Pareto-improving Immigration in the Presence of a Pay-as-you-go Social Security

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Abstract

The effect of accepting more immigrants on welfare in the presence of a pay-as-you-go social security system is analyzed theoretically and quantitatively in this study. First, it is shown that if intergenerational government transfers initially exist from the young to the old, the government can lead an economy to the (modified) golden rule level within a finite time in a Pareto-improving way by increasing the percentage of immigrants to natives (PITN). Second, by using the computational overlapping generation model, I calculate both the welfare gain of increasing the PITN from 15.5 percent to 25.5 percent in 80 years and the years needed to reach the (modified) golden rule level in a Pareto-improving way in a model economy. The simulation results show that the present discounted value of the Pareto-improving welfare gain of increasing the PITN is 23 percent of initial GDP. It takes 112 years for the model economy to reach the golden rule level in a Pareto-improving way.

1 Introduction

Immigration is becoming one of the most debated issues in the current political situation. In the UK referendum on the countrys exit from the European Union, one of the issues that divided national opinion was the handling of immigration from other EU countries. A key issue debated in the US presidential election in 2016 was the treatment of illegal immigrants. This problem is becoming an important political issue in other European countries as well. Considering the dynamic general equilibrium effect on the capital-labor ratio, one of the questions is whether accepting immigrants, in principle, yield a Pareto-improving welfare gain to a host country.

From a dynamic viewpoint, a rising number of immigrants has two economic effects on a host country. First, as an increasing number of immigrants arrive each year, the dependency ratio of the social security system decreases. When the capital-labor ratio and factor prices are fixed, this decreasing dependency ratio will bring about a welfare gain in the presence of a pay as you go (PYGO) social security system. Given that the social security benefits in the future represent a great liability for the government, the welfare gain from this channel can be substantial. Second, an increasing number of immigrants implies a rising population growth rate. With a production function that exhibits a diminishing marginal product of capital (MPK), starting from a dynamically efficient initial steady state, the neoclassical growth model (Solow, 1956) predicts that an increasing population growth rate leads to lower capital stock per capita, wages, and GDP per capita (the capital-shallowing effect). Thus, given these two conflicting effects, it is not clear how an increase in the number of immigrants can generate a welfare gain to a host country in the long run.

The literature also presents an unclear picture. In the public finance literature, Storesletten (2000) argues, using the computational overlapping-generations model (Auerbach and Kotlikoff, 1987), that accepting a particular type of immigrants (skilled immigrants who are of an age such that they will not be able to claim social security benefits because they do not satisfy the minimum requirement of the duration of social security tax payments) will increase social welfare as the baby boomer generation goes into retirement. By contrast, Fehr et al. (2004) show that no such welfare gain will ensue. They rather argue that the economic force that leads to a lower capital-labor ratio and lower GDP per capita is very strong; hence, accepting more immigrants cannot bring about a welfare gain. Bonin et al. (2000) conclude, analyzing the case of Germany, that immigration can decrease the fiscal burden of future resident generations in Germany. Analyzing the case of Spain, Collado et al. (2004) argue that accepting more immigrants brings about a positive welfare gain to Spain. Schou (2006) concludes, analyzing the effect of immigration on the fiscal sustainability in Denmark, that the effect is negative. Blake and Mayhew (2006) analyze the issue of immigration and pension sustainability in the UK. They argue that any realistic increase in labour productivity by the indigenous population will not be sufficient to sustain the pension system and that more immigration is inevitable.

Given the mixed results in the literature regarding the effect of accepting more immigrants on social welfare, whether accepting more immigrants Pareto-improves welfare in the presence of a PYGO social security system is a natural question from a theoretical standpoint. The capital-shallowing effect seems to be very strong, as emphasized by Fehr et al. (2004), when the economy experiences a higher population growth rate due to an increased inflow of immigrants. However, if this is so, one may wonder why previous studies have arrived at such different results regarding the effect on welfare of accepting more immigrants.

Motivated by this question, I theoretically and qualitatively analyze the effect on welfare of accepting more immigrants and increasing the population growth rate. I conclude that accepting more immigrants and increasing the population growth rate Pareto-improves welfare and, to a large extent, solves in a Pareto-improving way the problem of the under-accumulation of capital caused by implementing a PYGO social security system if a certain condition is met. More specifically, I first show, using the overlapping-generations model developed by Diamond (1965), that increasing the percentage of immigrants to natives (PITN) in an economy, with or without tax distortion, is Pareto-improving if there exist upward intergenerational transfers in the sense that the a young individuals pre-tax labor income, which is equal to labor supply times the marginal product of labor, is greater than the sum of resources that the individual consumes when he or she is young and the amount of resources that are transferred to future periods (the marginal product of labor (MPL) condition). In the presence of a PYGO social security system, some of the pre-tax labor income of a young individual in period t is used for the consumption of the old in period t. Thus, the above MPL condition is likely to be satisfied. In addition, the presence of government debt will make the MPL condition more likely. Second, I analytically show that, when this MPL condition is satisfied, the government can lead the economy to the (modified) golden rule level in a Pareto-improving way within a finite time by putting the government budget surplus, which is obtained by increasing the PITN, into savings. Note that when the economy reaches the golden rule level, the problem of under-accumulation of capital caused by PYGO social security is solved for all practical purposes. Third, I quantify the Pareto-improving welfare gain yielded by increasing the PITN in the presence of a PYGO social security system and calculate the number of years required to reach the (modified) golden rule level in a Pareto-improving way, using the computational overlapping-generations model developed by Auerbach and Kotlikoff (1987).[1] I consider a moderate increase in the PITN, such that the PITN starts to increase from 15.5 percent, reaches 25.5 percent in the 80th year, and remains constant at 25.5 percent in later years.[2]

With this rate of increase in the PITN and in the model that mimics the important dimensions of the US economy, my simulation shows it takes a minimum of 112 years for the model economy to reach the golden rule level in a Pareto-improving way. On the new balanced growth path, capital stock per efficient unit of labor increases by 102 percent and publicly provided private goods per capita increase by 36 percent. When the target capital stock is set at the modified golden rule level with a 3 percent intergenerational discount rate, it takes 65 years to reach the modified golden rule level in a Pareto-improving way with capital stock per efficient unit of labor increasing by 18 percent. The present discounted value (PDV) of Pareto-improved utility, measured by the expenditure function of natives and their descendants, which does not include the increased utility of immigrants and their descendants, comprises 23 percent of initial GDP.[3] When the time to reach the target PITN is shortened to 42 years, the economy reaches the modified golden rule level in the 59th year and the PDV of the Pareto improvement comprises 28 percent of initial GDP.

Finally, I conduct robustness checks by changing several parameter valuesthe share of the surplus for the government savings, replacement rate, time preference rate, risk aversion, initial government debt (asset) level, level of immigrants earnings, and

[1]The Matlab code used for this simulation is available from the journal's website and from the author.

[2]A 15.5 percent initial PITN is obtained by using census 2000 data from the author's calculation. See section 4.2 for a more detailed discussion.

[3]If I include the increased utility of immigrants and their descendants, the welfare gain of accepting immigrants becomes bigger.Thus, my estimate is likely to underestimate the welfare gain of accepting more immigrants. I calculate this number to analyze to what extent accepting more immigrants is Pareto-improving from the point of the natives.

the consumption of public services by each immigrant, for example. Those robustness checks show that the results of the simulation do not change substantially in magnitude for different parameter values. Both the theoretical results and the computational results suggest the robustness of the welfare gain of increasing the PITN in the presence of a PYGO social security system.

The above results can be intuitively understood as follows. Consider a situation where the government has a PYGO social security system initially and accepts immigrants at a fixed ratio to the number of natives. Now, assume that the government starts to accept additional immigrants, in addition to the initially planned quota of immigrants, in period t. When the government collects the social security tax from the additional immigrants in period t, the host country residents welfare is affected through two channels. First, the government does not need to use the tax revenue collected from newly admitted immigrants since the social security system is a pay-as-you-go service. Thus, the revenue from this social security tax is "free money"for the government. The government can invest this social security tax revenue in capital stock. Second, in period t+1 , the government does not need to use the return from investment made in period t . Since the social security benefit of the additionally accepted immigrants in period t can be partially or fully financed by the social security tax levied on the additionally accepted immigrants children, the government can reinvest, in period t+1, some of the return from the investment made in period t. Starting from a dynamically efficient allocation, this investment will grow faster than the population growth rate.

Note that in the case of the standard pre-funding argument of a PYGO social security, the double burden problem happens because the government does not have this kind of free money. For the government to have money to invest, the young cohort needs to pay two taxes: one for the benefit of the old and another for investment. However, when the government accepts additional immigrants, the government can use the available free money to solve the double burden problem of pre-funding social security.

There are several questions about the above intuition, however. First, it is not clear how the above argument holds when the capital-labor ratio is endogenous. When more immigrants are accepted, the capital-labor ratio and income per capita will decrease initially. In such a situation, it seems difficult to Pareto-improve welfare by accepting more immigrants. Second, it is not clear how to Pareto-improve welfare in the absence of lump-sum taxes and transfers. When immigrants are accepted, factor prices are changed, and there are winners and losers. In the absence of lump-sum taxes and transfers, it seems to be difficult to Pareto-improve welfare. Third, it might take quite a long time for the economy to increase capital stock per capita, although accepting additional immigrants can raise welfare in the long run. The following theoretical and simulation analysis answers these questions as well. The remainder of the paper is organized as follows. Section 2 reviews the related literature and clarifies the research contributions. Section 3 describes the theoretical analysis. Subsection 3.1 obtains the MPL condition and explains the underlying intuition of the MPL condition by analyzing a case where the government cannot use the lump-sum tax and transfer and where immigrants and natives have identical productivities and preferences. The MPL condition plays a critical role in other cases. Subsection 3.2 analyzes a case where the government cannot use the lump-sum tax and transfer. It also shows that the government can lead the economy to the golden rule in a Pareto-improving way by accepting more immigrants if the MPL condition is satisfied. Subsection 3.3 analyzes a case where immigrants and natives have different productivities and preferences. Section 4 presents a simulation-based analysis using the computational overlapping-generations model. Section 5 concludes the paper.

2 Literature review and research contributions

This study is related to several strands of the literature. First, the theoretical results presented in previous studies of social security pre-funding show that it is not possible to Pareto-improve all generations by pre-funding or transiting from a PYGO social security system to a funded system (Genakoplos et al., 1998; Belan and Pestieau, 1999). As a result, previous authors have focused on steady-state welfare-maximizing social security reform or policies that aim to balance the social security burden across cohorts in the presence of negative demographic shocks (Feldstein and Samwick, 1998; Kotlikoff et al., 1998; Nishiyama and Smetters, 2007). Second, this work is also related to public finance studies analyzing the effect of immigration in the presence of social security in a dynamic general equilibrium model (Storesletten, 1995, 2000; Collado et al., 2004; Schou, 2006; Attanasio et al., 2007).

Third, a large body of research has calculated the costs and benefits of accepting immigrants. The first generation of this research (i.e., static cost/benefit analyses) does not consider the fact that social security benefits are partially paid for by the children of immigrants (Borjas, 1999; Passel, 1994; Akbari, 1989; Bonin et al., 2000; Storesletten, 2003), whereas the second-generation research recognizes that the children of immigrants also contribute to the PYGO social security system (Lee and Miller, 1998; Auerbach and Oreopoulos, 1999). Fourth, there are theoretical studies that analyze the effect of immigration in the presence of a PYGO social security system. Razin and Sadka (2000) shows that accepting immigrants is unlikely to improve welfare when capital accumulation is endogenous. Kemnitz (2008) analyzes the effect of immigration when labor union exists. Casarico and Devillanova (2003) analyze when skill of natives can be upgraded. Preston (2014) conducts a comprehensive theoretical analysis. He concludes that the effect of accepting immigration depends on the several parameters and that it is difficult to draw a general conclusion.

This study contributes to the body of knowledge on this topic in several ways. First, I identify the conditions under which it is Pareto-improving to accept more immigrants (the MPL condition). This MPL condition can be checked by the data and has a natural economic interpretation. Thus, it can improve the situation described by Preston (2014). Second, I show that the government can lead the economy to the (modified) golden rule level by accepting more immigrants within a finite time in a Pareto-improving way. This finding is in sharp contrast to those of previous studies of social security reform, which argue that increasing capital stock by pre-funding social security in a Pareto-improving way is difficult (Genakoplos et al., 1998; Belan and Pestieau, 1999). Third, I quantify the welfare gain of the Pareto improvement predicted by the theoretical model. Consistent with the results of Storesletten (2000), I show a non-trivial welfare gain from increasing immigration into the United States and demonstrate that my results are robust for different parameter values. The theoretical results and robustness of the simulation results show that increasing immigration into the United States should be considered an important policy option.[4]

3 The model

The model proposed in this study uses the standard overlapping generation model with a neoclassical production function developed by Diamond (1965). Each individual lives for two periods. When individuals are in the first period, they work and are called

[4]There are several studies that analyzed the political factor of immigration policy such as Epstein and Nitzan (2006) and Razin et al. (2011). Taking political factors into the current analysis is beyond the scope of the current paper.

"young." When they are in the second period, they are retired and are called "old." I assume that immigrants arrive in the host country only when they are young and that the host government prohibits inward migration when immigrants are old. I define individuals born at the beginning of period t in the host country as natives of cohort t, regardless of the nationality of their parents. Immigrants who move to the host country at the beginning of period t are considered to be immigrants of cohort t. Let j be the index indicating nationality. If an individual is a native, $j = n$ and if he or she is an immigrant, $j = m$. Let N_t^j be the number of the young of type j in period t. Let $(c_t^{y,j}\ c_{t+1}^{o,j})$ be consumption in the young period and the old period of a type j $(j = n, m)$ individual of cohort t. Let $g_t^{y,j}$ be the amount of publicly provided private goods, such as education and government-provided health care services for the young, for each young individual of type j, which is consumed in period t. Let $g_t^{o,j}$ be the amount of publicly provided private goods such as Medicaid and publicly provided nursing homes for each old individual of type j in period t. Let $g_t^{ind,j}$ be the amount of age-independent publicly provided private goods consumed by a young individual and an old individual of type j.[5] I assume that the utility function of the cohort of type j is

$$
\begin{aligned}
U^j(c_t^{yj}, l_t^j, g_t^{yj}, g_t^{ind}, c_{t+1}^{oj}, g_{t+1}^{oj}, g_{t+1}^{ind,j}) &= u^{yj}(c_t^{y,j}, l_t^j) + v^{yj}(g_t^{yj}, g_t^{ind,j}) \\
&+ \frac{1}{1+\rho}[u^{oj}(c_{t+1}^{o,j}) + v^{oj}(g_{t+1}^{o,j}, g_{t+1}^{ind,j})] \ . \qquad (1)
\end{aligned}
$$

Further, I assume that $u^{ij}(c_t^{i,j}, l_t^j)$ and $v^i(g_t^{ij}, g_t^{ind,j})$ $(i = y, o; j = n, m)$ are strictly increasing and concave functions. I assume the additive separability of publicly provided private goods to ensure that the provision of such goods does not affect the consumption and saving decisions of individuals.

On the production side, let $F(L_t, K_t)$ be a production function, where L_t and K_t are the total amount of labor and the total capital stock used in period t. Let δ be the capital depreciation rate. I assume that $F(L_t, K_t)$ displays constant returns to scale and that both the MPL and MPK are diminishing. I also assume that the standard Inada condition is satisfied.

The economy is at the steady state initially and the initial economy is dynamically

[5]In this study, I ignore non-rival public goods, whose presence would favor immigration because accepting immigrants means that the costs of such goods are shared by more individuals without decreasing their consumption.

efficient.[6] Furthermore, for the welfare analysis of accepting more immigrants, I make the following additional assumptions:

AS1: The government uses a PYGO social security system at the initial steady state.

AS2: For a one-unit supply of labor by a native, ϕ^n efficient units of labor are supplied. For a one-unit supply of labor by an immigrant, ϕ^m efficient units of labor are supplied where $\phi^m \leq \phi^n$. For normalization, I assume that $\phi^n = 1$.

AS3: The descendants of immigrants integrate with the native population and earn the same income as natives.

AS4: Immigrants and their children stay permanently in the host country.

AS5: The fertility rate of immigrants is equal to or higher than the fertility rate of natives.

AS6: If immigrants and natives have the same productivities, then the government treats them in the same way in the tax and social security system.

I need AS1 to assess the effect of increasing the number of immigrants in the presence of a PYGO social security system. AS3 and AS4 are relaxed in the simulation analysis. AS6 needs more discussion. Clearly, if the government can treat immigrants in a discriminatory way in the tax and public pension systems, the utility of both natives and immigrants could be increased. The wage rate of immigrants in their countries of origin is typically lower than that in the host country. Thus, it is possible to Pareto-improve the welfare of both natives and immigrants if (a) the host government places a high tax burden on incoming immigrants in such a way that the net wage rate of immigrants in the host country is still higher than that in their countries of origin and (b) the government redistributes to natives the tax revenue collected from immigrants. AS6 precludes such an obvious case from occurring.

At the initial steady state, I assume that the amount of publicly provided private goods per person $(g_t^{yj}, g_t^{oj}, g_t^{ind,j})$ is constant and that $(g_t^{y,j}, g_t^{o,j}, g_t^{ind,j}) = (g^{y,j}, g^{o,j}, g^{ind,j})$. Let F_K and F_L be the partial derivative of the production function with respect to capital and labor. Let w_t^j and r_t be the wage rate of an individual of type j and the interest rate in period t. Let s_t^j and a_t be the amount of savings made by cohort t of type j and the total amount of government savings divided by the number of cohorts t. Then, w_t^j

[6] In the literature, it is well known that if the market interest rate is lower than the population growth rate, it is possible to Pareto-improve welfare (dynamic inefficiency). Since this study's interest is not such a dynamic inefficiency problem, I postulate that at the initial steady state, the market interest rate is higher than the population growth rate (Cass (1972)).

and r_t are determined as follows:

$$w_t^n = F_L(L_t, K_t), w_t^m = \phi^m F_L(L_t, K_t), r_t = F_K(L_t, K_t) - \delta$$
$$\text{where } L_t = \sum_{j=n,m} \phi^j l_t^j N_t^j \text{ and } K_t = \sum_{j=n,m} s_{t-1}^j N_{t-1}^j + a_{t-1} \sum_{j=n,m} N_{t-1}^j. \tag{2}$$

The resource constraint in period t is as follows:

$$F(L_t, K_t) + (1-\delta)K_t$$
$$= \sum_{j=n,m} \{c_t^{y,j} + s_t^j + g^{y.j} + g^{ind.j} + a_t\} N_t^j + \sum_{j=n,m} N_{t-1}^j \{c_t^{o,j} + g^{o.j} + g^{ind.j}\}. \tag{3}$$

Let the fertility rates of natives and immigrants be π_n and π_m, respectively. Given π_n and π_m, N_t^n can be written as follows:

$$N_t^n = (1+\pi_m) \times N_{t-1}^m + (1+\pi_n) \times N_{t-1}^n. \tag{4}$$

Let α_t be the immigrants-to-natives ratio (INR) of cohort t.[7] The immigration policy is expressed in terms of α_t. For example, a one-time increase in the INR means that $\alpha_0 = \alpha^*$, $\alpha_1 = \widetilde{\alpha}$, and $\alpha_t = \alpha^*$ for $t \geq 2$, where $\widetilde{\alpha} > \alpha^*$. Permanently increasing the INR means that $\alpha_0 = \alpha^*$ and $\alpha_t = \widetilde{\alpha}$ for all $t \geq 1$, where $\widetilde{\alpha} > \alpha^*$. The sum of natives and immigrants in cohort t is

$$\begin{aligned}\sum_{j=n,m} N_t^j &= N_t^n \times (1+\alpha_t) \\ &= \{(1+\pi_m) \times N_{t-1}^m + (1+\pi_n) \times N_{t-1}^n\} \times (1+\alpha_t) \\ &= \{(1+\pi_m) \times \alpha_{t-1} + (1+\pi_n)\} \times N_{t-1}^n (1+\alpha_t).\end{aligned} \tag{5}$$

Note that the total size of cohort $t-1$ is $N_{t-1}^n(1+\alpha_{t-1})$. Thus, at the steady-state immigration policy α^*, the growth rate of cohorts is $(1+\pi_m) \times \alpha^* + (1+\pi_n) - 1$. We

[7]Note that INR of cohort t is the ratio of the number of immigrants of cohort t over the number of natives of cohort t while PITN at period t is the percentage of the total number of immigrants at period t over the total number of natives at period t, not cohort t. In the simulation analysis, I use PITN because calculating PITN involves less measurement errors. But for the theoretical analysis, use INR is more convenient.

define $R(\alpha)$ as follows:

$$R(\alpha) \equiv (1+\pi_m) \times \alpha + (1+\pi_n). \tag{6}$$

We can thus interpret $R(\alpha)$ as one plus the cohort population growth rate when the immigration policy α is implemented. To avoid a situation in which the total population in period t becomes zero or negative, I assume

$$R(\alpha_{t-1}) \equiv (1+\pi_m) \times \alpha_{t-1} + (1+\pi_n) > 0 \;\text{ for } \alpha_{t-1} \geq 0. \tag{7}$$

At the golden rule level, the government sets the MPK equal to the population growth rate and depreciation rate. Thus, I assume that

$$R(\alpha_{t-1}) - 1 + \delta > 0. \tag{8}$$

By using $R(\alpha_{t-1})$, N_t^n and N_t^m can be denoted as

$$N_t^n = R(\alpha_{t-1})N_{t-1}^n \;\text{ and } N_t^m = R(\alpha_{t-1})N_{t-1}^n \alpha_t. \tag{9}$$

3.1 An Economy with Lump-sum Tax and Identical Productivities and Preferences: Deriving the MPL Condition

In this subsection, we derive the MPL condition. This MPL condition plays an important role when we analyze an economy where there are with distorting taxes and natives and immigrants have different productivities and preferences. To clarify the economic meaning of the MPL condition, in this subsection I assume that the government can use the lump-sum tax and transfer and that natives and immigrants have identical productivities and preferences. Those two assumptions are not needed for my results, but they help illustrate the underlying intuition of the MPL condition. Readers can skip this subsection and go directly to subsections 3.2–3.4 to find the analysis of the economy where immigrants and natives show different productivities, consumption of public services, and preferences and where the lump-sum tax is not available.

Let b_{t+1}^j be the social security benefit for type j of cohort t in period $t+1$ and τ_t^j the lump-sum tax in period t for type j of cohort t. The assumption of identical productivities and preferences for natives and immigrants and AS7 imply that $\phi^j, w_t^j, c_t^{yj}, c_{t+1}^{oj} l_t^j, s_t^j$, b_{t+1}^j and τ_t^j do not change for different values of j. Thus, we eliminate the superscript j from those variables and from the utility functions. Cohort t maximizes the lifetime utility function subject to the budget constraint. The budget constraint of cohort t of type j is

$$w_t l_t - \tau_t = c_t^y + s_t \text{ and } b_{t+1} + (1 + r_{t+1})s_t = c_{t+1}^o. \tag{10}$$

The government budget constraint in period t is

$$(\tau_t - g^y - g^{ind} - a_t) \sum_{j=n,m} N_t^j - (b_t + g^o + g^{ind} - (1 + r_t)a_{t-1}) \sum_{j=n,m} N_{t-1}^j = 0. \tag{11}$$

By using the individual budget constraint, the homogeneity of the production function, and equation (2), it is straightforward to show that the government budget constraint is equivalent to the following resource constraint:[8]

$$\begin{gathered} F(L_t, K_t) + (1 - \delta)K_t \geq \\ \{c_t^y + s_t + g^y + g^{ind} + a_t\} \sum_{j=n,m} N_t^j + \{c_t^o + g^o + g^{ind}\} \sum_{j=n,m} N_{t-1}^j \\ \text{where } L_t = \sum_{j=n,m} l_t N_t^j \text{ and } K_t = (s_{t-1} + a_{t-1}) \sum_{j=n,m} N_{t-1}^j. \end{gathered} \tag{12}$$

From (5), the above resource constraint can be rewritten as

$$\begin{gathered} F(L_t, K_t) + (1 - \delta)K_t \geq \\ \{c_t^y + s_t + g^y + g^{ind} + a_t\} N_{t-1}^n R(\alpha_{t-1})(1 + \alpha_t) + \{c_t^o + g^o + g^{ind}\} N_{t-1}^n (1 + \alpha_{t-1}). \end{gathered} \tag{13}$$

Before analyzing the effect of accepting more immigrants, we characterize the initial steady state. Let w^* and r^* be the wage rate and the interest rate at the initial steady state, respectively, where the immigration policy at the initial steady state is α^* for all t. Let s^* and l^* be the amount of savings of each individual and labor supply at the initial steady state, while a^* and b^* represent government savings (or debt if this value is negative) divided by the number of young individuals and the social security benefit at the initial steady state. The government will choose the steady-state lump-sum tax policy τ^* so that it satisfies its budget constraint. This approach implies that at b^* and

[8] By using the homogeneity of the production function, we have $F(L_t, K_t) + (1 - \delta)K_t = F_k K_t + F_L L_t + (1 - \delta)K_t$. Then, from (2), $F(L_t, K_t) = (r_t + \delta)K_t + w_t L_t + (1 - \delta)K_t$. By using the definition of K_t and L_t and the individual budget constraint, we have (12).

τ^*, the resource constraint must be satisfied. Conversely, when the resource constraint is satisfied, then the government budget constraint is also satisfied. c^{y*} and c^{o*} denote the steady-state consumption of a young individual and an old individual, respectively.

