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Intergenerationally Equitable Discounting and its Implications for Climate Policy

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Abstract

This paper investigates the properties of intergenerationally equitable discounting by using an axiomatically well-founded welfare function which was originally developed by Epstein (1983), and more recently extended by Bommier and Zuber (2008). In stead of seeking for the appropriate value of social rate of time preference, intergenerational equity is incorporated at axiomatic level. I show that the intergenerational-equity-consistent (IE-consistent) discount rate can be higher or lower than the standard no-time-preference case without appealing to uncertainty. The relationship between IE-consistent discount rates and risk of world extinction is also examined with an emphasis on the case where the hazard rate is endogenously determined. With an application to climate change, I show that endogenous hazard rate can increase the discount rate, which implies relatively less stringent carbon abatement as the optimal climate policy.

Keywords: Intergenerational Equity, Discounting, Climate Change, Uncertainty, Endogenous Hazard Rate

JEL Classification: D91, Q54, Q56

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1 Introduction

The first comprehensive study of economic implications of climate change was launched by William R. Cline. In his pioneering work, Cline (1992) conducted the cost-benefit analysis of pollution abatement policies, and Cline (1993) summarized the findings that aggressive abatement is worthwhile even though the future is much richer, because potential massive damages warrant the costs. On the other hand, William D. Nordhaus, who has been studying the economics of climate change for decades, tackled this problem based on his Dynamic Integrated Model of Climate and the Economy (DICE), and reached a quite different conclusion. The main message of his series of influential studies is that any radical reduction of carbon emissions in the near future should be avoided despite the serious threats to the global economy posed by climate change. In fact, the most well-known policy implication given by Nordhaus (1994) is that controls on carbon-intensive economic activities should be put into effect in an increasing but gradual manner, starting several decades from now. More recently, Stern (2007) tried to revive the spirit of Cline's approach with insights from new scientific findings and concluded that strong and immediate actions to curb global carbon emissions can be justified based on their framework. Although the framework and assumptions adopted by Stern were severely criticized by many economists such as Dasgupta (2007), Weitzman (2007), and Nordhaus (2007), his contribution to the literature rekindled the debate over appropriateness of the existing approaches.

One of the most controversial issues in the literature is how to incorporate the idea of intergenerational equity into the policy evaluation framework. Since the impact of climate change easily stretches across generations, it is inappropriate to evaluate economic impacts of alternative climate policies solely based on the view point of the present generation. In the literature, such intergenerational consideration has been boiled down to the choice of one particular parameter value: social rate of time preference. Following Sidgwick (1907) and Pigou (1920), some economists have been arguing that the interest of future generations should be taken into account by setting the rate of social time preference as small as possible (Ramsey, 1928). But others including Nordhaus (2008) claim that the discount rate must be based on actual behavior in markets rather than any idealized philosophy about the treatment of future generations.

More generally, not only the rate of social time preference, but also the discount rate as a whole is important in evaluating climate policies. The discount rate is defined as

$$\rho_t(x) := \frac{\partial W(x)}{\partial c_t} \Big/ \frac{\partial W(x)}{\partial c_{t+1}} - 1, \tag{1.1}$$

where $x = (c_1, c_2, c_3, \ldots)$ is a consumption path and $W(x)$ is a welfare function. The commonly used form of welfare function is the additive and separable one:

$$W(x) = \sum_{t=1}^{\infty} \left(\frac{1}{1+\delta} \right)^{t-1} v(c_t). \tag{1.2}$$

Given this specification, the discount rate is computed as

$$\begin{aligned} \rho_t(x) &= \frac{v'(c_t)}{v'(c_{t+1})}(1+\delta) - 1 \\ &\approx \delta + \eta_t(x) g_t(x) \end{aligned} \tag{1.3}$$

where

$$\eta_t(x) := -\frac{v''(c_t)c_t}{v'(c_t)}, \quad g_t(x) := \frac{c_{t+1} - c_t}{c_t}.$$

Here, $\eta_t(x)$ is the elasticity of the marginal utility of consumption. Higher value of $\eta_t(x)$ implies less value of additional consumption for rich people. In (1.3), this parameter is multiplied by $g_t(x)$, the consumption growth rate. This means that when consumption is expected to grow, the value of future consumption is discounted at higher rate. Hence, the second term of (1.3) as a whole represents the aversion to consumption inequality.

The rate of social time preference is captured by the other term, δ, and this is the parameter that plays a crucial role in evaluating climate policies. Since the planning horizons of climate policies are typically very long, even a small difference in δ could make a huge difference. Actually, most of the discrepancy among the conclusions in the past studies can be explained by the difference in their choice of this parameter. Cline (1992) and Stern (2007) set $\delta \approx 0$ in consideration of intergenerational equity, and consequently reached almost the same conclusions. On the other hand, Nordhaus (1994) chose $\delta = 0.03$ to ensure consistency between the model and real-world economic data, leading to very different policy implications. As an illustration, the optimal paths of emission reduction rate based on a simplified version of DICE model for each case are plotted in Figure 1 below. Obviously, there is a huge difference between the two cases. If we stick with the choice of $\delta = 0$, which is usually considered necessary to meet the requirement of intergenerational equity, seemingly a bit too ambitious carbon abatement is required compared with the case of $\delta = 0.03$.

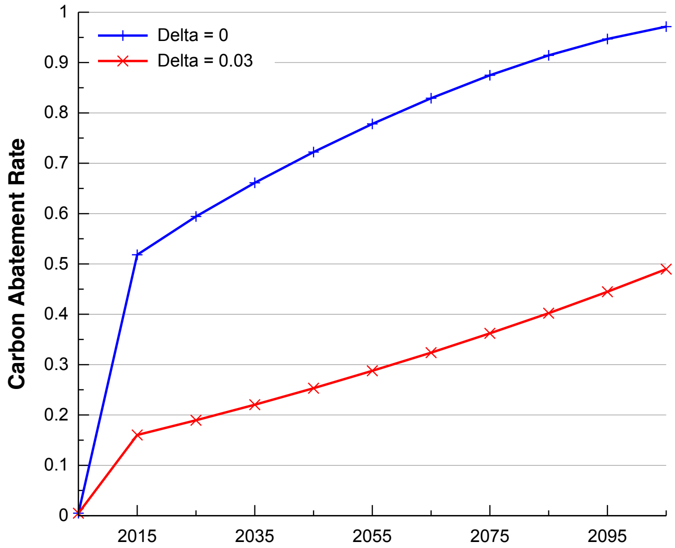

Figure 1: Optimal Abatement Rates for $\delta = 0$ and $\delta = 0.03$

The question is whether there is any way of reconciling those two approaches without violating the requirement of intergenerational equity. To answer this question, it seems necessary to reconsider assumptions adopted in the standard argument on discounting. In particular, the functional form of welfare function should be examined. As is clear from (1.1), discount rate largely depends on the specification of welfare function. The discussion over discounting in the literature implicitly premises that the welfare function can appropriately be specified as (1.2), and this is the very reason why the intergenerational equity immediately implies the choice of $\delta = 0$. If the essence of intergenerational equity is not all captured by the standard model, other forms of welfare functions could provide different ways of incorporating concerns for future generations.

The origin of the additive and separable discounted welfare function is traced back to Samuelson (1937), and its axiomatic foundation was later given by Koopmans (1960). Koopmans (1960) demonstrated that the set of axioms assumed behind the use of the standard welfare function necessarily implies impatience of the underlying preference. In line with this argument, Diamond (1965) showed that impartial treatment among generations is not compatible with very weak axioms imposed on the orderings over infinite utility streams. This type of unavoidable impatience was also reconfirmed by Epstein (1983) in the context of preference over uncertain consumption streams. More recently, however, Bommier and Zuber (2008) derived a recursive form of welfare function which satisfies the axiom of anonymity, and showed that preference for catastrophe avoidance justifies higher discount rates.

In this paper, I investigate the properties of intergenerationally equitable discounting by using an axiomatically well-founded welfare function which was originally developed by Epstein (1983), and later extended by Bommier and Zuber (2008). In stead of choosing $\delta = 0$ in the standard welfare function, I incorporate intergenerational equity at axiomatic level. This approach makes it possible to derive the intergenerationally equitable discount rates in a more flexible manner. Moreover, the risk of world extinction, which is positive but usually tiny, is also incorporated. As Yaari (1965) suggested, exogenous hazard rate of world ending can be a basis for higher discounting. I examine this argument under a fairly general framework of intergenerationally equitable discounting. In particular, I take a close look at the case where the risk of world extinction is endogenously determined. This is of particular relevance to climate change because, as is discussed by Weitzman (2009), increasing temperature could cause some climatic catastrophe in the future.

