Stereotypes, Segregation, and Ethnic Inequality

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Abstract

Disparities in economic conditions among different ethnic, racial, or religious groups continue to be serious concerns in most economies. Evidence shows a strong negative relationship between inequality across ethnic groups and economic development. Relative standings of different groups are rather persistent, although some groups initially in disadvantaged positions successfully caught up with then-advantaged groups. Two obstacles, costly skill investment and negative stereotypes or discrimination in the labor market, seem to distort investment and sectoral choices between formal and informal sectors, give rise to skill and labor market segregations by ethnicity, and slow down the progress of disadvantaged groups.

How do these obstacles affect skill investment and sectoral choices of different groups and the dynamics of their economic outcomes and inter-group inequality? Is affirmative action necessary to significantly improve conditions of subordinate groups, or redistributive policies sufficient? In order to tackle these questions, this paper develops a dynamic model of statistical discrimination and examines how initial economic standings of groups and initial institutionalized discrimination affect subsequent dynamics and long-run outcomes.

Keywords: ethnic inequality; statistical discrimination; dual economy; labor market segregation; skill investment.

JEL Classification Numbers: J15, J24, J31, O15, O17

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1 Introduction

Disparities in economic conditions among different ethnic, racial, or religious groups continue to be serious concerns in most economies. Evidence shows a strong negative relationship between inequality across ethnic groups and economic development (Alesina, Michalopoulos, and Papaioannou, 2016).[1] Relative standings of different groups are rather persistent (Borjas, 1994, 2006; Darity, Dietrich, and Guilkey, 2001),[2] although some groups initially in disadvantaged positions successfully caught up with then-dominant groups. Two obstacles seem to slow down the progress of subordinate groups. One is costly skill investment: the quality of public schools is low in many, especially developing, countries, thus people rely on costly private schools, supplementary study materials, and tutoring to acquire skills (Baker et al., 2001; Bray and Kwok, 2003).[3] The other is negative stereotypes or discrimination in the labor market, which compels many individuals of subordinate groups to invest less in skill, or to choose occupations or sectors where performance is less affected by such handicaps but earnings tend to be lower, such as informal-sector jobs and neighborhood jobs (Telles, 1993; Bayard et al., 1999; van de Walle and Gunewardena, 2001).[4] Even skilled workers often avoid mainstream jobs and sectors and run small businesses instead.

How do these obstacles affect skill investment and sectoral choices of different groups and the dynamics of their economic outcomes and inter-group inequality? Is affirmative action necessary to significantly improve conditions of disadvantaged groups, or redistributive policies sufficient? In order to tackle these questions, this paper develops a dynamic model of statistical discrimination and examines how initial economic standings of groups and initial institutionalized discrimination affect subsequent dynamics and long-run outcomes.

The model. The analysis is based on a discrete-time small-open OLG model. There exists

[1] Alesina, Michalopoulos, and Papaioannou (2016) construct measures of within-country differences in well-being across ethnic groups, called "ethnic inequality", by combining ethnographic and linguistic maps on the location of groups with satellite images of light density at night, and find a strong negative relationship between ethnic inequality and per capita GDP even after controlling for the overall degree of spatial inequality and inequality across administrative regions.

[2] Borjas (1994, 2006) shows that wages of a U.S. worker in 1940 and 1980 are significantly related to the average wage of immigrants of the worker's ethnic group in 1910, after individual characteristics are controlled for (blacks are not in the data). About 22% of the intergroup wage gap in the immigrant generation persists into the third generation. Darity, Dietrich, and Guilkey (2001) find that the occupational status of a U.S. worker in 1980 and 1990 is significantly related to human capital endowments and the degree of favorable or unfavorable treatment in the labor market in the period between 1880 and 1910 of his/her group.

[3] Baker et al. (2001) find that about 40% of seven and eight graders in a large sample from 41 developed and developing countries participate weekly in private supplementary tutoring, such as tutoring sessions and cram schools, to study mathematics. Further, at the national level, they find that the average participation rate is significantly negatively related to the percentage of public expenditure on education in GNP. Bray and Kwok (2003) briefly review existing studies, which show that the use of private tutoring is extensive even among primary school students in developing countries.

[4] Telles (1993) finds, in Brazilian metropolitan areas, that minorities (except Asians) are overrepresented in low-wage informal-sector jobs in which being minorities has less negative effects on earnings. For the U.S. economy, Bayard et al. (1999) find that greater racial and ethnic wage disparities for men than for women can be explained largely by more severe occupational and industry-level segregation among men. For the Vietnam economy, van de Walle and Gunewardena (2001) show that, compared to the majority Kinh, returns to education are lower but returns to land are higher for minorities, suggesting that minorities choose to exert more efforts on farming in which performance is less affected by their disadvantaged positions.

a continuum of two-period-lived individuals who belong to one of two ethnic (racial, religious) groups. In childhood, an individual receives a transfer from her parent and spends it on assets and skill investment to become a skilled worker. No credit market exists for skill investment, so she cannot invest if the transfer is not enough. Since she can spend wealth on assets too, she invests in skill only if it is affordable *and* profitable. In adulthood, she chooses a sector to work, obtains income from assets and work, and spends it on consumption and a transfer to her single child.

There exist the formal sector with advanced technology and the informal sector with backward technology, where the latter may not be active in equilibrium. They respectively correspond to modern/formal and traditional/informal sectors in developing economies, while in advanced economies, typical 'informal-sector' jobs are neighborhood jobs at small businesses.

In real economy, labor and product markets of the formal sector tend to be ethnically more mixed than the informal sector (Åslund and Skans, 2010; Glitz, 2014). In the integrated formal sector, a *subordinate group*, who are typically a minority but could be a majority in a historically disadvantaged position, are prone to face disadvantages in production or suffer greater disutility of work, because prevalent language, customs, taste, and culture are different from theirs, or they face taste-based discrimination. Hence, the effect of skill investment on human capital in the formal sector, where skilled workers have comparative advantages, is assumed to be smaller for the subordinate group, while the investment raises human capital in the informal sector equally for both groups. Main implications of the model, however, *remain intact without this assumption* (although the dynamics are affected): it is imposed for analytical simplicity as well as for reality.

In the formal sector, due to complex production processes and organizational structures, evaluating each worker's contribution to output tends to be difficult. Accurate evaluation is particularly difficult at least initially, if a worker and her evaluators belong to different groups due to the above inter-group differences (Pinkerson, 2006; Fryer, Pager, and Spenkuch, 2013). Qualifications of a job applicant too tend to be assessed less precisely when interviewers are from other groups (Stoll, Raphael, and Holzer, 2004). Hence, the wage is assumed to depend partly on her human capital and partly on its signal, the average human capital (average wage) of her group in the sector (in the spirit of classic models of statistical discrimination by Aigner and Cain, 1977, and Lundberg and Startz, 1983), *and* the signal's importance decreases with the group's share in the sector's skilled workers (a proxy for evaluators or interviewers): as the proportion of evaluators or interviewers from her own group is higher, her human capital is revealed more precisely (Åslund, Hensvik, and Skans, 2014; Pinkerson, 2006). In the informal sector, typically, each worker's contribution is easy to measure, thus wage equals human capital.[5,6]

[5]As with many models of statistical discrimination, it is implicitly supposed that education *level* (i.e. years of education) is not a good signal of a worker's human capital. This implies that the model is concerned with an economy where the quality of public schools is low or varies greatly across schools and thus many people expend on supplementary study materials and tutoring or attend private schools, which, as mentioned at the beginning, is the case in many countries (Baker et al., 2001; Bray and Kwok, 2003). Skill investment of the model may be interpreted as spendings on these activities.

[6]The wage equations can be derived from maximization problems of firms hiring physical capital too (footnote 20). Introducing productivity growth does not affect qualitative results, if the cost of skill investment is assumed to

Wealth in the initial period is unequally distributed, and the inequality is transmitted intergenerationally through transfers. Hence, generally, individuals are heterogeneous in accessibility to skill investment, and those without enough wealth do not invest even if it is profitable. Their descendants, however, may become accessible if enough wealth is accumulated. (Similarly, offspring of those with enough wealth may become inaccessible if wealth decreases enough.)

An important property of the model is that skill investment and sectoral choices of individuals within *and* across groups could be *interrelated*, because a worker's wage in the formal sector depends on her group's average human capital in the sector (termed the group's *reputation*) and on the reputation's importance in the wage (termed *the degree of prejudice* toward the group), which decreases with the group's share in the sector's skilled workers. Hence, the dynamics of wealth and economic positions of people too could be interrelated. The paper examines how the initial distribution of wealth within and across groups affects the dynamics of skill investment, sectoral choices, intra and intergroup disparities, and the steady-state outcome.

Main results. First, sectoral choices and skill investment may not be socially optimal. Even if unskilled workers are less productive in the formal sector,[7] they may choose the sector due to a positive effect from skilled workers through the reputation. Individuals with sufficient wealth may not carry out productive investment due to the negative effect from unskilled workers. For a similar reason, it is possible that all skilled workers of a group choose the informal sector and all unskilled workers choose the *formal* sector, even if *skilled* workers have comparative advantages and are more productive in the formal sector. This result may explain the fact that skilled people of subordinate groups often avoid formal-sector jobs and run small businesses in their communities or in industries they are concentrated in.

Second, multiple equilibria could exist regarding skill investment and sectoral choices of skilled workers: both the non-poor of a group invest (or skilled workers of a group choose the formal sector) and do not could be equilibria. One source of multiplicity is strategic complementarity within a group: to take the investment as an example, as more people invest and get skilled, prejudice toward the group eases, formal-sector wages reflect individual human capital more closely, and the return to investment rises. Another source of multiplicity is strategic substitutability across groups: as more people of a group invest, prejudice toward the other group intensifies and the opponent's return to investment falls; If the substitutability is strong enough, *either* of the groups invest and the other do not are equilibria.[8]

Third, the dynamics and long-run outcomes of groups, particularly of the subordinate group,

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grow proportionately.

[7]Individuals, particularly the minority, could be less productive in the formal sector if the quality of formal institutions and thus the sector's productivity are low, if non-statistical discrimination exists, or if the disutility of work is greater in the sector (human capital may be measured *net* of the disutility).

[8]The setting that there is a continuum of individuals also plays a role in generating multiple equilibria: because each individual's decision has a negligible impact on aggregate variables, it is often the case that unilateral deviation is not profitable. The setting that the utility of an individual depends on a transfer to her child, but not directly on the child's utility, too seems to raise the possibility of multiple equilibria: although analytically intractable, in a model with perfectly altruistic individuals, a parent takes into account the negative effect of no skill investment on her child's utility and thus would be affected less by decisions of other individuals.

depend greatly on initial conditions and could be quite different from a "prejudice-free" economy. Since *good* (*bad*) *reputation tends to beget good* (*bad*) *reputation*, a group starting with a good (bad) initial condition, i.e. a high (low) fraction of them can afford skill investment initially, tend to be in a good (bad) position in the long run, according with empirical evidence such as Borjas (1994, 2006) and Bertocchi and Arcangelo (2012). Initial conditions could greatly affect not only the dynamics of skill investment but also *the dynamics of sectoral choices and labor market segregation*. It is possible that, if the initial condition of the subordinate group is bad (good), in the long run, all of them are unskilled (skilled) *and* in the informal (formal) sector and thus the labor market is ethnically segregated (integrated). The strong dependence on initial conditions arises because, unlike a "prejudice-free" economy in which the dynamics of an individual lineage are affected only by its own initial condition, the dynamics are affected by initial conditions of the groups too through the dependence of formal-sector wages on group-level variables, reputation and the degree of prejudice.

Fourth, when multiple equilibria exist regarding skill investment or sectoral choices of skilled workers, which is the case when prejudice is severe or the efficacy of investment is low, *given the initial distribution of wealth*, *the initial realization of equilibrium* could affect the dynamics greatly.[9] When multiple equilibrium choices exist for the subordinate group, it is possible that, if the group's non-poor *happen not to* (*happen to*) invest [or happen to choose the informal (formal) sector] initially, the number of the group's skilled workers falls (grows) over time and the group are totally unskilled (skilled) eventually. When multiple equilibria exist for both groups, the long-run outcome of the dominant group too is sensitive to the initial realization. The dominant staring with a much better condition than the subordinate could end up with the *smaller* fraction of skilled workers, if they happen not to and the subordinate happen to invest initially. The result suggests that, if institutionalized discrimination limiting a group's access to investment or skilled jobs in the formal sector affects the initial realization of equilibrium, it could have a lasting impact on their well-beings well after its abolishment, consistent with the finding of the persistent effect of initial discrimination by Darity, Dietrich, and Guilkey (2001). Income or wealth redistribution does little to change the situation, while affirmative action treating them favorably in skill investment or formal-sector employment, such as tuition or wage subsidies, can be very effective.

Organization of the paper. Section 2 reviews the related literature and details contributions of the paper in the literature. Section 3 presents and analyzes the model's static part. Section 4 presents the full-fledged model and Section 5 analyzes the dynamics. Section 6 examines a general case by lifting one assumption that excludes situations of severe prejudice and low relative efficacy of skill investment in the formal sector. Section 7 concludes. Appendix A presents two propositions, and Appendix B contains proofs of lemmas and propositions.

[9] It is assumed that the initial coordination among individuals of a group continues for subsequent periods: if the group's non-poor happen to invest initially, they invest subsequently too. The assumption would be reasonable since children tend to mimic parental behaviors in real society. Kim and Loury (2009) make a similar assumption in their model (see footnote 32).

2 Related Literature

Literature on statistical discrimination. Models of statistical discrimination examine the situation where employers cannot observe workers' skills and thus use two kinds of signals, race and a signal imperfectly correlated with individual skill, such as a test and an on-the-job monitoring, to screen workers (see Fang and Moro, 2010, for a survey). The first type of models such as Coate and Loury (1993) explain skill and earnings disparities among groups with equal endowment based on multiple equilibria. Employers assign individuals to two kinds of jobs, jobs requiring skill investment for good performance and those not, based on the signals. Since one's return to investment increases with investments by others of her race, multiple equilibria with different shares of skilled workers could exist. The second type, by contrast, assume that the non-race signal is nosier for the subordinate group to explain the disparities. Lundberg and Startz (1983), drawing on Phelps (1972) and Aigner and Cain (1977), develop a model where wage equals expected marginal productivity conditional on the signals. The return to investment is lower for the subordinate group due to the noisier signal and thus they invest less even if groups' endowment is identical.

Recent major progress in the literature are twofold. One is the extension to a *dynamic* setting. This is particularly important to the first type of models, where employers' self-confirming beliefs about groups' skill levels select an equilibrium, because a static model does not explain how such beliefs are formed. Kim and Loury (2009) develop a continuous-time OLG model in which employers' beliefs are formed based on objective information on groups' present and future skill levels (reputations) and are updated with changing investments. If the initial reputation of a group is high (low), the group converges to the high (low) reputation steady state, while if it is intermediate, the group could converge to either steady state, i.e. self-confirming expectations determine the final state as in static models.

The other is the consideration of *inter-group interactions*. In the above models, different groups do not interact and thus behaviors and welfare of one group do not affect those of other groups.[10] Chaudhuri and Sethi (2008) present a static model of the first type in which the investment cost depends on both individual ability and the fraction of skilled peers, which equals a weighted average of the fractions in one's own group and in the overall population and the constant weight on own group is interpreted as the degree of segregation. In a special case, they show that, in an economy where inter-group inequality exists under complete segregation, complete integration eliminates inequality and raises (lowers) shares of skilled workers of both groups, if the fraction of the initially disadvantaged group is low (high). Lundberg and Startz (2007) construct a random search model with a second-type element where searchers observe imperfect signals of potential partners' abilities. In a one-sided search model where homogenous white searchers observe more

[10] Hence, the models cannot provide economic rationales for institutionalized discrimination used to be enforced by dominant groups in many countries. Moro and Norman (2004) construct a static general equilibrium model of the Coate and Loury type, in which productivities of two types of jobs are interrelated. When the two jobs are complementary, an increased share of skilled workers has a negative (positive) effect on the wage of good (bad) jobs, and thus the return to investment of a dominant group is negatively affected by investment of the disadvantaged group, giving dominant groups an incentive for the discrimination.

accurate signals of whites than of blacks, there could exist an equilibrium where they trade only with whites with good signal, even if both groups have identical ability distribution. In a two-sided search model where searchers are heterogenous in ability and race (and signals observed by black searchers reveal abilities of both races equally), they numerically show that there could exist an equilibrium of racially segregated transactions where high ability whites (blacks) accept only whites (blacks) with good signal.

Contributions of the paper in the literature. This paper shares with the second type of models such as Lundberg and Startz (1983) the feature that the importance of own group's average human capital (reputation) in wage, is different among groups (footnote 17). The existing works assume that the importance (the degree of prejudice) is *constant* and greater for a subordinate group, while, in this paper, it decreases with the share of own group in formal-sector skilled workers (a proxy for evaluators of performance or job interviewers): as the proportion of evaluators or interviewers from own group is higher, individual human capital is revealed more precisely. Unlike these works, the model is *dynamic* and inter-group disparities could change over time, thus making the reputation's importance depend on the endogenous variable would be crucial. Such formulation yields a different kind of inter-group interactions from works such as Lundberg and Startz (2007) and Chaudhuri and Sethi (2008), and the interactions generate a different type of multiple equilibria from works such as Coate and Loury (1993).

