Appendix C (Online Appendix) Proof of lemmas and propositions of the general case

Proof of Lemma 4. The result is proved by examining under what conditions each case is realized.

(i) [Case 1: $e^*_{2L} = 0$ and the indifference condition holds for those with $a \geq e^*_{2N}$] As explained in Appendix A, $\frac{H_{2N}}{H_{2L}}$ is determined by (28) independently of the distribution of wealth, as in the unconstrained case with $e^*_{2L} = 0$. Thus, this case exists iff the condition for $e^*_{2L} = 0$ in Section 4 holds, i.e., when $s \leq \underline{s}$ or $s \geq \overline{s}$, and, from (A3) in Appendix A, the following is true

$$\frac{H_{2N}}{H_{2L}} \leq \frac{[\delta_N(1-s)e^*_{2N}]^\gamma(1-F(e^*_{2N}))}{(\bar{l})^\gamma F(e^*_{2N})}, \tag{C1}$$

which can be expressed as

$$\frac{H_{2N}}{H_{2L}}\left[(1-\alpha)T_2{}^\alpha T_N{}^{1-\alpha}\gamma\delta_N(1-s)\left(\frac{H_{2L}}{H_{2N}}\right)^\alpha\right]^{-\frac{\gamma}{1-\gamma}}(\bar{l})^\gamma \leq \frac{1-F(e^*_{2N})}{F(e^*_{2N})} \text{ (from (21))}$$

$$\Leftrightarrow \left\{\left(\frac{1-\alpha}{\alpha}\frac{1-\gamma}{(\bar{l})^\gamma}\right)^{1-\gamma}[(1-\alpha)\gamma\delta_N(1-s)T_2{}^\alpha T_N{}^{1-\alpha}]^\gamma\right\}^{\frac{1}{1-\gamma}}\left[(1-\alpha)T_2{}^\alpha T_N{}^{1-\alpha}\gamma\delta_N(1-s)\right]^{-\frac{\gamma}{1-\gamma}}(\bar{l})^\gamma \leq \frac{1-F(e^*_{2N})}{F(e^*_{2N})} \text{ (from (28))}$$

$$\Leftrightarrow \frac{1-\alpha}{\alpha}(1-\gamma) \leq \frac{1-F(e^*_{2N})}{F(e^*_{2N})}. \tag{C2}$$

Because the RHS of the above equation decreases with e^*_{2N} and thus increases with s from (21) and (28), for given $F(\cdot)$, there exists a critical $s \in (0,1)$ such that the condition holds for greater s or the condition holds for any s, if the RHS of the equation at $s = 1$ is strictly greater than the LHS, i.e., $\frac{1-\alpha}{\alpha}(1-\gamma) < \frac{1-F(0)}{F(0)} \Leftrightarrow F(0) < \frac{\alpha}{1-\gamma(1-\alpha)}$. ($e^*_{2N} \to 0$ as $s \to 1$ from (21) and (28).) For given s, the condition tends to hold when the proportion of those with adequate wealth for education is high, i.e., $F(e^*_{2N})$ is low. Thus, the critical s, which is denoted by $s^+(F) \in [0,1)$, increases as the proportion of those with adequate wealth is lower. ($s^+(F)$ is set to be 0 when the proportion is high enough that (C2) holds for any s.) Hence, the economy is in Case 1 if $F(0) < \frac{\alpha}{1-\gamma(1-\alpha)}$ and either $s \in [s^+(F), \underline{s}]$ (when $s^+(F) < \underline{s}$) or $s \in [\max\{\overline{s}, s^+(F)\}, 1]$.

[Case 2: $e^*_{2L} = 0$ and the indifference condition holds for those with $a = \widehat{a}_0 < e^*_{2L}$] This case exists iff the condition for $e^*_{2L} = 0$, $\gamma\delta_L s\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}(\bar{l})^{\gamma-1} \leq 1$ (in the proof of Lemma 1), holds and the condition for $\widehat{a}_0 < e^*_{2N}$ holds, which, from (A5) in Appendix A, equals

$$\frac{H_{2N}}{H_{2L}} > \frac{[\delta_N(1-s)e^*_{2N}]^\gamma(1-F(e^*_{2N}))}{(\bar{l})^\gamma F(e^*_{2N})}. \tag{C3}$$

This equation holds with equality when $\widehat{a}_0 = e^*_{2N}$ from (A5) and, as the proportion of those with adequate wealth rises (i.e., $F(a)$ for given a decreases), $\widehat{a}_0$ increases and converges to e^*_{2N} from the proof of Lemma 5 (ii). Hence, the above equation with " = " divides this case and Case 1, which, from the proof for Case 1, can be expressed as

$$\frac{1-\alpha}{\alpha}(1-\gamma) = \frac{1-F(e^*_{2N})}{F(e^*_{2N})}. \tag{C4}$$

From the proof for Case 1, when $s \leq \underline{s}$ or $s \geq \overline{s}$, the critical s, $s^+(F)$, if exists (thus $F(0) < \frac{\alpha}{1-\gamma(1-\alpha)}$ must hold), increases as the proportion of those with adequate wealth falls, and given $F(\cdot)$, the economy is in Case 2 (Case 1) for $s < (\geq) s^+(F)$, while if $F(0) \geq \frac{\alpha}{1-\gamma(1-\alpha)}$, (C2) does not hold for any s and thus Case 2 is realized for any s. Hence, when $s \leq \underline{s}$ or $s \geq \overline{s}$, Case 2 is realized if $F(0) \geq \frac{\alpha}{1-\gamma(1-\alpha)}$ or if $s \in [0, \min\{s^+(F), \underline{s}\}]$ when $s^+(F) > 0$ or $s \in [\overline{s}, s^+(F))$ when $s^+(F) > \overline{s}$.

Unlike Case 1, the condition for $e^*_{2L} = 0$ holds for some ranges of $s \in (\underline{s}, \overline{s})$ too. In particular, the smallest (largest) critical s satisfying $\gamma\delta_L s\alpha {T_2}^\alpha {T_N}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}(\bar{l})^{\gamma-1} = 1$, if exists, is larger than $\underline{s}$ (smaller than $\overline{s}$) and increases (decreases) as the proportion of those with adequate wealth falls. (It is not clear if there exist more than two critical values of s.) This is because $\frac{H_{2N}}{H_{2L}}$ decreases as the proportion falls from Lemma 5 (ii) and thus $\frac{H_{2N}}{H_{2L}}$ for given s is lower than Case 1.

Denote the smallest (largest) critical s by $\underline{s}(F)$ ($\overline{s}(F)$). Then, if $\underline{s}(F)$ and $\overline{s}(F)$ exist, which is the case when the proportion of those with adequate wealth is high enough (because $\underline{s}(F)$ and $\overline{s}(F)$ respectively converge to $\underline{s}$ and $\overline{s}$ as the proportion rises), the economy is in Case 2 at least for $s \in [0, \min\{s^+(F), \underline{s}(F)\})$ when $s^+(F) > 0$ and for $s \in (\overline{s}(F), s^+(F)]$ when $s^+(F) > \overline{s}$. (If critical values other than $\underline{s}(F)$ and $\overline{s}(F)$ exist, some ranges of $s \in [\underline{s}(F), \overline{s}(F)]$ too belong to this case.)

When the proportion of those with adequate wealth is low enough, $\overline{s}(F)$ and $\underline{s}(F)$ do not exist and the economy is in Case 2 for any s. This is because, as the proportion falls, $\frac{H_{2N}}{H_{2L}}$ decreases and converges to 0 from the proof of Lemma 5 (ii) and thus $\gamma\delta_L s\alpha {T_2}^\alpha {T_N}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}(\bar{l})^{\gamma-1} < 1$ for any s.

(ii) [Case 3: $e^*_{2L} > 0$ and the indifference condition holds for those with $a \geq e^*_{2N}$] As explained in Appendix A, $\frac{H_{2N}}{H_{2L}}$(thus e^*_{2N} and e^*_{2L}) is determined by (29) independently of the distribution of wealth, as in the unconstrained case with $e^*_{2L} > 0$. Thus, this case exists iff the condition for $e^*_{2L} > 0$ in Section 4 holds, i.e., when $s \in (\underline{s}, \overline{s})$, and, from (A8) in Appendix A, the following is true

$$\frac{H_{2N}}{H_{2L}} \leq \frac{[\delta_N(1-s)e^*_{2N}]^\gamma(1-F(e^*_{2N}))}{(\bar{l}+\delta_L s e^*_{2L})^\gamma[F(e^*_{2N})-F(e^*_{2L})]+\int_0^{e^*_{2L}}(\bar{l}+\delta_L sa)^\gamma dF(a)+(\bar{l})^\gamma F(0)}. \tag{C5}$$

As the proportion of those with adequate wealth falls (i.e., $F(a)$ for given a increases), the RHS of this equation decreases, thus the condition holds with equality when the proportion is lowest in this case (for given s). Hence, the economy is in this case if $s \in (\underline{s}, \overline{s})$ and the proportion of those with adequate wealth is high enough that the above condition is satisfied.

[Case 4: $e^*_{2L} > 0$ and the indifference condition holds for those with $a = \widehat{a} \in [e^*_{2L}, e^*_{2N})$] This case exists iff the condition for $e^*_{2L} > 0$, $\gamma\delta_L s\alpha {T_2}^\alpha {T_N}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}(\bar{l})^{\gamma-1} > 1$ (in the proof of Lemma 1) holds (thus $s \in (\underline{s}(F), \overline{s}(F))$ must hold) and the condition for $\widehat{a} \in [e^*_{2L}, e^*_{2N})$ holds, which equals, from (A11) in Appendix A,

$$\frac{H_{2N}}{H_{2L}} \in \left(\frac{[\delta_N(1-s)]^\gamma(e^*_{2N})^\gamma(1-F(e^*_{2N}))}{(\bar{l}+\delta_L s e^*_{2L})^\gamma(F(e^*_{2N})-F(e^*_{2L}))+\int_0^{e^*_{2L}}(\bar{l}+\delta_L sa)^\gamma dF(a)+(\bar{l})^\gamma F(0)}, \frac{[\delta_N(1-s)]^\gamma\left[(e^*_{2N})^\gamma(1-F(e^*_{2N}))+\int_{e^*_{2L}}^{e^*_{2N}} a^\gamma dF(a)\right]}{\int_0^{e^*_{2L}}(\bar{l}+\delta_L sa)^\gamma dF(a)+(\bar{l})^\gamma F(0)}\right]. \tag{C6}$$

As the proportion of those with adequate wealth rises, $\widehat{a}$ rises from the proof of Lemma 5 (ii). Thus, when the proportion is supremum in this case, $\widehat{a} \to e^*_{2N}$ and $\frac{H_{2N}}{H_{2L}} \to \frac{[\delta_N(1-s)]^\gamma(e^*_{2N})^\gamma(1-F(e^*_{2N}))}{(\bar{l}+\delta_L s e^*_{2L})^\gamma(F(e^*_{2N})-F(e^*_{2L}))+\int_0^{e^*_{2L}}(\bar{l}+\delta_L sa)^\gamma dF(a)+(\bar{l})^\gamma F(0)}$ from (A11). Hence, $\frac{H_{2N}}{H_{2L}} = \frac{[\delta_N(1-s)]^\gamma(e^*_{2N})^\gamma(1-F(e^*_{2N}))}{(\bar{l}+\delta_L s e^*_{2L})^\gamma(F(e^*_{2N})-F(e^*_{2L}))+\int_0^{e^*_{2L}}(\bar{l}+\delta_L sa)^\gamma dF(a)+(\bar{l})^\gamma F(0)}$ divides this case and Case 3. Given s, the proportion of those with adequate wealth is lower (i.e., $F(a)$ for given a is higher) than Case 3, because $\widehat{a} \to e^*_{2N}$ ($\widehat{a} = e^*_{2N}$) when the proportion is supremum (lowest) in this case (in Case 3).

At $s = \underline{s}$, $\overline{s}$ and thus $e^*_{2L} = 0$, the equation becomes $\frac{H_{2N}}{H_{2L}} = \frac{[\delta_N(1-s)e^*_{2N}]^\gamma(1-F(e^*_{2N}))}{(\bar{l})^\gamma F(e^*_{2N})}$, the same as Case 1. That is, the dividing line and $s = s^+(F)$ intersect at $s = \underline{s}$, $\overline{s}$.

[Case 5: $e_{2L}^* > 0$ and the indifference condition holds for those with $a = \widetilde{a} < e_{2L}^*$] This case exists iff the condition for $e_{2L}^* > 0$, $\gamma \delta_L s \alpha T_2{}^{\alpha} T_N{}^{1-\alpha} \left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha} (\bar{l})^{\gamma-1} > 1$ (in the proof of Lemma 1), holds (thus $s \in (\underline{s}(F), \overline{s}(F))$ must hold) and the condition for $\widetilde{a} < e_{2L}^*$ holds, which equals, from (A14) in Appendix A,

$$\frac{H_{2N}}{H_{2L}} > \frac{[\delta_N(1-s)]^{\gamma}\left[(e_{2N}^*)^{\gamma}(1-F(e_{2N}^*)) + \int_{e_{2L}^*}^{e_{2N}^*} a^{\gamma} dF(a)\right]}{\int_0^{e_{2L}^*} (\bar{l} + \delta_L s a)^{\gamma} dF(a) + (\bar{l})^{\gamma} F(0)}. \tag{C7}$$

As the proportion of those with adequate wealth rises, $\widetilde{a}$ rises from the proof of Lemma 5 (ii). Thus, when the proportion is supremum in this case, $\widetilde{a} \to e_{2L}^*$ and $\frac{H_{2N}}{H_{2L}} \to \frac{[\delta_N(1-s)]^{\gamma}\left[(e_{2N}^*)^{\gamma}(1-F(e_{2N}^*)) + \int_{e_{2L}^*}^{e_{2N}^*} a^{\gamma} dF(a)\right]}{\int_0^{e_{2L}^*} (\bar{l} + \delta_L s a)^{\gamma} dF(a) + (\bar{l})^{\gamma} F(0)}$ from (A14). Hence, $\frac{H_{2N}}{H_{2L}} = \frac{[\delta_N(1-s)]^{\gamma}\left[(e_{2N}^*)^{\gamma}(1-F(e_{2N}^*)) + \int_{e_{2L}^*}^{e_{2N}^*} a^{\gamma} dF(a)\right]}{\int_0^{e_{2L}^*} (\bar{l} + \delta_L s a)^{\gamma} dF(a) + (\bar{l})^{\gamma} F(0)}$ divides this case and Case 4. Given s, the proportion of those with adequate wealth is lower (i.e., $F(a)$ for given a is higher) than Case 4, because $\widetilde{a} \to e_{2L}^*$ ($\widehat{a} = e_{2L}^*$) holds when the proportion is supremum (lowest) in this case (in Case 4).

