt}$ increases. From the summaries of the dynamics, eventually the economy could transit to the case satisfying $\frac{F_{ht}}{F_{mt}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$, $F_{ht} \geq F_h^\flat$, and $F_{ht} < F_h^\flat(B_t)$ only when the economy first transit to either a case satisfying $\frac{F_{ht}}{F_{mt}} \in \left(\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right],(\frac{F_h}{F_m})_{hm}\right)$, $P(F_{ht},F_{mt},B^*(F_{ht},F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, and $P(F_{ht},F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [I](a) i B or ii of (VI)) or one satisfying $\frac{F_h}{F_m} \geq (\frac{F_h}{F_m})_{hm}$, $\widehat{P}(F_{ht}+F_{mt},\widehat{B}^*(F_{ht}+F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, and $\widehat{P}(F_{ht}+F_{mt},B_t)A_T > \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (case [I](a) i B or ii of (V)), where $B_t > B^*(F_{ht},F_{mt})$ must hold. Since $B^*(F_{ht},F_{mt})$ increases over time in the region $\frac{F_{ht}}{F_{mt}} \in \left(\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right],(\frac{F_h}{F_m})_{hm}\right)$, $B_t > B^*(F_{ht},F_{mt})$ must continue to hold and thus $B_t$ must continue to decrease after the economy "jumps" to this region. The decrease in $B_t$, however, implies rightward shifts of $F_{mt} = \phi(F_{ht}, B_t)F_{ht}$, $P(F_{ht}, F_{mt}, B_t)A_T = \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, and $\widehat{P}(F_{ht}+F_{mt}, B_t)A_T = \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$ (see Figure 1), hence the economy cannot transit to these cases.

By contrast, when $\frac{F_{h0}}{F_{m0}} < \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right]$, $F_{h0} \geq F_h^\flat$, and $F_{h0} < F_h^\flat(B_t)$ ((i)(b) of the result), a scenario in which the economy cycles among different cases cannot be ruled out. From the summaries of the dynamics, the economy could cycle between this case and a case satisfying $\frac{F_{ht}}{F_{mt}} \in \left[\widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_m\right], \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]\right]$, $P(F_{ht}, F_{mt}, B^*(F_{ht}, F_{mt}))A_T \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e_m$, $F_{ht} > F_h^\flat$, and $B_t > B^*(F_{ht}, F_{mt})$. The economy cannot cycle between this case and a case in the region $\frac{F_{ht}}{F_{mt}} > \widetilde{w_m}^{-1}\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e_h\right]$, since $F_{ht}$ increases in this region.