Further, the paper models *sectoral choices* between the formal and the informal sectors, where reputation could affect wage only in the former, and the credit constraint in skill investment, both of which are not considered in other works but are important real-economy elements, especially in developing countries, as stated in the introduction. The credit constraint generates occupational mobilities of lineages through intergenerational transmission of wealth and the interesting group dynamics described in the introduction, whereas modeling the sectoral choice enables the analysis of the *dynamics of labor market segregation*.

Regarding several elements, the paper employs a simpler setting: there is no non-race signal, which implicitly supposes that one's contribution to production cannot be observed initially but is fully revealed later; the investment cost is homogeneous; and the generational structure is simpler than the dynamic model of Kim and Loury (2009). However, because of the simpler setting, it can consider the above-mentioned new elements and examine how *transitional dynamics* as well as steady states depend on the initial condition using phase diagrams. Further, it can identify conditions under which multiple equilibria exist, the dynamics are different from a "prejudice-free" economy, inter-group disparities are eradicated in the long run, etc.

Other related studies. Studies that examine the dynamics of inter-group inequality based on models without statistical discrimination too are closely related.[11] Lundberg and Startz (1998),

[11] Bowles, Loury, and Sethi (2014), building on Loury (1977), construct a discrete-time OLG model with two groups, where the cost of skill investment is modeled as in Chaudhuri and Sethi (2008). They prove that, when the degree of segregation is sufficiently high, the long-run group equality cannnot be attained even with very small initial inequality. In a special case, they show a dynamic version of the result of Chaudhuri and Sethi (2008) mentioned earlier. Yuki (2009) examines the dynamics of disparities between educated and uneducated workers in a one-group

based on Loury (1977) and the 'ethnic capital' model of Borjas (1992), examine a dynamic two-group economy in which human capital is the engine of growth and there exist spillovers from coworkers in production and from elder neighbors and, for the minority, from elders of the majority in skill development. Individuals are *assumed* to be segregated by ethnicity both in the workplace and in residence. There are no spillovers from the minority to the majority and inter-group inequality disappears in the long run. Using a version of the model with heterogeneous innate ability and without the third spillover, they examine the effect of workplace desegregation, i.e. allowing the minority to move to majority-dominated jobs by paying a mobility cost, on the dynamics. They examine the effect of *one-time* workplace desegregation, while this paper examines the *dynamics* of labor market segregation in an economy where workers can freely choose sectors.

The modeling of skill investment and intergenerational transmission of wealth draws on Galor and Zeira (1993) and Yuki (2008, 2016), in which, as in this paper, skill investment is constrained by intergenerational transfers motivated by impure altruism.

3 Static part of the model

This section presents and analyzes the static part of the model. The dynamic part is presented in the next section. Consider a small open economy (interest rate r is exogenous) populated by a continuum of individuals who belong to one of two ethnic (racial, religious) groups. Results in this section can be applied to traits that are not intergenerationally transmitted, such as gender and native region, as well. Individuals decide whether or not to invest in skill, then choose a sector to work. The cost of skill investment c_h must be self-financed, so they must have enough wealth.

There exist the formal sector with advanced technology and the informal sector with backward technology, where the latter may not be active in equilibrium. They respectively correspond to modern/formal and traditional/informal sectors in developing economies, while in advanced economies, typical 'informal-sector' jobs would be neighborhood jobs at small businesses. In real economy, labor and product markets of the formal sector tend to be ethnically more mixed than the other sector (Åslund and Skans, 2010; Glitz, 2014), probably due to differences in needed skills, scales of operations, and enforcement of law.[12]

Two assumptions are made based on this fact. First, skill investment raises human capital from h_u (u is for unskilled) to h_s (s is for skilled) in the informal sector, while, in the formal sector, it raises human capital of ethnic (racial, religious) group i from $A_{ui}h_u$ to $A_{si}h_s$, where the

and one-sector economy where innate ability is heterogeneous and wage is determined as in this paper (education is the signal).

[12] Formal-sector firms need workers with highly specialized skills and scales of operations tend to be large. Thus, to assign jobs to workers with appropriate skills efficiently, labor markets tend to be anonymous and ethnically integrated. Further, the sector tends to be regulated by laws prohibiting overt employment discrimination. By contrast, in the informal sector with the contrasting features, employment is largely through personal connections and thus labor markets are more segregated. Also, products of the formal sector are supplied to national markets, while those of the informal sector, especially services, are mainly for local markets of particular groups. Åslund and Skans (2010) and Glitz (2014) find that the tendency for a minority worker to work with coethnics is stronger in smaller establishments in Sweden and Germany, respectively. Glitz (2014) also finds that ethnic segregation is stronger in sectors such as agriculture, construction, and the low-skill service sector.

relative human capital A_{ki} ($k=u,s$) is weakly greater for the dominant group (*group 1*) than the subordinate group (*group 2*):

$$A_{k1} \geq A_{k2}. \tag{1}$$

Given skill, human capital in the sector is weakly lower for the subordinate group, who are typically the minority but could be the majority in a historically disadvantaged position.[13] The assumption captures the following facts: in the integrated formal sector, they are prone to face disadvantages in production or suffer greater disutility of work (human capital may be measured *net* of the disutility), because prevalent language, customs, taste, and culture are different from theirs; if taste-based discrimination exists, they are not assigned relevant tasks and end up in lower productivity.[14]

As is made clear later, the assumption (1) is imposed for analytical simplicity as well as for reality, and *main implications remain intact without it.* By contrast, the next assumption is crucial.

In the formal sector, due to complex production processes and organizational structures, evaluating each worker's contribution to output tends to be difficult. Accurate evaluation is particularly difficult at least initially, if a worker and her evaluators belong to different groups due to the above-mentioned inter-group differences (Pinkerson, 2006; Fryer, Pager, and Spenkuch, 2013).[15] Qualifications of a job applicant too tend to be assessed less precisely when interviewers are from the other group (Stoll, Raphael, and Holzer, 2004).[16] Hence, the wage is assumed to depend partly on her human capital and partly on its signal, the average human capital (average wage) of her group in the sector (in the spirit of classic models by Aigner and Cain, 1977, and Lundberg and Startz, 1983),[17] *and* the signal's importance decreases with the group's share in the sector's skilled workers (a proxy for evaluators or interviewers): as the proportion of evaluators or interviewers from her own group is higher, her human capital is revealed more precisely or sooner (Åslund, Hensvik, and Skans, 2014; Pinkerson, 2006).[18]

[13]Skill and human capital are *different*: skill is ability, while human capital is the contribution of skill to output, and workers of given skill can be different in human capital levels depending on sectoral choices and ethnicity.

[14]Taste-based discrimination seems to affect labor market outcomes even in advanced nations. For the U.S., Charles and Guryan (2008) find that white-black wage gaps in a state are related to the degree of bias by whites in the left tail of the bias distribution in the state, consistent with the model of Becker (1971).

[15]Pinkerson (2006) finds that the effect of AFQT scores, a measure of skills not observed directly by employers, on wages increases with experience for black men but does not change for white men in the U.S., which, considering that managers are overwhelmingly white, is consistent with the view that accurate evaluation takes time when evaluators and workers belong to different groups. By contrast, Arcidiacono, Bayer, and Hizmo (2010) find similar effects of AFQT scores on wages of the two groups. Fryer, Pager, and Spenkuch (2013) develop a test for the presence of racial discrimination in the labor market and find, based on New Jersey data, that the effect of discrimination on offered wages is at least one-third of the black-white wage gap and empirical findings are most naturally explained by a model in which employers statistically discriminate on the basis of race when hiring from the market but learn about their employees' productivity over time.

[16]Stoll, Raphael, and Holzer (2004) find that establishments where blacks are in charge of hiring are significantly more likely to employ blacks than those with white hiring agents as a result of the higher application rate of blacks and the higher hiring rate of black applicants in the former establishments in the U.S, which is despite *much stricter* hiring requirements and screening methods there. This finding is consistent with the view that qualifications of (particularly, able) black applicants tend to be assessed more accurately in the former establishments.

[17]Unlike this model, a worker's contribution to output is never revealed and thus her wage equals a weighted average of her group's average human capital and her non-race signal, and the importance of the race signal (corresponding to β_i in the equations below) is *constant* and is *assumed* to be greater for the subordinate group in their models.

[18]Åslund, Hensvik, and Skans (2014), based on data set covering 70,000 Swedish establishments, find that workers

The wage of an individual with skill level k ($k=u,s$) of group i is given by:

$$(1-\beta_i)A_{ki}h_k+\beta_i E[Ah|i], \tag{2}$$

where $\beta_i \in [0,1]$ measures the importance of the average human capital, $E[Ah|i]$, and decreases with the share:[19]

$$\beta_i=\beta\left(\frac{p_{si}H_iN_i}{p_{si}H_iN_i+p_{sj}H_jN_j}\right),\ j\neq i,\ \beta'(\cdot)<0,\ \beta(1)=0. \tag{3}$$

H_i is the fraction of skilled workers in group i, N_i is the group's population, and p_{si} is the probability that a skilled worker of the group chooses the formal sector. The size of β_i reflects the degree of the incomplete information and is named *the degree of prejudice* toward the group. If the sector's skilled workers are all from her group, $\beta(1)=0$ for simplicity. The average human capital, termed the group's *reputation*, equals (p_{ui} is the probability for an unskilled worker):

$$E[Ah|i]=\frac{p_{si}H_iA_{si}h_s+p_{ui}(1-H_i)A_{ui}h_u}{p_{si}H_i+p_{ui}(1-H_i)}. \tag{4}$$

In the informal sector, typically, each worker's contribution is easy to measure, thus wage equals human capital, h_k ($k=u,s$).

As with many works in the literature, it is implicitly supposed that education *level* (i.e. years of education) is not a good signal of a worker's human capital, implying that the model is concerned with an economy where the quality of public schools is low or varies greatly across schools and thus many people expend on supplementary study materials and tutoring or attend private schools, which, as mentioned in the introduction, is the case in many countries (see footnote 3 for details). Skill investment of the model may be interpreted as spendings on these activities.

The wage equations can be derived from profit maximization problems of firms that hire workers *and* physical capital.[20] Further, productivity growth can be incorporated without affecting results qualitatively, as long as the cost of skill investment c_h is assumed to grow proportionately.

The following assumptions are imposed on A_{ki} ($k=u,s$) and the function $\beta(\cdot)$.

Assumption 1 *(i)* $A_{si}\geq A_{ui}$

(ii) $A_{si}>1$ *and* $A_{si}h_s-(1+r)c_h-\max\{A_{ui},1\}h_u\geq 0 \Leftrightarrow A_{si}\geq \frac{(1+r)c_h+\max\{A_{ui},1\}h_u}{h_s}$

(iii) $\beta(0)+\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}\leq 1 \Leftrightarrow A_{si}\geq \frac{h_u}{h_s}A_{ui}+\frac{h_s-h_u}{(1-\beta(0))h_s}$.

The first assumption states that skilled workers have comparative advantages (weakly) in the formal sector, which would be justified from the fact that the sector adopts more advanced technology and thus workers' skills are more important. The second assumption states that skilled workers are

who share an origin (native or immigrant) with their managers earn higher wages, especially when they are immigrants from the same country. Further, they find *no* direct effect of manager-worker similarity on wages *after* worker fixed effects are controlled for, suggesting that managers sharing an origin can detect workers with good objectively unobservable qualities better and reward them with higher wages. This finding and Pinkerson (2006) in footnote 15 are consistent with the setting the signal's importance decreases with the group's share in the skilled workers (a proxy for evaluators or interviewers).

[19] An interpretation of β_i is that evaluators cannot recognize her skill during the first β_i fraction of time but can identify it after that. Alternatively, $(1-\beta_i)A_{ki}h_k$ may be construed as the amount of her contribution to output recognized precisely by them.

[20] Suppose that firms with an identical CRS technology hire both factors in each sector. By normalizing the wage rate per human capital of the informal sector (which depends on the TFP and the interest rate) to 1, the same wage equations are obtained. The relative human capital in the formal sector A_{ki} increases with the sector's relative TFP.

more productive in the formal sector and the net *social return* to skill investment is non-negative. An old version of the paper (Yuki, 2012), which does not impose this assumption, shows that the result is different from the "prejudice-free" economy only when this condition holds. The last assumption means that the net private return to choosing the formal sector is weakly higher for skilled workers even when the degree of prejudice is severest, i.e. $\beta_i = \beta(0)$ (the assumption can be expressed as $[(1-\beta(0))A_{si}-1]h_s \geq [(1-\beta(0))A_{ui}-1]h_u$). The first two assumptions are maintained throughout the paper, while the last one is relaxed in Section 6.

3.1 Sectoral choices and skill investment

Since workers are freely mobile between the sectors, they choose the one(s) with higher earnings. The next lemma presents equilibrium values of p_{si} and p_{ui} for given p_{sj} $(j \neq i)$ of the other group, when $H_i > 0$ and $p_{sj}H_j > 0$, in which case $\beta_i > 0$ holds from (3).[21] Only equilibria that are stable with respect to small perturbations to equilibrium p_{si} and p_{ui} are considered.[22] A reader who is not interested in the exposition of results could directly go to "Graphical illustrations..." part.

Lemma 1 (Sectoral choices) *Suppose $H_i > 0$ and $p_{sj}H_j > 0$ for $j \neq i$.*

(i) When $A_{ui} \geq 1$, $p_{si} = p_{ui} = 1$.

(ii) When $A_{ui} < 1$, $p_{si} = 1$. $p_{ui} = 0$ for $\beta_i \leq \frac{(1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$, $p_{ui} = \frac{\beta_i A_{si}h_s + (1-\beta_i)A_{ui}h_u - h_u}{(1-A_{ui})h_u} \frac{H_i}{1-H_i} \in (0,1)$ for $\beta_i \in \left(\frac{(1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}, \frac{1}{H_i}\frac{(1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}\right)$, and $p_{ui} = 1$ for higher β_i.

When $(A_{si} \geq)\, A_{ui} \geq 1$, that is, when both types of workers weakly prefer the formal sector under $\beta_i = 0$, they choose the sector. Intuitively, the reason is that, with $\beta_i > 0$ and $p_{si} > 0$, unskilled workers benefit from the presence of skilled workers in the sector through the group's reputation $E[Ah|i]$ and thus strictly prefer the sector, and the net return from choosing the sector is higher for skilled workers from Assumption 1 (iii). When $(A_{si} >)\, 1 > A_{ui}$,[23] skilled workers select the formal sector, while choices of unskilled workers depend on β_i: since the positive effect from skilled workers increases with β_i, they select the formal (informal) sector when β_i is large (small), and when β_i is intermediate, they are indifferent between the sectors and $p_{ui} \in (0,1)$ is increasing in β_i.

Taking into account the dependence of wages on sectoral choices, an individual decides on skill investment. As detailed in the next section, she can spend wealth on assets too. Thus, she invests in skill only if it is financially accessible *and* profitable. Let F_i be the proportion of individuals who have enough wealth to cover the investment cost c_h in group i. H_i cannot exceed F_i but may not equal F_i. Let p_{hi} be the probability that an individual with enough wealth does invest. To simplify the analysis, the following assumption is imposed on p_{hi}.

Assumption 2 *If individuals are indifferent among multiple p_{hi}, the highest value is realized.*

[21] Clearly, when $H_i = 0$ or $p_{sj}H_j = 0$ for $j \neq i$, $p_{si} = 1$ and $p_{ui} = 1 (= 0)$ if $A_{ui} > (<) 1$ and any $p_{ui} \in [0,1]$ if $A_{ui} = 1$.

[22] An equilibrium is defined to be *stable* regarding the perturbations if there exists a neighborhood of equilibrium p_{si} and p_{ui} such that, from any p_{si} and p_{ui} in the neighborhood, they have tendencies to return to equilibrium values in a simple dynamics in which p_{ki} increases (decreases) when the net return to choosing the formal sector for type k workers is positive (negative).

[23] Note that $A_{ui} < 1$ is possible if the quality of formal institutions and thus the formal sector's productivity are low (as explained in footnote 20, A_{ui} increases with the sector's relative productivity in a model with an explicit production function), if the discrimination exists, or if the disutility is greater in the sector.