The initial steady-state economy with the steady-state immigration policy α^* is characterized as follows:

$$(s^*, l^*) = \arg\max_{s,l} u^y(w^*l - \tau^* - s, l) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho}[u^o((1+r^*)s + b^*) + v^o(g^o, g^{ind})] \tag{14}$$

$$\text{where } w^* = \frac{\partial F(L^*, K^*)}{\partial L},\ r^* + \delta = \frac{\partial F(L^*, K^*)}{\partial K} \tag{15}$$

$$L^* = l^* R(\alpha^*) N_0^{n*}(1+\alpha^*) \text{ and } K^* = (s^* + a^*) \times N_0^{n*}(1+\alpha^*) \tag{16}$$

$$\begin{aligned} F(K^*, L^*) + (1-\delta)K^* = \{&c^{y*} + s^* + g^y + g^{ind} + a^*\}R(\alpha^*)N_0^{n*}(1+\alpha^*) \\ &+\{c^{o*} + g^o + g^{ind}\}N_0^{n*}(1+\alpha^*) \end{aligned} \tag{17}$$

$$c^{y*} = w^*l^* - \tau^* - s^* \text{ and } c^{o*} = (1+r^*)s^* + b^* \tag{18}$$

$$N_0^{n*} \text{ is some positive number.}$$

The utility level at the initial steady state is defined as follows:

$$u^* \equiv u^y(c^{y*}) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho}[u^o(c^{o*}) + v^o(g^o, g^{ind})]. \tag{19}$$

3.1.1 Welfare Effect of Increasing the INR

In this subsection, I examine, starting from period 1, whether increasing the INR permanently Pareto-improves welfare. Increasing the INR permanently is defined as $\alpha_0 = \alpha^*$ and $\alpha_t = \alpha$, where $\alpha > \alpha^*$ for $t \geq 1$. For the analysis, consider the following constrained maximization problem (CMP), which is a function of α:

$$\text{CMP}: \quad V(\alpha) = \max_{\{c_t^y, c_t^o, s_t,\ a_t | t=1,2,..\}} \frac{1}{1+\rho}[u^o(c_1^o) + v^o(g_o, g^{ind})]$$

$$\text{s.t. } u^y(c_t^y, l_t) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho}[u^o(c_{t+1}^o) + v^o(g^o, g^{ind})] \geq u^* \text{ for } t = 1, 2, ... \quad (20)$$

$$F(L_t, K_t) + (1-\delta)K_t \geq$$
$$\{c_t^y + s_t + g^y + g^{ind} + a_t\} N_{t-1}^n R(\alpha_{t-1})(1+\alpha_t) + \{c_t^o + g^o + g^{ind}\}\ N_{t-1}^n(1+\alpha_{t-1}) \text{ for } t = 1, 2, ... \quad (21)$$

$$K_t = (s_{t-1} + a_{t-1}) N_{t-1}^n (1+\alpha_{t-1}) \text{ for } t = 2, ... \text{ and } s_0 = s^* \text{ and } a_0 = a^*$$
$$L_t = l_t N_{t-1}^n R(\alpha_{t-1})(1+\alpha_t) \text{ and (9)} \quad (22)$$
$$\alpha \text{ and } \alpha^* \text{ are given.}$$

The above CMP deserves several comments. First, $V(\alpha)$ is the utility of cohort 0 in period 1 when the government accepts immigrants at a constant ratio α^* at the initial steady state and starts to accept immigrants at the ratio α from period 1. Because the consumption of cohort 0 in the young period is already determined in period 1, it is excluded from the above programming problem. Second, the first constraint is related to Pareto improvement and this requires that all cohorts except cohort 0 must have at least the same utility as they would have at the initial steady state. Note that in the first constraint, there are α and α^*. From period 1, the immigration policy in period 0 is predetermined. Such a variable is denoted as α^*, while the policy determined in period 1 or later is denoted as α.

Note that N_t^n is determined by α and N_{t-1}^n, but N_{t-1}^n is also affected by α and N_{t-2}^n. This fact implies that a change in immigration policy α affects all N_t^n for $t = 1, 2, ...$ To simplify the calculation, it is useful to divide the resource constraint by N_{t-1}^n when $t = 1$ and by $N_{t-1}^n(1+\alpha)$ when $t = 2, 3, 4....$[9] Then, (21) becomes as follows:

$$F(R(\alpha^*)(1+\alpha)l_1, (s^* + a^*)(1+\alpha^*)) + (1-\delta)(s^* + a^*)(1+\alpha^*) \geq$$
$$\{c_1^y + s_1 + g^y + g^{ind} + a_1\} R(\alpha^*)(1+\alpha) + \{c_1^o + g^o + g^{ind}\}\ (1+\alpha^*) \text{ for } t = 1 \quad (23)$$

$$F(R(\alpha)l_t, s_{t-1} + a_{t-1}) + (1-\delta)(s_{t-1} + a_{t-1}) \geq$$
$$\{c_t^y + s_t + g^y + g^{ind} + a_t\} \times R(\alpha) + \{c_t^o + g^o + g^{ind}\} \text{ for } t = 2, 3, 4, ... \quad (24)$$

[9]We divide the resource constraint in period 1 by N_{t-1}^n, not by $N_{t-1}^n(1+\alpha)$, because the old population in period 1 is $N_{t-1}^n(1+\alpha^*)$, not $N_{t-1}^n(1+\alpha)$.

Let γ_t and λ_t be the Lagrangian multipliers of the minimum utility constraint (20) and the resource constraints (23) and (24). Let γ_t^* and λ_t^* be the Lagrangian multipliers when $\alpha = \alpha^*$. Then, we have the following observation.

Observation 1

When $\alpha = \alpha^*$ the solution of CMP is

$$c_t^y = c^{y*},\ c_0^o\ = c^{o*}\ ,\ s_t = s^*,\ a_t = a^*\ l_t = l^* \text{for } t = 1, 2, ... \tag{25}$$

$$\lambda_1^* = \frac{1}{1+\rho} u_c^o(c_1^o) \text{ and } \lambda_{t+1}^* = \frac{R(\alpha^*)}{1+r^*} \lambda_t^* \tag{26}$$

$$\gamma_t^* = \frac{1}{u_c^y(c^{y*}, l^*)} \lambda_t^* R(\alpha^*) \text{ and for } t = 1, 2, ... \tag{27}$$

For the proof of Observation 1, see Appendix B1.

Observation 1 implies that when the INR α is fixed at α^*, the initial steady-state allocation is Pareto-efficient and that it is not possible to have Pareto improvement from the initial steady-state holding $\alpha = \alpha^*$. Now, suppose that the government increases the INR from α^*. Whether such an increase in the INR Pareto-improves welfare can be analyzed by calculating $dV/d\alpha$ and evaluating it at $\alpha = \alpha^*$ because we put the minimum utility constraint (20) into the programming problem and because the utility level of cohort 0 is u^* at α^*. According to the envelope theorem, $dV/d\alpha|_{\alpha=\alpha^*}$ is equal to

$$dV/d\alpha|_{\alpha=\alpha^*} = \left\{ R(\alpha^*)\lambda_1^* + \sum_{t=2}^{\infty} \lambda_t^* R'(\alpha^*) \right\}$$
$$\times \left\{ \underbrace{F_L(R(\alpha^*), s^* + a^*)l^*}_{\text{contribution of the young}} - \underbrace{(c^{y*} + g^y + g^{ind} + s^* + a^*)}_{\text{resource used for the young or future cohorts}} \right\}. \tag{28}$$

where $R(\alpha)$is one plus population growth rate and defined in (6.

The first bracket is positive because the Lagrangian multiplier of the resource constraint is positive and the marginal effect of increasing α on one plus the population growth rate is positive. In the second bracket, the first term is the MPL multiplied by labor supply, which represents an individual's contribution to the economy when he or she is young at the initial steady state. It is also the pre-tax labor income of a young individual. When a young individual contributes $F_L l^*$ to the economy, the government has three choices when deciding how to distribute this contribution. The first choice is to let the young individual consume this contribution. The second choice is to transfer this contribution to future periods and let this young individual or his/her descendants consume in future periods. The third choice is to transfer this contribution to old individuals. $c^{y*}+ g^{y} + g^{ind}$ is the amount of the resource consumed by the current young individual when he or she is young. s^{*} is the amount of the resource consumed by this individual when he or she becomes old. a^{*} is the amount of the resource transferred to future periods. Note that $c^{y*}+ g^{y} + g^{ind} + s^{*} + a^{*}$ does not include c^{o*} and g^{o}. Thus, $\{F_L l^{*} - c^{y*} - g^{y} - g^{ind} - s^{*} - a^{*}\}$ is the amount of the resource transferred to old individuals at the initial steady state. We call this amount the "upward intergenerational transfer."

Definition: When $F_L l^{*} - c^{y*} - g^{y} - g^{ind} - s^{*} - a^{*}$ is positive, the MPL condition is satisfied and the amount $F_L l^{*} - c^{y*} - g^{y} - g^{ind} - s^{*} - a^{*}$ represents the upward intergenerational transfer.

Proposition 1 (MPL condition version) *If there exist upward intergenerational transfers, in the sense that the pre-tax labor income of a young individual is greater than the sum of the resources that he or she consumes when young and the amount of resources transferred to future periods, then accepting more immigrants Pareto-improves the welfare of all generations.*[10]

Because $c^{y*} + s^{*}$ is the after-tax income of the young by definition, $F_L l^{*} - c^{y*} - s^{*}$ is the amount of tax paid by a young individual. Moreover, $g^{y} + g^{ind} + a^{*}$ is the sum of the government resources provided to a current young individual and the amount of resources transferred to future cohorts per young individual. Thus, we have the following corollary:

Corollary (Tax expenditure version) *Alternatively, if the amount of tax paid by the young is greater than the amount of government resources provided to a current young individual and the amount of resources transferred to future cohorts per young individual at the initial steady state, accepting more immigrants Pareto-improves welfare.*

This MPL condition plays a critical role in the analysis not only of the case where the government has access to the lump-sum tax but also of the case where the government

[10]Note that this condition does not change even in the presence of public goods because an increase in the number of immigrants does not affect the consumption of public goods owing to their "public" nature.

has no such access. In addition, in the simulation analysis, this MPL condition is crucial. The intuition of Proposition 1 is as follows. When the MPL condition is satisfied, upward intergenerational transfers are made at the initial steady state. $F_L l^* - c^{y*} - g^y - g^{ind} - s^* - a^*$ is transferred from a young individual to an old individual at the initial steady state. However, when additional immigrants are accepted in period t, $F_L l^* - c^{y*} - g^y - g^{ind} - s^* - a^*$ will become"free money" for the government since the public pension granted to this group of additional immigrants is paid for by their children. Thus, the government can invest this free money and use it later to increase the welfare of all later generations.

The tax expenditure condition also has an important implication for the cost/benefit analysis of accepting immigrants. Note that for the government expenditure part, the social security benefit and publicly provided private goods for the old are not included in the tax expenditure condition. Only the tax that young immigrants pay and the resources used for the young or future cohorts should be included.[11][12] Graphically, Proposition 1 is explained in Appendix A1.

[11]When immigrants and natives have different preferences and productivities, then the sum of the redistribution from immigrants to natives and the MPL condition become important. See section 3.3.

[12]Traditional studies of the fiscal effect of accepting more immigrants calculate the present value of government expenditure such as the consumption of publicly provided private goods and the social security benefit that immigrants receive and tax revenue (including income tax and social security tax) they pay. However, if the social security benefit that retired immigrants receive is included in the cost/benefit calculation, then the social security tax that their children pay should also be included because the social security benefits are financed by the social security tax that the children of natives and immigrants pay. Of course, if the social security tax that the children of immigrants pay is included, then the social security benefit that the children of the immigrants receive should also be included. Again, if the social security benefit the children of immigrants receive is included, then the social security tax that the grandchildren of immigrants receive should be included. Note that in the PYGO social security system, the social security benefit that immigrants receive is roughly balanced by the social security tax paid by their children. This fact implies that in the cost/benefit calculation, the social security benefit that immigrants receive is roughly canceled out by the social security tax their children pay. Thus, in the cost/benefit calculation, only the tax that young immigrants pay and the publicly provided private goods for young immigrants should be included.

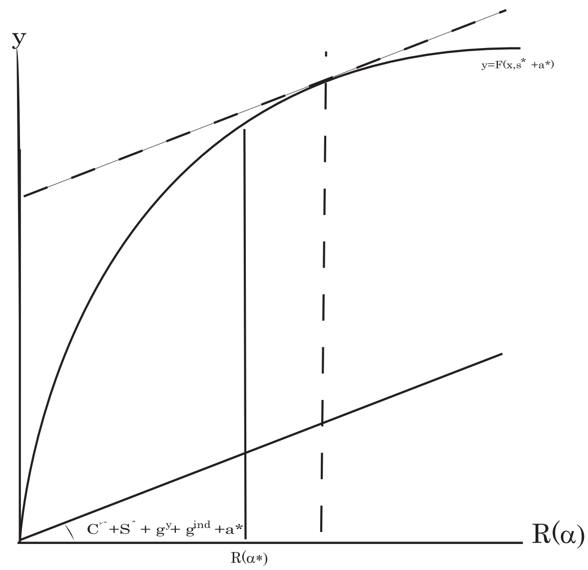

Figure A1: The population of the young and resources used for one old person when the number of population of the old is one.

The horizontal axis measures $R(\alpha)$. The curve through the origin is $y = F(R(\alpha), s^* + a^*)$and the straight line is $y = (c^{y*} + s^* + g^y + g^{ind} + a^*)R(\alpha)$. The vertical distance of the straight line through the origin measures the total resources used for the young divided the number of old people. The curved line measures the total output divided the number of the old. Thus, the difference between the curved line and the straight line measures the resources used per one old person at $R(\alpha)$. This figure shows that increasing $R(\alpha)$ will increase the resources available per old person without decreasing the resources used for the young.

3.2 Presence of Distorting Taxes and Implementation of Pareto Improvement

In the preceding subsection, I assumed that the lump-sum tax is available. Under a neoclassical production function that displays a diminishing marginal product, when more immigrants are accepted, the pre-tax wage will decrease and pre-tax interest rate will increase. In the absence of the lump-sum transfer, it is unclear whether it is possible to Pareto-improve all generations.

In this subsection, I show that it is possible to Pareto-improve welfare without changing the incentives of individuals, by increasing the INR in the absence of the lump-sum transfer as long as the MPL condition is satisfied. In addition, I show that a relatively simple adjustment of taxes (wage tax and interest tax) and social security benefit achieves this Pareto improvement when the government increases the INR. The technique presented in this subsection was developed by Dixit and Norman (1980) to show the superiority of free trade over autarky. I hence apply their technique to a dynamic economic model.

For the analysis, let $\widetilde{\alpha}$ be a time-invariant new immigration policy from period 1 where $\widetilde{\alpha} > \alpha^*$. As in the previous section, I assume that the economy is dynamically efficient at the initial steady state. This fact implies that

$$F_K(R(\alpha^*), s^* + a^*) > \delta + R(\alpha^*) - 1. \tag{29}$$

As for taxes, I assume that the government uses a capital income tax and a wage tax at the initial steady state. I assume that those taxes do not need to be the second best optimal. Let τ_{wt} and τ_{rt} be the wage tax rate and capital income tax rate in period t. Then, the individual budget constraint (10) is modified as follows:

$$w_t l_t (1 - \tau_{wt}) = c_t^y + s_t \text{ and } b_{t+1} + (1 + (1 - \tau_{rt+1}) r_{t+1}) s_t = c_{t+1}^o \ .$$

Let τ_w^* and τ_r^* be the wage tax rate and capital income tax rate at the initial steady state. At the initial steady state, the above budget constraints become

$$w^* l^* (1 - \tau_w^*) = c^{y^*} + s^* \text{ and } b^* + (1 + (1 - \tau_r^*) r^*) s^* = c^{o*}.$$

I assume that at the initial steady state the social security benefit is proportional to pre-tax earnings:

$$b^* = \Omega \times w^* l^*. \tag{30}$$

When the government increases the INR, the wage rate falls and interest rate increases initially. To achieve Pareto improvement by increasing the INR, first I assume that the government sets the tax rates such that the after-tax wage and interest rate after an increase in the INR are equal to the after-tax wage rate and interest rate at the initial steady state. Then, the wage tax rate and interest tax rate for period t are set as follows:

$$w_t(1-\tau_{wt}) = w^*(1-\tau_w^*) \text{ and } r_t(1-\tau_{rt}) = r^*(1-\tau_r^*). \tag{31}$$

Second, I assume that the government re-scales Ω so that the social security benefit becomes proportional to after-tax earnings, not pre-tax earnings, and that an individual receives the same benefit when the wage rate, the wage tax rate, and labor supply are at the same level as at the initial steady state. This fact implies that

$$b_{t+1} = \frac{\Omega}{1-\tau_w^*} w_t(1-\tau_{wt}) l_t. \tag{32}$$

Note that $b_{t+1} = \Omega w^* l_t$ when $w_t(1-\tau_{wt}) = w^*(1-\tau_w^*)$.

When the government sets taxes and social security benefit in this way, saving and labor supply behavior does not change because the budget constraint of a consumer in any period t is the same as at the initial steady state. This fact implies that the equilibrium social security benefit in any period t is the same as at the initial steady state. If the government provides at least the same level of publicly provided private goods, the utility levels of all cohorts are at least the same as at the initial steady state.

As for the extent of the increased immigration, motivated by Proposition 1, I assume that the MPL condition is satisfied at the initial steady state:

$$l^* F_L(l^* R(\alpha^*), s^* + a^*) > c^{y*} + g^y + g^{ind} + s^* + a^*. \tag{33}$$

The result presented in the preceding subsection shows that a marginal increase in the number of immigrants Pareto-improves welfare if the MPL condition is satisfied. However, this result does not imply that an unlimited acceptance of immigrants always Pareto-improves welfare. I impose two conditions on the extent to which immigrants are accepted. The first condition is regarding the MPL condition when the new immigration policy $\widetilde{\alpha}$ is implemented. I assume that when $\widetilde{\alpha}$ is operative and when government savings per young individual are held constant, the marginal increase in output due to a one-unit increase in the population is greater than or equal to the resources an individual receives when he or she is young at the initial steady state. This fact implies that

$$l^* F_L(l^*(1+\widetilde{\alpha})((1+\pi_n)N_{t-1}^n+(1+\pi_m)N_{t-1}^m),(s^*+a^*)(N_{t-1}^n+N_{t-1}^m)) \geq c^{y*}+g^y+g^{ind}+s^*+a^* \tag{34}$$

for $t = 1, 2, 3....$ Note that $(1 + \pi_m) \times N_{t-1}^m + (1 + \pi_n)N_{t-1}^n$ is the population of young natives in period t. For $t = 2, 3, 4...$, by using the homogeneity of the production function and $N_{t-1}^m = \widetilde{\alpha} N_{t-1}^n$, (34) can be written as

$$l^* F_L(l^*(R(\widetilde{\alpha}), s^* + a^*) \geq c^{y*} + s^* + g^y + g^{ind} + a^* \text{ for } t = 2, 3, 4... \tag{35}$$

The second condition is regarding the golden rule level. Note that we have assumed that at the initial steady state, (29) holds. As R in (29) increases, both the right hand side and the left hand side of (29) increase. However, the golden rule may be binding even if the balance of government savings per young individual is held constant. This binding is more likely when the MPK increases to a lower extent than the increase of the population growth rate holding capital stock per capita constant. In this case, the following golden rule is satisfied at $\widetilde{\alpha}$:

$$F_K(R(\widetilde{\alpha}), s^* + a^*) = \delta + R(\widetilde{\alpha}) - 1. \tag{36}$$

On other other hand, when capital stock per capita at the initial steady state is sufficiently lower than the golden rule level, the MPL condition (35) binds first instead of the golden rule (36) when the government accepts more immigrants. In this case, the following condition holds at $\widetilde{\alpha}$ when the balance of government savings per young individual is held at a^*:

$$F_K(R(\widetilde{\alpha}), s^* + a^*) > \delta + R(\widetilde{\alpha}) - 1. \tag{37}$$

If (36) holds, then the government does not need to increase its savings to reach the golden rule and the analysis becomes trivial. Thus, in the following analysis, I assume that (37) holds instead of (36).

When the government accepts more immigrants, it can increase its savings balance as shown below, implying that, at some point, the MPK may become equal to the golden rule level of capital stock per capita as a_t increases from a^*. However, when the MPK equals the golden rule level, it is clearly better to use the entire government surplus to increase the supply of publicly provided private goods rather than to increase the balance of government savings. Thus, I assume that as long as the MPK is higher than the golden rule level, the government uses some of its budget surplus to increase its savings and the remainder to increase the supply of publicly provided private goods. When the MPK reaches the golden rule level, the government uses the entire surplus to increase the supply of publicly provided private goods. Thus, we have the following MPK condition:

$$F_K(R(\widetilde{\alpha}), s^* + a_t) \geq \delta + R(\widetilde{\alpha}) - 1 \quad \text{where } a_t > a^*. \tag{38}$$

Now, we examine whether the social security benefit and taxes determined by (32) and (31) are feasible from the point of view of the government budget constraint. To check the feasibility of such taxes, consider the net government budget surplus for period 1, SP_1:

$$SP_1 = (w_1\tau_{w1}l^* - g^y - g^{ind} - a^*) \sum_{j=n,m} N_1^j + (r_1\tau_{r1}s^* - b^* - g^o - g^{ind}) \sum_{j=n,m} N_0^j + (1+r_1)a^* \sum_{j=n,m} N_0^j. \tag{39}$$

Note that τ_{w1} and τ_{r1} are defined in (31). Further, I assume that the government will save at least the same amount of government savings per young individual as at the initial steady state. By substituting τ_{w1} and τ_{r1} into SP_1 and using the homogeneity of the production function, we have (see Appendix B2)

$$SP_1 = N_1^n \int_{1+\alpha^*}^{1+\widetilde{\alpha}} F_L(N_1^n l^* z, (s^* + a^*)N_0^n(1+\alpha^*))l^* - c^{y*} - s^* - g^y - g^{ind} - a^*]dz, \tag{40}$$

where $N_1^n = N_0^n R(\alpha^*)$.

Thus, from the MPL conditions (33) and (34), the inside of the integration is positive for $z \in [1+\alpha^*, 1+\widetilde{\alpha}]$. This fact means that this tax plan is feasible in period 1. The government can use some of its budget surplus to increase the supply of publicly provided private goods and save the remainder. Let a_1 be the balance of government savings per young individual at the end of period 1, where $a_1 > a^*$. What will be the net budget surplus in period 2, SP_2?

Note that the natives and immigrants in cohort 1 will save the same amount as those in the cohort at the initial steady state, because under the proposed tax policy they face the same after-tax wage rate and interest rate as at the initial steady state. This fact implies that $s_1 = s^*$ for both natives and immigrants in cohort 1. Assume that in period 2, the government will save at least a^* per young individual. Thus, SP_2 becomes

$$SP_2 = \sum_{j=n,m} N_2^j \times (w_2\tau_{w2} - g^y - g^{ind} - a^*) + \sum_{j=n,m} N_1^j \times (r_2\tau_{r2}s^* - b^* - g^o - g^{ind}) + (1+r_2)a_1 \sum_{j=n,m} N_1^j. \tag{41}$$

Again, the government sets τ_{w2} and τ_{r2} such that the after-tax wage rate and after-tax interest rate become the same as at the initial steady state. Thus, SP_2 becomes (see Appendix B3)

$$\begin{aligned} SP_2 = {} & N_1^n(1+\widetilde{\alpha}) \int_{s^*+a^*}^{s^*+a_1} [F_K(l^*R(\widetilde{\alpha}), z) + 1 - \delta] dz \\ & + (1+\widetilde{\alpha})N_1^n \int_{\alpha^*}^{\widetilde{\alpha}} R'(\alpha) \left[F_L(l^*R(z),\ (s^*+a^*))l^* \right. \\ & \left. -\{c^{y*} + s^* + g^y + g^{ind} + a^*\}\right] dz. \end{aligned} \tag{42}$$

The first term of (42) measures the welfare gain that arises from the additional savings that the government accumulates at the end of period 1. The second term measures the welfare gain that arises from the increased population growth rate in the presence of the PYGO social security system. From the MPK condition (38) and the condition on the population growth rate, the inside of the first integration is positive. Moreover, from (35), the inside of the second integration is positive. Thus, SP_2 is positive and the government can implement the proposed tax policy. Again, at the end of period 2, the government can use some of the above surplus to increase the supply of publicly provided private goods and allocate the rest to increase the balance of government savings. Similarly, the government surplus for period t becomes

$$SP_t = N^n_{t-1}(1+\widetilde{\alpha})\int_{s^*+a^*}^{s^*+a_{t-1}}[F_K(l^*R(\widetilde{\alpha}), z)+1-\delta]dz$$
$$+(1+\widetilde{\alpha})N^n_{t-1}\int_{\alpha^*}^{\widetilde{\alpha}}R'(\alpha)[F_L(l^*R(z),\ (s^*+a^*))l^*$$
$$-\{c^{y*}+s^*+g^y+g^{ind}+a^*\}]dz, \tag{43}$$

where a_{t-1} is the balance of government savings per young individual at the end of period $t-1$. This fact implies that $SP_t > 0$ for all $t = 1, 2, \ldots$. Thus, we have the following proposition:

Proposition 2. *Consider an economy in which the wage and interest taxes are used at the initial steady state. If the MPL condition is satisfied at the initial steady state, accepting more immigrants with tax rule (31) and social security benefit rule (32) Pareto-improves the welfare of all generations.*

3.2.1 Government Savings and the Golden Rule

In this subsection, I examine the path of capital stock and the balance of government savings when the government increases the INR. To examine the government savings path, I need to specify how much of the government surplus, SP_t, is placed into the additional government savings. For the analysis, I assume that the surplus that arises from the increased balance of government savings in period $t-1$, which is the first integration of SP_t, is put into the additional government savings added to a^* in period t. Note that the government could use some part of the second integration in SP_t, the surplus generated directly from the increased immigration. Thus, my assumption is a conservative value for government savings. However, I show that even with this conservative level, the economy reaches the golden rule level of capital stock per capita within a finite time in a Pareto-improving way. Note that the total number of young individuals in period t is $N^n_{t-1}R(\widetilde{\alpha})(1+\widetilde{\alpha})$. Thus, the balance of government savings per young individual at the end of period t minus a^* for $t \geq 2$ becomes

$$\begin{aligned} a_t - a^* &= \frac{N^n_{t-1}(1+\widetilde{\alpha})}{N^n_{t-1}R(\widetilde{\alpha})(1+\widetilde{\alpha})} \int_{s^*+a^*}^{s^*+a_{t-1}} [F_K(l^*R(\widetilde{\alpha}), z) + 1 - \delta] dz \\ &= \frac{1}{R(\widetilde{\alpha})} \int_{s^*+a^*}^{s^*+a_{t-1}} [F_K(l^*R(\widetilde{\alpha}), z) + 1 - \delta] dz \\ &\equiv Q(a_{t-1}). \end{aligned} \tag{44}$$

Note that from (38), we have

$$\begin{aligned} a_t - a^* &\geq \frac{N^n_{t-1}(1+\widetilde{\alpha})}{N^n_{t-1}R(\widetilde{\alpha})(1+\widetilde{\alpha})} \int_{s^*+a^*}^{s^*+a_{t-1}} R(\widetilde{\alpha}) dz \\ &= a_{t-1} - a^*. \end{aligned} \tag{45}$$

Thus, a_t is increasing over time as long as it is determined according to equations (44) and (38). Now, consider the graph of $a_t = Q(a_{t-1}) + a^*$, where a_t is measured on the vertical axis and a_{t-1} on the horizontal axis. $Q(a_{t-1}) + a^*$ is equal to a^*at $a_{t-1} = a^*$. $Q'(a_{t-1})$ is

$$Q'(a_{t-1}) = \frac{1}{R(\widetilde{\alpha})} \left(F_K(l^*R(\widetilde{\alpha}), s^* + a_{t-1}) + 1 - \delta \right).$$

Thus, $Q(a_{t-1})$ is increasing and concave due to the diminishing MPK. The slope of $Q(a_{t-1}) + a^*$ at $a_{t-1} = a^*$ is

$$Q'(a^*) = \frac{1}{R(\widetilde{\alpha})} \left(F_K(l^*R(\widetilde{\alpha}), s^* + a^*) + 1 - \delta \right). \tag{46}$$

Because of the assumption regarding the golden rule (37), $Q'(a_{t-1})$ at $a_{t-1} = a^*$ is strictly greater than 1. On the other hand, owing to the diminishing MPK, F_k approaches zero as a_{t-1} rises from a^*. Thus, $a_t = Q(a_{t-1}) + a^*$ and the 45-degree line intersect at $a_{t-1} = a^*$ and a^{***}, where $a^{***} > a^*$. Let a^{**} be the point where $Q'(a^{**})$ $= 1$, implying that at a^{**},

$$\frac{\partial F(l^*R(\widetilde{\alpha}), s^* + a^{**})}{\partial K} + 1 - \delta = R(\widetilde{\alpha}). \tag{47}$$

In other words, at a^{**}, the golden rule is satisfied. Note that the government can choose a_1 so that $a_1 > a^*$ because the surplus in period 1 is strictly positive. From the graph of $a_t = Q(a_{t-1}) + a^*$, we see that a_t keeps increasing from a small $a_1 > a^*$.