The paper is structured as follows. Section 2 derives a class of welfare functions which represent a ranking over possible outcomes in a way consistent with intergenerational equity. Based on the result of section 2, section 3 provides the intergenerationally equitable discount rates and analyzes their properties. The difference from the commonly-used discount rate is studied. The relationship between uncertainty and intergenerationally equitable discounting is also investigated. Section 4 applies this welfare function to the evaluation of climate policies and derives some implications for optimal carbon abatement paths. Section 5 concludes.

2 Welfare Function with Intergenerational Equity

Consider a society with a sequence of generations or cohorts. Let $c_t \in \mathbb{R}^n$ be the quantity of n-dimensional goods consumed by t-th generation. With $X := \{x = (c_1, c_2, c_3, \ldots) : c_t \in [0, \bar{c}]^n \ \forall t\}$, we denote the set of bounded consumption streams. We restrict our attention to the case where the world ends at some point in the future and consumption will never occur thereafter. To be more precise, the outcome set in our analysis is given by

$$X^* := \{x = (c_1, c_2, c_3, \ldots) \in X : c_t = 0 \ \ \forall t \geq T \ \text{ for some } T < \infty\}.$$

It is worth noting that X^* includes practically every outcome which could be realized in the real world. Since the end-time T can be arbitrarily large, consumption can be positive up until farthest in the future one could think of. The assumption of "world end" is introduced here just for the instrumental purpose of excluding positive consumption in the infinite future. Or perhaps we could reasonably assume the world actually ends in some very distant future. As is indicated by Sackmann *et al.* (1993), the Earth is likely to have a dire consequence in the next several billion years as the Sun gradually goes through its evolutionary life-cycle.

Also notice that the introduction of world end is consistent with the assumption of bounded consumption set. If we suppose the world lasts forever, it is not very reasonable to assume a finite upper bound on consumption set because the economy can grow without a limit. Once the possibility of world ending is embraced, however, the infinitely large level of consumption can be readily ruled out at least from a practical point of view. As long as the upper bound $\bar{c}$ is taken to be sufficiently large, we could safely say that any realistic consumption stream would not hit the bound before the world ends.

Let $M(X^*)$ be the set of probability measures on the measurable space $(X^*, R(X^*))$, where $R(X^*)$ denotes the Borel σ-algebra of X^*. The social preference or ranking $\succsim$ is a binary relation defined on $M(X^*)$. As usual, indifference and strict preference are denoted by $\sim$ and $\succ$ respectively. Let $p_x \in M(X^*)$ denote the probability measure that assigns probability 1 to an outcome $x \in X^*$. For each $c_1 \in [0, \bar{c}]^n$ and $p \in M(X^*)$, we define (c_1, p) as a probability measure with which the consumption level c_1 occurs with certainty for the first generation while consumption for later generations is realized according to p.

To derive a welfare function which represents the ranking $\succsim$, we first assume $\succsim$ satisfies the following basic axioms:

Monotonicity: If $x > x'$, then $p_x \succ p_{x'}$.

Stationarity: There exists $\hat{c}_1 \in [0, \bar{c}]^n$ such that for all $p, q \in M(X^*)$, $(\hat{c}_1, p) \succsim (\hat{c}_1, q)$ if and only if $p \succsim q$.

Independence: For all $c_1, c_1' \in [0, \bar{c}]^n$ and $p, q \in M(X^*)$, $(c_1, p) \succsim (c_1, q)$ if and only if $(c_1', p) \succsim (c_1', q)$.

Expected Utility: There exists a continuous function $U : X^* \to \mathbb{R}$ such that $\int_{X^*} U(x)dp \geq \int_{X^*} U(x)dq$ if and only if $p \succsim q$, and U is unique up to positive affine transformations.

These axioms are almost the same as the ones in Epstein (1983). The only difference is that the sensitivity axiom is now replaced by the monotonicity axiom. This stronger assumption gives a more precise structure to the form of welfare function.

In order to make sure that every generation is treated equally, we need to introduce some axiom of intergenerational equity. Following Bommier and Zuber (2008), let $\pi : \mathbb{N} \to \mathbb{N}$ be a finite permutation over the set of natural numbers and Π denote the set of all such permutations. For each $x = (c_1, c_2, \ldots) \in X^*$ and $\pi \in \Pi$, define $x(\pi)$ as

$$x(\pi) := (c_{\pi(1)}, c_{\pi(2)}, \ldots) \in X^*.$$

Our axiom of intergenerationl equity is then given by

Intergenerational Equity: For any $x \in X^*$ and $\pi \in \Pi$, $p_x \sim p_{x(\pi)}$.

Note that this axiom ensures intergenerational equity in a bit stronger sense than the one in Bommier and Zuber (2008). Their anonymity axiom only requires impartial treatment among those generations who born before the world end. In our axiom, on the other hand, equal treatment between pre-world-ending generations and post-world-ending generations are required as well.

The following proposition is a straightforward extention of theorem 1 in Epstein (1983):

Proposition 1. *The ranking $\succsim$ sasisfies the axioms above if and only if the von Neuman-Morgenstern utility function U can be expressed in the form*

$$U(x) = \sum_{t=1}^{\infty} v(c_t) \prod_{\tau=1}^{t-1} (1 - Kv(c_\tau)) \qquad \forall x \in X^*, \tag{2.1}$$

where $v : [0, \bar{c}]^n \to \mathbb{R}_+$ is an increasing function with $v(0) = 0$ and $K < 1/v(\bar{c})$.

Proof. See Appendix A.1. □

The functional form (2.1) is basically the same as the one derived by Bommier and Zuber (2008). The corresponding welfare function is now given by

$$W(p) = \int_{X^*} U(x)dp \qquad \forall p \in M(X^*).$$

Since the concern for future generations is already incorporated at axiomatic level, the ranking over alternative outcomes provided by this function meets the requirement of intergenerational equity. Hence, the discount rate derived from this welfare function is always consistent with intergenerational equity. We call it *IE-consistent discount rate*.

Notice that the parameter K can be positive or negative, and such choice does not change the fact that the ranking is consistent with intergenerational equity. This point makes a sharp contrast to the additive and separable form of welfare function, where impartial treatment among generations automatically implies $\delta = 0$. Bommier and Zuber (2008) linked K with the aversion to catastrophe. I will present another interpretation of this new parameter shortly. It should also be emphasized here that K is bounded above by $1/v(\bar{c})$. Bearing in mind that $v(c)$ is increasing in c and $\bar{c}$ can be a very large number, this boundedness implies that K must be very small if it is positive.

3 IE-consistent Discount Rates

We are now ready to derive the IE-consistent discount rate and investigate its properties. In what follows, we restrict our attention to the case with $n = 1$ for simplicity. Also, we assume $v''(c) < 0$ for any $c \in [0, \bar{c}]$ and write

$$U_T(x) := \sum_{t=1}^{T} v(c_t) \prod_{\tau=1}^{t-1} (1 - Kv(c_\tau)),$$

for each $T \in \mathbb{N}$.

3.1 Deterministic Case

We first analyze the case where consumption streams are non-stochastic. In this case, it will be useful to use $\rho_t^0(x) := \eta_t(x)g_t(x)$ as a benchmark. This is the discount rate which is considered to be intergenerationally equitable for the standard welfare function. We are interested in if and in what condition the IE-consistent discount rate can be higher or lower than this benchmark rate.

Before presenting the result, notice that the welfare function for non-stochastic consumption streams is equivalent to the von Neuman-Morgenstern utility function:

$$W(p_x) = \int_{X^*} U(z)dp_x = \sum_{t=1}^{\infty} v(c_t) \prod_{\tau=1}^{t-1} (1 - Kv(c_\tau)). \tag{3.1}$$

Given this welfare function, a bit tedious computation yields the following result:

Proposition 2. *Given a deterministic consumption path $x \in X^*$, the IE-consistent discount rate $\rho_t^{de}(x)$ is given by*

$$\rho_t^{de}(x) = \frac{v'(c_t)}{v'(c_{t+1})} \frac{1 - Kv(c_{t+1})}{1 - Kv(c_t)} - 1,$$

and thus, as long as $c_t < c_{t+1}$, $\rho_t^{de}(x)$ is higher than $\rho_t^0(x)$ if and only if K is negative.

Proof. See Appendix A.2. □

The first thing to be noted here is that higher discount rates are justifiable even when the impartial treatment among generation is required. Hence, rephrased in the context of standard welfare function, the proposition says that positive value of δ in equation (1.3) does not necessarily imply the violation of intergenerational equity. On the other hand, however, negative value of δ is also justifiable. In that case, the discount rate is even lower than the benchmark rate, which is usually considered as the lowest discount rate. While the choice of $\delta = 0$ can also be supported as intergenerationally equitable discounting, such zero time-preference is not required in general.