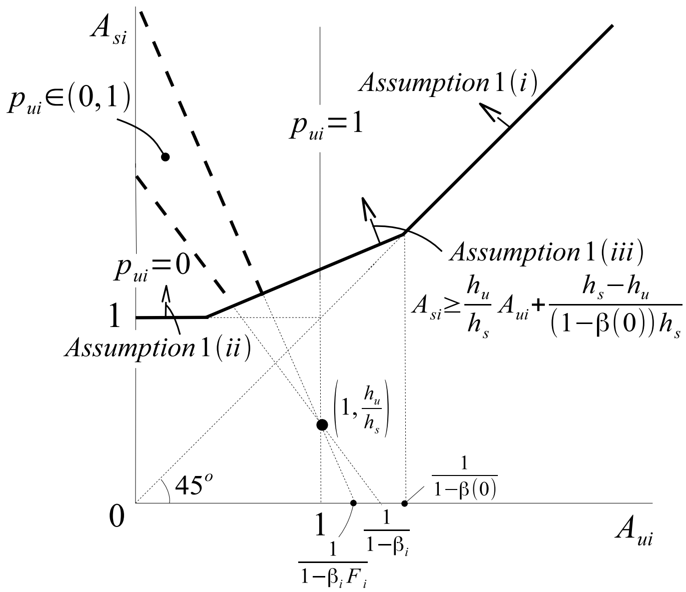

Figure 1: Sectoral choices of unskilled workers when $h_s-(1+r)c_h \geq h_u$ and $p_{sj}H_j>0$ for $j \neq i$

For example, when $p_{si}=p_{ui}=0$ and $h_s-(1+r)c_h-h_u=0$, $p_{hi}=1$ holds. The next lemma presents equilibrium $H_i=p_{hi}F_i$ for given H_j and p_{sj} $(j \neq i)$ when $F_i>0$. Only equilibria that are stable with respect to a small perturbation to equilibrium p_{hi} are considered.

Lemma 2 (Skill investment) *Suppose* $F_i>0$.

(*i*) *When* $h_s-(1+r)c_h \geq h_u$, $H_i=F_i$.

(*ii*) *When* $h_s-(1+r)c_h < h_u$, $H_i=F_i$ *when* $p_{sj}H_j=0$ *for* $j \neq i$. *When* $p_{sj}H_j>0$ (*thus* $\beta_i>0$),

(*a*) *If* $A_{ui} \geq 1$, $H_i=F_i$ *if* $\beta(0) \leq 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$; *otherwise, both* $H_i=F_i$ *and* $H_i=0$ *are equilibria* ($H_i=0$ *is the equilibrium*) *when* $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)<(\geq)1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$.

(*b*) *If* $A_{ui}<1$, $H_i=F_i$ (*no stable equilibria exist*) *when* $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)<(\geq)1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$.

When $h_s-(1+r)c_h \geq h_u$, i.e. the investment is socially productive (and privately profitable) in the informal sector, every individual with enough wealth invests, because, in the case where choosing the formal sector is more profitable, the net private return to investment is weakly higher than $h_s-(1+r)c_h-h_u$ from Assumption 1 (i) (when $p_{sj}H_j=0$ for $j \neq i$) and (iii) (when $p_{sj}H_j>0$).

In contrast, when $h_s-(1+r)c_h < h_u$, the decision depends on A_{ui} and β_i. When $A_{ui} \geq 1$ and $p_{sj}H_j>0$ for $j \neq i$, since all workers choose the formal sector from Lemma 1 (i), the net return equals $(1-\beta_i)[A_{si}h_s-A_{ui}h_u]-(1+r)c_h$ and decreases with β_i. Hence, if β_i is small enough that the net return is positive at $H_i=F_i$, i.e. $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)<1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$ is an equilibrium, while if $\beta(0)>1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ and thus the return is negative at $H_i=0$, $H_i=0$ is an equilibrium. Since $\beta(0)>\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)$, *both* $H_i=F_i$ *and* $H_i=0$ *are equilibria* when $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)<1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}<\beta(0)$ due to *strategic complementarity*: as more individuals invest and become skilled workers, the degree of prejudice β_i falls and formal-sector wages get closer to human capital, raising the return. The result when $A_{ui}<1$ and $p_{sj}H_j>0$ can be explained

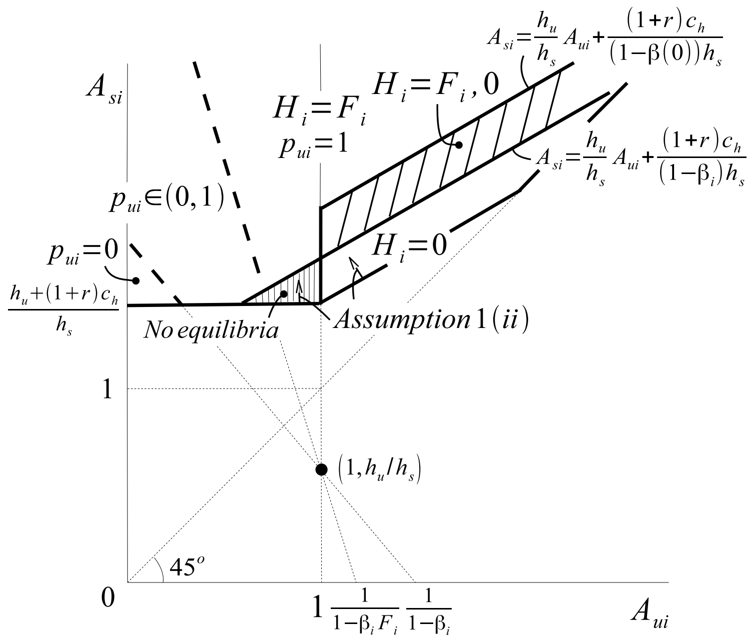

Figure 2: Skill investment and sectoral choices of unskilled workers when $h_s-(1+r)c_h<h_u$ and $p_{sj}H_j>0$

similarly. In this case, $H_i=0$ is not an equilibrium (given $H_i=0$, no unskilled workers choose the formal sector and thus the investment is profitable from Assumption 1 (ii), $A_{si}h_s-(1+r)c_h\geq h_s$), thus *no stable equilibria exist* if $\beta(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j})\geq 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, i.e. β_i is too high for the return to be positive at $H_i=F_i$.

Graphical illustrations of skill investment and sectoral choices. Skill investment and sectoral choices of a group *given* choices by the other group are summarized in Proposition A1 of Appendix A by combining the lemmas. Figure 1 illustrates sectoral choices of unskilled workers when $h_s-(1+r)c_h\geq h_u$ (i.e. the net return to investment is non-negative in the informal sector) and $p_{sj}H_j>0$, $j\neq i$, on the (A_{ui},A_{si}) plane, based on Proposition A1 (i). ($H_i=F_i$, i.e. everyone with enough wealth becomes a skilled worker, and $p_{si}=1$, i.e. all skilled workers choose the formal sector, in this case.)[24] A_{ui} and A_{si} must satisfy Assumption 1, thus only the upper left region of the bold solid lines is feasible. Their choices are determined by the two bold broken lines. $p_{ui}=0$ ($=1$) in the region at the left (right) side of the left (right) broken line and $p_{ui}\in(0,1)$ in the middle region. $p_{ui}>0$ is possible with $A_{ui}<1$ (i.e. unskilled workers are less productive in the formal sector) because of the positive effect from skilled workers on the unskilled wage in the formal sector.

Figure 2 shows investment and sectoral choices of unskilled workers when $h_s-(1+r)c_h<h_u$ (and $p_{sj}H_j>0$), based on Proposition A1 (ii).[25] ($p_{si}=1$ always.) When $A_{ui}\geq 1$, $p_{ui}=1$, and $H_i=F_i$

[24] For a reader who skips the lemmas: F_i is the proportion of group i individuals who have enough wealth to invest in skill, i.e. those who have wealth greater than c_h.

[25] The figure is drawn assuming $h_s-(1-\beta(0))(1+r)c_s\leq h_u$. When $h_s-(1-\beta(0))(1+r)c_s>h_u$, the bold solid line for Assumption 1 (ii) is located below the line for Assumption 1 (iii) when $A_{ui}>1$, as in Figure 1.

($H_i = 0$) is the only equilibrium in the region on or above $A_{si} = \frac{h_u}{h_s} A_{ui} + \frac{(1+r)c_h}{(1-\beta(0))h_s}$ (on or below $A_{si} = \frac{h_u}{h_s} A_{ui} + \frac{(1+r)c_h}{(1-\beta_i)h_s}$), while *both* $H_i = F_i$ *and* $H_i = 0$ *are equilibria* between the lines, the area with slanting lines. When $A_{ui} < 1$, $H_i = F_i$ holds above $A_{si} = \frac{h_u}{h_s} A_{ui} + \frac{(1+r)c_h}{(1-\beta_i)h_s}$ and no equilibria exist on or below it (the area with vertical lines). Sectoral choices when $A_{ui} < 1$ are as in Figure 1.

Sectoral choices and skill investment may *not* be socially optimal when $\beta_i > 0$. When $A_{ui} < 1$, because of the positive effect from skilled workers, some or all of unskilled workers choose the less productive formal sector at the right side of the left broken line in the figures. When $A_{ui} \geq 1$, an individual may not carry out the productive investment in the region below $A_{si} = \frac{h_u}{h_s} A_{ui} + \frac{(1+r)c_h}{(1-\beta(0))h_s}$ due to the negative effect of unskilled workers on the private return to investment.

Choices of both groups are determined by applying the proposition to the two groups simultaneously. Cases where the investment is always profitable for both groups can be easily known from the proposition (see Figure 2 too).

Corollary 1 (Cases in which the investment is always profitable for both groups) $H_i = F_i$ *for any* i *and* F_i, *when* $h_s - (1+r)c_h \geq h_u$ *and when* $h_s - (1+r)c_h < h_u$ *and* $\beta(0) \leq 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ *for any* i.

In remaining cases, the determination of H_i is not simple, which is examined later with an additional assumption.

3.2 Wages

Wages depend on skill investment and sectoral choices. Denote the unskilled wage of group i by w_{ui} and the skilled wage *net of the investment cost* by w_{si}. Then, when $H_i = F_i$ and $p_{sj} H_j > 0$ for $j \neq i$, i.e. $\beta_i > 0$,

if $p_{ui} = 1$,
$$w_{ui} = (1-\beta_i) A_{ui} h_u + \beta_i [F_i A_{si} h_s + (1-F_i) A_{ui} h_u] = A_{ui} h_u + \beta_i F_i (A_{si} h_s - A_{ui} h_u), \quad (5)$$
$$w_{si} = (1-\beta_i) A_{si} h_s + \beta_i [F_i A_{si} h_s + (1-F_i) A_{ui} h_u] - (1+r)c_h = A_{si} h_s - \beta_i (1-F_i)(A_{si} h_s - A_{ui} h_u) - (1+r)c_h; \quad (6)$$

if $p_{ui} \in (0,1)$,
$$w_{ui} = (1-\beta_i) A_{ui} h_u + \beta_i \frac{F_i A_{si} h_s + p_{ui}(1-F_i) A_{ui} h_u}{F_i + p_{ui}(1-F_i)} = h_u, \quad (7)$$
$$w_{si} = (1-\beta_i) A_{si} h_s + \beta_i \frac{F_i A_{si} h_s + p_{ui}(1-F_i) A_{ui} h_u}{F_i + p_{ui}(1-F_i)} - (1+r)c_h = h_u + (1-\beta_i)(A_{si} h_s - A_{ui} h_u) - (1+r)c_h; \quad (8)$$

and if $p_{ui} = 0$, $w_{ui} = h_u$ and $w_{si} = A_{si} h_s - (1+r)c_h$. When $H_i = 0$ or $\beta_i = 0$, $w_{ui} = \max\{A_{ui}, 1\} h_u$ and $w_{si} = A_{si} h_s - (1+r)c_h$ (when $H_i > 0$).

Note that, when $H_i = F_i$, wages in most cases depend on F_i directly *and* F_i and F_j ($j \neq i$) *indirectly through* β_i.

4 Dynamic part of the model

Based on the previous section, this section presents the dynamic part of the model. Consider an OLG economy composed of a continuum of two-period-lived individuals. The distribution of wealth over the initial generation of each group is given, while wealth distributions of subsequent generations are determined endogenously.

4.1 Lifetime of an individual

Childhood: In childhood, an individual receives a transfer from her parent (if she belongs to the initial generation, it is given) and spends it on two options, assets (yields interest rate r) and skill investment (costs c_h), to maximize future income. Consider an individual of generation t (born in period $t-1$ and becomes an adult in period t) born into a lineage of group i, who receives b_{it} units of transfer and allocates it between asset a_{it} and skill investment v_{it}. When $H_{it}=F_{it}$, i.e. everyone with enough wealth becomes a skilled worker, the allocation is determined by b_{it}:

$$a_{it}=b_{it}, \qquad v_{it}=0, \quad \text{if} \quad b_{it}<c_h, \tag{9}$$

$$a_{it}=b_{it}-c_h, \quad v_{it}=c_h, \quad \text{if} \quad b_{it}\geq c_h. \tag{10}$$

By contrast, when $H_{it}=0$, i.e. nobody becomes a skilled worker, $a_{it}=b_{it}$ and $v_{it}=0$.

Adulthood: In adulthood, she chooses a sector to work, obtains income from assets and work, and spends it on consumption c_{it} and a transfer to her single child b_{it+1}. Her utility maximization problem is:

$$\max u_{it} = (c_{it})^{1-\gamma_b}(b_{it+1})^{\gamma_b}, \quad s.t. \;\; c_{it}+b_{it+1} = w_{it}+(1+r)a_{it}, \tag{11}$$

where $\gamma_b \in (0,1)$ and w_{it} is her gross wage. By solving the problem, her consumption and transfer rules equal

$$c_{it} = (1-\gamma_b)\{w_{it}+(1+r)a_{it}\}, \tag{12}$$

$$b_{it+1} = \gamma_b\{w_{it}+(1+r)a_{it}\}. \tag{13}$$

Generational change: At the beginning of the next period, current adults pass away, current children become adults, and new children are born into the economy. Since each adult has one child, the total (adult) population of the group is time-invariant and equals N_i.

4.2 Dynamics of individual transfers

The dynamic equation linking the received transfer b_{it} to the transfer given to the next generation b_{it+1} is derived from the transfer rule (13). For a current unskilled worker, it is obtained by substituting $w_{it}=w_{uit}$ and $a_{it}=b_{it}$ into (13):

$$b_{it+1} = \gamma_b\{w_{uit}+(1+r)b_{it}\}. \tag{14}$$

The assumption $\gamma_b(1+r) < 1$ is made so that the fixed point of the equation for given w_{uit}, $b^*(w_{uit}) \equiv \frac{\gamma_b}{1-\gamma_b(1+r)}w_{uit}$, exists. The fixed point becomes crucial in later analyses.

For a current skilled worker, who exists only when $H_{it}=F_{it}$, the dynamic equation is

$$b_{it+1} = \gamma_b\{w_{sit}+(1+r)b_{it}\}, \tag{15}$$

which is obtained by substituting $w_{it}=w_{sit}+(1+r)c_h$ and $a_{it}=b_{it}-c_h$ into (13).

The equations show that the dynamics of transfers within a lineage depend on the dynamics of wages and H_{it}, which in turn are determined by the time evolution of F_{it} and F_{jt} ($j\neq i$).

4.3 Aggregate dynamics

The time evolution of F_{it} (the fraction of group i individuals who have enough wealth to invest in skill, i.e. $b_{it} \geq c_h$) is determined by the dynamics of individual transfers. That is, the individual and aggregate dynamics are interrelated.

More specifically, when $H_{it} = F_{it}$, if offspring of some unskilled workers become accessible to the investment through wealth accumulation, $F_{it+1} > F_{it}$, while, if some of present skilled workers cannot leave their offspring enough transfers to cover the investment cost, $F_{it+1} < F_{it}$.

The former takes places iff there exist lineages satisfying $b_{it} < c_h$ and $b_{it+1} \geq c_h$. From (14), the following condition must hold for such lineages to exist:

$$b^*(w_{uit}) \equiv \frac{\gamma_b}{1-\gamma_b(1+r)} w_{uit} > c_h. \tag{16}$$

The latter occurs iff lineages satisfying $b_{it} \geq c_h$ and $b_{it+1} < c_h$ exist. From (15), the necessary condition is

$$b^*(w_{sit}) \equiv \frac{\gamma_b}{1-\gamma_b(1+r)} w_{sit} < c_h. \tag{17}$$

Since $b^*(w_{sit}) \geq b^*(w_{uit})$, the above equations do not hold simultaneously. If (16) holds, $F_{it+1} \geq F_{it}$, while if (17) is true, $F_{it+1} \leq F_{it}$. ($F_{it+1} = F_{it}$ is possible depending on the distribution of transfers, but, if the condition continues to hold for many periods, F_{it} does change at some point.) When neither equations are satisfied, $F_{it+1} = F_{it}$. The dynamics when $H_{it} = 0$ depend on the relative value of $b^*(w_{uit})$ to c_h only.

Regarding the value of $b^*(w_{uit})$, the following is assumed.

Assumption 3 $h_u \leq \frac{1-\gamma_b(1+r)}{\gamma_b} c_h$

This implies that $b^*(w_{uit}) \leq c_h$ when $p_{ui,t} < 1$, that is, offspring of unskilled workers can never afford the investment if the unskilled wage stays at the level in the informal sector, h_u. The assumption rules out the trivial case in which $h_u > \frac{1-\gamma_b(1+r)}{\gamma_b} c_h$ and thus F_{it} always increases.

Since the dynamics of individual transfers of group i depend, through skill investment and wages, on the evolution of F_{jt} ($j \neq i$) as well, the dynamics of F_{it} and F_{jt} are interrelated. The next section examines the joint dynamics of the variables, thereby analyzing the dynamics of variables of interest.