Before it reaches a^{***}, it reaches a^{**} within a finite time, suggesting that the economy achieves the golden rule level of capital stock per capita within a finite time.

Note first that in this analysis, I assume that only the first integration of SP_t, namely the surplus that arises from the increased balance of government savings in period $t-1$, is placed into government savings in period t. However, the second integration of SP_t, namely the surplus that arises directly from the increase in immigration, can also be placed into government savings. Thus, the government can shorten the time to reach the golden rule level by adding the surplus that arises directly from increased immigration into government savings. Second, we note that the government can induce the economy to reach the modified golden rule level within a finite time in a Pareto-improving way because the modified golden rule level is lower than the golden rule level.

Proposition 3. (Reaching the (modified) golden rule level) *Suppose that a PYGO social security system is used initially and that the MPL condition is satisfied. Then, by accepting more immigrants, the government can induce the economy to reach the (modified) golden rule level within a finite time in a Pareto-improving way.*

3.3 Intra-redistributional Channel and Difference in Productivities and Preferences

When immigrants earn less or consume more publicly provided private goods than natives, accepting more immigrants could decrease the welfare of natives, because it means that more resources are taken from natives and used by immigrants. This is termed the intra-redistributional channel of accepting more immigrants. A similar redistribution could also occur when the preferences of immigrants and natives differ and when the labor supply or savings of immigrants differ from those of natives. To analyze this redistributional channel of accepting more immigrants, reconsider a permanent change to the immigration policy such that $\widetilde{\alpha} > \alpha^*$. Let $(c^{y,j*}, c^{o,j*}, s^{j*}, l^{j*})$ be consumption in the young period, consumption in the old period, and the savings and labor supply of type $j (j = n, m)$, respectively at the initial steady state. Since I assume that the preferences and productivities of natives and immigrants differ, I use a superscript j for the variables in the steady-state situation. Further, let $g^{i,j}$ be the publicly provided private goods of type i for type j nationality, where $i = y, o, ind$ and $j = n, m$. To guarantee that accepting more immigrants Pareto-improves welfare, I assume that the following condition is satisfied:

$$\text{Modified MPL condition}: R'(\alpha)\underbrace{[F_L l^{n*} - (c^{y,n*} + g^{y,n} + g^{ind,n} + s^{n*} + a^*)]}_{\text{upward intergenerational transfer of natives}}$$
$$+R'(\alpha)\alpha\underbrace{[F_L \phi^m l^{m*} - (c^{y,m*} + g^{y,m} + g^{ind,m} + s^{m*} + a^*)]}_{\text{upward intergenerational transfer of immigrants}}$$
$$+\text{intra-redistribution}(\alpha) > 0 \text{ for all } \alpha \in [\alpha^*, \widetilde{\alpha}], \tag{48}$$
$$\text{where}$$
$$\text{intra-redistribution}(\alpha) = \underbrace{R(\alpha)F_L\phi^m l^{m*} + (F_K + 1 - \delta)(s^{m*} + a^*)}_{\text{contribution of young and old immigrants}}$$
$$-\underbrace{[R(\alpha)(c^{y,m*} + s^{m*} + g^{y,m} + g^{ind,m} + a^*) + c^{o,m*} + g^{o,m} + g^{ind,m}]}_{\text{resource used for young and old immigrants}}$$

In the first and second lines, the inside of the bracket represents the upward intergenerational transfer made by a native and an immigrant, respectively. Thus, the sum of the first and second lines is essentially the same as the MPL condition in the preceding section. The intra-redistribution channel captures the intra-redistribution between immigrants and natives. The first bracket of the intra-redistribution is the amount a young immigrant earns minus the sum of the amount a young immigrant receives and the government saving for a young immigrant. The second bracket is the amount an old immigrant earns plus the return from the government savings for young immigrant minus the amount an old immigrant receives. When the productivities and preferences of immigrants and natives are the same, the intra-redistribution is equal to zero and this is equivalent to the MPL condition. Thus, the modified MPL condition above states that the total upward intergenerational transfer is large enough to offset the intra-redistribution from immigrants to natives. From (48), we therefore have the following propositions.[13]

Proposition 4 *Assume that natives and immigrants do not have the same productivities and preferences. If the modified MPL condition is satisfied, the welfare of all generations may be Pareto-improved by accepting more immigrants.* For the proof, see Appendix B4.

Proposition 5 *If the modified MPL condition is satisfied, the government can*

[13] In Appendix B5, I show that the modified MPL condition becomes the MPL condition when immigrants and natives have the same productivities and preferences.

induce the economy to reach the (modified) golden rule level within a finite time in a Pareto-improving way by accepting more immigrants.

4 Quantifying the Welfare Gain of Accepting More Immigrants in the Presence of a PYGO Social Security System

Propositions 1 and 2 show that accepting more immigrants can Pareto-improve the welfare of all generations if there are upward intergenerational transfers. Furthermore, Proposition 3 shows that if the government can place some of the welfare gain into its savings, it may induce the economy to reach the golden rule level of capital in a Pareto-improving way within a finite time. This result is in sharp contrast to the findings in the literature on social security reform that one generation must bear a double burden in order to increase the capital stock of the economy in the presence of a PYGO social security system (Geanakoplos, Mitchell and Zeldes (1998)). In addition, Propositions 4 and 5 show that even if an immigrant earns less than a native, accepting more immigrants Pareto-improves welfare if the intra-redistribution is lower than the intergenerational transfer from the young to the old.

A number of issues arise regarding accepting Propositions 1–5, however. First, these propositions are based on a two-period overlapping generation model. In a realistic multi-period overlapping generation model, an economy might not be able to reach the (modified) golden rule level of capital stock per capita within a finite time in a Pareto-improving way by accepting more immigrants. Second, although Proposition 3 shows that the economy reaches the (modified) golden rule level of capital stock per capita within a finite time, in practice it might take a long time, as long as 1000 years, to do so. Third, Propositions 1–5 are silent on the quantitative effect on welfare. Given that it might take quite a long time to reach the golden rule level, the welfare gain of accepting more immigrants may be very small. Finally, Propositions 4 and 5 say nothing about the degree to which the difference in productivity between immigrants and natives is allowed with respect to Pareto improvement when more immigrants are accepted.

This section addresses these four issues by using the computational overlapping generation model developed by Auerbach and Kotlikoff (1987). I assume that the model economy consists of overlapping generations in which each generation lives for 80 periods and that the probability of death increases with each passing period. I also assume that the model economy is similar to the US economy in several dimensions.[14] In the analysis, I assume both that the model economy is initially on the balanced growth path and that the PITN is similar to the percentage indicated by data taken from the US Census in 2000. Then, I examine whether the welfare of all the generations in the model economy can be Pareto-improved by increasing the PITN by a reasonable amount. In addition, I investigate how long it takes for the model economy to reach the (modified) golden rule level in a Pareto-improving way and quantify the Pareto-improving welfare gain. To check the robustness of the results, I also recalculate the model by changing the following parameter values: the replacement rate, initial government debt (assets) level, level of immigrants' earnings, immigrants' consumption of publicly provided private goods, probability of immigrants returning to their countries of origin, constant relative risk aversion (CRRA), and time preference rate.

4.1 The Model Economy: Auerbach–Kotlikoff Model with Immigration

Agents appear from age 1 in the model , which corresponds to age 20 in real life. They work from age 1 until age 45, which corresponds to 65 in real life. From the beginning of age 46, they retire. At each age, they die with some probability and can live until age 80. Let i be the index of age. For $i \geq 2$, let p_i be the probability that an agent is alive at age i, given that he or she is alive until age $i-1$. The lack of data forces the assumption that p_i is the same for natives and immigrants. To simplify the notation, I assume that $p_1 = 1$.[15] An agent who enters the model in period t maximizes the following utility function:

$$\max \sum_{i=1}^{45} \beta^i \prod_{q=1}^{i} p_q \left\{ \frac{[(c_{t-1+i}^{i,j})^{\zeta}(1-l_{t-1+i}^{i,j})^{1-\zeta}]^{1-\gamma}}{1-\gamma} + g_{t-1+i}^{i,j} \right\} + \sum_{i=46}^{80} \beta^i \prod_{q=1}^{i} p_q \left\{ \frac{[c_{t-1+i}^{i,j}]^{\zeta(1-\gamma)}}{1-\gamma} + g_{t-1+i}^{i,j} \right\}, \tag{49}$$

where $c_{t-1+i}^{i,j}$ and $l_{t-1+i}^{i,j}$ are the amounts of private consumption and labor supply of a type j agent of age i in period $t-1+i$. $g_{t-1+i}^{i,j}$ is the amount of publicly provided private goods used by a type j agent of age i in period $t-1+i$. I assume that the amount of $g_{t-1+i}^{i,j}$ is chosen by the government. In this formulation, as in Storesletten

[14]However, the model economy is different in several important dimensions as well. For example, it does not include certain aspects of open economies such as international trade and capital mobility and does not incorporate the accumulation of human capital for either natives or immigrants.

[15]Infant and child mortality is defined in (55).

(2000), I postulate that immigrants assume they will stay in the host country until the end of their lives.[16] β is the time preference rate and γ is the coefficient of constant relative risk aversion. Let $s_t^{i,j}$ be the savings of type j agents of age i at time t. The budget constraint of an agent at age i is

$$s_{t-2+i}^{i-1,j}(1 + r_{t-1+i}(1 - \tau_{r,t-1+i})) + (1 - \tau_{w,t-1+i})w_{t-1+i}H^{i,j} \times l_{t-1+i}^{i,j} = c_{t-1+i}^{i,j} + s_{t-1+i}^{i,j} \text{ for } 1 \leq i \leq 45 \tag{50}$$

$$s_{t-2+i,}^{i-1,j}(1 + r_{t-1+i}(1 - \tau_{r,t-1+i})) + b_{t-1+i}^{i,j} = c_{t-1+i}^{i,j} + s_{t-1+i}^{i,j} \text{ for } 46 \leq i \leq 80 \tag{51}$$

$$s_{t-1+i}^{i,j} = 0 \text{ for } i = 0 \text{ and } s_{t-1+i}^{i,j} \geq 0 \text{ for } 1 \leq i \leq 80, \tag{52}$$

where $H^{i,j}$ is the efficient unit of human capital of type j agents at age i and $H^{i,j} > 0$ for $1 \leq i \leq 45$ and $H^{i,j} = 0$ for $i \geq 46$. w_t is the wage rate for one efficient unit of labor in period t. I assume that an individual cannot have a negative savings balance. Once an individual dies, the government imposes a 100 percent inheritance tax.[17] $b_{t-1+i}^{i,j}$ is the social security benefit for type j agents of age i given in period $t-1+i$. For $i \leq 45$, $b_{t-1+i}^{i,j} = 0$ and for $i \geq 46$, $b_{t-1+i}^{i,j}$ is determined as follows:

$$b_{t-1+i}^{i,j} = 12 \times RR \times AIME^j(t) \text{ and } AIME^j(t) = \frac{\sum_{i=1}^{45}(1+\mu)^{45-i}w_{t-1+i}l_{t-1+i}^{i,j}H^{i,j}}{45 \times 12},$$

where RR is the replacement rate and $AIME^j(t)$ is the average income monthly index of the cohort who become age 1 at time t.

Next, I assume that the economy's aggregate production can be described by the Cobb–Douglas production function:

$$Y_t = K_t^{\theta}(E_t L_t)^{1-\theta} \text{ and } \mu = (E_{t+1} - E_t)/E_t, \tag{53}$$

where θ is the capital share and E_t represents the level of technology. μ is the income per capita growth rate, while L_t is the efficient unit of labor supply in period t.

Let $1 - \widehat{p}_i$ be the probability that an immigrant returns to his or her home country

[16] Alternatively, we could postulate that immigrants enjoy the same wage level in their home countries as in the host country if they decide to return. The two assumptions generate the same results.

[17] This assumption have little effect on my results. As long as the inheritance is distributed equally among those who are alive, the effect of changing the inheritance tax rate affects the balance of total savings in the model economy and the effect is similar to the effect of changing the time preference rate. As my robustness checks show in section 4.4, changing the time preference rate has little effect on my results.

at the beginning of age i, given that he or she stays in the host country at age $i-1$ for $i \geq 2$. To simplify the notation, I assume that $\widehat{p}_1 = 1$. Let N_t^{ij} be the number of agents of age i of type j at time t. Then, $N_t^{im} = p_i \widehat{p}_i N_t^{i-1,m}$ and $N_t^{in} = p_i N_t^{i-1,n}$. L_t is defined as follows:

$$L_t = \sum_{i=1}^{45} \sum_{j=n,m} H^{i,j} N_t^{i,j} l_t^{i,j} \ . \tag{54}$$

I assume that all immigrants arrive in the host country at age 1. Let σ_n be the growth rate of the number of natives of age 1 at the steady state and σ_m be the same growth rate for immigrants. The total number of natives is $\sum_{i=1}^{80} N_t^{in} = N_t^{1,n} \sum_{i=1}^{80} (1+\sigma_n)^{-(i-1)} \times \Pi_{q=1}^{i} p_q$. The total number of immigrants is $N_t^{1,m} \sum_{i=1}^{80} (1+\sigma_m)^{-(i-1)} \times \Pi_{q=1}^{i} p_q \widehat{p}_q$. This fact implies that the PITN is constant if and only if the growth rates of $N_t^{1,n}$ and $N_t^{1,m}$ are the same. Thus, to calculate the steady state, I assume that $\sigma_n = \sigma_m = \sigma$ and that the children of immigrants become natives if their parents stay in the host country until the children attain adulthood. Then, $N_t^{1,n}$ is determined by the fertility rates of both immigrants and natives and the return rate of the former. Let $\eta_j^{i,j}$ be the age-specific fertility rate of type j agents at age i. The number of children born to type j parents of age i is $N_t^{i,j} \eta^{i,,j}$. Let d be the infant and child mortality rate. Then, the number of natives of age 1 at $t+20$, $N_{t+20}^{1,n}$, is determined as follows:[18]

$$N_{t+20}^{1,n} = (1-d) \times \left\{ \sum_{i=1}^{80} \eta^{i,m} N_t^{i,m} \Pi_{x=1}^{20} \widehat{p}_{i+x} + \sum_{i=1}^{80} \eta^{i,n} N_t^{i,n} \right\}. \tag{55}$$

The PITN at the steady state (see Appendix A6) becomes

$$\begin{aligned} \frac{\sum_{i=1}^{80} N_t^{i,m}}{\sum_{i=1}^{80} N_t^{i,n}} \times 100 = & \frac{\frac{1}{1-d} - \sum_{i=1}^{80} \frac{1}{(1+\sigma)^{i+19}} \eta^{i,n} \times \Pi_{q=1}^{i} p_q}{\sum_{i=1}^{80} \frac{1}{(1+\sigma)^{i+19}} \eta^{i,m} \times \Pi_{q=1}^{i} p_q \widehat{p}_q \Pi_{x=1}^{20} \widehat{p}_{i+x}} \\ & \times \frac{\sum_{i=1}^{80} (1+\sigma)^{-(i-1)} \times \Pi_{q=1}^{i} p_q \widehat{p}_q}{\sum_{i=1}^{80} (1+\sigma)^{-(i-1)} \times \Pi_{q=1}^{i} p_q} \times 100. \end{aligned} \tag{56}$$

Equation (56) says that the steady-state PITN is determined once the age 1 population growth rate, σ, the return rate of immigrants, and the fertility rates of natives and immigrants are set. Conversely, we can choose σ so that the resulting PITN is consistent

[18]This comes from the assumption that the age 1 in our model corresponds to age 20 in real life.

with the data.[19]

The capital stock in period t is the sum of the balance of individual and government savings. Let a_{t-1} be the balance of government assets (or debt if this is negative) per capita at the end of period $t-1$. Then, the total capital stock in period t is

$$K_t = \sum_{i=1}^{80} \sum_{j=n,m} N_{t-1,i}^{i,j} s_{t-1}^{i,j} + a_{t-1} \sum_{i=1}^{80} \sum_{j=n,m} N_{t-1}^{i,j}. \tag{57}$$

The (efficient unit) wage rate at time t, w_t, and the pre-tax interest rate at time t, r_t, are determined as

$$w_t = (1-\theta) K_t^{\theta} E_t^{1-\theta} L_t^{-\theta} \qquad \text{and} \qquad r_t = \theta K_t^{\theta-1} E_t^{1-\theta} L_t^{1-\theta}. \tag{58}$$

Now, consider the initial balanced growth path where the capital/labor ratio (in efficient units) stays constant. Let $w^*(1+\mu)^t$ and r^* be the wage rate and interest rate in period t on the initial balanced growth path. $s^{*i,j}(1+\mu)^t$ and $b^{*i,j}(1+\mu)^t$ are the savings and social security benefit for type j agents of age i in period t on the initial balanced growth path. Let $a^*(1+\mu)^t$ and $g^{*i,j}(1+\mu)^t$ be the government assets (debt) per capita and publicly provided private goods for type j agents of age i in period t on the initial balanced growth path. Because immigrants may use more public services (e.g., children's education), here I use a superscript j. Let w^* and r^* be the efficiency unit wage rate and interest rate on the initial balanced growth path determined from (57) and (58). Then, the government budget constraint on the initial balanced growth path is

$$\begin{aligned}(1+\mu)^{t-1} \Bigg\{ & \tau_{wt} w^*(1+\mu) L_t + \tau_{rt} r^* \sum_{i=2}^{80} \sum_{j=n,m} p_i N_{t-1}^{i-1,j} s^{*i-1,j} + (1+r^*) \sum_{i=2}^{80} \sum_{j=n,m} (1-p_i) N_{t-1}^{i-1,j} s_{t-1}^{*i-1,j} \\ & - a^*(1+\mu) \sum_{i=1}^{80} \sum_{j=n,m} N_t^{i,j} + (1+r^*) a^* \sum_{i=1}^{80} \sum_{j=n,m} N_{t-1}^{i,j} - \sum_{i=1} \sum_{j=n,m} N_{t,i}^{i,j} g^{*i,j}(1+\mu) \\ & - \sum_{i=46}^{80} \sum_{j=n,m} b^{*i,j}(1+\mu) \times N_t^{i,j} - \kappa \sum_{i=46}^{80} \sum_{x=11}^{45} b^{*i,rm,x}(1+\mu) \times N_t^{i,rm,x} \Bigg\} = 0, \end{aligned} \tag{59}$$

where τ_w^* and τ_r^* are the wage tax rate and capital income tax rate on the initial balanced growth path. $b^{*i,rm,x}$ and $N_t^{i,rm,x}$ are, respectively, the social security benefit

[19] The RHS is an increasing function of σ at the parameter values consistent with the US data.

and number of immigrants of age i who returned to the home country at age x, but who are still eligible to claim social security benefits. κ is the share of those returned immigrants that actually claim social security benefits. US social security does not require residence as long as the individual is eligible for benefits. I assume that an immigrant is eligible to receive social security benefits as long as he or she pays social security contributions for at least 10 years. The first term of (59) is the revenue from the wage tax and the second term is the revenue from the capital income tax. The third term is the revenue from the inheritance tax. In my simulation, as noted earlier, I assume that the government imposes a 100 percent inheritance tax rate. The fourth term is the revenue from issuing government bonds and the fifth term is the expenditure on the principal and interest of these bonds. The sixth and seventh terms are the expenditure on the publicly provided private goods and social security benefits of natives and immigrants who reside in the host country. The last term is the social security benefits of eligible immigrants who returned to the home country.

When the government increases the PITN, the wage rate decreases and interest rate increases. To Pareto-improve welfare, I assume that the wage tax rate, interest tax rate, and social security benefit are adjusted as in the theoretical analysis. More specifically, I assume that the government keeps the after-tax wage rate and interest rate at the same level as on the initial balanced growth path. This fact implies that the wage tax rate τ_{wt} and interest tax rate τ_{kt} are set as follows:

$$w_t(1-\tau_{wt}) = w^*(1+\mu)^t \times (1-\tau_w^*) \text{ and } r_t(1-\tau_{rt}) = r^*(1-\tau_r^*). \qquad (60)$$

The social security benefit is adjusted as follows:

$$b_{t-1+i}^{i,j} = 12 \times RR \times AIME^j(t) \text{ and } AIME^j(t) = \frac{\sum_{i=1}^{45}(1+\mu)^{45-i} w_{t-1+i}(1-\tau_{wt}) l_{t-1+i}^{i,j} H^{i,j}}{(1-\tau_w^*) \times 45 \times 12}. \qquad (61)$$

When the tax and social security benefit formulae are adjusted according to (60) and (61), the individual budget constraint is the same as on the initial balanced growth path.[20] Thus, consumption, labor supply, and savings do not change. Then, a government budget surplus would exist even if the government were to spend the same amount of publicly provided private goods per person as on the initial balanced growth path,

[20]Note that when $w_t(1-\tau_{wt}) = w_t^*(1-\tau_w^*)$, $AIME^j(t) = \frac{\sum_{i=1}^{45}(1+\mu)^{45-i} w^*(1+\mu)^{t-1+i} l_{t-1+i}^{i,j} H^{i,j}}{45 \times 12}$.

as discussed in the previous section. The government can use this surplus to increase the balance of government savings or increase the level of publicly provided private goods. Let V be the distributional parameter that indicates what percentage of the budget surplus is saved. The government can keep increasing its savings balance until the economy reaches the golden rule level or modified golden rule level. As long as the MPK is greater than or equal to the (modified) golden rule level, the balance of government savings in period t is determined from the following equation:

$$(1+\mu)^t a_t \sum_{i=1}^{80} \sum_{j=n,m} N_t^{i,j} = V \times SP_t$$

, where SP_t is the government budget surplus in period t. If the MPK is equal to the (modified) golden rule level, V becomes 0. SP_t is defined as follows:

$$SP_t = \tau_{wt} w_t L_t + (1+\mu)^{t-1} \Bigg\{ \tau_{rt} r_t \sum_{i=1}^{80} \sum_{j=n,m} p_i N_{t-1}^{i-1,j} s^{*i-1,j} + (1+r_t) \sum_{i=1}^{80} \sum_{j=n,m} (1-p_i) N_{t-1}^{i-1,j} s^{*i-1,j}$$
$$(1+r_t) \times a_{t-1} \sum_{i=1}^{80} \sum_{j=n,m} N_{t-1}^{i,j} - \sum_{i=1}^{80} \sum_{j=n,m} N_t^{i,j} \times g^{*,i,j}(1+\mu)$$
$$- \sum_{i=46}^{80} \sum_{j=n,m} b^{*i,j}(1+\mu) \times N_t^{i,j} - \kappa \sum_{i=46}^{80} \sum_{x=11}^{45} b^{*i,rm,x}(1+\mu) \times N_t^{i,rm,x} \Bigg\}. \tag{62}$$

The rest of the surplus available for increasing publicly provided private goods is $(1-V) \times SP_t$, which is distributed equally across the entire population (including immigrants) in period t.[21] Then, after increasing the PITN, the amount of publicly provided private goods, g_t^{ij}, becomes

$$g_t^{ij} = g^{*i,j}(1+\mu)^t + \frac{(1-V) \times SP_t}{\sum_{i=1}^{80}(N_t^{i,n} + N_t^{i,m})}. \tag{63}$$

For the (modified) golden rule level, given the intergenerational discount rate for the modified golden rule level, we say that the economy reaches the (modified) golden rule level if the MPK equals the sum of the growth rate of the efficiency unit of labor, the depreciation rate, and the intergenerational discount rate for the modified golden rule level. In the case of the golden rule level, the intergenerational discount rate for

[21] Because the government could distribute a greater share of this surplus to natives than to immigrants, this assumption underestimates the welfare gain to the former.

the modified golden rule level is set to zero.

