Fertility, childbearing age, and education

Online Appendix

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Chen Wu and Kazuhiro Yuki*

Online Appendix C Proofs of Propositions A.1, A.2, and A.3

Solutions and conditions of the utility maximization problem

The objective function of the period 2 maximization problem now becomes

$$ln\{gA(1+\rho e)^{\alpha}h_1[1-g_\delta\delta-\lambda(\theta_q+\theta_e e_{c2})n_2]\}+ln[n_1(1+\rho_c e_{c1})^{\alpha_c}+n_2(1+g_{\rho_c}\rho_c e_{c2})^{\alpha_c}]. \tag{1}$$

The first order conditions are

$$\frac{\partial U}{\partial e_{c2}}=n_2\left[\frac{-\lambda\theta_e}{1-g_\delta\delta-\lambda(\theta_q+\theta_e e_{c2})n_2}+\frac{\alpha_c g_{\rho_c}\rho_c(1+g_{\rho_c}\rho_c e_{c2})^{\alpha_c-1}}{n_1(1+\rho_c e_{c1})^{\alpha_c}+n_2(1+g_{\rho_c}\rho_c e_{c2})^{\alpha_c}}\right]\le 0, \tag{2}$$

$$\frac{\partial U}{\partial n_2}=\frac{-\lambda(\theta_q+\theta_e e_{c2})}{1-g_\delta\delta-\lambda(\theta_q+\theta_e e_{c2})n_2}+\frac{(1+g_{\rho_c}\rho_c e_{c2})^{\alpha_c}}{n_1(1+\rho_c e_{c1})^{\alpha_c}+n_2(1+g_{\rho_c}\rho_c e_{c2})^{\alpha_c}}\le 0. \tag{3}$$

From $g_\delta\le 1$, $\frac{\partial U}{\partial n_2}=0$ could hold. Thus, when $n_2>0$, from $\frac{\partial U}{\partial n_2}=0$,

$$n_1(1+\rho_c e_{c1})^{\alpha_c}+n_2(1+g_{\rho_c}\rho_c e_{c2})^{\alpha_c}=\frac{(1+g_{\rho_c}\rho_c e_{c2})^{\alpha_c}}{\lambda(\theta_q+\theta_e e_{c2})}[1-g_\delta\delta-\lambda(\theta_q+\theta_e e_{c2})n_2]. \tag{4}$$

By substituting this equation into (65),

$$\frac{\partial U}{\partial e_{c2}}=\frac{\lambda n_2}{1-g_\delta\delta-\lambda(\theta_q+\theta_e e_{c2})n_2}\left[-\theta_e+\frac{\alpha_c g_{\rho_c}\rho_c(\theta_q+\theta_e e_{c2})}{1+g_{\rho_c}\rho_c e_{c2}}\right]. \tag{5}$$

When $n_2>0$, $\frac{\partial U}{\partial e_{c2}}=0$ from $g_{\rho_c}\ge 1$. Thus,

$$e_{c2}=\frac{\alpha_c g_{\rho_c}\rho_c\theta_q-\theta_e}{(1-\alpha_c)\theta_e g_{\rho_c}\rho_c}. \tag{6}$$

When $n_2>0$, from (66),

*Chen Wu
School of Economics, Yunnan University, Yunnan, China
e-mail: wuchen@ynu.edu.cn
Kazuhiro Yuki
Faculty of Economics, Kyoto University, Kyoto, Japan
e-mail: yuki@econ.kyoto-u.ac.jp

$$\frac{\partial U}{\partial n_2}|_{n_2=0} = \frac{-\lambda(\theta_q+\theta_e e_{c2})}{1-g_\delta \delta} + \frac{(1+g_{\rho_c}\rho_c e_{c2})^{\alpha_c}}{n_1(1+\rho_c e_{c1})^{\alpha_c}} > 0. \tag{7}$$

From (67) and (69), the equation corresponding to (33) of the time-invariant case is

$$n_2 = \frac{(1-g_\delta\delta)g_{\rho_c}\rho_c(1-\alpha_c)\left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)}{\theta_e(1-\alpha_c)}\right]^{\alpha_c} - \lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)(1+\rho_c e_{c1})^{\alpha_c}n_1}{2\lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)\left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)}{\theta_e(1-\alpha_c)}\right]^{\alpha_c}}. \tag{8}$$

