Language education and economic outcomes in a bilingual society

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April 2024

Abstract

Poor economic performance of subordinate (typically, minority) groups and large disparities between these groups and the dominant ethnic group are major concerns in most countries. In many of these countries, the mother tongue of the dominant group serves as the common language in national business and intergroup communications. Determining the appropriate emphasis on teaching a local ethnic language relative to the common language is a crucial issue in the education of students from subordinate groups.

This paper develops a model to examine the issue theoretically. The analysis shows that balanced education of the two languages is valuable for skill development. By contrast, with respect to consumption, earnings net of educational expenditure, and their between-group inequalities, the analysis reveals that balanced bilingual education is desirable *only when* the country has favorable educational and technological conditions (i.e., when sectoral productivities and the effectiveness of education are reasonably high) and *only for* those with adequate wealth. Language education focused solely on the common language maximizes the economic outcomes of those with little wealth and under adverse conditions, the outcomes for all. The paper discusses the policy implications of these results. It also examines the implications of the asymmetric language positions of the groups for sectoral choices and within-group inequalities.

Keywords: language policy, bilingual education, ethnic inequality, human capital, economic development

JEL classification numbers: I25, J15, J24, O15, Z13

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1 Introduction

Poor economic performance of subordinate (typically, minority) groups and large disparities between these groups and the dominant group are major concerns in many countries. Ethnic inequality has serious consequences for development (Alesina, Michalopoulos, and Papaioannou, 2016).

Discrimination, inequality in endowments such as wealth, and unequal political power are identified as the primary factors contributing to the worrying situation of subordinate groups. However, limited attention is given to the language barriers they face. In many countries, the mother tongue of the dominant ethnic group serves as the common language in national business and inter-group communications, such as Spanish in Bolivia, Mandarin in China, Amharic in Ethiopia, Filipino in the Philippines, Turkish in Turkey, and Vietnamese in Vietnam.

Determining the appropriate emphasis on teaching a local ethnic language relative to the common language, as well as deciding which language should be used for instruction in other subjects, are crucial issues in the education of students from subordinate groups. Even today, mother-tongue education remains limited in many countries (Walter and Benson, 2012).[1] Recently, some countries have greatly increased the emphasis on mother-tongue education, while others have maintained or intensified the emphasis on common-language education.[2]

Empirical research suggests that acquiring proficiency in a common language, while challenging for non-native speakers, offers significant rewards: Azam, Chin, and Prakash (2013) show that returns to fluently speaking English are comparable to returns to secondary education and half as large as returns to college education in India. By contrast, acquiring proficiency in one's mother tongue is less demanding, but its use is limited to the local or ethnic business and community.

There is a broad consensus among language and education experts that emphasizing mother-tongue education, at least in primary education, is important for students to develop adequate language and non-language skills (Ball, 2011; Heugh, 2011). In contrast, there is little knowledge about what combination of the two types of education is desirable in terms of future *earnings and consumption,* and what kind of educational and economic policies should be conducted when *both educational and economic outcomes* of students are considered. The main purpose of this paper is to develop a simple model to examine these pivotal issues theoretically. The paper also examines the implications of the fact that the common language is the mother tongue of the dominant group, but not of the subordinate group, for sectoral choices and within-group inequalities.

Model: The model economy comprises two ethnic groups, the dominant group and the subordinate group, and is divided into three sectors, the *national sector* and two group-specific *local sectors.* In the real world, national- (local-) sector jobs correspond to many positions in companies operating nationwide (locally) and jobs involving communications with other groups (locals).

Working in the national sector requires the skill to use the common language, which is the ethnic language of the dominant group, while working in the local sector of an ethnic group requires the skill to use the ethnic language of that group. This implies that the skills required in the two sectors are the same for the dominant group but different for the subordinate group.

Each person has a wealth endowment to expend on education to develop these skills. The subordinate group *cannot* choose the allocation of the expenditure for developing the two skills. This allocation is fixed, reflecting the fact that the relative weight of common-language education and mother-tongue education is mostly determined by the government in basic education. The level

[1]Walter and Benson (2012) find that as many as 40 percent of students in the world do not have access to education in a language they speak or understand.

[2]The former group of countries includes Bolivia (Haboud and Limerick, 2017), Ethiopia (Benson et al., 2012) and the Philippines (Tupas and Martin, 2017), while the latter group includes China (Gao and Wang, 2017), Turkey (Faltis, 2014), and Vietnam (Nguyen and Nguyen, 2019).

of common-language skill of the group is zero without education, while the level of mother-tongue skill is *positive* even without education (i.e., a portion of the skill is acquired at home).[3]

Individuals must self-finance education due to the absence of a credit market.[4] The distribution of wealth is such that some people may not have enough wealth to make an optimal educational investment. After education, individuals choose a sector to work in, receive earnings, and consume.

Results: The paper explores the implications of the asymmetric language positions of the groups for sectoral choices and within-group inequalities, and the effects of the relative weights of the two types of education on skill, earnings, and consumption, as well as their between-group inequalities. The main results can be summarized as follows.

First, while individuals from the dominant group are indifferent between the national and local sectors, those from the subordinate group with relatively large wealth choose the national sector and those with less wealth choose the local sector. This is because costly education is a prerequisite for working in the national sector only for the latter group. As a result of the contrasting sectoral choices, a change in within-group wealth inequality tends to have a greater impact on within-group inequalities in earnings and consumption for the subordinate group than for the dominant group.

Second, regarding the development of the mother-tongue skill of the subordinate group, a balanced allocation of resources between teaching the common language and the mother tongue is crucial for those with limited wealth who choose the local sector, and also beneficial for others.[5,6] This result is consistent with the aforementioned consensus among specialists.

Third, with regards to the consumption and earnings net of educational expenditure of the subordinate group, balanced bilingual education is desirable *only when* the country has favorable educational and technological conditions (i.e., the effectiveness of education for the group and sectoral productivities are sufficiently high) and the share of the non-poor is not too small, and *only for* those with sufficient wealth. Language education solely focused on the common language is *always* optimal for those with *little* wealth, and under adverse conditions, optimal for *all*.[7] In the real society, these conditions are closely related to a country's level of economic and social development. Thus, the result implies that, in general, if the level of development is low, teaching only the common language is preferable in terms of the economic outcomes; otherwise, balanced bilingual education is desirable for all except the very poor. Interestingly, the shape of the relationship between the weight on teaching the mother tongue and consumption/net earnings is *bimodal* except for the very poor and countries with bad conditions: as the weight increases, these variables *decrease* when the weight is low, then increase, and decrease again when the weight is high.[8] The crucial assumption for these results is that the level of the mother-tongue skill is positive even without education: if education is essential for the skill, balanced dual education *always* maximizes

[3]For analytical tractability, nonlanguage skills are not considered. However, footnotes in later sections argue that the main results would be unchanged even if a nonlanguage skill is an input in human capital production functions.

[4]This setting reflects the fact that, in most countries, students rely on family wealth to cover expenses such as study materials and commuting costs, even when public schools do not charge tuitions.

[5]A balanced allocation is crucial for future local-sector workers. If the allocation is heavily biased toward the education of either language, they do not invest in education. While this result may appear implausible given that most students receive some education even in poor countries, it is important to note that the model abstracts from non-investment motives for attending school, such as consumption motives (e.g., the pleasure of learning or attending school) and social motives (e.g., pressure from family or community to attend school), for analytical tractability.

[6]For the less affluent segment of national-sector workers, the skill level is highest under bilingual education with a strong emphasis on the mother tongue. For others, the highest skill level is attained under a more balanced education.

[7]The analysis also suggests that implementing such language education is *always better* than introducing teaching of the mother tongue *on a small scale*: the latter does not enhance the skills of future local-sector workers and reduces consumption and net earnings for all.

[8]This is proved analytically when the proportion of those with relatively large wealth is sufficiently high. Numerical simulations suggest that this is also the case when the proportion is lower.

the economic outcomes.

Lastly, as individuals from the dominant group are unaffected by the relative weights of the two types of education students from the subordinate group receive, the above results directly apply to inter-group disparities in skill, net earnings, and consumption. For example, switching from language education focused solely on the common language to balanced bilingual education can reduce inter-group economic inequalities (excluding the very poor) only in favorable educational and technological environments.

Policy implications: The results suggest that policies aimed at achieving positive educational and economic outcomes for the subordinate group and reducing the inter-group inequalities should be tailored to the educational and technological conditions or the level of development of the country. Under favorable conditions, typically associated with high levels of economic and social development, the government should adopt balanced bilingual education *along with* redistributive measures that enable those with little wealth to spend sufficiently more on education, thereby reaping economic benefits from balanced education. By contrast, under unfavorable conditions, generally associated with low levels of development, the government should opt for bilingual education with a *smaller* (*but not too small*) emphasis on the mother tongue than under more favorable conditions (along with redistribution toward the very poor).

Finally, the result that changes in within-group wealth inequality tend to have a greater impact on economic inequalities for the subordinate group suggests that redistributive policies that increase access to education for the poor would be more critical for this group.

It should be noted that the model does not take into account the potential effects that the choice of languages in education may have on social capital, political participation, national unity, and public goods provision. Policy implementation in the real world must also consider these effects.

Relation to Yuki (2022): To the best of the author's knowledge, this paper, along with Yuki (2022), is the first attempt to theoretically examine how the relative emphasis on teaching the common language and the mother tongue influence the skills, net earnings, and consumption of individuals with different family incomes. The main differences from Yuki (2022) are as follows. First, the common language is the mother tongue of the dominant group in this paper, whereas ethnic groups are symmetric and the common language is not a mother tongue of any group in Yuki (2022). Such a setting is relevant to many sub-Saharan Africa countries in which the common language is the language of the former colonizer. Second, because groups are symmetric, Yuki (2022) does not examine the effects of the education-language policy on between-group inequalities. It does not explore the implications of asymmetric language positions of the groups for sectoral choices and within-group inequalities either. Third, the human capital production functions in this model exhibit decreasing returns to educational expenditure, which are standard and more plausible than Yuki (2022)'s linear functions with an upper bound on expenditure. Mainly because of the functional forms, the model and some of the results of this paper are more transparent.

Organization of the paper: Section 2 reviews related works. Section 3 presents the model. Section 4 examines the case where everyone has enough wealth for education, and Section 5 considers the general case where the educational investment of some individuals is constrained by wealth. Section 6 discusses the policy implications of the results. Section 7 concludes. Appendix A explains the determination of endogenous variables in the general case, and Appendix B presents proofs of the results for the unconstrained case. Online Appendix C contains proofs for the general case.

2 Related Literature

Aside from Yuki (2022), several works theoretically examine related issues. Pool (1991) considers the choice of official languages in a multilingual society in which earnings are exogenous, learning a non-native language is costly, and translations among different official languages are costly and financed by tax. He shows that an efficient and fair choice of official languages exists if appropriate inter-group redistribution is implemented. Lazear (1999) develops a model in which individuals, who vary in the cost of learning non-native languages, decide whether to master languages of other groups, each person randomly matches with another, and goods are produced only when the pair can use the same language. He derives several implications of the model and empirically examines them. Ortega and Tangerås (2008) model a society with two language groups in which the dominant group determines the type(s) of schools (bilingual or monolingual in either language) accessible to each group, individuals decide whether to attend school, and goods are produced from bilateral random matching as in Lazear (1999). They show that the dominant group choose laissez-faire or restrict access to schools using the language of the subordinate group, while the subordinate group prefer schools using their mother tongue.

Besides addressing different issues, the present work is distinct from these works in several aspects. First, in this paper, individuals within each group are heterogeneous in the amount of wealth available for education, whereas in the aforementioned works, they are either homogenous (Pool; Ortega and Tangerås) or heterogenous in the costs of learning non-native languages, which may capture differences in innate ability (Lazear). This work adopts a different setting because it mainly focuses on developing countries in which family wealth is a critical determinant of educational investment even at the basic education level, in contrast to the primary focus on developed countries in the existing works. Second, unlike the previous works, this paper does not account for the effect of the size of language groups, such as network externalities in language usage. Finally, unlike Ortega and Tangerås (2008), educational institutions are given rather than determined endogenously and strategic interactions among agents are not considered.

Many studies in education and linguistics examine the effect of education-language policy on the academic achievement of students. A general consensus among researchers is that emphasizing mother-tongue education, at least in primary education, is important for skill developmen (Ball, 2011; Heugh, 2011). A small number of works in economics also empirically investigate the effects on educational outcomes. Jain (2017) examines the effect on academic outcomes using data from South India, where primary education is largely conducted in the official language of a state. By comparing districts in which the official language matched the district's language and those in which it did not, he finds that mismatched districts had lower literacy and college graduation rates, but after states were reorganized based on linguistic lines, the previously mismatched districts caught up with others. Ramachandran (2017) finds that the reform in Ethiopia that introduced mother-tongue instruction in primary education has positive effects on reading skills and years of schooling. These findings are consistent with the results of the model on educational outcomes.

Very few studies examine labor market outcomes. Angrist and Lavy (1997) find that the policy change in Morocco during the 1980s, which replaced French with Arabic as the medium of instruction in post-primary education, greatly lowered returns to schooling. Cappellari and Di Paolo (2018) analyze the effects of the 1983 bilingual-education reform in Catalonia, which significantly increased the weight of Catalan in mandatory education, and find a positive effect on earnings. In line with the model's result on earnings, these findings suggest that a large increase in the emphasis on mother-tongue education leads to lowers wages in a developing country (Morocco) and higher wages in a developed region (Catalonia). Chakraborty and Bakshi (2016) find that the policy change in the Indian state of West Bengal, which abolished English education in primary schools, significantly decreased wages. This aligns with the result that education heavily biased toward the mother-tongue skill results in low earnings. While these findings are consistent with the model's results, they are very limited in number, and further research is needed. The results of this paper would be helpful in guiding future empirical works and interpreting their results.

3 Model

3.1 Production

Consider a bilingual society that is populated by two ethnic groups, groups 1 and 2, and has three sectors, the *national sector* and two group-specific *local sectors*. The local sector of each group produces final goods specific to their group, using intermediate goods produced by the national sector and the group's labor. Meanwhile, the national sector produces intermediate goods using labor from both ethnic groups.

In the real world, national-sector jobs, which largely overlaps with modern or formal sector jobs in developing countries, correspond to many positions in companies operating nationwide and jobs involving communications with other groups, all requiring proficiency in a common language. Local-sector jobs represent many positions in locally-operating businesses and jobs involving communications with local customers, such as those in retail, food service, and personal care, necessitating proficiency in the local ethnic language. Assuming the local sectors as sectors producing group-specific final goods reflects the fact that these services are dominant in the final stage of the production process.

The production function of the local sector of group i ($i = 1, 2$) is[9]

$$Y_i = (T_i H_{iL})^{\alpha} (Y_{iN})^{1-\alpha}, \alpha \in (0, 1), \tag{1}$$

where H_{iL} is the total human capital of the sector's workers (whose determination is explained later), T_i is the sector's constant total factor productivity (TFP), and Y_{iN} is the amount of intermediate goods used. The production function implies that both human capital and intermediate goods are essential, but substitutable to some extent in the production of the final goods.

The production function of the national sector is

$$Y_N = T_N(H_{1N} + H_{2N}), \tag{2}$$

where H_{iN} is the total human capital of group i workers in the sector and T_N is the sector's TFP. Workers from the two groups are perfectly substitutable in the production of intermediate goods.

Markets are perfectly competitive. Let the intermediate good be the numeraire. Then, from (2), the wage rate *per unit of human capital* for workers in the national sector is

$$w_N = T_N. \tag{3}$$

Denote the relative price of the final good of group i by P_i and the wage rate per unit of human capital for local-sector workers of group i by w_{iL}. Since the profit of the final-good producer is $P_i Y_i - w_{iL} H_{iL} - Y_{iN}$, from the first-order conditions of the profit-maximization problem,

$$P_i \frac{\partial Y_i}{\partial H_{iL}} = w_{iL} \Leftrightarrow P_i \frac{\alpha Y_i}{H_{iL}} = w_{iL}, \tag{4}$$

$$P_i \frac{\partial Y_i}{\partial Y_{iN}} = 1 \Leftrightarrow P_i \frac{(1-\alpha) Y_i}{Y_{iN}} = 1. \tag{5}$$

From these equations,

$$w_{iL} = \frac{\alpha}{1-\alpha} \frac{Y_{iN}}{H_{iL}}. \tag{6}$$

[9]The alternative interpretation of the production part of the model is that people consume two kinds of final goods, goods produced by the national sector and goods produced by the local sector of their group, both of which use labor only. In this interpretation, (1) is the utility function and the local sector's production function is $Y_i = T_i H_{iL}$.

Because the final goods are group specific, the goods are not traded between the groups. Thus, a group's demand for intermediate goods must equal the amount of intermediate goods produced by the group's workers:[10]

$$Y_{iN} = T_N H_{iN}. \tag{7}$$

By substituting the above equation into (1) and (6), the output of the final goods and the wage rate of local-sector workers can be expressed as functions of H_{iN} and H_{iL}:

$$Y_i = (T_i H_{iL})^{\alpha}(T_N H_{iN})^{1-\alpha}, \tag{8}$$

$$w_{iL} = \frac{\alpha}{1-\alpha}\frac{T_N H_{iN}}{H_{iL}}. \tag{9}$$

From (5), (7), and (8), the relative price of the final good is also expressed as a function of the human capital variables.

$$\begin{aligned} P_i &= \frac{1}{1-\alpha}\frac{Y_{iN}}{Y_i} \\ &= \frac{1}{1-\alpha}\left(\frac{T_N H_{iN}}{T_i H_{iL}}\right)^{\alpha}. \end{aligned} \tag{10}$$

3.2 Education

The national sector requires the common-language skill, while the local sector of an ethnic group requires the group's ethnic-language skill. Assume that the common language is group 1's mother tongue. In the actual society, group 1 typically corresponds to the majority or historically dominant group. The assumption implies that the skill requirements of the two sectors are the same for group 1, whereas they differ for group 2.[11]

The assumption that only the common (local) language is used in the national (local) sector would exaggerate reality, yet it captures the fact that the language essential for tasks varies depending on occupations and sectors in multilingual societies. Using survey data from China, Dovì (2019) shows that proficiency in Mandarin is not statistically related to employment probabilities in rural areas, but it is strongly associated with employment probabilities in urban areas, where modern-sector jobs are concentrated. Azam, Chin, and Prakash (2013) argue that proficiency in English, which serves as a lingua franca in India, is essential for many management and technical jobs in the modern sector, as well as many positions in the government and the education sector. Hellerstein and Neumark (2008) discover that proficiency in English, rather than education level, explains a large part of (establishment-level) workplace segregation between Hispanics and whites in the U.S. Further, they find that (one-digit) occupation can account for Hispanic-white segregation to a similar extent as English proficiency, owing to a large overlap in the distributions of occupations and English skill among Hispanics.