5 Analyses

This section examines the evolution of F_{it}, skill composition, sectoral choices, wages, and intergroup inequality with phase diagrams. As might be inferred from Figures 1 and 2, various dynamics arise depending on values of exogenous variables such as A_{si} and A_{ui}, but most of them are qualitatively similar. Thus, analyses focus on cases that are representative and yield clear-cut results.

For simplicity, the elasticity of β_i with respect to F_i (in absolute value) is assumed to be less than 1 in dynamic analyses, although all of main results hold without the assumption.

Assumption 4 $\beta(x) + \beta'(x)x(1-x) > 0$ *for any* $x \in [0, 1) \Leftrightarrow \frac{\partial(\beta_i F_i)}{\partial F_i} > 0$ *always.*

5.1 When skill investment is always profitable

First, consider the case where $H_i = F_i$ always holds for any group i. From Corollary 1, this is true when $h_s - (1+r)c_h \geq h_u$, or when $h_s - (1+r)c_h < h_u$ and $\beta(0) + \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u} \leq 1$ ($\Leftrightarrow A_{si} \geq \frac{h_u}{h_s} A_{ui} + \frac{(1+r)c_h}{(1-\beta(0))h_s}$) for any i (see Figure 2). This is the case in which skill investment is socially productive even in

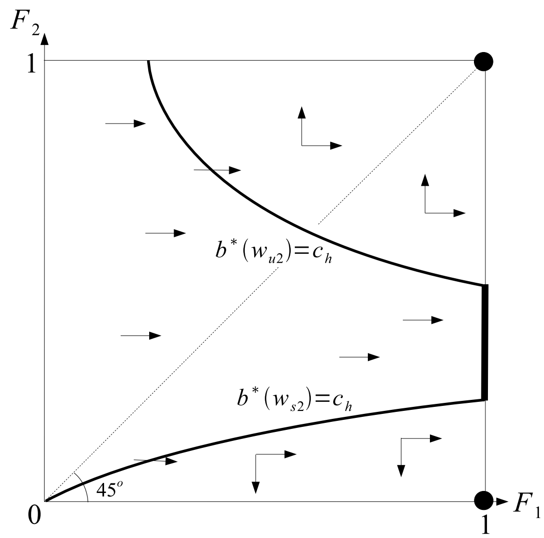

Figure 3: Dynamics when $H_{it}=F_{it}$ always and group 2's disadvantages are moderate

the informal sector, investment is very productive in the formal sector, or the maximum degree of prejudice $\beta(0)$ is low.

5.1.1 When disadvantages of the subordinate group are moderate

Consider an economy where a dominant group (*group 1*) and a subordinate group (*group 2*) exist, i.e. $A_{k1} > A_{k2}$ $(k = u, s)$. Suppose that institutionalized disadvantages the latter group face are moderate enough (or the relative productivity of the formal sector is high enough, see footnote 20) that $A_{ki} > 1$ for any $k = u, s$ and $i = 1, 2$, i.e. all workers are more productive in the formal sector. Then, $p_{si} = p_{ui} = 1$, i.e. all workers choose the formal sector, from Proposition A1.

As for the dynamics of F_{1t}, it is assumed that institutionalized advantages of the dominant group are large enough that $A_{u1}h_u > \frac{1-\gamma_b(1+r)}{\gamma_b}c_h$, that is, even with the lowest wage $A_{u1}h_u$, descendants of the group's unskilled workers can afford the investment eventually. Then, F_{1t} increases over time and $H_1^* = F_1^* = 1$ in the long run (superscript $*$ indicates the steady state value). In contrast, A_{u2} and A_{s2} are lower and the following is assumed: $A_{u2}h_u < \frac{1-\gamma_b(1+r)}{\gamma_b}c_h$, i.e. with the lowest wage, descendants of unskilled workers of the subordinate group remain unskilled; $\left[1-\beta\left(\frac{N_2}{N_1+N_2}\right)\right]A_{u2}h_u + \beta\left(\frac{N_2}{N_1+N_2}\right)A_{s2}h_s > \frac{1-\gamma_b(1+r)}{\gamma_b}c_h$, i.e. with the highest wage (from Assumption 4), they can afford the investment eventually; $A_{s2}h_s > \frac{c_h}{\gamma_b}$ and $(1-\beta(0))A_{s2}h_s + \beta(0)A_{u2}h_u < \frac{c_h}{\gamma_b}$, i.e. with the highest (lowest) wage, descendants of skilled workers of the group can (cannot) stay skilled. Then, $b^*(w_{u2}) = c_h$ and $b^*(w_{s2}) = c_h$ exist and equal:

$$(w_{u2} =) A_{u2}h_u + \beta\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)F_2(A_{s2}h_s - A_{u2}h_u) = \frac{1-\gamma_b(1+r)}{\gamma_b}c_h, \tag{18}$$

$$(w_{s2} =) A_{s2}h_s - \beta\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)(1-F_2)(A_{s2}h_s - A_{u2}h_u) = \frac{c_h}{\gamma_b}, \tag{19}$$

which are obtained by plugging (5) and (6) with $\beta_2 = \beta\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)$ into (16) and (17), respectively.[26]

[26] Since w_{u2} for given $\frac{F_2}{F_1}$ increases with F_2 and $b^*(w_{u2}) > (<) c_h$ at $(F_1, F_2) = (1,1)$ (as $F_2 \to 0$ on $\frac{F_2}{F_1} = 1$) from the

The dynamics of F_{1t} and F_{2t} can be analyzed graphically by placing $b^*(w_{u2})=c_h$ and $b^*(w_{s2})=c_h$ on the (F_1,F_2) plane (Figure 3).[27] $b^*(w_{u2})=c_h$ is negatively sloped and $b^*(w_{s2})=c_h$ is positively sloped from (18) and (19) (since β_2 increases (decreases) with F_1 (F_2) and $\beta_2 F_2$ increases with F_2). The direction of motion of F_{2t} (F_{1t}) is represented by vertical (horizontal) arrows. Since w_{s2} decreases and w_{u2} increases with β_2 and thus F_1, in the region at the right (left) side of $b^*(w_{s2})=c_h$, $b^*(w_{s2})<(>)c_h$ and F_{2t} decreases (non-decreases) over time, while in the region at the right (left) side of $b^*(w_{u2})=c_h$, $b^*(w_{u2})>(<)c_h$ and F_{2t} increases (non-increases).

Unlike the economy in which reputation does not affect wages, i.e. $\beta_{it}=0$ always, where F_{1t} increases and F_{2t} is constant over time, the fate of the subordinate group could differ greatly depending on F_{20}.

When the initial distribution of wealth is such that a sufficiently large portion of the subordinate group can afford the investment, to be accurate, when $b^*(w_{u2})>c_h$ at $(F_1,F_2)=(1,F_{20})$, $H_2^*=F_2^*=1$ as well as $H_1^*=F_1^*=1$ in the long run. As an illustration, suppose that F_{10} is not so high that $b^*(w_{u20})<c_h$ holds. Then, as $H_{1t}=F_{1t}$ grows over time, the influence of the dominant group in wage determination becomes stronger and wages of the subordinate group are affected more by their reputation, i.e. β_{2t} increases. As a result, the unskilled (skilled) wage of group 2 rises (falls) over time. Since $H_{2t}=F_{20}$ is not low and thus their reputation (average human capital) is not bad, the wage of skilled workers stays high enough for their descendants to remain skilled, while the unskilled wage grows to the point that the investment becomes affordable to some of their offspring at some point, i.e. $b^*(w_{u2t})>c_h$. $H_{2t}=F_{2t}$ and the reputation start to rise, and the improved reputation further stimulates the upward mobility of unskilled workers. In the long run, everyone becomes a skilled worker.

By contrast, when F_{20} is small enough that $b^*(w_{s2})<c_h$ at $(F_1,F_2)=(1,F_{20})$, $(H_1^*,H_2^*)=(1,0)$ in the long run. Since group 2's initial reputation is low and its effect on wages strengthens over time, the wage of skilled workers falls to the point that their offspring become unable to afford the investment at some point. F_{2t} start to decrease and the deteriorated reputation and stronger prejudice (higher β_{2t}) spur the downward mobility. In the long run, all of group 2 are unskilled. (When F_{20} is in the intermediate range, $(H_1^*,H_2^*)=(1,F_{20})$.)

As long as (F_{10},F_{20}) is located at the left side of the two loci, group 2's average skill and wage levels relative to group 1 fall at first. However, if F_{20} is sufficiently high, they start to rise at some point and both groups become totally skilled eventually. Otherwise, the relative levels continue to fall and, if F_{20} is sufficiently low, the two groups are totally segregated by skill levels in the long run. The initial condition affects the long-run fate of the subordinate group through reputation:

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assumptions on w_{u2} (see eq. 18), there exists $F_2\in(0,1)$ on $\frac{F_2}{F_1}=1$ satisfying $b^*(w_{u2})=c_h$. $b^*(w_{s2})=c_h$ exists for any $F_1\in(0,1]$ ($b^*(w_{s2})>c_h$ at $F_1=0$) since, for $F_1\neq 0$, w_{s2} increases with F_2 and $b^*(w_{s2})<(>)c_h$ at $F_2=0$ $(=1)$ from the assumptions on w_{s2} (see eq. 19).

[27] $b^*(w_{u2})=c_h$ intersects with $F_1=1$ at $F_2\in(0,1)$ and with $F_2=1$ at $F_1\in(0,1)$ from the assumptions on w_{u2}. $b^*(w_{s2})=c_h$ crosses $F_1=1$ at $F_2\in(0,1)$ from the assumptions on w_{s2}, does not cross $F_2=0$ from $(1-\beta(0))A_{s2}h_s+\beta(0)A_{u2}h_u<\frac{c_h}{\gamma_b}$, and not $F_1=0$ from $A_{s2}h_s>\frac{c_h}{\gamma_b}$. (Thus, it approaches $(F_1,F_2)=(0,0)$.) $b^*(w_{u2})=c_h$ and $b^*(w_{s2})=c_h$ do not cross from $w_{s2}>w_{u2}$ for $F_2>0$. In the figure, $b^*(w_{s2})=c_s$ is below the 45^0 line, but if $[1-\beta(\frac{N_2}{N_1+N_2})]A_{s2}h_s+\beta(\frac{N_2}{N_1+N_2})A_{u2}h_u<\frac{c_s}{\gamma_b}$, it crosses the line.

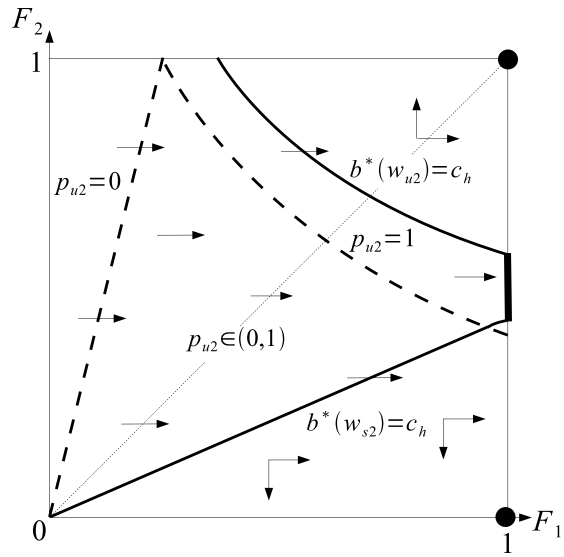

Figure 4: Dynamics when $H_{it} = F_{it}$ always and group 2's disadvantages are severe

good (*bad*) *reputation begets good* (*bad*) *reputation.*

5.1.2 When disadvantages of the subordinate group are severe

Next consider an economy where institutionalized disadvantages of the subordinate group are severe enough (or the relative productivity of the formal sector is low enough) that $A_{u2} < 1 < A_{s2}$, i.e. unskilled workers of the group are less productive (*net* of the disutility of work) in the formal sector. Then, $p_{si} = p_{u1} = 1$ $(i = 1, 2)$, while $p_{u2} = 0$ for $\beta_2 \leq \frac{(1-A_{u2})h_u}{A_{s2}h_s - A_{u2}h_u}$, $p_{u2} = \frac{\beta_2 A_{s2}h_s + (1-\beta_2)A_{u2}h_u - h_u}{(1-A_{u2})h_u} \frac{F_2}{1-F_2} \in (0,1)$ for $\beta_2 \in \left(\frac{(1-A_{u2})h_u}{A_{s2}h_s - A_{u2}h_u}, \frac{1}{F_2}\frac{(1-A_{u2})h_u}{A_{s2}h_s - A_{u2}h_u}\right)$, and $p_{u2} = 1$ for higher β_2 from Proposition A1. The boundary between $p_{u2} = 0$ and $p_{u2} \in (0,1)$ and the one between $p_{u2} \in (0,1)$ and $p_{u2} = 1$ are given by:

$$\beta\left(\frac{F_2 N_2}{F_1 N_1 + F_2 N_2}\right) = \frac{(1-A_{u2})h_u}{A_{s2}h_s - A_{u2}h_u}, \tag{20}$$

$$\beta\left(\frac{F_2 N_2}{F_1 N_1 + F_2 N_2}\right) F_2 = \frac{(1-A_{u2})h_u}{A_{s2}h_s - A_{u2}h_u}. \tag{21}$$

Assumptions related to the dynamics of F_{1t} and F_{2t} are same as the previous case except that $(1-\beta(0))A_{s2}h_s + \beta(0)A_{u2}h_u < \frac{c_h}{\gamma_b}$ is slightly changed to $(1-A_{u2})h_u + (1-\beta(0))A_{s2}h_s + \beta(0)A_{u2}h_u < \frac{c_h}{\gamma_b}$ (and $A_{u2}h_u < \frac{1-\gamma_b(1+r)}{\gamma_b}c_h$ now follows from Assumption 3). As before, $b^*(w_{u2}) = c_h$ and $b^*(w_{s2}) = c_h$ when $p_{u2} = 1$ are given by (18) and (19), respectively. When $p_{u2} \in (0,1)$, $b^*(w_{s2}) = c_h$ equals

$$(w_{s2} =) h_u + \left[1 - \beta\left(\frac{F_2 N_2}{F_1 N_1 + F_2 N_2}\right)\right](A_{s2}h_s - A_{u2}h_u) = \frac{c_h}{\gamma_b}, \tag{22}$$

which is obtained by substituting (8) into (17) (with $<$ replaced by $=$).[28]

Figure 4 illustrates the dynamics of F_{1t}, F_{2t}, and p_{u2t}. On the (F_1, F_2) plane, the dividing line between $p_{u2} = 0$ and $p_{u2} \in (0,1)$ is a positively-sloped straight line that is located above the 45^0 line and approaches the origin ($p_{u2} = 0$ at $F_2 = 0$). The dividing line between $p_{u2} \in (0,1)$ and $p_{u2} = 1$

[28] $b^*(w_{s2}) = c_h$ when $p_{u2} \in (0,1)$ exists since the LHS of (22) is strictly higher (lower) than the RHS at lowest (highest) β_2, i.e. at β_2 satisfying (20) ($\beta_2 = \beta(0)$), from $A_{s2}h_s > \frac{c_h}{\gamma_b}$ and $h_u + (1-\beta(0))(A_{s2}h_s - A_{u2}h_u) < \frac{c_h}{\gamma_b}$.

is a negatively-sloped curve (since $\beta_2 F_2$ increases with F_2).[29] The two lines are located at the left side of $b^*(w_{u2}) = c_h$ (from Assumption 3) and intersect at $F_2 = 1$. $b^*(w_{s2}) = c_h$ when $p_{u2} \in (0,1)$ is a positively-sloped straight line approaching the origin.[30]

The dynamics of F_{1t} and F_{2t} are qualitatively same as before: when F_{20} is large [small] enough that $b^*(w_{u2}) > c_h [b^*(w_{s2}) < c_h]$ at $(F_1, F_2) = (1, F_{20})$, F_{2t} starts to rise [fall] eventually and $(F_1^*, F_2^*) = (H_1^*, H_2^*) = (1,1)[(1,0)]$ in the long run.

What is new is that sectoral choices by unskilled workers of the subordinate group change over time. Suppose F_{10} is small enough that $p_{u20} = 0$, i.e. they choose the informal sector initially. Then, as long as $p_{u2t} = 0$, group 2's wages equal human capital and are constant. After $H_{1t} = F_{1t}$ and thus β_{2t} become high enough that $p_{u2t} \in (0,1)$, i.e. some of them start choosing the formal sector, induced by increasing β_{2t}, more and more of them choose the sector over time despite such choice is *inefficient*, i.e. $A_{u2} < 1$. This deteriorates the group's reputation (average human capital) in the sector and, together with the increasing importance of reputation (increasing β_{2t}), lowers the group's skilled wage, while the unskilled wage remains constant at h_u. That is, average earnings of the subordinate group *fall*.