Thus, the objective function of the period 1 maximization problem becomes

$$U = ln\{Ah_1[1-\delta-\eta e-(\theta_q+\theta_e e_{c1})n_1]\}$$

$$+ln\left\{gA(1+\rho e)^\alpha h_1 \frac{(1-g_\delta\delta)g_{\rho_c}\rho_c(1-\alpha_c)\left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)}{\theta_e(1-\alpha_c)}\right]^{\alpha_c} + \lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)(1+\rho_c e_{c1})^{\alpha_c}n_1}{2g_{\rho_c}\rho_c(1-\alpha_c)\left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)}{\theta_e(1-\alpha_c)}\right]^{\alpha_c}}\right\}$$

$$+ln\left\{\frac{(1-g_\delta\delta)g_{\rho_c}\rho_c(1-\alpha_c)\left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)}{\theta_e(1-\alpha_c)}\right]^{\alpha_c} + \lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)(1+\rho_c e_{c1})^{\alpha_c}n_1}{2\lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)}\right\} \tag{9}$$

The first order conditions are

$$\frac{\partial U}{\partial n_1} = \frac{-(\theta_q+\theta_e e_{c1})}{1-\delta-\eta e-(\theta_q+\theta_e e_{c1})n_1} + \frac{2\lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)(1+\rho_c e_{c1})^{\alpha_c}}{(1-g_\delta\delta)g_{\rho_c}\rho_c(1-\alpha_c)\left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)}{\theta_e(1-\alpha_c)}\right]^{\alpha_c} + \lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)(1+\rho_c e_{c1})^{\alpha_c}n_1} \leq 0, \tag{10}$$

$$\frac{\partial U}{\partial e_{c1}} = n_1\left\{\frac{-\theta_e}{1-\delta-\eta e-(\theta_q+\theta_e e_{c1})n_1} + \frac{2\lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)\alpha_c\rho_c(1+\rho_c e_{c1})^{\alpha_c-1}}{(1-g_\delta\delta)g_{\rho_c}\rho_c(1-\alpha_c)\left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)}{\theta_e(1-\alpha_c)}\right]^{\alpha_c} + \lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)(1+\rho_c e_{c1})^{\alpha_c}n_1}\right\} \leq 0, \tag{11}$$

$$\frac{\partial U}{\partial e} = \frac{-\eta}{1-\delta-\eta e-(\theta_q+\theta_e e_{c1})n_1} + \frac{\alpha\rho}{1+\rho e} \leq 0. \tag{12}$$

When $n_1 > 0$, from $\frac{\partial U}{\partial n_1} = 0$,

$$(1-g_\delta\delta)g_{\rho_c}\rho_c(1-\alpha_c)\left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)}{\theta_e(1-\alpha_c)}\right]^{\alpha_c} + \lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)(1+\rho_c e_{c1})^{\alpha_c}n_1$$

$$= \frac{2\lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)(1+\rho_c e_{c1})^{\alpha_c}}{\theta_q+\theta_e e_{c1}}[1-\delta-\eta e-(\theta_q+\theta_e e_{c1})n_1].$$

By substituting this equation into (74),

$$\frac{\partial U}{\partial e_{c1}} = \frac{n_1}{1-\delta-\eta e-(\theta_q+\theta_e e_{c1})n_1}\left[-\theta_e + \frac{\alpha_c\rho_c(\theta_q+\theta_e e_{c1})}{1+\rho_c e_{c1}}\right]. \tag{13}$$

Thus, when $n_1 > 0$, $e_{c1} = \frac{\alpha_c\rho_c\theta_q-\theta_e}{(1-\alpha_c)\theta_e\rho_c}$ from $\frac{\partial U}{\partial e_{c1}} = 0$.

When $n_1 = (>)0$, from (73),

$$\frac{\partial U}{\partial n_1}|_{n_1=0} = \frac{-(\theta_q+\theta_e e_{c1})}{1-\delta-\eta e} + \frac{2\lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)(1+\rho_c e_{c1})^{\alpha_c}}{(1-g_\delta\delta)g_{\rho_c}\rho_c(1-\alpha_c)\left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)}{\theta_e(1-\alpha_c)}\right]^{\alpha_c}} \leq (>)0. \tag{14}$$

When $e = (>)0$, from (75),

$$\frac{\partial U}{\partial e}\Big|_{e=0} = \frac{-\eta}{1-\delta-(\theta_q+\theta_e e_{c1})n_1} + \alpha\rho \le (>)0. \quad (15)$$

Case (3): When $n_1 > 0$ and $e = 0$

In this case, values of variables are $e = 0$, $e_{c1} = \frac{\alpha_c\rho_c\theta_q-\theta_e}{(1-\alpha_c)\theta_e\rho_c}$ and $e_{c2} = \frac{\alpha_c g_{\rho_c}\rho_c\theta_q-\theta_e}{(1-\alpha_c)\theta_e g_{\rho_c}\rho_c}$,

$$n_1 = \frac{(1-\alpha_c)\rho_c\left[(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}2\lambda-(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right]}{3\lambda(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}, \quad (16)$$

and $n_2 = \frac{(1-\alpha_c)\rho_c\left[2(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}-(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\lambda\right]}{3\lambda(\rho_c\theta_q-\theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)}$. (17)