When $e_{2L}^* = 0$, the equation becomes $\frac{H_{2N}}{H_{2L}} = \frac{[\delta_N(1-s)]^{\gamma}\left[(e_{2N}^*)^{\gamma}(1-F(e_{2N}^*)) + \int_0^{e_{2N}^*} a^{\gamma} dF(a)\right]}{(\bar{l})^{\gamma} F(0)}$, which is different from $\frac{H_{2N}}{H_{2L}} = \frac{[\delta_N(1-s)e_{2N}^*]^{\gamma}(1-F(e_{2N}^*))}{(\bar{l})^{\gamma} F(e_{2N}^*)}$, i.e., $s = s^+(F)$. Hence, the dividing line between Case 4 and Case 5 does not intersect with $s = s^+(F)$ and the dividing line between Case 3 and Case 4 at $s = \underline{s}, \overline{s}$. This implies that when s is close to $\underline{s}$ or $\overline{s}$, Case 5 is not realized. ■

Proof of Lemma 5. (i) As explained in in Appendix A, $\frac{H_{2N}}{H_{2L}}$ (thus e_{2N}^* and e_{2N}^*) is determined independently of the distribution of wealth by (28) [(29)] when $e_{2L}^* = (>)0$. If the proportion of those with adequate wealth falls (i.e., $F(a)$ increases for given a) so that the numerator of (A3) [(A8)] in Appendix A decreases and the denominator increases when $e_{2L}^* = (>)0$, p_{2N} must increase for the equation to hold.

(ii) [Case 2: $e_{2L}^* = 0$ and the indifference condition holds for $a = \widehat{a}_0 < e_{2L}^*$] Because $T_N(\delta_N(1-s)\widehat{a}_0)^{\gamma} - \frac{1}{1-\alpha}\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^{\alpha} \widehat{a}_0$ increases with $\widehat{a}_0$ from $\widehat{a}_0 < e_{2N}^*$, the relationship between $\frac{H_{2N}}{H_{2L}}$ and $\widehat{a}_0$ satisfying (A4) in Appendix A is positive. Because e_{2N}^* decreases with $\frac{H_{2N}}{H_{2L}}$ from (21), the relationship between $\frac{H_{2N}}{H_{2L}}$ and $\widehat{a}_0$ satisfying (A5) in Appendix A is negative. When the proportion of those with adequate wealth falls (i.e., $F(a)$ increases for given a) so that the numerator of (A5) decreases and the denominator increases, $\frac{H_{2N}}{H_{2L}}$ satisfying (A5) must decrease for given $\widehat{a}_0$. Hence, $\frac{H_{2N}}{H_{2L}}$ and $\widehat{a}_0$ decrease from (A4) and (A5). From the equations, when the proportion falls to the point that $F(0) \to 1$, $\frac{H_{2N}}{H_{2L}} \to 0$ and $\widehat{a}_0 \to 0$, while when it rises sufficiently, $\widehat{a}_0 \to e_{2N}^*$, which is the threshold of Case 1 (note that e_{2N}^* decreases with $\frac{H_{2N}}{H_{2L}}$).

[Case 4: $e_{2L}^* > 0$ and the indifference condition holds for $a = \widehat{a} \in [e_{2L}^*, e_{2N}^*)$] Because $T_N(\delta_N(1-s)\widehat{a})^{\gamma} - \frac{1}{1-\alpha}\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^{\alpha} \widehat{a}$ increases with $\widehat{a}$ from $\widehat{a} < e_{2N}^*$, the relationship between $\frac{H_{2N}}{H_{2L}}$ and $\widehat{a}$ satisfying (A10) in Appendix A is positive. Because e_{2N}^* decreases with $\frac{H_{2N}}{H_{2L}}$ from (21) and e_{2L}^* increases with $\frac{H_{2N}}{H_{2L}}$ from (24), the relationship between $\frac{H_{2N}}{H_{2L}}$ and $\widehat{a}$ satisfying (A11) in Appendix A is negative. When the proportion of those with adequate wealth falls so that the numerator of (A11) decreases and the denominator increases, $\frac{H_{2N}}{H_{2L}}$ satisfying (A11) must decrease for given $\widehat{a}$. Hence, $\frac{H_{2N}}{H_{2L}}$ and $\widehat{a}$ decrease from (A10) and (A11). From the equations, when the proportion rises sufficiently, $\widehat{a} \to e_{2N}^*$, which is the threshold of Case 3 (note that e_{2N}^* decreases and e_{2L}^* increases with $\frac{H_{2N}}{H_{2L}}$). By contrast, when the proportion and thus $\frac{H_{2N}}{H_{2L}}$ fall sufficiently, either $\widehat{a} \to e_{2L}^*$, which is the threshold of Case 5, or the condition for $e_{2L}^* = 0$ holds with equality, i.e., $\gamma\delta_L s\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}(\bar{l})^{\gamma-1} = 1$, and the economy shifts to Case 2.

[Case 5: $e_{2L}^* > 0$ **and the indifference condition holds for** $a = \widetilde{a} < e_{2L}^*$**]** The relationship between $\frac{H_{2N}}{H_{2L}}$ and $\widetilde{a}$ satisfying (A13) in Appendix A is positive, while the relationship between $\frac{H_{2N}}{H_{2L}}$ and $\widetilde{a}$ satisfying (A14) is negative because e_{2N}^* decreases with $\frac{H_{2N}}{H_{2L}}$ from (21). When the proportion of those with adequate wealth falls so that the numerator of (A14) decreases and the denominator increases, $\frac{H_{2N}}{H_{2L}}$ satisfying (A14) must decrease for given $\widetilde{a}$. Hence, $\frac{H_{2N}}{H_{2L}}$ and $\widetilde{a}$ decrease from (A13) and (A14). From the equations, when the proportion rises sufficiently, $\widetilde{a} \to e_{2L}^*$ (note that e_{2N}^* decreases with $\frac{H_{2N}}{H_{2L}}$ and $e_{2L}^* < e_{2N}^*$), whereas when the proportion and thus $\frac{H_{2N}}{H_{2L}}$ fall sufficiently, the condition for $e_{2L}^* = 0$ holds with equality, i.e., $\gamma\delta_L s\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}(\bar{l})^{\gamma-1} = 1$, and the economy shifts to Case 2. ■

Proof of Proposition 3. The result on human capital is from Lemma 5 and (12), (13), (21), and (24). **(i)** Because $\frac{H_{2N}}{H_{2L}}$ does not depend on the distribution of wealth from Lemma 5 (i), net earnings and consumption too do not depend on the distribution.

(ii) From Appendix A, consumption of those who have relatively large wealth and choose the national sector is given by (30) for those with $a \geq e_{2N}^*$ and by (A6) for those with $a < e_{2N}^*$, while consumption of those who have relatively small wealth and choose the local sector is given by (A12) for those with $a \geq e_{2L}^*$ (Case 4), and for those with $a < e_{2L}^*$ by (A7) (Case 2) and (A9) (Cases 4 and 5). Net earnings in unit of the final good equal consumption minus wealth.

Because $\frac{H_{2N}}{H_{2L}}$ decreases as the proportion of those with adequate wealth falls from Lemma 5 (ii), from these equations, consumption and net earnings of those who choose the local sector decrease and of those who choose the national sector increase. Hence, consumption and earnings inequalities between any pairs of national and local sector workers increase. ■

Proof of Lemma 6. As explained in in Appendix A, in Cases 1 and 3, $\frac{H_{2N}}{H_{2L}}$ is determined by (29) when $e_{2L}^* > 0$ and by (28) when $e_{2L}^* = 0$, same as when everyone has enough wealth for education. Thus, Lemma 2 applies.

In Case 2, as shown in the proof of Lemma 5 (ii), the relationship between $\frac{H_{2N}}{H_{2L}}$ and $\widehat{a}_0$ satisfying (A4) in Appendix A is positive, and the relationship between $\frac{H_{2N}}{H_{2L}}$ and $\widehat{a}_0$ satisfying (A5) is negative. For given $\widehat{a}_0$, an increase in s lowers $\frac{H_{2N}}{H_{2L}}$ satisfying (A4). From (A5) and (21), for given $\widehat{a}_0$, an increase in s lowers $\frac{H_{2N}}{H_{2L}}$ satisfying (A5). Therefore, an increase in s lowers $\frac{H_{2N}}{H_{2L}}$.

In Case 4, as shown in the proof of Lemma 5 (ii), the relationship between $\frac{H_{2N}}{H_{2L}}$ and $\widehat{a}$ satisfying (A10) in Appendix A is positive, and the relationship satisfying (A11) is negative. For given $\widehat{a}$, an increase in s lowers $\frac{H_{2N}}{H_{2L}}$ satisfying (A10), because the derivative of the expression inside the curly bracket of the RHS of the equation with respect to s equals

$$\frac{1}{s^2}\left\{\gamma s\left[(\gamma\delta_L s)^\gamma \alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}} - \frac{\bar{l}}{\delta_L}\right\} > 0 \quad \text{from (24).}$$

From (A11), (21), and (24), for given $\widehat{a}$, an increase in s lowers $\frac{H_{2N}}{H_{2L}}$ satisfying (A11), because the derivative of se_{2L}^* with respect to s equals

$$e_{2L}^* + s\frac{\partial e_{2L}^*}{\partial s} = e_{2L}^* + \frac{1}{\delta_L s}\left(-\left\{\left[\alpha\delta_L\gamma\delta_L s T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}} - \bar{l}\right\} + \frac{1}{1-\gamma}\left[\alpha\gamma\delta_L s T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}\right)$$

$$= \frac{1}{(1-\gamma)\delta_L s}\left[\alpha\gamma\delta_L s {T_2}^{\alpha} {T_N}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}} > 0.$$

Therefore, an increase in s lowers $\frac{H_{2N}}{H_{2L}}$.

In Case 5, as shown in the proof of Lemma 5 (ii), the relationship between $\frac{H_{2N}}{H_{2L}}$ and $\widetilde{a}$ satisfying (A13) in Appendix A is positive, and the relationship satisfying (A14) is negative. For given $\widetilde{a}$, an increase in s lowers $\frac{H_{2N}}{H_{2L}}$ satisfying (A13). From (A14) and (21), for given $\widetilde{a}$, an increase in s lowers $\frac{H_{2N}}{H_{2L}}$ satisfying (A14). Therefore, an increase in s lowers $\frac{H_{2N}}{H_{2L}}$. ■

Proof of Lemma 7. Only the proof of the result on the consumption is presented, because net earnings in unit of the final good equal consumption minus wealth. **(i) [Case 1: the indifference condition holds for** $a \geq e^*_{2N}$**]** Because c_2 for any a is given by (30) from Appendix A, Lemma 3 (i) applies and thus c_2 decreases with s.

[Case 2: the indifference condition holds for $a = \widehat{a}_0 < e^*_{2L}$**]** Because $\frac{H_{2N}}{H_{2L}}$ decreases with s from Lemma 6, c_2 for $a < \widehat{a}_0$ decreases with s from (A7) in Appendix A. From (30) and (A6) in Appendix A, $\dfrac{dc_2}{ds}$ for $a \geq \widehat{a}_0$ is proportional to $-\left[\frac{\gamma}{1-s} + \alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right]$. In the following, $\frac{\gamma}{1-s} + \alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} > 0$ is shown.

Totally differentiating (A4) gives

$$\left[\frac{\gamma}{\widehat{a}_0}T_N(\delta_N(1-s)\widehat{a}_0)^{\gamma} - \frac{1}{1-\alpha}\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^{\alpha}\right]d\widehat{a}_0 = \frac{\gamma}{1-s}T_N(\delta_N(1-s)\widehat{a}_0)^{\gamma}ds + \alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\left[\frac{1}{1-\alpha}\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^{\alpha}\widehat{a}_0 + \frac{1}{1-\alpha}\frac{H_{2N}}{H_{2L}}T_N(\bar{l})^{\gamma}\right]d\frac{H_{2N}}{H_{2L}}, \tag{C8}$$

where $\frac{\gamma}{\widehat{a}_0}T_N(\delta_N(1-s)\widehat{a}_0)^{\gamma} - \frac{1}{1-\alpha}\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^{\alpha} > 0$ from $\widehat{a}_0 < e^*_{2N}$.