This point should be clarified by noting

$$\rho_t^{de}(x) \approx \delta_t(x; K) + \eta_t(x) g_t(x),$$

where

$$\delta_t(x; K) := \log \left[\frac{1 - Kv(c_{t+1})}{1 - Kv(c_t)} \right].$$

Here, $\delta_t(x; K)$ is a counterpart to the time preference δ in the standard discount rate. This means, in the IE-consistent discount rate, the "time preference" can be positive or negative and even vary over time. The parameter K plays an important role, which is illustrated in Figure 2 below. This figure displays the trajectories of $\delta_t(x; K)$ for different values of K in each case of increasing and decreasing consumption path.

To obtain some interpretation of K, observe

$$\frac{\partial \rho_t^{de}(x)}{\partial K} = -\frac{v'(c_t)}{v'(c_{t+1})} \frac{v(c_{t+1}) - v(c_t)}{(1 - Kv(c_t))^2},$$

and hence

$$\frac{\partial \rho_t^{de}(x)}{\partial K} \lesseqgtr 0 \quad \Leftrightarrow \quad c_t \lesseqgtr c_{t+1}.$$

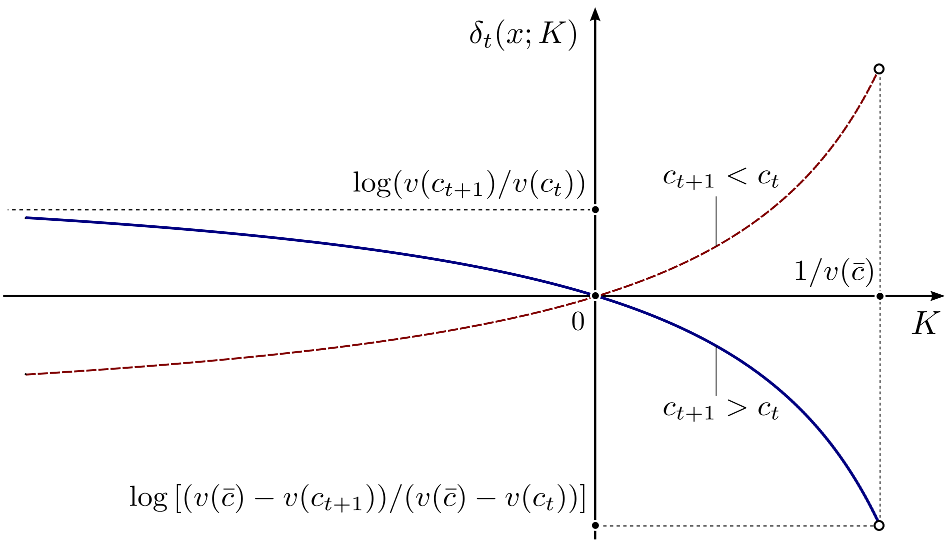

Figure 2: $\delta_t(x; K)$ for different K

When consumption increases over time, higher value of K implies lower value of the discount rate. If the level of consumption is expected to become lower in the future, on the other hand, higher value of K means higher value of discount rate. In other words, higher (lower) value of K scales down (scales up) the aversion to consumption inequality. Therefore, one possible interpretation of K is that it is another parameter which controls the aversion to consumption inequality among generations.

It should also be noticed that K provides a linkage between discount rates and the level of consumption enjoyed by *present* generation. As is clear from (1.3), the discount rate in the standard model does not depend on c_t as long as both the elasticity of the marginal utility of consumption, η_t, and the growth rate of consumption, g_t, are fixed. In the IE-consistent discount rate, however, not only the growth rate of, but also the current level of consumption matters.

To better illustrate this point, suppose both η_t and g_t do not depend on today's consumption c_t. Then

$$\frac{\partial \delta_t(x;K)}{\partial c_t} = \frac{Kv'(c_{t+1})}{1 - Kv(c_{t+1})} \rho_t^{de}(x),$$

which is negative if and only if $K < 0$. Thus negative value of K implies that the discount rate declines as consumption level rises. In other words, "people become less envy as they become richer." When K is positive, on the other hand, higher level of today's consumption implies a higher discount rate for consumption of future generations. This could be rephrased as "people become more averse to inequality of consumption when they become wealthier." If the former case (*i.e.*, $K < 0$) sounds more reasonable than the latter, the welfare function (3.1) provides a justification for higher discount rate as long as increasing consumption is expected.

3.2 Uncertain World End

The discussion in the previous subsection suggests the IE-consistent discount rate can be higher than the benchmark rate without appealing to any kind of uncertainty. This argument alone, however, might not be able to fill the gap between the cases of $\delta = 0$ and $\delta = 0.03$. As is illustrated in Figure 2, the term $\delta_t(x; K)$ can not be arbitrary large. In fact, it is bounded above by $\log(v(c_{t+1})/v(c_t))$ when the level of consumption grows. Hence, the relationship between uncertainty and the IE-consistent discount rate is still worth investigating. As Yaari (1965) showed and Bommier and Zuber (2008) reiterated, uncertainty provides another basis for higher discount rates. If society is subject to an exogenous risk of extinction in each period, it is reasonable to discount future consumptions even when impartial treatment among generations is required. One of the interesting results in this context is that once extinction risk is incorporated, the role of K is reversed.

In order to investigate this point in more detail, let $f : \mathbb{N} \to [0, 1]$ be a probability density function of extinction date. This extinction risk is assumed to be exogenous at this stage of the analysis. Then the corresponding sequence $\{h_t\}_{t=1}^{\infty}$ of hazard rates is constructed recursively by

$$h_T = f_T \Big/ \prod_{s=1}^{T-1} (1 - h_s),$$

where $\prod_{s=1}^{0}(1 - h_s) := 1$. We assume $h_t \in (0, 1)$ for all $t \in \mathbb{N}$ and let H be the set of all such sequences of hazard rates. When uncertainty exists only in the extinction date, the welfare function is given by

$$\begin{aligned} W(p) &= \sum_{T=1}^{\infty} U_T(x) f_T \\ &= \sum_{T=1}^{\infty} \sum_{t=1}^{T} v(c_t) \prod_{\tau=1}^{t-1} (1 - Kv(c_\tau)) \, h_T \prod_{s=1}^{T-1} (1 - h_s), \end{aligned}$$

where $p \in M(X^*)$ is the probability measure which corresponds to the density f. Our next proposition clarifies the relationship between the IE-consistent discount rate, extinction risk, and the parameter K:

Proposition 3. *Given a consumption path $x \in X^*$ and a sequence $\{h_t\}_{t=1}^{\infty} \in H$ of hazard rates of extinction date, the IE-consistent discount rate $\rho_t^{ex}(x)$ is given by*

$$\rho_t^{ex}(x) = \frac{v'(c_t)}{v'(c_{t+1})} \frac{1 - Kv(c_{t+1})}{1 - Kv(c_t)} \left(1 + \Phi_t^{-1}(x; K)\right) - 1 \tag{3.2}$$

where

$$\Phi_t(x; K) := \sum_{T=t+1}^{\infty} \frac{h_T}{h_t} \left(\frac{1 - h_t}{1 - h_T}\right) \prod_{\tau=t+1}^{T} \{[1 - Kv(c_\tau)]\,(1 - h_\tau)\} > 0,$$

and thus uncertainty of extinction date increases discount rates.

Proof. See Appendix A.3. □

The role of uncertain extinction date is all captured by the term $\Phi_t(x; K)$ in (3.2). Since $\Phi_t(x; K)$ is always positive, introduction of uncertainty of this kind increases the discount rate in general. Hence, even higher value of discount rate is justifiable compared with the deterministic case studied above. This argument will be highlighted by considering the case where the hazard rate is constant over time. If $h_t = h \in (0, 1)$ for all $t \in \mathbb{N}$, then $\Phi_t(x; K)$ may be written as

$$\Phi_t(x; K) = \sum_{T=t+1}^{\infty} (1 - h)^{T-t} \prod_{\tau=t+1}^{T} (1 - Kv(c_\tau)),$$

which is decreasing in h. Thus, as depicted in Figure 3 below, higher constant hazard rate implies higher discount rate.