By substituting these values of n_1, e_{c1}, and e_{c2} into $\frac{\partial U}{\partial n_2} > 0$ when $n_2 = 0$, (70),

$$\frac{-\lambda\frac{g_{\rho_c}\rho_c\theta_q-\theta_e}{g_{\rho_c}\rho_c(1-\alpha_c)}}{1-g_\delta\delta} + \frac{\left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)}{(1-\alpha_c)\theta_e}\right]^{\alpha_c}}{\left[\frac{\alpha_c(\rho_c\theta_q-\theta_e)}{(1-\alpha_c)\theta_e}\right]^{\alpha_c}}\frac{3\lambda(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{(1-\alpha_c)\left[(1-\delta)\rho_c(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}2\lambda-(1-g_\delta\delta)g_{\rho_c}\rho_c(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right]} > 0.$$

By substituting the values of e and e_{c1} into $\frac{\partial U}{\partial n_1} > 0$ when $n_1 = 0$, (77),

$$\frac{-\frac{\rho_c\theta_q-\theta_e}{\rho_c(1-\alpha_c)}}{1-\delta} + \frac{2\lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)\left[\frac{\alpha_c(\rho_c\theta_q-\theta_e)}{(1-\alpha_c)\theta_e}\right]^{\alpha_c}}{(1-g_\delta\delta)g_{\rho_c}\rho_c(1-\alpha_c)\left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)}{(1-\alpha_c)\theta_e}\right]^{\alpha_c}} > 0 \Leftrightarrow \lambda > \frac{(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{2(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}. \quad (18)$$

Because g_{ρ_c} is not large and g_δ is not small, (81) always holds.

By substituting the values of n_1 and e_{c1} into $\frac{\partial U}{\partial e} \le 0$ when $e = 0$, (78),

$$\frac{-\eta}{1-\delta-\frac{\rho_c\theta_q-\theta_e}{\rho_c(1-\alpha_c)}\frac{(1-\alpha_c)\left[(1-\delta)\rho_c(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}2\lambda-(1-g_\delta\delta)g_{\rho_c}\rho_c(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right]}{3\lambda(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}} + \alpha\rho \le 0.$$

Hence, the conditions for this case to be realized are

$$\lambda < \frac{2(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}, \quad (19)$$

$$\rho \le \frac{3\lambda\eta(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{\alpha\left[(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\lambda+(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right]}. \quad (20)$$

Case (4): When $n_1 > 0$ and $e > 0$

In this case, values of variables are $e_{c1} = \frac{\alpha_c\rho_c\theta_q-\theta_e}{(1-\alpha_c)\theta_e\rho_c}$, $e_{c2} = \frac{\alpha_c g_{\rho_c}\rho_c\theta_q-\theta_e}{(1-\alpha_c)\theta_e g_{\rho_c}\rho_c}$,

$$n_1 = \frac{(1-\alpha_c)\rho_c\left\{2[(1-\delta)\rho+\eta](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\lambda-(1+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right\}}{(3+\alpha)\lambda\rho(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}, \quad (21)$$

$$n_2 = \frac{(1-\alpha_c)\rho_c\left[(2+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}\left(\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}-[(1-\delta)\rho+\eta]\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}\lambda\right]}{(3+\alpha)\lambda\rho\left(\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)}, \tag{22}$$

and $e = \frac{\alpha\rho\left[(1-\delta)\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}\lambda+(1-g_\delta\delta)g_{\rho_c}\left(\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}\right]-3\lambda\eta\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}}{(3+\alpha)\lambda\rho\eta\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}}.$ (23)

By substituting these values of n_1, e_{c1}, and e_{c2} into $\frac{\partial U}{\partial n_2} > 0$ when $n_2 = 0$, (70),

$$\frac{-\lambda\frac{g_{\rho_c}\rho_c\theta_q-\theta_e}{g_{\rho_c}\rho_c(1-\alpha_c)}}{1-g_\delta\delta} + \frac{\left[\frac{\alpha_c\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)}{(1-\alpha_c)\theta_e}\right]^{\alpha_c}}{\left[\frac{\alpha_c\left(\rho_c\theta_q-\theta_e\right)}{(1-\alpha_c)\theta_e}\right]^{\alpha_c}}\frac{(3+\alpha)\lambda\rho\left(\rho_c\theta_q-\theta_e\right)\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}}{(1-\alpha_c)\left[2[(1-\delta)\rho+\eta]\rho_c\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}\lambda-(1+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}\rho_c\left(\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}\right]} > 0.$$