Each individual has a wealth endowment a, which can be spent on education to develop the skills. Let e be the amount of educational spending. Group 1 individuals develop the skill to use their mother tongue, whereas group 2 individuals develop both the mother-tongue and common-language skills. However, they *cannot* choose the allocation of spending for developing the two

[10]Note, however, that some of the intermediate goods used for producing the final goods of group i, Y_{iN}, are produced by the other group. The proportion of Y_{iN} that is produced by themselves is $\frac{H_{iN}}{H_{1N}+H_{2N}}$, which is smaller as H_{iN} becomes smaller.

[11]As mentioned in the introduction, Yuki (2022) develops a related (but less standard) model in which ethnic groups are symmetric in every respect, and the common language is not the mother tongue of any ethnic group. Such a setting would be relevant to many sub-Saharan Africa countries in which the common language is the language of the former colonizer. By contrast, the present model would be relevant to countries in which the mother tongue of the dominant ethnic group serves as the common language, such as Bolivia, China, Ethiopia, and Turkey.

types of skills. This allocation is fixed, reflecting the fact that the government mostly determines the relative weight of common-language education and mother-tongue education in primary and lower-secondary education.

The human capital production function of group 1 individuals is

$$h_1 \equiv h_{1N} = h_{1L} = (\bar{l} + e)^{\gamma}, \gamma \in (0,1),\ \bar{l} > 0 \tag{11}$$

where h_{1N} and h_{1L} are individual human capital in the national and local sectors, respectively, which are the same, and $\bar{l}$ is a constant. The level of human capital is positive without education, reflecting that the mother-tongue skill is developed partly at home.

The human capital production functions of group 2 individuals are

$$h_{2N} = [\delta_N(1-s)e]^{\gamma}, s \in [0,1], \tag{12}$$

$$h_{2L} = (\bar{l} + \delta_L s e)^{\gamma}, \tag{13}$$

where $s \in [0,1]$ is the share of e allocated to developing the mother-tongue skill, and δ_N (δ_L) is the effectiveness of teaching the common language (mother tongue) for skill development.[12] The function for the local sector is similar to that for group 1, while the function for the national sector is different: the level of the common-language skill is zero without education because the common language is not the mother tongue of group 2.[13]

A person with wealth a can spend at most $e = a$ on education due to the absence of a credit market to finance education.[14] The next section analyzes the case in which no one is bound by the wealth constraint on educational investment; that is, everyone has enough wealth to make optimal investment. However, this case is not relevant to many developing countries in which students must rely on limited family wealth to pay for study materials, commuting costs, uniforms, and supplementary education even when public schools do not charge tuitions. Hence, Section 5 examines the general case in which some people may not have sufficient wealth for optimal investment. Further, it analyzes important issues that are not present in the unconstrained case.

After making an educational choice, each person chooses a sector to work in and receives earnings, which, together with the remaining wealth $a - e$, are spent on final goods for consumption.

4 Unconstrained Case

This section considers the case in which everyone has enough wealth to make optimal educational investment. Although this case may not be applicable to many developing countries, it is easier to examine and is helpful in understanding the general case in the next section.

[12] $\delta_N < \delta_L$ would be reasonable considering the higher cost-effectiveness of mother-tongue education in skill development (Vaillancourt and Grin, 2000), although the results do not depend on it. The effectiveness of education for group 1 is normalized to 1 without loss of generality.

[13] For analytical tractability, the model abstracts from nonlanguage skills. When a nonlanguage skill is also an input of human capital production functions, the functions should be such that the language skill and nonlanguage skill are complementary (i.e., the language skill stimulates the development of the nonlanguage skill and vice versa); and human capital is positive even without education in the nonlanguage skill (in particular, manual and social skills are largely acquired outside school). A natural extension of the original production functions satisfying these properties is $h_1 = (\bar{l}+qe)^{\gamma}[\bar{l}_n+(1-q)e]^{\gamma_n}$ for group 1, $h_{2N} = [\delta_N q(1-s)e]^{\gamma}[\bar{l}_n+(1-q)e]^{\gamma_n}$ and $h_{2L} = (\bar{l}+\delta_L qse)^{\gamma}[\bar{l}_n+(1-q)e]^{\gamma_n}$ for group 2, where $q \in [0,1]$ is a fixed proportion of e allocated to language education, $\gamma_n \in (0,1)$, and $\bar{l}_n > 0$ is a constant. As explained in footnote 17 in Section 4, the main results would not change under such a specification.

[14] Introducing a government that partially finances education complicates the analysis but would not affect the results qualitatively.

4.1 Group 1

First, we examine how group 1 variables are determined. Because group 1's human capital is the same in the national and local sectors, the wage rate per unit of human capital in both sectors must be equal. Thus, from (3) and (9),

$$w_N = w_{1L} = T_N = \frac{\alpha}{1-\alpha}\frac{T_N H_{1N}}{H_{1L}}. \tag{14}$$

Hence, the ratio of the group's human capital in the national sector to that in the local sector is constant:

$$\frac{H_{1N}}{H_{1L}} = \frac{1-\alpha}{\alpha}.$$

By substituting this equation into (10), the relative price of the final good for group 1 is

$$P_1 = \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\left(\frac{T_N}{T_1}\right)^\alpha. \tag{15}$$

Because wealth endowment can alternatively be spent on final-good consumption, the income (net of the cost of education) maximization problem for a group 1 individual is

$$\max_e\{w_N h_1 - P_1 e\} = \max_e\{T_N(\bar{l}+e)^\gamma - P_1 e\}. \tag{16}$$

From the first-order condition,[15]

$$\gamma T_N(\bar{l}+e)^{\gamma-1} = P_1 \Leftrightarrow \bar{l}+e = \left(\frac{\gamma T_N}{P_1}\right)^{\frac{1}{1-\gamma}}.$$

Thus, the optimal educational spending, denoted e_1^*, equals

$$e_1^* = \left[\gamma(\alpha T_1)^\alpha((1-\alpha)T_N)^{1-\alpha}\right]^{\frac{1}{1-\gamma}} - \bar{l}. \tag{17}$$

By substituting (15) and (17) into (16), earnings net of the cost of education equal

$$\begin{aligned} &T_N\left[\gamma(\alpha T_1)^\alpha((1-\alpha)T_N)^{1-\alpha}\right]^{\frac{\gamma}{1-\gamma}} - \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\left(\frac{T_N}{T_1}\right)^\alpha\left\{\left[\gamma(\alpha T_1)^\alpha((1-\alpha)T_N)^{1-\alpha}\right]^{\frac{1}{1-\gamma}} - \bar{l}\right\} \\ &= \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\left(\frac{T_N}{T_1}\right)^\alpha\left\{(1-\gamma)\left[\gamma^\gamma(\alpha T_1)^\alpha((1-\alpha)T_N)^{1-\alpha}\right]^{\frac{1}{1-\gamma}} + \bar{l}\right\}. \end{aligned} \tag{18}$$

The consumption of a group 1 worker with wealth a equals, from (15) and (18),

$$\begin{aligned} c_1^*(a) &= \frac{w_N h_1^* - P_1 e_1^*}{P_1} + a \\ &= (1-\gamma)\left[\gamma^\gamma(\alpha T_1)^\alpha((1-\alpha)T_N)^{1-\alpha}\right]^{\frac{1}{1-\gamma}} + \bar{l} + a. \end{aligned} \tag{19}$$

4.2 Group 2

4.2.1 Individuals choosing the national sector

Now, we analyze the determination of group 2 variables. First, consider those who become national-sector workers. Their income-maximization problem is

$$\max_e\{w_N h_{2N} - P_2 e\} = \max_e\{T_N[\delta_N(1-s)e]^\gamma - P_2 e\} \quad \text{(from (3))}. \tag{20}$$

[15] For $e > 0$ to be optimal, it is assumed that T_N and T_1 are large enough that $\gamma T_N \bar{l}^{\gamma-1} - P_1 > 0 \Leftrightarrow \gamma(\alpha T_1)^\alpha[(1-\alpha)T_N]^{1-\alpha}\bar{l}^{\gamma-1} > 1$ holds.

The first-order condition is

$$\gamma T_N \frac{[\delta_N(1-s)e]^\gamma}{e} - P_2 = 0.$$

From the above equation, their educational spending equals

$$\begin{aligned} e_{2N}^* &= \left\{\frac{\gamma T_N[\delta_N(1-s)]^\gamma}{P_2}\right\}^{\frac{1}{1-\gamma}} \\ &= \left\{(1-\alpha){T_2}^\alpha {T_N}^{1-\alpha}\gamma[\delta_N(1-s)]^\gamma \left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right\}^{\frac{1}{1-\gamma}} \quad \text{(from (10))}. \end{aligned} \tag{21}$$

Since their human capital is zero without education, $e_{2N}^* > 0$ holds, unless $s = 1$.

By substituting (10) and (21) into (20), their earnings net of the cost of education equal

$$w_N h_{2N}^* - P_2 e_{2N}^* = (1-\gamma)\left\{T_N\left[(1-\alpha)\gamma\delta_N(1-s)\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^{-\alpha}\right]^\gamma\right\}^{\frac{1}{1-\gamma}}. \tag{22}$$

4.2.2 Individuals choosing the local sector

Next, consider those who become local-sector workers. Their income-maximization problem is

$$\max_e\{w_{2L}h_{2L} - P_2 e\} = \max_e\{w_{2L}(\bar{l} + \delta_L s e)^\gamma - P_2 e\}. \tag{23}$$

From the first-order condition, when $\gamma\delta_L s w_{2L}\bar{l}^{\gamma-1} - P_2 > 0$—i.e., when positive e is optimal—

$$\gamma\delta_L s w_{2L}(\bar{l} + \delta_L s e)^{\gamma-1} = P_2 \Leftrightarrow \bar{l} + \delta_L s e = \left(\frac{\gamma\delta_L s w_{2L}}{P_2}\right)^{\frac{1}{1-\gamma}}.$$

Thus, their educational spending equals

$$\begin{aligned} e_{2L}^* &= \frac{1}{\delta_L s}\left[\left(\frac{\gamma\delta_L s w_{2L}}{P_2}\right)^{\frac{1}{1-\gamma}} - \bar{l}\right] \\ &= \frac{1}{\delta_L s}\left[\left(\frac{\gamma\delta_L s\alpha Y_2}{H_{2L}}\right)^{\frac{1}{1-\gamma}} - \bar{l}\right] \quad \text{(from (5))} \\ &= \frac{1}{\delta_L s}\left\{\left[\alpha\gamma\delta_L s {T_2}^\alpha {T_N}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}} - \bar{l}\right\}. \end{aligned} \tag{24}$$

By substituting (4), (10), and (24) into (23), their earnings net of the cost of education equal

$$\begin{aligned} w_{2L}h_{2L}^* - P_2 e_{2L}^* &= P_2\left[\frac{\alpha Y_2}{H_{2L}}(\bar{l}+\delta_L s e_{2L}^*)^\gamma - e_{2L}^*\right] \\ &= \frac{1}{1-\alpha}\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^\alpha\left\{(1-\gamma)\left[(\gamma\delta_L s)^\gamma \alpha {T_2}^\alpha {T_N}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}} + \frac{\bar{l}}{\delta_L s}\right\}. \end{aligned} \tag{25}$$

By contrast, when $\gamma\delta_L s w_{2L}\bar{l}^{\gamma-1} - P_2 \le 0$, it is optimal not to spend on education; i.e., $e_{2L}^* = 0$. In this case, from (9), their earnings equal

$$w_{2L}\bar{l}^\gamma = \frac{\alpha}{1-\alpha}T_N\frac{H_{2N}}{H_{2L}}\bar{l}^\gamma. \tag{26}$$

4.2.3 Indifference condition

Since everyone has enough wealth to receive optimal education for either sector, they are indifferent between the sectors, which implies that the net earnings of the two sectors are equal.

Thus, when $e^*_{2L} = 0$, from (22) and (26), the following must hold:

$$(1-\gamma)\left\{T_N\left[(1-\alpha)\gamma\delta_N(1-s)\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^{-\alpha}\right]^{\gamma}\right\}^{\frac{1}{1-\gamma}} = \frac{\alpha}{1-\alpha}T_N\frac{H_{2N}}{H_{2L}}\bar{l}^{\gamma} \tag{27}$$

$$\Leftrightarrow \frac{H_{2N}}{H_{2L}} = \left\{\left(\frac{1-\alpha}{\alpha}\frac{1-\gamma}{\bar{l}^{\gamma}}\right)^{1-\gamma}\left[(1-\alpha)\gamma\delta_N(1-s)T_2{}^{\alpha}T_N{}^{1-\alpha}\right]^{\gamma}\right\}^{\frac{1}{1-\gamma(1-\alpha)}}. \tag{28}$$

When $e^*_{2L} > 0$, from (22) and (25), the indifference condition is

$$(1-\gamma)\left\{T_N\left[(1-\alpha)\gamma\delta_N(1-s)\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^{-\alpha}\right]^{\gamma}\right\}^{\frac{1}{1-\gamma}} = \frac{\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^{\alpha}}{1-\alpha}\left\{(1-\gamma)\left[(\gamma\delta_L s)^{\gamma}\alpha T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}} + \frac{\bar{l}}{\delta_L s}\right\} \tag{29}$$

Once $\frac{H_{2N}}{H_{2L}}$ is determined from the indifference condition, e^*_{2N}, e^*_{2L} when it is positive, and P_2 are determined from (21), (24), and (10), respectively.[16]

Finally, consumption of a group 2 individual with wealth (endowment) a is determined from

$$\begin{aligned} c^*_2(a) &= \frac{w_N h^*_{2N} - P_2 e^*_{2N}}{P_2} + a \\ &= (1-\alpha)\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^{-\alpha}(1-\gamma)\left\{T_N\left[(1-\alpha)\gamma\delta_N(1-s)\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^{-\alpha}\right]^{\gamma}\right\}^{\frac{1}{1-\gamma}} + a \quad \text{(from (10) and (22))} \\ &= (1-\gamma)\left\{(1-\alpha)\left[\gamma\delta_N(1-s)\right]^{\gamma}T_2{}^{\alpha}T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right\}^{\frac{1}{1-\gamma}} + a. \end{aligned} \tag{30}$$

4.3 Results

Based on the above analysis, this subsection aims to answer the following questions regarding group 2 individuals: what is the desirable combination of the two types of education in terms of future earnings and consumption, and what is the desirable combination in terms of the mother-tongue skill? Additionally, it examines the effect of the education policy for the subordinate group on between-group inequalities in skills, earnings, and consumption.

4.3.1 Effect of s on e^*_{2L}

The previous subsection showed that educational spending of those who subsequently choose the local sector is either zero or positive. The next lemma shows that whether $e^*_{2L} = 0$ or $e^*_{2L} > 0$ depends on the weight on teaching the mother tongue, s.

Lemma 1 *Suppose that all group 2 individuals have sufficient wealth endowment to achieve optimal education and that T_2 and T_N are not extremely low. Then,*

[16] Further, once $\frac{H_{2N}}{H_{2L}}$ is determined, H_{2N}, H_{2L}, and the number of workers in each sector, L_{2N} and $L_{2L} = L_2 - L_{2N}$, are determined from $H_{2N} = [\delta_N(1-s)e^*_{2N}]^{\gamma}L_{2N}$ and $H_{2L} = (\bar{l} + \delta_L s e^*_{2L})^{\gamma}L_{2L}$. In particular, when $e^*_{2L} = 0$, $\frac{L_{2N}}{L_{2L}} = \frac{1-\alpha}{\alpha}(1-\gamma)$, while when $e^*_{2L} > 0$, $\frac{L_{2N}}{L_{2L}} = \left[\frac{H_{2N}}{H_{2L}}\left(\frac{\alpha}{1-\alpha}\frac{\delta_L s}{\delta_N(1-s)}\right)^{\gamma}\right]^{\frac{1}{1-\gamma}}$.