The parameter K is as important here as in the non-stochastic case and, interestingly, it works in quite a different way. Notice first

$$\rho_t^{ex}(x) \approx \Phi_t^{-1}(x; K) + \rho_t^{de}(x),$$

and recall that $\rho_t^{de}(x)$ is decreasing in K when the level of consumption is expected to rise. The first term, $\Phi_t^{-1}(x; K)$, on the other hand, is increasing in K because $\Phi_t(x; K)$ is decreasing in K. Thus, while negative value of K increases the discount rate through the deterministic version of the IE-consistent discount

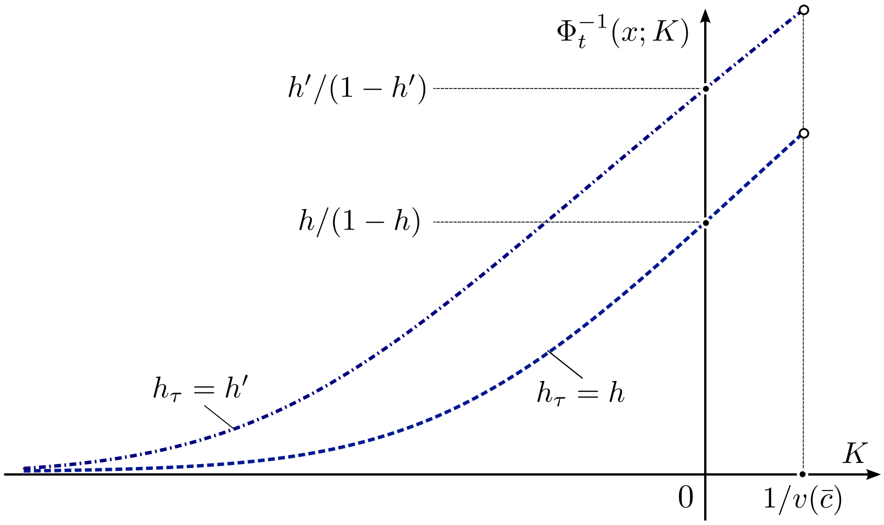

Figure 3: $\Phi_t^{-1}(x;K)$ for different K with $h' > h$

rate, it suppress the role of uncertainty in the first term. In particular, if K is negative and $(1 - Kv(c_\tau))(1 - h_\tau) > 1$ for all $\tau > T$ for some $T > t$, then $\Phi_t^{-1}(x;K) = 0$, which means the impact of uncertainty completely disappears. This could easily happen when the extinction risk is very small and consumption grows over time.

While proposition 3 clarifies higher discount rates are justifiable when the hazard rate is positive, it is not obvious what would happen when the hazard rate at one point of time increases while the one at another point decreases. In particular, it will be of interest to see the impact on the discount rate when a part of extinction risk can be transferred from present generation to future generation. More formally, given a sequence $\{h_t\}_{t=1}^{\infty} \in H$ of hazard rates, define another sequence $\{\tilde{h}_t\}_{t=1}^{\infty} \in H$ as

$$\tilde{h}_\tau = \begin{cases} h_\tau - \gamma & \text{for } \tau = t \\ h_\tau + \gamma & \text{for } \tau = t+1 \\ h_\tau & \text{otherwise,} \end{cases}$$

for some $\gamma > 0$. Here $\{\tilde{h}_t\}_{t=1}^{\infty}$ represents a sequence of hazard rates after a part of risk is transferred from generation t to generation $t+1$.

Letting $\rho_t^{ex}(x)$ and $\tilde{\rho}_t^{ex}(x)$ be the discount rates respectively corresponding to $\{h_t\}_{t=1}^{\infty}$ and $\{\tilde{h}_t\}_{t=1}^{\infty}$, we have the following proposition:

Proposition 4. *There exists* $\gamma > 0$ *such that*

$$\tilde{\rho}_t^{ex}(x) < \rho_t^{ex}(x). \tag{3.3}$$

Moreover, if

$$\frac{1-h_{t+1}}{h_{t+1}}\frac{1-h_t}{h_t} \geq h_t \tag{3.4}$$

then (3.3) holds for any $\gamma > 0$.

Proof. See Appendix A.4. □

Proposition 4 says the IE-consistent discount rate should be decreased when the extinction risk is transferred to future generations. Put differently, higher extinction risk in the future does not justify higher discount rate when such higher risk is attributed to some "risk transfer" from present generation. In such a case, consumption for future generation should rather be discounted at lower rate.

Note that the condition (3.4) is not very restrictive. In fact, it is satisfied by a wide rage of hazard rates. To be more specific, suppose $h_\tau \leq 0.5$ for $\tau = t, t+1$. Then

$$\frac{1-h_{t+1}}{h_{t+1}}\frac{1-h_t}{h_t} \geq 1 > h_t.$$

Hence, if the hazard rate is lower than or equal to 0.5 in both periods, the condition is easily met. Since world extinction risk is usually far smaller than half a chance, we could say any reasonable transfer of extinction risk into the future decreases the discount rate.

This result has an interesting implication for such problems as climate change. There exist some ways of reducing carbon concentration in the atmosphere which seemingly transfer the extinction risk from present generation to future generations. The carbon capture and sequestration technology is one example. By capturing emitted carbon dioxide and storing them deep into the ocean, we will be able to avoid climatic catastrophe in the near term. Such technology, however, may not be completely reliable, and the vast amount of carbon stored in the ocean may leak out at some point in the future. The resulting abrupt increase of carbon concentration will be likely to increase the risk of world ending at that point. Although the overall impact of such technology is ambiguous, our result indicates that lower discount rates should be used if it increases the extinction risk in future.

3.3 Endogenous Uncertainty

In the case of climate change, it seems more reasonable to assume the extinction risk is endogenously determined. With an application to climate change in mind, I introduce endogeneity of extinction risk as follows. First, it is natural to assume that if the level M_t of carbon concentration in the atmosphere rises, the hazard rate of world extinction increases:

$$h_t(x) = h(M_t) \quad \text{with} \quad h'(M) \in (0, +\infty). \tag{3.5}$$

While the carbon dioxide in the atmosphere decays at rate of $\xi \in (0, 1)$, human-induced carbon emission E_t accumulates in the atmosphere:

$$M_{t+1} = (1 - \xi)M_t + \kappa E_t \quad \text{with} \quad \kappa > 0. \tag{3.6}$$

Finally, we assume carbon emission is proportional to the level of consumption:

$$E_t = \sigma_t c_t \quad \text{with} \quad \sigma_t > 0, \tag{3.7}$$

where σ_t is an index of emission intensity of consumption. We here assume $\sigma_{t+1} \leq \sigma_t$, meaning that the energy efficiency improves over time. Note that under these assumptions, both hazard rate $h_t(x)$ and probability density $f_t(x)$ are determined by the past consumption level $(c_1, c_2, \ldots, c_{t-1})$.

Before presenting the main result of this subsection, I provide the following lemma:

Lemma 1. *Suppose a sequence $\{h_t(x)\}_{t=1}^{\infty} \in H$ of hazard rates is endogenously determined by (3.5), (3.6), and (3.7). Then, given a consumption path $x \in X^*$, the IE-consistent discont rate $\rho_t^{en}(x)$ can be written as*

$$\rho_t^{en}(x) = \rho_t^{ex}(x) + \Theta_t(x),$$

where

$$\Theta_t(x) = U_{t+1}(x)\frac{\partial f_{t+1}(x)}{\partial c_t} \bigg/ \frac{\partial W(x)}{\partial c_{t+1}} + \sum_{T=t+2}^{\infty} U_T(x)\left(\frac{\partial f_T(x)}{\partial c_t} - (1 + \rho_t^{ex}(x))\frac{\partial f_T(x)}{\partial c_{t+1}}\right) \bigg/ \frac{\partial W(x)}{\partial c_{t+1}}.$$

Proof. See Appendix A.5. □

Notice that $\rho_t^{ex}(x)$ in the lemma is nothing but the IE-consistent discount rate when hazard rate is exogenous. Hence, we can say the endogeneity of extinction risk increases the discount rate if $\Theta_t(x) > 0$ and vice-versa. Since the first term in $\Theta_t(x)$ is positive, the real question is whether the second term can be negative or not. This may be rephrased as

$$\Upsilon_t(x,T) := \frac{\partial f_T(x)}{\partial c_t} - (1+\rho_t^{ex}(x))\frac{\partial f_T(x)}{\partial c_{t+1}} \gtreqless 0, \tag{3.8}$$

for each $T \geq t+2$.

Inequality (3.8) can be interpreted as follows. Now that the hazard rates are endogenously determined by the past consumption history, both c_t and c_{t+1} change the hazard rates $h(M_T)$, and hence $f_T(x)$, for $T \geq t+2$. The impacts of c_t and c_{t+1} on $h(M_T)$ are different in general, and such difference determines the sign of $\Upsilon_t(x,T)$. If c_t increases future hazard rates more than c_{t+1}, which happens when today's production technology is more pollution-intensive than the one in the future, then $\Upsilon_t(x,T)$ is likely to be positive, and thus $\rho_t^{en}(x) > \rho_t^{ex}(x)$. This is consistent with the result of previous subsection. Increase of future hazard rates (not transfer) usually raises the IE-consistent discount rate. Hence, when today's consumption increases future hazard rates more than tomorrow's consumption does, future consumption will be even more discounted than in the case of exogenous hazard rate.