By substituting the values of e and e_{c1} into $\frac{\partial U}{\partial n_1} > 0$ when $n_1 = 0$, (77),

$$\frac{-\frac{\rho_c\theta_q-\theta_e}{\rho_c(1-\alpha_c)}}{1-\delta-\eta\frac{\alpha\rho\left[(1-\delta)\rho_c\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}\lambda+(1-g_\delta\delta)g_{\rho_c}\rho_c\left(\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}\right]-3\lambda\eta\rho_c\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}}{(3+\alpha)\lambda\rho\eta\rho_c\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}}}$$

$$+\frac{2\lambda\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)\left[\frac{\alpha_c\left(\rho_c\theta_q-\theta_e\right)}{(1-\alpha_c)\theta_e}\right]^{\alpha_c}}{(1-g_\delta\delta)(1-\alpha_c)g_{\rho_c}\rho_c\left[\frac{\alpha_c\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)}{(1-\alpha_c)\theta_e}\right]^{\alpha_c}} > 0 \Leftrightarrow \lambda > \frac{(1+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}\left(\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}}{2[(1-\delta)\rho+\eta]\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}}. \tag{24}$$

Because g_{ρ_c} is not large and g_δ is not small, (87) always holds.

By substituting the values of n_1 and e_{c1} into $\frac{\partial U}{\partial e} > 0$ when $e = 0$, (78),

$$\frac{-\eta}{1-\delta_1-\frac{\rho_c\theta_q-\theta_e}{\rho_c(1-\alpha_c)}\frac{(1-\alpha_c)\left[2[(1-\delta)\rho+\eta]\rho_c\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}\lambda-(1+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}\rho_c\left(\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}\right]}{(3+\alpha)\lambda\rho\left(\rho_c\theta_q-\theta_e\right)\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}}} + \alpha\rho > 0.$$

Hence, the conditions for this case to be realized are

$$\lambda < \frac{(2+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}\left(\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}}{[(1-\delta)\rho+\eta]\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}}, \tag{25}$$

$$\lambda < \frac{\rho(1-g_\delta\delta)g_{\rho_c}\left(\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}}{[3\eta-(1-\delta)\rho\alpha]\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}} \Leftrightarrow \rho > \frac{3\lambda\eta\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}}{\alpha\left[(1-\delta)\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}\lambda+(1-g_\delta\delta)g_{\rho_c}\left(\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}\right]}. \tag{26}$$

Proof of Proposition A.1

(1) $n_1 = \frac{(1-\alpha_c)\rho_c\left[(1-\delta)\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}2\lambda-(1-g_\delta\delta)g_{\rho_c}\left(\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}\right]}{3\lambda\left(\rho_c\theta_q-\theta_e\right)\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}}$ from (79).

$$\frac{dn_1}{d\rho_c} = \frac{(1-\alpha_c)\theta_e}{3\left(\rho_c\theta_q-\theta_e\right)^2\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{2-\alpha_c}}\left\{\frac{1}{\lambda}g_{\rho_c}(1-g_\delta\delta)\left(\rho_c\theta_q-\theta_e\right)^{1-\alpha_c}\left[\alpha_c\rho_c\theta_q\left(g_{\rho_c}-1\right)+\left(\rho_c\theta_q-\theta_e\right)\right]-2(1-\delta)\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{2-\alpha_c}\right\} < \frac{2(1-\alpha_c)^2(1-\delta)\rho_c\theta_q\theta_e}{3\left(\rho_c\theta_q-\theta_e\right)^2\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)}\left(1-g_{\rho_c}\right) \le 0$$

from (81) and $g_{\rho_c} \ge 1$.

$\frac{dn_1}{dg_{\rho_c}} = \frac{(1-\alpha_c)\rho_c(1-g_\delta\delta)}{3\lambda\left(\rho_c\theta_q-\theta_e\right)^{\alpha_c}\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)^{2-\alpha_c}}\left(\theta_e-\alpha_c g_{\rho_c}\rho_c\theta_q\right) < 0$ from $\rho_c > \frac{\theta_e}{\alpha_c\theta_q}$ and $g_{\rho_c} \ge 1$.

$\frac{dn_1}{d\delta} = \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)\lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}\left[g_{\rho_c}g_\delta(\rho_c\theta_q-\theta_e)^{1-\alpha_c} - 2\lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right] <$

$\frac{(1-\alpha_c)\rho_c g_{\rho_c}}{3(1-\delta)\lambda(\rho_c\theta_q-\theta_e)^{\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}(g_\delta - 1) \le 0$ from (81) and $g_\delta \le 1$.

$\frac{dn_1}{d\theta_q} = \frac{(1-\alpha_c)\rho_c^2}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}}\Big\{-2(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c} + \frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q -$

$\theta_e)^{1-\alpha_c}\left[\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e) + (1-\alpha_c)g_{\rho_c}(\rho_c\theta_q-\theta_e)\right]\Big\} < \frac{2(1-\alpha_c)^2(1-\delta)\rho_c^2\theta_e}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)}(1-g_{\rho_c}) \le 0$ from (81) and $g_{\rho_c} \ge 1$.