Thereafter, the dynamics of p_{u2t} and the wages differ greatly depending on the initial condition. When F_{20} is sufficiently high, $p_{u2t} = 1$ after some point and wage dynamics become qualitatively same as the previous economy. The labor market is integrated in the long run in the sense that everyone works in the formal sector. By contrast, when F_{20} is small, the skilled wage of group 2 falls to the point that $b^*(w_{s2t}) < c_h$ and F_{2t} starts to fall at some point. The fall of $H_{2t} = F_{2t}$, like the growth of $H_{1t} = F_{1t}$, raises the importance of reputation β_{2t}, but it also worsens the group's reputation. While the positive effect on β_{2t} is stronger, p_{u2t} continues to rise, but eventually the negative effect dominates and p_{u2t} starts to *fall*. In the long run, all of the subordinate group are unskilled *and* in the informal sector, thus *the labor market is segregated completely by ethnicity.* Inefficient sectoral choices of the group's unskilled workers make the outcome sensitive to the initial condition and quite different from a "prejudice-free" economy: if their choices are optimal, i.e. $p_{u2t} = 0$, F_{2t} is constant as under $\beta_{2t} = 0$.

5.1.3 When there are no institutionalized disadvantages

Finally, consider an economy where the majority (group 1) and the minority (group 2) exist, none of whom face institutionalized disadvantages, i.e. $N_1 > N_2$ and $A_{k1} = A_{k2} \equiv A_k$ $(k = u, s)$. $A_u > 1$ is assumed so that $p_{si} = p_{ui} = 1$ $(i = 1, 2)$ holds from Proposition A1. As for the dynamics of F_{it}, suppose that A_k is not very high and thus the same assumptions as the ones for the subordinate group in the first economy (Figure 3) hold. Then, $b^*(w_{ui}) = c_h$ and $b^*(w_{si}) = c_h$ $(i = 1, 2)$ exist and are given by (18) and (19) with $A_{k1} = A_{k2} = A_k$. This economy may be seen as an approximation to

[29] Since $\beta(\frac{N_2}{N_1+N_2}) > \frac{(1-A_{u2})h_u}{A_{s2}h_s - A_{u2}h_u}$, i.e. $p_{u2} > 0$ on $\frac{F_2}{F_1} = 1$ and $p_{u2} = 1$ at $(F_1, F_2) = (1,1)$, from $[1 - \beta(\frac{N_2}{N_1+N_2})] A_{u2} h_u + \beta(\frac{N_2}{N_1+N_2}) A_{s2} h_s > \frac{1-\gamma_b(1+r)}{\gamma_b} c_h$ and Assumption 3, the dividing line between $p_{u2} = 0$ and $p_{u2} \in (0,1)$ is above the 45^0 line and the one between $p_{u2} \in (0,1)$ and $p_{u2} = 1$ exists and intersects with the 45^0 line (and with $F_2 = 1$).

[30] Unlike the figure, $b^*(w_{s2}) = c_h$ when $p_{u2} \in (0,1)$ may be located above the 45^0 line or it may not intersect with the dividing line between $p_{u2} \in (0,1)$ and $p_{u2} = 1$, although main results are not affected.

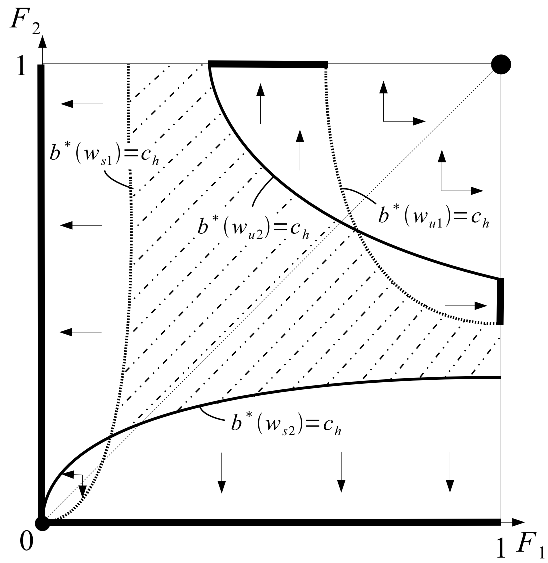

Figure 5: Dynamics when $H_{it} = F_{it}$ always and no institutionalized disadvantages exist

an economy where both institutionalized disadvantages of the subordinate group and advantages of the dominant group are small.

Figure 5 illustrates the dynamics of F_{1t} and F_{2t}. Since $N_1 > N_2$ and thus $\beta_1 < \beta_2$, the region with $b^*(w_{u1}) > c_h$ ($b^*(w_{s1}) < c_h$) is smaller than the region with $b^*(w_{u2}) > c_h$ ($b^*(w_{s2}) < c_h$). Unlike Figures 3 and 4, $\left[1-\beta\left(\frac{N_1}{N_1+N_2}\right)\right]A_s h_s + \beta\left(\frac{N_1}{N_1+N_2}\right)A_u h_u < \frac{c_s}{\gamma_b}$ is assumed and thus $b^*(w_{si}) = c_s$ intersects with the 45^0 line.[31]

Now, the groups' fate depends on both F_{10} and F_{20}. When (F_{10}, F_{20}) is above $b^*(w_{u1}) = c_h$ ($b^*(w_{u2}) = c_h$), F_{1t} (F_{2t}) rises over time and $H_1^* = 1 (H_2^* = 1)$ in the long run. Particularly, when both F_{10} and F_{20} are high, i.e. when $b^*(w_{ui}) > c_h$ at $(F_1, F_2) = (F_{10}, F_{20})$ for at least one group i and, for $j \neq i$, $b^*(w_{uj}) > c_h$ at $F_i = 1$ and $F_j = F_{j0}$, everyone is skilled in the long run. By contrast, when (F_{10}, F_{20}) is at the left side of $b^*(w_{s1}) = c_h$ (below $b^*(w_{s2}) = c_h$), F_{1t} (F_{2t}) falls over time and $H_1^* = 0$ ($H_2^* = 0$). In particular, when (F_{10}, F_{20}) satisfies $b^*(w_{s1}) < c_h$ and $b^*(w_{s2}) < c_h$, it is possible that *nobody is skilled in the long run*: a bad impression each group has about the other affects the skilled wage negatively, thus F_{it} falls and the impression deteriorates further. (In the area with chained lines, F_{1t} and F_{2t} are constant.) Long-run outcomes tend to be more sensitive to the initial conditions for the minority since their wages are affected more by prejudice and reputation.

5.1.4 Summary and discussions

Analyses have shown that the dynamics and long-run outcomes of groups, particularly of the subordinate group, depend greatly on groups' initial conditions and could be quite different from a "prejudice-free" economy. Since good (bad) reputation tends to beget good (bad) reputation, a group starting with a good (bad) initial condition, i.e. a high (low) fraction of them can afford

[31] A minor assumption, $b^*(w_{si}) = c_h$ and $b^*(w_{uj}) = c_h$ ($i \neq j$) do not intersect, too is imposed.

investment initially, tend to be in a good (bad) condition in the long run. In the first economy, if the initial condition of the subordinate group is bad (good), all of them are unskilled (skilled) in the long run. In the second economy, if the condition is bad (good), all of them are unskilled (skilled) and work in the informal (formal) sector, hence the labor market becomes ethnically segregated (integrated) eventually. In the third economy where institutionalized disadvantages do not exist, the dynamics and long-run outcomes of both groups, particularly of the minority, tend to be affected greatly by initial conditions. The strong dependence on initial conditions arises because, unlike a "prejudice-free" economy in which the dynamics of an individual lineage are affected only by the lineage's initial condition, they are affected by initial conditions of *the groups* too through the dependence of formal-sector wages on reputation (average human capital) and the degree of reputation (average human capital).

Empirical findings show the dependence of individual and group level outcomes on initial conditions of groups: Borjas (1994, 2006) finds that wages of a U.S. worker in 1940 and 1980 are significantly related to the average wage of immigrants of the worker's ethnic group in 1910 (blacks are not in the data) after individual characteristics are controlled for, and about 22% of the intergroup wage gap in the immigrant generation persists into the third generation; Bertocchi and Arcangelo (2012) find for US states that the initial gap (in 1940) in educational attainment between blacks and whites affects the racial gap in subsequent years positively and the subsequent income growth negatively.

Note that the main implication that long-run outcomes of groups, particularly of the group whose earnings are affected more by prejudice and reputation, depend greatly on initial conditions *remains unchanged even if* $A_{k1} = A_{k2}$ $(k = u, s)$ *holds*, although the dynamics are affected. That is, the implication holds even when *neither group* face disadvantages in production, suffer greater disutility of work, or face non-statistical discrimination in the formal sector. $A_{k1} > A_{k2}$ is assumed in the first two economies for analytical simplicity as well as for reality: when $A_{k1} = A_{k2}$ holds, as in Figure 5, critical loci exist for both groups and thus analyses become complicated. The same is true for implications of analyses below.

5.2 When skill investment is not always profitable

Section 5.1 examined the case where $H_i = F_i$ always holds. Now consider the case in which $H_i = 0$ could hold at least for the subordinate group. This is true when $h_s - (1+r)c_h < h_u$, $A_{ui} \geq 1$, and, at least for $i = 2$, $\beta(0) + \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u} > 1$ ($\Leftrightarrow A_{si} < \frac{h_u}{h_s}A_{ui} + \frac{(1+r)c_h}{(1-\beta(0))h_s}$) from Proposition A1 (ii) (see Figure 2). This is the case where investment is unproductive in the informal sector, and investment is not very productive in the formal sector (at least for group 2) or the degree of prejudice is high.

Investment decisions of the groups are interrelated, thus equilibrium combinations of H_1 and H_2 are varied depending on A_{si} and A_{ui}. To reduce the combinations, the following is assumed.

Assumption 5 $A_{s1}h_s - A_{u1}h_u \geq A_{s2}h_s - A_{u2}h_u$.

That is, the social return to investment in the formal sector is weakly higher for the dominant group, which is reasonable since the subordinate group tend to have greater disadvantages in skilled

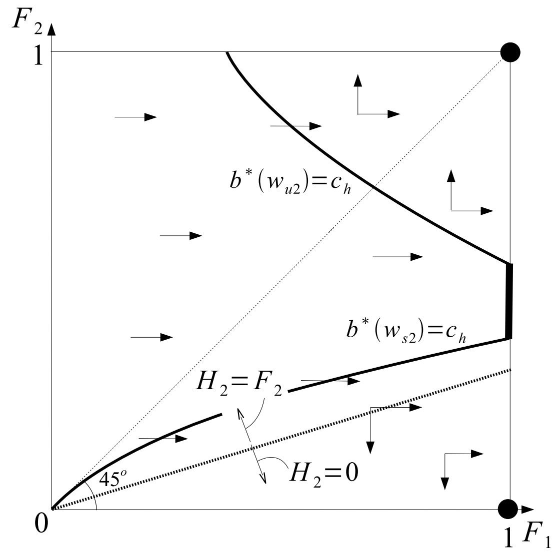

Figure 6: Dynamics when $H_{2t}=F_{2t}$ is selected in the region where both $H_{2t}=0$ and $H_{2t}=F_{2t}$ are equilibria

jobs requiring high interpersonal ability (e.g. management jobs). The next proposition presents equilibrium (H_1,H_2).

Proposition 1 (Equilibrium ($\mathbf{H_1}$,$\mathbf{H_2}$) when the investment is not always profitable) *Assume* $h_s-(1+r)c_h<h_u$, *(*$A_{k1}\geq$*)*$A_{k2}\geq 1$ *(*$k=u,s$*), and* $\beta(0)+\frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}>1$.

(i) If $(1-\beta(0))[A_{s1}h_s-A_{u1}h_u]\geq(1+r)c_h \Leftrightarrow \beta(0)+\frac{(1+r)c_h}{A_{s1}h_s-A_{u1}h_u}\leq 1$, $H_1=F_1$ *and both* $H_2=F_2$ *and* $H_2=0$ *are equilibria (*$H_2=0$ *is the equilibrium) when* $\beta\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)<(\geq)\ 1-\frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$.

(ii) Otherwise, $(H_1,H_2)=(0,F_2),(F_1,0)$*, and, when* $\beta\left(\frac{F_iN_i}{F_iN_i+F_jN_j}\right)<1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ *for any* $i,j=1,2$ *(*$j\neq i$*),* $(H_1,H_2)=(F_1,F_2)$ *as well.*

Multiple equilibria exist unless $\beta(0)+\frac{(1+r)c_h}{A_{s1}h_s-A_{u1}h_u}\leq 1$ and $\beta\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)\geq 1-\frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$.

5.2.1 When investment is always profitable for the dominant group

If investment is always (weakly) profitable for the dominant group, i.e. $(1-\beta(0))[A_{s1}h_s-A_{u1}h_u]\geq(1+r)c_h$ ($\Leftrightarrow A_{s1}\geq\frac{h_u}{h_s}A_{u1}+\frac{(1+r)c_h}{(1-\beta(0))h_s}$) from Proposition 1 (i) (see Figure 2 of Section 3.1), $H_1=F_1$ is always true, while both $H_2=F_2$ and $H_2=0$ are equilibria ($H_2=0$ is the equilibrium) when $\frac{F_2N_2}{F_2N_2+F_1N_1}$ is strictly greater (smaller) than the value satisfying

$$\beta\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)=1-\frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}. \tag{23}$$

As explained after Lemma 2, multiple equilibria arise due to strategic complementarity within the subordinate group: as more of them invest in skill and become skilled workers (i.e. take management positions), prejudice toward the group β_2 eases and the investment becomes more profitable. As for sectoral choices, since $A_{ui}\geq 1$, $p_{si}=1$ and $p_{ui}=1$ $(i=1,2)$ from Proposition A1.

Suppose that assumptions related to the dynamics of F_{1t} and F_{2t} are same as the first economy in Section 5.1, except that $(1-\beta(0))A_{s2}h_s+\beta(0)A_{u2}h_u<\frac{c_h}{\gamma_b}$ now follows from $A_{u2}h_u<\frac{1-\gamma_b(1+r)}{\gamma_b}c_h$.

Thus, F_{1t} always increases. When multiple equilibria exist, assume that the initial coordination continues for subsequent periods: for example, if $H_{20} = F_{20}$ happens to hold, then $H_{2t} = F_{2t}$ for any $t > 0$. This assumption would be reasonable considering that children tend to mimic parental behaviors in real society.[32]

Then, if $\frac{F_{20}N_2}{F_{10}N_1+F_{20}N_2}$ is high enough that $\beta\left(\frac{F_{20}N_2}{F_{10}N_1+F_{20}N_2}\right) < 1-\frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$, i.e. both $H_{20}=F_{20}$ and $H_{20}=0$ are equilibria, *and* $H_{20}=0$ *happens to hold initially, the subordinate group never make productive investment*, F_{1t} rises and F_{2t} falls (since $H_{2t} = 0$ and $A_{u2}h_u < \frac{1-\gamma_b(1+r)}{\gamma_b}c_h$) over time, and $H_1^* = F_1^* = 1$ and $H_2^* = F_2^* = 0$.

Otherwise (thus $H_{20}=F_{20}$ if $\beta\left(\frac{F_{20}N_2}{F_{10}N_1+F_{20}N_2}\right)<1-\frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$), the dynamics are as illustrated in Figure 6. The dividing line between $H_2 = F_2$ and $H_2 = 0$ is a positively-sloped straight line approaching the origin, and $H_2 = 0$ holds below the line. The dynamics of F_{2t} when $H_{2t} = F_{2t}$ are qualitatively same as the first economy of Section 5.1 (Figure 3), while when $H_{2t} = 0$, F_{2t} decreases over time. Hence, if F_{20} is not so small that $b^*(w_{s2}) \geq c_h$ at $(F_1,F_2) = (1,F_{20})$, *given the initial distribution of wealth*, the fate of the subordinate differs greatly depending on which equilibrium *happens to* hold initially: if $H_{20} = F_{20}$, $H_2^* = 1$ (if $b^*(w_{u2}) > c_h$ at $(F_1,F_2) = (1,F_{20})$) or $H_2^* = F_{20}$ (otherwise) from the figure, whereas if $H_{20} = 0$, $H_2^* = F_2^* = 0$.[33] The initial realization of the good (bad) equilibrium results in the better (worse) long-run outcome than under $\beta_i = 0$.

5.2.2 When investment is not always profitable for both groups

If the investment is not always profitable for both groups, i.e. $(1-\beta(0))[A_{s1}h_s - A_{u1}h_u] < (1+r)c_h$ ($\Leftrightarrow$ $A_{s1} < \frac{h_u}{h_s}A_{u1} + \frac{(1+r)c_h}{(1-\beta(0))h_s}$) from Proposition 1 (ii) (see Figure 2), equilibria are $(H_1,H_2) = (0,F_2),(F_1,0)$, and, when $\beta\left(\frac{F_iN_i}{F_iN_i+F_jN_j}\right) < 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ for any i,j, $(H_1,H_2) = (F_1,F_2)$ too. ($p_{si} = p_{ui} = 1$ holds.) $(H_1,H_2) = (0,F_2),(F_1,0)$ are equilibria since strategic substitutability is at work between the groups: as more individuals of one group invest, prejudice toward the other group intensifies and their return to investment falls. As in the previous economy, assumptions related to the dynamics of F_{it} are same as the first economy in Section 5.1, and the initial coordination is maintained when multiple equilibria exist.