4.2 Policy Experiments and Parameter Values for the Simulation

Policy Experiments

To calculate the PITN, I take data from the US Census in 2000. These data show that the PITN above the age of 20 is 15.5 percent. Thus, I assume that the PITN at the initial balanced growth path is 15.5 percent. The target PITN is derived by examining past US Census data, which show a ratio of 5 percent in 1970, increasing to 18.3 percent in 2010.[22] Since the PITN increased by more than 10 percentage points in the 40 years from 1970 to 2010, I assume that a 10 percentage point increase in the PITN over the next 80 years is tolerable. Therefore, I set the target PITN to 25.5 percent. In the slow benchmark case, the PITN reaches its target in the 80th year and remains at that level thereafter. By contrast, in the intermediate and fast cases, the PITN hits its target level in the 62nd and 42nd years, respectively.[23] Finally, I assume that all immigrants arrive at age 1. Figure 1 illustrates the PITN over time.[24]

Fertility, Mortality, the Return Rate of Immigrants, and Other Parameter Values

Parameter values regarding fertility, mortality, the return rate of immigrants, government expenditure, government debt, taxes, preferences, and the production function are standard. To save space, I discuss them in Appendix A2.

[22]Data in 1970 are derived from the US Census, while those in 2010 are taken from the CPS. Since 2000, data on foreign-born US residents have only been provided by the American Community Survey and the CPS, not the Census, which raises a concern about the comparability of the CPS and Census data. However, for 2000, when both Census and CPS data are available, the PITN in the former is 12.42 percent compared with 11.54 percent in the latter, a minor difference. **?** also conclude that the difference between estimates based on Census and CPS data are trivial.

[23]In the fast and intermediate cases, the PITN keeps increasing even after reaching its target level to allow for a smooth transition. See Figure 1 for more details.

[24]More specifically, each of the three cases is calculated as follows. Let $f_{15.5}$ be the steady-state age 1 INR when the PITN is 15.5 percent. Define $f_{25.5}$ in a similar fashion. Then, the age 1 INR in period t, f_t, in the slow case is $f_t = f_{25.5}$ for all t. For the intermediate case, $f_t = f_{25.5} + 0.1 \times f_{25.5} \times (\frac{t}{15})$ for $1 \leq t \leq 15$, $f_t = 1.1 \times f_{25.5} - 0.1 \times f_{25.5} \times (t - 15)$ for $16 \leq t \leq 30$ and $f_t = f_{25.5}$ for $31 \leq t$. For the fast case, $f_t = f_{25.5} + 0.4 \times f_{25.5} \times (\frac{t}{15})$ for $1 \leq t \leq 15$, $f_t = 1.4 \times f_{25.5} - 0.4 \times f_{25.5} \times (t - 15)$ for $16 \leq t \leq 30$ and $f_t = f_{25.5}$ for $31 \leq t$.

Note that the steady-state age 1 INR is defined in the first term on the RHS of equation (40). Thus, once the PITN is determined, the steady-state age 1 INR is calculated from equation (40).

4.3 Results

Figures A3–A5 respectively show the balance of assets, consumption, and leisure for the lifecycle of an individual on the initial balanced growth path in the benchmark analysis. At the age of 46, leisure consumption becomes 1 because of mandatory retirement. On the initial balanced growth path, the capital/output ratio is 2.98, which is higher than the values used in Storesletten (2000) (2.4) and Nishiyama and Smetters (2007) (2.7), but lower than the value (3.2) used in standard business cycle research (Cooley and Prescott, 1995). To check how my results are affected by the capital/output ratio, I change the time preference rate and examine how the results change in the robustness checks presented in section 4.4.

Table 1 shows the parameter values and welfare effect of increasing the PITN for different values of V in equation (62), which is the share of the government surplus placed into savings (see column (3)). With respect to the rate at which immigrants are accepted, I consider the three benchmark cases introduced above (i.e., it takes 80, 62, and 42 years for the PITN to reach its target level).

Columns (4)–(9) present the values calculated within the simulation. Column (4) shows how many years it takes for the economy to reach the (modified) golden rule level. When it does not reach the (modified) golden rule level within 300 years, this is indicated by * or **. * indicates that the capital/labor ratio (efficient unit) in the 300th year is higher than that on the initial balanced growth path and that it is still increasing. ** indicates that the capital/labor ratio in the 300th year is lower than that on the initial balanced growth path. For example, when $V = 100\%$, it takes 112 years to reach the golden rule level in the slow acceptance case (row (16)).

Column (5) shows how much the capital stock per efficient unit of labor increases at the golden rule level compared with the level on the initial balanced growth path. When $V = 100\%$, the capital stock per efficient unit of labor increases by 102 percent.

Column (6) shows how much publicly provided private goods increase compared with the level on the initial balanced growth path. At $V = 100\%$, publicly provided private goods increase by about 36 percent. To calculate columns (5) and (6), I evaluate at the year in which the economy reaches the golden rule level if it reaches within 300 years, and at the 300th year otherwise.

Column (7) shows the rate at which the utility, measured by the expenditure function, of the cohort born when the economy reaches the golden rule level increases compared with the utility of the same cohort if it were on the initial balanced growth path. When the economy does not reach the golden rule level, I calculate the utility of the cohort born in the 300th year. In the expenditure function, the price vector on the initial balanced growth path is used to evaluate utility. Note that all welfare gain is distributed through an increase in publicly provided private goods.

Column (8) shows the share of the PDV of the increased publicly provided private goods relative to initial GDP. For example, in row (16), the PDV of the increased publicly provided private goods is 12 percent of initial GDP. To discount the increased publicly provided private goods in future periods, I apply a 5 percent discount rate instead of the equilibrium interest rate in order to eliminate the effect of the discount rate when comparing different cases.

Column (9) measures the welfare gain of natives and their descendants rather that of immigrants and their descendants. Specifically, it measures the degree to which the utility of natives and their descendants, not including immigrants and their descendants, is Pareto-improved by accepting more immigrants compared with the utility level on the initial balanced growth path. As before, it evaluates utility by using the expenditure function. I apply the price vector on the initial balanced growth path for the expenditure function and the equivalent variation to measure the difference in the utility levels for these two cases. To discount the welfare gain of future cohorts, I again use a 5 percent discount rate. Note that in all cases in Table 1, I assume that the government does not discriminate between immigrants and natives with respect to the distribution of the welfare gain. The welfare gain is distributed equally among immigrants and natives in the forms of publicly provided private goods. Row (16) in column (9) shows that the PDV of the Pareto-improving welfare gain of natives and their descendants comprises 11 percent of initial GDP.

In rows (2) and (3), (5) and (6), and (8) and (9), I shorten the years needed to reach the target PITN and increase the rate at which the PITN increases. When the number of years needed to reach the target PITN is reduced to 42, instead of 80, and V=100, the PDV of the welfare gain of increasing the PITN comprises 13 percent of initial GDP (row (18)).

Table 1 shows that the number of years needed to reach the golden rule level decreases as V increases because the government saves more for future cohorts. In contrast, the PDV of increased utility, measured as the share of initial GDP, increases as V decreases as long as V is greater than or equal to 50 percent. The PDV of increased utility, measured as the share of initial GDP, is highest when V=50. In this case, the quantified Pareto improvement ranges from 21 to 26 percent of initial GDP.

In Table 1, I set the intergenerational discount rate for the modified golden rule level to 0 percent and the target level of capital stock to the golden rule level. However, targeting capital stock at the golden rule level does not necessarily maximize the PDV of the welfare gain. In Table A2, I examine the effect on welfare of increasing the PITN for different target levels of capital stock by changing the value of the intergenerational discount rate for the modified golden rule level when V=100. Table A2 shows that the welfare gain is maximized when the intergenerational discount rate is 3 percent.

In Table 2, which I consider to be the representative case of my simulation, I recalculate all the rows of Table 1 by setting the intergenerational discount rate for the modified golden rule level to 3 percent. In this table, the Pareto-improving welfare gain of increasing the PITN is more than 20 percent of initial GDP and the capital stock per efficient unit of labor increases by 18 percent as long as V is greater than or equal to 50 percent.

Figures 2 and A6 show the MPK and capital/output ratio over time for different values of V when the target capital stock is the modified golden rule level (3 percent intergenerational discount rate). The MPK increases initially because of the acceptance of more immigrants; however, as the government savings balance increases, the capital stock per efficient unit of labor begins to rise and the MPK continues to decrease until the economy reaches the golden rule level. The capital/output ratio also displays a consistent pattern.

Figure 3 compares the utility levels of all cohorts on the initial balanced growth path with those of all cohorts for different values of the share for government savings (V) when the target capital stock is set at the golden rule level. For example, when V=100, all the surplus is placed into savings until the economy reaches the golden rule level and only distributed to individuals after the economy reaches the golden rule level. This fact implies that the utility of the 65th cohort, which dies in the 65th year, starts to experience higher utility than that on the initial balanced growth path. In all the cases considered in Tables 1–4, the simulation results show that all cohorts are Pareto-improved; this finding confirms my theoretical results.[25]

[25] The presented simulation results differ from those of some previous studies. For example, Fehr, Jokisch and Kotlikoff (2004) report that the welfare gain of doubling immigrants is very small. Lee and Miller (1998) similarly argue that the fiscal impact of accepting an additional 100,000 immigrants is very small. Several factors in those studies generate different results. Fehr, Jokisch and Kotlikoff

Figure 4 shows the importance of the government savings balance for the new equilibrium path. It presents the ratio between the interest income from the government savings balance and the social security benefit payment in each period. When the economy reaches the golden rule level, the interest income from the government savings balance comprises 70 percent of the social security benefit payments. Thus, on the new equilibrium path, the interest income from the government savings balance contributes a substantial amount.

4.4 Robustness Checks

Tables 3 and 4 present the robustness checks of the results in Table 2. Rows (1)–(6) in Table 3 check whether the results in Table 2 are sensitive to the initial government debt (assets) level. As argued in A2, different authors assume different levels of government debt (assets) on the initial balanced growth path. In rows (1)–(3) in Table 5, I set the initial government debt to 10 percent of private capital instead of 0 percent. In rows (4)–(6), I assume that the initial government debt level is -10 percent of private capital.

Rows (7)–(12) in Table 3 check whether the results in Table 2 are sensitive to the replacement rate. The theoretical analysis implies that higher intergenerational redistribution will result in the acceptance of more immigrants, yielding a higher welfare gain. Thus, it is predicted that as the replacement rate decreases, so does the welfare gain of accepting more immigrants. Rows (7)–(12) confirm this prediction. Decreasing the replacement rate by 10 percentage points decreases the welfare gain by 7 percentage points in terms of the percentage of initial GDP.

Rows (13)–(21) in Table 3 present sensitivity checks on immigrants' earnings. Following Storesletten (1995), I set the immigrant wage rate to 84.3 percent of that of natives. By deriving data from the CPS 2000 June supplement, I find that immigrants' earnings are 91 percent of those of natives. Rows (13)–(15) assume that the wage rate of immigrants is 89.3 percent of that of natives, rather than 84.3 percent. Rows (16)–

(2004) analyze immigration policy in an open economy setting, whereas I use a closed economy setting. In an open economy setting, the effect of additional government savings is offset by the mobility of capital. In my simulation, the government can use the budget surplus, which is obtained by accepting more immigrants, for savings and the amount of savings affects overall welfare. Likewise, Lee and Miller (1997) consider a much smaller increase in the number of immigrants than I do. By contrast, my simulation results are consistent with those presented by Auerbach and Oreopoulos (1999). They find that if the initial fiscal imbalance is not adjusted, then the welfare loss of halving the number of new immigrants is substantial. This finding is consistent with my theoretical and simulation results.

(18) and (19)–(21) assume that the wage rate of immigrants is 79.3 and 75 percent of that of natives. The results in rows (19)–(21) show that if immigrants earn 25 percent less than natives, the welfare gain of accepting more immigrants is 17 percent of initial GDP instead of 23 percent.

Rows (22)–(27) in Table 3 show the sensitivity checks on the consumption of publicly provided private goods by immigrants. In Tables 1 and 2, I assumed that immigrants and natives consume publicly provided private goods equally. In contrast, in rows (22)–(24) of Table 3, I assume that young immigrants consume 20 percent more publicly provided private goods than young natives. The PDV of the welfare gain comprises 17 percent of initial GDP instead of 23 percent. Rows (25)–(27) assume that immigrants of all ages consume 20 percent more publicly provided private goods than natives. In this case, the PDV of the welfare gain is 15 percent of initial GDP.

In Table 4, I present robustness checks by changing the parameter values of the utility function and the return rate of immigrants. Rows (1)–(6) examine the sensitivity of the results regarding the parameter values for CRRA. In Tables 1–3, I assumed that the CRRA is 3. Auerbach and Kotlikoff (1987) and Storesletten (2000) assume that the CRRA is 4, while Nishiyama and Smetters (2007) assume that it is 2. Rows (1)–(3) in Table 4 assume that the CRRA is equal to 4 and rows (5) and (6) assume that it is 2. The results presented in rows (1)–(6) show that those in Table 2 are not sensitive to the CRRA value. Rows (7)–(12) examine the sensitivity of the results to the time preference rate. Although it is common to assume that the time preference rate is greater than 1, readers might think that the results in Tables 1 and 2 are sensitive to the assumption that the time preference rate is greater than 1. Theoretically, lowering the time preference rate would result in lowering savings and would decrease the capital/labor ratio on the initial balanced growth path. Since increasing the number of immigrants would increase capital stock and the production function displays the diminishing MPK, lowering the initial capital stock as a result of assuming a lower time preference rate would increase the welfare gain of increasing the PITN. The results in rows (7)–(12) confirm this theoretical prediction, but show that the magnitude of those changes is very small. For example, changing the time preference rate from 1.011 to 0.99 increases the PDV of the welfare gain from 23.14 to 23.52 percent of the initial GDP.

Finally, rows (13)–(15) in Table 4 present robustness checks by assuming that the return rate of immigrants is 0. They show that the results presented in Table 2 change little and are robust.

5 Conclusion

This study examined both theoretically and quantitatively the effect on welfare of increasing the PITN in the presence of a PYGO social security system. The results for both the theoretical and the quantitative analysis show that the welfare gain of accepting more immigrants is robust and non-trivial. If intergenerational government transfers exist from the young to the old, the government can lead an economy to the (modified) golden rule level within a finite time in a Pareto-improving way by increasing the PITN. The PDV of the welfare gain of increasing the PITN from 15.5 to 25.5 percent amounts to about 20 percent of initial GDP. In the shortest case, the economy reaches the golden rule level in the 112th year in a Pareto-improving way. The presented analysis suggests that accepting more immigrants may be an important tool for policymakers when addressing the economic problems caused by the existence of a PYGO social security system.

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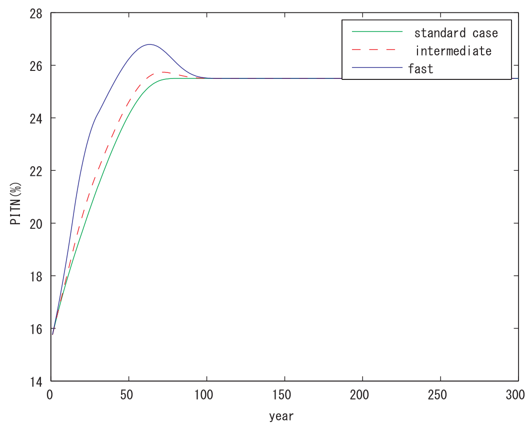
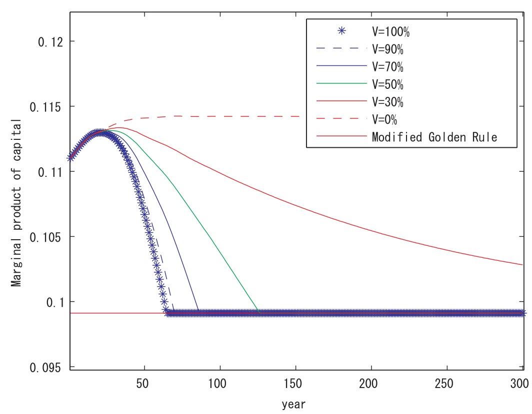
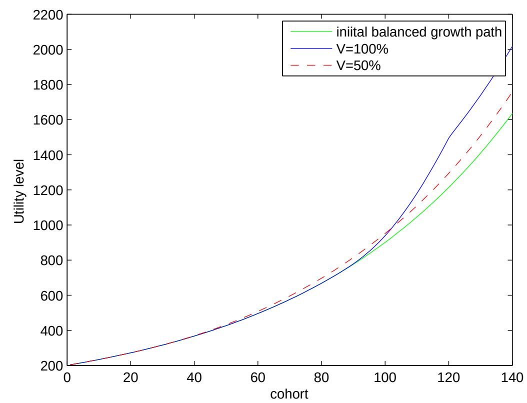


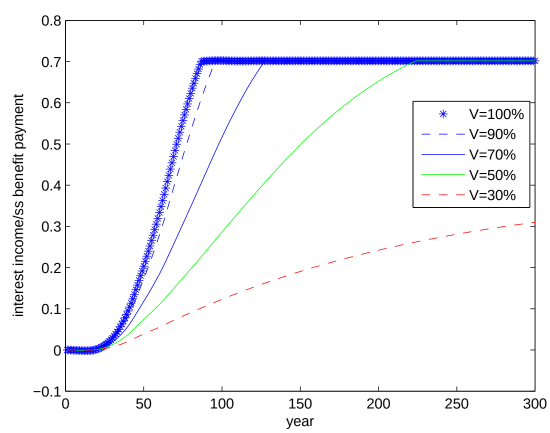
The PITN over time. The initial PITN is set at 15.5 percent and the target PITN is set at 25.5 percent.



The MPK over time for different values of the share of the surplus used for government savings (V). The target capital stock is the modified golden rule level with a 3 percent intergenerational discount rate.



Utility level of different cohorts for different values of the share of the surplus used for government savings (V). The target capital stock level is the modified golden rule level with a 3 percent intergenerational discount rate for the modified golden rule level.



The ratio between the interest income from the government savings balance and the social security benefit payment in each period. It is assumed that the PITN reaches its target level in the 80th year. The intergenerational discount rate for the modified golden rule level is set at 3 percent.

Table 1
The effect of increasing the PITN
(The target capital stock is the golden rule level)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target PITN	share of the surplus for the gov. savings (V)	year reaching the golden rule	% increase of capital stock per efficient unit labor at the golden rule	%change of publicly provided private goods per capita at the golden rule	% change of welfare of cohort born at the golden rule	share of the sum of the PDV of increased publicly provided private goods in the initial GDP	share of the sum of PDV of welfare gain of all natives and their descendants in the initial GDP
share of the surplus for the gov. savings =0%								
1	80	0%	300**	-4.37%	5.00%	0.53%	15.57%	17.72%
2	62	0%	300**	-4.37%	5.00%	0.53%	16.74%	18.94%
3	42	0%	300**	-4.37%	5.00%	0.53%	20.20%	22.47%
share of the surplus for the gov. savings =30%								
4	80	30%	300*	11.89%	14.69%	1.41%	17.86%	19.59%
5	62	30%	300*	12.00%	14.76%	1.42%	19.20%	20.94%
6	42	30%	300*	73.43%	30.82%	2.88%	25.32%	26.09%
share of the surplus for the gov. savings =50%								
7	80	50%	300*	71.21%	30.27%	2.79%	19.66%	20.79%
8	62	50%	300*	71.82%	30.42%	2.81%	21.10%	22.18%
9	42	50%	300*	73.43%	30.82%	2.88%	25.32%	26.09%
share of the surplus for the gov. savings =70%								
10	80	70%	184	102.43%	35.86%	3.77%	18.48%	18.92%
11	62	70%	182	102.43%	35.86%	3.77%	19.75%	20.09%
12	42	70%	119	50.99%	30.14%	3.17%	25.87%	25.56%
share of the surplus for the gov. savings =90%								
13	80	90%	127	102.43%	35.86%	3.77%	14.52%	13.75%
14	62	90%	125	102.43%	35.86%	3.77%	15.47%	14.56%
15	42	90%	120	102.43%	35.86%	3.77%	18.21%	16.80%
share of the surplus for the gov. savings =100%								
16	80	100%	112	102.43%	35.86%	3.77%	12.23%	10.69%
17	62	100%	110	102.43%	35.86%	3.77%	13.04%	11.33%
18	42	100%	106	102.43%	35.86%	3.77%	15.38%	13.14%

Notes

1. In all rows, the initial PITN is 15.5% and target PITN is 25.5%. The replacement rate is 60 %, CRRA=3 and the time preference rate is 1.011. The equilibrium capital to output ratio on the initial balanced growth path is 2.98.

2. In all rows, wage rate of immigrants is 84.3 % of that of natives.

3. * indicates that the capital stock per efficient unit labor does not reach the golden rule level within 300 years. Its value at the 300th year is higher than at the initial balanced growth path and keeps increasing at the 300th year. The percent change of the capital stock per efficient unit of labor is evaluated at the 300th year.

4. ** indicates that capital stock per efficient unit labor does not reach the golden rule level within 300 years and the capital stock per efficient unit labor at the 300th year is lower than at the initial balanced growth path. The percent change of the capital stock per efficient unit of labor is evaluated at 300th year.

Table 2
The effect of increasing the PITN for different values of V
(The target capital stock is the modified golden rule level)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target PITN	share of the surplus for the gov. savings (V)	year reaching the golden rule	% increase of the capital stock per efficient unit labor at the golden rule	%change of publicly provided private goods per capita at the golden rule	% change of welfare of cohort born at the golden rule	share of the PV of increased publicly provided private goods in the initial GDP	share of the PDV of welfare gain of all natives and their descendants in the initial GDP
share of the surplus for the gov. savings =0%								
1	80	0%	300**	-4.37%	5.00%	0.53%	15.57%	17.72%
2	62	0%	300**	-4.37%	5.00%	0.53%	16.74%	18.94%
3	42	0%	300**	-4.37%	5.00%	0.53%	20.20%	22.47%
share of the surplus for the gov. savings =30%								
4	80	30%	300*	11.89%	14.69%	1.41%	17.86%	19.59%
5	62	30%	300*	12.00%	14.76%	1.42%	19.20%	20.94%
6	42	30%	300*	12.32%	14.96%	1.45%	23.18%	24.80%
share of the surplus for the gov. savings =50%								
7	80	50%	127	18.19%	18.51%	1.95%	19.86%	21.06%
8	62	50%	124	18.19%	18.51%	1.95%	21.31%	22.47%
9	42	50%	116	18.19%	18.51%	1.95%	25.58%	26.45%
share of the surplus for the gov. savings =70%								
10	80	70%	87	18.19%	18.51%	1.95%	21.31%	22.11%
11	62	70%	78	18.19%	18.51%	1.95%	27.15%	27.49%
12	42	70%	78	18.19%	18.51%	1.95%	27.15%	27.49%
share of the surplus for the gov. savings =90%								
13	80	90%	70	18.19%	18.51%	1.95%	22.32%	22.85%
14	62	90%	68	18.19%	18.51%	1.95%	23.81%	24.24%
15	42	90%	63	18.19%	18.51%	1.95%	28.14%	28.13%
share of the surplus for the gov. savings =100%								
16	80	100%	65	18.19%	18.51%	1.95%	22.71%	23.14%
17	62	100%	63	18.19%	18.51%	1.95%	24.19%	24.51%
18	42	100%	59	18.19%	18.51%	1.95%	28.51%	28.37%

Notes

1. In all rows, the initial PITN is 15.5% and target PITN is 25.5%. The replacement rate is 60 %, CRRA=3 and the time preference rate is 1.011. The equilibrium capital to output ratio on the initial balanced growth path is 2.98.

2. In all rows, wage rate of immigrants is 84.3 % of that of natives.

3. The intergenerational discount rate for the modifed golden rule is 3%. At the modified golden rule level, the marginal product of capital is equal to the sum of the growth rate of efficient unit of labor, the depreciation rate and the intergenerational discount rate for the modified golden rule.

3. * indicates that the capital stock per efficient unit labor does not reach the modified golden rule level within 300 years. Tts value at the 300th year is higher than at the initial balanced growth path and keeps increasing at the 300th year. The percent change of capital stock per efficient unit of labor is evaluated at the 300th year.

4. ** indicates that the capital stock per efficient unit labor does not reach the modified golden rule level within 300 years and the capital stock per efficient unit labor at the 300th year is lower than at the initial balanced growth path. The percent change of the capital stock per efficient unit of labor is evaluated at the 300th year.