$\frac{dn_1}{d\theta_e} = \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}}\Big\{2(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c} - \frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\big[\alpha_c(g_{\rho_c}\rho_c\theta_q -$

$\theta_e) + (1-\alpha_c)(\rho_c\theta_q-\theta_e)\big]\Big\} > \frac{2(1-\alpha_c)^2(1-\delta)\rho_c^2\theta_q}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)}(g_{\rho_c}-1) \ge 0$ from (81) and $g_{\rho_c} \ge 1$.

(2) $n_2 = \frac{(1-\alpha_c)\rho_c\left[2(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} - (1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\lambda\right]}{3\lambda(\rho_c\theta_q-\theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)}$ from (80).

Since

$\frac{dn_2}{d\rho_c} = \frac{(1-\alpha_c)\theta_e}{3(\rho_c\theta_q-\theta_e)^{2-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\Big\{(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\left[\alpha_c(\rho_c\theta_q-\theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)\right] -$

$2\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{2-\alpha_c}\Big\}$,

$\frac{dn_2}{d\rho_c} < (\ge) 0$ when $\lambda < (\ge) \frac{2(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{2-\alpha_c}}{(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\left[\alpha_c(\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)\right]}$.

Since

$\frac{dn_2}{dg_{\rho_c}} = \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\left[(1-\delta)\alpha_c\rho_c\theta_q(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} - 2\frac{1}{\lambda}\theta_e(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right]$, $\frac{dn_2}{dg_{\rho_c}} <$

$(\ge) 0$ when $\lambda < (\ge) \frac{2\theta_e(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{(1-\delta)\alpha_c\rho_c\theta_q(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}$.

Since $\frac{dn_2}{d\delta} = \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)}\left[(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} - 2\frac{1}{\lambda}g_{\rho_c}g_\delta(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right]$,

$\frac{dn_2}{d\delta} < (\ge) 0$ when $\lambda < (\ge) \frac{2g_{\rho_c}g_\delta(\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}$.

Since

$\frac{dn_2}{d\theta_q} = \frac{(1-\alpha_c)\rho_c^2}{3(\rho_c\theta_q-\theta_e)^{2-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\Big\{(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\left[\alpha_c g_{\rho_c}(\rho_c\theta_q-\theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)\right] -$

$2\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}^2(\rho_c\theta_q-\theta_e)^{2-\alpha_c}\Big\}$,

$\frac{dn_2}{d\theta_q} < (\ge) 0$ when $\lambda < (\ge) \frac{2(1-g_\delta\delta)g_{\rho_c}^2(\rho_c\theta_q-\theta_e)^{2-\alpha_c}}{(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\left[\alpha_c g_{\rho_c}(\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)\right]}$.

Since

$\frac{dn_2}{d\theta_e} = \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)^{2-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\Big\{2\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{2-\alpha_c} - (1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\big[\alpha_c(\rho_c\theta_q -$

$\theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)\big]\Big\}$,

$\frac{dn_2}{d\theta_e} > (\leq) 0$ when $\lambda < (\geq) \frac{2(1-g_\delta \delta) g_{\rho_c} (\rho_c\theta_q - \theta_e)^{2-\alpha_c}}{(1-\delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}[\alpha_c(\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)]}$.

Proof of Proposition A.2

(1) $n_1 = \frac{(1-\alpha_c)\rho_c\{2[(1-\delta)\rho+\eta](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\lambda-(1+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\}}{(3+\alpha)\lambda\rho(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}$ from (84).

$\frac{dn_1}{d\rho_c} = \frac{(1-\alpha_c)\theta_e}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}}\left\{\frac{1}{\lambda}(1+\alpha)(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\left[(g_{\rho_c}\rho_c\theta_q-\theta_e)-(1-\alpha_c)\rho_c\theta_q(g_{\rho_c}-1)\right]-2\left[(1-\delta)+\frac{\eta}{\rho}\right](g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}\right\} < \frac{2(1-\alpha_c)^2\rho_c\theta_q\theta_e\left[(1-\delta)+\frac{\eta}{\rho}\right]}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)}(1-g_{\rho_c}) \leq 0$ from (87) and $g_{\rho_c} \geq 1$.

$\frac{dn_1}{dg_{\rho_c}} = \frac{(1+\alpha)(1-\alpha_c)\rho_c(1-g_\delta\delta)}{(3+\alpha)\lambda(\rho_c\theta_q-\theta_e)^{\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}}(\theta_e-\alpha_c g_{\rho_c}\rho_c\theta_q) < 0$ from $\rho_c > \frac{\theta_e}{\alpha_c\theta_q}$ and $g_{\rho_c} \geq 1$.