Then, if only the subordinate (dominant) group *happen to* make productive investment initially, i.e. $H_{10} = 0$ and $H_{20} = F_{20}$ ($H_{10} = F_{10}$ and $H_{20} = 0$), F_{1t} rises and F_{2t} is constant (falls) and $H_1^* = 0$ ($H_1^* = 1$) and $H_2^* = F_{20}$ ($H_2^* = 0$). Since this type of equilibria exist for *any* F_{10} and F_{20}, it is possible that the dominant group with a *much better* initial condition than the subordinate group, i.e. $F_{10} >> F_{20}$, end up with the *smaller* fraction of skilled workers, i.e. $H_1^* = 0 < H_2^* = F_{20}$ ($F_1^* = 1 > F_2^* = F_{20}$, though). (If $\beta\left(\frac{F_{i0}N_i}{F_{i0}N_i+F_{j0}N_j}\right) < 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ for any

[32]Relatedly, in a dynamic model of statistical discrimination, Kim and Loury (2009) assume that, when there exist equilibrium paths to both good and bad steady states, an initial consensus on the final state shared by group members picks one path and the consensus is maintained over generations.

[33]When $A_{u2} < 1$ and $\beta(0) + \frac{(1+r)c_h}{A_{s2}h_s - A_{u2}h_u} > 1$ (not considered in the proposition or Corollary 1), a figure similar to Figure 6 depicts the dynamics. Differences are that no stable equilibria existin the region on or below $\beta(\frac{F_2N_2}{F_1N_1+F_2N_2}) = 1 - \frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$, and in the region above it, $p_{u2} < 1$ is possible like Figure 4. Thus, if $\beta(\frac{F_{20}N_2}{F_{10}N_1+F_{20}N_2}) < 1 - \frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$ and $\beta(\frac{F_{20}N_2}{N_1+F_{20}N_2}) \geq 1 - \frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$, F_{1t} rises and $H_{it} = F_{it}$ at first, but after the economy crosses the line, the stable equilibrium fails to exist.

i, j and $(H_{10}, H_{20}) = (F_{10}, F_{20})$ happens to hold, the dynamics are similar to those illustrated in Figure 6.[34]) Unlike the previous economy, *given* the initial condition, the long-run outcome of *the dominant group too* is sensitive to the initial realization of equilibrium.

5.2.3 Summary and discussions

To summarize, when skill investment is unproductive in the informal sector, and investment's productivity in the formal sector is low or the degree of prejudice is high, multiple equilibria could exist regarding skill investment, and *given the initial distribution of wealth*, *the initial realization of equilibrium* could affect the dynamics greatly. When the investment is profitable for the dominant group, it can be the case that, if the subordinate group with enough wealth *happen not to* (*to*) invest initially, F_{2t} falls (rises) over time and all of the group are unskilled (skilled) in the long run. When investment is not profitable with high β_i for the dominant group too, given the initial condition, the long-run outcome of *the dominant group too* is sensitive to the initial realization of equilibrium. The dominant group with a much better initial condition than the subordinate group could end up with the *smaller* fraction of skilled workers, if the dominant (subordinate) group happen not to (happen to) invest initially. A similar result holds for the majority and the minority when there are no institutionalized disadvantages, i.e. $A_{k1} = A_{k2}$ $(k = u, s)$.

The results suggest that, in an economy where prejudice is severe ($\beta(0)$ is high) or the effectiveness of skill investment is low, if the initial realization of equilibrium is affected by institutionalized discrimination against a group limiting their access to investment opportunities, such discrimination could have a *lasting impact* on their welfare well after its abolishment. Income or wealth redistribution raising F_i does little to change the situation, while affirmative action treating them favorably in skill investment, such as a tuition subsidy, could be very effective. To be successful, their investment cost c_h must be lowered so that $(1-\beta(0))[A_{si}h_s - A_{ui}h_u] \geq (1+r)c_h$ holds for any group and thus $H_i = F_i$ becomes the unique equilibrium (Corollary 1). The redistribution becomes effective only after such policy is implemented.

6 General case

So far, Assumption 1 (iii), $A_{si} \geq \frac{h_u}{h_s} A_{ui} + \frac{h_s - h_u}{(1-\beta(0))h_s} \Leftrightarrow \beta(0) + \frac{h_s - h_u}{A_{si}h_s - A_{ui}h_u} \leq 1$, is imposed. It does not hold when $\beta(0)$ is high or the relative effectiveness of investment in the formal sector, $\frac{A_{si}h_s - A_{ui}h_u}{h_s - h_u}$, is low. Thus it is dropped now. Under the assumption, the net return to the formal sector is weakly higher for skilled workers even when the degree of prejudice is severest ($\beta_i = \beta(0)$) and thus $p_{si} \geq p_{ui}$ always holds. Without it, $p_{si} = 0$ *and* $p_{ui} = 1$, i.e. all skilled workers choose the informal sector and all unskilled workers choose the *formal* sector, is possible, even if *skilled* workers have comparative advantages and are more productive in the formal sector. This may explain the fact

[34]Differences are positively-sloped $\beta(\frac{F_1 N_1}{F_1 N_1 + F_2 N_2}) = 1 \frac{(1+r)c_h}{A_{s1}h_s - A_{u1}h_u}$ exists above the 45^0 line, and $(H_1, H_2) = (F_1, F_2)$ exists only in the region between it and $\beta(\frac{F_2 N_2}{F_1 N_1 + F_2 N_2}) = 1 - \frac{(1+r)c_h}{A_{s2}h_s - A_{u2}h_u}$ (the dotted bold line). The economy is in the region and F_{1t} rise at first. If $\beta(\frac{F_{20} N_2}{N_1 + F_{20} N_2}) \geq 1 - \frac{(1+r)c_h}{A_{s2}h_s - A_{u2}h_u}$, it crosses the dotted line eventually, after which the equilibrium switches to $(H_{1t}, H_{2t}) = (0, F_{2t})$ *or* $(F_{1t}, 0)$. If a group with lower F_{it} switches to $H_{it} = 0$, F_{2t} keeps falling and $H_2^* = F_2^* = 0$, $H_1^* = 1$.

that skilled people of subordinate groups often avoid formal-sector jobs and run small businesses in their communities or in industries they are concentrated in. Further, multiple equilibria could exist on *sectoral choices of skilled workers* as well as skill investment. Hence, the initial realization of *sectoral choices* too could have lasting impacts on the dynamics.

6.1 Sectoral choices and skill investment

To analyze the model without Assumption 1 (iii), this subsection examines sectoral choices and skill investment when the assumption *does not hold.* Assumption 1 (iii) is replaced by:

Assumption 6 $\beta(0)+\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}>1 \Leftrightarrow A_{si}<\frac{h_u}{h_s}A_{ui}+\frac{h_s-h_u}{(1-\beta(0))h_s}$.

As in Section 3.1, Assumptions 3 through 5 are not imposed in this subsection.

The following lemma on sectoral choices is parallel to Lemma 1 under the old assumption. A reader who is not interested in the exposition of results could directly go to "Graphical illustrations..." part.

Lemma 3 (Sectoral choices under Assumption 6) *Suppose* $H_i>0$ *and* $p_{sj}H_j>0$, $j\neq i$.

(*i*) *When* $A_{ui}\geq 1$, $p_{ui}=1$. $p_{si}=1$ *if* $\beta(0)\leq\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$; *both* $p_{si}=1$ *and* $p_{si}=0$ *if* $\beta(0)>\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ *and* $\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<\frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$; *otherwise,* $p_{si}=0$.

(*ii*) *When* $A_{ui}<1$,

(*a*) *If* $(1-\beta_i)A_{si}h_s-h_s>(1-\beta_i)A_{ui}h_u-h_u$ *with* $p_{si}=1 \Leftrightarrow \beta_i=\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$, $p_{si}=1$ *and* p_{ui} *is determined as in Lemma 1 (ii).*

(*b*) *Otherwise,* $p_{si}=p_{ui}=1$ *if* $\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<\frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$, *or else, no stable equilibrium exists.*

When $A_{ui}\geq 1$, unskilled workers always choose the formal sector as before, while choices of skilled workers now depend on the net return to the formal sector: if it is positive at $p_{si}=1$, i.e. $\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<\frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$, $p_{si}=1$ as before, whereas if it is negative at $p_{si}=0$, i.e. $\beta(0)>\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$, $p_{si}=0$.[35] That is, when the (maximum) degree of prejudice $\beta(0)$ is sufficiently high, all skilled workers choose the informal sector and all unskilled workers choose the *formal* sector, even if *skilled* workers have comparative advantages and are more productive in the formal sector. Since $\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<\beta(0)$, *both* $p_{si}=1$ *and* $p_{si}=0$ *are equilibria* for some combinations of A_{si} and A_{ui} due to strategic complementarity among skilled workers (i.e. their net return increases with p_{si} chosen by others). When $A_{ui}<1$ and the net return to the formal sector at $p_{si}=1$ of skilled workers is weakly lower than unskilled workers, i.e. $\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)\geq 1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$, if the return for the skilled (at $p_{si}=p_{ui}=1$) is positive, $p_{si}=p_{ui}=1$, otherwise (thus $p_{si}<1$), *no stable equilibrium exists*: $p_{si}=0$ cannot be an equilibrium from $A_{si}>1>A_{ui}$, while an equilibrium with $p_{si}\in(0,1)$ is not stable due to the strategic complementarity. (When $A_{ui}<1$ and the net return at $p_{si}=1$ is higher for the skilled, choices are same as the corresponding case of Lemma 1.)

The next lemma corresponding to Lemma 2 under Assumption 1 (iii) presents equilibrium values of H_i.

[35] An equilibrium with $p_{si}\in(0,1)$ is not stable because the net return for the skilled increases with p_{si}.

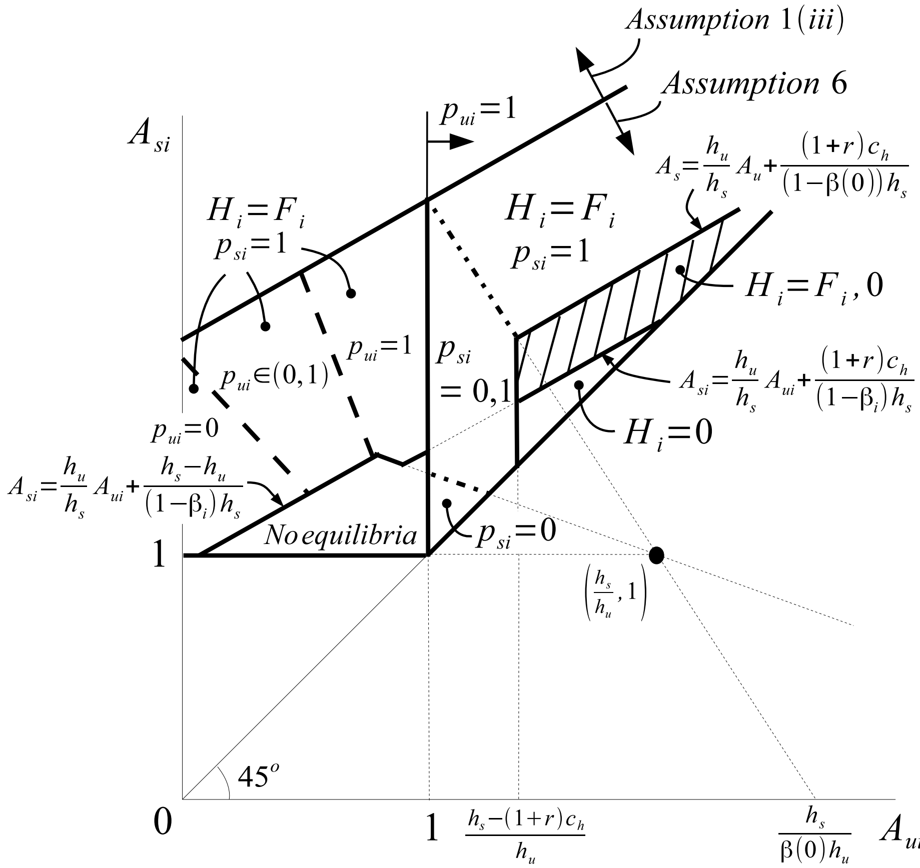

Figure 7: Investment and sectoral choices under Assumption 6 when $h_s-(1+r)c_h \geq h_u$ and $p_{sj}H_j>0$ $(j\neq i)$

Lemma 4 (Skill investment under Assumption 6) *Suppose* $F_i>0$.

(i) *When* $h_s-(1+r)c_h\geq h_u$, $H_i=F_i$ *if* $p_{sj}H_j=0$ *for* $j\neq i$. *If* $p_{sj}H_j>0$,

(a) *When* $A_{ui}\geq 1$,

1. *If* $h_s-(1+r)c_h\geq A_{ui}h_u$, $H_i=F_i$.
2. *If* $h_s-(1+r)c_h < A_{ui}h_u$, $H_i=F_i$ *when* $\beta(0)\leq 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$; *both* $H_i=F_i$ *and* $H_i=0$ *are equilibria when* $\beta(0)>1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}>\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)$; *or else,* $H_i=0$.

(b) *When* $A_{ui}<1$, *if* $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)<\min\left\{\max\left[\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)},1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}\right],1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\right\}$, $H_i=F_i$, *otherwise, no stable equilibrium exists.*

(ii) *When* $h_s-(1+r)c_h<h_u$, *Lemma 2* (ii) *applies* (*except case* $\beta(0)\leq 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ *does not exist*).

When $h_s-(1+r)c_h\geq h_u$, $H_i=F_i$ always under the old assumption, while *inefficient* $H_i=0$ *can be an equilibrium and stable equilibria may not exist* under the new assumption. When $A_{ui}\geq 1$, $p_{si}=0$ or 1 and $p_{ui}=1$ from Lemma 3 (i). Hence, if the net return to investment is non-negative even under $p_{si}=0$ and $p_{ui}=1$ (the return is lower than under $p_{si}=p_{ui}=1$), i.e. $h_s-(1+r)c_h\geq A_{ui}h_u$, $H_i=F_i$ holds; otherwise, when the net return at $H_i=0$ is negative even under $p_{si}=p_{ui}=1$, $H_i=0$ holds, while when the return at $H_i=F_i$ is positive under $p_{si}=p_{ui}=1$, $H_i=F_i$ is true (and both

$H_i = 0$ and $H_i = F_i$ are equilibria when both conditions hold due to the strategic complementarity). When $A_{ui} < 1$, no stable equilibria exist if stable p_{si} and p_{ui} do not exist or if stable H_i does not exist (otherwise, $H_i = F_i$).[36]

Graphical illustrations of skill investment and sectoral choices under Assumption 6. Investment and sectoral choices of a group for given choices by the other group under Assumption 6 are summarized in Proposition A2 of Appendix A. Figure 7 shows the choices when $h_s - (1+r)c_h \geq h_u$ and $p_{sj}H_j > 0$ ($j \neq i$). Although the figure looks complicated, the choices when $A_{ui} > \frac{h_s-(1+r)c_h}{h_u}$ are same as when $h_s - (1+r)c_h < h_u$ and $A_{ui} \geq 1$ under Assumption 1 (iii) illustrated in Figure 2 of Section 3.1, and the choices when $A_{ui} < 1$ are similar to the corresponding case of Figure 2. What is really new occurs when $A_{ui} \in \left[1, \frac{h_s-(1+r)c_h}{h_u}\right]$, in which $H_i = F_i$, $p_{ui} = 1$, and, depending on A_{si} and A_{ui}, $p_{si} = 0$, both $p_{si} = 0$ and $p_{si} = 1$, or $p_{si} = 1$. That is, it is possible that all skilled workers choose the informal sector and all unskilled workers choose the *formal* sector, even if *skilled* workers have comparative advantages and are more productive in the formal sector. The choices when $h_s - (1+r)c_h < h_u$ are almost same as the corresponding case under Assumption 1 (iii).[37]

6.2 Analyses

Qualitatively new dynamics arise when [1] $A_{ui} \in \left[1, \frac{h_s-(1+r)c_h}{h_u}\right]$ (thus $h_s - (1+r)c_h \geq h_u$) and $\beta(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$ hold for at least one group i (the region below the upper dashed double-dotted line, i.e. the region in which $p_{si} = 0$ is an equilibrium, in Figure 7), and [2] either $A_{uj} \geq 1$ or Assumption 1 (iii) holds for the other group j.[38] This is the case in which skill investment is socially productive, $\beta(0)$ is high, and A_{ui} is at an intermediate level for at least one group.