Does this mean endogeneity of extinction risk provides yet another justification for higher discount rates? Our next proposition indicates that it is not always the case. To see this, we restrict our attention to the case where the atmospheric carbon concentration is stabilized over the planning horizon. Notice that along a stabilized path with $M_t = M$ for all t, the corresponding hazard rate is also constant over time at the level of $h(M)$.

Proposition 5. *If*

$$\frac{1}{1+\rho_t^{de}(x)}\frac{\sigma_t}{\sigma_{t+1}} < 1, \tag{3.9}$$

then there exists $h^* > 0$ such that for any stabilized path with $h(M) < h^*$,

$$\rho_t^{en}(x) < \rho_t^{ex}(x).$$

Proof. See Appendix A.6. □

This proposition says that the IE-consistent discount rate under endogenous hazard rates can be lower than in the case of exogenous hazard rates. Note that the conditions in the proposition may be decomposed into three different factors: *time effect*, *technology effect*, and *risk effect*. Time effect, which is captured by the term $1/(1+\rho_t^{de}(x))$, purely comes from the fact that today's consumption is preferred to tomorrow's consumption even when there is no uncertainty. Recall that $\rho_t^{de}(x)$ is the IE-consistent discount rate in the deterministic case. Inequality (3.9) indicates that when the discount rate for non-stochastic consumption streams is high enough in the first place (*i.e.*, time effect is large), consideration of endogenous risk decreases the discount rate compared with the case of exogenous hazard rates. This effect is governed by the specification of the individual utility function v and the parameter K.

Technology effect, on the other hand, is represented by the term σ_t/σ_{t+1}. Since σ_t is an index of pollution intensity, the ratio σ_t/σ_{t+1} represents how fast alternative clean technologies develop. If cleaner technologies become available and consumption with fewer emission is possible at $t+1$, the emission intensity of c_{t+1} decreases, which means $\sigma_{t+1} < \sigma_t$. When σ_t/σ_{t+1} is relatively large (*i.e.*, technology effect is large), the condition (3.9) is less likely to be satisfied. If technological development is not so promising, however, the condition is more likely to be satisfied, and thus the discount rate is more likely to be lower. This effect is determined by social circumstances regarding environmental technologies.

The last factor, risk effect, is naturally represented by the stabilized hazard rate $h(M)$. According to the proposition above, when time effect is so large or technology effect is so small that the condition (3.9) is satisfied, sufficiently small hazard rate implies lower discount rate. If the stabilized hazard rate is relatively large (*i.e.*, risk effect is large), however, even when time effect is large and technology effect is small, endogenous hazard rate can imply higher discount rate. This effect is largely determined by the relationship between the level of pollution and the risk of climatic catastrophe, relevance of which belongs to the field of natural science.

The overall effect of endogenous risk depends on the assumptions about the welfare function (ρ^{de}), the prospect of technological development (σ), and the physical characteristic of the problem (h). Relative importance among these factors can not be determined without conducting quantitative analysis. Hence, if we are to derive more concrete implications for climate policy, it is necessary to consult with some climate-economy modeling, which is to be explored in the next section.

4 Implications for Climate Policy

In this section, we further investigate the implications of intergenerationally equitable discounting for climate policy by conducting a numerical simulation. Our model is a variant of DICE model with tiny risk of world extinction which is endogenously determined.

4.1 The Model

First, technology is specified in the form of Cobb-Douglas production function

$$Y_t = \frac{\psi_t}{1+d_t} K_t^{\epsilon} N_t^{1-\epsilon} \quad \text{where} \quad d_t = \bar{d}(Z_t - \tau^*)^2 \tag{4.1}$$

where Y_t is output, K_t is capital, N_t is population, and Z_t is temperature. Notice we are assuming that increasing temperature has a negative impact on production. Carbon emission E_t is produced according to the equation

$$E_t = \sigma_t(1 - \mu_t)Y_t, \tag{4.2}$$

where μ_t is control rate or abatement rate of the emission. Emitted carbon dioxide is accumulated in the atmosphere through the equation

$$M_{t+1} = (1 - \xi)M_t + \kappa E_t. \tag{4.3}$$

We simplify the climate module used by Nordhaus (2008) and assume instead

$$\begin{aligned} Z_{t+1} = \tau_0 + \tau_1 Z_t + \tau_2 \log(M_t) \\ + \tau_3 F_t + \tau_4 \log(1 + \tau_5 F_t) \end{aligned} \tag{4.4}$$

where F_t is aerosol emission, which is exogenously given. The dynamics of capital accumulation is governed by

$$K_{t+1} = (1 - \upsilon)K_t + I_t \tag{4.5}$$

where I_t is investment. The resource constraint is given as

$$Y_t = C_t + I_t + A_t \quad \text{where} \quad A_{j,t} = \alpha_t(\mu_t)^{\zeta} Y_t, \tag{4.6}$$

where C_t is consumption and A_t represents abatement cost of carbon emission.

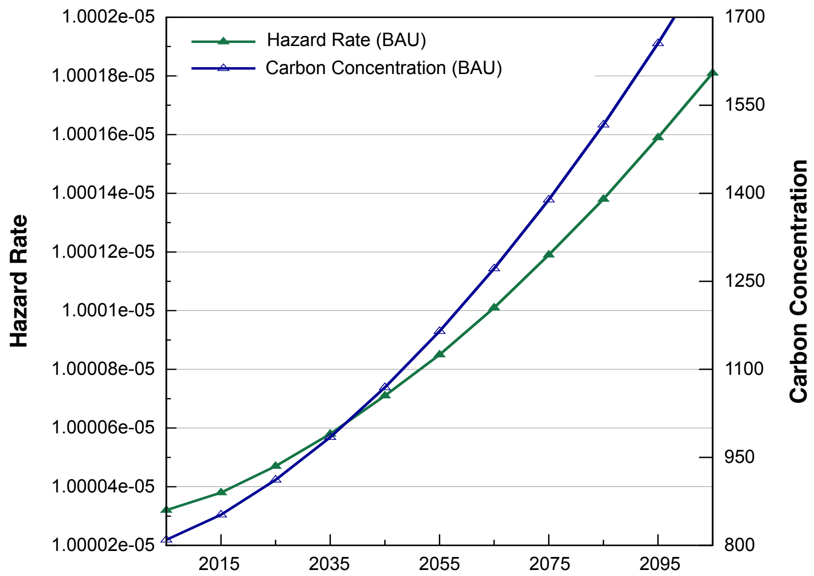

Figure 4: Hazard Rate along BAU path

A key feature of our model is the specification of welfare function:

$$W(x) = \sum_{T=1}^{\infty} \sum_{t=1}^{T} v(c_t) \prod_{\tau=1}^{t-1} (1 - Kv(c_\tau))\, h(M_T) \prod_{s=1}^{T-1} (1 - h(M_t)) \tag{4.7}$$

where

$$v(c_t) = \omega_0 + \frac{(\omega_1 + c_t)^{1-\omega_2}}{1 - \omega_2} \quad \text{with} \quad c_t = C_t/N_t. \tag{4.8}$$

The parameters ω_0, ω_1, and ω_2 are introduced to control the value of $\eta_t(x)$ and make sure $v(0) = 0$. Following Fisher and Narain (2003), the hazard rate function is specified as

$$h(M_t) = \bar{h} + (1 - \bar{h}) \left[\frac{2}{1 + e^{-\beta(M_t - M_{1750})}} - 1 \right], \tag{4.9}$$

where $\bar{h}$ is the extinction risk which does not depend on the climatic condition. The relationship between the hazard rate and the concentration along the Business as Usual (BAU) scenario of DICE model is illustrated in Figure 4.