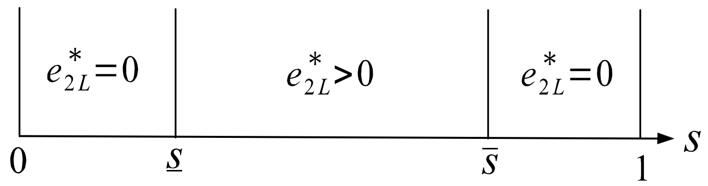


Figure 1: Lemma 1 (i)

(*i*) *Two critical values of s exist, denoted as $\underline{s}, \overline{s} \in (0,1)$ ($\underline{s} < 1-\gamma(1-\alpha) < \overline{s}$), such that those who choose the local sector do not spend on education, i.e., $e^*_{2L} = 0$, for $s \leq \underline{s}$ and $s \geq \overline{s}$, while their educational spending is positive, i.e., $e^*_{2L} > 0$, for $s \in (\underline{s}, \overline{s})$.*

(*ii*) *$\underline{s}$ ($\overline{s}$) decreases (increases) with T_2, T_N, δ_N, and δ_L.*

Figure 1 shows the result. When the share of educational spending allocated to developing the skill useful in the local sector, i.e, the mother-tongue skill, is very low or *very high*, i.e., for $s \leq \underline{s}$ and $s \geq \overline{s}$, those who subsequently choose the local sector do not invest in education, i.e., $e^*_{2L} = 0$, while when the allocation is balanced, i.e., for $s \in (\underline{s}, \overline{s})$, they invest in education, i.e., $e^*_{2L} > 0$.[17]

The result can be understood by examining the marginal return to educational investment at $e = 0$ (in units of the final good) for those who choose the local sector, which equals $\frac{w_{2L}}{P_2}\frac{\partial h_{2L}}{\partial e}|_{e=0} - 1 = \alpha T_2{}^{\alpha} T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha} \gamma\delta_L s\bar{l}^{\gamma-1} - 1$ from (23), (8), and (10). When s is very low, the marginal return is negative, because the marginal effect of educational spending on human capital for the local sector ($\gamma\delta_L s\bar{l}^{\gamma-1}$) is very small and this dominates the sector's high wage rate due to large $\frac{H_{2N}}{H_{2L}}$.[18] By contrast, when s is very high, the marginal return is negative, because $\frac{w_{2L}}{P_2}$ becomes very low due to small $\frac{H_{2N}}{H_{2L}}$ and this dominates the large marginal effect of e on h_{2L}. Higher sectoral TFPs (total factor productivities), T_2, T_N, and higher education efficacy for the group, δ_N, δ_L, increase the range of s over which educational investment pays off because the wage rate is higher.[19]

The result that future local-sector workers do not spend on education when s is very low or very high may initially seem implausible, given that even in poor countries, the vast majority of students receive some education. However, the difference between the model and the real world arises from the fact that, for tractability, the model abstracts from motives for attending school other than the investment motive. These include consumption motives, such as the enjoyment of learning or attending school, and social motives, such as pressure from family or community to attend school. Despite this limitation, the result reveals an important source of poor academic performance among students in many countries. According to the result, students who go into the local sector have weak incentives to study, and thus perform poorly, either because what they learn has little relevance to their future work in the local sector (when s is very low), or because the sector's wage rate is low due to deficient skills among workers in the complementary national sector (when s is very high).

[17] When the human capital production functions include a nonlanguage skill as an input and are represented by the equations in footnote 13 of Section 3, unlike in the original specification, $e^*_{2L} > 0$ at very low s; i.e., $\underline{s}$ does not exist, when T_N, T_2, δ_N, and δ_L are high. This is because the return to educational expenditure on the nonlanguage skill does not depend on s and is positive even at $e = 0$. However, the intuitions behind the lemmas (except the result on $\underline{s}$) and the propositions below remain unchanged; hence, the main results would not change qualitatively.

[18] This is because a small s leads to a large h_{2N} and a small h_{2L}. The next lemma, Lemma 2, formally shows that $\frac{H_{2N}}{H_{2L}}$ is large when s is small.

[19] An increase in T_N and δ_N raises $\frac{w_{2L}}{P_2}$, because workers in the local sector and the intermediate good produced in the national sector are complementary in the production of the final good.

4.3.2 Effect of s on the net earnings and consumption of group 2

Next, the effect of s on net earnings (in units of the final good) and consumption of group 2 individuals is examined. The previous subsection showed that these variables for national-sector workers depend negatively on s and $\frac{H_{2N}}{H_{2L}}$ (equation (30)). How does $\frac{H_{2N}}{H_{2L}}$ depend on s?

Lemma 2 *Suppose that all individuals in group 2 have sufficient wealth to obtain optimal education. Then, $\frac{H_{2N}}{H_{2L}}$ decreases with s.*

Given $\frac{H_{2N}}{H_{2L}}$, an increase in s makes the local sector more attractive: as s increases, h_{2N} decreases, thereby reducing net earnings in the national sector, while h_{2L} rises, leading to increased net earnings in the local sector when $e^*_{2L} > 0$, and these variables remain unchanged when $e^*_{2L} = 0$. In contrast, given s, a *decrease* in $\frac{H_{2N}}{H_{2L}}$ makes the national sector more attractive: a decrease in $\frac{H_{2N}}{H_{2L}}$ lowers P_2 (due to an increase in $\frac{Y_2}{Y_{2N}}$) and increases net earnings in the national sector, $\frac{w_N}{P_2}h_{2N}-e=\frac{T_N}{P_2}[\delta_N(1-s)e^*_{2N}]^\gamma-e^*_{2N}$, while reducing $\frac{w_{2L}}{P_2}=\alpha\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^{1-\alpha}$ and net earnings in the local sector. Hence, $\frac{H_{2N}}{H_{2L}}$ must decrease with s for workers to remain indifferent between the sectors.

Thus, the direct effect of s on the net earnings and consumption of national-sector workers is negative, while the indirect effect through $\frac{H_{2N}}{H_{2L}}$ is positive. Which effect dominates? The following lemma examines the total effect separately when $e^*_{2L} = 0$ and when $e^*_{2L} > 0$.

Lemma 3 *Suppose that all individuals in group 2 have sufficient wealth to acquire optimal education.*

(i) When $e^*_{2L} = 0$, earnings net of the cost of education and consumption of group 2 individuals decrease with s.

(ii) When $e^*_{2L} > 0$, if T_N, T_2, δ_N, and δ_L are low, the net earnings and consumption of group 2 individuals decrease with s; otherwise, they decrease with s for small s, increase with s for intermediate s, and decrease with s for large s.

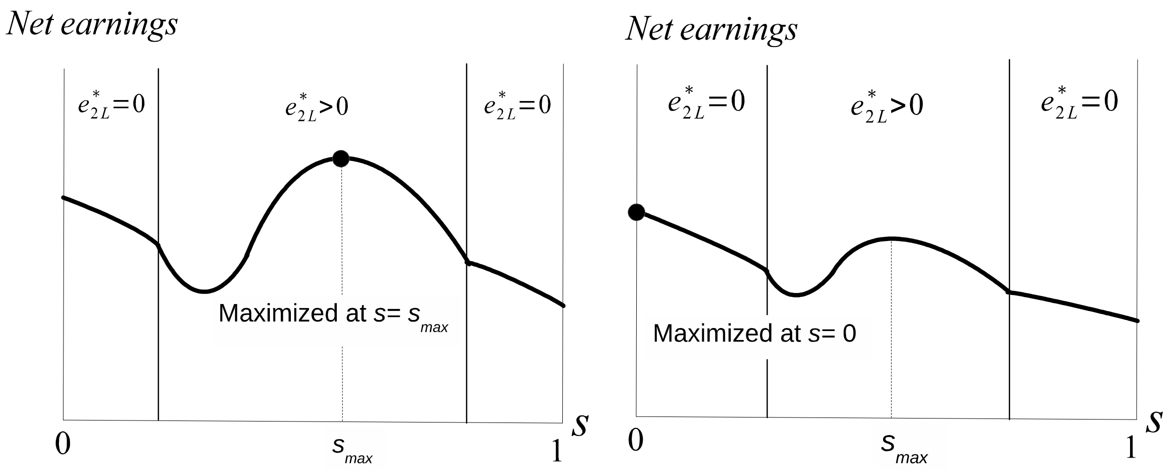
When future local-sector workers do not invest in education, i.e., $e^*_{2L} = 0$, net earnings and consumption *decrease* as the proportion of educational expenditure allocated to teaching the mother tongue increases, i.e., s increases. Earnings in the local sector decrease because $\frac{w_{2L}}{P_2}$ falls due to a decrease in $\frac{H_{2N}}{H_{2L}}$ while h_{2L} remains unchanged. Since individuals are indifferent between the sectors, the same applies to net earnings in the national sector and consumption.

By contrast, when $e^*_{2L} > 0$, if sectoral TFPs (T_2, T_N) and the effectiveness of education for the subordinate group (δ_N, δ_L) are low, net earnings and consumption *decrease* with the weight on teaching the mother tongue; otherwise, the shape of the relationship between the weight and these variables is *bimodal*: they decrease with s for small s, increase with s for intermediate s, and decrease with s for large s. An increase in s negatively affects net earnings in the national sector, $\frac{w_N}{P_2}h_{2N}-e=\frac{T_N}{P_2}[\delta_N(1-s)e^*_{2N}]^\gamma-e^*_{2N}$, as well as consumption through a decrease in h_{2N}, while it positively affects through decreases in $\frac{H_{2N}}{H_{2L}}$ and thus P_2. If T_N, T_2, δ_N, and δ_L are low, or if s is small or large, the former effect dominates the latter effect and net earnings and consumption decrease with s; otherwise, the latter effect dominates and they increase with s.

The lemma implicitly assumes that only one of $e^*_{2L} = 0$ and $e^*_{2L} > 0$ holds for any s, which is not true, as was shown in Lemma 1 (Figure 1). By taking into account how s affects whether $e^*_{2L} = 0$ or $e^*_{2L} > 0$, the next proposition examines the effect of s on net earnings and consumption.

Proposition 1 *Suppose that everyone in group 2 have sufficient wealth to obtain optimal education.*

(i) If T_N, T_2, δ_N, and δ_L are low, the net earnings and consumption of group 2 individuals decrease with s for any s and are maximized at $s = 0$.



(a) Relatively high T_N, T_2, δ_N, and δ_L (b) Relatively low T_N, T_2, δ_N, and δ_L

Figure 2: Relationship between s and net earnings (in units of the final good) when T_N, T_2, δ_N, and δ_L are not low in the unconstrained case (Proposition 1 (ii))

(*ii*) *Otherwise, the net earnings and consumption decrease with s for small s and large s (at least for $s \leq \underline{s}$ and $s \geq \min\{\alpha, \overline{s}\}$) and increase with s for intermediate s. If T_N, T_2, δ_N, and δ_L are sufficiently high, they are maximized at an intermediate s, where the s maximizing these variables increases with T_N, T_2, δ_N, and δ_L; otherwise, they are maximized at $s = 0$.*

If sectoral TFPs (T_2, T_N) and the efficacy of education for the subordinate group (δ_N, δ_L) are low, the net earnings and consumption of the group *decrease* with the weight on teaching the mother tongue; thus, the highest economic outcomes are attained when educational expenditure is entirely allocated to teaching the common language.

Otherwise, their net earnings and consumption decrease with s for small s and large s (at least when $e^*_{2L} = 0$) and increase with s for intermediate s. If T_2, T_N, δ_N, and δ_L are sufficiently high, these variables are highest at an intermediate s (i.e., a balanced allocation of expenditure to the two types of education maximizes the economic outcomes); otherwise, they are highest at $s = 0$. In other words, for balanced bilingual education to be economically beneficial to the subordinate group, sectoral TFPs and the effectiveness of education must be sufficiently high. Further, the s maximizing the economic outcomes increases with the exogenous variables; that is, as the TFPs increase and education becomes more effective for the subordinate group, a greater emphasis on teaching the mother tongue becomes desirable. Figure 2 illustrates the relationship between s and net earnings (in units of the final good) for this case: (a) when T_2, T_N, δ_N, and δ_L are relatively high, and (b) when they are relatively low. (Similar figures can be drawn for consumption, which equals net earnings plus wealth.) In both cases, the shape of the graph is similar, with net earnings peaking at $s = 0$ when $e^*_{2L} = 0$ and at $s = s_{\max}$ when $e^*_{2L} > 0$. However, in (a), the value at $s = s_{\max}$ exceeds that at $s = 0$, while the value at $s = 0$ is higher in (b).

In the real world, the sectoral TFPs and the effectiveness of education are closely related to a country's level of economic and social development. Hence, the result implies that, in general, if the development level is low, net earnings and consumption are highest when language education solely focuses on the common language; otherwise, they are highest under balanced bilingual education.[20] Angrist and Lavy (1997) show that a policy change in Morocco in the 1980s, which replaced French with Arabic as the medium of instruction in postprimary education, greatly reduced returns to schooling.[21] Cappellari and Di Paolo (2018) find that the 1983 reform in Catalonia, which substantially increased the weight on Catalan in mandatory education (from a very low weight to one slightly higher than Spanish), had a positive effect on earnings. Consistent with the result, the findings suggest that a large increase in the weight on mother-tongue education lowered wages in a developing country (Morocco) and raised wages in a developed region (Catalonia).

Even among developing countries, there are large disparities in the productivity of primary education, which explains most of the gaps in test scores (Singh, 2020).[22] The above result implies that net earnings and consumption are highest under balanced bilingual education only when the country has an effective education system; with an ineffective education system, they are highest under language education exclusively focused on the common language.

What is crucial to the results is the assumption that human capital in the local sector is positive without education, i.e., $\bar{l} > 0$. If $\bar{l} = 0$, *regardless of* the values of T_2, T_N δ_N, and δ_L, $e^*_{2L} > 0$ holds for any positive s, and net earnings and consumption are highest at intermediate $s = \alpha$, increasing (decreasing) with s for smaller (greater) s. This assumption renders education unprofitable for future local-sector workers when s is very low or very high, causing net earnings and consumption to decrease with s for low s. Consequently, these variables can be highest at $s = 0$.

4.3.3 Effect of s on the mother-tongue skill of group 2

One of the primary concerns for language and education experts is how the relative weight of the two types of education affects the skill development of students. In particular, the mother-tongue skill is an essential skill in daily life.[23] Hence, the following proposition examines the effect of s on human capital for the local sector.[24]

Proposition 2 *Suppose that all individuals in group 2 have sufficient wealth to obtain optimal education.* h_{2L} *is highest at* $s = 1-(1-\alpha)\gamma \in (\underline{s}, \overline{s})$, *which is greater than the* s *maximizing net earnings and consumption, increases (decreases) with* s *for smaller (greater)* s, *and is lowest and equals* $h_{2L} = \bar{l}^{\gamma}$ *for* $s \leq \underline{s}$ *and* $s \geq \overline{s}$ (at $s = 0, 1$) *for those who choose the local (national) sector.*

Human capital for the local sector is highest at an intermediate s (i.e., $s = 1-(1-\alpha)\gamma \in (\underline{s}, \overline{s})$), which is greater than the s that maximizes net earnings and consumption. For those who choose the local sector, h_{2L} equals $h^*_{2L} \equiv (\bar{l} + \delta_L s e^*_{2L})^{\gamma}$ and is lowest when s is small or large enough that

[20] When the model is applied to countries at various levels of development, it is necessary to take into account the fact that a significant portion of the cost of education is the cost of hiring teachers and other staff, which increases with the wage. This can be considered by replacing e in human capital production functions with e deflated by the unit cost of education, which increases with $w_N = T_N$. As long as the unit cost changes less than proportionately to the wage, which is plausible given nonlabor cost is non-negligible, the qualitative results remain the same.

[21] Although French is not the ethnic language of any group, at that time, it was the dominant language in many parts of the modern sector, such as public administration, foreign trade, and science and technology. Consequently, children of workers in these areas had a significant advantage in acquiring French language skills. Hence, the model's result would apply to this environment, if one considers these children as the dominant group in the model.

[22] Based on panel data from four developing countries with widely differing test scores in primary school students (Ethiopia, India, Peru, and Vietnam), Singh (2020) shows that cross-country differences in the productivity of primary education explain most of the differences in student achievement.

[23] Note, however, that the model abstracts from the utility of the mother-tongue skill in daily life; the skill affects utility only through earnings.

[24] A previous version of the paper (Yuki, 2021) analyzes the effect on human capital for the national sector as well. Since its importance is low, the analysis is omitted to save space.

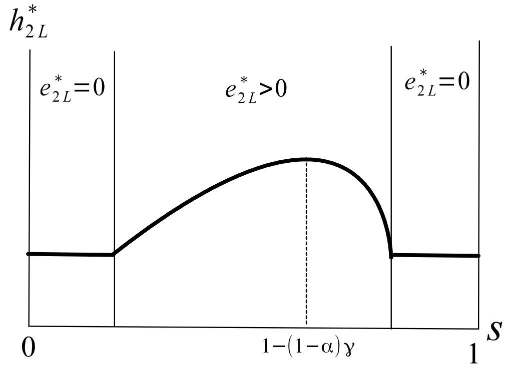


Figure 3: Relationship between s and h^*_{2L} in the unconstrained case

$e^*_{2L} = 0$ (i.e., $s \leq \underline{s}$ and $s \geq \overline{s}$), while for national-sector workers, h_{2L} equals $(\bar{l} + \delta_L s e^*_{2N})^\gamma$ and is lowest at $s = 0, 1$. Figure 3 illustrates the relationship between s and h^*_{2L}.

The result implies that balanced bilingual education is crucial for developing the mother-tongue skill. This result aligns with the general consensus among experts on language and education that mother-tongue education is important for skill development, at least in primary education (Ball, 2011; Heugh, 2011). Further, it is consistent with previous economic research (Jain, 2017; Ramachandran, 2017) showing that significant introduction of mother-tongue education leads to increased academic skills and years of education.

4.3.4 Effect of s on inter-group inequalities

Finally, as the dominant group is not affected by the relative weight of the two types of education that the subordinate group receives, the above results directly apply to between-group inequalities in skill, net earnings, and consumption. This is summarized in the next corollary, where $T_1 \geq T_2$ is assumed to ensure that individuals from the dominant group have higher net earnings and consumption for a given level of wealth.

Corollary 1 *Suppose that $T_1 \geq T_2$ and everyone has sufficient wealth to obtain optimal education.*

(*i*) *When T_N, T_2, δ_N, and δ_L are small, inter-group inequalities in net earnings and consumption are lowest, but the inequality in human capital for the local sector is highest at $s = 0$.*

(*ii*) *When T_N, T_2, δ_N, and δ_L are sufficiently large, the inter-group inequalities are lowest when s is in the intermediate range.*

When sectoral TFPs and the efficacy of education for the subordinate group are low, language education solely focused on the common language yields the lowest inter-group inequalities in net earnings and consumption, but the highest inequality in the mother-tongue skill. In contrast, balanced bilingual education leads to a lower gap in the mother-tongue skill, but at the cost of higher gaps in other dimensions. Balanced education achieves the lowest inequalities in all dimensions only when the TFPs and the effectiveness of education are sufficiently high.