For example, consider an economy in which the former set of conditions hold for the subordinate group (group 2), and either $A_{u1} > \frac{h_s-(1+r)c_h}{h_u} (> 1)$ and $A_{s1} \geq \frac{h_u}{h_s}A_{u1} + \frac{(1+r)c_h}{(1-\beta(0))h_s}$ or $A_{u1} \geq 1$ and Assumption 1 (iii) hold for the dominant group (group 1). $H_i = F_i$ and $p_{ui} = 1$ for $i = 1, 2$, and $p_{s1} = 1$, while p_{s2} is 0 or 1 from Figures 7 and 1. The dividing line between the region $p_{s2} = 0, 1$ and the region $p_{s2} = 0$ is, from Proposition A2 (i)(a):

$$\beta\Big(\frac{F_2N_2}{F_1N_1+F_2N_2}\Big)(1-F_2) = \frac{(A_{s2}-1)h_s}{A_{s2}h_s - A_{u2}h_u}. \tag{24}$$

Suppose that assumptions on the dynamics are same as the first economy in Section 5.1 (because disadvantages of the subordinate group or advantages of the dominant group are moderate, or the

[36] Stable H_i fails to exist when $\beta(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}) \geq 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ since the net return under $H_i = 0$ is *higher* than under $H_i > 0$ due to the dependence of p_{ui} on H_i: given $H_i = 0$, $p_{ui} = 0$ from $A_{ui} < 1$ and thus $H_i = 0$ is not an equilibrium from Assumption 1 (ii), while, given $H_i \in (0, F_i]$, the return is non-positive from $\beta(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}) \geq 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ ($p_{ui} > 0$ must hold from $\beta(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}) \geq 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u} > \frac{(1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$, Lemma 3 (ii), and Lemma 1 (ii), and $p_{si} = 1$ must be from Lemma 3 (ii)) and thus $H_i \in (0, F_i]$ is not stable.

[37] The only difference is that, when $A_{ui} \geq 1$, the region where $H_i = F_i$ is the unique equilibrium does not exist. Note that $H_i = F_i$ is possible under Assumption 6 too: unlike Figure 2, if $h_s - (1-\beta(0))(1+r)c_s < h_u$, the line dividing the regions satisfying Assumption 1 (iii) and Assumption 6 is above the line for Assumption 1 (ii) in Figure 2.

[38] The dynamics are similar to Section 5 in other cases. When $h_s - (1+r)c_h < h_u$, analyses of the corresponding case there go through. When $h_s - (1+r)c_h \geq h_u$, if either $A_{ui} \geq 1$ and $\beta(0) \leq \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$ (the region on or above the upper dashed double-dotted line in Figure 7) or $A_{ui} > \frac{h_s-(1+r)c_h}{h_u}$ for any i, analyses in Section 5 apply, while when Assumption 6 and $A_{ui} < 1$ for some i and thus stable equilibria may fail to exist depending on F_i, analyses in footnote 33 of Section 5.2 apply.

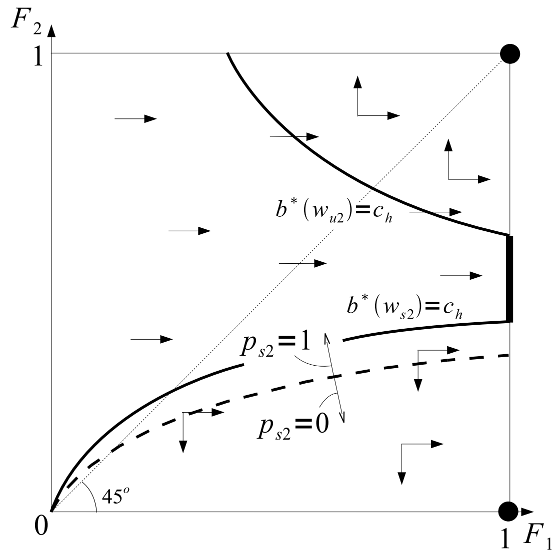

Figure 8: Dynamics when $p_{s2t}=1$ is selected in the region where both $p_{s2t}=0$ and $p_{s2t}=1$ are equilibria

relative productivity of the formal sector is not low), implying that F_{1t} rises over time and, when $p_{s2}=1$, $b^*(w_{s2})=c_h$ (eq. 19) and $b^*(w_{u2})=c_h$ (eq. 18) exist. Further, assume $h_s<\frac{c_h}{\gamma_b}$ and thus F_{2t} falls when $p_{s2t}=0$. When multiple equilibria exist, initial coordination continues for subsequent periods as in Section 5.2.

Then, if $\beta\left(\frac{F_{20}N_2}{F_{10}N_1+F_{20}N_2}\right)(1-F_{20})<\frac{(A_{s2}-1)h_s}{A_{s2}h_s-A_{u2}h_u}$ holds, i.e. both $p_{s20}=0$ and $p_{s20}=1$ are equilibria, *and* $p_{s20}=0$ *happens to hold*, F_{1t} rises and F_{2t} falls over time and $H_1^*=1$ and $H_2^*=0$. Although skilled workers of the subordinate group are more productive in the *formal* sector, they choose the informal sector to avoid the negative effect from their fellow unskilled workers. The sector's wage, however, is not high enough for their descendants to remain skilled and the group end up being totally unskilled in the long run.

Instead, if $p_{s20}=1$ *happens to* hold under the *same* situation, the dynamics of F_{1t} and F_{2t} are as illustrated in Figure 8.[39]The skilled workers efficiently choose the formal sector and earn the higher wage than the previous case. In particular, if F_{20} is high enough that $b^*(w_{u2})>c_h$ at $(F_1,F_2)=(1,F_{20})$, F_{2t} starts to rise at some point and $H_1^*=H_2^*=1$ in the long run. The group's unskilled workers benefit from the presence of the skilled workers in the formal sector, enabling the upward mobility of their descendants.

Given the initial distribution of wealth, the long-run fate of the subordinate group differs greatly depending on the initial realization of *sectoral choices* by the group's skilled workers. The same is true for *the dominant group too* when $A_{ui}\in\left[1,\frac{h_s-(1+r)c_h}{h_u}\right]$ and $\beta(0)>\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ hold for both groups. The result suggests that initial institutionalized discrimination against a group

[39]From (24), the dividing line (the dashed line) is positively sloped and approaches the origin (note $\beta(0)>\frac{(A_{s2}-1)h_s}{A_{s2}h_s-A_{u2}h_u}$). (F_{10},F_{20}) is above the line since $p_{s2}=1$ is possible only in the region above it.

limiting their access to skilled jobs in the formal sector, which affects the initial realization of equilibrium, could have a lasting negative impact on their well-beings well after its abolishment. This is consistent with the finding by Darity, Dietrich, and Guilkey (2001) that the occupational status of a U.S. worker in 1980 and 1990 is significantly related to human capital endowments *and* the degree of favorable or unfavorable treatment in the labor market in the period between 1880 and 1910 of his/her ethnic group. Affirmative action treating them favorably in the sector, such as a wage subsidy to the sector's (particularly skilled) jobs that makes $p_{si}=1$ the unique equilibrium, could be very effective to change the situation.[40]

7 Conclusions

Disparities in economic conditions among different ethnic, racial, or religious groups continue to be serious concerns in most economies. Evidence shows a strong negative relationship between inequality across ethnic groups and economic development. Relative standings of different groups are rather persistent, although some groups initially in disadvantaged positions successfully caught up with then-advantaged groups. Two obstacles, costly skill investment and negative stereotypes or discrimination in the labor market, seem to distort investment and sectoral decisions and slow down the progress of the disadvantaged.

This paper has developed a dynamic model of statistical discrimination in which these obstacles affect skill investment and sectoral choices of individuals of two groups and examined how initial economic standings of the groups and initial institutionalized discrimination affect subsequent dynamics and long-run outcomes. There exist the formal sector that is ethnically mixed and group reputation affects wages due to statistical discrimination and the informal sector with the contrasting features, where the latter may not be active in equilibrium.

Main results are summarized as follows. First, sectoral choices and skill investment may not be socially optimal since choices of different individuals within and across groups could be interrelated. Second, multiple equilibria could exist regarding investment and sectoral choices of skilled workers: both the non-poor of a group invest (skilled workers choose the formal sector) and do not could be equilibria. Third, the dynamics and long-run outcomes of groups, particularly of the subordinate group, depend greatly on initial conditions and could be quite different from a "prejudice-free" economy. Since good (bad) reputation tends to beget good (bad) reputation, a group starting with a good (bad) initial condition tend to be in a good (bad) position in the long run. Fourth, when multiple equilibria exist, which is the case when the effect of stereotypes is strong or the efficacy of investment is low, given initial conditions, the initial realization of equilibrium could affect the dynamics and long-run outcomes greatly. If the initial realization is determined by institutionalized discrimination limiting a group's access to investment or skilled jobs in the formal sector, the discrimination could have a lasting impact on their welfare well after its abolishment.

[40] A subsidy to all formal-sector jobs could lower incentives for skill investment and result in $H_i=0$. A subsidy targeting skilled workers can be implemented, if skill is revealed eventually (see footnote 19 for an interpretation of the wage function).

Redistribution does little to change the situation, while affirmative action could have a large impact.

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Appendix A: Propositions A1 and A2

The next proposition summarizes skill investment and sectoral choices of group i given choices by the other group when Assumption 1 (iii) is imposed, by combining Lemmas 1 and 2 of Section 3.

Proposition A1 (Group i's investment and sectoral choices given choices by group j)

(i) *When* $h_s-(1+r)c_h \geq h_u$, $H_i=F_i$ *and* $p_{si}=1$.

(a) *When* $p_{sj}H_j>0$ *for* $j\neq i$, *if* $A_{ui}\geq 1$, $p_{ui}=1$; *and if* $A_{ui}<1$, $p_{ui}=0$ *for* $\beta_i \leq \frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, $p_{ui}=\frac{\beta_i A_{si}h_s+(1-\beta_i)A_{ui}h_u-h_u}{(1-A_{ui})h_u}\frac{F_i}{1-F_i}\in(0,1)$ *for* $\beta_i\in\left(\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u},\frac{1}{F_i}\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}\right)$, *and* $p_{ui}=1$ *for higher* β_i.

(b) *When* $p_{sj}H_j=0$, $p_{ui}=1(=0)$ *if* $A_{ui}>(<)1$ *and any* $p_{ui}\in[0,1]$ *if* $A_{ui}=1$.

(ii) *When* $h_s-(1+r)c_h<h_u$, $p_{si}=1$ *and, when* $p_{sj}H_j=0$, $H_i=F_i$. *When* $p_{sj}H_j>0$,

(a) *If* $A_{ui}\geq 1$, $H_i=F_i$ *if* $\beta(0)\leq 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, *otherwise, both* $H_i=F_i$ *and* $H_i=0$ *are equilibria* ($H_i=0$ *is the equilibrium*) *when* $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)<(\geq)1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$.

(b) *If* $A_{ui}<1$, $H_i=F_i$ (*no stable equilibria exist*) *when* $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)<(\geq)1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$.

(c) p_{ui} *is determined as in* $(i)(a)$ $[(i)(b)]$ *when* $p_{sj}H_j>0$ *and* $H_i=F_i$ [*in other cases*].

The following proposition summarizes skill investment and sectoral choices of group i given choices by the other group under Assumption 6, by combining Lemmas 3 and 4 of Section 6.

Proposition A2 (Investment and sectoral choices of group i under Assumption 6)

(i) *When* $h_s-(1+r)c_h\geq h_u$, *if* $p_{sj}H_j=0$ *for* $j\neq i$, $H_i=F_i$, $p_{si}=1$, *and Proposition A1* $(i)(b)$ *applies for* p_{ui}. *If* $p_{sj}H_j>0$,

(a) *When* $A_{ui}\geq 1$, $p_{ui}=1$.

1. *When* $h_s-(1+r)c_h\geq A_{ui}h_u$, $H_i=F_i$. $p_{si}=1$ *if* $\beta(0)\leq\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$; *otherwise, both* $p_{si}=1$ *and* $p_{si}=0$ *are equilibria if* $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)<\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}$, *or else* $p_{si}=0$.
2. *When* $h_s-(1+r)c_h<A_{ui}h_u$, $p_{si}=1$. *If* $\beta(0)\leq 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$; *otherwise, both* $H_i=0$ *and* $H_i=F_i$ *are equilibria if* $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)<1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, *or else* $H_i=0$.

(b) *When* $A_{ui}<1$,

1. *If* $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)<1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$, $p_{si}=1$, *and Proposition A1* $(i)(a)$ *applies for* p_{ui}.
2. *Otherwise, if* $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)<\min\left\{\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)},1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\right\}$, $H_i=F_i$ *and* $p_{si}=p_{ui}=1$; *or else, no stable equilibrium exists.*

(ii) *When* $h_s-(1+r)c_h<h_u$, *Proposition A1* (ii) *applies* (*no case* $\beta(0)\leq 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$).

Appendix B: Proofs of lemmas and propositions

Proof of Lemma 1. (i) If $p_{si}>0$, $p_{si}=p_{ui}=1$ is the only stable equilibrium because

$$\begin{aligned}(1-\beta_i)A_{si}h_s+\beta_i E[Ah|i]-h_s &\geq (1-\beta(0))A_{si}h_s+\beta(0)E[Ah|i]-h_s &(25)\\ &\geq (1-\beta(0))A_{ui}h_u+\beta(0)E[Ah|i]-h_u &(26)\\ &\geq (1-\beta_i)A_{ui}h_u+\beta_i E[Ah|i]-h_u &(27)\\ &>[(1-\beta_i)A_{ui}-1]h_u+\beta_i A_{ui}h_u=(A_{ui}-1)h_u\geq 0, &(28)\end{aligned}$$

where (26) is from Assumption 1 (iii) and (28) is from $p_{si}>0$. If $p_{si}=0$, $p_{ui}=0$ must be from $(1-\beta_i)A_{si}h_s+\beta_i E[Ah|i]-h_s\geq(1-\beta_i)A_{ui}h_u+\beta_i E[Ah|i]-h_u$. But $p_{si}=p_{ui}=0$ is not an equilibrium from $A_{si}>1$.

(ii) As shown in (i), $(1-\beta_i)A_{si}h_s+\beta_i E[Ah|i]-h_s\geq(1-\beta_i)A_{ui}h_u+\beta_i E[Ah|i]-h_u$. Thus, if $p_{si}=0$, $p_{ui}=0$ must hold, but not an equilibrium, as shown in (i). If $p_{si}\in(0,1)$, (27) holds with $>$ and $p_{ui}=0$ must hold, not an equilibrium from $A_{si}>1$. Thus, if an equilibrium exists, $p_{si}=1$ and the net return to the formal sector for the unskilled is $(1-\beta_i)A_{ui}h_u+\beta_i E[Ah|i]-h_u=[(1-\beta_i)A_{ui}-1]h_u+\beta_i\frac{H_iA_{si}h_s+p_{ui}(1-H_i)A_{ui}h_u}{H_i+p_{ui}(1-H_i)}$, which decreases with p_{ui}. Hence, $p_{ui}=0$ if it is non-positive with $p_{ui}=0$, i.e. $\beta_i\leq\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$; $p_{ui}=1$ if non-negative with $p_{ui}=1$, i.e. $[(1-\beta_i)A_{ui}-1]h_u+\beta_i[H_iA_{si}h_s+(1-H_i)A_{ui}h_u]\geq 0$; otherwise, $p_{ui}\in(0,1)$ and p_{ui} is determined from the zero return condition. Such p_{ui} and $p_{si}=1$ is an equilibrium when $p_{ui}=0$ from $A_{si}>1$ and when $p_{ui}>0$ from $(1-\beta_i)A_{si}h_s+\beta_i E[Ah|i]-h_s>(1-\beta_i)A_{ui}h_u+\beta_i E[Ah|i]-h_u$. If $p_{ui}=0$ $(=1)$ and the return for the unskilled is negative (positive), the equilibrium is stable since the one for the skilled is positive. Otherwise, it is stable since the return for the skilled (unskilled) is positive (falls with p_{ui}). ■

Proof of Lemma 2. $H_i=F_i$ when $p_{sj}H_j=0$ for $j\neq i$ is obvious from Assumption 1 (ii).

(Existence/nonexistence of $H_i>0$ when $p_{sj}H_j>0$) (i) Given $H_i>0$, $p_{si}=1$ from Lemma 1. Thus, when $p_{ui}>0$, $(1-\beta_i)[A_{si}h_s-A_{ui}h_u]-(1+r)c_h>(1-\beta(0))[A_{si}h_s-A_{ui}h_u]-(1+r)c_h\geq h_s-h_u-(1+r)c_h\geq 0$ from Assumption 1 (iii) (thus $p_{hi}=1$). When $p_{ui}=0$, since $A_{si}>1$, $(1-\beta_i)A_{si}h_s+\beta_i E[Ah|i]-(1+r)c_h=A_{si}h_s-(1+r)c_h>h_s-(1+r)c_h\geq h_u$. Thus, $H_i=p_{hi}F_i=F_i$ is the only equilibrium with $H_i>0$, which is clearly stable.

(ii) Given $H_i>0$, $p_{si}=1$ from Lemma 1 and $\beta_i=\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)$. When $p_{ui}=0$, i.e. $\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)\leq\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$ from Lemma 1 (ii) (occurs only in (b)), $A_{si}h_s-(1+r)c_h\geq h_u$ (from Assumption 1 (ii)) and thus $p_{hi}=1$ from Assumption 2. When $p_{ui}>0$, i.e. $\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)>\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, the net return is $\big[1-\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)\big](A_{si}h_s-A_{ui}h_u)-(1+r)c_h$ and $p_{hi}=1(=0)$ if $\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)\leq(>)\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ from Assumption 2. Note $\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\geq\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$ from $A_{si}h_s-(1+r)c_h\geq h_u$. Thus, if $\beta\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)>1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, the net return is negative for any $H_i\in(0,F_i]$ and no equilibrium with $H_i>0$ exists, while if $\beta\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$ is the only stable equilibrium with $H_i>0$ since the return increases with H_i (if $\beta(0)>1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, equilibrium with $H_i\in(0,F_i)$ exists but is not stable). Similarly, if $\beta\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)=1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$ is the only equilibrium with $H_i>0$ but is not stable.