Table 3
Robustness checks
The role of the initial government debt level, the replacement rate, immigrants earnings and the use of public services by immigrants
(The target capital stock is the modified golden rule level)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target PITN	capital to output ratio at the initial balanced growth path	years taken to reach the modified goldren rule level	% increase of capital stock per efficient unit labor at the new balanced growth path	%change of publicly provided private goods per capita at the new balanced growth path	% change of welfare of the cohort born at the new balanced growth path	share of the sum of PDV of increased publicly provided private goods in the initial GDP	share of the sum of the PDV of welfare gain of all natives and their descendants in initial GDP
initial government debt ratio (% of private capital)= 10 %								
1	80	2.91	63	22.39%	21.87%	2.29%	28.48%	27.22%
2	62	2.91	61	22.39%	21.87%	2.29%	30.31%	28.82%
3	42	2.91	57	22.39%	21.87%	2.29%	35.66%	33.30%
initial government debt ratio(% of private capital)= -10 %								
4	80	3.04	67	14.71%	15.75%	1.67%	18.30%	19.68%
5	62	3.04	65	14.71%	15.75%	1.67%	19.51%	20.87%
6	42	3.04	61	14.71%	15.75%	1.67%	23.01%	24.16%
replacement rate =0.55								
7	80	3.03	68	14.87%	15.79%	1.67%	18.05%	19.32%
8	62	3.03	66	14.87%	15.79%	1.67%	19.24%	20.48%
9	42	3.03	61	14.87%	15.79%	1.67%	22.67%	23.70%
replacement rate =0.5								
10	80	3.1	71	11.51%	13.07%	1.39%	13.68%	15.38%
11	62	3.1	69	11.51%	13.07%	1.39%	14.572%	16.30%
12	42	3.1	65	11.51%	13.07%	1.39%	17.159%	18.84%
level of human capital of immigrants is 89.3 % of that of natives								
13	80	2.98	62	18.20%	18.96%	2.01%	25.22%	25.93%
14	62	2.98	60	18.20%	18.96%	2.01%	26.871%	27.48%
15	42	2.98	56	18.20%	18.96%	2.01%	31.702%	31.86%
level of human capital of immigrants is 79.3 % of that of natives								
16	80	2.98	69	18.18%	18.05%	1.89%	20.09%	20.24%
17	62	2.98	67	18.18%	18.05%	1.89%	21.392%	21.43%
18	42	2.98	62	18.18%	18.05%	1.89%	25.165%	24.74%
level of human capital of immigrants is 74.3 % of that of natives								
19	80	2.98	73	18.18%	17.58%	1.83%	17.32%	17.20%
20	62	2.98	71	18.18%	17.58%	1.83%	18.430%	18.20%
21	42	2.98	67	18.18%	17.58%	1.83%	21.629%	20.96%
young adult immigrants consume publicly provided private goods 20% higher than young adlut natives								
22	80	2.97	74	18.46%	17.70%	1.90%	17.27%	17.01%
23	62	2.97	72	18.46%	17.70%	1.90%	18.343%	17.96%
24	42	2.97	68	18.46%	17.70%	1.90%	21.409%	20.56%
immigrants of all ages consume publicly provided private goods 20% higher than natives								
25	80	2.96	77	18.84%	16.89%	1.86%	15.71%	15.25%
26	62	2.96	75	18.84%	16.89%	1.86%	16.674%	16.09%
27	42	2.96	71	18.84%	16.89%	1.86%	19.410%	18.36%

Notes
1. In all rows, V is 100 percent and the inter-generational social discount factor for the modified golden rule is set at 3%. At the modified golden rule level, the marginal product of capital is equal to the sum of the growth rate of efficient unit of labor, the depreciation rate and the inter-generational discount rate for the modified golden rule.
2. In all rows, the initial PITN is 15.5% and the target PITN is 25.5%. CRRA=3 and the time preference
4. The wage rate of immigrants is 84.3 % of that of natives in all rows except in rows (13) to (18).
5. Immigrants consume the same amount of publicly provided private goods as natives in rows (22) to (27).
6. The replacement is 0.6 in all rows except rows (7) to (12).

Table 4
Robustness checks (2)

The role of the CRRA, the time preference rate and the return rate of immigrant

(The target capital stock is the modified golden rule level)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target INR	capital to output ratio at the initial balanced growth path	years taken to reach the modified goldren rule level	% increase of the capital stock per efficient unit labor at the new balanced growth path	%change of publicly provided private goods per capita at the new balanced growth path	% change of welfare of cohort born at the new balanced growth path	share of the sum of the PDV of increased publicly provided private goods in the initial GDP	share of sum of the PDV of welfare gain of all natives and their descendants in initial GDP
CRRA=4								
1	80	2.634	74	41.87%	33.40%	3.28%	32.25%	22.18%
2	62	2.634	72	41.87%	33.40%	3.28%	34.04%	23.26%
3	42	2.634	69	41.87%	33.40%	3.28%	39.10%	26.15%
CRRA=2								
4	80	3.44	63	11.52%	12.70%	1.47%	16.26%	25.82%
5	62	3.44	61	11.52%	12.70%	1.47%	17.52%	27.70%
6	42	3.44	56	11.52%	12.70%	1.47%	21.32%	33.23%
the time prefernece rate=1								
7	80	2.67	72	39.41%	32.00%	3.17%	32.35%	23.15%
8	62	2.67	71	39.41%	32.00%	3.17%	34.184%	24.31%
9	42	2.67	67	39.41%	32.00%	3.17%	39.396%	27.43%
the time preference rate=0.99								
10	80	2.42	74	61.43%	46.18%	4.38%	43.96%	23.52%
11	62	2.42	73	61.43%	46.18%	4.38%	46.130%	24.51%
12	42	2.42	70	61.43%	46.18%	4.38%	52.171%	27.09%
the retrun rate of immigrant is 0%								
13	80	2.97	63	17.04%	17.86%	1.87%	24.00%	24.25%
14	62	2.97	61	17.04%	17.86%	1.87%	25.56%	25.68%
15	42	2.97	61	14.87%	15.79%	1.67%	22.67%	23.70%

Notes

1. In all rows, V is 100 percent. The parameter values of the wage rate of immigrants, the replacement rate, and the consumpiton of publcly provided private goods by immigrants are the same as in Table 2.

2. In all rows, the initial PITN is 15.5% and the target PITN is 25.5%.

3. The inter-generational discount rate for the modifed golden rule is 3% except in rows (4) to (6). In rows (4) to (6), the economy's capital stock is already above the modified golden rule level with a 3% inter-generational discount rate. In rows (4) to (6), I set the inter-generational discount rate at 2% instead of 3%. At the modified golden rule level, the marginal product of capital is equal to the sum of the growth rate of efficient unit of labor, the depreciation rate and the inter-generational discount rate for the modified golden rule.

Appendices A (The following appendices are placed on the journal's website as supplementary material, but not included in the main paper.)

Appendix A1

Consider a simple case in which the depreciation rate is 100 percent ($\delta = 1$) and immigration policy α is implemented at the initial steady state. Assume that the old population is equal to 1. In period 0, the following resource constraint must hold:

$$F(R(\alpha^*)l^*, s^* + a^*) - (c^{o*} + g^o + g^{ind}) - R(\alpha^*)(c^{y*} + s^* + a^* + g^y + g^{ind}) = 0. \quad (64)$$

Note that $F(R(\alpha^*)l^*, s^* + a^*)$ is the GDP per old individual when one plus the population growth rate is equal to $R(\alpha^*)$. Now, consider the graph of $(y, R(\alpha))$, where the vertical axis represents the GDP per old individual and horizontal axis represents one plus the population growth rate, which is $R(\alpha)$. This implies that $y = F(R(\alpha)l^*, s^* + a^*)$. Next, draw another graph defined by $(y, R(\alpha))$, where $y = (c^{y*} + s^* + a^* + g^y + g^{ind})R(\alpha)$. The slope of this line, which passes through the origin, is $c^{y*} + s^* + a^* + g^y + g^{ind}$. At $R(\alpha) = R(\alpha^*)$, the vertical distance of this line represents the total resources used for the young divided by the number of old individuals when one plus the population growth rate is equal to $R(\alpha^*)$. Thus, the difference between $y = F(R(\alpha)l^*, s^* + a^*)$ and $y = (c^{y*} + s^* + a^* + g^y + g^{ind})R(\alpha)$ at $R(\alpha) = R(\alpha^*)$ represents the resources used for one old individual at the initial steady state.

Now, suppose that a social planner increases the population growth rate by accepting more immigrants permanently. Hence, $R(\alpha)$ increases by $R'(\alpha^*)$ from $R(\alpha^*)$. If the slope of $y = F(R(\alpha), s^* + a^*)$ at $R(\alpha) = R(\alpha^*)$ is greater than $c^{y*} + s^* + a^* + g^y + g^{ind}$, the social planner can maintain the same allocation of resources per young individual (private consumption, savings, government-provided private goods, and age-independent public goods) and increase the allocation of resources to each old individual. Clearly, this constitutes Pareto improvement. Note that when the government accepts immigrants, the surplus is equal to $R'(\alpha^*)\{l^* F_L((R(\alpha)l^*, s^* + a^*) - (c^{y*} + s^* + a^* + g^y + g^{ind})\}$ in every period (see Figure A1).

Appendix A2

Fertility, Mortality, and the Return Rate of Immigrants

To ascertain age-specific fertility rates for natives and immigrants, I use data taken from the CPS 2000 June supplement, which the Census bureau also uses to calculate its fertility information. Figure A2 illustrates average births by age, showing that immigrant women have a greater number of births than native women at all age levels. From this figure, I calculate $\eta^{i,n}$ and $\eta^{i,m}$, the age-specific birth rates for native and immigrant women (see Table A1). For the adult mortality profile, p_i, I take the values from Nishiyama and Smetters (2007). I set d, the sum of infant and child mortality, to be 1.7 percent, by using data derived from the Vital Statistics of the United States for 1993 (**?**). For the return rate of immigrants to their home countries, I use the official Census estimate provided by **?**. They estimate that the return rate of immigrants for the first 10 years, second 10 years, and third 10 years is 19 percent, 9 percent, and 7 percent, respectively. Those return proportions correspond to annual rates of 2.09 percent, 0.94 percent, and 0.72 percent, respectively. Thus, I set $\widehat{p_i} = 0.9715$ for $2 \leq i < 10$, $\widehat{p_i} = 0.99062$ for $11 \leq i \leq 20$, $\widehat{p_i} = 0.9927$ for $21 \leq i \leq 30$, and $\widehat{p_i} = 1$ for $i \geq 31$.

Setting the age-specific fertility rate, infant and child mortality rate, and initial PITN (at 15.5 percent) allows us to calculate the annual population growth rate according to equation (56) under the assumption that the PITN is at the steady state. The annual growth rate of the population of age 1 is thus 0.39 percent.[26] This finding implies that at the initial steady state, the government accepts immigrants such that the annual growth rate of immigrants of age 1 becomes 0.39 percent.

Age-specific Government Expenditure

I assume that age-specific government expenditure, $g^{*i,j}$, is the same for natives and immigrants. Empirical studies show no systematic difference in the use of public ser-

[26] Annual CPS data on immigrants and natives from 1995 to 2010 show that the median annual growth rate of the total population (immigrant population) aged from 20 to 40 is 0.12 (1.85) percent. In the theoretical model, the model assumes that the age 1 population growth rates of immigrants and natives are the same. Thus, the theoretically predicted growth rate of the age 1 population, 0.39 percent, is between the growth rate of immigrants aged 20–40 and that of the total population aged 20–40.

vices by these two groups.[27] Thus, I assume that for $1 \leq i \leq 24$, $g^{*i,j} = g_y$, for $25 \leq i \leq 44$, $g^{*i,j} = g_m$, and for $45 \leq i \leq 80$, $g^{*i,j} = g_o$. Further, I assume that g_y, gm, and g_0 are 24.5, 13.4, and 23.2 percent of GDP per capita on the initial balanced growth path following Storesletten (1995) and Auerbach, Kotlikoff, Hagememann, and Nicoletti (1989). Moreover, to check the robustness of my results, I also assume that immigrants consume 20 percent more publicly provided private goods than natives in the robustness checks.

Utility Function, Production Function, and Human Capital Profile

Previous studies have not thus far estimated precisely the coefficient for relative risk aversion, γ. Auerbach and Kotlikoff (1987) and Storesletten (2000) assume that γ is 4, while Nishiyama and Smetters (2007) set γ equal to 2. In the presented analysis, I assume that $\gamma = 3$ and check the robustness with $\gamma = 2$ and $\gamma = 4$. For the time preference rate, β, following Hurd (1989) and Storesletten (2000), I assume that $\beta = 1.011$. A higher β implies higher savings and a higher capital/output ratio. To check the sensitivity of my results, I also set $\beta = 1$ and $\beta = 0.99$. I assume that the leisure share in the utility function, ζ, is 0.33.

Further, I assume that the depreciation rate of capital, δ, is equal to 0.047, while the capital share in the production function, θ, is set to $\theta = 0.4$ and for technological progress, I assume that the income per capita growth rate, μ, is 0.015.

For the human capital profile of natives, H_i^s, I take the value from Auerbach and Kotlikoff (1987):

$$H^{i,n} = \exp(4.47 + 0.033 \times i - 0.00067 \times i^2) \quad \text{for } 1 \leq i \leq 45 \tag{65}$$

$$H^{i,n} = 0 \quad \text{for } 46 \leq i. \tag{66}$$

For the human capital profile of immigrants, Storesletten (1995) shows that immigrants' earnings are, on average, 15.7 percent lower than those of natives.[28] Similarly, data

[27]**?** show that immigrants have a higher participation rate in welfare programs than natives. **?** find that these differences are explained by the higher participation rate in welfare programs among refugees and retired immigrants and that there is no difference for working immigrants. Thus, for the theoretical part of the analysis, I assume that g^i is independent of birthplace. For the computational part of the analysis, I relax this assumption for the robustness checks.

[28]Figure 2.2 of Storesletten (1995) shows that at ages 20, 25, 30, 35, 40, and 45, the wage rate of immigrants is lower than that of natives by 15, 20, 17.8, 16.4, 12, and 13 percent, respectively. By averaging those rates, I obtain a working value of 15.7 percent.

taken from the CPS June 2000 supplement show that immigrants' earnings are 10 percent lower than those of natives. These estimates allow me to assume that the efficient unit of human capital of immigrants is 84.3 percent of that of natives and that $H^{i,m} = 0.843 \times H_i^n$ in the benchmark calculation. To examine the robustness of my results, I change the level of human capital from 84.3 percent to 89.3 percent, 79.3 percent, or 74.3 percent and recheck the results.

Taxes and Government Debt

For the capital income tax, I take the value from Nishiyama and Smetters (2007) and assume that $\tau_k = 0.28$. For the level of social security benefit, a higher replacement rate means a greater intergenerational redistribution of income, which leads to greater welfare gain as a result of increasing the immigrant population. Following Auerbach and Kotlikoff (1987), in the benchmark case I set the replacement to 0.6 and check the robustness of my results by varying it to 0.55 and 0.5.

As for the initial level of government debt or assets, different authors set different levels. Storesletten (2000) considers only government debt and assumes that the initial level is 50 percent of initial GDP. Given his estimated initial capital/output ratio of 2.4, this implies that government debt is about 20 percent of private capital. Nishiyama and Smetters (2007) consider not only government debt, but also government assets by using BEA information on the government's fixed capital. They assume that at the initial steady state, the government has positive net assets of 10 percent of total private capital. This naturally leads to a higher capital/output ratio on the initial balanced growth path. Herein, as a benchmark case, I assume that the initial level of government debt or assets is 0 percent of private capital and that government debt is 10 percent or -10 percent of private capital.

Table A1 Number of birth at each age for native and immigrnt in the simulation

age	number of birth of immigrant	number of birth of native
1	0.1934279	0.1743087
2	0.0520106	0.0474751
3	0.0550358	0.0505587
4	0.0569373	0.0527624
5	0.0578227	0.054141
6	0.0577998	0.0547487
7	0.0569765	0.0546402
8	0.0554604	0.0538698
9	0.0533594	0.052492
10	0.0507811	0.0505614
11	0.0478333	0.0481323
12	0.044624	0.0452592
13	0.0412605	0.0419967
14	0.0378511	0.0383992
15	0.0345031	0.034521
16	0.0313245	0.0304168
17	0.0284231	0.026141
18	0.0259064	0.0217481
19	0.0238824	0.0172925
20	0.0224589	0.0128286
21	0.0217435	0.0084112
22	0.0218439	0.0040944
23-80	0	0

Note
1. The calculaton is based on Figure 3. Let TB(i,j) be the vertical axis of group j of Figure 3 where j is native or immigrant. Then, the number of births of age i of group j in the model is calcluated as follows. When i=1, the number of births of age i of group j in the model is TB(20,j)/2. When 2<=i<=22, the number of births is (TB(19+i,j)-TB(18+i,j))/2.

Table A2

The effect of increasing the PITN for different inter-generational discount rates

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target PITN	inter-generational discount rate for the modifed golden rule	year reaching the modified golden rule	% increase of the capital stock per efficient unit labor at the	%change of publicly provided private goods per capita at the golden	% change of welfare of the cohort born at the golden rule	share of the PDV of increased publicly provided private goods in	share of the PDV of welfare gain of all natives and their descendants
1	80	0.0%	112	102.43%	35.86%	3.77%	12.23%	10.69%
2	80	0.5%	103	82.39%	35.07%	3.69%	15.39%	13.87%
3	80	1.0%	95	65.46%	33.04%	3.48%	18.10%	16.75%
4	80	1.5%	87	50.99%	30.14%	3.17%	20.26%	19.22%
5	80	2.0%	80	38.52%	26.63%	2.80%	21.80%	21.17%
6	80	2.5%	73	27.69%	22.71%	2.39%	22.64%	22.51%
7	80	3.0%	65	18.19%	18.51%	1.95%	22.71%	23.14%
8	80	3.5%	56	9.82%	14.12%	1.49%	21.86%	22.89%
9	80	4.0%	45	2.40%	9.64%	1.01%	19.80%	21.43%

Note

1.At the modified golden rule level, the marginal product of capital is equal to the sum of the growth rate of efficient unit of labor, the depreciation rate and the inter-generational discount rate for the modified golden rule.

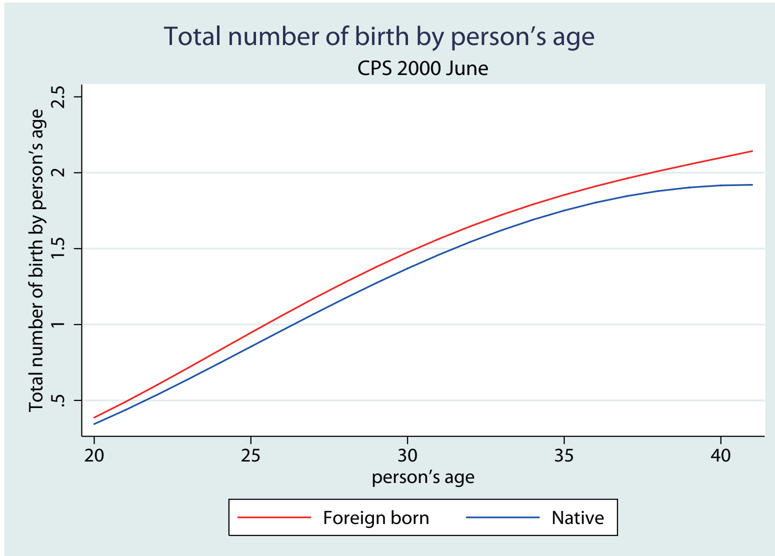


Figure A2: Total births by age. The data source is the CPS June 2000 supplement. Total births by age are regressed on a sixth-order polynomial function of age separately for natives and immigrants. The predicted values are then plotted.

Appendices B (The following appendices are for the purpose of refereeing.)

Appendix B1

Notice that in the programming problem, the objective function is concave and the constrained set is convex. Thus, if some allocation satisfies the first-order condition, it is also the solution of the programming problem. Now set up the Lagrangian function as follows:

$$\begin{aligned}
L &= \frac{1}{1+\rho}[u^o(c_1^o) + v^o(g^o, q)] \\
&+ \sum_{t=1}^{\infty} \gamma_t\{u^y(c_t^y, l_t) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho}[u^o(c_{t+1}^o) + v^o(g^o, g^{ind}) - u^*\} \\
&+ \sum_{t=1}^{\infty} \lambda_t\{F(R(\alpha^*)l_t, (s_{t-1} + a_{t-1})) + (1-\delta)(s_{t-1} + a_{t-1}) \\
&\quad - (c_t^o + g^o + g^{ind}) - R(\alpha^*) \times (c_t^y + s_t + a_t + g^y + g^{ind}) \}
\end{aligned}$$

$$\text{where } a_0 = a^* \text{ and } s_0 = s^* \qquad (67)$$

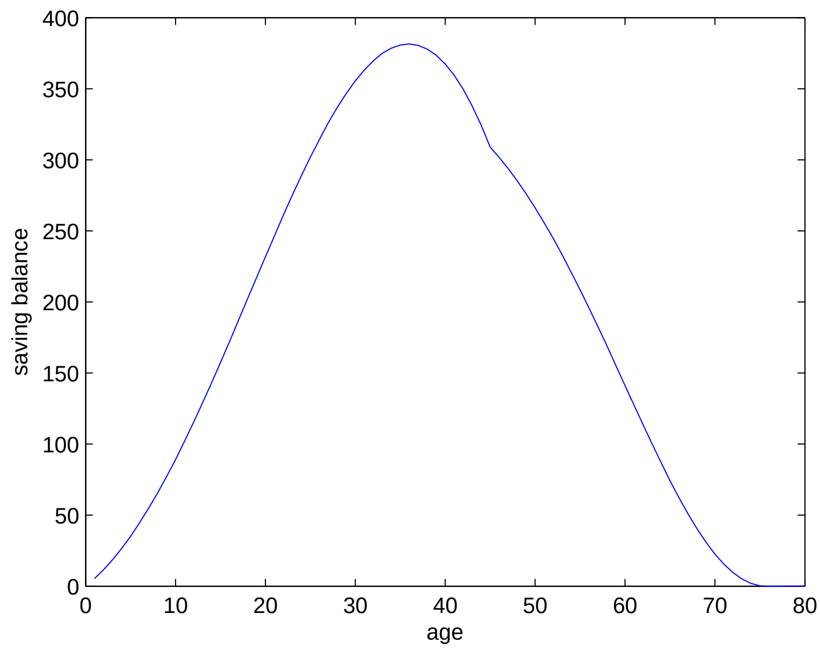


Figure A3: Balance of assets over the lifecycle on the initial balanced growth path

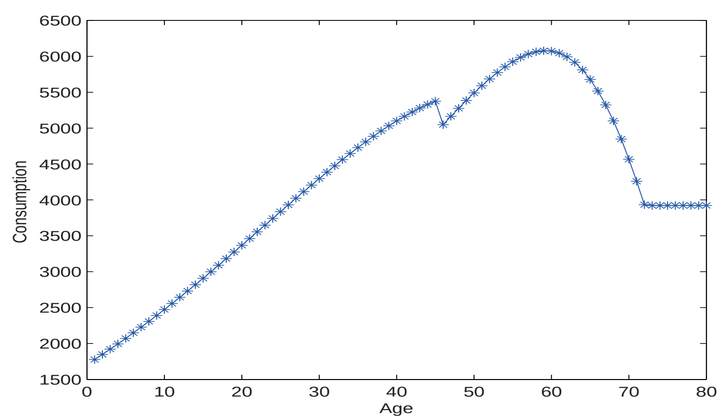


Figure A4: Lifecycle consumption path on the initial balanced growth path.

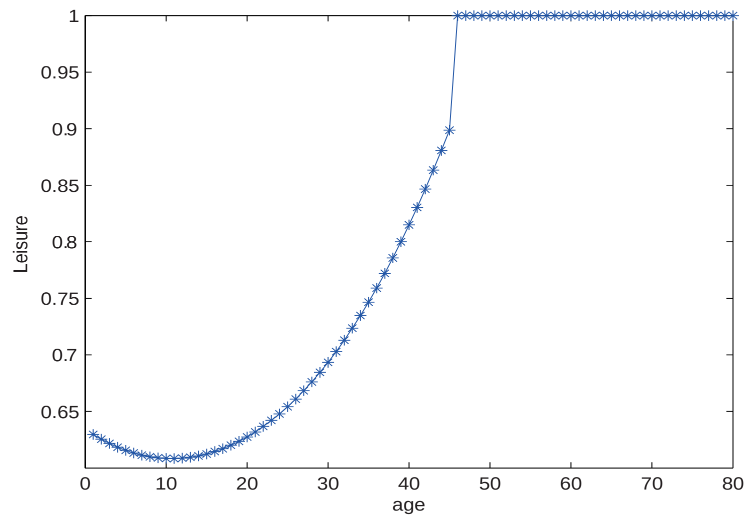


Figure A5: Lifecycle leisure consumption on the initial balanced growth path.

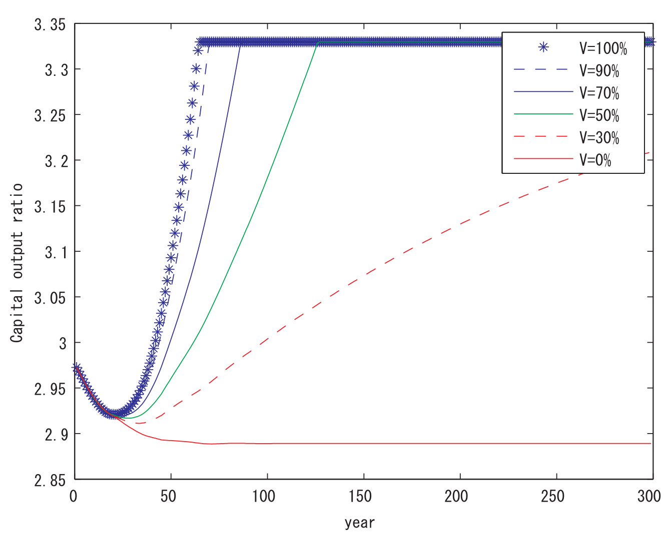


Figure A6: Capital/output ratio over time for different shares of the surplus used for government savings (V). In those simulations, the number of years needed for the PITN to reach its target level is set at 80. The target capital stock is the modified golden rule level. The intergenerational discount rate for the modified golden rule level is set at 3 percent.