5 General Case

This section examines the general case in which some individuals may lack sufficient wealth to make optimal educational investment. This case is particularly relevant to many developing countries, where students often rely on limited family wealth to cover study materials, transportation,

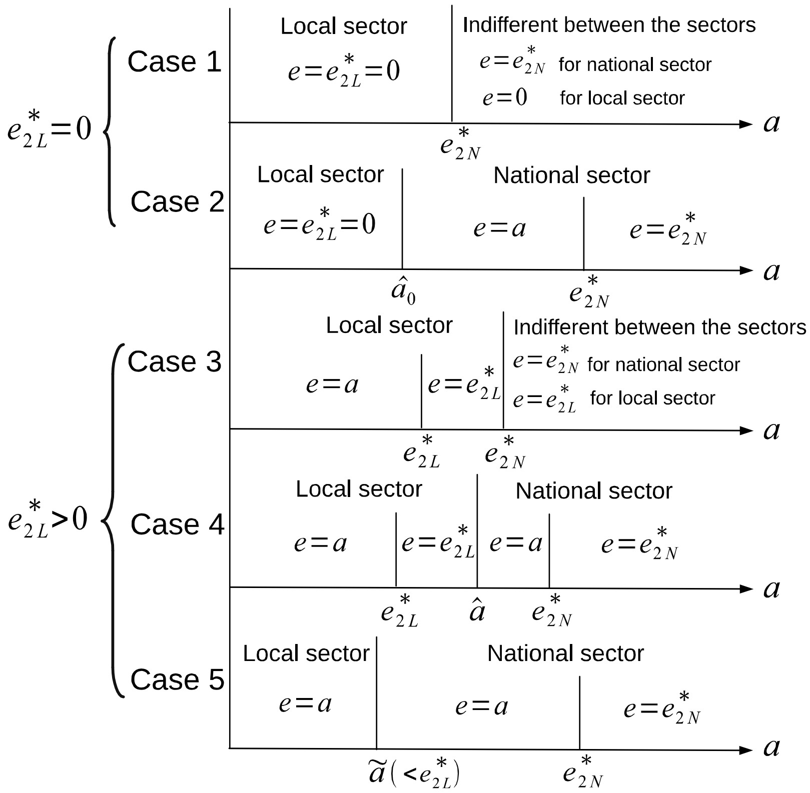


Figure 4: Dependence of educational and sectoral choices of group 2 individuals on wealth

uniforms, and extra education, even when public schools are tuition-free. The crucial difference from the unconstrained case is that educational spending and sectoral choices may vary depending on individual wealth. This suggests that wealth distribution generally matters. Thus, the section first analyzes how wealth distribution affects the net earnings and consumption of individuals and intra-group inequalities. Then, as in the previous section, it explores the effect of the relative weight of the two types of education received by the subordinate group on their net earnings, consumption, and skill, as well as inter-group inequalities. Let $F(a)$ be the distribution function of wealth for group 2, which is differentiable for $a > 0$ and allows for a mass at $a = 0$; i.e., $F(0) > 0$.[25]

5.1 Group 1

Since the human capital of a group 1 individual is the same in the two sectors, as before, the wage rates per unit of human capital of the sectors are equal, and everyone is indifferent between them. However, unlike the unconstrained case, those with wealth $a < e_1^*$ cannot make optimal educational investment (i.e., $e = a < e_1^*$), leading to variations in net earnings among those with different levels of wealth (except among those with $a \geq e_1^*$). Appendix A shows that the levels of net earnings and consumption of the group's individuals do not depend on the distribution of wealth.

5.2 Group 2

The asymmetry in the human capital production functions of group 2—without education, human capital for the national sector is zero, but human capital for the local sector is positive—implies

[25] As shown just below, the individual outcomes of group 1 are not affected by the distribution of wealth.

that only individuals with a specific level (or levels) of wealth are indifferent between the sectors. Those with greater wealth, who invest more in education, have a comparative advantage in the national sector and opt for it, while those with lower wealth choose the local sector.

To be more accurate, five qualitatively distinctive cases can arise, depending on whether $e^*_{2L} = 0$ or $e^*_{2L} > 0$ and the level(s) of wealth at which the indifference condition holds. Figure 4 illustrates how educational and sectoral choices depend on wealth for each case. (Readers not interested in details may skip the next three paragraphs.)

As shown in the figure, in Cases 1 and 2, $e^*_{2L} = 0$ holds, so those who strictly prefer the local sector (i.e., those with $a < e^*_{2N}$ in Case 1 and those with $a < \widehat{a}_0[< e^*_{2N}]$ in Case 2) do not spend on education (i.e., $e = e^*_{2L} = 0$). In Case 1, those with $a \geq e^*_{2N}$ are indifferent between the sectors, and those who choose the national (local) sector spend $e = e^*_{2N}$ $(e = 0)$; in Case 2, those with $a > \widehat{a}_0$ strictly prefer the national sector and spend $e = a$ if $a < e^*_{2N}$, and $e = e^*_{2N}$ otherwise.

In Cases 3, 4, and 5, $e^*_{2L} > 0$ holds, so those who strictly prefer the local sector (i.e., those with $a < e^*_{2N}$ in Case 3, those with $a < \widehat{a} \in [e^*_{2L}, e^*_{2N})$ in Case 4, and those with $a < \widetilde{a}[< e^*_{2L}]$ in Case 5) choose positive e. ($e^*_{2N} > e^*_{2L}$ is proved in Appendix B.) In Cases 3 and 4, they spend $e = a$ if $a < e^*_{2L}$ and $e = e^*_{2L}$ otherwise; in Case 5, they spend $e = a$. Regarding individuals with wealth above the threshold, in Case 3, they are indifferent between the sectors, and those who choose the national (local) sector spend $e = e^*_{2N}$ $(e = e^*_{2L})$; in Cases 4 and 5, they strictly prefer the national sector and spend $e = a$ if $a < e^*_{2N}$, and $e = e^*_{2N}$ otherwise.

Appendix A explains how the threshold levels of wealth and other endogenous variables such as $\frac{H_{2N}}{H_{2L}}$, net earnings, and consumption are determined in each case.

Under what conditions are each of the five cases realized? The next lemma shows that it depends on the distribution of wealth and the weight on teaching the mother tongue. The proofs of the lemmas and propositions for the general case are provided in Online Appendix C.

Lemma 4 (*i*) *When $s \leq \underline{s}$ or $s \geq \overline{s}$, $e^*_{2L} = 0$. If a relatively high (low) proportion of individuals in group 2 have sufficient wealth for education (i.e., when $F(e^*_{2N})$ is relatively low [high]), Case 1 (Case 2) is realized.*[26]

(*ii*) *When $s \in (\underline{s}, \overline{s})$, as the proportion of those with relatively large wealth becomes lower (i.e., as $F(a)$ becomes higher for a given a), the realized equilibrium changes in the following order: Case 3, Case 4, Case 5 (except when s is close to $\underline{s}$ or $\overline{s}$), and Case 2.*[27]

Figure 5 illustrates the lemma. When $s \leq \underline{s}$ or $s \geq \overline{s}$, $e^*_{2L} = 0$, and either Case 1 or Case 2 is realized. Given s, Case 1 (Case 2) occurs when a relatively high (low) proportion of individuals in group 2 have sufficient wealth for education (i.e., when the cumulative distribution $F(a)$ at $a = e^*_{2N}$ is relatively low [high]). When $s \in (\underline{s}, \overline{s})$, all the cases except Case 1 can occur. Given s, as the proportion of those with relatively large wealth becomes lower (i.e., as $F(a)$ becomes higher for a given a), the realized equilibrium changes in the following order: Case 3, Case 4, Case 5

[26]The proof of the proposition also shows, as illustrated in Figure 5, that given the distribution of wealth, Case 2 (Case 1) is realized when s is low (high), and, as the proportion falls, the region of Case 2 (Case 1) expands (shrinks).

[27]Other results reflected in Figure 5 are as follows. Given the distribution of wealth, Case 2 is realized when s is relatively low or high, except when the proportion of those with relatively large wealth is extremely low, in which case Case 2 is realized for any s. More precisely, when $s \in (\underline{s}, \overline{s})$, Case 2 is realized at least for $s \in (\underline{s}, \underline{s}(F))$ and for $s \in (\overline{s}(F), \overline{s})$, where $\underline{s}(F)$ $(\overline{s}(F))$ is the critical s below (above) which $e^*_{2L} = 0$ holds, and $\underline{s}(F)$ increases $(\overline{s}(F)$ decreases) as the proportion of those with relatively large wealth falls. (It is not analytically clear whether other critical values exist between $\underline{s}(F)$ and $\overline{s}(F)$, but numerical simulations with truncated lognormal distributions suggest other thresholds do not exist.) The dividing line between Case 3 and Case 4 and the one between Case 1 and Case 2 intersect at $s = \underline{s}$, $\overline{s}$, while the one between Case 4 and Case 5 does not intersect with them.

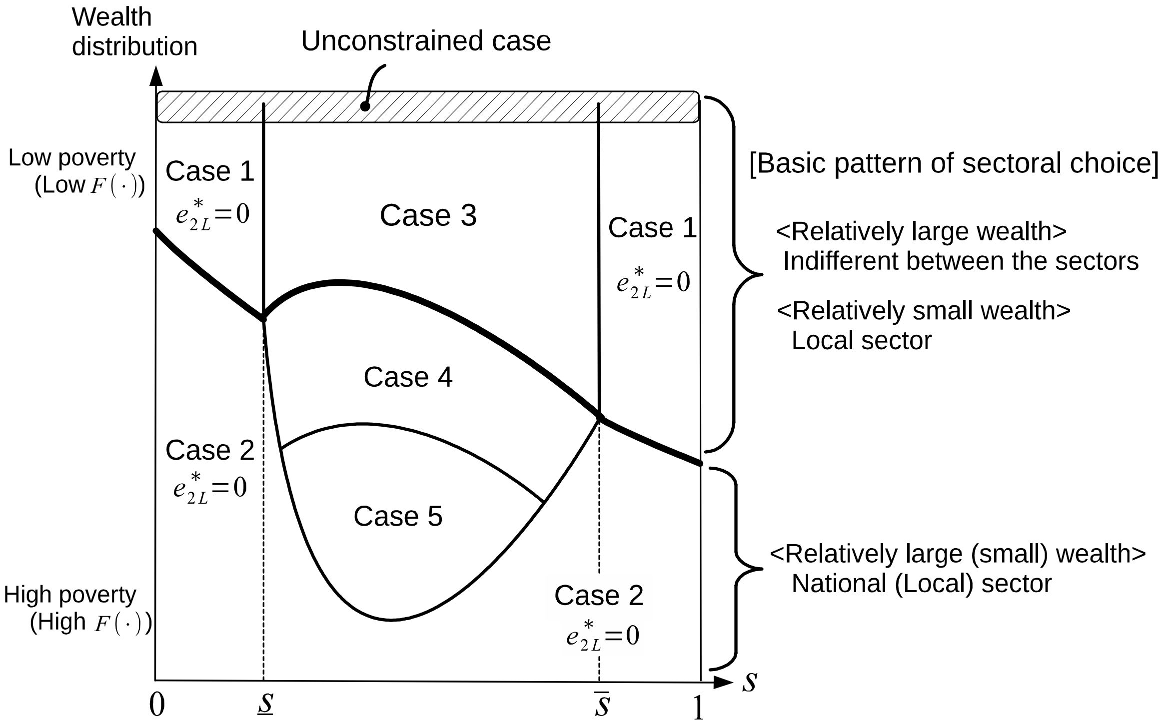


Figure 5: Wealth distribution and realized cases (Lemma 4) with Basic pattern of sectoral choice

(except when s is close to $\underline{s}$ or $\overline{s}$), and Case 2.[28] Note that the unconstrained case examined in the previous section is a special case of Cases 1 and 3, where everyone has enough wealth to make optimal educational investment (i.e., $a \geq e^*_{2N} > e^*_{2L}$ holds for everyone).[29]

The figure also shows the basic pattern of sectoral choice, based on Figure 4. In Cases 1 and 3, where the proportion of individuals with sufficient wealth for education is relatively high, those with relatively large wealth are indifferent between the sectors, while those with smaller wealth choose the local sector. In the remaining cases, those with relatively large (small) wealth choose the national (local) sector.

5.3 Effects of the distribution of wealth

As mentioned earlier, the distribution of wealth does not affect the net earnings and consumption of group 1 individuals with given levels of wealth. What about group 2 individuals? Since their net earnings and consumption depend on $\frac{H_{2N}}{H_{2L}}$, the next lemma examines how the wealth distribution affects $\frac{H_{2N}}{H_{2L}}$.

[28]The shapes of the dividing lines between Case 3 and Case 4, between Case 4 and Case 5, and between Case 2 and Cases 4 and 5 may vary from those depicted in the figure, yet the results do not depend on the specific shapes.

[29]The occurrence of each case depends also on sectoral productivities, T_N, T_2, and the effectiveness of education for group 2, δ_N, δ_L. When s is low or high enough that $e^*_{2L} = 0$, Case 1 (Case 2) occurs when these variables are relatively low (high). When s is intermediate and thus $e^*_{2L} > 0$, analytical results are not obtained, but numerical simulations suggest that as levels of these variables increase, the realized case changes from Case 3 to Case 4, and then to Case 5. Intuitively, given a wealth distribution, higher values of T_N, T_2, δ_N, or δ_L make educational investment more profitable and raise e^*_{2N} and e^*_{2L}, leading to a lower proportion of individuals capable of making optimal investment.

Lemma 5 [30]

(*i*) *If the proportion of individuals with relatively large wealth in group 2 is high enough* ($F(a)$ *is low enough for a given* a) *that Case 1 or Case 3 occurs, a change in the proportion of such individuals does not affect* $\frac{H_{2N}}{H_{2L}}$.

(*ii*) *Otherwise, as the proportion of those with relatively large wealth decreases,* $\frac{H_{2N}}{H_{2L}}$ *becomes smaller.*

If the proportion of individuals with relatively large wealth in group 2 is high enough ($F(a)$ is low enough for a given a) that Cases 1 or 3 is realized, the distribution of wealth does not affect $\frac{H_{2N}}{H_{2L}}$, as in group 1. As the proportion of such individuals decreases, the percentage of those who can financially access the national sector decreases. However, a higher share of them, who are indifferent between the sectors, choose the sector (footnote 30), thereby keeping $\frac{H_{2N}}{H_{2L}}$ unchanged.

By contrast, if the proportion of relatively wealthy individuals is not high and thus Cases 2, 4, or 5 occurs, $\frac{H_{2N}}{H_{2L}}$ decreases as the proportion of such individuals decreases. In these cases, those with large wealth uniformly choose the national sector. Hence, a decline in the fraction of people accessible to the national sector results in a decrease in $\frac{H_{2N}}{H_{2L}}$, although a decrease in the wealth threshold satisfying the indifference condition (footnote 30) mitigates the decrease in $\frac{H_{2N}}{H_{2L}}$.

The result that the distribution of wealth affects $\frac{H_{2N}}{H_{2L}}$ in Cases 2, 4, and 5 implies that the net earnings and consumption of group 2 individuals are influenced by the distribution in these cases. The next proposition shows how the distribution of wealth affects the levels of these variables and their inequalities between national- and local-sector workers.

Proposition 3 *Suppose that the proportion of individuals with relatively large wealth in group 2 decreases.*

(*i*) *If the proportion of such individuals is high enough that either Cases 1 or 3 is realized, as in group 1, the net earnings and consumption of individuals with given wealth do not change. The change in the proportion only directly affects the inequalities in these variables.*

(*ii*) *Otherwise* (*thus, Cases 2, 4, or 5 is realized*), *the net earnings and consumption of those with relatively large* (*small*) *wealth who choose the national* (*local*) *sector increase* (*decrease*). *A decrease in the proportion exacerbates the earnings and consumption disparities between any pair of national- and local-sector workers with given levels of wealth.*

If the proportion of those with relatively large wealth in the subordinate group is high enough that Cases 1 or 3 is realized, as in the dominant group, a change in the proportion does not affect $\frac{H_{2N}}{H_{2L}}$. Thus, the net earnings and consumption of individuals with given wealth remain unchanged. The change in wealth distribution only directly affects the distributions of these variables.

In contrast, if the proportion of such individuals is not high and thus Cases 2, 4, or 5 is realized, a decrease in the proportion lowers $\frac{H_{2N}}{H_{2L}}$ and raises $\frac{w_{2N}}{P_2}$ (lowers $\frac{w_{2L}}{P_2}$). As a result, *for given levels of wealth*, the net earnings and consumption of those who choose the national (local) sector increase (decrease), and the earnings and consumption gaps between any pair of national- and local-sector workers widen. Hence, the change in wealth distribution affects the distributions of these variables not only directly but also indirectly by altering their values at given levels of wealth.

This result has an interesting implication for within-group inequalities in net earnings and consumption. While inequality in wealth among the dominant group only directly affects the inequalities in these variables, wealth inequality among the subordinate group has both direct and

[30] The proof of the lemma also shows that, as the proportion of individuals with relatively large wealth decreases, a higher proportion of those with $a \geq e^*_{2N}$, who are indifferent between the sectors, choose the national sector in (i), while the wealth threshold satisfying the indifference condition becomes smaller in (ii).

indirect effects on the inequalities, as long as the proportion of those with relatively large wealth is not high. Hence, an increase in wealth inequality tends to raise earnings and consumption inequalities within the subordinate group *more* than within the dominant group.[31]

Why is the indirect effect present only for group 2? This is because sectoral choices depend on individual wealth only for that group: since education is a prerequisite for the subordinate group to work in the national sector, individuals from the subordinate group with relatively large wealth opt for the national sector and those with limited wealth choose the local sector, whereas individuals from the dominant group are indifferent between the sectors. A change in wealth distribution alters the proportion of those who can afford education sufficient for the national sector, thereby affecting $\frac{H_{2N}}{H_{2L}}$, wage rates, net earnings, and consumption, unless the proportion of such individuals is more than enough for the amount of national-sector jobs.

5.4 Effects of the weight on teaching the mother tongue

This subsection examines the question analyzed in the previous section: for the subordinate group, what is the desirable combination of the two types of education in terms of future net earnings and consumption, and what is the desirable combination in terms of the mother-tongue skill? The important difference from the unconstrained case is that people with varying levels of wealth choose different levels of educational spending and different sectors. Hence, answers to the question may differ among those with different levels of wealth.

The next lemma shows that $\frac{H_{2N}}{H_{2L}}$ decreases with s, as in the unconstrained case.