$\frac{dn_1}{d\delta} = \frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}\left[\frac{1}{\lambda}(1+\alpha)g_{\rho_c}g_\delta(\rho_c\theta_q-\theta_e)^{1-\alpha_c}-2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right] \leq$

$\frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}\left[(1+\alpha)g_{\rho_c}g_\delta(\rho_c\theta_q-\theta_e)^{1-\alpha_c}-2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right] < 0$

from $\lambda \geq 1$ and the assumption that $g_{\rho_c} \geq 1$ is not large.

$\frac{dn_1}{d\theta_q} = \frac{(1-\alpha_c)\rho_c^2}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}}\left\{-2\left[(1-\delta)+\frac{\eta}{\rho}\right](g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}+\frac{1}{\lambda}(1+\alpha)(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\left[\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)+(1-\alpha_c)g_{\rho_c}(\rho_c\theta_q-\theta_e)\right]\right\} < \frac{2(1-\alpha_c)^2\left[(1-\delta)+\frac{\eta}{\rho}\right]\rho_c^2\theta_e}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)}(1-g_{\rho_c}) \leq 0$ from (87) and $g_{\rho_c} \geq 1$.

$\frac{dn_1}{d\theta_e} = \frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}}\left\{2\left[(1-\delta)+\frac{\eta}{\rho}\right](g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}-\frac{1}{\lambda}(1+\alpha)(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\left[\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)+(1-\alpha_c)(\rho_c\theta_q-\theta_e)\right]\right\} > \frac{2(1-\alpha_c)^2\left[(1-\delta)+\frac{\eta}{\rho}\right]\rho_c^2\theta_q}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)}(g_{\rho_c}-1) \geq 0$ from (87) and $g_{\rho_c} \geq 1$.

(2) $n_2 = \frac{(1-\alpha_c)\rho_c\left[(2+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}-[(1-\delta)\rho+\eta](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\lambda\right]}{(3+\alpha)\lambda\rho(\rho_c\theta_q-\theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)}$ from (85).

Since

$\frac{dn_2}{d\rho_c} = \frac{(1-\alpha_c)\theta_e}{(3+\alpha)(\rho_c\theta_q-\theta_e)^{2-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\left\{\left[(1-\delta)+\frac{\eta}{\rho}\right](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\left[\alpha_c(\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)\right]-(2+\alpha)\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{2-\alpha_c}\right\}$,

$\frac{dn_2}{d\rho_c} < (\geq) 0$ when $\lambda < (\geq) \frac{(2+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{2-\alpha_c}}{[(1-\delta)\rho+\eta](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}[\alpha_c(\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)]}$.

Since

$\frac{dn_2}{dg_{\rho_c}} = \frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\left[\left[(1-\delta)+\frac{\eta}{\rho}\right]\alpha_c\rho_c\theta_q(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}-(2+\alpha)\frac{1}{\lambda}\theta_e(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right]$, $\frac{dn_2}{dg_{\rho_c}} < (\geq) 0$ when $\lambda < (\geq) \frac{(2+\alpha)\rho\theta_e(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{[(1-\delta)\rho+\eta]\alpha_c\rho_c\theta_q(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}$.

Since $\frac{dn_2}{d\delta} = \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)}\left[(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}-(2+\alpha)\frac{1}{\lambda}g_{\rho_c}g_\delta(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right]$, $\frac{dn_2}{d\delta} < (\geq) 0$

when $\lambda < (\geq) \frac{(2+\alpha)g_{\rho_c}g_\delta(\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}$.

Since

$\frac{dn_2}{d\theta_q} = \frac{(1-\alpha_c)\rho_c^2}{(3+\alpha)(\rho_c\theta_q-\theta_e)^{2-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\left\{-(2+\alpha)\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}^2(\rho_c\theta_q-\theta_e)^{2-\alpha_c} + \left[(1-\delta)+\frac{\eta}{\rho}\right](g_{\rho_c}\rho_c\theta_q - \right.$

$\left.\theta_e)^{1-\alpha_c}\left[\alpha_c g_{\rho_c}(\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)\right]\right\}$,

$\frac{dn_2}{d\theta_q} < (\geq) 0$ when $\lambda < (\geq) \frac{(2+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}^2(\rho_c\theta_q-\theta_e)^{2-\alpha_c}}{[(1-\delta)\rho+\eta](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}[\alpha_c g_{\rho_c}(\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)]}$.

Since

$\frac{dn_2}{d\theta_e} = \frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)^{2-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\left\{(2+\alpha)\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{2-\alpha_c} - \left[(1-\delta)+\frac{\eta}{\rho}\right](g_{\rho_c}\rho_c\theta_q - \right.$

$\left.\theta_e)^{1-\alpha_c}\left[\alpha_c(\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)\right]\right\}$,

$\frac{dn_2}{d\theta_e} > (\leq) 0$ when $\lambda < (\geq) \frac{(2+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{2-\alpha_c}}{[(1-\delta)\rho+\eta](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}[\alpha_c(\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)]}$.