Totally differentiating (A5) gives

$$\frac{\gamma}{1-s}\frac{H_{2N}}{H_{2L}}ds + d\frac{H_{2N}}{H_{2L}} - \frac{[\delta_N(1-s)]^{\gamma}\gamma(e^*_{2N})^{\gamma-1}(1-F(e^*_{2N}))}{(\bar{l})^{\gamma}F(\widehat{a}_0)}de^*_{2N} + \frac{[\delta_N(1-s)]^{\gamma}\left\{F(\widehat{a}_0)(\widehat{a}_0)^{\gamma} + \left[(e^*_{2N})^{\gamma}(1-F(e^*_{2N})) + \int_{\widehat{a}_0}^{e^*_{2N}}(a)^{\gamma}dF(a)\right]\right\}dF(\widehat{a}_0)}{(\bar{l})^{\gamma}[F(\widehat{a}_0)]^2}d\widehat{a}_0 = 0, \tag{C9}$$

where, by totally differentiating (21),

$$de^*_{2N} = -\left[\frac{\gamma}{1-s}ds + \alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}d\frac{H_{2N}}{H_{2L}}\right]\frac{e^*_{2N}}{1-\gamma}. \tag{C10}$$

When the first and third equations are substituted into the second one and divided by ds, the resulting equation consists of the term associated with $\frac{\gamma}{1-s}\frac{H_{2N}}{H_{2L}} + \frac{d\frac{H_{2N}}{H_{2L}}}{ds}$, the one associated with $\frac{\gamma}{1-s}\frac{H_{2N}}{H_{2L}} + \alpha\frac{d\frac{H_{2N}}{H_{2L}}}{ds}$, and the one associated with $\frac{\gamma}{1-s}\frac{H_{2N}}{H_{2L}} + \frac{\alpha}{T_N(\delta_N(1-s)\widehat{a}_0)^{\gamma}}\frac{1}{1-\alpha}\left[\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^{\alpha}\widehat{a}_0 + \frac{H_{2N}}{H_{2L}}T_N(\bar{l})^{\gamma}\right]\frac{d\frac{H_{2N}}{H_{2L}}}{ds}$. Since $\frac{\gamma}{1-s}\frac{H_{2N}}{H_{2L}} + \alpha\frac{d\frac{H_{2N}}{H_{2L}}}{ds}$ is the largest from $\frac{d\frac{H_{2N}}{H_{2L}}}{ds} < 0$ (Lemma 6) and (A4), $\frac{\gamma}{1-s}\frac{H_{2N}}{H_{2L}} + \alpha\frac{d\frac{H_{2N}}{H_{2L}}}{ds} > 0$. Therefore, c_2 for $a \geq \widehat{a}_0$ decreases with s.

(ii) [Case 3: the indifference condition holds for those with $a \geq e^*_{2N}$**]** In Case 3, as explained in Appendix A, $\frac{H_{2N}}{H_{2L}}$ is determined by (29) as in the unconstrained case. Since c_2 for $a \geq e^*_{2L}$ is given by (30) as in the unconstrained case from Appendix A, Lemma 3 (ii) applies.

Since c_2 for $a < e^*_{2L}$ is given by (A9) in Appendix A,

$$\frac{dc_2}{ds} \propto (1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} + \gamma\frac{\delta_L a}{\bar{l} + \delta_L s a}. \tag{C11}$$

Because $\frac{d\frac{H_{2N}}{H_{2L}}}{ds} < 0$ from Lemma 6, when a is sufficiently small, $\dfrac{dc_2}{ds} < 0$ for any s in this case. For any $a < e^*_{2L}$,

$$(1-\alpha)\left(\tfrac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{\delta_L a}{\bar{l}+\delta_L s a} < (1-\alpha)\left(\tfrac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{\delta_L e^*_{2L}}{\bar{l}+\delta_L s e^*_{2L}}$$

$$=\frac{\frac{1-\alpha}{s}\left\{\frac{1-\gamma-s}{1-s}\left[(1-\gamma)^{1-\gamma}[(1-\alpha)\delta_N(1-s)]^\gamma\left(\tfrac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}}-\left[\alpha(\delta_L s)^\gamma\left(\tfrac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}\right\}}{\alpha\left[(1-\gamma)^{1-\gamma}[(1-\alpha)\delta_N(1-s)]^\gamma\left(\tfrac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}}+(1-\alpha)\left[\alpha(\delta_L s)^\gamma\left(\tfrac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}}+\gamma\frac{\delta_L e^*_{2L}}{\bar{l}+\delta_L s e^*_{2L}} \tag{C12}$$

(from (A20) in the proof of Lemma 3),

where, from (24) and (A17) in the proof of Lemma 3),

$$\frac{\delta_L e^*_{2L}}{\bar{l}+\delta_L s e^*_{2L}}=\frac{\frac{1}{s}\left\{\left[\alpha\gamma\delta_L s {T_2}^\alpha {T_N}^{1-\alpha}\left(\tfrac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}-\bar{l}\right\}}{\left[\alpha\gamma\delta_L s {T_2}^\alpha {T_N}^{1-\alpha}\left(\tfrac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}}$$

$$=\frac{\frac{1}{s}\left[\alpha\gamma\delta_L s {T_2}^\alpha {T_N}^{1-\alpha}\left(\tfrac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}-(1-\gamma)(\gamma^\gamma {T_2}^\alpha {T_N}^{1-\alpha})^{\frac{1}{1-\gamma}}\left\{\left[(1-\alpha)[\delta_N(1-s)]^\gamma\left(\tfrac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}}-\left[\alpha(\delta_L s)^\gamma\left(\tfrac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}\right\}}{\left[\alpha\gamma\delta_L s {T_2}^\alpha {T_N}^{1-\alpha}\left(\tfrac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}}. \tag{C13}$$

Let $B_0\equiv\left[(1-\alpha)[\delta_N(1-s)]^\gamma\left(\tfrac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}}$, $B_1\equiv\left[\alpha(\delta_L s)^\gamma\left(\tfrac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}$, and $B_2\equiv\left[\alpha\gamma\delta_L s {T_2}^\alpha {T_N}^{1-\alpha}\left(\tfrac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}$. By substituting (C13) into (C12), $(1-\alpha)\left(\tfrac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{\delta_L e^*_{2L}}{\bar{l}+\delta_L s e^*_{2L}}$ is proportional to

$$\begin{aligned}
& B_2\tfrac{1-\alpha}{s}\left(\tfrac{1-\gamma-s}{1-s}B_0-B_1\right)+\gamma[\alpha B_0+(1-\alpha)B_1]\left[\tfrac{1}{s}B_2-(1-\gamma)(\gamma^\gamma {T_2}^\alpha {T_N}^{1-\alpha})^{\frac{1}{1-\gamma}}(B_0-B_1)\right]\\
=\ & \tfrac{\gamma}{s}B_2\left[-\tfrac{1-\alpha}{1-s}B_0+\alpha B_0+(1-\alpha)B_1\right]+(B_0-B_1)\left\{B_2\tfrac{1-\alpha}{s}-(1-\gamma)(\gamma^\gamma {T_2}^\alpha {T_N}^{1-\alpha})^{\frac{1}{1-\gamma}}\gamma\,[\alpha B_0+(1-\alpha)B_1]\right\}\\
=\ & (\gamma {T_2}^\alpha {T_N}^{1-\alpha})^{\frac{1}{1-\gamma}}\left\{\gamma B_1\left[-\left(\tfrac{1-\alpha}{1-s}-1\right)B_0-(1-\alpha)(B_0-B_1)\right]+(B_0-B_1)[\gamma(1-\alpha)B_1-(1-\gamma)\alpha B_0]\right\}\\
=\ & -(\gamma {T_2}^\alpha {T_N}^{1-\alpha})^{\frac{1}{1-\gamma}}\tfrac{1}{1-s}B_0[\alpha(1-\gamma)(1-s)\,(B_0-B_1)+\gamma\,(s-\alpha)\,B_1],
\end{aligned} \tag{C14}$$

where the last two equalities are from $B_2=\delta_L s(\gamma {T_2}^\alpha {T_N}^{1-\alpha})^{\frac{1}{1-\gamma}}B_1$. Noting that the expression inside the square bracket of (C14) is same as that of (A21) in the proof of Lemma 3 (ii), the proof of the lemma applies.

Hence, $\dfrac{dc_2}{ds}<0$ when $s\geq\alpha$ (also when s is close to 0 or $s<\alpha$ and close to α), and $\dfrac{dc_2}{ds}<0$ for any s in this case when $a(<e^*_{2L})$ is sufficiently small or when T_N, T_2, δ_N, and δ_L are sufficiently low that $(1-\alpha)\left(\tfrac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{\delta_L e^*_{2L}}{\bar{l}+\delta_L s e^*_{2L}}\equiv G\leq 0$ for any s. Further, when T_N, T_2, δ_N, and δ_L

are sufficiently large that $G > 0$ and thus $\dfrac{dc_2}{ds} > 0$ hold for not very small and not large s (Figure A2) when $a \geq e^*_{2L}$, $\dfrac{dc_2}{ds} > 0$ holds for such range of s when $a < e^*_{2L}$ as well, if a is sufficiently large that $(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} + \gamma\frac{\delta_L a}{\bar{l}+\delta_L sa} = G - \gamma\left(\frac{\delta_L e^*_{2L}}{\bar{l}+\delta_L se^*_{2L}} - \frac{\delta_L a}{\bar{l}+\delta_L sa}\right) > 0$.

[Case 4: the indifference condition holds for $a = \widehat{a} \in [e^*_{2L}, e^*_{2N})$**]**

(Results for $a \geq \widehat{a}$**)** From (30) and (A6) in Appendix A, $\dfrac{dc_2}{ds}$ for $a \geq \widehat{a}$ is proportional to $-\left[\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right]$. In the following, it is proved that $\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} > 0$ and thus $\dfrac{dc_2}{ds} < 0$ for $a \geq \widehat{a}$, when $s \geq \frac{\alpha}{\alpha+(1-\alpha)\gamma}$ or when T_N, T_2, and δ_N are sufficiently low. It is also proved that there exist ranges of s $(\leq \alpha)$ satisfying $\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} < 0$ and thus $\dfrac{dc_2}{ds} > 0$ for $a \geq \widehat{a}$, when T_N, T_2, and δ_N are sufficiently high.

Totally differentiating (A10) in Appendix A, one of the two equations determining $\widehat{a}$ and $\frac{H_{2N}}{H_{2L}}$, gives

$$-\frac{\gamma}{1-s}\left\{\frac{1-s\gamma}{s}\left[(\gamma\delta_L s)^\gamma\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{(1+\gamma)s-1}{s\gamma}\frac{\bar{l}}{\delta_L s}+\widehat{a}\right\}ds$$
$$-\alpha\left\{\frac{1-\alpha\gamma}{\alpha}\left[(\gamma\delta_L s)^\gamma\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{\bar{l}}{\delta_L s}+\widehat{a}\right\}\frac{d\frac{H_{2N}}{H_{2L}}}{\frac{H_{2N}}{H_{2L}}}+\frac{\gamma}{\widehat{a}}\left\{(1-\gamma)\left[(\gamma\delta_L s)^\gamma\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{\bar{l}}{\delta_L s}-\frac{1-\gamma}{\gamma}\widehat{a}\right\}d\widehat{a}=0, \tag{C15}$$

where (A10) is used to derive the term associated with ds and the expression associated with $d\widehat{a}$ is positive from (A10) and $\widehat{a} < e^*_{2N}$.

This equation can be expressed as

$$-\left\{\frac{1-\alpha\gamma}{\alpha}\left[(\gamma\delta_L s)^\gamma\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{\bar{l}}{\delta_L s}+\widehat{a}\right\}\left[\frac{\gamma}{1-s}\frac{\frac{1-s\gamma}{s}\left[(\gamma\delta_L s)^\gamma\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{(1+\gamma)s-1}{s\gamma}\frac{\bar{l}}{\delta_L s}+\widehat{a}}{\frac{1-\alpha\gamma}{\alpha}\left[(\gamma\delta_L s)^\gamma\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{\bar{l}}{\delta_L s}+\widehat{a}}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right]$$
$$+\frac{\gamma}{\widehat{a}}\left\{(1-\gamma)\left[(\gamma\delta_L s)^\gamma\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{\bar{l}}{\delta_L s}-\frac{1-\gamma}{\gamma}\widehat{a}\right\}\frac{d\widehat{a}}{ds}=0. \tag{C16}$$