The optimal carbon abatement path $\{\mu_t\}_{t=1}^{\infty}$ is defined as a part of the solution to the problem

$$\max_{C_t, \mu_t, I_t} W(x) \quad \text{subject to (4.1) through (4.9)},$$

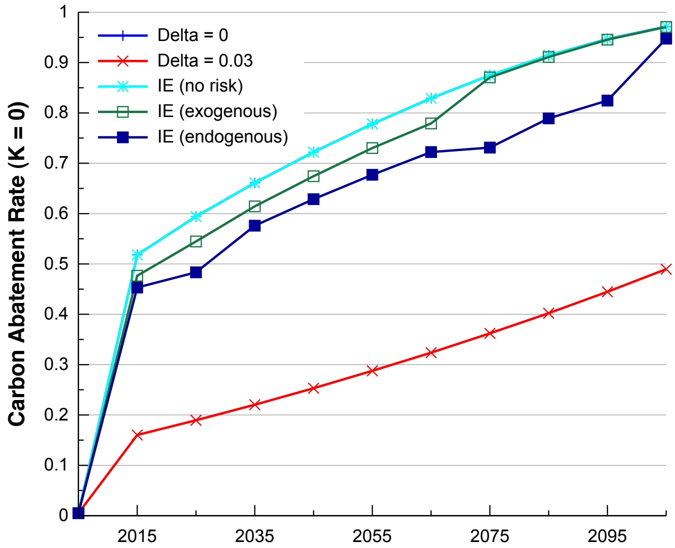

Figure 5: Optimal Carbon Abatement Paths ($K = 0$)

given K_1, M_1, and Z_1. The model is calibrated based on Ikefuji *et el.* (2010), and is solved with GAMS. The data and calibrated parameter values used in this analysis are all listed in Table 1 of Appendix B.

4.2 Result

The result of our simple simulation is well summarized in Figure 5 above. In this figure, the optimal abatement paths for each case of deterministic consumption, exogenous hazard rate, and endogenous hazard rate are plotted. The exogenous hazard rate was chosen such that the probability density in the initial period is comparable to the case of endogenous one. The parameter K was chosen to be 0 here. As is illustrated in the Figure 6 and Figure 7 of Appendix C, small changes, either positive or negative, of the value of K do not affect the qualitative nature of the result.

First, in the case of no-risk of world extinction, the optimal abatement rate is the same as the one in the benchmark case with $\delta = 0$. This is just what is expected from the theoretical analysis in the preceding section. When $K = 0$, the IE-consistent discount rate is exactly the same as the benchmark rate. Manipulation of K alone does not change the result much. Once the exogenous risk of world extinction is incorporated, however, the optimal carbon reduction rate becomes a bit lower and gets closer to the case of $\delta = 0.03$. Again, this result is consistent with the analysis in the previous section. Uncertainty over extinction date always increases the discount rate.

What is more of interest is the optimal abatement path under endogenous risk. The figure above reveals that endogeneity of hazard rate can work as a downward driving force for the optimal carbon abatement rate. This is an indication that the IE-consistent discount rate is higher than the one in the case of exogenous hazard rate. Hence, endogenous risk of climatic catastrophe makes it even easier to justify higher rates of discounting. Having said that, there still remains a huge gap between the normative approach with intergenerational equity and the descriptive approach with $\delta = 0.03$. Therefore, the commonly used discount rate of $\delta = 0.03$ is not readily justifiable as long as impartial treatment among generations is required. Although the proposition in section 3.3 cannot directly be applied to this model, our simulation result implies that time effect is small, technology effect is large, or risk effect is large in the context of climate change.

These implications of course depend on several assumptions. The scale of abatement rate is not completely independent of the choice of various parameters. Our simulation result should at best be considered as a qualitative analysis in its nature. Moreover, relatively large uncertainty exists in the specification of some exogenous trend and functions. The endogenous hazard function $h(M)$, for instance, does not have rigorous scientific background. To discuss the appropriateness of this functional form is beyond the scope of a brief paper.

5 Conclusion

In this paper, I investigated the properties of intergenerationally equitable discounting by using a welfare function which was originally developed by Epstein (1983), and more recently extended by Bommier and Zuber (2008). In stead of seeking for the appropriate value of social rate of time preference, intergenerational equity was incorporated at axiomatic level. I showed that the IE-consistent discount rate can be higher or lower than the standard no-time-preference case without appealing to uncertainty. The relationship between IE-consistent discount rates and risk of world extinction was also examined with an emphasis on the case where the hazard rate is endogenously determined. With an application to climate change, I showed that endogenous hazard rate can increase the discount rate, which implies relatively less stringent carbon abatement as the optimal climate policy.

A Proofs of Propositions

A.1 Proof of Proposition 1

First, let $\tilde{U} : X^* \to \mathbb{R}$ be a von Neuman-Morgenstern utility function with which the expected utility form represents the ranking $\succsim$. Define $U : X^* \to \mathbb{R}$ by

$$U(x) := -\tilde{U}(0) + \tilde{U}(x) \qquad \forall x \in X^*,$$

so that $U(0) = 0$. Since $\tilde{U}$ is unique up to positive affine transformation, U is another von Neuman-Morgenstern utility function for the same social ranking.

Next the independence axiom implies

$$\begin{aligned}
\int_{X^*} U(c_1, x)dp \geq \int_{X^*} U(c_1, x)dq \quad &\Leftrightarrow \quad (c_1, p) \succsim (c_1, q) \\
&\Leftrightarrow \quad (0, p) \succsim (0, q) \\
&\Leftrightarrow \quad \int_{X^*} U(0, x)dp \geq \int_{X^*} U(0, x)dq,
\end{aligned}$$

for any $p, q \in M(X^*)$ and $c_1 \in [0, \bar{c}]^n$. This means there must be mappings $a : [0, \bar{c}]^n \to \mathbb{R}$ and $b : [0, \bar{c}]^n \to \mathbb{R}_{++}$ such that

$$U(c_1, x) = a(c_1) + b(c_1)U(0, x), \tag{A.1}$$

for any $x \in X^*$ and $c_1 \in [0, \bar{c}]^n$. On the other hand, the stationarity axiom implies

$$\begin{aligned}
\int_{X^*} U(0, x)dp \geq \int_{X^*} U(0, x)dq \quad &\Leftrightarrow \quad (0, p) \succsim (0, q) \\
&\Leftrightarrow \quad (\hat{c}_1, p) \succsim (\hat{c}_1, q) \\
&\Leftrightarrow \quad p \succsim q \\
&\Leftrightarrow \quad \int_{X^*} U(x)dp \geq \int_{X^*} U(x)dq,
\end{aligned}$$

for any $p, q \in M(X^*)$, and hence there exist constant numbers $\hat{a} \in \mathbb{R}$ and $\hat{b} \in \mathbb{R}_{++}$ such that

$$U(0, x) = \hat{a} + \hat{b}U(x), \tag{A.2}$$

for any $x \in X^*$. Define mappings $v : [0, \bar{c}]^n \to \mathbb{R}$ and $B : [0, \bar{c}]^n \to \mathbb{R}_{++}$ as

$$\begin{aligned}
v(c_1) &:= a(c_1) + \hat{a}b(c_1) \\
B(c_1) &:= \hat{b}b(c_1).
\end{aligned}$$

Then (A.1) and (A.2) mean

$$U(c_1, x) = v(c_1) + B(c_1)U(x), \tag{A.3}$$

for any $x \in X^*$ and $c_1 \in [0, \bar{c}]^n$. Notice that $v(0) = 0$ and $U(c_1, 0) = v(c_1)$ because $U(0) = 0$. Recursive application of equation (A.3) yields

$$U(x) = \sum_{t=1}^{\infty} v(c_t) \prod_{\tau=1}^{t-1} B(c_\tau), \tag{A.4}$$

for each $x = (c_1, c_2, \ldots) \in X^*$ where $\prod_{\tau=1}^{0} B(c_\tau) := 1$.

Take a pair of non-zero consumption bundles $c, c' \in [0, \bar{c}]^n \setminus \{0\}$ and consider the following three alternative outcomes $z, z', z'' \in X^*$:

$$\begin{aligned} z &= (c, c', 0, 0, \ldots) \\ z' &= (c', c, 0, 0, \ldots) \\ z'' &= (0, c, c', 0, \ldots). \end{aligned}$$

Then the axiom of intergenerational equity requires $p_z \sim p_{z'}$ and $p_z \sim p_{z''}$. Observe first that since both c and c' are non-zero vectors, the monotonicity axiom implies $v(c) > 0$ and $v(c') > 0$. Hence

$$\begin{aligned} p_z \sim p_{z'} \quad &\Leftrightarrow \quad \int_{X^*} U(x) dp_z = \int_{X^*} U(x) dp_{z'} \\ &\Leftrightarrow \quad U(z) = U(z') \\ &\Leftrightarrow \quad v(c) + B(c)v(c') = v(c') + B(c')v(c) \\ &\Leftrightarrow \quad \frac{1 - B(c)}{v(c)} = \frac{1 - B(c')}{v(c')}. \end{aligned}$$

This means there exists a constant $K \in \mathbb{R}$ such that $K = (1 - B(c))/v(c)$ or

$$B(c) = 1 - Kv(c), \tag{A.5}$$

for any $c, c' \in [0, \bar{c}]^n \setminus \{0\}$. Notice here that since $B(c) > 0$ and $v(\bar{c}) \geq v(c)$ for any $c \in [0, \bar{c}]$,

$$K = \frac{1 - B(c)}{v(c)} < \frac{1}{v(c)} \leq \frac{1}{v(\bar{c})},$$

meaning $1/v(\bar{c})$ is the upper bound of K.