The first order conditions are:

$$
\begin{aligned}
c_1^o &: \frac{1}{1+\rho}u^{o\prime}(c_1^o) = \lambda_1; c_{t+1}^o : \gamma_t \frac{1}{1+\rho}u^{o\prime}(c_{t+1}^o) = \lambda_{t+1}; \\
c_t^y &: \gamma_t \frac{\partial u^y(c_t^y, l_t)}{\partial c_t^y} = \lambda_t R(\alpha^*); l_t : \gamma_t \frac{\partial u^y(c_t^y, l_t)}{\partial l_t} = \lambda_t \frac{\partial F}{\partial L} R(\alpha^*) \\
\gamma_t &: u^y(c_t^y, l_t) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho}[u^o(c_{t+1}^o) + v^o(g^o, g^{ind}) - u^* = 0; \\
s_t, a_t &: \lambda_{t+1}\left\{\frac{\partial F}{\partial K} + 1 - \delta\right\} = \lambda_t R(\alpha^*) \\
\lambda_t &: F(R(\alpha^*), s_{t-1} + a_{t-1}) + (1-\delta)(s_{t-1} + a_{t-1}) \\
&-(c_t^o + g^o + g^{ind}) - R(\alpha^*)(c_t^y + s_t + a_t + g^o + g^{ind}) \ = 0 \qquad (68)
\end{aligned}
$$

Those first order conditions imply that

$$
\begin{aligned}
\left(\frac{\partial u^y(c_t^y, l_t)}{\partial l_t}\right) / \left(\frac{\partial u^y(c_t^y, l_t)}{\partial c_t^y}\right) &= \frac{\partial F}{\partial L} \\
\frac{\partial F}{\partial K} + 1 - \delta &= \left(\frac{\partial u^y(c_t^y, l_t)}{\partial c_t^y}\right) / \left(\frac{1}{1+\rho}u^{o\prime}(c_{t+1}^o)\right)
\end{aligned}
$$

On the other hand, at the initial steady state, the initial steady state allocation, $(c^{y*}, c^{o*}, s^*, l^*, a^*)$ satisfy the following allocation:

$$
\begin{aligned}
-\left(\frac{\partial u^y(c^{y*}, l^*)}{\partial l}\right) / \left(\frac{\partial u^y(c^{y*}, l^*)}{\partial c^y}\right) = w^* = \frac{\partial F(R(\alpha^*)l^*, s^* + a^*)}{\partial L} \\
\frac{\partial F(R(\alpha^*)l^*, s^* + a^*)}{\partial K} + 1 - \delta = 1 + r^* = \left(\frac{\partial u^y(c^{y*}, l^*)}{\partial c_t^y}\right) / \left(\frac{1}{1+\rho}u^{o\prime}(c^{o*})\right) \\
u^y(c^{y*}, l^*) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho}[u^o(c^{o*}) + v^o(g^o, g^{ind}) = u^* \\
F(R(\alpha^*)l^*, s^* + a^*) + (1-\delta)(s^* + a^*) = R(\alpha^*)(c^{y^*} + s^* + a^* + g^o + g^{ind}) + (c^{o*} + g^o + g^{ind}) \qquad (69)
\end{aligned}
$$

Now, we set c_t^y, c_{t+1}^o, l_t, s_t, a_t, λ_t, γ_t as follows

$$
\begin{aligned}
c_t^o = c^{o*}; c_t^y = c^{y*}; s_t = s^*; l_t = l^*; a_t = a^*; \lambda_1 = \frac{1}{1+\rho}u^{o\prime}(c^{o*}) \\
\lambda_{t+1} = \lambda_t \frac{R(\alpha^*)}{1+r^*}; \gamma_t \frac{1}{1+\rho}u^{o\prime}(c^{o*}) = \lambda_{t+1} \qquad (70)
\end{aligned}
$$

If c_t^y, $c_{t+1}^o, s_t, a_t, \lambda_t, \gamma_t$ are set in this way, it clearly satisfies the first-order conditions of the programming problem. Thus, the initial steady state allocation is Pareto-efficient. Q.E.D.

Appendix B2

Using the definitions of τ_{wt} and τ_{rt}, we have

$$w_t \tau_{wt} = w_t - (1-\tau_w^*)w^*$$
$$r_t \tau_{rt} = r_t - (1-\tau_r^*)r^*$$

Then, the government budget surplus at period 1 is

$$SP_1 = (w_1 - (1-\tau_w^*)w^*)l^* \sum_{j=n,m} N_1^j + (r_1 - (1-\tau_r^*)r^*)s^* \sum_{j=n,m} N_0^j - (b^* + g^o + g^{ind}) \sum_{j=n,m} N_0^j - (g^y + g^{ind} + a^*) \sum_{j=n,m} N_1^j + a^*(1+r_1) \sum_{j=n,m} N_0^j \quad (71)$$

Note that $N_1^m = N_1^n \widetilde{\alpha}$ where $\widetilde{\alpha} > \alpha^*$ and that N_1^n is pre-determined where $N_1^n = N_1^0(1+\pi_n) + N_0^m(1+\pi_m)$ and $N_0^m = N_0^n \alpha^*$.

$$SP_1 = w_1 l^* \sum_{j=n,m} N_1^j - w^*(1-\tau_w^*)l^* \sum_{j=n,m} N_1^j + r_1 s^* N_0^n(1+\alpha^*) - r^* s^*(1-\tau_r^*)N_0^n(1+\alpha^*) - N_0^n(1+\alpha^*)(b^* + g^o + g^{ind}) - (g^y + g^{ind} + a^*)\{N_1^n(1+\alpha^*) + N_1^n(\widetilde{\alpha} - \alpha^*)\} + N_0^n(1+\alpha^*)a^*(1+r_1) \quad (72)$$

Also notice that $\sum_{j=n,m} N_1^j = N_1^n(1+\widetilde{\alpha}) = N_1^n(1+\alpha^* + \widetilde{\alpha} - \alpha^*)$ and $\sum_{j=n,m} N_0^j = N_0^n(1+\alpha^*)$. Thus, SP_1 becomes

$$SP_1 = w_1 l^* \sum_{j=n,m} N_1^j - w^*(1-\tau_w^*)l^*\{N_1^n(1+\alpha^*) + N_1^n(\widetilde{\alpha} - \alpha^*)\} + r_1 s^* N_0^n(1+\alpha^*) - r^* s^*(1-\tau_r^*)N_0^n(1+\alpha^*) - N_0^n(1+\alpha^*)(b^* + g^o + g^{ind}) - (g^y + g^{ind} + a^*)\{N_1^n(1+\alpha^*) + N_1^n(\widetilde{\alpha} - \alpha^*)\} + N_0^n(1+\alpha^*)a^*(1+r_1) \quad (73)$$

At the steady state, as for the government budget constraint, we have

$$(\tau_w^* w^* l^* - g^y - g^{ind} - a^*) N_1^n (1+\alpha^*) + (\tau_r^* r^* s^* - b^* - g^o - g^{ind}) N_0^n (1+\alpha^*) + N_0^n (1+\alpha^*) a^* (1+r^*) = 0 \tag{74}$$

By using the government budget constraint at the initial steady state, we can rewrite SP_1 as follows:

$$\begin{aligned} SP_1 = {} & w_1 l^* \sum_{j=n,m} N_1^j - w^* l_1^* N_1^n (1+\alpha^*) - w^* l^* (1-\tau_w^*) N_1^n (\widetilde{\alpha} - \alpha^*) - N_0^n (1+\alpha^*) a^* (1+r^*) \\ & + r_1 s^* N_0^n (1+\alpha^*) - r^* s^* N_0^n (1+\alpha^*) - (g^y + g^{ind} + a^*) N_1^n (\widetilde{\alpha} - \alpha^*) + N_0^n (1+\alpha^*) a^* (1+r_1) \end{aligned}$$

$N_0^n (1+\alpha^*) a^*$ can be canceled out in the above equation. Thus we have

$$\begin{aligned} SP_1 = {} & w_1 l^* \sum_{j=n,m} N_1^j - w^* l_1^* N_1^n (1+\alpha^*) - w^* l^* (1-\tau_w^*) N_1^n (\widetilde{\alpha} - \alpha^*) \\ & + r_1 s^* N_0^n (1+\alpha^*) - r^* s^* N_0^n (1+\alpha^*) - (g^y + g^{ind} + a^*) N_1^n (\widetilde{\alpha} - \alpha^*) + N_0^n (1+\alpha^*) a^* (r_1 - r^*) \end{aligned}$$

From the homogeneity of the production function and Euler's theorem, we have

$$w_1 l^* \sum_{j=n,m} N_1^j + r_1 (s^* + a^*) N_0^n (1+\alpha^*) = F(l^* \sum_{j=n,m} N_1^j,\ (s^* + a^*) N_0^n (1+\alpha^*)) - \delta (s^* + a^*) N_0^n (1+\alpha^*)$$

At the initial steady-state, we also have

$$w^* l^* N_1^n (1+\alpha^*) + r^* (s^* + a^*) N_0^n (1+\alpha^*) = F(l^* N_1^n (1+\alpha^*), (s^* + a^*) N_0^n (1+\alpha^*)) - \delta (s^* + a^*) N_0^n (1+\alpha^*)$$

Thus, SP_1 becomes

$$\begin{aligned} SP_1 = {} & F(l^* \sum_{j=n,m} N_1^j,\ (s^* + a^*) N_0^n (1+\alpha^*)) - \delta (s^* + a^*) N_0^n (1+\alpha^*) \\ & - \{ F(l_1^* N_1^n (1+\alpha^*), (s^* + a^*) N_0^n (1+\alpha^*)) \\ & - \delta (s^* + a^*) N_0^n (1+\alpha^*) \} - w^* l^* (1-\tau_w^*) N_1^n (\widetilde{\alpha} - \alpha^*) - N_1^n (g^y + g^{ind} + a^*) (\widetilde{\alpha} - \alpha^*) \end{aligned} \tag{75}$$

Note that $N_1^m = \widetilde{\alpha} N_1^n$. Thus,

$$
\begin{aligned}
SP_1 &= F(l^*(N_1^n(1+\widetilde{\alpha}), (s^*+a^*)N_0^n(1+\alpha^*)) - F(l^*\{N_1^n(1+\alpha^*)\}, (s^*+a^*)N_0^n(1+\alpha^*)) \\
&- \{w^* l^*(1-\tau_w^*) - (g^y + g^{ind} + a^*)\} N_1^n(\widetilde{\alpha} - \alpha^*) \\
&= \int_{1+\alpha^*}^{1+\widetilde{\alpha}} [F_L(N_1^n l^* z, (s^*+a^*)N_0^n(1+\alpha^*))N_1^n l^* - w^*(1-\tau_w)N_1^n l^* - (g^y + g^{ind} + a^*)N_1^n] dz \\
&= N_1^n \int_{1+\alpha^*}^{1+\widetilde{\alpha}} F_L(N_1^n l^* z, (s^*+a^*)N_0^n(1+\alpha^*))l^* - w^* l^*(1-\tau_w) - (g^y + g^{ind} + a^*)] dz
\end{aligned}
\tag{76}
$$

Note that $w^* l^*(1-t_w) = c^{y*} + s^*$. Thus, we have

$$
= N_1^n \int_{1+\alpha^*}^{1+\widetilde{\alpha}} F_L(N_1^n l^* z, (s^*+a^*)N_0^n(1+\alpha^*))l^* - c^{y*} - s^* - g^y - g^{ind} - a^*)] dz \tag{77}
$$

Note that $N_1^n = R(\alpha^*)N_0^n$ and using the homogeneity of F_L, we have

$$
SP_1 = N_1 \int_{1+\alpha^*}^{1+\widetilde{\alpha}} F_L(R(\alpha^*) l^* z, (s^*+a^*)(1+\alpha^*))l^* - c^{y*} - s^* - g^y - g^{ind} - a^*)] dz
$$

Appendix B3

Note that $\sum_{j=n,m} N_2^j = N_2^n(1+\widetilde{\alpha})$ and $\sum_{j=n,m} N_1^j = N_1^n(1+\widetilde{\alpha})$. Thus, SP_2 becomes as follows:

$$
\begin{aligned}
SP_2 &= w_2 \tau_{w2} l^* N_2^n(1+\widetilde{\alpha}) + r_2 \tau_{r2} s^* N_1^n(1+\widetilde{\alpha}) - N_1^n(1+\widetilde{\alpha}) \times (b^* + g^o + g^{ind}) \\
&\quad - N_2^n(1+\widetilde{\alpha}) \times (g^y + g^{ind} + a^*) + (1+r_2)a_1 N_1^n(1+\widetilde{\alpha})
\end{aligned}
\tag{78}
$$

Using the definitions of τ_{w2} and τ_{r2}, we have

$$
\begin{aligned}
SP_2 &= w_2 l^* N_2^n(1+\widetilde{\alpha}) - w^*(1-\tau_w^*) l^* N_2^n(1+\widetilde{\alpha}) \\
&\quad + r_2 s^* N_1^n(1+\widetilde{\alpha}) - r^*(1-\tau_r^*) s^* N_1^n(1+\widetilde{\alpha}) - N_1^n(1+\widetilde{\alpha})(b + g^o + g^{ind}) \\
&\quad - N_2^n(1+\widetilde{\alpha})(g^y + g^{ind} + a^*) + (1+r_2)a_1 N_1^n(1+\widetilde{\alpha})
\end{aligned}
\tag{79}
$$

By changing the order in the above equation, SP_2 becomes

$$\begin{aligned}
SP_2 &= w_2 l^* N_2^n(1+\widetilde{\alpha}) + r_2 s^* N_1^n(1+\widetilde{\alpha}) \\
&- w^* l^* N_2^n(1+\widetilde{\alpha}) - r^* s^* N_1^n(1+\widetilde{\alpha}) \\
&\tau_w^* w^* l^* N_2^n(1+\widetilde{\alpha}) + \tau_r^* r^* s^* N_1^n(1+\widetilde{\alpha}) - N_1^n(1+\widetilde{\alpha})(b+g^o+g^{ind}) \\
&- N_2^n(1+\widetilde{\alpha})(g^y+g^{ind}+a^*) + (1+r_2)a_1 N_1^n(1+\widetilde{\alpha}) \\
&= w_2 l^* N_2^n(1+\widetilde{\alpha}) + r_2 s^* N_1^n(1+\widetilde{\alpha}) + (1+r_2)a_1 N_1^n(1+\widetilde{\alpha}) \\
&- w^* l^* N_2^n(1+\widetilde{\alpha}) - r^* s^* N_1^n(1+\widetilde{\alpha}) \\
&+ (1+\widetilde{\alpha})\{\tau_w^* w^* l^* N_2^n + \tau_r^* r^* s^* N_1^n - N_1^n(b+g^o+g^{ind}) - N_2^n(g^y+g^{ind}+a^*)\}
\end{aligned} \tag{80}$$

Now we calculate $\tau_w^* w^* l^* N_2^n + \tau_r^* r^* s^* N_1^n - N_1^n(b+g^o+g^{ind}) - N_2^n(g^y+g^{ind}+a^*)$. Note that $N_2^n = N_1^n((1+\pi_m)\widetilde{\alpha}+1+\pi_n) = N_1^n((1+\pi_m)\alpha^*+1+\pi_n+(1+\pi_m)\widetilde{\alpha}-(1+\pi_m)\alpha^*)$. Thus,

$$\begin{aligned}
&\tau_w^* w^* l^* N_2^n + \tau_r^* r^* s^* N_1^n - N_1^n(b+g^o+g^{ind}) - (g^y+g^{ind}+a^*)N_2^n \\
&= \tau_w^* w^* l^* N_1^n(R(\alpha^*) + (1+\pi_m)(\widetilde{\alpha}-\alpha^*)) \\
&+ \tau_r^* r^* s^* N_1^n - N_1^n(b+g^o+g^{ind}) \\
&- (g^y+g^{ind}+a^*)N_1^n(R(\alpha^*) + (1+\pi_m)(\widetilde{\alpha}-\alpha^*))
\end{aligned} \tag{81}$$

At the initial steady state, we have

$$(\tau_w^* w^* l^* - g^y - g^{ind} - a^*)R(\alpha^*)N_0^n(1+\alpha^*) + (\tau_r^* r^* s^* + (1+r^*)a^* - b^* - g^o)N_0^n(1+\alpha^*) = 0 \tag{82}$$

By dividing by $N_0^n(1+\alpha^*)$, we have

$$(\tau_w^* w^* l^* - g^y - g^{ind} - a^*)R(\alpha^*) + (\tau_r^* r^* s^* + (1+r^*)a^* - b^* - g^o) = 0 \tag{83}$$

Thus, (81) becomes

$$= \tau_w^* w^* l^* N_1^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - (g^y+g^{ind}+a^*)N_1^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - (1+r^*)a^* N_1^n \tag{84}$$

Therefore, SP_2 becomes

$$
\begin{aligned}
SP_2 = {} & w_2 l^* N_2^n(1+\widetilde{\alpha}) + r_2 s^* N_1^n(1+\widetilde{\alpha}) + (1+r_2)a_1 N_1^n(1+\widetilde{\alpha}) \\
& - w^* l^* N_2^n(1+\widetilde{\alpha}) - r^* s^* N_1^n(1+\widetilde{\alpha}) \\
& + (1+\widetilde{\alpha})\{\tau_w^* w^* l^* N_1^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) \\
& - (g^y + g^{ind} + a^*)N_1^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - (1+r^*)a^* N_1^n\} \qquad (85)
\end{aligned}
$$

Now, we decompose $w^* l^* N_2^n(1+\widetilde{\alpha})$ in the second line in the above equitation. Since $N_2^n = N_1^n R(\widetilde{\alpha}) = N_1^n(R(\alpha^*) + (1+\pi_m)(\widetilde{\alpha}-\alpha^*))$, SP_2 becomes

$$
\begin{aligned}
SP_2 = {} & w_2 l^* N_2^n(1+\widetilde{\alpha}) + r_2 s^* N_1^n(1+\widetilde{\alpha}) + (1+r_2)a_1 N_1^n(1+\widetilde{\alpha}) \\
& - (1+\widetilde{\alpha})w^* l^* N_1^n\{R(\alpha^*) + (1+\pi_m)(\widetilde{\alpha}-\alpha^*)\} - r^* s^* N_1^n(1+\widetilde{\alpha}) \\
& + (1+\widetilde{\alpha})\{\tau_w^* w^* l^* N_1^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - (g^y + g^{ind} + a^*)N_1^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - (1+r^*)a^* N_1^n\} \qquad (86)
\end{aligned}
$$

Re-arranging the second line in the above equation, we have

$$
\begin{aligned}
SP_2 = {} & w_2 l^* N_2^n(1+\widetilde{\alpha}) + r_2 s^* N_1^n(1+\widetilde{\alpha}) + (1+r_2)a_1 N_1^n(1+\widetilde{\alpha}) \\
& - (1+\widetilde{\alpha})w^* l^* N_1^n R(\alpha^*) - (1+\widetilde{\alpha})w^* l^* N_1^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - r^* s^* N_1^n(1+\widetilde{\alpha}) \\
& + (1+\widetilde{\alpha})\{\tau_w^* w^* l^* N_1^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - (g^y + g^{ind} + a^*)N_1^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - (1+r^*)a^* N_1^n\} \qquad (87)
\end{aligned}
$$

Next, we re-arrange $r_2 s^* N_1^n(1+\widetilde{\alpha}) + (1+r_2)a_1 N_1^n(1+\widetilde{\alpha})$ and $(1+r^*)a^* N_1^n$. Then, we have

$$
\begin{aligned}
SP_2 = {} & w_2 l^* N_2^n(1+\widetilde{\alpha}) + r_2(s^* + a_1)N_1^n(1+\widetilde{\alpha}) + a_1 N_1^n(1+\widetilde{\alpha}) \\
& - (1+\widetilde{\alpha})w^* l^* N_1^n R(\alpha^*) - r^*(s^* + a^*)N_1^n(1+\widetilde{\alpha}) - (1+\widetilde{\alpha})w^* l^* N_1^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) \\
& + (1+\widetilde{\alpha})\{\tau_w^* w^* l^* N_1^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - (g^y + g^{ind} + a^*)N_1^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - a^* N_1^n\} \qquad (88)
\end{aligned}
$$

Using the homogeneity of the production function, we have

$$w_2 l^* N_2^n(1+\widetilde{\alpha}) + r_2(s^*+a_1)N_1^n(1+\widetilde{\alpha}) = F(l^* N_2^n(1+\widetilde{\alpha}), (s^*+a_1)N_1^n(1+\widetilde{\alpha})) - \delta(s^*+a_1)N_1^n(1+\widetilde{\alpha})$$

and

$$\begin{aligned} & w^* l^*(1+\alpha^*)N_1^n R(\alpha^*) + r^*(s^*+a^*)N_1^n(1+\alpha^*) \\ &= F(l^*(1+\alpha^*)N_1^n R(\alpha^*),\ (s^*+a^*)N_1^n(1+\alpha^*)) \\ &\quad -\delta(s^*+a^*)N_1^n(1+\alpha^*) \end{aligned} \tag{89}$$

Thus, SP_2 becomes

$$\begin{aligned} SP_2 &= F(l^* N_2^n(1+\widetilde{\alpha}), (s^*+a_1)N_1^n(1+\widetilde{\alpha})) - \delta(s^*+a_1)N_1^n(1+\widetilde{\alpha}) + a_1 N_1^n(1+\widetilde{\alpha}) \\ &- \frac{1+\widetilde{\alpha}}{1+\alpha^*}\{F(l^*(1+\alpha^*)N_1^n R(\alpha^*),\ (s^*+a^*)N_1^n(1+\alpha^*)) - \delta(s^*+a^*)N_1^n(1+\alpha^*)\} \\ &-(1+\widetilde{\alpha})w^* l^* N_1^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) \\ &+(1+\widetilde{\alpha})\{\tau_w^* w^* l^*(1+\pi_m)(\widetilde{\alpha}-\alpha^*)N_1^n - (g^y+g^{ind}+a^*)N_1^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - a^* N_1^n\} \end{aligned} \tag{90}$$

Combining the third line and fourth line, we have

$$\begin{aligned} SP_2 &= F(l^* N_2^n(1+\widetilde{\alpha}), (s^*+a_1)N_1^n(1+\widetilde{\alpha})) - \delta(s^*+a_1)N_1^n(1+\widetilde{\alpha}) + a_1 N_1^n(1+\widetilde{\alpha}) \\ &-F(l^*(1+\widetilde{\alpha})N_1^n R(\alpha^*),\ (s^*+a^*)N_1^n(1+\widetilde{\alpha})) + \delta(s^*+a^*)N_1^n(1+\widetilde{\alpha}) \\ &+(1+\widetilde{\alpha})N_1^n\{-(1-\tau_w)w^* l^*(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - (g^y+g^{ind}+a^*)(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - a^* N_1^n\} \end{aligned} \tag{91}$$

Notice that $\delta s^* N_1^n(1+\widetilde{\alpha})$ is canceled out in the above equation. Rearranging the term $(a_1-a^*)N_1^n(1+\widetilde{\alpha})$, we have

$$\begin{aligned} SP_2 = {} & F(l^* N_2^n(1+\widetilde{\alpha}), (s^*+a_1)N_1^n(1+\widetilde{\alpha})) + (a_1-a^*)N_1^n(1+\widetilde{\alpha}) - \delta(a_1-a^*)N_1^n(1+\widetilde{\alpha}) \\ & - F(l^*(1+\widetilde{\alpha})N_1^n R(\alpha^*),\ (s^*+a^*)N_1^n(1+\widetilde{\alpha})) \\ & + (1+\widetilde{\alpha})N_1^n\{-(1-\tau_w)w^* l^*(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - (g^y+g^{ind}+a^*)(1+\pi_m)(\widetilde{\alpha}-\alpha^*)\} \end{aligned} \tag{92}$$

Subtracting and adding $F(l^* N_2^n(1+\widetilde{\alpha}), (s^*+a^*)N_1^n(1+\widetilde{\alpha}))$, SP_2 becomes

$$
\begin{aligned}
SP_2 &= F(l^* N_2^n(1+\widetilde{\alpha}), (s^*+a_1)N_1^n(1+\widetilde{\alpha})) + (1-\delta)(a_1-a^*)N_1^n(1+\widetilde{\alpha}) \\
&- F(l^* N_2^n(1+\widetilde{\alpha}), (s^*+a^*)N_1^n(1+\widetilde{\alpha})) \\
&+ F(l^* N_2^n(1+\widetilde{\alpha}), (s^*+a^*)N_1^n(1+\widetilde{\alpha})) - F(l^*(1+\widetilde{\alpha})N_1^n R(\alpha^*),\ (s^*+a^*)N_1^n(1+\widetilde{\alpha})) \\
&+ (1+\widetilde{\alpha})N_1^n\{-(1-\tau_w)w^* l^*(1+\pi_m)(\widetilde{\alpha}-\alpha^*) - (g^y+g^{ind}+a^*)(1+\pi_m)(\widetilde{\alpha}-\alpha^*)\}
\end{aligned} \tag{93}
$$

For the first and the second line in the above equation, it can be re-written as

$$
\begin{aligned}
&= F(l^* N_1^n R(\widetilde{\alpha})((1+\widetilde{\alpha}), (s^*+a_1)N_1^n(1+\widetilde{\alpha})) + (1-\delta)(a_1-a^*)N_1^n(1+\widetilde{\alpha}) \\
&- F(l^{**} N_1^n R(\widetilde{\alpha})(1+\widetilde{\alpha}), (s^*+a^*)N_1^n(1+\widetilde{\alpha})) \\
&= N_1^n(1+\widetilde{\alpha})\{F(l^* R(\widetilde{\alpha}), s^*+a_1) + (1-\delta)(a_1-a^*) \\
&- F(l^{**}(1+\widetilde{\alpha}), s^*+a^*)\} \\
&= N_1^n(1+\widetilde{\alpha}) \int_{s^*+a^*}^{s^*+a_1} [F_K(l^* R(\widetilde{\alpha}), z) + (1-\delta)]\, dz
\end{aligned} \tag{94}
$$

Next, we focus on the third and fourth lines of (93) . Note that $N_2^n = N_1^n R(\widetilde{\alpha})$. Thus, the third and fourth line of (93) can be re-written as

$$
\begin{aligned}
&\int_{\alpha^*}^{\widetilde{\alpha}} [F_L(l^*(1+\widetilde{\alpha})N_1^n R(z),\ (s^*+a^*)N_1^n(1+\widetilde{\alpha}))l^*(1+\widetilde{\alpha})N_1^n(1+\pi_m) \\
&- (1+\widetilde{\alpha})N_1^n(1+\pi_m)\{(1-\tau_w)w^* l^* + g^y + g^{ind} + a^*\}]dz \\
&= (1+\widetilde{\alpha})N_1^n(1+\pi_m) \int_{\alpha^*}^{\widetilde{\alpha}} [F_L(l^*(1+\widetilde{\alpha})N_1^n R(z),\ (s^*+a^*)N_1^n(1+\widetilde{\alpha}))l^* \\
&- ((1-\tau_w)w^* l^* + g^y + g^{ind} + a^*)]dz
\end{aligned} \tag{95}
$$

Since theF_L is homogenous degree of zero, the above equation becomes

$$
\begin{aligned}
&= (1+\widetilde{\alpha})N_1^n(1+\pi_m) \int_{\alpha^*}^{\widetilde{\alpha}} [F_L(l^* R(z),\ (s^*+a^*))l^* \\
&- \{(1-\tau_w)w^* l^* + g^y + g^{ind} + a^*\}]dz
\end{aligned}
$$

Note that $(1-\tau_w)w^* l^* = c^{y*} + s^*$ and $R'(\alpha) = 1 + \pi_m$. Therefore, SP_2 becomes as follows:

$$SP_2 = N_1^n(1+\widetilde{\alpha})\int_{s^*+a^*}^{s^*+a_1}[F_K(l^*R(\widetilde{\alpha}),z))+1-\delta]dz$$
$$(1+\widetilde{\alpha})N_1^n\int_{\alpha^*}^{\widetilde{\alpha}}R'(z)\left[F_L(l^*R(z),\ s^*+a^*)l^*\right.$$
$$\left.-\{c^{y*}+s^*+g^y+g^{ind}+a^*\}\right]dz$$

Appendix B4

To save space, I will show that $SP_t > 0$ for $t = 2, 3, ...$ For $t = 1$, the same proof is applied as in the appendix B2.