Totally differentiating (A11) in Appendix A, the other equation determining $\widehat{a}$ and $\frac{H_{2N}}{H_{2L}}$, and dividing the resulting equation by ds gives

$$A_{\widehat{a}}\frac{d\widehat{a}}{ds}=-\left\{\begin{array}{c}\frac{H_{2N}}{H_{2L}}\left[\frac{\gamma}{1-s}+\gamma\delta_L\frac{(\bar{l}+\delta_L se^*_{2L})^{\gamma-1}e^*_{2L}(F(\widehat{a})-F(e^*_{2L}))+\int_0^{e^*_{2L}}(\bar{l}+\delta_L sa)^{\gamma-1}adF(a)}{(\bar{l}+\delta_L se^*_{2L})^\gamma(F(\widehat{a})-F(e^*_{2L}))+\int_0^{e^*_{2L}}(\bar{l}+\delta_L sa)^\gamma dF(a)+(\bar{l})^\gamma F(0)}\right]+\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\\ -\frac{\gamma}{e^*_{2N}}\frac{H_{2N}}{H_{2L}}\frac{(e^*_{2N})^\gamma(1-F(e^*_{2N}))}{(e^*_{2N})^\gamma(1-F(e^*_{2N}))+\int_{\widehat{a}}^{e^*_{2N}}a^\gamma dF(a)}\frac{de^*_{2N}}{ds}\\ +\frac{\gamma\delta_L s}{\bar{l}+\delta_L se^*_{2L}}\frac{H_{2N}}{H_{2L}}\frac{(\bar{l}+\delta_L se^*_{2L})^\gamma(F(\widehat{a})-F(e^*_{2L}))}{(\bar{l}+\delta_L se^*_{2L})^\gamma(F(\widehat{a})-F(e^*_{2L}))+\int_0^{e^*_{2L}}(\bar{l}+\delta_L sa)^\gamma dF(a)+(\bar{l})^\gamma F(0)}\frac{de^*_{2L}}{ds}\end{array}\right\}$$

$$=-\frac{H_{2N}}{H_{2L}}\left[\begin{array}{c}\frac{1}{\alpha}\left\{\alpha\left[\frac{\gamma}{1-s}+\gamma\delta_L\frac{(\bar{l}+\delta_L se_{2L}^*)^{\gamma-1}e_{2L}^*(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma-1}adF(a)}{(\bar{l}+\delta_L se_{2L}^*)^{\gamma}(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\right]+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right\}\\ +\frac{\gamma}{1-\gamma}\frac{(e_{2N}^*)^{\gamma}(1-F(e_{2N}^*))}{(e_{2N}^*)^{\gamma}(1-F(e_{2N}^*))+\int_{\widehat{a}}^{e_{2N}^*}a^{\gamma}dF(a)}\left[\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right]\\ +\frac{\gamma}{1-\gamma}\frac{1-\alpha}{\alpha}\frac{(\bar{l}+\delta_L se_{2L}^*)^{\gamma}(F(\widehat{a})-F(e_{2L}^*))}{(\bar{l}+\delta_L se_{2L}^*)^{\gamma}(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\\ \times\left(\frac{\alpha}{1-\alpha}\frac{1-\gamma}{s}\frac{\frac{\gamma}{1-\gamma}\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\bar{l}}{\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right)\end{array}\right], \tag{C17}$$

where, $A_{\widehat{a}}$ is a positive term, and, to derive the last equality, the following equations and (24) are used.

$$\frac{de_{2N}^*}{ds}=-\left[\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right]\frac{e_{2N}^*}{1-\gamma}\quad\text{(from (21))}, \tag{C18}$$

$$\begin{aligned}\frac{de_{2L}^*}{ds}&=\frac{1}{\delta_L s}\left(\frac{1}{s}\left\{\frac{\gamma}{1-\gamma}\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\bar{l}\right\}+\frac{1-\alpha}{1-\gamma}\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right)\text{ (from (24))},\\ &=\frac{1}{\alpha\delta_L s}\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}\frac{1-\alpha}{1-\gamma}\left\{\frac{\alpha}{1-\alpha}\frac{1-\gamma}{s}\frac{\frac{\gamma}{1-\gamma}\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\bar{l}}{\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right\}.\end{aligned} \tag{C19}$$

From the equation that is obtained by substituting (C16) into (C17) and eliminating $\frac{d\widehat{a}}{ds}$, $\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}>0$ if $\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}$ is higher than other similar expressions in the equation. In the following, it is proved that this is the case when $s\geq\frac{\alpha}{\alpha+(1-\alpha)\gamma}$ or when T_N, T_2, and δ_N are sufficiently low.

$$\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\geq\frac{\gamma}{1-s}\frac{\frac{1-s\gamma}{s}\left[(\gamma\delta_L s)^{\gamma}\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{(1+\gamma)s-1}{s\gamma}\frac{\bar{l}}{\delta_L s}+\widehat{a}}{\frac{1-\alpha\gamma}{\alpha}\left[(\gamma\delta_L s)^{\gamma}\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{\bar{l}}{\delta_L s}+\widehat{a}}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\text{ iff}$$

$$\frac{\frac{1-s\gamma}{s}\left[(\gamma\delta_L s)^{\gamma}\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{(1+\gamma)s-1}{s\gamma}\frac{\bar{l}}{\delta_L s}+\widehat{a}}{\frac{1-\alpha\gamma}{\alpha}\left[(\gamma\delta_L s)^{\gamma}\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{\bar{l}}{\delta_L s}+\widehat{a}}\leq 1$$

$$\Leftrightarrow\left(\frac{1}{s}-\frac{1}{\alpha}\right)\left[(\gamma\delta_L s)^{\gamma}\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}-\frac{1-s}{s\gamma}\frac{\bar{l}}{\delta_L s}\leq 0$$

$$\Leftrightarrow(\alpha-s)J-\alpha(1-s)\bar{l}\leq 0,\ \text{ where } J\equiv\left[\gamma\delta_L s\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}} \tag{C20}$$

$$\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\geq\alpha\left[\frac{\gamma}{1-s}+\gamma\delta_L\frac{(\bar{l}+\delta_L se_{2L}^*)^{\gamma-1}e_{2L}^*(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma-1}adF(a)}{(\bar{l}+\delta_L se_{2L}^*)^{\gamma}(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\right]+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}$$

iff

$$\alpha\left[\frac{\gamma}{1-s}+\gamma\delta_L\frac{(\bar{l}+\delta_L se_{2L}^*)^{\gamma-1}e_{2L}^*(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma-1}adF(a)}{(\bar{l}+\delta_L se_{2L}^*)^{\gamma}(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\right]\leq\frac{\gamma}{1-s}$$

$$\Leftrightarrow\delta_L\frac{(\bar{l}+\delta_L se_{2L}^*)^{\gamma-1}e_{2L}^*(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma-1}adF(a)}{(\bar{l}+\delta_L se_{2L}^*)^{\gamma}(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\leq\frac{1-\alpha}{\alpha}\frac{1}{1-s}$$

$$\Leftrightarrow(\bar{l}+\delta_L se_{2L}^*)^{\gamma-1}\left[\alpha(1-s)\delta_L e_{2L}^*-(1-\alpha)(\bar{l}+\delta_L se_{2L}^*)\right](F(\widehat{a})-F(e_{2L}^*))$$

$$+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma-1}\left[\alpha(1-s)\delta_L a-(1-\alpha)(\bar{l}+\delta_L sa)\right]dF(a)-(1-\alpha)(\bar{l})^{\gamma}F(0)\leq 0$$

$$\Leftrightarrow(\bar{l}+\delta_L se_{2L}^*)^{\gamma-1}\left[(\alpha-s)\delta_L e_{2L}^*-(1-\alpha)\bar{l}\right](F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma-1}\left[(\alpha-s)\delta_L a-(1-\alpha)\bar{l}\right]dF(a)-(1-\alpha)(\bar{l})^{\gamma}F(0)\leq 0$$

$$\Leftrightarrow(\bar{l}+\delta_L se_{2L}^*)^{\gamma-1}\frac{1}{s}\left\{(\alpha-s)J-\alpha(1-s)\bar{l}\right\}(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma-1}\left[(\alpha-s)\delta_L a-(1-\alpha)\bar{l}\right]dF(a)-(1-\alpha)(\bar{l})^{\gamma}F(0)\leq 0, \tag{C21}$$

where (24) is used to derive the last equation, and, as for the second term, $(\alpha-s)\delta_L a-(1-\alpha)\bar{l}\leq(\alpha-s)\frac{1}{s}(J-\bar{l})-(1-\alpha)\bar{l}=\frac{1}{s}\left[(\alpha-s)J-\alpha(1-s)\bar{l}\right]$ from (24).

$$\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\geq\frac{\alpha}{1-\alpha}\frac{1-\gamma}{s}\frac{\frac{\gamma}{1-\gamma}\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\bar{l}}{\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\text{ iff}$$

$$\frac{\alpha}{1-\alpha}\frac{1-\gamma}{s}\frac{\frac{\gamma}{1-\gamma}\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\bar{l}}{\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}}\leq\frac{\gamma}{1-s}$$

$$\Leftrightarrow\gamma\left(\frac{\alpha}{1-\alpha}\frac{1}{s}-\frac{1}{1-s}\right)\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{\alpha}{1-\alpha}\frac{1-\gamma}{s}\bar{l}\leq 0$$

$$\Leftrightarrow\gamma(\alpha-s)J+\alpha(1-s)(1-\gamma)\bar{l}\leq 0. \tag{C22}$$

From (C20)−(C22), $\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}$ is higher than the other expressions if (C22) holds. Since $e_{2L}^*>0\Leftrightarrow J>\bar{l}$ from (24), this is true if $\gamma(\alpha-s)+\alpha(1-s)(1-\gamma)\leq 0\Leftrightarrow s\geq\frac{\alpha}{\alpha+(1-\alpha)\gamma}$. Further, (C22) is true for $s>\alpha$ when T_N, T_2, δ_N and δ_L are sufficiently low from the next lemma.

Lemma C1 *(i)* $T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}$ *increases with* T_N, T_2, *and* δ_N. *(ii)* $\frac{H_{2N}}{H_{2L}}$ *decreases with* δ_L.

Proof. (i) Suppose the contrary. Then, an increase in T_N, T_2, or δ_N lowers $T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}$, which implies that $\frac{H_{2N}}{H_{2L}}$ decreases. Then, $\widehat{a}$ must decrease, since (A10) in Appendix A can be expressed as follows.

$$(1-\alpha)T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}(\delta_N(1-s)\widehat{a})^{\gamma}-\widehat{a}=(1-\gamma)\left[(\gamma\delta_L s)^{\gamma}\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{\bar{l}}{\delta_L s}. \tag{C23}$$

Because a decrease in $T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}$ lowers e_{2L}^* from (24) and a decrease in $\frac{H_{2N}}{H_{2L}}$ and an increase in T_N, T_2, or δ_N raises e_{2N}^* from (21), for (A11) to hold, $\widehat{a}$ must increase, a contradiction. Therefore, $T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}$ increases with T_N, T_2, and δ_N.

(ii) The result holds because for given $\widehat{a}$, an increase in δ_L lowers $\frac{H_{2N}}{H_{2L}}$ satisfying (C23) (the LHS of the equation increases with δ_L) and $\frac{H_{2N}}{H_{2L}}$ satisfying (A11). ■

Therefore, $\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} > 0$ and thus $\dfrac{dc_2}{ds} < 0$ for $a \geq \widehat{a}$ when $s \geq \frac{\alpha}{\alpha+(1-\alpha)\gamma}$, and if T_N, T_2, δ_N and δ_L are sufficiently low, when $s > \alpha$. $\gamma(\alpha-s)J+\alpha(1-s)(1-\gamma)\bar{l} \leq 0$ when δ_L is sufficiently low, because $\gamma(\alpha-s)J+\alpha(1-s)(1-\gamma)\bar{l} = (\alpha-s)\left(\delta_L s e_{2L}^*+\bar{l}\right)-\alpha(1-s)\bar{l} < (\alpha-s)(\delta_L s e_{2N}^*+\bar{l})-\alpha(1-s)\bar{l}$, where e_{2N}^* increases with δ_L from (21) and Lemma C1.

Similarly, $\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} < 0$ if $\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}$ is smaller than other expressions in the equation obtained by substituting (C16) into (C17), which is the case when (C21) holds with ">". Noting that $e_{2L}^* = \frac{1}{\delta_L s}(J-\bar{l})$ from (24) and (C21) holds with ">" only if $s < \alpha$, the LHS of (C21) increases with J, because the derivative of the LHS of the equation with respect to J is proportional to $-\frac{(1-\gamma)}{J}\left\{(\alpha-s)J-\alpha(1-s)\bar{l}\right\}+(\alpha-s) > 0$. Therefore, from Lemma C1, there exist ranges of s ($< \alpha$) satisfying $\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} < 0$ and thus $\dfrac{dc_2}{ds} > 0$ for $a \geq \widehat{a}$, when T_N, T_2, and δ_N are sufficiently high.

(Results for $a < e_{2L}^*$) From (A9) in Appendix A, $\dfrac{dc_2}{ds}$ for $a < e_{2L}^*$ is proportional to $(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{\delta_L a}{\bar{l}+\delta_L s a}$. Since $\frac{d\frac{H_{2N}}{H_{2L}}}{ds} < 0$ from Lemma 6, $\dfrac{dc_2}{ds}<0$ for any s in this case when a is sufficiently small.

For any $a < e_{2L}^*$,

$$(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{\delta_L a}{\bar{l}+\delta_L s a} < (1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{\delta_L e_{2L}^*}{\bar{l}+\delta_L s e_{2L}^*}. \tag{C24}$$

In the following, it is proved that $(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{\delta_L e_{2L}^*}{\bar{l}+\delta_L s e_{2L}^*} < 0$ and thus $\dfrac{dc_2}{ds} < 0$, when $s \geq \frac{1}{2-\alpha}$ or when T_N, T_2, δ_N and δ_L are sufficiently low.