(3) $e = \frac{\alpha\rho\left[(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\lambda+(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right]-3\lambda\eta(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{(3+\alpha)\lambda\rho\eta(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}$ from (86).

$\frac{de}{d\rho_c} = \frac{\alpha(1-g_\delta\delta)g_{\rho_c}(1-\alpha_c)\theta_q\theta_e}{(3+\alpha)\lambda\eta(\rho_c\theta_q-\theta_e)^{\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}}(g_{\rho_c}-1) \geq 0$ from $g_{\rho_c} \geq 1$.

$\frac{de}{dg_{\rho_c}} = \frac{\alpha(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{(3+\alpha)\lambda\eta(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}}(\alpha_c g_{\rho_c}\rho_c\theta_q-\theta_e) > 0$ from $\rho_c > \frac{\theta_e}{\alpha_c\theta_q}$ and $g_{\rho_c} \geq 1$.

$\frac{de}{d\theta_q} = \frac{\alpha(1-\alpha_c)(1-g_\delta\delta)g_{\rho_c}\theta_e}{(3+\alpha)\lambda\eta(\rho_c\theta_q-\theta_e)^{\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}}(g_{\rho_c}-1) \geq 0$ from $g_{\rho_c} \geq 1$.

$\frac{de}{d\theta_e} = \frac{\alpha(1-\alpha_c)(1-g_\delta\delta)g_{\rho_c}\rho_c\theta_q}{(3+\alpha)\lambda\eta(\rho_c\theta_q-\theta_e)^{\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}}(1-g_{\rho_c}) \leq 0$ from $g_{\rho_c} \geq 1$.

Proof of Proposition A.3

(1) $n_1 + n_2 = \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)}\left\{\left[2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}-(\rho_c\theta_q-\theta_e)^{\alpha_c}\right](1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} + \left[2(\rho_c\theta_q - \right.\right.$

$\left.\left.\theta_e)^{\alpha_c}-(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}\right]\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right\}$ from (79) and (80). n_1+n_2 decreases with λ, δ and g_δ from the assumption that $g_{\rho_c} \geq 1$ is not large.

$\frac{d(n_1+n_2)}{d\rho_c} = \frac{(1-\alpha_c)\theta_e}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\left(\left\{(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}\left[\alpha_c\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)+(1-\alpha_c)(\rho_c\theta_q-\theta_e)\right]-2(\rho_c\theta_q - \right.\right.$

$\left.\theta_e)^{1+\alpha_c}\right\}\frac{1}{\lambda}g_{\rho_c}(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c} + \left\{(\rho_c\theta_q-\theta_e)^{\alpha_c}\left[\alpha_c(\rho_c\theta_q-\theta_e)+(1-\alpha_c)\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)\right] - \right.$

$\left.\left.2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1+\alpha_c}\right\}(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\right) < 0$

from the assumption that $g_{\rho_c} \geq 1$ is not large.

Since

$\frac{d(n_1+n_2)}{dg_{\rho_c}} = \frac{(1-\alpha_c)\rho_c}{3\lambda(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\left\{\lambda(1-\delta)\alpha_c\rho_c\theta_q(\rho_c\theta_q-\theta_e)^{\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}-(1-g_\delta\delta)(\rho_c\theta_q - \right.$

$\theta_e)^{1-\alpha_c}\left[(\alpha_c g_{\rho_c}\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}+2\theta_e(\rho_c\theta_q-\theta_e)^{\alpha_c}\right]\Big\}$,

$\frac{d(n_1+n_2)}{dg_{\rho_c}}<(\geq)0$ when $\lambda<(\geq)\frac{(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-2\alpha_c}\left[(\alpha_c g_{\rho_c}\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}+2\theta_e(\rho_c\theta_q-\theta_e)^{\alpha_c}\right]}{(1-\delta)\alpha_c\rho_c\theta_q(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}$.

$\frac{d(n_1+n_2)}{d\theta_q}=\frac{(1-\alpha_c)\rho_c^2}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\Big(\Big\{(\rho_c\theta_q-\theta_e)^{\alpha_c}\left[\alpha_c(g_{\rho_c}\rho_c\theta_q-g_{\rho_c}\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)\right]-2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1+\alpha_c}\Big\}(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}+\Big\{(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}\left[\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-g_{\rho_c}\theta_e)\right]-2g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1+\alpha_c}\Big\}\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\Big)<0$ from the assumption that g_{ρ_c} is not large.