(Existence/nonexistence of $H_i = 0$ when $p_{sj}H_j > 0$) Given $H_i = 0$, if $A_{ui} < 1$, the net return is $A_{si}h_s - (1+r)c_h - h_u \geq 0$ and $H_i = 0$ is not an equilibrium (when the net return is 0, $p_{hi} = 1$ from Assumption 2). Given $H_i = 0$, if $A_{ui} \geq 1$, the net return is $max[(1-\beta_i)A_{si}h_s + \beta_i A_{ui}h_u, h_s] - (1+r)c_h - A_{ui}h_u = max\{(1-\beta(0))[A_{si}h_s - A_{ui}h_u], h_s - A_{ui}h_u\} - (1+r)c_h = (1-\beta(0))[A_{si}h_s - A_{ui}h_u] - (1+r)c_h \geq h_s - h_u - (1+r)c_h$ from Assumption 1 (iii) and $A_{ui} \geq 1$. Thus, in (i), $H_i = 0$ is not an equilibrium when $A_{ui} \geq 1$ too (note Assumption 2). In (ii), the net return is negative (non-negative) when $\beta(0) > (\leq) 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$. Hence, $H_i = 0$ is (is not) an equilibrium if $\beta(0) > (\leq) 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ in (ii)(a). Equilibrium $H_i = 0$ is stable in (ii)(a) since $\beta(0) > 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ implies a negative net return with small $H_i > 0$ (since $A_{ui} \geq 1$, given $H_i > 0$, $p_{si} = p_{ui} = 1$ from Lemma 1 (i)). ■

Proof of Proposition A1. (i) $H_i = F_i$ from Lemma 2 (i), thus Lemma 1 applies with $H_i = F_i$. (ii)(a)/(b) The value of H_i is from Lemma 2 (ii)(a)/(b). ■

Proof of Proposition 1. When $(1-\beta(0))[A_{s2}h_s - A_{u2}h_u] \geq (1+r)c_h$, the same condition holds for group 1 from Assumption 5, which is the case covered in Section 5.1. When $A_{u2} < 1$ and $(1-\beta(0))[A_{s2}h_s - A_{u2}h_u] < (1+r)c_h$, given $p_{s1}H_1 > 0$, stable H_2 does not exist for some F_2 from Proposition A1 (ii)(b). Given $p_{s1}H_1 = H_1 = 0$, $H_2 = F_2$ from the proposition, but $(H_1, H_2) = (0, F_2)$ is an equilibrium for any F_1 and F_2 only if $A_{u1} \geq 1$, $(1-\beta(0))[A_{s1}h_s - A_{u1}h_u] < (1+r)c$ (see Figure 2), *and* $H_1 = 0$ *happens to* hold ($H_1 = F_1$ too could hold depending on F_1 and F_2) from Assumption 5 and Proposition A1 (ii)(a). Equilibria that are stable for any F_1 and F_2 may not exist and thus this case is not considered in the proposition (briefly discussed in footnote 33).

$p_{si} = 1$ when $H_i > 0$ $(i = 1, 2)$ from Proposition A1 (ii). Then, from Proposition A1 (ii)(a), given $p_{s1}H_1 = H_1 = 0$, $H_2 = F_2$, and given $H_1 > 0$, both $H_2 = F_2$ and $H_2 = 0$ ($H_2 = 0$) when $\beta(\frac{F_2N_2}{F_2N_2 + H_1N_1}) < (\geq)\ 1 - \frac{(1+r)c_h}{A_{s2}h_s - A_{u2}h_u}$. (i) In this case, from Proposition A1 (ii)(a), $H_1 = F_1$ always. Hence, $H_1 = F_1$ and both $H_2 = F_2$ and $H_2 = 0$ are equilibria ($H_2 = 0$ is the equilibrium) when $\beta(\frac{F_2N_2}{F_2N_2 + F_1N_1}) < (\geq)\ 1 - \frac{(1+r)c_h}{A_{s2}h_s - A_{u2}h_u}$. (ii) Given H_2, the value of H_1 is determined in the same way as H_2. Hence, $(H_1, H_2) = (0, F_2), (F_1, 0)$, and, when $\beta(\frac{F_iN_i}{F_iN_i + F_jN_j}) < 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ for any $i, j = 1, 2$ $(j \neq i)$, $(H_1, H_2) = (F_1, F_2)$ as well. ■

Proof of Lemma 3. An equilibrium with $p_{si} \in (0,1)$ and $p_{ui} = 1$, if exists, is not stable, because the return to the formal sector for the skilled becomes positive whenever p_{si} increases. An equilibrium with $p_{si} \in (0,1)$ and $p_{ui} = 0$ does not exist from $A_{si} > 1$. An equilibrium with $p_{si}, p_{ui} \in (0,1)$, which satisfies $(1-\beta_i)A_{si}h_s + \beta_i E[Ah|i] - h_s = (1-\beta_i)A_{ui}h_u + \beta_i E[Ah|i] - h_u = 0$, is not stable because, whenever p_{si} increases and p_{ui} non-increases, the return for the skilled becomes positive and p_{si} does not have a tendency to return to the original value: the effect of p_{si} on the return for the skilled is greater than for the unskilled from $A_{si}h_s > A_{ui}h_u$ and the effect of p_{ui} on the return is same for both types of workers.

Thus, if a stable equilibrium exists, $p_{si} = 0$ or 1. $p_{si} = p_{ui} = 0$ is not an equilibrium from the proof of Lemma 1 (i). $p_{si} = 0$ and $p_{ui} \in (0,1)$ is not an equilibrium when $A_{ui} \neq 1$ and not stable when $A_{ui} = 1$ (the return for the unskilled is positive when p_{si} rises). $p_{si} = 0$ and $p_{ui} = 1$ is not an equilibrium when $A_{ui} < 1$. When $A_{ui} \geq 1$, it is a stable equilibrium if the return for the skilled is negative, i.e. $(1-\beta(0))A_{si}h_s + \beta(0)A_{ui}h_u - h_s < 0$, since the one for the unskilled is positive (when

$A_{ui}>1$) or it increases when p_{si} rises (when $A_{ui}=1$). (When $\beta(0)=\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$, not stable since the return for the skilled increases with p_{si}.)

As for possible equilibria with $p_{si}=1$, if $(1-\beta_i)A_{si}h_s-h_s>(1-\beta_i)A_{ui}h_u-h_u$ with $p_{si}=1 \Leftrightarrow \beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, the proof of Lemma 1 (i) and (ii) can be applied with a slight modification, thus the result of the lemma holds. If $(1-\beta_i)A_{si}h_s-h_s\le(1-\beta_i)A_{ui}h_u-h_u$ with $p_{si}=1$, $p_{si}\le p_{ui}$ and thus $p_{ui}=1$ must hold. $p_{si}=p_{ui}=1$ is a stable equilibrium when $(1-\beta_i)A_{si}h_s+\beta_iE[Ah|i]-h_s=(1-\beta_i)A_{si}h_s+\beta_i[H_iA_{si}h_s+(1-H_i)A_{ui}h_u]-h_s>0$ with $p_{si}=1\Leftrightarrow\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<\frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$, since the returns for both types are positive. When $\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)=\frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$, it is not stable because the return for the skilled falls with a decrease in p_{si}. (When $(1-\beta_i)A_{si}h_s-h_s=(1-\beta_i)A_{ui}h_u-h_u$ with $p_{si}=1$, the additional reason is that the effect of p_{si} on the return for the skilled is greater than for the unskilled and the effect of p_{ui} on the returns are same.)

To summarize, when $A_{ui}\ge1$, $p_{ui}=1$, and since $\frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}\ge\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, $p_{si}=1$ if $\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<\frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$ and $p_{si}=0$ if $\beta(0)>\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$. Hence, because $\beta(0)>\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$ (from Assumption 6), $\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}\le\frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$, and $\beta(0)>\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)$ hold, the stable equilibrium(a) is $p_{si}=p_{ui}=1$ when $\beta(0)\le\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$; both $p_{si}=p_{ui}=1$ and $p_{si}=0$, $p_{ui}=1$ when $\beta(0)>\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ and $\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<\frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$; and $p_{si}=0$ and $p_{ui}=1$ otherwise.

When $A_{ui}<1$, if $\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, the result of Lemma 1 (ii) applies, otherwise, the stable equilibrium is $p_{si}=p_{ui}=1$ (no stable equilibrium exists) if $\beta\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<(\ge)\frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$. ■

Proof of Lemma 4. (Proof when $p_{sj}H_j=0$ for $j\neq i$) The proof of Lemma 2 applies.

(Existence/nonexistence of $H_i>0$ when $p_{sj}H_j>0$) (i) If $(1-\beta_i)A_{si}h_s-h_s>(1-\beta_i)A_{ui}h_u-h_u$ with $p_{si}=1$ and $H_i=F_i$, and $p_{si}=1$ for given $H_i=F_i$ hold, i.e. $\beta\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$ (from Lemma 3 (i) and (ii)), the corresponding part of Lemma 2 (i) applies and thus $H_i=F_i$ is the only stable equilibrium with $H_i>0$.[41] Instead, if $(1-\beta_i)A_{si}h_s-h_s\le(1-\beta_i)A_{ui}h_u-h_u$ with $p_{si}=1$ and $H_i=F_i$ and $p_{si}=1$ for given $H_i=F_i$ hold, i.e. $\beta\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)\in\big[\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u},\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}\big)$ (from Lemma 3 (i) and (ii)), $p_{si}=p_{ui}=1$ from the lemma and the net return to investment with $H_i=F_i$ equals $\big[1-\beta\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)\big](A_{si}h_s-A_{ui}h_u)-(1+r)c_h$. Hence, if the return is positive, i.e. if $\beta\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$ is the only stable equilibrium with $H_i>0$, otherwise, no stable equilibrium with $H_i>0$ exists. (If the return is positive, an equilibrium with $H_i\in(0,F_i)$ too may exist, and if it is zero, $H_i=F_i$ is the only equilibrium with $H_i>0$, both of which are not stable.) Finally, when $A_{ui}\ge1$, if $p_{si}=0$ for $H_i=F_i$ holds, i.e. $\beta(0)>\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ (from Lemma 3 (i)), the net return is $h_s-(1+r)c_h-A_{ui}h_u$, thus, if it is non-negative, $H_i=F_i$ is the only stable equilibrium with $H_i>0$ (note Assumption 2), otherwise, no stable equilibrium with $H_i>0$ exists.

[41] To be exact, if $s(0)>\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$, for given F_i, there exists $H_i^\dagger\in(0,F_i)$ such that $s\big(\frac{H_i^\dagger N_i}{H_i^\dagger N_i+p_{sj}H_jN_j}\big)=\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{(1-H_i^\dagger)(A_{si}h_s-A_{ui}h_u)}$, and when $A_{ui}\ge1$, $p_{si}=0$ and $p_{ui}=1$ is the only equilibrium for $H_i\in(0,H_i^\dagger]$. However, such H_i ($p_{hi}\in(0,1)$) is not an equilibrium since the net return is $h_s-(1+r)c_h-A_{ui}h_u$. The same reasoning applies to the next case and the corresponding cases of (ii).

To summarize, when $A_{ui} \geq 1$, $H_i = F_i$ if $h_s - (1+r)c_h \geq A_{ui}h_u$ and $\beta(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$ or if $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right) < \min\left\{\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}, \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\right\}$. Note that, when $h_s-(1+r)c_h \geq A_{ui}h_u$, $\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u} \leq \min\left\{\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}, \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\right\}$ and thus $H_i = F_i$ always. Hence, when $A_{ui} \geq 1$, $H_i = F_i$ if $h_s-(1+r)c_h \geq A_{ui}h_u$ or if $h_s-(1+r)c_h < A_{ui}h_u$ and $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right) < 1 - \frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$. When $A_{ui} < 1$, if $1 - \frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u} \geq \frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}$, $H_i = F_i$ when $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right) < 1 - \frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$; otherwise, $H_i = F_i$ when $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right) < \min\left\{\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}, 1 - \frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\right\}$.

(ii) If $(1-\beta_i)A_{si}h_s - h_s > (1-\beta_i)A_{ui}h_u - h_u$ with $p_{si} = 1$ and $H_i = F_i$ and $p_{si} = 1$ for $H_i = F_i$ hold, i.e. $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right) < 1 - \frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$ holds (see the proof of (i)), the corresponding part of the proof of Lemma 2 (ii) applies. In particular, since $1 - \frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u} < 1 - \frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$, $H_i = F_i$ is a stable equilibrium in the same cases as Lemma 2 (ii), except that now $\beta(0) \leq 1 - \frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ is not possible from Assumption 6. No equilibrium with $H_i > 0$ exists in the remaining cases: if $(1-\beta_i)A_{si}h_s - h_s \leq (1-\beta_i)A_{ui}h_u - h_u$ with $p_{si} = 1$ and $H_i = F_i$ and $p_{si} = 1$ hold, $p_{ui} = 1$ and the net return is $(1-\beta_i)[A_{si}h_s - A_{ui}h_u] - (1+r)c_h \leq h_s - (1+r)c_h - h_u < 0$; and when $A_{ui} \geq 1$, if $p_{si} = 0$, $p_{ui} = 1$ and the net return is $h_s - (1+r)c_h - A_{ui}h_u \leq h_s - (1+r)c_h - h_u < 0$.

(Existence/nonexistence of $H_i = 0$ when $p_{sj}H_j > 0$) Given $H_i = 0$, if $A_{ui} < 1$ (thus $p_{ui} = 0$), the corresponding part of the proof of Lemma 2 applies and thus $H_i = 0$ is not an equilibrium in (i)(b) and when $A_{ui} < 1$ in (ii). Given $H_i = 0$, if $A_{ui} \geq 1$, the net return is $max\{(1-\beta_i)A_{si}h_s + \beta_iA_{ui}h_u, h_s\} - (1+r)c_h - A_{ui}h_u = max\{(1-\beta(0))[A_{si}h_s - A_{ui}h_u], h_s - A_{ui}h_u\} - (1+r)c_h < h_s - (1+r)c_h - h_u$. Thus, $H_i = 0$ is always an equilibrium when $A_{ui} \geq 1$ in (ii), while it is (is not) an equilibrium if $max\{(1-\beta(0))[A_{si}h_s - A_{ui}h_u], h_s - A_{ui}h_u\} - (1+r)c_h < (\geq) 0$ in (i)(a). That is, $H_i = 0$ if $(1-\beta(0))[A_{si}h_s - A_{ui}h_u] < h_s - A_{ui}h_u \Leftrightarrow \beta(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ and $h_s - (1+r)c_h < A_{ui}h_u$, or if $\beta(0) \leq \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ and $\beta(0) > \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$. Since $\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u} > \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u} \Leftrightarrow h_s - (1+r)c_h < A_{ui}h_u$, $H_i = 0$ if $h_s - (1+r)c_h < A_{ui}h_u$ and $\beta(0) > \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ in (i)(a). ■

Proof of Proposition A2. (When $p_{sj}H_j = 0$ for $j \neq i$) Since H_i, p_{si}, and p_{ui} are determined independent of $\beta(\cdot)$, the corresponding result of Proposition A1 applies.

(When $p_{sj}H_j > 0$) (i)(a)1 From Lemmas 4 (i)(a)1 and 3 (i). (a)2 From Lemmas 4 (i)(a)2 and 3 (i). Note that $p_{si} = 1$ when $H_i = F_i$, since, if $p_{si} = 0$, $H_i = 0$ from $h_s - (1+r)c_h - A_{ui}h_u < 0$. (b)1 From Lemmas 4 (i)(b) and 3 (ii)(a) and Proposition A1 (i)(a). (b)2 From Lemmas 4 (i)(b) and 3 (ii)(b). (Since $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right) \geq 1 - \frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$, $\max\left[\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}, 1 - \frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}\right] = \frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}$.)

(ii) H_i is determined as in Lemma 4 (ii). When $H_i = 0$, p_{si} and p_{ui} are determined independent of $\beta(\cdot)$ and Proposition A1 applies. When $H_i = F_i$, from Lemma 4 (ii) and Lemma 2 (ii), $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right) < \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$. Hence, when $A_{ui} \geq 1$, $p_{si} = p_{ui} = 1$ from Lemma 3 (i), since $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right) < \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u} < \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ and $H_i = 0$ holds if $p_{si} = 0$. When $A_{ui} < 1$, from Lemma 3 (ii)(a), p_{si} and p_{ui} are determined as in Lemma 1 (ii) from $\beta\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right) < 1 - \frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u} < 1 - \frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$. ■