On the other hand, noting $v(0) = 0$ and $U(z) > 0$,

$$
\begin{aligned}
p_z \sim p_{z''} \quad &\Leftrightarrow \quad \int_{X^*} U(x) dp_z = \int_{X^*} U(x) dp_{z''} \\
&\Leftrightarrow \quad U(z) = U(z'') = v(0) + B(0)U(z) \\
&\Leftrightarrow \quad B(0) = 1,
\end{aligned}
$$

and thus

$$B(0) = 1 - Kv(0). \tag{A.6}$$

Finally, apply (A.5) and (A.6) to (A.4) and obtain

$$U(x) = \sum_{t=1}^{\infty} v(c_t) \prod_{\tau=1}^{t-1} (1 - Kv(c_\tau)),$$

for each $x = (c_1, c_2, \dots) \in X^*$.

A.2 Proof of Proposition 2

Let $B(c) := 1 - Kv(c)$ and note that

$$
\begin{aligned}
\frac{\partial U(x)}{\partial c_t} &= v'(c_t) \prod_{\tau=1}^{t-1} B(c_\tau) + \frac{B'(c_t)}{B(c_t)} \sum_{k=t}^{\infty} v(c_{k+1}) \prod_{\tau=1}^{k} B(c_\tau) \\
&= \frac{v'(c_t)}{B(c_t)} \left\{ \prod_{\tau=1}^{t} B(c_\tau) - K \sum_{k=t}^{\infty} v(c_{k+1}) \prod_{\tau=1}^{k} B(c_\tau) \right\} \\
&= \frac{v'(c_t)}{B(c_t)} \left\{ \underbrace{(1 - Kv(c_{t+1}))}_{= B(c_{t+1})} \prod_{\tau=1}^{t} B(c_\tau) - K \sum_{k=t+1}^{\infty} v(c_{k+1}) \prod_{\tau=1}^{k} B(c_\tau) \right\} \\
&= \frac{v'(c_t)}{B(c_t)} \left\{ \prod_{\tau=1}^{t+1} B(c_\tau) - K \sum_{k=t+1}^{\infty} v(c_{k+1}) \prod_{\tau=1}^{k} B(c_\tau) \right\} \\
&= \frac{v'(c_t)}{v'(c_{t+1})} \frac{B(c_{t+1})}{B(c_t)} \underbrace{\frac{v'(c_{t+1})}{B(c_{t+1})} \left\{ \prod_{\tau=1}^{t+1} B(c_\tau) - K \sum_{k=t+1}^{\infty} v(c_{k+1}) \prod_{\tau=1}^{k} B(c_\tau) \right\}}_{= \frac{\partial U(x)}{\partial c_{t+1}}} \\
&= \frac{v'(c_t)}{v'(c_{t+1})} \frac{B(c_{t+1})}{B(c_t)} \frac{\partial U(x)}{\partial c_{t+1}}.
\end{aligned}
$$

Hence

$$\begin{aligned}\rho_t^{de}(x) &= \frac{\partial U(x)}{\partial c_t} \Big/ \frac{\partial U(x)}{\partial c_{t+1}} - 1 \\ &= \frac{v'(c_t)}{v'(c_{t+1})} \frac{B(c_{t+1})}{B(c_t)} - 1 \\ &= \frac{v'(c_t)}{v'(c_{t+1})} \frac{1 - Kv(c_{t+1})}{1 - Kv(c_t)} - 1.\end{aligned}$$

A.3 Proof of Proposition 3

Notice first

$$\frac{\partial U_T(x)}{\partial c_t} = \begin{cases} 0 & \text{for } T \leq t-1 \\ \frac{v'(c_t)}{B(c_t)} \prod_{\tau=1}^{t} B(c_\tau) & \text{for } T = t \\ \frac{v'(c_t)}{v'(c_{t+1})} \frac{B(c_{t+1})}{B(c_t)} \frac{\partial U_T(x)}{\partial c_{t+1}} & \text{for } T \geq t+1. \end{cases} \tag{A.7}$$

Hence

$$\begin{aligned}\frac{\partial W(x)}{\partial c_t} &= \sum_{T=t}^{\infty} f_T \frac{\partial U_T(x)}{\partial c_t} \\ &= f_t \frac{\partial U_t(x)}{\partial c_t} + \sum_{T=t+1}^{\infty} f_T \frac{\partial U_T(x)}{\partial c_t} \\ &= \frac{v'(c_t)}{B(c_t)} f_t \prod_{\tau=1}^{t} B(c_\tau) + \frac{v'(c_t)}{v'(c_{t+1})} \frac{B(c_{t+1})}{B(c_t)} \sum_{T=t+1}^{\infty} f_T \frac{\partial U_T(x)}{\partial c_{t+1}} \\ &= \frac{v'(c_t)}{v'(c_{t+1})} \frac{B(c_{t+1})}{B(c_t)} \left(1 + \Phi_t^{-1}(x;K)\right) \underbrace{\sum_{T=t+1}^{\infty} f_T \frac{\partial U_T(x)}{\partial c_{t+1}}}_{= \frac{\partial W(x)}{\partial c_{t+1}}},\end{aligned}$$

where

$$\Phi_t(x;K) := \sum_{T=t+1}^{\infty} f_T \frac{\partial U_T(x)}{\partial c_{t+1}} \Big/ \frac{v'(c_{t+1})}{B(c_{t+1})} f_t \prod_{\tau=1}^{t} B(c_\tau). \tag{A.8}$$

Therefore

$$\rho_t^{ex}(x) = \frac{\partial W(x)}{\partial c_t} \bigg/ \frac{\partial W(x)}{\partial c_{t+1}} - 1$$
$$= \frac{v'(c_t)}{v'(c_{t+1})} \frac{1 - Kv(c_{t+1})}{1 - Kv(c_t)} \left(1 + \Phi_t^{-1}(x; K)\right) - 1.$$

For more precise form of $\Phi_t(x)$, apply (A.7) recursively and obtain

$$\frac{\partial U_T(x)}{\partial c_t} = \frac{v'(c_t)}{v'(c_{t+1})} \frac{B(c_{t+1})}{B(c_t)} \frac{\partial U_T(x)}{\partial c_{t+1}}$$
$$= \frac{v'(c_t)}{B(c_t)} \prod_{\tau=1}^{T} B(c_\tau)$$

for $T \geq t+1$. This, together with (A.7) for $T = t$, means

$$\frac{\partial U_T(x)}{\partial c_t} = \frac{v'(c_t)}{B(c_t)} \prod_{\tau=1}^{T} B(c_\tau) \qquad \forall T \geq t$$

Plugging this into (A.8) yields

$$\Phi_t(x; K) = \sum_{T=t+1}^{\infty} f_T \frac{v'(c_{t+1})}{B(c_{t+1})} \prod_{\tau=1}^{T} B(c_\tau) \bigg/ \frac{v'(c_{t+1})}{B(c_{t+1})} f_t \prod_{\tau=1}^{t} B(c_\tau)$$
$$= \sum_{T=t+1}^{\infty} \frac{f_T}{f_t} \prod_{\tau=t+1}^{T} B(c_\tau)$$
$$= \sum_{T=t+1}^{\infty} \frac{h_T \prod_{s=1}^{T-1}(1-h_s)}{h_t \prod_{s=1}^{t-1}(1-h_s)} \prod_{\tau=t+1}^{T} B(c_\tau)$$
$$= \sum_{T=t+1}^{\infty} \frac{h_T}{h_t} \left(\frac{1-h_t}{1-h_T}\right) \prod_{s=t+1}^{T} (1-h_s) \prod_{\tau=t+1}^{T} B(c_\tau)$$
$$= \sum_{T=t+1}^{\infty} \frac{h_T}{h_t} \left(\frac{1-h_t}{1-h_T}\right) \prod_{\tau=t+1}^{T} \left\{[1 - Kv(c_\tau)]\,(1-h_\tau)\right\},$$

which is decreasing in K. This means that higher value of K increase the term $(1 + \Phi_t^{-1}(x; K))$.