Lemma 6 $\frac{H_{2N}}{H_{2L}}$ *decreases with* s.

Hence, an increase in s has a positive (negative) direct effect on the net earnings and consumption of local- (national-) sector workers through human capital, while it has a negative (positive) indirect effect by decreasing $\frac{H_{2N}}{H_{2L}}$ and thus lowering $\frac{w_{2L}}{P_2}$ (raising $\frac{w_{2N}}{P_2}$).

The next lemma examines the total effect on net earnings and consumption when $e^*_{2L} = 0$ and when $e^*_{2L} > 0$ separately.

Lemma 7 (i) *When* $e^*_{2L}=0$*, the net earnings and consumption of group 2 workers decrease with* s.

(ii) *Suppose* $e^*_{2L} > 0$.

(a) *If* T_N, T_2, δ_N, *and* δ_L *are low, the result is the same as in* (i).[32]

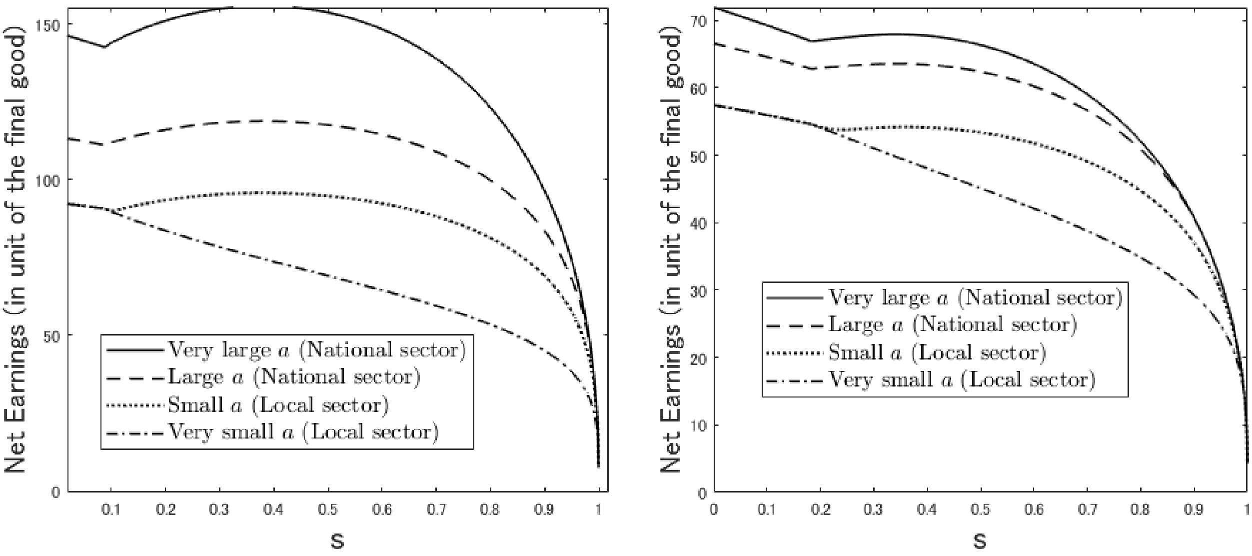
(b) *Otherwise, the net earnings and consumption of group 2 individuals with very small wealth, who choose the local sector, decrease with* s *for any* s*, and those of wealthier individuals decrease with* s *for large* s*. If* T_N, T_2, δ_N, *and* δ_L *are sufficiently high, these variables increase with* s *over some range of* s *for those with relatively large wealth.*[33]

As in the unconstrained case, the net earnings and consumption of the subordinate group decline with s when $e^*_{2L} = 0$, and when $e^*_{2L} > 0$ and T_N, T_2, δ_N, and δ_L are low. In contrast, when $e^*_{2L} > 0$ and the TFPs or the effectiveness of education for the group are not low, the result varies depending on wealth. The net earnings and consumption of those with little wealth decrease with s for any s. For others, these variables decrease with s for large s, but if T_N, T_2, δ_N, and δ_L are sufficiently high, they *increase* with s over some range of s when the wealth level is relatively large.

[31]This claim cannot be verified analytically, but numerical simulations suggest that this is the case.

[32]For those who have relatively large wealth and thus choose the national sector in Case 4, the result is analytically proved only for large s, but numerical simulations suggest that it holds for any s.

[33]In Cases 4 and 5, the result is not analytically proved for those who choose the local sector and for δ_L, but numerical exercises suggest that it holds for these workers with relatively large a and for high enough δ_L.



(a) Relatively high T_N, T_2, δ_N, and δ_L (b) Relatively low T_N, T_2, δ_N, and δ_L

Figure 6: Numerical example of the relationship between s and net earnings (in units of the final good) (Proposition 4 (ii))

The result for those with little wealth, who choose the local sector, can be explained as follows. An increase in s has a positive direct effect on the net earnings and consumption of local-sector workers through h_{2L}, while it has a negative indirect effect by decreasing $\frac{H_{2N}}{H_{2L}}$ and thus $\frac{w_{2L}}{P_2}$. The positive direct effect *increases with wealth* because one with greater wealth spends more on education and thus benefits more from the increased emphasis on teaching the mother tongue. When she has little wealth to spend on education, the positive effect is small and is dominated by the negative effect for any s; thus, net earnings and consumption always decrease with s.

Based on the lemma, the following proposition examines the effect of s on net earnings and consumption by taking into account how s affects whether $e^*_{2L} = 0$ or $e^*_{2L} > 0$. Thereafter, it is assumed that the value of $\bar{l}$ is not very large compared to the wealth levels of most individuals.[34]

Proposition 4

(*i*) *If T_N, T_2, δ_N, and δ_L are low, or if the proportion of those with relatively large wealth is very low, net earnings and consumption of group 2 workers decrease with s and are highest at $s = 0$.*

(*ii*) *Otherwise,*

(*a*) *The net earnings and consumption of group 2 individuals with little wealth, who choose the local sector, decrease with s and are highest at $s = 0$. For those with greater wealth, these variables decrease with s for small s and large s (at least when $e^*_{2L} = 0$).*

(*b*) *If T_N, T_2, δ_N, and δ_L are sufficiently high, the net earnings and consumption of those with sufficiently large wealth increase with s over some range of intermediate s (i.e., when $e^*_{2L} > 0$).*

[34]The assumption is necessary to prove the existence of an intermediate s that maximizes net earnings and consumption in (ii)(a) of the next proposition. Numerical simulations suggest that, when $\frac{a}{\bar{l}}$ is low for many individuals and δ_L is small, *irrespective of* levels of T_N, T_2, and δ_N, these variables decrease with s and are highest at $s = 0$.

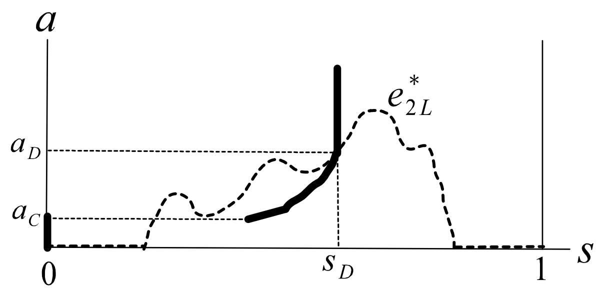


Figure 7: Relationship between wealth and the s maximizing net earnings and consumption for those who choose the local sector (Proposition 4 (ii))

Further, if T_N, T_2, δ_N, and δ_L are high enough, their net earnings and consumption are highest at an intermediate s, where the optimal s for local-sector workers weakly increases with a.

As in the unconstrained case, if T_N, T_2, δ_N, and δ_L are low, the net earnings and consumption of the subordinate group decrease with s and are highest at $s = 0$. The same is true if the proportion of those with relatively large wealth is so low that Case 2 is realized for any s (Figure 5).

Otherwise, the result depends on wealth. The net earnings and consumption of those with little wealth, who choose the local sector, decrease with s and are highest when language education solely focuses on the common language. For those with greater wealth, these variables decrease with s for small and large s (at least when $e_{2L}^* = 0$), while for intermediate s (i.e., when $e_{2L}^* > 0$), the relationship with s is generally unclear. However, for those with sufficiently large wealth, the variables increase with s over some range of intermediate s, if T_N, T_2, δ_N, and δ_L are sufficiently high. Further, if the exogenous variables are high enough, their net earnings and consumption are highest at an intermediate s. In such a case, the s that maximizes the variables weakly *increases* with wealth for those choosing the local sector. That is, the economically-optimal weight on mother-tongue education is higher for the wealthier. This is because one with greater wealth spends more on education and thus benefits more from the increased emphasis on teaching the mother tongue.

The sectoral TFPs and the effectiveness of education are closely related to a country's level of development. Hence, the result suggests that, in general, net earnings and consumption are maximized under language education solely focused on the common language when the development level is low, whereas when the development level is sufficiently high, the economic outcomes are highest under balanced bilingual education, *except* for the very poor. This implication is consistent with empirical findings (Angrist and Lavy, 1997; Cappellari and Di Paolo, 2018). The productivity of education varies widely even among developing countries (Singh, 2020). The result implies that if a country has an ineffective education system, teaching only the common language is economically optimal; otherwise, balanced bilingual education is optimal *except* for the very poor.

Figure 6 shows a numerical example of the relationship between s and net earnings for individuals with varying levels of wealth when T_N, T_2, δ_N, and δ_L are relatively high (panel a) and relatively low (panel b). The net earnings of those with very small a decrease as s increases, while the net earnings of the wealthier, among whom those with greater (smaller) a choose the national (local) sector, decrease for small and large s, and increase for intermediate s, as in the unconstrained case. Their net earnings peak at an intermediate s in panel (a) and at $s = 0$ in panel (b).[35]

[35]In both panels, $\alpha = 0.5$, $\gamma = 0.45$, $\bar{l} = 90$, and the distribution of wealth follows a truncated log-normal distribution with maximum 200, mean 20, and variance 80. The values of a for the four types of individuals are

Focusing on those who choose the local sector, Figure 7 illustrates the relationship between wealth and the s maximizing net earnings and consumption when T_N, T_2, δ_N, and δ_L are high enough, which is represented by a thick solid line.[36] The thick line shows that the s maximizing the economic outcomes weakly increases with wealth. s_D denotes the value of s that maximizes these variables when educational spending is not constrained by wealth. When $a \geq a_D$, optimal investment $e = e^*_{2L}$ can be made at $s = s_D$, and thus the variables are highest at $s = s_D$. When $a \in [a_C, a_D)$ and thus $e = e^*_{2L}$ cannot be chosen at $s = s_D$, $e = a < e^*_{2L}$, and the s maximizing the variables increases with a. When $a < a_C$, the variables are highest at $s = 0$ and $e = e^*_{2L} = 0$.

The following proposition examines the effect of s on human capital for the local sector.

Proposition 5 *For individuals choosing the local sector, h_{2L} is maximized at an intermediate s (i.e., when $e^*_{2L} > 0$), except when the proportion of those with relatively large wealth is very low, in which case $e^*_{2L} = 0$ and $h_{2L} = \bar{l}^{\gamma}$ for any s. For those choosing the national sector, h_{2L} is highest at an intermediate or large $s \in (1-\gamma, 1)$. The s that maximizes h_{2L} weakly decreases with a and is greater than the s that maximizes net earnings and consumption.*

Similar to the unconstrained case, human capital for the local sector, i.e., the mother-tongue skill, is highest at an intermediate s (i.e., when $e^*_{2L} > 0$) for individuals choosing the local sector, except when the proportion of those with relatively large wealth is very low and thus Case 2 is realized for any s (Figure 5), in which case $e^*_{2L} = 0$ and h_{2L} is at the lowest level for any s. In contrast, for those choosing the national sector, h_{2L} is highest at an intermediate or large s (at least greater than $1-\gamma$ and less than 1), where the skill-maximizing s, particularly for those with relatively small wealth, could lie in the right region of $e^*_{2L} = 0$ in Figure 7. The result suggests that balanced bilingual education is beneficial for developing the mother-tongue skill for most students. Regardless of sectoral choice, the skill-maximizing s weakly decreases with a. That is, as wealth increases, a *reduced* emphasis on teaching the mother tongue becomes optimal for skill development, *contrary* to the result on net earnings and consumption for local-sector workers. The skill-maximizing s exceeds the s maximizing the economic outcomes for everyone.

Finally, based on the propositions, the following corollary presents the result on between-group inequalities in human capital for the local sector, net earnings, and consumption.

Corollary 2 *Suppose $T_1 \geq T_2$.*

(i) If T_N, T_2, δ_N, and δ_L are low, or if the proportion of individuals with relatively large wealth is very low, inter-group inequalities in net earnings and consumption are lowest, but the inequality in human capital for the local sector is highest at $s = 0$.

(ii) Otherwise, if T_N, T_2, δ_N, and δ_L are sufficiently high, the inter-group inequalities are lowest when s falls within the intermediate range (i.e., when $e^*_{2L} > 0$),[37] with the exception that the inequalities in net earnings and consumption of the very poor are lowest at $s = 0$.

If T_N, T_2, δ_N, and δ_L are low, or if the share of those with relatively large wealth is very low, language education solely focused on the common language minimizes inter-group inequalities in net earnings and consumption, but it maximizes the inequality in the mother-tongue skill. Otherwise, if the sectoral TFPs and the effectiveness of education for the subordinate group are sufficiently

$a = 100, 30, 15$, and 1. In panel (a), $T_N = T_2 = 30, \delta_N = 15$, and $\delta_L = 25$, in panel (b), $T_N = T_2 = 15, \delta_N = 10$, and $\delta_L = 20$. In this example, those with $a = 100$ and $a = 30$ choose the national sector for any s, while those with smaller a choose the local sector (except that those with $a = 15$ choose the national sector when s is very close to 1).

[36] s_D is located in the region in which e^*_{2L} increases with s. As in the figure, one cannot rule out the possibility that there exist multiple values of s locally maximizing e^*_{2L}, although simulations suggest that such s is unique.

[37] To be precise, for those choosing the national sector, particularly those with relatively small wealth, the inequality in human capital for the local sector could be lowest at a high s, i.e., in the right region of $e^*_{2L} = 0$ in Figure 7.

high, a balanced allocation of the budget to teaching the two languages minimizes inter-group inequalities in most dimensions, but the inter-group gaps in the net earnings and consumption of the very poor are lowest when the budget is solely spent on teaching the common language.

6 Policy Implications

6.1 Implications for language education

Proposition 5 implies that balanced bilingual education is valuable for developing the mother-tongue skill of the subordinate group.[38] This is consistent with the views of language and education specialists (Ball, 2011; Heugh, 2011) and empirical findings in economics (Jain, 2017; Ramachandran, 2017).

In contrast, according to Proposition 4, while balanced bilingual education maximizes the net earnings and consumption of most individuals from the subordinate group when sectoral TFPs and the effectiveness of education for the group are sufficiently high (and the proportion of those with relatively large wealth is not too low), language education solely focused on the common language always maximizes the economic outcomes of the very poor, and when the TFPs and education effectiveness are low (or when the share of relatively wealthy people is very low), it maximizes the outcomes for all. This result implies that, in general, when a country's level of development is low, net earnings and consumption are highest when only the common language is taught, whereas they are highest under balanced bilingual education (except for the very poor) when the development level is sufficiently high. Empirical studies, albeit very limited in number, are mostly consistent with this implication (Angrist and Lavy, 1997; Cappellari and Di Paolo, 2018).

These results suggest that policies leading to good educational and economic outcomes for the subordinate group vary depending on the educational and technological conditions or the level of development of the country. Suppose that the government chooses the education policy s, along with a redistributive policy (tuition subsidy or income transfer), financed by a lump-sum tax, to maximize a social welfare function that depends on both the educational outcomes (mother-tongue skill h_{2L}) and economic outcomes (consumption, which equals the sum of net earnings, wealth, and means-tested transfer minus tax) of individuals.

When the educational and technological conditions are favorable (and the share of those with relatively large wealth is not too low), which is likely when the development level is not low, the welfare-maximizing government would adopt balanced bilingual education *alongside* a redistributive policy that enables those with little wealth to invest sufficiently in education.[39] Redistribution toward the wealth-constrained very poor is socially desirable because it not only significantly raises their consumption but also maximizes it under the balanced education, which yields positive economic outcomes for the wealthier and positive educational outcomes for all.

By contrast, when the educational and technological conditions are adverse (or when the share of relatively wealthy individuals is very low), the policy tools cannot lead to good educational and

[38]As shown in the proposition, the optimal weights on the two types of education differs depending on individual wealth. This also generally applies to net earnings and consumption discussed next.

[39]The mother-tongue skill h_{2L} is maximized at an intermediate s (i.e., when $e^*_{2L} > 0$) for those choosing the local sector and at an intermediate or large s (i.e., in the right region of $e^*_{2L} = 0$) for those choosing the national sector, where the skill-maximizing s weakly decreases with a, as shown in Proposition 5. *Without redistribution*, consumption is maximized at $s_l = 0$ for the very poor and at an intermediate s for the wealthier, whose value weakly increases with a for those choosing the local sector from Proposition 4 (ii). A redistributive policy that, at the expense of the rich, induces the wealth-constrained very poor to spend sufficiently more on education not only significantly raises their consumption for a given s (with an increase greater than the magnitude of consumption decrease of the unconstrained rich) but also maximizes it at an intermediate s_l. Hence, the education policy with an intermediate s together with such a redistributive policy would maximize social welfare.

economic outcomes for everyone. In such situations, the government seeking to balance educational outcomes against economic ones would choose bilingual education with a *smaller* (*but not too small*) weight on the mother tongue than under more favorable conditions (e.g., s slightly greater than the threshold s at the left border of the region $e^*_{2L} > 0$), alongside redistribution toward the very poor.[40] This policy achieves superior educational outcomes compared to language education solely focused on the common language, with a relatively small loss of consumption.

The propositions also imply that introducing teaching of the mother tongue *on a small scale* (i.e., s is small enough that $e^*_{2L} = 0$) is *always inferior* to teaching only the common language: it fails to improve the mother-tongue skill for those choosing the local sector and reduces consumption for all.

While existing empirical findings generally align with the model's results, they are very limited in number, warranting further research. The results and implications provided by the model would be helpful in guiding future empirical works and interpreting their findings.