$\frac{d(n_1+n_2)}{d\theta_e}=\frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\Big(\Big\{2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1+\alpha_c}-(\rho_c\theta_q-\theta_e)^{\alpha_c}\left[\alpha_c(\rho_c\theta_q-\theta_e)+(1-\alpha_c)\left(g_{\rho_c}\rho_c\theta_q-\theta_e\right)\right]\Big\}(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}+\Big\{2(\rho_c\theta_q-\theta_e)^{1+\alpha_c}-(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}\left[\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)+(1-\alpha_c)(\rho_c\theta_q-\theta_e)\right]\Big\}\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\Big)>0$

from the assumption that g_{ρ_c} is not large.

(2) $n_1+n_2=\frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)}\Big\{\left[2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}-(\rho_c\theta_q-\theta_e)^{\alpha_c}\right]\left[(1-\delta)+\frac{\eta}{\rho}\right](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}+\left[(2+\alpha)(\rho_c\theta_q-\theta_e)^{\alpha_c}-(1+\alpha)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}\right]\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\Big\}$ from (84) and (85). n_1+n_2 decreases with λ, δ and g_δ from the assumption that $g_{\rho_c}\geq 1$ is not large.

$\frac{d(n_1+n_2)}{d\rho_c}=\frac{(1-\alpha_c)\theta_e}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\Big(\Big\{(1+\alpha)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}\left[\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)+(1-\alpha_c)(\rho_c\theta_q-\theta_e)\right]-(2+\alpha)(\rho_c\theta_q-\theta_e)^{1+\alpha_c}\Big\}\frac{1}{\lambda}g_{\rho_c}(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c}+\Big\{(\rho_c\theta_q-\theta_e)^{\alpha_c}\left[\alpha_c(\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)\right]-2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1+\alpha_c}\Big\}\left[(1-\delta)+\frac{\eta}{\rho}\right](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\Big)<0$

from the assumption that g_{ρ_c} is not large.

Since

$\frac{d(n_1+n_2)}{dg_{\rho_c}}=\frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\Big\{\left[(1-\delta)+\frac{\eta}{\rho}\right]\alpha_c\rho_c\theta_q(\rho_c\theta_q-\theta_e)^{\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}-\frac{1}{\lambda}(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\left[(1+\alpha)(\alpha_c g_{\rho_c}\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}+(2+\alpha)\theta_e(\rho_c\theta_q-\theta_e)^{\alpha_c}\right]\Big\}$, $\frac{d(n_1+n_2)}{dg_{\rho_c}}<(\geq)0$ when

$\lambda<(\geq)\frac{(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-2\alpha_c}\rho\left[(1+\alpha)(\alpha_c g_{\rho_c}\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}+(2+\alpha)\theta_e(\rho_c\theta_q-\theta_e)^{\alpha_c}\right]}{[(1-\delta)\rho+\eta]\alpha_c\rho_c\theta_q(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}$.

$\frac{d(n_1+n_2)}{d\theta_q}=\frac{(1-\alpha_c)\rho_c^2}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\Big(\Big\{(\rho_c\theta_q-\theta_e)^{\alpha_c}\left[\alpha_c(g_{\rho_c}\rho_c\theta_q-g_{\rho_c}\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)\right]-2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1+\alpha_c}\Big\}\left[(1-\delta)+\frac{\eta}{\rho}\right](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}+\Big\{(1+\alpha)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}\left[\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-g_{\rho_c}\theta_e)\right]-(2+\alpha)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1+\alpha_c}\Big\}\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}\Big)<0$ from the assumption that g_{ρ_c} is not large.

$\frac{d(n_1+n_2)}{d\theta_e} = \frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^2}\Big(\Big\{(2+\alpha)\big(\rho_c\theta_q-\theta_e\big)^{1+\alpha_c}-(1+\alpha)\big(g_{\rho_c}\rho_c\theta_q-\theta_e\big)^{\alpha_c}\big[\alpha_c\big(g_{\rho_c}\rho_c\theta_q-\theta_e\big)+(1-\alpha_c)\big(\rho_c\theta_q-\theta_e\big)\big]\Big\}\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}\big(\rho_c\theta_q-\theta_e\big)^{1-\alpha_c}+\Big\{2\big(g_{\rho_c}\rho_c\theta_q-\theta_e\big)^{1+\alpha_c}-\big(\rho_c\theta_q-\theta_e\big)^{\alpha_c}\big[\alpha_c\big(\rho_c\theta_q-\theta_e\big)+(1-\alpha_c)\big(g_{\rho_c}\rho_c\theta_q-\theta_e\big)\big]\Big\}\Big[(1-\delta)+\frac{\eta}{\rho}\Big]\big(g_{\rho_c}\rho_c\theta_q-\theta_e\big)^{1-\alpha_c}\Big) > 0$ from the assumption that g_{ρ_c} is not large. ■