When $s \geq \alpha$ or when T_N, T_2, δ_N and δ_L are low enough that $(\alpha-s)\left[\gamma\delta_L s\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}-\alpha(1-s)\bar{l} < 0$ holds (Lemma C1),[C1] $\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} > \frac{\gamma}{1-s}\dfrac{\frac{1-s\gamma}{s}\left[(\gamma\delta_L s)^{\gamma}\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{(1+\gamma)s-1}{s\gamma}\frac{\bar{l}}{\delta_L s}+\widehat{a}}{\frac{1-\alpha\gamma}{\alpha}\left[(\gamma\delta_L s)^{\gamma}\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{\bar{l}}{\delta_L s}+\widehat{a}}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}$ from (C20).

$$\frac{\gamma}{1-s}\frac{\frac{1-s\gamma}{s}\left[(\gamma\delta_L s)^{\gamma}\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{(1+\gamma)s-1}{s\gamma}\frac{\bar{l}}{\delta_L s}+\widehat{a}}{\frac{1-\alpha\gamma}{\alpha}\left[(\gamma\delta_L s)^{\gamma}\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{\bar{l}}{\delta_L s}+\widehat{a}} > \alpha\left[\frac{\gamma}{1-s}+\gamma\delta_L\frac{(\bar{l}+\delta_L s e_{2L}^*)^{\gamma-1}e_{2L}^*(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L s a)^{\gamma-1}a\,dF(a)}{(\bar{l}+\delta_L s e_{2L}^*)^{\gamma}(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L s a)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\right]$$

holds too, because ($J\equiv\left[\gamma\delta_L s\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}$)

[C1] The inequality holds when δ_L is sufficiently low, because for $s < \alpha$, $(\alpha-s)\left[\gamma\delta_L s\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}-\alpha(1-s)\bar{l} = (\alpha-s)\left(\delta_L s e_{2L}^*+\bar{l}\right)-\alpha(1-s)\bar{l} < (\alpha-s)(\delta_L s e_{2N}^*+\bar{l})-\alpha(1-s)\bar{l}$, where e_{2N}^* increases with δ_L from (21) and Lemma C1.

$$\frac{\gamma}{1-s}\frac{\frac{1-s\gamma}{s}\frac{J}{\gamma\delta_L s}+\frac{(1+\gamma)s-1}{s\gamma}\frac{\bar{l}}{\delta_L s}+\widehat{a}}{\frac{1-\alpha\gamma}{\alpha}\frac{J}{\gamma\delta_L s}+\frac{\bar{l}}{\delta_L s}+\widehat{a}} > \alpha\left[\frac{\gamma}{1-s}+\gamma\delta_L\frac{(\bar{l}+\delta_L se_{2L}^*)^{\gamma-1}e_{2L}^*(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma-1}adF(a)}{(\bar{l}+\delta_L se_{2L}^*)^{\gamma}(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\right]$$

$$\Leftrightarrow \frac{1}{1-s}\frac{(1-s\gamma)\frac{J}{\gamma\delta_L s}+\frac{(1+\gamma)s-1}{\gamma}\frac{\bar{l}}{\delta_L s}+s\widehat{a}}{\frac{1-\alpha\gamma}{\alpha}\frac{J}{\gamma\delta_L s}+\frac{\bar{l}}{\delta_L s}+\widehat{a}} > \alpha\left[\frac{s}{1-s}+\frac{(\bar{l}+\delta_L se_{2L}^*)^{\gamma-1}\delta_L se_{2L}^*(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma-1}\delta_L sadF(a)}{(\bar{l}+\delta_L se_{2L}^*)^{\gamma}(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\right]$$

$$\Leftarrow \frac{1}{1-s}\frac{(1-s\gamma)\frac{J}{\gamma\delta_L s}+\frac{(1+\gamma)s-1}{\gamma}\frac{\bar{l}}{\delta_L s}+s\widehat{a}}{\frac{1-\alpha\gamma}{\alpha}\frac{J}{\gamma\delta_L s}+\frac{\bar{l}}{\delta_L s}+\widehat{a}} \geq \alpha\left(\frac{s}{1-s}+\frac{\delta_L se_{2L}^*}{\bar{l}+\delta_L se_{2L}^*}\right) = \alpha\left(\frac{s}{1-s}+\frac{J-\bar{l}}{J}\right) \text{ (from (24))}$$

$$\Leftrightarrow \frac{1}{1-s}\left\{\left[(1-s\gamma)\frac{J}{\gamma\delta_L s}+\frac{(1+\gamma)s-1}{\gamma}\frac{\bar{l}}{\delta_L s}+s\widehat{a}\right]-\left[(1-\alpha\gamma)\frac{J}{\gamma\delta_L s}+\alpha\frac{\bar{l}}{\delta_L s}+\alpha\widehat{a}\right]\right\} \geq -\frac{(1-\alpha\gamma)\frac{J}{\gamma\delta_L s}+\alpha\frac{\bar{l}}{\delta_L s}+\alpha\widehat{a}}{J}\bar{l}$$

$$\Leftrightarrow (\widehat{a}-e_{2L}^*)\left[(\alpha-s)J-\alpha(1-s)\bar{l}\right] \leq 0 \text{ (from (24)).} \tag{C25}$$

Hence, either $\alpha\left[\frac{\gamma}{1-s}+\gamma\delta_L\frac{(\bar{l}+\delta_L se_{2L}^*)^{\gamma-1}e_{2L}^*(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma-1}adF(a)}{(\bar{l}+\delta_L se_{2L}^*)^{\gamma}(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\right]+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}$ or $\frac{\alpha}{1-\alpha}\frac{1-\gamma}{s}\frac{\frac{\gamma}{1-\gamma}\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\bar{l}}{\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}$ is lowest among the terms of the equation obtained by substituting (C16) into (C17). From the equation, the lowest term must be negative.

If the latter is lowest, $\frac{\alpha}{1-\alpha}\frac{1-\gamma}{s}\frac{\frac{\gamma}{1-\gamma}\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\bar{l}}{\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}<0$. Thus,

$$\begin{aligned}(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{\delta_L e_{2L}^*}{\bar{l}+\delta_L se_{2L}^*} &< -\frac{1-\gamma}{s}\frac{\frac{\gamma}{1-\gamma}J+\bar{l}}{J}+\gamma\frac{\frac{1}{s}\left(J-\bar{l}\right)}{J}\\ &= -\frac{\frac{\bar{l}}{s}}{J}<0.\end{aligned}$$

Otherwise, $\alpha\left[\frac{\gamma}{1-s}+\gamma\delta_L\frac{(\bar{l}+\delta_L se_{2L}^*)^{\gamma-1}e_{2L}^*(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma-1}adF(a)}{(\bar{l}+\delta_L se_{2L}^*)^{\gamma}(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\right]+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}<0$. Thus,

$$\begin{aligned}(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{\delta_L e_{2L}^*}{\bar{l}+\delta_L se_{2L}^*} &< -(1-\alpha)\left[\frac{\gamma}{1-s}+\gamma\delta_L\frac{(\bar{l}+\delta_L se_{2L}^*)^{\gamma-1}e_{2L}^*(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma-1}adF(a)}{(\bar{l}+\delta_L se_{2L}^*)^{\gamma}(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\right]+\frac{\gamma}{s}\frac{J-\bar{l}}{J}\\ &< -\gamma\left(\frac{1-\alpha}{1-s}-\frac{1}{s}\right)-\frac{\gamma}{s}\frac{\bar{l}}{J}\\ &= \frac{\gamma}{s(1-s)J}\left\{[1-(2-\alpha)s]J-(1-s)\bar{l}\right\},\end{aligned}$$

which is negative when $s \geq \frac{1}{2-\alpha}$ or when T_N, T_2, δ_N and δ_L are sufficiently low (Lemma C1).

Therefore, $(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{\delta_L e_{2L}^*}{\bar{l}+\delta_L se_{2L}^*}<0$ and thus $\frac{dc_2}{ds}<0$ for $a<e_{2L}^*$ when $s \geq \frac{1}{2-\alpha}$ or when T_N, T_2, δ_N and δ_L are sufficiently low.

(Results for $a \in [e_{2L}^*, \widehat{a})$**)** Finally, from (A12) in Appendix A, $\frac{dc_2}{ds}<0$ for $a \in [e_{2L}^*, \widehat{a})$ if $\left[(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\frac{\gamma}{s}\right]\frac{J}{\gamma}-\frac{\bar{l}}{s}<0$. The result can be proved following a similar step as the above proof of $\frac{dc_2}{ds} < 0$ for $a < e^*_{2L}$. In particular, when $\frac{\alpha}{1-\alpha}\frac{1-\gamma}{s}\frac{\frac{\gamma}{1-\gamma}\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\bar{l}}{\left[\alpha\gamma\delta_L sT_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}} + \alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} < 0$,

$$(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\frac{\gamma}{s} < -\frac{1-\gamma}{s}\frac{\frac{\gamma}{1-\gamma}J+\bar{l}}{J}+\frac{\gamma}{s} = -\frac{1-\gamma}{s}\frac{\bar{l}}{J} < 0,$$

and when $\alpha\left[\frac{\gamma}{1-s}+\gamma\delta_L\frac{(\bar{l}+\delta_L se^*_{2L})^{\gamma-1}e^*_{2L}(F(\widehat{a})-F(e^*_{2L}))+\int_0^{e^*_{2L}}(\bar{l}+\delta_L sa)^{\gamma-1}adF(a)}{(\bar{l}+\delta_L se^*_{2L})^{\gamma}(F(\widehat{a})-F(e^*_{2L}))+\int_0^{e^*_{2L}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\right]+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} < 0$,

$$\begin{aligned}\left[(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\frac{\gamma}{s}\right]\frac{J}{\gamma}-\frac{\bar{l}}{s} &< \left\{-(1-\alpha)\left[\frac{1}{1-s}+\delta_L\frac{(\bar{l}+\delta_L se^*_{2L})^{\gamma-1}e^*_{2L}(F(\widehat{a})-F(e^*_{2L}))+\int_0^{e^*_{2L}}(\bar{l}+\delta_L sa)^{\gamma-1}adF(a)}{(\bar{l}+\delta_L se^*_{2L})^{\gamma}(F(\widehat{a})-F(e^*_{2L}))+\int_0^{e^*_{2L}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\right]+\frac{1}{s}\right\}J-\frac{\bar{l}}{s}\\ &< \frac{1}{s}\left\{\frac{1}{1-s}\left[-(2-\alpha)s+1\right]J-\bar{l}\right\},\end{aligned} \tag{C26}$$

which is negative when $s \geq \frac{1}{2-\alpha}$ or when T_N, T_2, δ_N, δ_L are sufficiently low from Lemma C1.

[Case 5: the indifference condition holds for $a = \widetilde{a} < e^*_{2L}$]

(Results for $a \geq \widetilde{a}$) From (30) and (A6) in Appendix A, $\frac{dc_2}{ds}$ for $a \geq \widetilde{a}$ is proportional to $-\left[\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right]$. In the following, it is proved that $\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} > 0$ and thus $\frac{dc_2}{ds} < 0$ holds for $a \geq \widetilde{a}$, when $s \geq \alpha$ or when T_N, T_2, δ_N and δ_L are sufficiently small, and $\frac{dc_2}{ds} > 0$ holds for not large s ($< \alpha$) when T_N, T_2, and δ_N are sufficiently large.

In order to prove the result, the following lemma is used.

Lemma C2 *(i) $\frac{H_{2N}}{H_{2L}}$ and $\widetilde{a}$ increase with T_N, T_2, and δ_N. (ii) $\frac{H_{2N}}{H_{2L}}$ decreases with δ_L.*

Proof. (i) Suppose the contrary. Then, an increase in T_N, T_2, or δ_N lowers $\frac{H_{2N}}{H_{2L}}$, which implies that $\widetilde{a}$ decreases from (A13) in Appendix A. Because an increase in T_N, T_2, or δ_N together with a decrease in $\frac{H_{2N}}{H_{2L}}$ raises e^*_{2N} from (21), for (A14) in the appendix to hold, $\widetilde{a}$ must increase, a contradiction. Therefore, $\frac{H_{2N}}{H_{2L}}$ and $\widetilde{a}$ increase with T_N, T_2, and δ_N.

(ii) The result holds because for given $\widetilde{a}$, an increase in δ_L lowers both $\frac{H_{2N}}{H_{2L}}$ satisfying (A13) and $\frac{H_{2N}}{H_{2L}}$ satisfying (A14). ■

Totally differentiating (A13), one of the two equations determining $\widetilde{a}$ and $\frac{H_{2N}}{H_{2L}}$, gives

$$\gamma\frac{l}{\widetilde{a}(\bar{l}+\delta_L s\widetilde{a})}\frac{H_{2N}}{H_{2L}}d\widetilde{a} = \frac{\gamma}{1-s}\frac{l+\delta_L\widetilde{a}}{\bar{l}+\delta_L s\widetilde{a}}\frac{H_{2N}}{H_{2L}}ds + d\frac{H_{2N}}{H_{2L}}. \tag{C27}$$

Totally differentiating (A14), the other equation determining $\widetilde{a}$ and $\frac{H_{2N}}{H_{2L}}$, gives

$$\begin{aligned}&\gamma\left(\frac{1}{1-s}+\frac{1}{s}\frac{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma-1}\delta_L sadF(a)}{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\right)\frac{H_{2N}}{H_{2L}}ds + d\frac{H_{2N}}{H_{2L}} - \frac{[\delta_N(1-s)]^{\gamma}\gamma e^*_{2N}{}^{\gamma-1}(1-F(e^*_{2N}))}{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}de^*_{2N}\\ &+\frac{[\delta_N(1-s)]^{\gamma}\left\{\left[\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)\right]\widetilde{a}^{\gamma}+\left[e^*_{2N}{}^{\gamma}(1-F(e^*_{2N}))+\int_{\widetilde{a}}^{e^*_{2N}}a^{\gamma}dF(a)\right](\bar{l}+\delta_L s\widetilde{a})^{\gamma}\right\}dF(\widetilde{a})}{\left[\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)\right]^2}d\widetilde{a} = 0,\end{aligned} \tag{C28}$$

where, from (C10),

$$de_{2N}^* = -\left[\frac{\gamma}{1-s}ds + \alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1} d\frac{H_{2N}}{H_{2L}}\right]\frac{e_{2N}^*}{1-\gamma}. \tag{C29}$$

If the first and third equations are substituted into the second one and divided by ds, the resulting equation consists of the term associated with $\frac{\gamma}{1-s}\frac{\bar{l}+\delta_L\widetilde{a}}{\bar{l}+\delta_L s\widetilde{a}}\frac{H_{2N}}{H_{2L}} + \frac{d\frac{H_{2N}}{H_{2L}}}{ds} = \gamma\left(\frac{1}{1-s}+\frac{\delta_L\widetilde{a}}{\bar{l}+\delta_L s\widetilde{a}}\right)\frac{H_{2N}}{H_{2L}} + \frac{d\frac{H_{2N}}{H_{2L}}}{ds}$, the one associated with $\gamma\left(\frac{1}{1-s}+\frac{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma-1}\delta_L a dF(a)}{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\right)\frac{H_{2N}}{H_{2L}} + \frac{d\frac{H_{2N}}{H_{2L}}}{ds}$, and the one associated with $\frac{\gamma}{1-s} + \alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} = \alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\left(\frac{1}{\alpha}\frac{\gamma}{1-s}\frac{H_{2N}}{H_{2L}} + \frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right)$. The first expression is greater than the second one because

$$\frac{\widetilde{a}}{\bar{l}+\delta_L s\widetilde{a}} > \frac{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma-1}a dF(a)}{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)} \Leftrightarrow \int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma-1}(\widetilde{a}-a)\bar{l}dF(a)+(\bar{l})^{\gamma}F(0)\widetilde{a} > 0.$$

Hence, when $\frac{1}{\alpha}\frac{\gamma}{1-s}\frac{H_{2N}}{H_{2L}} \geq \frac{\gamma}{1-s}\frac{\bar{l}+\delta_L\widetilde{a}}{\bar{l}+\delta_L s\widetilde{a}}\frac{H_{2N}}{H_{2L}} \Leftrightarrow (\alpha-s)\delta_L\widetilde{a} \leq (1-\alpha)\bar{l}$, $\frac{\gamma}{1-s} + \alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} > 0$. $(\alpha-s)\delta_L\widetilde{a} \leq (1-\alpha)\bar{l}$ holds when $s \geq \alpha$ or when

$$(\alpha-s)\delta_L e_{2L}^* \leq (1-\alpha)\bar{l}$$

$$\Leftrightarrow (\alpha-s)J - \alpha(1-s)\bar{l} \leq 0 \text{ (from (24)), where } J \equiv \left[\gamma\delta_L s\alpha {T_2}^{\alpha} {T_N}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}},$$

which is true when T_N, T_2, and δ_N are sufficiently small from Lemma C2.