We assume the same tax adjustment as in the preceding subsection.

$$(1-\tau_{wt})w_t = (1-\tau_w^*)w^* \text{ and } (1-\tau_{rt})r_t = (1-\tau_r^*)r^* \tag{96}$$

With this tax adjustment, labor supply and saving of each individual is the same as at the initial steady state. As in the proof in the appendix B2 and B3, I assume that the government will save at least the same amount of the government saving per each young individual as at the initial steady state. Note that

$$\begin{aligned} SP_t = {} & w_t\tau_{wt}l^{n*}N_t^n + \phi^m w_t\tau_{wt}l^{m*}N_t^m + \tau_{rt}r_ts^{n*}N_{t-1}^n + \tau_{rt}r_ts^{m*}N_{t-1}^m \\ & - N_t^n(g^{y,n}+g^{ind,m}+a^*) - N_t^m(g^{y,m}+g^{ind,m}+a^*) \\ & - N_{t-1}^n(b^{n*}+g^{o,n}+g^{ind,n}) - N_{t-1}^m(b^{m*}+g^{o,m}+g^{ind,m}) \\ & + (1+r_t)a_{t-1}(N_{t-1}^n+N_{t-1}^m) \end{aligned} \tag{97}$$

Using the definition of τ_{wt} and τ_{rt}, we have

$$w_t\tau_{wt} = w_t - (1-\tau_w^*)w^* \tag{98}$$
$$r_t\tau_{rt} = r_{t+1} - (1-\tau_r^*)r^* \tag{99}$$

Thus, SP_t becomes

$$\begin{aligned}
SP_t = & w_t l^{n*} N_t^n - w^* l^{n*} N_t^n + \tau_w^* w^* l^{n*} N_t^n + \phi^m w_t l^{m*} N_t^m - \phi^m w^* l^{m*} N_t^m + \phi^m \tau_w^* w^* l^{m*} N_t^m \\
& + r_t s^{n*} N_{t-1}^n - r^* s^{n*} N_{t-1}^n + \tau_r^* r^* s^{n*} N_{t-1}^n + \\
& + r_t s^{m*} N_{t-1}^m - r^* s^{m*} N_{t-1}^m + \tau_r^* r^* s^{m*} N_{t-1}^m \\
& - N_t^n \times (g^{y,n} + g^{ind,n} + a^*) - N_t^m (g^{y,m} + g^{ind,m} + a^*) \\
& - N_{t-1}^n (b^{n*} + g^{o,n} + g^{ind,n}) - N_{t-1}^m (b^{m*} + g^{o,m} + g^{ind,m}) \\
& + (1 + r_t) a_{t-1} (N_{t-1}^n + N_{t-1}^m)
\end{aligned} \tag{100}$$

By changing the order of the above equation, we have

$$\begin{aligned}
SP_t = & w_t l^{n*} N_t^n - w^* l^{n*} N_t^n + \phi^m w_t l^{m*} N_t^m - \phi^m w^* l^{m*} N_t^m \\
& + r_t s^{n*} N_{t-1}^n - r^* s^{n*} N_{t-1}^n + r_t s^{m*} N_{t-1}^m - r^* s^{m*} N_{t-1}^m \\
& + (1 + r_t) a_{t-1} (N_{t-1}^n + N_{t-1}^m) \\
& + \tau_w^* w^* l^{n*} N_t^n + \phi^m \tau_w^* w^* l^{m*} N_t^m \\
& + \tau_r^* r^* s^{n*} N_{t-1}^n + \tau_r^* r^* s^{m*} N_{t-1}^m \\
& - N_t^n \times (g^{y,n} + g^{ind,n} + a^*) - N_t^m (g^{y,m} + g^{ind,m} + a^*) \\
& - N_{t-1}^n (b^{n*} + g^{o,n} + g^{ind,n}) - N_{t-1}^m (b^{m*} + g^{o,m} + g^{ind,m})
\end{aligned} \tag{101}$$

Now we need to calculate the fourth line to seventh line:

$$\begin{aligned}
& \tau_w^* w^* l^{n*} N_t^n + \phi^m \tau_w^* w^* l^{m*} N_t^m + \tau_r^* r^* s^{n*} N_{t-1}^n + \tau_r^* r^* s^{m*} N_{t-1}^m \\
& - N_t^n (g^{y,n} + g^{ind,n} + a^*) - N_t^m (g^{y,m} + g^{ind,m} + a^*) \\
& - N_{t-1}^n (b^{n*} + g^{o,n} + g^{ind,n}) - N_{t-1}^m (b^{m*} + g^{o,m} + g^{ind,m})
\end{aligned} \tag{102}$$

Note that $N_t^n = N_{t-1}^n R(\widetilde{\alpha})$ and $N_t^m = N_t^n \widetilde{\alpha}$. Thus,the above equation becomes as follows:

$$\begin{aligned}
&\tau_w^* w^* l^{n*} N_{t-1}^n R(\widetilde{\alpha}) + \phi^m \tau_w^* w^* l^{n*} N_{t-1}^n J(\widetilde{\alpha}) \\
&+ \tau_r^* r^* s^{n*} N_{t-1}^n + \tau_r^* r^* s^{m*} N_{t-1}^n \widetilde{\alpha} \\
&- N_{t-1}^n R(\widetilde{\alpha})(g^{y,n} + g^{ind,n} + a^*) \\
&- N_{t-1}^n J(\widetilde{\alpha})(g^{y,m} + g^{ind,m} + a^*) \\
&- N_{t-1}^n (b^{n*} + g^{o,n} + g^{ind,n}) - N_{t-1}^n \widetilde{\alpha}(b^{m*} + g^{o,m} + g^{ind,m})
\end{aligned} \tag{103}$$

where $J(\alpha)$ is defined as follows:

$$J(\alpha) = (1 + \pi_n + \alpha(1 + \pi_m))\alpha$$

On the other hand, at the initial steady state, $N_t^n = N_{t-1}^n R(\alpha^*)$and $N_t^m = N_{t-1}^n \alpha^*$ Thus, .the government budget constraint at the initial steady state implies

$$\begin{aligned}
&\tau_w^* w^* l^{n*} R(\alpha^*) + \phi^m \tau_w^* w^* l^{m*} J(\alpha^*) \\
&+ \tau_r^* r^* s^{n*} + \tau_r^* r^* s^{m*} \alpha^* \\
&- R(\alpha^*)(g^{y,n} + g^{ind,n} + a^*) - J(\alpha^*)(g^{y,m} + g^{ind,m} + a^*) \\
&- (b^{n*} + g^{o,n} + g^{ind,n}) - \alpha^*(b^{m*} + g^{o,m} + g^{ind,m}) + (1 + r^*) a^* (1 + \alpha^*) = 0
\end{aligned} \tag{104}$$

Thus, (103) becomes

$$\begin{gathered}
\tau_w^* w^* l^{n*} N_{t-1}^n (\widetilde{\alpha} - \alpha^*)(1 + \pi_m) \\
+\phi^m \tau_w^* w^* l^{m*} N_{t-1}^n (J(\widetilde{\alpha}) - J(\alpha^*)) \\
+\tau_r^* r^* s^{m*} N_{t-1}^n (\widetilde{\alpha} - \alpha^*) \\
-N_{t-1}^n (g^{y,n} + g^{ind,n} + a^*)(\widetilde{\alpha} - \alpha^*)(1 + \pi_m) \\
-N_{t-1}^n (g^{y,m} + g^{ind,m} + a^*)(J(\widetilde{\alpha}) - J(\alpha^*)) \\
-N_{t-1}^n (b^{m*} + g^{o,m} + g^{ind,m})(\widetilde{\alpha} - \alpha^*) \\
-(1 + r^*) a^* N_{t-1}^n (1 + \alpha^*)
\end{gathered} \tag{105}$$

Therefore, SP_t becomes

$$
\begin{aligned}
SP_t &= w_t l^{n*} N_t^n - w^* l^{n*} N_t^n \\
&+\phi^m w_t l^{m*} N_t^m - \phi^m w^* l^{m*} N_t^m \\
&+r_t s^{n*} N_{t-1}^n - r^* s^{n*} N_{t-1}^n \\
&+r_t s^{m*} N_{t-1}^m - r^* s^{m*} N_{t-1}^m \\
&+(1+r_t) a_{t-1}(N_{t-1}^n + N_{t-1}^m) \\
&+\tau_w^* w^* l^{n*} N_{t-1}^n (1+\pi_m)(\widetilde{\alpha} - \alpha^*) \\
&+\phi^m \tau_w^* w^* l^{m*} N_{t-1}^n \{J(\widetilde{\alpha}) - J(\alpha^*)\} \\
&+\tau_r^* r^* s^{m*} N_{t-1}^n (\widetilde{\alpha} - \alpha^*) \\
&-N_{t-1}^n (g^{y,n} + g^{ind,n} + a^*)(1+\pi_m)(\widetilde{\alpha} - \alpha^*) \\
&-N_{t-1}^n (g^{y,m} + g^{ind,m} + a^*)(J(\widetilde{\alpha}) - J(\alpha^*)) \\
&-N_{t-1}^n (b^{m*} + g^{o,m} + g^{ind,m})(\widetilde{\alpha} - \alpha^*) \\
&-(1+r^*) a^* N_{t-1}^n (1+\alpha^*) \\
&= w_t l^{n*} N_t^n + \phi^m w_t l^{m*} N_t^m \\
&+r_t s^{n*} N_{t-1}^n + r_t s^{m*} N_{t-1}^m + r_t a_{t-1}(N_{t-1}^n + N_{t-1}^m) \\
&+a_{t-1}(N_{t-1}^n + N_{t-1}^m) \\
&-\{w^* l^{n*} N_t^n + \phi^m w^* l^{m*} N_t^m + r^* s^{n*} N_{t-1}^n + r^* s^{m*} N_{t-1}^m \\
&+r^* a^* N_{t-1}^n (1+\alpha^*)\} \\
&-a^* N_{t-1}^n (1+\alpha^*) \\
&+\tau_w^* w^* l^{n*} N_{t-1}^n (1+\pi_m)(\widetilde{\alpha} - \alpha^*) \\
&+\phi^m \tau_w^* w^* l^{m*} N_{t-1}^n \{J(\widetilde{\alpha}) - J(\alpha^*)\} \\
&+\tau_r^* r^* s^{m*} N_{t-1}^n (\widetilde{\alpha} - \alpha^*) \\
&-N_{t-1}^n (g^{y,n} + g^{ind,n} + a^*)(1+\pi_m)(\widetilde{\alpha} - \alpha^*) \\
&-N_{t-1}^n (b^{m*} + g^{o,m} + g^{ind,m})(\widetilde{\alpha} - \alpha^*) \\
&-N_{t-1}^n (g^{y,m} + g^{ind,m} + a^*)(J(\widetilde{\alpha}) - J(\alpha^*))
\end{aligned} \tag{106}
$$

We add and subtract $\delta(s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m))$ to and from SP_t. We also subtract and $\delta(s^{n*} N_{t-1}^n + s^{m*} \alpha^* N_{t-1}^n + a^* N_{t-1}^n (1+\alpha^*)$ to and from SP_t. Then, we have

$$
\begin{aligned}
SP_t = w_t l^{n*} N_t^n + \phi^m w_t l^{m*} N_t^m \\
+(r_t+\delta)(s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) + a_{t-1}(N_{t-1}^n + N_{t-1}^m) \\
-\delta(s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) \\
-\{w^* l^{n*} N_t^n + \phi^m w^* l^{m*} N_t^m \\
+r^* s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a^* N_{t-1}^n (1+\alpha^*)\} \\
-\delta(s^{n*}N_{t-1}^n + s^{m*}\alpha^* N_{t-1}^n + a^*(N_{t-1}^n(1+\alpha^*)) \\
+\delta(s^{n*}N_{t-1}^n + s^{m*}\alpha^* N_{t-1}^n + a^* N_{t-1}^n(1+\alpha^*)) \\
+\tau_w^* w^* l^{n*} N_{t-1}^n (1+\pi_m)(\widetilde{\alpha}-\alpha^*) \\
+\phi^m \tau_w^* w^* l^{m*} N_{t-1}^n \{J(\widetilde{\alpha}) - J(\alpha^*)\} \\
+\tau_r^* r^* s^{m*} N_{t-1}^n (\widetilde{\alpha}-\alpha^*) \\
-N_{t-1}^n (g^{y,n} + g^{ind,n} + a^*)(1+\pi_m)(\widetilde{\alpha}-\alpha^*) \\
-N_{t-1}^n (g^{y,m} + g^{ind,m} + a^*)\{J(\widetilde{\alpha}) - J(\alpha^*)\} \\
-N_{t-1}^n (b^{m*} + g^{o,m} + g^{ind,m})(\widetilde{\alpha}-\alpha^*) \\
-a^* N_{t-1}^n (1+\alpha^*)
\end{aligned} \tag{107}
$$

Note that the first three line of the above equation becomes

$$
\begin{aligned}
&F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) \\
&+ (1-\delta) a_{t-1}(N_{t-1}^n + N_{t-1}^m)) - \delta(s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m)
\end{aligned} \tag{108}
$$

Next, we focus on $w^* l^{n*} N_t^n + \phi^m w^* l^{m*} N_t^m + r^*(s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m + r^* a^*(N_{t-1}^n + \alpha^* N_{t-1}^n)) + \delta(s^{n*}N_{t-1}^n + s^{m*}\alpha^* N_{t-1}^n + a^* N_{t-1}^n(1+\alpha^*))$. Note that $N_t^n = R(\widetilde{\alpha})N, N_t^m =$

$N^n_{t-1}J(\widetilde{\alpha}), N^m_{t-1} = N^n_{t-1}\widetilde{\alpha}$. Thus, we have

$$\begin{aligned}
& w^* l^{n*} N^n_t + \phi^m w^* l^{m*} N^m_t + r^*(s^{n*} N^n_{t-1} + s^{m*} N^m_{t-1} + a^* N^n_{t-1}(1+\alpha^*)) \\
& + \delta(s^{n*} N^n_{t-1} + s^{m*} \alpha^* N^n_{t-1} + a^* N^n_{t-1}(1+\alpha^*)) \\
& = w^* l^{n*} N^n_{t-1} R(\widetilde{\alpha}) + \phi^m w^* l^{m*} N^n_{t-1} J(\widetilde{\alpha}) \\
& + r^* s^{n*} N^n_{t-1} + r^* s^{m*} N^n_{t-1}\widetilde{\alpha} + r^* a^* N^n_{t-1}(1+\alpha^*)) \\
& + \delta(s^{n*} N^n_{t-1} + s^{m*} \alpha^* N^n_{t-1} + a^* N^n_{t-1}(1+\alpha^*)) \\
& = w^* l^{n*} N^n_{t-1}\{R(\alpha^*) + (1+\pi_m)(\widetilde{\alpha} - \alpha^*)\} \\
& + \phi^m w^* l^{m*} N^n_{t-1} l^{m*}(J(\alpha^*) + J(\widetilde{\alpha}) - J(\alpha^*)) \\
& + r^* s^{n*} N^n_{t-1} + r^* s^{m*} N^n_{t-1}(\alpha^* + \widetilde{\alpha} - \alpha^*) + r^* a^* N^n_{t-1}(1+\alpha^*)) \\
& + \delta(s^{n*} N^n_{t-1} + s^{m*} \alpha^* N^n_{t-1} + r^* a^* N^n_{t-1}(1+\alpha^*)) \\
& = w^* N^n_{t-1}[R(\alpha^*) l^{n*} + \phi^m J(\alpha^*) l^m] \\
& + (r^* + \delta)[s^{n*} N^n_{t-1} + s^{m*} N^n_{t-1}\alpha^* + a^* N^n_{t-1}(1+\alpha^*)] \\
& + w^* l^{n*} N^n_{t-1}(1+\pi_m)(\widetilde{\alpha} - \alpha^*) + \phi^m w^* l^{m*} N^n_{t-1}(J(\widetilde{\alpha}) - J(\alpha^*)) \\
& + r^* s^{m*} N^n_{t-1}(\widetilde{\alpha} - \alpha^*)
\end{aligned} \tag{109}$$

On the other hand, from the homogeneity of the production function and Euler's theorem, (109) becomes

$$\begin{aligned}
& F(R(\alpha^*) l^{n*} + \phi^m J(\alpha^*) l^{m*}) N^n_{t-1}, s^{n*} N^n_{t-1} + s^{m*} N^n_{t-1}\alpha^* + a^* N^n_{t-1}(1+\alpha^*)) \\
& + w^* l^{n*} N^n_{t-1}(1+\pi_m)(\widetilde{\alpha} - \alpha^*) \\
& + \phi^m w^* l^{m*} N^n_{t-1} l^{m*}(J(\widetilde{\alpha}) - J(\alpha^*)) \\
& + r^* s^{m*} N^n_{t-1}(\widetilde{\alpha} - \alpha^*)
\end{aligned} \tag{110}$$

Similarly, we have the following relationship. Thus, SP_t becomes as follows:

$$
\begin{aligned}
SP_t = & F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) \\
& + (1-\delta) a_{t-1} N_{t-1}^n (1+\widetilde{\alpha}) - \delta(s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m) \\
& - F(R(\alpha^*) l^{n*} N_{t-1}^n + \phi^m J(\alpha^*) l^{m*} N_{t-1}^n, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^n \alpha^* + a^* N_{t-1}^n (1+\alpha^*)) \\
& + \delta(s^{n*} N_{t-1}^n + s^{m*} \alpha^* N_{t-1}^n + a^* N_{t-1}^n (1+\alpha^*)) \\
& - w^* l^{n*} N_{t-1}^n (1+\pi_m)(\widetilde{\alpha} - \alpha^*) \\
& - \phi^m w^* l^{m*} N_{t-1}^n \{J(\widetilde{\alpha}) - J(\alpha^*)\} \\
& - r^* s^{m*} N_{t-1}^n (\widetilde{\alpha} - \alpha^*) \\
& + \tau_w^* w^* l^{n*} N_{t-1}^n (1+\pi_m)(\widetilde{\alpha} - \alpha^*) \\
& + \phi^m \tau_w^* w^* l^{m*} N_{t-1}^n (J(\widetilde{\alpha}) - J(\alpha^*)) \\
& + \tau_r^* r^* s^{m*} N_{t-1}^n (\widetilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n (1+\pi_m)(g^{y,n} + g^{ind,n} + a^*)(\widetilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n (g^{y,m} + g^{ind,m} + a^*)\{J(\widetilde{\alpha}) - J(\alpha^*)\} \\
& - N_{t-1}^n (b^{m*} + g^{o,m} + g^{ind,m})(\widetilde{\alpha} - \alpha^*) \\
& - a^* N_{t-1}^n (1+\alpha^*)
\end{aligned} \tag{111}
$$

Combining 4th line and 14th line, we have

$$
\begin{aligned}
SP_t = F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) \\
+(1-\delta) a_{t-1} N_{t-1}^n (1+\widetilde{\alpha}) - \delta(s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m) \\
-F(R(\alpha^*) l^{n*} N_{t-1}^n + \phi^m J(\alpha^*) l^{m*} N_{t-1}^n, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^n \alpha^* + a^* N_{t-1}^n (1+\alpha^*)) \\
+\delta(s^{n*} N_{t-1}^n + s^{m*} \alpha^* N_{t-1}^n) - (1-\delta) a^* N_{t-1}^n (1+\alpha^*)) \\
-w^* l^{n*} N_{t-1}^n (1+\pi_m)(\widetilde{\alpha} - \alpha^*) \\
-\phi^m w^* l^{m*} N_{t-1}^n \{J(\widetilde{\alpha}) - J(\alpha^*)\} \\
-r^* s^{m*} N_{t-1}^n (\widetilde{\alpha} - \alpha^*) \\
+\tau_w^* w^* l^{n*} N_{t-1}^n (1+\pi_m)(\widetilde{\alpha} - \alpha^*) \\
+\phi^m \tau_w^* w^* l^{m*} N_{t-1}^n (J(\widetilde{\alpha}) - J(\alpha^*)) \\
+\tau_r^* r^* s^{m*} N_{t-1}^n (\widetilde{\alpha} - \alpha^*) \\
-N_{t-1}^n (1+\pi_m)(g^{y,n} + g^{ind,n} + a^*)(\widetilde{\alpha} - \alpha^*) \\
-N_{t-1}^n (g^{y,m} + g^{ind,m} + a^*)\{J(\widetilde{\alpha}) - J(\alpha^*)\} \\
-N_{t-1}^n (b^{m*} + g^{o,m} + g^{ind,m})(\widetilde{\alpha} - \alpha^*)
\end{aligned}
$$

Note that $-(1-\delta)a^* N_{t-1}^n (1+\alpha^*) = -(1-\delta)a^* N_{t-1}^n (1+\widetilde{\alpha}+\alpha^*-\widetilde{\alpha}) = -(1-\delta)a^* N_{t-1}^n (1+\widetilde{\alpha}) + (1-\delta) N_{t-1}^n (\widetilde{\alpha} - \alpha^*)$. Also note that $\delta s^{n*} N_{t-1}^n$ is canceled out from the second and fourth lines. Also note that in the second line $N_{t-1}^m = N_{t-1}^n (\alpha^* + \widetilde{\alpha} - \alpha^*)$. Thus,

$\delta s^{m*} N^n_{t-1} \alpha^*$ is canceled out from the second and fourth line. Thus, we have

$$
\begin{aligned}
SP_t = F(N^n_t l^{n*} + \phi^m N^m_t l^{m*}, s^{n*} N^n_{t-1} + s^{m*} N^m_{t-1} + a_{t-1}(N^n_{t-1} + N^m_{t-1})) \\
+(1-\delta) a_{t-1} N^n_{t-1}(1+\widetilde{\alpha}) \\
-F(R(\alpha^*) l^{n*} N^n_{t-1} + \phi^m J(\alpha^*) l^{m*} N^n_{t-1}, s^{n*} N^n_{t-1} + s^{m*} N^n_{t-1}\alpha^* + a^*(N^n_{t-1} + N^n_{t-1}\alpha^*)) \\
-(1-\delta) a^* N^n_{t-1}(1+\widetilde{\alpha}) + (1-\delta) a^* N^n_{t-1}(\widetilde{\alpha}-\alpha^*) \\
-\delta s^{m*} N^n_{t-1}(\widetilde{\alpha}-\alpha^*) \\
-w^* l^{n*} N^n_{t-1}(1+\pi_m)(\widetilde{\alpha}-\alpha^*) \\
-\phi^m w^* l^{m*} N^n_{t-1}\{J(\widetilde{\alpha}) - J(\alpha^*)\} \\
-r^* s^{m*} N^n_{t-1}(\widetilde{\alpha}-\alpha^*) \\
+\tau^*_w w^* l^{n*} N^n_{t-1}(1+\pi_m)(\widetilde{\alpha}-\alpha^*) \\
+\phi^m \tau^*_w w^* l^{m*} N^n_{t-1}(J(\widetilde{\alpha}) - J(\alpha^*)) \\
+\tau^*_r r^* s^{m*} N^n_{t-1}(\widetilde{\alpha}-\alpha^*) \\
-N^n_{t-1}(1+\pi_m)(g^{y,n} + g^{ind,n} + a^*)(\widetilde{\alpha}-\alpha^*) \\
-N^n_{t-1}(g^{y,m} + g^{ind,m} + a^*)\{J(\widetilde{\alpha}) - J(\alpha^*)\} \\
-N^n_{t-1}(b^{m*} + g^{o,m} + g^{ind,m})(\widetilde{\alpha}-\alpha^*)
\end{aligned}
$$

We subtract and add $F(N^n_t l^{n*} + \phi^m N^m_t l^{m*}, s^{n*} N^n_{t-1} + s^{m*} N^m_{t-1} + a^*(N^n_{t-1} + N^m_{t-1}))$ and $s^{m*} N^n_{t-1}(\widetilde{\alpha}-\alpha^*)$ from and to SP_t. We also combine $s^{m*} N^n_{t-1}(\widetilde{\alpha}-\alpha^*), r^* s^{m*} N^n_{t-1}(\widetilde{\alpha}-\alpha^*)$

and $\tau_r^* s^{m*} N_{t-1}^n(\widetilde{\alpha}-\alpha^*)$. Then, SP_t becomes as follows

$$
\begin{aligned}
SP_t = & F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) \\
& - F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m + a^*(N_{t-1}^n + N_{t-1}^m)) \\
& + (1-\delta)a_{t-1}N_{t-1}^n(1+\widetilde{\alpha}) - (1-\delta)a^*N_{t-1}^n(1+\widetilde{\alpha}) \\
& + F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m + a^*(N_{t-1}^n + N_{t-1}^m)) \\
& - F(N_t^n R(\alpha^*) l^{n*} + \phi^m N_t^n J(\alpha^*) l^{m*}, s^{n*}N_t^n + s^{m*}N_t^n\alpha^* + a^*(N_{t-1}^n + \alpha^* N_{t-1}^n)) \\
& + (1-\delta)a^*N_{t-1}^n(\widetilde{\alpha}-\alpha^*) + (1-\delta)s^{m*}N_{t-1}^n(\widetilde{\alpha}-\alpha^*) \\
& - (1-\tau_w^*)w^* l^{n*} N_t^n(1+\pi_m)(\widetilde{\alpha}-\alpha^*) \\
& - (1-\tau_w^*)\phi^m w^* l^{m*} N_t^n\{J(\widetilde{\alpha}) - J(\alpha^*)\} \\
& - (1+(1-\tau_r^*)r^*)s^{m*}N_t^n(\widetilde{\alpha}-\alpha^*) \\
& - N_t^n(1+\pi_m)(g^{y,n} + g^{ind,n} + a^*)(\widetilde{\alpha}-\alpha^*) \\
& - N_t^n(g^{y,m} + g^{ind,m} + a^*)\{J(\widetilde{\alpha}) - J(\alpha^*)\} \\
& - N_t^n(b^{m*} + g^{o,m} + g^{ind,m})(\widetilde{\alpha}-\alpha^*)
\end{aligned} \tag{112}
$$