Note that the model does not consider several potentially important effects of language choice in education. Teaching the mother tongue may contribute to the accumulation of social capital in the local community. It might also stimulate political participation and increase support for democracy (Albaugh, 2016). On the other hand, teaching the common language can facilitate national identity formation, thereby promoting national unity and stability. It may also reduce linguistic diversity and promote public goods provision and economic growth (Desmet, Ortuño-Ortín, and Wacziarg, 2012). When implementing policies in an actual society, these effects, as well as the effects considered in the model, must be taken into account.

6.2 Implications for between-group and within-group inequalities

Since the education policy toward the subordinate group does not affect outcomes for the dominant group, the above results have direct implications for inter-group inequalities in skill and consumption. If the educational and technological conditions are favorable (and the share of those with relatively large wealth is not too low), in other words, if the level of development is not low, balanced bilingual education, alongside redistribution toward the very poor, can reduce the inequalities; otherwise, balanced education leads to a narrower gap in the mother-tongue skill but a wider gap in consumption, compared to language education solely focused on the common language.

Proposition 3 has implications for within-group inequalities in net earnings and consumption. While inequality in wealth among the dominant group impacts inequalities in these variables only directly, wealth inequality among the subordinate group has both direct and indirect effects on the inequalities, as long as the proportion of relatively wealthy people is not high. Consequently, an increase in wealth inequality tends to cause greater increases in economic inequalities for the subordinate group compared to the dominant group.

Therefore, redistributive policies that improve the poor's access to education would be more important for the subordinate group: redistribution toward the group's poor not only reduces the between-group inequalities but is also more effective in alleviating the within-group inequalities.

[40]As previously mentioned in footnote 39, the mother-tongue skill is maximized at an intermediate s for those choosing the local sector and at an intermediate or large s for those choosing the national sector. At the same time, unlike under favorable conditions, consumption of everyone decreases with s and is highest at $s = 0$. Hence, the welfare-maximizing s is smaller compared to more favorable conditions. Unless the weights assigned to the educational outcomes and the poor who become local-sector workers are small in the social welfare function, social welfare is higher when $e > 0$ holds for them, and s is relatively close to 0, e.g., s slightly greater than the threshold s at the left border of the region $e^*_{2L} > 0$. Redistribution toward the very poor raises social welfare because the increase in their consumption is greater than the decrease in consumption of the rich.

7 Conclusion

Poor economic performance of subordinate (typically, minority) groups and large economic disparities between these groups and the dominant ethnic group are major concerns in most countries. The mother tongue of the dominant group is the common language in many of these countries. Determining the appropriate emphasis on common-language education versus mother-tongue education is a crucial issue in school education of students from subordinate groups.

This paper developed a model to study the issue theoretically. Then, it analyzed how the relative weight on teaching the common language and the mother tongue affects skill, earnings, consumption, and between-group inequalities. It also examined the implications of the groups' asymmetric language positions for sectoral choices and within-group inequalities. The main results are summarized as follows.

First, concerning the development of the mother-tongue skill of the subordinate group, balanced bilingual education is crucial for those with limited wealth and valuable for others.

Second, regarding the consumption and earnings net of educational spending of the subordinate group, balanced bilingual education is desirable only under good educational and technological conditions (i.e., sectoral productivities and the efficacy of education for the group are sufficiently high) and only for those with adequate wealth. Language education exclusively focused on the common language is always optimal for those with little wealth and, under bad conditions, optimal for all. In the real world, the conditions are closely related to a country's level of development. Thus, the results suggest that, generally, if the level of development is low, teaching only the common language is desirable in terms of economic outcomes; otherwise, balanced education is desirable for all except the very poor.

Third, since members of the dominant group are not affected by the education policy toward the subordinate group, the above results directly apply to inter-group inequalities in skill, net earnings, and consumption. For example, switching from language education focused solely on the common language to balanced bilingual education can reduce inter-group economic inequalities (except for the very poor) only under favorable educational and technological conditions.

The results imply that policies to foster positive educational and economic outcomes for the subordinate group and mitigate the inter-group inequalities vary depending on the aforementioned conditions. In favorable conditions, the government considering both educational and economic outcomes would adopt balanced bilingual education, coupled with a redistributive policy that supports educational investment by those with little wealth. In adverse conditions, the government would choose bilingual education with a smaller (but not too small) weight on the mother tongue compared to more favorable conditions (alongside redistribution toward the very poor).

Finally, a change in within-group wealth inequality tends to have greater effects on earnings and consumption inequalities for the subordinate group than for the dominant group. This result suggests that redistribution toward the poor would be more important for the subordinate group.

While the empirical findings of existing works largely align with the model's results, they are very limited in number, warranting further research. The results and implications of the model would be helpful in guiding future empirical works and interpreting their findings.

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Appendix A Determination of endogenous variables in the general case

Group 1

For group 1 individuals with $a \geq e_1^*$, consumption is (19) as before, while, for individuals with $a < e_1^*$, from (16) and (15), net earnings in units of the intermediate good equal

$$T_N(\bar{l}+a)^\gamma - \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\left(\frac{T_N}{T_1}\right)^\alpha a, \tag{A1}$$

and from (15) and the above equation, consumption equals

$$\begin{aligned} c_1(a) &= \frac{w_N h_1 - P_1 a}{P_1} + a \\ &= (\alpha T_1)^\alpha((1-\alpha)T_N)^{1-\alpha}(\bar{l}+a)^\gamma. \end{aligned} \tag{A2}$$

Net earnings in units of the final good equal consumption minus wealth.

Group 2

Net earnings in units of the final good equal consumption minus wealth.

(I) When $e_{2L}^* = 0$

Case 1: Indifference condition holds for $a \geq e_{2N}^*$. When $e_{2L}^* = 0$ and those with $a \geq e_{2N}^*$ are indifferent between choosing the national sector by expending $e = e_{2N}^*$ and choosing the local sector by expending $e = e_{2L}^* = 0$, the indifference condition is (27), same as when everyone has enough wealth for education and $e_{2L}^* = 0$. Because those with $a < e_{2N}^*$ expend $e = 0$ and choose the local sector, $H_{2N} = [\delta_N(1-s)e^*_{2N}]^\gamma p_{2N}(1-F(e^*_{2N}))L_2$ (L_2 is the group 2 population) and $H_{2L} = \bar{l}^\gamma\,[(1-p_{2N})(1-F(e^*_{2N}))+F(e^*_{2N})]L_2$, where p_{2N} is the proportion of those with $a \geq e^*_{2N}$ choosing the national sector. Hence,

$$\frac{H_{2N}}{H_{2L}} = \frac{[\delta_N(1-s)e^*_{2N}]^\gamma p_{2N}(1-F(e^*_{2N}))}{\bar{l}^\gamma\left[(1-p_{2N})(1-F(e^*_{2N}))+F(e^*_{2N})\right]}, \tag{A3}$$

where e^*_{2N} is given by (21) and thus determined by $\frac{H_{2N}}{H_{2L}}$.

Once $\frac{H_{2N}}{H_{2L}}$ is determined from (28), this equation determines p_{2N}. Since $e = e^*_{2L} = 0$ for any individual choosing the local sector, as in the unconstrained case, c_2 for any a is given by (30).

Case 2: Indifference condition holds for $a = \widehat{a}_0 < e^*_{2N}$. When $e^*_{2L} = 0$ and those with $a = \widehat{a}_0 < e^*_{2N}$ are indifferent between choosing the national sector by expending $e = \widehat{a}_0$ and choosing the local sector without education, the indifference condition is, from (20), (10), and (26),

$$T_N(\delta_N(1-s)\widehat{a}_0)^\gamma - \frac{1}{1-\alpha}\left(\frac{T_N H_{2N}}{T_2 H_{2L}}\right)^\alpha \widehat{a}_0 = \frac{\alpha}{1-\alpha}T_N\frac{H_{2N}}{H_{2L}}\bar{l}^\gamma. \tag{A4}$$

Since those with $a > \widehat{a}_0$ expend $e = \min\{a, e^*_{2N}\}$ and choose the national sector, and those with $a < \widehat{a}_0$ expend $e = 0$ and choose the local sector, $H_{2N} = \left\{[\delta_N(1-s)e^*_{2N}]^\gamma(1-F(e^*_{2N})) + \int_{\widehat{a}_0}^{e^*_{2N}}[\delta_N(1-s)a]^\gamma dF(a)\right\}L_2$ and $H_{2L} = \bar{l}^\gamma F(\widehat{a}_0)L_2$. Hence,

$$\frac{H_{2N}}{H_{2L}} = \frac{[\delta_N(1-s)]^\gamma\left[(e^*_{2N})^\gamma(1-F(e^*_{2N})) + \int_{\widehat{a}_0}^{e^*_{2N}} a^\gamma dF(a)\right]}{\bar{l}^\gamma F(\widehat{a}_0)}, \tag{A5}$$

where e^*_{2N} is given by (21).

$\frac{H_{2N}}{H_{2L}}$ and $\widehat{a}_0$ are obtained by solving (A4) and (A5), which implies that, unlike the previous case, $\frac{H_{2N}}{H_{2L}}$ (and thus individual net earnings and consumption) depends on the distribution of wealth.

Finally, c_2 for $a \geq e^*_{2N}$ is given by (30) as before, while c_2 for $a \in [\widehat{a}_0, e^*_{2N})$ is given by, from (20) and (10),

$$\begin{aligned} c_{2N}(a) &= \frac{w_N h_{2N} - P_2 a}{P_2} + a \\ &= (1-\alpha){T_2}^\alpha {T_N}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}(\delta_N(1-s)a)^\gamma, \end{aligned} \tag{A6}$$

and c_2 for $a < \widehat{a}_0$ equals, from (26), (9), and (10),

$$\begin{aligned} c_{2L}(a) &= \frac{w_{2L}h_{2L}}{P_2} + a \\ &= \alpha {T_2}^\alpha {T_N}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\bar{l}^\gamma + a. \end{aligned} \tag{A7}$$

(II) When $e^*_{2L} > 0$

Case 3: Indifference condition holds for $a \geq e^*_{2N}$. When $e^*_{2L} > 0$ and those with $a \geq e^*_{2N}$ are indifferent between choosing the national sector by spending $e = e^*_{2N}$ and choosing the local sector by spending $e = e^*_{2L}$, the indifference condition is (29), same as in the unconstrained case. Since those with $a < e^*_{2N}$ spend $e = \min\{a, e^*_{2L}\}$ and choose the local sector,[41] $H_{2N} = [\delta_N(1-s)e^*_{2N}]^\gamma p_{2N}(1-$

[41] This is obvious for $a \in [e^*_{2L}, e^*_{2N})$. The result for $a < e^*_{2L}$ can be proved as follows. Since $w_{2L}h_{2L} - P_2 a > w_N h_{2N} - P_2 a \Leftrightarrow w_{2L}(\bar{l} + \delta_L s a)^\gamma > w_N(\delta_N(1-s)a)^\gamma$ holds for those with $a = 0$ and those with $a = e^*_{2L}$, if there exist ranges of $a \in (0, e^*_{2L})$ over which $w_{2L}h_{2L} - P_2 a < w_N h_{2N} - P_2 a$ is true, there must exist at least two values of a satisfying $w_{2L}(\bar{l} + \delta_L s a)^\gamma = w_N(\delta_N(1-s)a)^\gamma$, which is not possible.

$F(e_{2N}^*))L_2$ and $H_{2L}=\Big\{(\bar{l}+\delta_L se_{2L}^*)^\gamma[(1-p_{2N})(1-F(e_{2N}^*))+F(e_{2N}^*)-F(e_{2L}^*)]+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^\gamma dF(a)+\bar{l}^{\,\gamma}F(0)\Big\}L_2$, where p_{2N} is the proportion of those with $a\geq e_{2N}^*$ choosing the national sector. Hence,

$$\frac{H_{2N}}{H_{2L}}=\frac{[\delta_N(1-s)e_{2N}^*]^\gamma p_{2N}(1-F(e_{2N}^*))}{(\bar{l}+\delta_L se_{2L}^*)^\gamma[(1-p_{2N})(1-F(e_{2N}^*))+F(e_{2N}^*)-F(e_{2L}^*)]+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^\gamma dF(a)+\bar{l}^{\,\gamma}F(0)}, \tag{A8}$$

where e_{2N}^* and e_{2L}^* are given by (21) and (24), respectively.

Once $\frac{H_{2N}}{H_{2L}}$ is determined from (29), the above equation determines p_{2N}.

c_2 for $a\geq e_{2L}^*$ is given by (30) from (29), while c_2 for $a<e_{2L}^*$ equals, from (23), (9), and (10),

$$\begin{aligned} c_{2L}(a) &= \frac{w_{2L}h_{2L}-P_2a}{P_2}+a \\ &= \alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}(\bar{l}+\delta_L sa)^\gamma. \end{aligned} \tag{A9}$$

Case 4: Indifference condition holds for $a=\widehat{a}\in[e_{2L}^*,e_{2N}^*)$. When those with $a=\widehat{a}\in[e_{2L}^*,e_{2N}^*)$ are indifferent between choosing the national sector with $e=\widehat{a}$ and choosing the local sector with $e=e_{2L}^*>0$, the indifference condition is, from (20), (10), and (25),

$$T_N(\delta_N(1-s)\widehat{a})^\gamma-\frac{1}{1-\alpha}\left(\frac{T_NH_{2N}}{T_2H_{2L}}\right)^\alpha\widehat{a}=\frac{1}{1-\alpha}\left(\frac{T_NH_{2N}}{T_2H_{2L}}\right)^\alpha\left\{(1-\gamma)\left[(\gamma\delta_L s)^\gamma\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{\bar{l}}{\delta_L s}\right\}. \tag{A10}$$

Since those with $a>\widehat{a}$ choose $e=\min\{a,e_{2N}^*\}$ and the national sector and those with $a<\widehat{a}$ choose $e=\min\{a,e_{2L}^*\}$ and the local sector, $H_{2N}=\Big\{[\delta_N(1-s)e_{2N}^*]^\gamma(1-F(e_{2N}^*))+\int_{\widehat{a}}^{e_{2N}^*}[\delta_N(1-s)a]^\gamma dF(a)\Big\}L_2$ and $H_{2L}=\Big\{(\bar{l}+\delta_L se_{2L}^*)^\gamma(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^\gamma dF(a)+\bar{l}^{\,\gamma}F(0)\Big\}L_2$. Hence,

$$\frac{H_{2N}}{H_{2L}}=\frac{[\delta_N(1-s)]^\gamma\left[(e_{2N}^*)^\gamma(1-F(e_{2N}^*))+\int_{\widehat{a}}^{e_{2N}^*}a^\gamma dF(a)\right]}{(\bar{l}+\delta_L se_{2L}^*)^\gamma(F(\widehat{a})-F(e_{2L}^*))+\int_0^{e_{2L}^*}(\bar{l}+\delta_L sa)^\gamma dF(a)+\bar{l}^{\,\gamma}F(0)}, \tag{A11}$$

where e_{2N}^* and e_{2L}^* are given by (21) and (24), respectively.

$\frac{H_{2N}}{H_{2L}}$ and $\widehat{a}$ are obtained by solving (A10) and (A11), which implies that, unlike the previous case, $\frac{H_{2N}}{H_{2L}}$ (and thus net earnings and consumption) depends on the distribution of wealth.

c_2 for $a\geq e_{2N}^*$ is given by (30), c_2 for $a\in[\widehat{a},e_{2N}^*)$ is given by (A6), c_2 for $a\in[e_{2L}^*,\widehat{a})$ equals, from (25) and (10),

$$\begin{aligned} c_{2L}^*(a) &= \frac{w_{2L}h_{2L}^*-P_2e_{2L}^*}{P_2}+a \\ &= (1-\gamma)\left[(\gamma\delta_L s)^\gamma\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}+\frac{\bar{l}}{\delta_L s}+a, \end{aligned} \tag{A12}$$

and c_2 for $a<e_{2L}^*$ is given by (A9).

Case 5: Indifference condition holds for $a=\widetilde{a}<e_{2L}^*$. When those with $a=\widetilde{a}<e_{2L}^*$ are indifferent between choosing the national sector by expending $e=\widetilde{a}$ and choosing the local sector by expending the same $\widetilde{a}$ on education, the indifference condition is, from (20), (23), and (9),

$$T_N(\delta_N(1-s)\widetilde{a})^\gamma=\frac{\alpha}{1-\alpha}\frac{T_NH_{2N}}{H_{2L}}(\bar{l}+\delta_L s\widetilde{a})^\gamma\Leftrightarrow\frac{H_{2N}}{H_{2L}}=\frac{1-\alpha}{\alpha}\left[\frac{\delta_N(1-s)\widetilde{a}}{\bar{l}+\delta_L s\widetilde{a}}\right]^\gamma. \tag{A13}$$

Since those with $a > \widetilde{a}$ expend $e = \min\{a, e_{2N}^*\}$ and choose the national sector and those with $a < \widetilde{a}$ expend $e = a$ and choose the local sector, $H_{2N} = \left\{[\delta_N(1-s)e_{2N}^*]^\gamma(1-F(e_{2N}^*)) + \int_{\widetilde{a}}^{e_{2N}^*}[\delta_N(1-s)a]^\gamma dF(a)\right\}L_2$ and $H_{2L} = \left[\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^\gamma dF(a) + \bar{l}^\gamma F(0)\right]L_2$. Hence,

$$\frac{H_{2N}}{H_{2L}} = \frac{[\delta_N(1-s)]^\gamma\left[e_{2N}^{*\gamma}(1-F(e_{2N}^*)) + \int_{\widetilde{a}}^{e_{2N}^*} a^\gamma dF(a)\right]}{\int_0^{\widetilde{a}}(\bar{l}+\delta_L sa)^\gamma dF(a) + \bar{l}^\gamma F(0)}, \tag{A14}$$

where e_{2N}^* is given by (21).

$\frac{H_{2N}}{H_{2L}}$ and $\widetilde{a}$ are obtained by solving (A13) and (A14). c_2 for $a \geq e_{2N}^*$ is given by (30), c_2 for $a \in [\widetilde{a}, e_{2N}^*)$ is given by (A6), and c_2 for $a < \widetilde{a}$ is given by (A9).