$(\alpha-s)\delta_L\widetilde{a} \leq (1-\alpha)\bar{l}$ holds when δ_L is sufficiently small as well, because it is true if $(\alpha-s)\delta_L e_{2N}^* \leq (1-\alpha)\bar{l}$, where e_{2N}^* decreases with $\frac{H_{2N}}{H_{2L}}$ and $\frac{H_{2N}}{H_{2L}}$ decreases with δ_L from Lemma C2.

From the above analysis, when $\frac{1}{1-s}+\frac{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma-1}\delta_L a dF(a)}{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)} \geq \frac{1}{\alpha}\frac{1}{1-s} \Leftrightarrow \frac{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma-1}\delta_L a dF(a)}{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)} \geq \frac{1-\alpha}{\alpha}\frac{1}{1-s}$, $\frac{\gamma}{1-s} + \alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} < 0$ and thus $\frac{dc_2}{ds} > 0$. Because $\frac{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma-1}\delta_L a dF(a)}{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}$ increases with $\widetilde{a}$ from

$$\left[\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)\right](\bar{l}+\delta_L s\widetilde{a})^{\gamma-1}\delta_L\widetilde{a}dF(\widetilde{a}) - \int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma-1}\delta_L a dF(a)(\bar{l}+\delta_L s\widetilde{a})^{\gamma}dF(\widetilde{a})$$
$$= (\bar{l}+\delta_L s\widetilde{a})^{\gamma-1}dF(\widetilde{a})\delta_L\left\{\left[\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)\right]\widetilde{a} - \left[\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma-1}a dF(a)\right](\bar{l}+\delta_L s\widetilde{a}\right\} > 0,$$

the inequality holds when T_N, T_2, and δ_N are sufficiently large from Lemma C2. The inequality and $\frac{dc_2}{ds} > 0$ could hold only for $s < \alpha$, since $\frac{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma-1}\delta_L a dF(a)}{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)} = \frac{1}{s}\frac{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma-1}\delta_L sa dF(a)}{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)} < \frac{1}{s}$.

(Results for $a < \widetilde{a}$) From (A9) in Appendix A, $\frac{dc_2}{ds}$ for $a < \widetilde{a}$ is proportional to $(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} + \gamma\frac{\delta_L a}{\bar{l}+\delta_L sa}$. Since $\frac{d\frac{H_{2N}}{H_{2L}}}{ds} < 0$ from Lemma 6, $\frac{dc_2}{ds} < 0$ for any s in this case when a is sufficiently small.

From the above analysis, either $\gamma\left(\frac{1}{1-s}+\frac{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma-1}\delta_L a dF(a)}{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^{\gamma}F(0)}\right)\frac{H_{2N}}{H_{2L}} + \frac{d\frac{H_{2N}}{H_{2L}}}{ds}$ or $\frac{1}{\alpha}\frac{\gamma}{1-s}\frac{H_{2N}}{H_{2L}} + \frac{d\frac{H_{2N}}{H_{2L}}}{ds}$ is the smallest among the similar expressions in (C27), (C28), and (C29) and thus is negative. This implies $\frac{\gamma}{1-s}\frac{H_{2N}}{H_{2L}} + \frac{d\frac{H_{2N}}{H_{2L}}}{ds} < 0$. Hence, for any $a < \widetilde{a}$,

$$(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds} + \gamma\frac{\delta_L a}{\bar{l}+\delta_L sa} < \gamma\left(-\frac{1-\alpha}{1-s} + \frac{\delta_L a}{\bar{l}+\delta_L sa}\right)$$

$$
\begin{aligned}
&= \tfrac{\gamma}{(1-s)(\bar{l}+\delta_L sa)}\left[-(1-\alpha)(\bar{l}+\delta_L sa)+(1-s)\delta_L a\right] \\
&= \tfrac{\gamma}{(1-s)(\bar{l}+\delta_L sa)}\left\{\left[-(2-\alpha)s+1\right]\delta_L a-(1-\alpha)\bar{l}\right\},
\end{aligned}
$$

which is negative when $s \geq \frac{1}{2-\alpha}$. When $s < \frac{1}{2-\alpha}$,

$$
\begin{aligned}
(1-\alpha)\left(\tfrac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{\delta_L a}{\bar{l}+\delta_L sa} &< \tfrac{\gamma}{(1-s)(\bar{l}+\delta_L sa)}\left\{\left[-(2-\alpha)s+1\right]\delta_L a-(1-\alpha)\bar{l}\right\} \\
&< \tfrac{\gamma}{(1-s)(\bar{l}+\delta_L sa)}\left\{\left[-(2-\alpha)s+1\right]\delta_L e_{2L}^{*}-(1-\alpha)\bar{l}\right\} \\
&= \tfrac{\gamma}{s(1-s)(\bar{l}+\delta_L sa)}\left\{\left[-(2-\alpha)s+1\right]J-(1-s)\bar{l}\right\} \text{ (from (24))},
\end{aligned}
$$

which is negative when T_N, T_2, and δ_N are sufficiently low from Lemma C2. $(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{\delta_L a}{\bar{l}+\delta_L sa}<0$ when δ_L is sufficiently small as well, because $[-(2-\alpha)s+1]\delta_L a-(1-\alpha)\bar{l}<[-(2-\alpha)s+1]\delta_L e_{2N}^{*}-(1-\alpha)\bar{l}$, where e_{2N}^{*} decreases with $\frac{H_{2N}}{H_{2L}}$ and $\frac{H_{2N}}{H_{2L}}$ decreases with δ_L from Lemma C2. ■

Proof of Proposition 4. Only the proof of the result on the consumption is presented, because net earnings in unit of the final good equal consumption minus wealth. **(i)** From Lemma 7 (i), consumption of any (group 2) individual decreases with s when $e_{2L}^{*}=0$. From (ii) of the lemma, if T_N, T_2, δ_N, and δ_L are low, it decreases with s when $e_{2L}^{*}>0$ too. Hence, from Lemma 4 and Figure 4, consumption of any individual decreases with s for any s, if the proportion of those with adequate wealth is low enough that Case 2 is realized for any s or if T_N, T_2, δ_N, and δ_L are low.

(ii) From Lemmas 4 and 7, consumption of any individual decreases with s for small s (when s is small enough that $e_{2L}^{*}=0$ holds) and large s.

(a) In Cases 1 and 2, c_2 decreases with s from Lemma 7 (i), and in Case 5, when a is sufficiently low, c_2 decreases with s from Lemma 7 (ii)(b). As for Cases 3 and 4, the proof of Lemma 7 (ii)(b) is valid as long as $e_{2L}^{*}>0$, which is not true when s is very high or very low, as shown in Lemma 4. Here, the result is proved by taking into account how s affects whether $e_{2L}^{*}>0$ or $e_{2L}^{*}=0$.

[**Case 3**] As for Case 3, the proof of Lemma 7 shows that c_2 for $a<e_{2L}^{*}$ decreases with s when a is sufficiently small. Because $e_{2L}^{*}=0$ when $s \geq \overline{s}$ or $s \leq \underline{s}$ from Lemma 4 (see Figure 5), for any positive a, $a \geq e_{2L}^{*}$ holds when $e_{2L}^{*}>0$ and s is close to $\overline{s}$ or $\underline{s}$. Hence, it must be proved that c_2 for $a \geq e_{2L}^{*}$ when $e_{2L}^{*}>0$ and s is close to $\overline{s}$ or $\underline{s}$ decreases with s.

The proof of Lemma 7 shows that c_2 for $a \geq e_{2L}^{*}$ decreases with s for $s \geq \alpha$. From Lemma 1, $\overline{s}>1-\gamma(1-\alpha)$. Because $\alpha<1-\gamma(1-\alpha)$, the consumption decreases with s for any $s \in [\alpha, \overline{s})$.

From (A23) in the proof of Lemma 3, when $s<\alpha$, $\dfrac{dc_2}{ds}<0$ iff

$$
\begin{aligned}
&(\gamma^{\gamma}{T_2}^{\alpha}{T_N}^{1-\alpha})^{\frac{1}{1-\gamma}}[(1-\alpha)\delta_N^{\gamma}]^{\frac{1-\alpha}{1-\gamma}}\left(\frac{(\alpha\delta_L)^{\frac{\gamma}{1-\gamma}}}{1-\gamma}\right)^{\alpha}\gamma\frac{s^{1+\alpha\frac{\gamma}{1-\gamma}}(1-s)^{(1-\alpha)\frac{\gamma}{1-\gamma}-\alpha}(\alpha-s)}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{1-\alpha}}<\frac{\bar{l}}{\delta_L(1-\gamma)} \\
\Leftrightarrow\ &\gamma^{\gamma}{T_2}^{\alpha}{T_N}^{1-\alpha}[(1-\alpha)\delta_N^{\gamma}]^{1-\alpha}\left[\tfrac{(\alpha\delta_L)^{\gamma}}{(1-\gamma)^{1-\gamma}}\right]^{\alpha}[\gamma(1-\gamma)]^{1-\gamma}\left[\frac{s^{1+\alpha\frac{\gamma}{1-\gamma}}(1-s)^{(1-\alpha)\frac{\gamma}{1-\gamma}-\alpha}(\alpha-s)}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{1-\alpha}}\right]^{1-\gamma}<\left(\frac{\bar{l}}{\delta_L}\right)^{1-\gamma} \\
\Leftrightarrow\ &(\delta_N)^{\gamma(1-\alpha)}(\delta_L)^{1-\gamma(1-\alpha)}\gamma(1-\gamma)^{(1-\gamma)(1-\alpha)}(\alpha T_1)^{\alpha}[(1-\alpha)T_N]^{1-\alpha}s^{1-\gamma(1-\alpha)}\frac{(1-s)^{\gamma-\alpha}(\alpha-s)^{1-\gamma}}{\alpha^{\alpha(1-\gamma)}\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{(1-\alpha)(1-\gamma)}}<(\bar{l})^{1-\gamma}.
\end{aligned}
\tag{C30}
$$

From (A15) in the proof of Lemma 1, $e_{2L}^{*}>0$ iff

$$
(\delta_N)^{\gamma(1-\alpha)}(\delta_L)^{1-\gamma(1-\alpha)}\gamma(1-\gamma)^{(1-\gamma)(1-\alpha)}(\alpha T_1)^{\alpha}[(1-\alpha)T_N]^{1-\alpha}s^{1-\gamma(1-\alpha)}(1-s)^{\gamma(1-\alpha)}>(\bar{l})^{1-\gamma}. \tag{C31}
$$

The LHS of (C30) equals that of (C31) times $\left[\frac{(1-s)^{-\alpha}(\alpha-s)}{\alpha^{\alpha}\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{(1-\alpha)}}\right]^{1-\gamma}$. $\frac{(1-s)^{-\alpha}(\alpha-s)}{\alpha^{\alpha}\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{(1-\alpha)}}$ decreases with s for $s<\alpha$ because

$$\begin{aligned}
&\frac{\alpha}{1-s}-\frac{1}{\alpha-s}+\frac{(1-\alpha)\,[\gamma(1-\alpha)+\alpha]}{\alpha-[\gamma(1-\alpha)+\alpha]s} \\
&=(1-\alpha)\left\{\frac{-(1+\alpha-s)}{(1-s)(\alpha-s)}+\frac{[\gamma(1-\alpha)+\alpha]}{\alpha-[\gamma(1-\alpha)+\alpha]s}\right\} \\
&=(1-\alpha)\frac{-(1+\alpha-s)\{\alpha-[\gamma(1-\alpha)+\alpha]s\}+(1-s)(\alpha-s)\,[\gamma(1-\alpha)+\alpha]}{(1-s)(\alpha-s)\{\alpha-[\gamma(1-\alpha)+\alpha]s\}} \\
&=(1-\alpha)\alpha\frac{-(1-s)+\gamma(1-\alpha)}{(1-s)(\alpha-s)\{\alpha-[\gamma(1-\alpha)+\alpha]s\}}<0.
\end{aligned}$$

Further, $\frac{(1-s)^{-\alpha}(\alpha-s)}{\alpha^{\alpha}\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{(1-\alpha)}}=1$ at $s=0$. Hence, $\frac{(1-s)^{-\alpha}(\alpha-s)}{\alpha^{\alpha}\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{(1-\alpha)}}<1$ for $s\in(0,\alpha)$. This implies that when $e^*_{2L}>0$ and s is close to $\underline{s}$, $\dfrac{dc_2}{ds}<0$.