Note that $N_t^n = N_{t-1}^n R(\widetilde{\alpha})$ and $N_t^m = N_{t-1}^n J(\widetilde{\alpha})$. Then, the first three lines of (112) can be re-written as follows:

$$
\begin{aligned}
& F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) \\
& - F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m + a^*(N_{t-1}^n + N_{t-1}^m)) \\
& + (1-\delta)a_{t-1}N_{t-1}^n(1+\widetilde{\alpha}) - (1-\delta)a^*N_{t-1}^n(1+\widetilde{\alpha}) \\
& = F(N_{t-1}^n R(\widetilde{\alpha}) l^{n*} + \phi^m N_{t-1}^n J(\widetilde{\alpha}) l^{m*}, s^{n*}N_{t-1}^n + s^{m*}\widetilde{\alpha}N_{t-1}^n + a_{t-1}(N_{t-1}^n + \widetilde{\alpha}N_{t-1}^n)) \\
& - F(N_{t-1}^n R(\widetilde{\alpha}) l^{n*} + \phi^m N_{t-1}^n J(\widetilde{\alpha}) l^{m*}, s^{n*}N_{t-1}^n + s^{m*}\widetilde{\alpha}N_{t-1}^n + a^*(N_{t-1}^n + \widetilde{\alpha}N_{t-1}^n)) \\
& + (1-\delta)a_{t-1}N_{t-1}^n(1+\widetilde{\alpha}) - (1-\delta)a^*N_{t-1}^n(1+\widetilde{\alpha}) \\
& = N_{t-1}^n\{F(R(\widetilde{\alpha})l^{n*} + \phi^m J(\widetilde{\alpha})l^{m*}, s^{n*} + s^{m*}\widetilde{\alpha} + a_{t-1}(1+\widetilde{\alpha})) \\
& - F(R(\widetilde{\alpha})l^{n*} + \phi^m J(\widetilde{\alpha})l^{m*}, s^{n*} + s^{m*}\widetilde{\alpha} + a^*(1+\widetilde{\alpha})) \\
& + (1-\delta)a_{t-1}(1+\widetilde{\alpha}) - (1-\delta)a^*(1+\widetilde{\alpha})\} \\
& = N_{t-1}^n \int_{s^{n*}+s^{m*}\widetilde{\alpha}+a^*(1+\widetilde{\alpha}))}^{s^{n*}+s^{m*}\widetilde{\alpha}+a_{t-1}(1+\widetilde{\alpha})} [F_K(R(\widetilde{\alpha})l^{n*} + \phi^m J(\widetilde{\alpha})l^{m*}, z) + (1-\delta)]dz
\end{aligned} \tag{113}
$$

The fourth line and the fifth line of (112)can be transformed as follows

$$\begin{aligned}
&F(N^n_{t-1}R(\widetilde{\alpha})l^{n*} + \phi^m N^n_{t-1}J(\widetilde{\alpha})l^{m*}, s^{n*}N^n_{t-1} + s^{m*}N^n_{t-1}\widetilde{\alpha} + a^*(N^n_{t-1} + N^n_{t-1}\widetilde{\alpha}))\\
&- F(N^n_{t-1}R(\alpha^*)l^{n*} + \phi^m N^n_t J(\alpha^*)l^{m*}, s^{n*}N^n_{t-1} + s^{m*}N^n_{t-1}\alpha^* + a^*(N^n_{t-1} + \alpha^* N^n_{t-1}))\\
&= N^n_{t-1}\{F(R(\widetilde{\alpha})l^{n*} + \phi^m J(\widetilde{\alpha})l^{m*}, s^{n*} + s^{m*}\widetilde{\alpha} + a^*(1+\widetilde{\alpha}))\\
&- F(R(\alpha^*)l^{n*} + \phi^m J(\alpha^*)l^{m*}, s^{n*} + s^{m*}\alpha^* + a^*(1+\alpha^*))\\
&= N^n_{t-1}\int_{\alpha^*}^{\widetilde{\alpha}}[F_L(R(z)l^{n*} + \phi^m J(z)l^{m*}, s^{n*} + s^{m*}z + a^*(1+z))(R'(z)l^{n*} + \phi^m J'(z)l^{m*})\\
&+ F_K(R(z)l^{n*} + \phi^m J(z)l^{m*}, s^{n*} + s^{m*}z + a^*(1+z))(s^{m*} + a^*)]dz \qquad (114)
\end{aligned}$$

. Then, SP_t becomes as follows:

$$\begin{aligned}
SP_t = N^n_{t-1}&\int_{s^{n*}+s^{m*}\widetilde{\alpha}+a^*(1+\widetilde{\alpha}))}^{s^{n*}+s^{m*}\widetilde{\alpha}+a_{t-1}(1+\widetilde{\alpha})}[F_K(R(\widetilde{\alpha})l^{n*} + \phi^m J(\widetilde{\alpha})l^{m*}, z) + (1-\delta)]dz\\
&+ N^n_{t-1}\int_{\alpha^*}^{\widetilde{\alpha}}[F_L(R(z)l^{n*} + \phi^m J(z)l^{m*}, s^{n*} + s^{m*}z + a^*(1+z))(R'(z)l^{n*} + \phi^m J'(z)l^{m*})\\
&+ F_K(R(z)l^{n*} + \phi^m J(z)l^{m*}, s^{n*} + s^{m*}z + a^*(1+z))(s^{m*} + a^*)]dz\\
&+ (1-\delta)N^n_{t-1}(a^* + s^{m*})(\widetilde{\alpha} - \alpha^*)\\
&- (1-\tau^*_w)w^*l^{n*}N^n_t(1+\pi_m)(\widetilde{\alpha} - \alpha^*)\\
&- (1-\tau^*_w)\phi^m w^* l^{m*}N^n_t\{J(\widetilde{\alpha}) - J(\alpha^*)\}\\
&- (1 + (1-\tau^*_r)r^*)s^{m*}N^n_t(\widetilde{\alpha} - \alpha^*)\\
&- N^n_t(1+\pi_m)(g^{y,n} + g^{ind,n})(\widetilde{\alpha} - \alpha^*)\\
&- N^n_t(g^{y,m} + g^{ind,m})\{J(\widetilde{\alpha}) - J(\alpha^*)\}\\
&- N^n_t(b^{m*} + g^{o,m} + g^{ind,m})(\widetilde{\alpha} - \alpha^*) \qquad (115)
\end{aligned}$$

Let $c^{o,m*}$ be the consumption of old immigrants at the initial steady state. From the individual budget constraint, $c^{o,m*} = b^m + (1 + (1-\tau^*_r)r^*)s^{m*}$. Thus, SP_t becomes

$$
\begin{aligned}
SP_t = {} & N_{t-1}^n \int_{s^{n*}+s^{m*}\widetilde{\alpha}+a^*(1+\widetilde{\alpha}))}^{s^{n*}+s^{m*}\widetilde{\alpha}+a_{t-1}(1+\widetilde{\alpha})} [F_K(R(\widetilde{\alpha})l^{n*} + \phi^m J(\widetilde{\alpha})l^{m*}, z) + (1-\delta)]dz \\
& + N_{t-1}^n \int_{\alpha^*}^{\widetilde{\alpha}} [\widetilde{F}_L \times (R'(z)l^{n*} + \phi^m J'(z)l^{m*}) \\
& + \widetilde{F}_K \times (s^{m*} + a^*)]dz \\
& + (1-\delta)N_{t-1}^n a^*(\widetilde{\alpha} - \alpha^*) \\
& (1-\delta)s^{m*}N_{t-1}^n(\widetilde{\alpha} - \alpha^*) \\
& - (1-\tau_w^*)w^* l^{n*} N_{t-1}^n (1+\pi_m)(\widetilde{\alpha} - \alpha^*) \\
& - (1-\tau_w^*)\phi^m w^* l^{m*} N_{t-1}^n \{J(\widetilde{\alpha}) - J(\alpha^*)\} \\
& - (1 + (1-\tau_r^*)r^*)s^{m*}N_{t-1}^n(\widetilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n(1+\pi_m)(g^{y,n} + g^{ind,n} + a^*)(\widetilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n(g^{y,m} + g^{ind,m} + a^*)\{J(\widetilde{\alpha}) - J(\alpha^*)\} \\
& - N_{t-1}^n(c^{o,m*} + g^{o,m} + g^{ind,m})(\widetilde{\alpha} - \alpha^*) \qquad (116) \\
\text{where } \widetilde{F}_L = {} & F_L(R(z)l^{n*} + \phi^m J(z)l^{m*}, s^{n*} + s^{m*}z + a^*(1+z)) \\
\widetilde{F}_K = {} & F_K(R(z)l^{n*} + \phi^m J(z)l^{m*}, s^{n*} + s^{m*}z + a^*(1+z))
\end{aligned}
$$

From fourth line to twelfth line, we can rearrange as follows:

$$
\begin{aligned}
& N_{t-1}^n \int_{\alpha^*}^{\widetilde{\alpha}} \{(1-\delta)(s^{m*} + a^*) \\
& - R'(\alpha)(1-\tau_w^*)w^* l^{n*} - (1-\tau_w^*)\phi^m w^* l^{m*} J^{'}(z) \\
& - R'(\alpha)(g^{y,n} + g^{ind,n} + a^*) \\
& - (g^{y,m} + g^{ind,m} + a^*)J'(z) \\
& - (c^{o,m*} + g^{o,m} + g^{ind,m})\}dz \qquad (117)
\end{aligned}
$$

Thus, SP_t becomes as follows:

$$SP_t = N^n_{t-1}\int_{s^{n*}+s^{m*}\widetilde{\alpha}+a^*(1+\widetilde{\alpha}))}^{s^{n*}+s^{m*}\widetilde{\alpha}+a_{t-1}(1+\widetilde{\alpha})}[F_K(R(\widetilde{\alpha})l^{n*}+\phi^m J(\widetilde{\alpha})l^{m*},z)+(1-\delta)]dz$$
$$+ N^n_{t-1}\int_{\alpha^*}^{\widetilde{\alpha}}\{\widetilde{F}_L \times (R'(z)l^{n*}+\phi^m J'(z)l^{m*})$$
$$+ (\widetilde{F}_K \times (s^{m*}+a^*)$$
$$- R'(\alpha)(1-\tau^*_w)w^*l^{n*} - (1-\tau^*_w)\phi^m w^* l^{m*}J^{'}(z)$$
$$- R'(\alpha)(g^{y,n}+g^{ind,n}+a^*) - (g^{y,m}+g^{ind,m}+a^*)J'(z) - (c^{o,m*}+g^{o,m}+g^{ind,m})\}dz \tag{118}$$

Note that $(1-\tau^*_w)w^*l^{n*}$ is the after-tax income of the native when the native is young at the initial steady state. From the individual budget constraint, this is equal to $c^{y,n*}+s^{n*}$. Similarly, $(1-\tau^*_w)\phi^m w^* l^{m*} = c^{y,m*}+s^{m*}$. Therefore, we have

$$SP_t = N^n_{t-1}\int_{s^{n*}+s^{m*}\widetilde{\alpha}+a^*(1+\widetilde{\alpha}))}^{s^{n*}+s^{m*}\widetilde{\alpha}+a_{t-1}(1+\widetilde{\alpha})}[F_K(R(\widetilde{\alpha})l^{n*}+\phi^m J(\widetilde{\alpha})l^{m*},z)+(1-\delta)]dz$$
$$+ N^n_{t-1}\int_{\alpha^*}^{\widetilde{\alpha}}\{R'(z)[\widetilde{F}_L l^{n*} - (c^{y,n*}+s^{n*}+g^{y,n}+g^{ind,n}+a^*)]$$
$$+ R'(z)z[\widetilde{F}_L\phi^m l^{m*} - (c^{y,m*}+s^{m*}.+g^{y,m}+g^{ind,m}+a^*)]$$
$$+ R(z)[\widetilde{F}_L\phi^m l^{m*} - (c^{y,m*}+s^{m*}.+g^{y,m}+g^{ind,m}+a^*)] \tag{119}$$
$$+ (\widetilde{F}_K+1-\delta)(s^{m*}+a^*) - (c^{o,m*}+g^{o,m}+g^{ind,m})\}dz \tag{120}$$

where $\widetilde{F}_L = F_L(R(z)l^{n*}+\phi^m J(z)l^{m*}, s^{n*}+s^{m*}z+a^*(1+z))$
$\widetilde{F}_K = F_K(R(z)l^{n*}+\phi^m J(z)l^{m*}, s^{n*}+s^{m*}z+a^*(1+z))$

Note that $J(z) = R(z)z$ and $J'(z) = R(z)+R'(z)z$. In the above equation, the first line is the effect of increasing the government savings. The second line is the MPL condition for the native. The third line is the MPL condition for immigrants. The fourth and firth lines measure the intra-redistributional effect.

Appendix B5

Now, to see the correctness of the above equation, check what will happen to (**??**) when natives and immigrants have the same productivities and the same preferences. First, note that $l^n = l^m$ and $g^{y,n} = g^{y,m}$, $s^{n*} = s^{m*}$ and $\phi^m = 1$ when natives and immigrants have the same preferences and productivities Thus, we have

$$
\begin{aligned}
SP_t = {} & N_{t-1}^n \int_{(s^{n*}+a^*)(1+\widetilde{\alpha}))}^{(s^{n*}+a_{t-1})(1+\widetilde{\alpha})} [F_K(R(\widetilde{\alpha})l^{n*}(1+\widetilde{\alpha}), z) + (1-\delta)]dz \\
& + N_{t-1}^n \int_{\alpha^*}^{\widetilde{\alpha}} R'(z)[\widetilde{F}_L l^{n*} - (c^{y,n*} + s^{n*} + g^{y,n} + g^{ind,n} + a^*)]dz \\
& + N_{t-1}^n \int_{\alpha^*}^{\widetilde{\alpha}} J'(z)[\widetilde{F}_L l^{n*} - (c^{y,n*} + s^{n*} + g^{y,n} + g^{ind,n} + a^*)]J'(z)dz \\
& + N_{t-1}^n \times (s^{n*} + a^*) \int_{\alpha^*}^{\widetilde{\alpha}} [\widetilde{F}_K + 1 - \delta]dz \\
& - N_{t-1}^n \int_{\alpha^*}^{\widetilde{\alpha}} (c^{o,n*n} + g^{o,n} + g^{ind,n})dz \qquad (121)
\end{aligned}
$$

$$
\text{where } \widetilde{F}_L = F_L(R(z)l^{n*} + J(z)l^{n*}, s^{n*} + s^{n*}z + a^*(1+z)) \text{ and}
$$
$$
\widetilde{F}_K = F_K(R(z)l^{n*} + J(z)l^{n*}, s^{n*} + s^{n*}z + a^*(1+z))
$$

The first line of (121) can be transformed as follows:

$$
\begin{aligned}
& N_{t-1}^n\{F(R(\widetilde{\alpha})l^{n*}(1+\widetilde{\alpha}), (s^{n*} + a_{t-1})(1+\widetilde{\alpha})) + (1-\delta)(s^{n*} + a_{t-1})(1+\widetilde{\alpha}) \\
& - F(R(\widetilde{\alpha})l^{n*}(1+\widetilde{\alpha}), (s^{n*} + a^*)(1+\widetilde{\alpha})) - (1-\delta)(s^{n*} + a^*)(1+\widetilde{\alpha})\} \\
& = N_{t-1}^n(1+\widetilde{\alpha})\{\{F(R(\widetilde{\alpha})l^{n*}, (s^{n*} + a_{t-1})) + (1-\delta)(s^{n*} + a_{t-1}) \\
& - F(R(\widetilde{\alpha})l^{n*}, (s^{n*} + a^*)) - (1-\delta)(s^{n*} + a^*)\} \\
& = N_{t-1}^n(1+\widetilde{\alpha}) \int_{s^{n*}+a^*}^{s^{n*}+a_{t-1}} [F_K(R(\widetilde{\alpha})l^{n*}, z) + (1-\delta)]dz
\end{aligned}
$$

Thus,2nd to 4th line of (121) can be transformed as follows:

$$
\begin{aligned}
& + N_{t-1}^n F(R(\widetilde{\alpha})l^{n*} + J(\widetilde{\alpha})l^{n*}, s^{n*} + s^{n*}\widetilde{\alpha} + a^*(1+\widetilde{\alpha})) \\
& - N_{t-1}^n F(R(\alpha^*)l^{n*} + J(\alpha^*)l^{n*}, s^{n*} + s^{n*}\alpha^* + a^*(1+\alpha^*))\} \\
& + N_{t-1}^n(1-\delta)(s^{n*} + a^*)(\widetilde{\alpha} - \alpha^*) \\
& - (1+\pi_m)N_{t-1}^n(c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})(\widetilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n(c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})(J(\widetilde{\alpha}) - J(\alpha^*)) \\
& - N_{t-1}^n(c^{o,n*n} + g^{o,n} + g^{ind,n})(\widetilde{\alpha} - \alpha^*)
\end{aligned}
$$

On the other hand, from the resource constraint at the initial steady state we have

$$
\begin{aligned}
&F(l^{n^*}R(\alpha^*)(1+\alpha^*)N_0, (s^*+a^*)(1+\alpha^*)N_0 + (1-\delta)(s^{n*}+a^*)N_0(1+\alpha^*)N_0 \\
&= (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})N_0 R(\alpha^*)(1+\alpha^*) \\
&+ (c^{o,n*} + g^o + g^{ind})(1+\alpha^*)
\end{aligned}
$$

We divide the above resource constraint by $N_0(1+\alpha^*)$ and multiply $N^n_{t-1}(\widetilde{\alpha}-\alpha^*)$. Then, we have

$$
\begin{aligned}
&F(l^{n^*}R(\alpha^*)N^n_{t-1}, (s^*+a^*)N_{t-1}(\widetilde{\alpha}-\alpha^*) + (1-\delta)(s^{n*}+a^*)N^n_{t-1}(\widetilde{\alpha}-\alpha^*) \\
&= (1+\alpha^*)R(\alpha^*)(c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})(\widetilde{\alpha}-\alpha^*) \\
&+ (c^{o,n*} + g^o + g^{ind})N^n_{t-1}(\widetilde{\alpha}-\alpha^*)
\end{aligned}
$$

Solving for $-(c^{o,n*} + g^o + g^{ind})N^n_{t-1}(\widetilde{\alpha}-\alpha^*)$, we have

$$
\begin{aligned}
-(c^{o,n*} + g^o + g^{ind})N^n_{t-1}(\widetilde{\alpha}-\alpha^*) &= -F(l^{n^*}R(\alpha^*)N^n_{t-1}, (s^*+a^*)N_{t-1})(\widetilde{\alpha}-\alpha^*) \\
&- (1-\delta)(s^{n*}+a^*)N^n_{t-1}(\widetilde{\alpha}-\alpha^*) \\
&+ (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})N^n_{t-1}R(\alpha^*)(\widetilde{\alpha}-\alpha^*) \\
&= -N^n_{t-1}\{F(l^{n^*}R(\alpha^*), s^*+a^*)(\widetilde{\alpha}-\alpha^*) \\
&+ (1-\delta)(s^{n*}+a^*)(\widetilde{\alpha}-\alpha^*) \\
&- (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})R(\alpha^*)(\widetilde{\alpha}-\alpha^*)\}
\end{aligned}
$$

Thus, SP_t becomes

$$
\begin{aligned}
SP_t &= N^n_{t-1}(1+\widetilde{\alpha})\int_{s^{n*}+a^*}^{s^{n*}+a_{t-1}} [F_K(R(\widetilde{\alpha})l^{n*}, z) + (1-\delta)]dz \\
&+ N^n_{t-1}\{F(R(\widetilde{\alpha})l^{n*} + J(\widetilde{\alpha})l^{n*}, s^{n*} + s^{n*}\widetilde{\alpha} + a^*(1+\widetilde{\alpha})) \\
&- F(R(\alpha^*)l^{n*} + J(\alpha^*)l^{n*}, s^{n*} + s^{n*}\alpha^* + a^*(1+\alpha^*)) \\
&- F(l^{n^*}R(\alpha^*), s^*+a^*)(\widetilde{\alpha}-\alpha^*) \\
&- (1+\pi_m)(c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})(\widetilde{\alpha}-\alpha^*) \\
&- (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})(J(\widetilde{\alpha}) - J(\alpha^*)) \\
&+ (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})R(\alpha^*)(\widetilde{\alpha}-\alpha^*)\} \qquad (122)
\end{aligned}
$$

Note that $(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})J(\alpha^*)$ and $(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})R(\alpha^*)\alpha^*$ are canceled out. Thus, SP_t becomes

$$\begin{aligned}
SP_t = N^n_{t-1}(1+\widetilde{\alpha})&\int_{s^{n*}+a^*}^{s^{n*}+a_{t-1}}[F_K(R(\widetilde{\alpha})l^{n*},z)+(1-\delta)]dz \\
&+N^n_{t-1}\{F(R(\widetilde{\alpha})l^{n*}+J(\widetilde{\alpha})l^{n*},s^{n*}+s^{n*}\widetilde{\alpha}+a^*(1+\widetilde{\alpha})) \\
&-F(R(\alpha^*)l^{n*}+J(\alpha^*)l^{n*},s^{n*}+s^{n*}\alpha^*+a^*(1+\alpha^*)) \\
&-F(l^{n^*}R(\alpha^*),s^*+a^*)(\widetilde{\alpha}-\alpha^*) \\
&-(1+\pi_m)(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})(\widetilde{\alpha}-\alpha^*) \\
&-(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})J(\widetilde{\alpha}) \\
&+(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})R(\alpha^*)\widetilde{\alpha}\}
\end{aligned} \tag{123}$$

Note that $-(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})J(\widetilde{\alpha})+(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})R(\alpha^*)\widetilde{\alpha}$ become equal to

$$-(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})(1+\pi_m)\widetilde{\alpha}(\widetilde{\alpha}-\alpha^*)$$

Thus, SP_t becomes

$$\begin{aligned}
SP_t = N^n_{t-1}(1+\widetilde{\alpha})&\int_{s^{n*}+a^*}^{s^{n*}+a_{t-1}}[F_K(R(\widetilde{\alpha})l^{n*},z)+(1-\delta)]dz \\
&+N^n_{t-1}\{F(R(\widetilde{\alpha})l^{n*}+J(\widetilde{\alpha})l^{n*},s^{n*}+s^{n*}\widetilde{\alpha}+a^*(1+\widetilde{\alpha})) \\
&-F(R(\alpha^*)l^{n*}+J(\alpha^*)l^{n*},s^{n*}+s^{n*}\alpha^*+a^*(1+\alpha^*)) \\
&-F(l^{n^*}R(\alpha^*),s^*+a^*)(\widetilde{\alpha}-\alpha^*) \\
&-(1+\pi_m)(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})(\widetilde{\alpha}-\alpha^*) \\
&-(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})(1+\pi_m)\widetilde{\alpha}(\widetilde{\alpha}-\alpha^*)\}
\end{aligned} \tag{124}$$

The second line of (124) becomes

$$N^n_{t-1}(1+\widetilde{\alpha})\{F(R(\widetilde{\alpha})l^{n*},s^{n*}+a^*)$$

The third and fourth line of the above equations become

$$
\begin{aligned}
&- N_{t-1}^{n}(1+\alpha^*)F(R(\alpha^*)l^{n*}, s^{n*}+a^*) \\
&- N_{t-1}^{n}F(l^{n^*}R(\alpha^*), s^*+a^*)(\widetilde{\alpha}-\alpha^*) \\
&= -N_{t-1}^{n}(1+\widetilde{\alpha})F(R(\alpha^*)l^{n*}, s^{n*}+a^*)
\end{aligned}
$$

The fifth and sixth line becomes

$$
-(1+\pi_m)N_{t-1}^{n}(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})(1+\widetilde{\alpha})(\widetilde{\alpha}-\alpha^*)
$$

Thus, SP_t becomes

$$
\begin{aligned}
SP_t = {} & N_{t-1}^{n}(1+\widetilde{\alpha})\int_{s^{n*}+a^*}^{s^{n*}+a_{t-1}}[F_K(R(\widetilde{\alpha})l^{n*}, z)+(1-\delta)]dz \\
&+ N_{t-1}^{n*}(1+\widetilde{\alpha})\{F(R(\widetilde{\alpha})l^{n*}, s^{n*}+a^*)-F(R(\alpha^*)l^{n*}, s^{n*}+a^*) \\
&- (1+\pi_m)(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})(\widetilde{\alpha}-\alpha^*)\} \\
&= N_{t-1}^{n}(1+\widetilde{\alpha})\int_{s^{n*}+a^*}^{s^{n*}+a_{t-1}}[F_K(R(\widetilde{\alpha})l^{n*}, z)+(1-\delta)]dz \\
&+ N_{t-1}^{n}(1+\widetilde{\alpha})\int_{\alpha^*}^{\widetilde{\alpha}}(1+\pi_m)l^{n*}F_L(R(z), s^{n*}+a^*)dz \\
&- N_{t-1}^{n}(1+\widetilde{\alpha})\int_{\alpha^*}^{\widetilde{\alpha}}(1+\pi_m)(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})dz
\end{aligned}
$$

Combining the second and the third line of the above equation, we have

$$
\begin{aligned}
SP_t = {} & N_{t-1}^{n}(1+\widetilde{\alpha})\int_{s^{n*}+a^*}^{s^{n*}+a_{t-1}}[F_K(R(\widetilde{\alpha})l^{n*}, z)+(1-\delta)]dz \\
&+ N_{t-1}^{n}(1+\widetilde{\alpha})\int_{\alpha^*}^{\widetilde{\alpha}}R'(z)[F_L(R(z), s^{n*}+a^*)l^{n*} \\
&- (c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})]dz
\end{aligned} \tag{125}
$$

This is SP_t when immigrant and native have the same productivities and preferences.