Appendix B Proofs of lemmas and propositions in the unconstrained case

Proof of Lemma 1. By plugging (4) and (10) into the condition for $e_{2L}^* = 0$, $\gamma\delta_L s w_{2L}\bar{l}^{\gamma-1} - P_2 \leq 0$,

$$\gamma\delta_L s\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\tfrac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\bar{l}^{\gamma-1} \leq 1$$

$$\Leftrightarrow \gamma\delta_L s\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left\{\left(\tfrac{1-\alpha}{\alpha}\tfrac{1-\gamma}{\bar{l}^\gamma}\right)^{1-\gamma}\left[(1-\alpha)\gamma\delta_N(1-s)T_2{}^\alpha T_N{}^{1-\alpha}\right]^\gamma\right\}^{\frac{1-\alpha}{1-\gamma(1-\alpha)}}\bar{l}^{\gamma-1} \leq 1 \text{ (from (28))}$$

$$\Leftrightarrow (\delta_L s)^{1-\gamma(1-\alpha)}[\delta_N(1-s)]^{\gamma(1-\alpha)}\gamma(1-\gamma)^{(1-\gamma)(1-\alpha)}(\alpha T_2)^\alpha((1-\alpha)T_N)^{1-\alpha}\bar{l}^{\gamma-1} \leq 1. \tag{A15}$$

Denote the higher (lower) s satisfying (A15) with equality by $\bar{s}$ ($\underline{s}$) [$\underline{s} < 1-\gamma(1-\alpha) < \bar{s}$], which exists when T_2 and T_N are not extemely low. The lemma is straightforward from the equation. ∎

Proof of Lemma 2. The result is clear from (28) when $e_{2L}^* = 0$. When $e_{2L}^* > 0$, the LHS (RHS) of (29) decreases (increases) with $\frac{H_{2N}}{H_{2L}}$ and s (with $\frac{H_{2N}}{H_{2L}}$). When $e_{2L}^* > 0$, the RHS also increases with s, because the derivative of the expression inside the curly bracket with respect to s equals

$$\frac{1}{s^2}\left\{\gamma s\left[(\gamma\delta_L s)^\gamma\alpha T_2{}^\alpha T_N{}^{1-\alpha}\left(\tfrac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}} - \frac{\bar{l}}{\delta_L}\right\} > 0,$$

where the inequality sign is from $e_{2L}^* > 0$ and (24). Therefore, an increase in s lowers $\frac{H_{2N}}{H_{2L}}$. ∎

Proof of Lemma 3. (i) When $e_{2L}^* = 0$, consumption of a group 2 person with a equals, from (30),

$$\begin{aligned} c_2^*(a) &= (1-\gamma)\left\{T_N(1-\alpha)\left[\gamma\delta_N(1-s)\right]^\gamma\left(\tfrac{T_N H_{2N}}{T_2 H_{2L}}\right)^{-\alpha}\right\}^{\frac{1}{1-\gamma}} + a \\ &= (1-\gamma)\left((1-\alpha)T_2{}^\alpha T_N{}^{1-\alpha}\left[\gamma\delta_N(1-s)\right]^\gamma\left\{\left(\tfrac{1-\alpha}{\alpha}\tfrac{1-\gamma}{\bar{l}^\gamma}\right)^{1-\gamma}\left[(1-\alpha)\gamma\delta_N(1-s)T_2{}^\alpha T_N{}^{1-\alpha}\right]^\gamma\right\}^{\frac{-\alpha}{1-\gamma(1-\alpha)}}\right)^{\frac{1}{1-\gamma}} + a \text{ (from (28))} \\ &= (1-\gamma)\left\{(1-\alpha)T_2{}^\alpha T_N{}^{1-\alpha}\left[\gamma\delta_N(1-s)\right]^{(1-\alpha)\gamma}\left[\tfrac{\alpha}{1-\alpha}\tfrac{\bar{l}^\gamma}{1-\gamma}\right]^\alpha\right\}^{\frac{1}{1-\gamma(1-\alpha)}} + a. \end{aligned} \tag{A16}$$

Hence, c_2 when $e_{2L}^* = 0$ decreases with s. The same is true for net earnings in units of the final good, because they equal consumption minus wealth.

(ii) Only the proof of the result on the consumption is presented, because net earnings in units of the final good equal consumption minus wealth.

[Condition for $\frac{dc_2}{ds} > (<) 0$] From (30), $\frac{dc_2}{ds}$ is proportional to $-\left[\frac{\gamma}{1-s} + \alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1} \frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right]$, where $\frac{d\frac{H_{2N}}{H_{2L}}}{ds} < 0$ from Lemma 2. Hence, in order to know the sign of $\frac{dc_2}{ds}$, $\frac{d\frac{H_{2N}}{H_{2L}}}{ds}$ needs to be calculated. The indifference condition, (29), can be expressed as

$$(\gamma^\gamma T_2{}^\alpha T_N{}^{1-\alpha})^{\frac{1}{1-\gamma}}\left\{\left[(1-\alpha)[\delta_N(1-s)]^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}} - \left[\alpha(\delta_L s)^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}\right\} = \frac{1}{1-\gamma}\frac{\bar{l}}{\delta_L s}. \tag{A17}$$

The derivative of the LHS$-$RHS of (A17) with respect to $\frac{H_{2N}}{H_{2L}}$ equals

$$-\frac{1}{1-\gamma}\frac{H_{2L}}{H_{2N}}(\gamma^\gamma T_2{}^\alpha T_N{}^{1-\alpha})^{\frac{1}{1-\gamma}}\left\{\alpha\left[(1-\alpha)[\delta_N(1-s)]^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}} + (1-\alpha)\left[\alpha(\delta_L s)^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}\right\} < 0. \tag{A18}$$

The derivative of the LHS$-$RHS of (A17) with respect to s equals

$$-\frac{\gamma}{1-\gamma}(\gamma^\gamma T_2{}^\alpha T_N{}^{1-\alpha})^{\frac{1}{1-\gamma}}\left\{\frac{1}{1-s}\left[(1-\alpha)[\delta_N(1-s)]^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}} + \frac{1}{s}\left[\alpha(\delta_L s)^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}\right\} + \frac{1}{1-\gamma}\frac{\bar{l}}{\delta_L s^2}$$

$$= \frac{1}{s(1-\gamma)}(\gamma^\gamma T_2{}^\alpha T_N{}^{1-\alpha})^{\frac{1}{1-\gamma}}\left\{\frac{1-\gamma-s}{1-s}\left[(1-\alpha)[\delta_N(1-s)]^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}} - \left[\alpha(\delta_L s)^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}\right\}. \tag{A19}$$

Hence,

$$\frac{d\frac{H_{2N}}{H_{2L}}}{ds} = \frac{\frac{1}{s}\frac{H_{2N}}{H_{2L}}\left\{\frac{1-\gamma-s}{1-s}\left[(1-\alpha)[\delta_N(1-s)]^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}} - \left[\alpha(\delta_L s)^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}\right\}}{\alpha\left[(1-\alpha)[\delta_N(1-s)]^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}} + (1-\alpha)\left[\alpha(\delta_L s)^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}}. \tag{A20}$$

Let $B_0 \equiv \left[(1-\alpha)[\delta_N(1-s)]^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}}$ and $B_1 \equiv \left[\alpha(\delta_L s)^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}$. Using (A20), $\dfrac{dc_2}{ds}$ is proportional to

$$-\left[\frac{\gamma}{1-s} + \alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right] = -\left[\frac{\gamma}{1-s} + \frac{\frac{\alpha}{s}\left(\frac{1-\gamma-s}{1-s}B_0 - B_1\right)}{\alpha B_0 + (1-\alpha)B_1}\right]$$

$$= -\frac{\frac{1}{s(1-s)}\{\alpha(1-\gamma)(1-s)B_0 + [\gamma(1-\alpha)s - \alpha(1-s)]B_1\}}{\alpha B_0 + (1-\alpha)B_1}$$

$$= -\frac{\frac{1}{s(1-s)}[\alpha(1-\gamma)(1-s)(B_0 - B_1) + \gamma(s-\alpha)B_1]}{\alpha B_0 + (1-\alpha)B_1}. \tag{A21}$$

Since $B_0 - B_1 > 0$ from (A17), $\dfrac{dc_2}{ds} < 0$ when $s \geq \alpha$.

When $s < \alpha$, $\dfrac{dc_2}{ds} < (>) 0$ iff the expression inside the square bracket of (A21) is positive (negative), that is,

$$\alpha(1-\gamma)(1-s)\left(\left[(1-\alpha)[\delta_N(1-s)]^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}} - \left[\alpha(\delta_L s)^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}\right) + \gamma(s-\alpha)\left[\alpha(\delta_L s)^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}$$

$$= \left(\frac{H_{2N}}{H_{2L}}\right)^{\frac{1-\alpha}{1-\gamma}}\left(\alpha(1-\gamma)(1-s)\left\{\left[(1-\alpha)[\delta_N(1-s)]^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\right]^{\frac{1}{1-\gamma}} - [\alpha(\delta_L s)^\gamma]^{\frac{1}{1-\gamma}}\right\} + \gamma(s-\alpha)[\alpha(\delta_L s)^\gamma]^{\frac{1}{1-\gamma}}\right) > (<) 0$$

$$\Leftrightarrow \alpha(1-\gamma)(1-s)\left[(1-\alpha)[\delta_N(1-s)]^{\gamma}\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\right]^{\frac{1}{1-\gamma}} > (<)[\alpha-(\gamma(1-\alpha)+\alpha)s][\alpha(\delta_L s)^{\gamma}]^{\frac{1}{1-\gamma}}$$

$$\Leftrightarrow \frac{H_{2N}}{H_{2L}} < (>)\frac{[\alpha(1-\gamma)(1-s)]^{1-\gamma}(1-\alpha)[\delta_N(1-s)]^{\gamma}}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{1-\gamma}\alpha(\delta_L s)^{\gamma}}. \quad \text{(A22)}$$

By substituting the RHS of the above equation into (A17), when $s<\alpha$, $\frac{dc_2}{ds} < (>) 0$ iff

$$(\gamma^{\gamma}T_2{}^{\alpha}T_N{}^{1-\alpha})^{\frac{1}{1-\gamma}}\left\{\begin{array}{l}\left[(1-\alpha)[\delta_N(1-s)]^{\gamma}\left(\frac{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{1-\gamma}\alpha(\delta_L s)^{\gamma}}{[\alpha(1-\gamma)(1-s)]^{1-\gamma}(1-\alpha)[\delta_N(1-s)]^{\gamma}}\right)^{\alpha}\right]^{\frac{1}{1-\gamma}}\\ -\left[\alpha(\delta_L s)^{\gamma}\left(\frac{[\alpha(1-\gamma)(1-s)]^{1-\gamma}(1-\alpha)[\delta_N(1-s)]^{\gamma}}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{1-\gamma}\alpha(\delta_L s)^{\gamma}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}\end{array}\right\} < (>)\frac{1}{1-\gamma}\frac{\bar{l}}{\delta_L s}$$

$$\Leftrightarrow (\gamma^{\gamma}T_2{}^{\alpha}T_N{}^{1-\alpha})^{\frac{1}{1-\gamma}}(1-s)^{-\alpha}\{(1-\alpha)[\delta_N(1-s)]^{\gamma}\}^{\frac{1-\alpha}{1-\gamma}}[\alpha(\delta_L s)^{\gamma}]^{\frac{\alpha}{1-\gamma}}\left\{\frac{\alpha-[\gamma(1-\alpha)+\alpha]s}{\alpha(1-\gamma)}\right\}^{\alpha}$$

$$\times\left\{1-(1-s)\frac{\alpha(1-\gamma)}{\alpha-[\gamma(1-\alpha)+\alpha]s}\right\} < (>)\frac{1}{1-\gamma}\frac{\bar{l}}{\delta_L s}$$

$$\Leftrightarrow (\gamma^{\gamma}T_2{}^{\alpha}T_N{}^{1-\alpha})^{\frac{1}{1-\gamma}}[(1-\alpha)\delta_N^{\gamma}]^{\frac{1-\alpha}{1-\gamma}}\left(\frac{\alpha^{\frac{\gamma}{1-\gamma}}}{1-\gamma}\right)^{\alpha}\gamma\frac{(\delta_L s)^{1+\alpha\frac{\gamma}{1-\gamma}}(1-s)^{(1-\alpha)\frac{\gamma}{1-\gamma}-\alpha}(\alpha-s)}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{1-\alpha}} < (>)\frac{\bar{l}}{1-\gamma}. \quad \text{(A23)}$$

If T_N, T_2, δ_N and δ_L are low enough that the LHS of (A23) at s maximizing $\frac{s^{1+\alpha\frac{\gamma}{1-\gamma}}(1-s)^{(1-\alpha)\frac{\gamma}{1-\gamma}-\alpha}(\alpha-s)}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{1-\alpha}}$ is smaller than the RHS, $\frac{dc_2}{ds}<0$ for any s; otherwise, there exist ranges of s satisfying $\frac{dc_2}{ds}>0$.

[**Relationship between s and c_2 when T_N, T_2, δ_N, and δ_L are sufficiently high**] The derivative of $\frac{s^{1+\alpha\frac{\gamma}{1-\gamma}}(1-s)^{(1-\alpha)\frac{\gamma}{1-\gamma}-\alpha}(\alpha-s)}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{1-\alpha}} = \left[\frac{s^{1-\gamma(1-\alpha)}(1-s)^{\gamma-\alpha}(\alpha-s)^{1-\gamma}}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{(1-\alpha)(1-\gamma)}}\right]^{\frac{1}{1-\gamma}}$ on the LHS of (A23) with respect to s equals $\frac{1}{1-\gamma}\left[\frac{s^{1-\gamma(1-\alpha)}(1-s)^{\gamma-\alpha}(\alpha-s)^{1-\gamma}}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{(1-\alpha)(1-\gamma)}}\right]^{\frac{1}{1-\gamma}}$ times

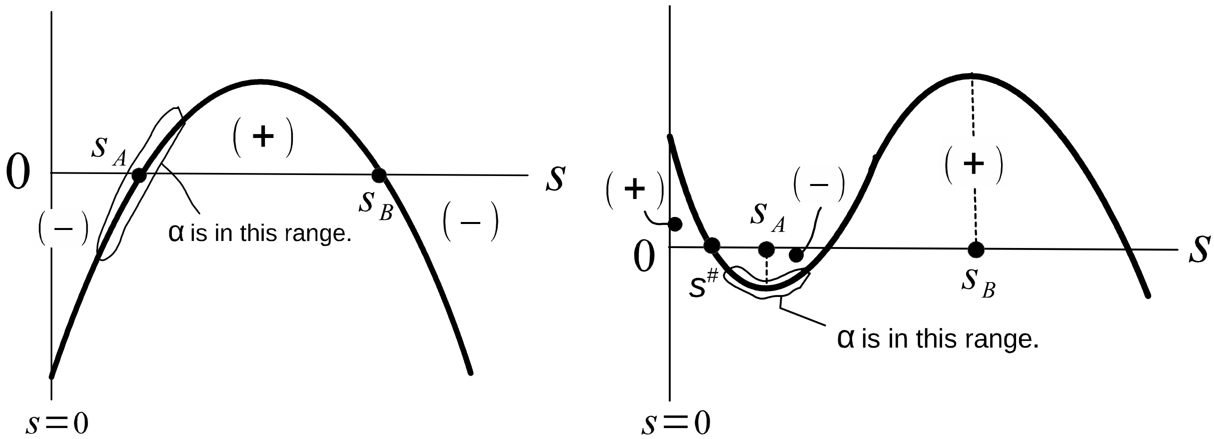
$$\begin{aligned}
&[1-\gamma(1-\alpha)]\frac{1}{s}-\frac{\gamma-\alpha}{1-s}-\frac{1-\gamma}{\alpha-s}+\frac{(1-\alpha)(1-\gamma)\,[\gamma(1-\alpha)+\alpha]}{\alpha-[\gamma(1-\alpha)+\alpha]s}\\
=\;&\frac{[1-\gamma(1-\alpha)]\,\{\alpha-[\gamma(1-\alpha)+\alpha]s\}+(1-\alpha)(1-\gamma)[\gamma(1-\alpha)+\alpha]\,s}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}\,s}-\frac{(\gamma-\alpha)(\alpha-s)+(1-\gamma)(1-s)}{(1-s)(\alpha-s)}\\
=\;&\frac{\alpha\,\{[1-\gamma(1-\alpha)]-[\gamma(1-\alpha)+\alpha]s\}}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}\,s}-\frac{(1-\alpha)\,(1+\alpha-\gamma-s)}{(1-s)(\alpha-s)}\\
=\;&\frac{\gamma(1-\alpha)+\alpha}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}s(1-s)(\alpha-s)}\left\{\alpha\left[\frac{1-\gamma(1-\alpha)}{\gamma(1-\alpha)+\alpha}-s\right](1-s)(\alpha-s)-(1-\alpha)(1+\alpha-\gamma-s)\left[\frac{\alpha}{\gamma(1-\alpha)+\alpha}-s\right]s\right\} \quad \text{(A24)}
\end{aligned}$$

Let $E_0 \equiv \frac{1-\gamma(1-\alpha)}{\gamma(1-\alpha)+\alpha}$, $E_1 \equiv 1+\alpha-\gamma$, and $E_2 \equiv \frac{\alpha}{\gamma(1-\alpha)+\alpha}$. Then, the derivative is expressed as $\frac{1}{1-\gamma}\left[\frac{s^{\alpha\gamma}(1-s)^{\gamma-\alpha}}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{(1-\alpha)(1-\gamma)}}\right]^{\frac{1}{1-\gamma}}\frac{\gamma(1-\alpha)+\alpha}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}(1-s)}$ times

$$\begin{aligned}
&\alpha(E_0-s)(1-s)(\alpha-s)-(1-\alpha)(E_1-s)(E_2-s)s\\
=\;&\alpha\left[E_0-(1+E_0)s+s^2\right](\alpha-s)-(1-\alpha)\left[E_1E_2-(E_1+E_2)s+s^2\right]s\\
=\;&\alpha\{\alpha E_0-[\alpha(1+E_0)+E_0]\,s+(1+E_0+\alpha)s^2-s^3\}-(1-\alpha)\left[E_1E_2s-(E_1+E_2)s^2+s^3\right]\\
=\;&-s^3+[\alpha(1+E_0+\alpha)+(1-\alpha)(E_1+E_2)]s^2-\{\alpha[\alpha(1+E_0)+E_0]+(1-\alpha)E_1E_2\}s+\alpha^2E_0. \quad \text{(A25)}
\end{aligned}$$

From the first line of the above equation, the derivative is 0 at $s=0$ and negative at $s=\alpha$.