[Case 4] The proof of Lemma 7 shows that c_2 for $a<e^*_{2L}$ decreases with s when a is sufficiently small. Since $e^*_{2L}=0$ when s is very low or high from Lemma 4, for any $a>0$, $a\geq e^*_{2L}$ when $e^*_{2L}>0$ and s is close to the threshold s below or above which $e^*_{2L}=0$. Thus, it must be proved that c_2 for $a\in[e^*_{2L},\widehat{a})$ when $e^*_{2L}>0$ and s is close to the threshold s decreases with s.

From the proof of Lemma 7, $\dfrac{dc_2}{ds}<0$ for $a\in[e^*_{2L},\widehat{a})$ when $s\geq\frac{1}{2-\alpha}$. When $s<\frac{1}{2-\alpha}$, from (C26) in the proof of the lemma, $\dfrac{dc_2}{ds}<0$ if $\frac{1}{1-s}[-(2-\alpha)s+1]\,J-\bar{l}\leq 0$. When $e^*_{2L}\to 0\Leftrightarrow J\to\bar{l}$ (from the proof of Lemma 1), $\dfrac{dc_2}{ds}<0$ because $\frac{1}{1-s}[-(2-\alpha)s+1]\,J-\bar{l}\to\frac{[-(1-\alpha)s+1-s]-(1-s)}{1-s}\bar{l}<0$. Hence, $\dfrac{dc_2}{ds}<0$ for $a\in[e^*_{2L},\widehat{a})$ when $e^*_{2L}>0$ and s is close to the threshold s.

(b) From Lemma 7 (ii)(b), when T_N, T_2, δ_N, and δ_L (in Case 3) are sufficiently high, there exist ranges of s over which consumption of those with relatively large wealth increases with s, if such ranges of s are *effective*, i.e., if $e^*_{2L}>0$ is true.

[Case 3 for intermediate s] When Case 3 is realized, as explained in Appendix A, $\frac{H_{2N}}{H_{2L}}$ is determined by (29) and c_2 for those with $a\geq e^*_{2L}$ is determined by (30) as in the unconstrained case. Hence, Proposition 1 (ii) applies and thus ranges of s over which $\dfrac{dc_2}{ds}>0$ holds are effective for such individuals when T_N, T_2, δ_N, and δ_L are sufficiently high. As for those with $a<e^*_{2L}$, from the proof of Lemma 7 (ii), $\dfrac{dc_2}{ds}>0$ for some ranges of s, if T_N, T_2, δ_N, and δ_L are sufficiently high, $e^*_{2L}>0$ is true, and a is sufficiently large that $(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{a}{\bar{l}+sa}=G-\gamma\left(\frac{e^*_{2L}}{\bar{l}+se^*_{2L}}-\frac{a}{\bar{l}+sa}\right)>0$, where from (C14) in the proof of the lemma, the sign of $G\equiv(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}+\gamma\frac{e^*_{2L}}{\bar{l}+se^*_{2L}}$ is same as that of (A21) in the proof of Lemma 3 (ii). Hence, the proofs of the lemma and Proposition 1 (ii) apply and ranges of s over which $\dfrac{dc_2}{ds}>0$ holds are effective when T_N, T_2, δ_N, and δ_L are high enough that the supremum of s satisfying $G>0$, $s_{\max}$, is sufficiently greater than $\underline{s}$.

When Case 3 is realized for intermediate s, c_2 of individuals with $a\geq e^*_{2L}$ when $e^*_{2L}>0$ and thus s is intermediate is given by (30), while their consumption at $s=0$, at which Case 1 or 2 is realized (Figure 5), equals or is smaller than the value of (30).[C2] Hence, c_2 when s is intermediate is greater than c_2 at $s=0$ if $(1-s)^{\gamma}\left(\frac{H_{2N}}{H_{2L}}|_{\text{intermediate } s}\right)^{-\alpha}>\left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{-\alpha}$.

[C2] When Case 2 is realized, c_2 at $s=0$ could be given by either (30) (when $a\geq e^*_{2N}$), (A6) (when $a\in[\widehat{a}_0,e^*_{2N})$), When Case 1 is realized at $s=0$, Proposition 1 (ii) applies and c_2 is highest at intermediate s, if T_N, T_2, δ_N, and δ_L are sufficiently high. When Case 2 is realized at $s=0$, unlike the unconstrained case, $\frac{H_{2N}}{H_{2L}}$ and $\widehat{a}_0$ at $s=0$ are determined by (A4) and (A5) in Appendix A, while $\frac{H_{2N}}{H_{2L}}$ when s is intermediate is determined by (29) as in the unconstrained case, which can be expressed as

$$(\gamma^\gamma T_2{}^\alpha T_N{}^{1-\alpha})^{\frac{1}{1-\gamma}}\left\{\left[(1-\alpha)[\delta_N(1-s)]^\gamma\left(\frac{H_{2N}}{H_{2L}}|_{\text{intermediate } s}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}}-\left[\alpha(\delta_L s)^\gamma\left(\frac{H_{2N}}{H_{2L}}|_{\text{intermediate } s}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}\right\}=\frac{1}{1-\gamma}\frac{\bar{l}}{\delta_L s}. \tag{C32}$$

By substituting $(1-s)^\gamma\left(\frac{H_{2N}}{H_{2L}}|_{\text{intermediate } s}\right)^{-\alpha}>\left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{-\alpha}$ into the above equation,

$$(\gamma^\gamma)^{\frac{1}{1-\gamma}}\left(\left[T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{-\alpha}(1-\alpha)\delta_N^\gamma\right]^{\frac{1}{1-\gamma}}-\left[T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{1-\alpha}\alpha(\delta_L s)^\gamma(1-s)^{\frac{\gamma}{\alpha}}\right]^{\frac{1}{1-\gamma}}\right)<\frac{1}{1-\gamma}\frac{\bar{l}}{\delta_L s}. \tag{C33}$$

This condition holds if the LHS of the equation is negative, i.e.,

$$\left[\left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{-1}(1-\alpha)\delta_N^\gamma\right]^{\frac{1}{1-\gamma}}-\left[\alpha(\delta_L s)^\gamma(1-s)^{\frac{\gamma}{\alpha}}\right]^{\frac{1}{1-\gamma}}<0. \tag{C34}$$

Because $\frac{H_{2N}}{H_{2L}}|_{s=0}$ does not depend on δ_L from (A4) and (A5), the above condition clearly holds when δ_L is sufficiently large. It can be proved that $\frac{H_{2N}}{H_{2L}}|_{s=0}$ increases with T_N, T_2, and δ_N from (A4) and (A5). Hence, the condition holds when T_N and T_2 are sufficiently large.

The condition holds when δ_N is sufficiently large if $\widehat{a}_0$ increases with δ_N, because $\left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{-1}\delta_N^\gamma$ must decrease with δ_N from the following equation, which is obtained from (A4) at $s=0$.

$$\frac{(\delta_N\widehat{a}_0)^\gamma}{\frac{H_{2N}}{H_{2L}}|_{s=0}}-\frac{1}{1-\alpha}\frac{1}{(T_N)^{1-\alpha}(T_2)^\alpha\left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{1-\alpha}}\widehat{a}_0=\frac{\alpha}{1-\alpha}(\bar{l})^\gamma \tag{C35}$$

If $\widehat{a}_0$ decreases with δ_N,the condition holds when δ_N is sufficiently large, because $\left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{-1}\delta_N^\gamma$ must decrease with δ_N from the following equation, which is obtained from (A5) at $s=0$.

$$1=\frac{\left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{-1}\delta_N^\gamma\left[(e_{2N}^*)^\gamma(1-F(e_{2N}^*))+\int_{\widehat{a}_0}^{e_{2N}^*}(a)^\gamma dF(a)\right]}{(\bar{l})^\gamma F(\widehat{a}_0)}, \tag{C36}$$

where e_{2N}^* increases with $\left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{-1}\delta_N^\gamma\left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{1-\alpha}$ from (21).

Hence, when Case 2 is realized at $s=0$, c_2 is highest at intermediate s, when T_N, T_2, δ_N, and δ_L are sufficiently high.

[Case 4 for intermediate s] When Case 4 is realized for intermediate s, from the proof of Lemma 7 (ii), c_2 of those with $a\geq\widehat{a}$ increases with s for some ranges of $s(<\alpha)$, if T_N, T_2, and δ_N are sufficiently high that (C21) in the proof holds with ">", which is the case only when $(\alpha-s)J-\alpha(1-s)\bar{l}>0$, where $J\equiv\left[\gamma s\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}$, and $e_{2L}^*>0$ is true. Since $e_{2L}^*>0\Leftrightarrow J>\bar{l}$ from the proof of Lemma 1, $e_{2L}^*>0$ is true when $(\alpha-s)J-\alpha(1-s)\bar{l}>0$.

When Case 4 is realized for intermediate s, from Appendix A, c_2 for those with $a\geq e_{2N}^*$ when s is intermediate is determined by (30), while their consumption at $s=0$, at which Case 1 or 2

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or (A7) (when $a<\widehat{a}_0$), because e_{2L}^* when s is intermediate could be smaller than e_{2N}^* or $\widehat{a}_0$ at $s=0$. Because $w_N h_{2N}^*-P_2e_{2N}^*>w_Nh_{2N}-P_2a$ for $a\in[\widehat{a}_0,e_{2N}^*)$ and $w_Nh_{2N}^*-P_2e_{2N}^*>w_{2L}h_{2L}$ for $a<\widehat{a}_0$ (note $e_{2L}^*=0$), c_2 of (30) is greater than that of (A6) or (A7).

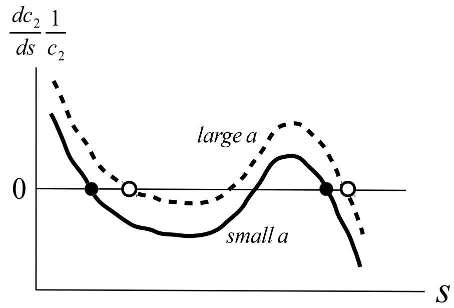

Figure C1: Relationship between s and $\frac{dc_2}{ds}\frac{1}{c_2}$ when $a < e^*_{2L}$ for large a and small a

is realized (Figure 5), equals or is smaller than the value of (30) (footnote C2). Hence, c_2 when s is intermediate is greater than c_2 at $s = 0$ if $(1-s)^\gamma \left(\frac{H_{2N}}{H_{2L}}|_{\text{intermediate } s}\right)^{-\alpha} > \left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{-\alpha}$. Given s and other parameters, $\frac{H_{2N}}{H_{2L}}|_{\text{intermediate } s}$ in Case 4 is smaller than the one in Case 3 from Lemmas 4 and 5, where the proportion of those with adequate wealth for education is higher in Case 3. Similarly, given s and other parameters, $\frac{H_{2N}}{H_{2L}}|_{s=0}$ in Case 2 when the distribution of wealth $F(a)$ is that of Case 4 is smaller than when $F(a)$ is that of Case 3 and $\left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{-\alpha}$ in Case 1. Thus, $(1-s)^\gamma \left(\frac{H_{2N}}{H_{2L}}|_{\text{intermediate } s}\right)^{-\alpha} > \left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{-\alpha}$ is true when Case 4 is realized for intermediate s, if $(1-s)^\gamma \left(\frac{H_{2N}}{H_{2L}}|_{\text{intermediate } s}\right)^{-\alpha}$ in Case 3 is greater than $\left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{-\alpha}$ in Case 2 when $F(a)$ is that of Case 4. From the proof of Case 3 above, this is true when T_N, T_2, δ_N, and δ_L are high enough.

From Appendix A, c_2 for those with $a \in [\widehat{a}, e^*_{2N})$ when s is intermediate is determined by (A12), which equals (30) at $a = e^*_{2N}$. Hence, when T_N, T_2, δ_N, and δ_L are sufficiently high that c_2 of those with $a = e^*_{2N}$ when $e^*_{2L} > 0$ is highest at intermediate s, it is also true for sufficiently large $a \in [\widehat{a}, e^*_{2N})$.

[**Case 5 for intermediate s**] When Case 5 is realized for intermediate s, from the proof of Lemma 7 (ii), c_2 of those with $a \geq \widetilde{a}$ increases with s for some ranges of $s(< \alpha)$, if T_N, T_2, and δ_N are sufficiently high that $\frac{1}{s}\frac{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma-1}\delta_L sadF(a)}{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^\gamma F(0)} \geq \frac{1-\alpha}{\alpha}\frac{1}{1-s}$. Because $\frac{\widetilde{a}}{\bar{l}+\delta_L s\widetilde{a}} > \frac{1}{s}\frac{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma-1}\delta_L sadF(a)}{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^{\gamma}dF(a)+(\bar{l})^\gamma F(0)}$ from the proof of Lemma 7 (ii), $\frac{\widetilde{a}}{\bar{l}+\delta_L s\widetilde{a}} > \frac{1-\alpha}{\alpha}\frac{1}{1-s}$ holds. Since $e^*_{2L} > \widetilde{a}$, this implies $\frac{e^*_{2L}}{\bar{l}+\delta_L s e^*_{2L}} > \frac{1-\alpha}{\alpha}\frac{1}{1-s}$ and thus $e^*_{2L} > 0$ is true.