The sign of the derivative for $s\in(0,\alpha)$ can be known by examining the shape of the above cubic function. The derivative of the cubic function with respect to s equals



(a) Shape of the quadratic function, (A26)

(b) Shape of the cubic function, (A25)

Figure A1: Shapes of (A25) and (A26)

$$-3s^2 + 2\left[\alpha(1{+}E_0{+}\alpha) + (1{-}\alpha)(E_1{+}E_2)\right]s - \left\{\alpha[\alpha(1{+}E_0){+}E_0] + (1{-}\alpha)E_1E_2\right\}, \qquad \text{(A26)}$$

which is negative at $s = 0$, while the sign at $s = \alpha$ is ambiguous.

The derivative of the quadratic function with respect to s equals $-6s{+}2\left[\alpha(1{+}E_0{+}\alpha) + (1{-}\alpha)(E_1{+}E_2)\right]$, which is positive at $s = 0$. It is also positive at $s = \alpha$ because

$$\begin{aligned}
& -6\alpha{+}2\left[\alpha(1{+}E_0{+}\alpha){+}(1{-}\alpha)(E_1{+}E_2)\right] \\
= \;& -6\alpha{+}2\left\{\alpha\frac{\alpha[\gamma(1{-}\alpha){+}\alpha]{+}(1{+}\alpha)}{[\gamma(1{-}\alpha){+}\alpha]}{+}(1{-}\alpha)\frac{(1{+}\alpha{-}\gamma)[\gamma(1{-}\alpha){+}\alpha]{+}\alpha}{\gamma(1{-}\alpha){+}\alpha}\right\} \\
= \;& -6\alpha + 2\frac{[1{-}(1{-}\alpha)\gamma][\alpha{+}(1{-}\alpha)\gamma]{+}2\alpha}{\gamma(1{-}\alpha){+}\alpha} \\
= \;& 2\frac{(1{-}\alpha)(1{-}\gamma)[3\alpha{+}(1{-}\alpha)\gamma]}{\gamma(1{-}\alpha){+}\alpha} > 0.
\end{aligned}$$

Hence, both $s = 0$ and $s = \alpha$ are located at the upward-sloping portion of the graph of the quadratic function. Figure A1 (a) shows a graph of the quadratic function. Based on this figure, Figure A1 (b) illustrates a graph of the cubic function. The sign of the derivative of $\frac{s^{1+\alpha\frac{\gamma}{1-\gamma}}(1-s)^{(1-\alpha)\frac{\gamma}{1-\gamma}-\alpha}(\alpha-s)}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{1-\alpha}}$ is the same as the sign of the cubic function for $s \in (0, \alpha]$ (the shapes are different because the derivative equals $\frac{1}{1-\gamma}\left[\frac{s^{\alpha\gamma}(1-s)^{\gamma-\alpha}}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{(1-\alpha)(1-\gamma)}}\right]^{\frac{1}{1-\gamma}}\frac{\gamma(1-\alpha)+\alpha}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}(1-s)}$ times the cubic function), while, as shown above, the sign of the derivative at $s = 0$ is zero.

Therefore, there exists $s^\sharp \in (0, \alpha)$ such that the derivative of $\frac{s^{1+\alpha\frac{\gamma}{1-\gamma}}(1-s)^{(1-\alpha)\frac{\gamma}{1-\gamma}-\alpha}(\alpha-s)}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{1-\alpha}}$ equals 0, and the derivative is 0 at $s = 0$, is positive for $s \in (0, s^\sharp)$, and is negative for $s \in (s^\sharp, \alpha)$.

Based on this result, Figure A2 illustrates graphs of the LHS and the RHS of (A23) when T_N, T_2, δ_N, and δ_L are high enough that they intersect. As shown above, when $s < \alpha$, $\frac{dc_2}{ds} < (>)0$ iff the LHS is smaller (greater) than the RHS. Therefore, $\frac{dc_2}{ds} < 0$ when s is small, $\frac{dc_2}{ds} > 0$ when s is intermediate, and $\frac{dc_2}{ds} < 0$ again when s is large (note $\frac{dc_2}{ds} < 0$ when $s \geq \alpha$). ■

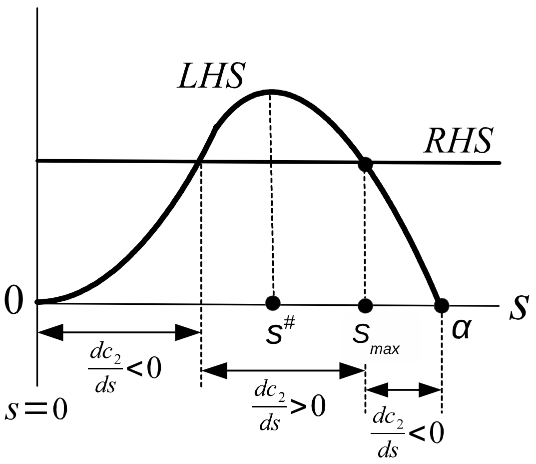


Figure A2: The determination of the sign of $\dfrac{dc_2}{ds}$ for $s < \alpha$

Proof of Proposition 1. **(i)** From Lemma 3 (i), net earnings and consumption of group 2 individuals decrease with s when $e^*_{2L} = 0$. From (ii) of the lemma, if T_N, T_2, δ_N, and δ_L are low enough, they decrease with s when $e^*_{2L} > 0$ too and thus they decrease with s for any s. Even when T_N, T_2, δ_N, and δ_L are high enough that net earnings and consumption increase with s for intermediate s when $e^*_{2L} > 0$ (Lemma 3 (ii)), they decrease with s for any s, if such range of s is not effective, i.e. $e^*_{2L} > 0$ is not true. From (A23) in the proof of Lemma 3, the supremum of s satisfying $\dfrac{dc_2}{ds} > 0$, which is $s_{\max}$ in Figure A2 and is smaller than α, increases with T_N, T_2, δ_N, and δ_L. From Lemma 1, $e^*_{2L} = 0$ iff $s \leq \underline{s}$ and $s \geq \overline{s}$ ($\underline{s} < 1 - \gamma(1-\alpha) < \overline{s}$), where $\underline{s}$ decreases ($\overline{s}$ increases) with T_N, T_2, δ_N, and δ_L. Hence, if these exogenous variables are low enough that $s_{\max} \leq \underline{s}$, consumption and net earnings decrease with s for any s.

(ii) Only the proof of the result on the consumption is presented, because net earnings in units of the final good equal consumption minus wealth. From the proof of (i), if T_N, T_2, δ_N, and δ_L are high enough that $s_{\max} > \underline{s}$, consumption increases with s when $e^*_{2L} > 0$ and s is intermediate. From Lemma 3, c_2 decreases with s for small s, increases with s for intermediate s, and decreases with s for large s, where, from Lemmas 1 and 3, c_2 decreases with s at least for $s \leq \underline{s}$ and $s \geq \min\{\alpha, \overline{s}\}$.

Hence, c_2 is maximized either at $s = s_{\max} < \alpha$ or at $s = 0$. From (30), c_2 at an intermediate s is greater (smaller) than c_2 at $s = 0$, where $e^*_{2L} = 0$, iff

$$(1-s)^{\gamma}\left(\frac{H_{2N}}{H_{2L}}|_{\text{intermediate } s}\right)^{-\alpha} > (<) \left(\frac{H_{2N}}{H_{2L}}|_{s=0}\right)^{-\alpha}$$

$$\Leftrightarrow (1-s)^{\gamma}\left(\frac{H_{2N}}{H_{2L}}|_{\text{intermediate } s}\right)^{-\alpha} > (<) \left\{\left(\frac{1-\alpha}{\alpha}\frac{1-\gamma}{\bar{l}^{\gamma}}\right)^{1-\gamma}[(1-\alpha)\gamma\delta_N {T_2}^{\alpha} {T_N}^{1-\alpha}]^{\gamma}\right\}^{-\frac{\alpha}{1-\gamma(1-\alpha)}} \quad \text{(from (28))}$$

$$\Leftrightarrow \frac{H_{2N}}{H_{2L}}|_{\text{intermediate } s} < (>) (1-s)^{\frac{\gamma}{\alpha}}\left\{\left(\frac{1-\alpha}{\alpha}\frac{1-\gamma}{\bar{l}^{\gamma}}\right)^{1-\gamma}[(1-\alpha)\gamma\delta_N {T_2}^{\alpha} {T_N}^{1-\alpha}]^{\gamma}\right\}^{\frac{1}{1-\gamma(1-\alpha)}} \quad \text{(A27)}$$

From (A22) in the proof of Lemma 3, s satisfies $\frac{dc_2}{ds} = 0$ and thus could be equal to $s_{\max}$ iff $\frac{H_{2N}}{H_{2L}} = \frac{[\alpha(1-\gamma)(1-s)]^{1-\gamma}(1-\alpha)[\delta_N(1-s)]^{\gamma}}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{1-\gamma}\alpha(\delta_L s)^{\gamma}}$. Hence, from this equation and (A27), the condition for c_2 at $s = s_{\max}$ to be greater (smaller) than c_2 at $s = 0$ is given by

$$(1-s)^{\frac{\gamma}{\alpha}}\left\{\left(\frac{1-\alpha}{\alpha}\frac{1-\gamma}{\bar{l}^{\gamma}}\right)^{1-\gamma}[(1-\alpha)\gamma\delta_N {T_2}^{\alpha} {T_N}^{1-\alpha}]^{\gamma}\right\}^{\frac{1}{1-\gamma(1-\alpha)}} >(<)\frac{[\alpha(1-\gamma)(1-s)]^{1-\gamma}(1-\alpha)[\delta_N(1-s)]^{\gamma}}{\{\alpha-[\gamma(1-\alpha)+\alpha]s\}^{1-\gamma}\alpha(\delta_L s)^{\gamma}}$$

$$\Leftrightarrow \alpha s^{\gamma}(1-s)^{\frac{\gamma(1-\alpha)}{\alpha}}\left\{\left(\frac{1-\alpha}{\alpha}\frac{1-\gamma}{\bar{l}^{\gamma}}\right)^{1-\gamma}[(1-\alpha)\gamma(\delta_N)^{\gamma(1-\alpha)}(\delta_L)^{1-\gamma(1-\alpha)} {T_2}^{\alpha} {T_N}^{1-\alpha}]^{\gamma}\right\}^{\frac{1}{1-\gamma(1-\alpha)}} >(<)(1-\gamma)^{1-\gamma}(1-\alpha)\left\{\frac{\alpha(1-s)}{\alpha-[\gamma(1-\alpha)+\alpha]s}\right\}^{1-\gamma}. \tag{A28}$$

Hence, if T_N, T_2, δ_N, and δ_L are is sufficiently large, c_2 at $s = s_{\max}$ is greater than c_2 at $s = 0$, otherwise c_2 at $s = 0$ is greater. From Figure A2 of the proof of Lemma 3 (ii), $s_{\max}$ is the largest of two values of s at which the LHS and the RHS of (A23) are equal. As T_N, T_2, δ_N and δ_L become higher, the graph of the LHS shifts upward and thus $s_{\max}$ increases. ■

Proof of Proposition 2. [**Those who become local-sector workers**] From Lemma 1, h^*_{2L} is lowest when $s \leq \underline{s}$ and $s \geq \bar{s}$. When $s \in (\underline{s}, \bar{s})$ and thus $e^*_{2L} > 0$, from (13) and (24),

$$\begin{aligned}\frac{dh^*_{2L}}{ds} &\propto \frac{1}{s} + (1-\alpha)\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\\ &= \frac{1}{s}\left[1+\frac{(1-\alpha)\left(\frac{1-\gamma-s}{1-s}B_0-B_1\right)}{\alpha B_0+(1-\alpha)B_1}\right] \quad \text{(from (A20) in the proof of Lemma 3)}\\ &= \frac{1}{s}\frac{\alpha B_0+(1-\alpha)\frac{1-\gamma-s}{1-s}B_0}{\alpha B_0+(1-\alpha)B_1}\\ &= \frac{1}{s(1-s)}\frac{(1-s)-(1-\alpha)\gamma}{\alpha B_0+(1-\alpha)B_1}B_0 >(<)0 \text{ for } s<(>)1-(1-\alpha)\gamma,\end{aligned} \tag{A29}$$

where $B_0 \equiv \left[(1-\alpha)[\delta_N(1-s)]^{\gamma}\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}}$ and $B_1 \equiv \left[\alpha(\delta_L s)^{\gamma}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}}$.

Hence, h^*_{2L} is maximized at $s = 1-(1-\alpha)\gamma \in (\underline{s}, \bar{s})$.

[**Those who become national-sector workers**] When $e^*_{2L} > 0$, from (21), $\frac{dh_{2L}}{ds} = \frac{d(\bar{l}+\delta_L s e^*_{2N})^{\gamma}}{ds}$ is proportional to

$$\begin{aligned}&\frac{1}{s}-\frac{1}{1-\gamma}\left[\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right]\\ &= \frac{1}{(1-\gamma)s(1-s)}\left((1-\gamma)(1-s)-\left\{\gamma s+\frac{\alpha[(1-\gamma-s)B_0-(1-s)B_1]}{\alpha B_0+(1-\alpha)B_1}\right\}\right) \quad \text{(from (A20))}\\ &= \frac{1}{(1-\gamma)s(1-s)[\alpha B_0+(1-\alpha)B_1]}[(1-\gamma-s)(1-\alpha)+\alpha(1-s)]B_1\\ &= \frac{1}{(1-\gamma)s(1-s)[\alpha B_0+(1-\alpha)B_1]}[1-(1-\alpha)\gamma-s]B_1.\end{aligned} \tag{A30}$$

When $e^*_{2L} = 0$, $\frac{d(\bar{l}+\delta_L s e^*_{2N})^{\gamma}}{ds}$ is proportional to

$$\begin{aligned}\frac{1}{s}-\frac{1}{1-\gamma}\left[\frac{\gamma}{1-s}+\alpha\left(\frac{H_{2N}}{H_{2L}}\right)^{-1}\frac{d\frac{H_{2N}}{H_{2L}}}{ds}\right] &= \frac{1}{(1-\gamma)s(1-s)}\left\{(1-\gamma)(1-s)-\left[\gamma s-\frac{\alpha\gamma s}{1-\gamma(1-\alpha)}\right]\right\} \quad \text{(from (28))}\\ &= \frac{1}{(1-\gamma)s(1-s)[1-\gamma(1-\alpha)]}\{(1-\gamma-s)[1-\gamma(1-\alpha)]+\alpha\gamma s\}\\ &= \frac{1}{s(1-s)[1-\gamma(1-\alpha)]}[1-(1-\alpha)\gamma-s].\end{aligned} \tag{A31}$$

Hence, in both cases, $\frac{d(\bar{l}+\delta_L se_{2N}^*)^\gamma}{ds} \gtreqless 0$ for $s \lesseqgtr 1-(1-\alpha)\gamma$. From (21) and (28), $se_{2N}^* = 0$ and thus $h_{2L} = \bar{l}^\gamma$ at $s = 0, 1$.

From (21) and (30), e_{2N}^* and c_2 are linear functions of $\left[(1-s)^\gamma \left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}}$. Hence, from $\frac{d(se_{2N}^*)}{ds} \propto \frac{1}{s} + \frac{de_{2N}^*}{ds}$, the s maximizing se_{2N}^* and thus h_{2L} is greater than the s maximizing c_2. ■

Proof of $e_{2N}^* > e_{2L}^*$. From (21) and (24),

$$e_{2N}^* = \left[(1-\alpha){T_2}^\alpha {T_N}^{1-\alpha}\gamma(\delta_N(1-s))^\gamma \left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}} > e_{2L}^* = \frac{1}{\delta_L s}\left\{\left[\alpha\gamma\delta_L s {T_2}^\alpha {T_N}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{1-\alpha}\right]^{\frac{1}{1-\gamma}} - \bar{l}\right\}$$

$$\Leftrightarrow \left[\gamma {T_2}^\alpha {T_N}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}} \delta_L s\left\{[(1-\alpha)(\delta_N(1-s))^\gamma]^{\frac{1}{1-\gamma}} - \left[\alpha(\delta_L s)^\gamma \frac{H_{2N}}{H_{2L}}\right]^{\frac{1}{1-\gamma}}\right\} + \bar{l} > 0. \qquad \text{(A32)}$$

Thus, $e_{2N}^* > e_{2L}^*$ if $(1-\alpha)(\delta_N(1-s))^\gamma \geq \alpha(\delta_L s)^\gamma \frac{H_{2N}}{H_{2L}}$.

Because the indifference condition, (29), can be expressed as

$$\left[\gamma^\gamma {T_2}^\alpha {T_N}^{1-\alpha}\left(\frac{H_{2N}}{H_{2L}}\right)^{-\alpha}\right]^{\frac{1}{1-\gamma}}\left(\{(1-\alpha)[\delta_N(1-s)]^\gamma\}^{\frac{1}{1-\gamma}} - \left[\alpha(\delta_L s)^\gamma\left(\frac{H_{2N}}{H_{2L}}\right)\right]^{\frac{1}{1-\gamma}}\right) = \frac{1}{1-\gamma}\frac{\bar{l}}{\delta_L s}, \qquad \text{(A33)}$$

$(1-\alpha)[\delta_N(1-s)]^\gamma > \alpha(\delta_L s)^\gamma \frac{H_{2N}}{H_{2L}}$ must hold. Therefore, $e_{2N}^* > e_{2L}^*$ is always true. ■