When Case 5 is realized for intermediate s, from Appendix A, c_2 for those with $a \geq e^*_{2N}$ when s is intermediate is determined by (30), while their consumption at $s = 0$, at which Case 2 is realized (Figure 5), equals or is smaller than the value of (30) (footnote C2). Hence, c_2 when s is intermediate is greater than c_2 at $s = 0$ if $(1-s)^\gamma \left(\frac{H_{2N}}{H_{2L}}|_{\text{intermediate } s}\right)^{-\alpha} > \left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{-\alpha}$. The rest of the proof is similar to the case in which Case 4 is realized for intermeidate s.

From Appendix A, c_2 for those with $a \in [\widetilde{a}, e^*_{2N})$ when s is intermediate is determined by (A12), which equals (30) at $a = e^*_{2N}$. Hence, when T_N, T_2, δ_N, and δ_L are sufficiently high that c_2 of those with $a = e^*_{2N}$ is highest at intermediate s, it is also true for sufficiently large $a \in [\widetilde{a}, e^*_{2N})$.

[**s maximizing c_2 of local sector workers**] When c_2 is maximized at intermediate s, s maximizing c_2 of national sector workers does not depend on a from (30) and (A6) in Appendix A, and s maximizing c_2 of local sector workers when $a \geq e^*_{2L}$ does not depend on a from (A12) in Appendix A. By contrast, c_2 of local sector workers when $a < e^*_{2L}$, which is realized in Cases 3−5,

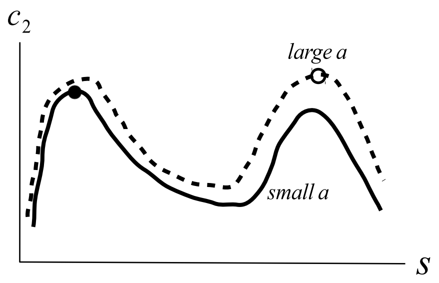

Figure C2: Relationship between s and c_2 when $a < e^*_{2L}$ for large a and small a

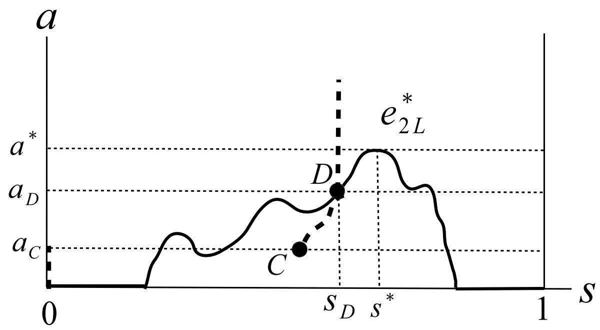

Figure C3: s maximizing c_2 of local sector workers

equals $\alpha {T_2}^\alpha {T_N}^{1-\alpha} \left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha} (\bar{l} + \delta_L sa)^\gamma$ from (A9). The derivative of consumption with respect to s equals $\left[(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1} \frac{d\frac{H_{2N}}{H_{2L}}}{ds} + \gamma \frac{\delta_L a}{\bar{l}+\delta_L sa}\right] c_2$, where $\frac{d\frac{H_{2N}}{H_{2L}}}{ds} < 0$ from Lemma 6. Thus, given s, $(1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1} \frac{d\frac{H_{2N}}{H_{2L}}}{ds} + \gamma \frac{\delta_L a}{\bar{l}+\delta_L sa}$ increases with a, which implies that s maximizing c_2 locally increases with a. Figure C1 illustrates the relationship between s and $\frac{dc_2}{ds}\frac{1}{c_2}$ for small a and large a. In this example, there are two values of s maximizing c_2 locally, denoted by small circles, both of which are higher when a is higher. Further, it cannot be the case that c_2 when a is large is maximized at the lowest of the two local maximizers and c_2 when a is small is maximized at the highest of the two local maximizers, which implies that s maximizing globally c_2 when $a < e^*_{2L}$ also increases with a. The reason is that the ratio of c_2 when a is large to c_2 when a is small increases with s from (A9). The following example would help understand this. Figure C2 illustrates the relationship between s and c_2 for two values of a. In the figure, because the ratio increases with s, when a is small, c_2 is highest at the lowest of the two values of s maximizing c_2 locally, while when a is large, c_2 is highest at the highest of the two local maximizers.

The above argument is incomplete because for given a, whether $a < e^*_{2L}$ or $a \geq e^*_{2L}$ depends on s. Figure C3 illustrates the relationship between s and e^*_{2L}. (As in the figure, it cannot be ruled out the possibility that the relationship is non-monotonic and thus there exist multiple values of s maximizing e^*_{2L} locally.) In the region below the e^*_{2L} profile, $a < e^*_{2L}$ and thus $e = a$ hold, and in the region on or above the profile, $a \geq e^*_{2L}$ and thus $e = e^*_{2L}$ hold. In the figure, s maximizing c_2 of

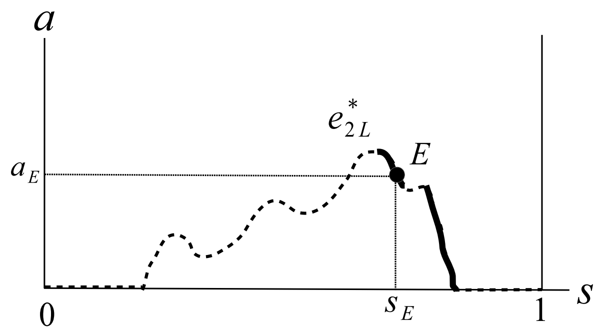

Figure C4: Relationship between s and e^*_{2L}

local sector workers when $a \geq e^*_{2L}$ is denoted s_D, which is smaller than s maximizing e^*_{2L}, s^*, from (24) and (A12). The segment CD of the thick dotted line passing through point D is the locus of s maximizing c_2 when $a \in [a_C, a_D)$. When $a < a_C$, c_2 is maximized at $s = 0$. It is now proved that, for given a, s maximizing c_2 of local sector workers is s on the thick dotted line. This is obvious when $a < a_D$ and $a \geq a^*$. When $a \in [a_D, a^*)$, s maximizing c_2 is s_C because for given s, c_2 when $a \geq e^*_{2L}$ is higher than c_2 when $a < e^*_{2L}$, and c_2 when $a \geq e^*_{2L}$ is highest at $s = s_C$. Therefore, s maximizing c_2 of local sector workers increases with a when $a \in [a_C, a_D)$. ■

Proof of Proposition 5. **[Those who become local-sector workers]** If the proportion of individuals with relatively large wealth is very low, from Lemma 4 (ii)(d) (see Figure 5), $e^*_{2L} = 0$ and thus $h_{2L} = (\bar{l})^\gamma$ hold for any s. Otherwise, from Lemma 4 (Figure 5), $e^*_{2L} = 0$ and thus $h_{2L} = (\bar{l})^\gamma$ hold when s is very low or very high, which implies that h_{2L} is highest at an intermediate s.

The last part of the result is proved as follows. Figure C4 illustrates the relationship between s and e^*_{2L}. (As in the figure, it cannot be ruled out the possibility that the relationship is non-monotonic and thus there exist multiple values of s maximizing e^*_{2L} locally.) In the region below the e^*_{2L} profile, $a < e^*_{2L}$ and thus $e = a$ hold, and in the region on or above the profile, $a \geq e^*_{2L}$ and thus $e = e^*_{2L}$ hold. Because $h_{2L} = (\bar{l} + \delta_L s e)^\gamma$ when $a < e^*_{2L}$ increases with s from $e = a$, for each a such that $a < e^*_{2L}$ holds for some s, s that maximizes h_{2L} when $a \leq e^*_{2L}$ is on a segment of the e^*_{2L} profile represented by a thick solid line. By contrast, h_{2L} when $a > e^*_{2L}$ increases (decreases) with s when $\frac{d(se^*_{2L})}{ds} \propto \frac{1}{s} + \frac{de^*_{2L}}{ds} > (<) 0$. Hence, s that maximizes h_{2L} when $a > e^*_{2L}$ must satisfy $\frac{de^*_{2L}}{ds} < 0$ and thus is on the same thick solid line. Suppose, without loss of generality, that such s is s_E in the figure. Then, if one has $a \geq a_E$, her h_{2L} is maximized at $s = s_E$, while if $a < a_E$, s maximizing h_{2L} is on a portion of the thick solid line below the wealth level and thus $s > s_E$. As a decreases, such portion of the line shortens, thus s maximizing h_{2L} weakly increases.

From Figures C3 and C4, the s that maximizes h_{2L} is greater than the s that maximizes net earnings and consumption.

[Those who become national-sector workers] For those who have enough wealth for optimal education, $\frac{dh_{2L}}{ds} = \frac{d(\bar{l}+\delta_L s e^*_{2N})^\gamma}{ds} = \gamma(\bar{l} + \delta_L s e^*_{2N})^{\gamma-1}\delta_L \frac{d(se^*_{2N})}{ds}$. From (21),

$$\frac{d(se^*_{2N})}{ds} = e^*_{2N} + s\frac{de^*_{2N}}{ds} = e^*_{2N}\left\{1 + \frac{s}{1-\gamma}\left[-\frac{\gamma}{1-s} - \alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right]\right\}$$

$$= \frac{e^*_{2N}}{1-\gamma}\left\{\frac{1-\gamma-s}{1-s} - \alpha s\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right\}, \tag{C37}$$

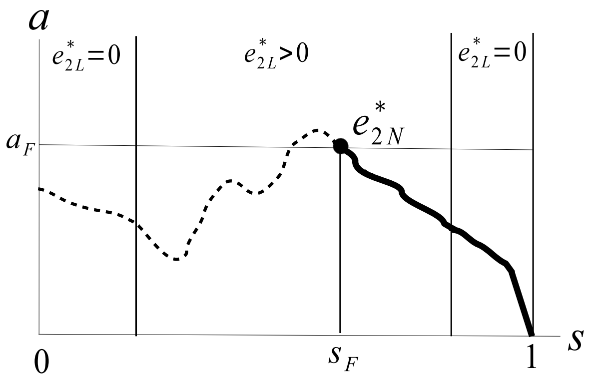

Figure C5: Relationship between s and e^*_{2N}

which is positive when $s \leq 1-\gamma$. Hence, se^*_{2N} and thus $(\bar{l}+\delta_L se^*_{2N})^\gamma$ are maximized at $s \in (1-\gamma, 1)$. (Note that $e^*_{2N} = 0$ at $s = 1$.) Denote such s by s_F and a that equals e^*_{2N} at $s = s_F$ by a_F. For those with $a \geq a_F$, h_{2L} is maximized at $s = s_F \in (1-\gamma, 1)$.

For those with $a < a_F$, the relationship between s and e^*_{2N} needs to be examined. For this purpose, the result on c_2 of Lemma 7 and Proposition 4 can be used. This is because e^*_{2N} is proportional to $\left[(1-s)^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}}$ from (21) and c_2 when it is given by (30) or (A6) in Appendix A—c_2 for any a in Case 1, c_2 for $a \geq \widehat{a}_0$ in Case 2, c_2 for $a \geq e^*_{2L}$ in Case 3, c_2 for $a \geq \widehat{a}$ in Case 4, and c_2 for $a \geq \widetilde{a}$ in Case 5—is a linear function of $\left[(1-s)^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}}$ (when c_2 is given by (30)) or of $(1-s)^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}$ (when c_2 is given by (A6)).

When T_N, T_2, δ_N, and δ_L are small or when the proportion of those with relatively large wealth is very low, c_2 decreases with s for any a from Proposition 4 (i) and thus e^*_{2N} decreases with s. When a figure like Figure C4 is drawn for this case, e^*_{2N} is downward-sloping and equals 0 at $s = 1$. From a similar logic to the proof for local-sector workers, for those with $a < a_F$, s maximizing $h_{2L} = (\bar{l} + \delta_L se_{2N})^\gamma$ is on a portion of the e^*_{2N} profile below the wealth level and thus $s > s_F$. As a decreases, the segment of the profile shortens and thus s maximizing h_{2L} weakly increases.

When T_N, T_2, δ_N, and δ_L are not small and the proportion of those with relatively large wealth is not very low, c_2 and e^*_{2N} decrease with s when $e^*_{2L} = 0$ and they decrease with s for large s when $e^*_{2L} > 0$ from Lemma 7 (ii)(b). Further, $\frac{de^*_{2N}}{ds} < 0$ at $s = s_F$ from $\frac{d(se^*_{2N})}{ds} \propto \frac{1}{s} + \frac{de^*_{2N}}{ds}$. Based on these results, Figure C5 illustrates the relationship between s and e^*_{2N}. (As in the figure, it cannot be ruled out the possibility that the relationship is non-monotonic and there exist multiple values of s maximizing e^*_{2N} locally.) From a similar logic to the proof for local-sector workers, for those with $a < a_F$, s maximizing $h_{2L} = (\bar{l} + \delta_L se_{2N})^\gamma$ is on a portion of the e^*_{2N} profile below the wealth level and thus $s > s_F$, and s maximizing h_{2L} weakly decreases with a.

As mentioned above, both c_2 of national-sector workers and e^*_{2N} are linear functions of $\left[(1-s)^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}}$. Hence, from $\frac{d(se^*_{2N})}{ds} \propto \frac{1}{s} + \frac{de^*_{2N}}{ds}$ and Figure C5, the s that maximizes h_{2L} is greater than the s that maximizes e^*_{2N} and thus net earnings and consumption. ■