A.4 Proof of Proposition 4

Note

$$
\begin{aligned}
\widetilde{\Phi}_t(x;K) &= \sum_{T=t+1}^{\infty} \frac{\tilde{h}_T}{\tilde{h}_t}\left(\frac{1-\tilde{h}_t}{1-\tilde{h}_T}\right)\prod_{\tau=t+1}^{T}\left\{B(c_\tau)(1-\tilde{h}_\tau)\right\} \\
&= \frac{\tilde{h}_{t+1}}{\tilde{h}_t}(1-\tilde{h}_t)B(c_\tau) \\
&\quad + \underbrace{\frac{h_t}{\tilde{h}_t}\frac{1-\tilde{h}_t}{1-h_t}\frac{1-\tilde{h}_{t+1}}{1-h_{t+1}}}_{=:\,\phi(h)}\sum_{T=t+2}^{\infty}\frac{h_T}{h_t}\left(\frac{1-h_t}{1-h_T}\right)\prod_{\tau=t+1}^{T}\{B(c_\tau)(1-h_\tau)\} \\
&= \frac{\tilde{h}_{t+1}}{\tilde{h}_t}(1-\tilde{h}_t)B(c_\tau) - \phi(h)\frac{h_{t+1}}{h_t}(1-h_t)B(c_\tau) \\
&\quad + \phi(h)\underbrace{\sum_{T=t+1}^{\infty}\frac{h_T}{h_t}\left(\frac{1-h_t}{1-h_T}\right)\prod_{\tau=t+1}^{T}\{B(c_\tau)(1-h_\tau)\}}_{=\ \Phi_t(x;K)} \\
&= \phi(h)\Phi_t(x;K) + \left(1-\frac{h_{t+1}}{\tilde{h}_{t+1}}\frac{1-\tilde{h}_{t+1}}{1-h_{t+1}}\right)\frac{\tilde{h}_{t+1}}{\tilde{h}_t}B(c_{t+1})(1-\tilde{h}_t)
\end{aligned}
$$

Since $\tilde{h}_{t+1} = h_{t+1} + \gamma > h_{t+1}$, the second term is positive for any $\gamma > 0$. As for the first term, observe

$$
\begin{aligned}
\phi(h) - 1 &= \frac{h_t}{\tilde{h}_t}\frac{1-\tilde{h}_t}{1-h_t}\frac{1-\tilde{h}_{t+1}}{1-h_{t+1}} - 1 \\
&= \frac{h_t(1-\tilde{h}_t)(1-\tilde{h}_{t+1}) - \tilde{h}_t(1-h_t)(1-h_{t+1})}{\tilde{h}_t(1-h_t)(1-h_{t+1})} \\
&= \frac{h_t\gamma\left((1-h_{t+1})\left\{\frac{1}{h_t}-(1-h_t)\right\}-\gamma\right)}{\tilde{h}_t(1-h_t)(1-h_{t+1})},
\end{aligned}
$$

which is positive if

$$
\gamma < (1-h_{t+1})\left\{\frac{1}{h_t}-(1-h_t)\right\}. \tag{A.9}
$$

Hence there is positive value of γ such that $\phi(h) > 1$. For such γ,

$$\begin{aligned}\widetilde{\Phi}_t(x;K) &= \phi(h)\Phi_t(x;K) + \left(1 - \frac{h_{t+1}}{\tilde{h}_{t+1}}\frac{1-\tilde{h}_{t+1}}{1-h_{t+1}}\right)\frac{\tilde{h}_{t+1}}{\tilde{h}_t}B(c_{t+1})(1-\tilde{h}_t) \\ &> \Phi_t(x;K),\end{aligned}$$

and thus

$$\begin{aligned}\tilde{\rho}_t^{ex}(x) &= \frac{v'(c_t)}{v'(c_{t+1})}\frac{1-Kv(c_{t+1})}{1-Kv(c_t)}\left(1+\widetilde{\Phi}_t^{-1}(x;K)\right) - 1 \\ &< \frac{v'(c_t)}{v'(c_{t+1})}\frac{1-Kv(c_{t+1})}{1-Kv(c_t)}\left(1+\Phi_t^{-1}(x;K)\right) - 1 \\ &= \rho_t^{ex}(x),\end{aligned}$$

as desired. Moreover, since $\tilde{h}_t = h_t - \gamma > 0$, condition (A.9) is always satisfied as long as

$$\begin{aligned} & h_t \leq (1-h_{t+1})\left\{\frac{1}{h_t} - (1-h_t)\right\} \\ \Leftrightarrow \quad & \frac{1-h_{t+1}}{h_{t+1}}\frac{1-h_t}{h_t} \geq h_t.\end{aligned}$$

A.5 Proof of Lemma 1

Since $\partial f_T(x)/\partial c_t = 0$ for all $T \leq t$,

$$\begin{aligned}\frac{\partial W(x)}{\partial c_t} &= \sum_{T=1}^{\infty}\left(\frac{\partial U_T(x)}{\partial c_t}f_T(x) + U_T\frac{\partial f_T(x)}{\partial c_t}\right) \\ &= \sum_{T=t}^{\infty}\frac{\partial U_T(x)}{\partial c_t}f_T(x) + \sum_{T=t+1}^{\infty}U_T\frac{\partial f_T(x)}{\partial c_t}. \qquad \text{(A.10)}\end{aligned}$$

Thus it is immediate from the proof of proposition 3 that

$$\begin{aligned}\sum_{T=t}^{\infty}\frac{\partial U_T(x)}{\partial c_t}f_T(x) &= (1+\rho_t^{ex}(x))\sum_{T=t+1}^{\infty}\frac{\partial U_T(x)}{\partial c_{t+1}}f_T(x) \\ &= (1+\rho_t^{ex}(x))\left(\sum_{T=t+1}^{\infty}\frac{\partial U_T(x)}{\partial c_{t+1}}f_T(x) + \sum_{T=t+2}^{\infty}U_T\frac{\partial f_T(x)}{\partial c_{t+1}}\right) \\ &\qquad - (1+\rho_t^{ex}(x))\sum_{T=t+2}^{\infty}U_T\frac{\partial f_T(x)}{\partial c_{t+1}} \\ &= (1+\rho_t^{ex}(x))\frac{\partial W(x)}{\partial c_{t+1}} - (1+\rho_t^{ex}(x))\sum_{T=t+2}^{\infty}U_T\frac{\partial f_T(x)}{\partial c_{t+1}}\end{aligned}$$

Putting this back into (A.10) yields

$$\begin{aligned}\frac{\partial W(x)}{\partial c_t} &= (1+\rho_t^{ex}(x))\frac{\partial W(x)}{\partial c_{t+1}} \\ &\quad + \sum_{T=t+1}^{\infty} U_T \frac{\partial f_T(x)}{\partial c_t} - (1+\rho_t^{ex}(x)) \sum_{T=t+2}^{\infty} U_T \frac{\partial f_T(x)}{\partial c_{t+1}}.\end{aligned}$$

Therefore

$$\begin{aligned}\rho_t^{en}(x) &= \frac{\partial W(x)}{\partial c_t} \Big/ \frac{\partial W(x)}{\partial c_{t+1}} - 1 \\ &= \rho_t^{ex}(x) + \Theta_t(x),\end{aligned}$$

where

$$\begin{aligned}\Theta_t(x) &= \frac{\left(\sum_{T=t+1}^{\infty} U_T(x)\frac{\partial f_T(x)}{\partial c_t} - (1+\rho_t^{ex}(x)) \sum_{T=t+2}^{\infty} U_T(x)\frac{\partial f_T(x)}{\partial c_{t+1}}\right)}{\frac{\partial W(x)}{\partial c_{t+1}}} \\ &= U_{t+1}(x)\frac{\partial f_{t+1}(x)}{\partial c_t} \Big/ \frac{\partial W(x)}{\partial c_{t+1}} \\ &\quad + \sum_{T=t+2}^{\infty} U_T(x) \left(\frac{\partial f_T(x)}{\partial c_t} - (1+\rho_t^{ex}(x))\frac{\partial f_T(x)}{\partial c_{t+1}}\right) \Big/ \frac{\partial W(x)}{\partial c_{t+1}}.\end{aligned}$$

A.6 Proof of Proposition 5

First notice

$$\begin{aligned}M_T &= (1-\xi)M_{T-1} + \kappa\sigma_{T-1}c_{T-1} \\ &= (1-\xi)^{T-1}M_1 + \sum_{k=1}^{T-1}(1-\xi)^{T-1-k}\kappa\sigma_k c_k\end{aligned}$$

and hence

$$\begin{aligned}\frac{\partial h_T(x)}{\partial c_t} &= h'(M_T)\frac{\partial M_T}{\partial c_t} \\ &= h'(M_T)\kappa\sigma_t(1-\xi)^{T-t-1}.\end{aligned}$$

Thus

$$\begin{aligned}
\frac{\partial f_T(x)}{\partial c_t} &= \prod_{s=1}^{T-1}(1-h_s(x))\left(\frac{\partial h_T(x)}{\partial c_t} - \sum_{l=t+1}^{T-1}\frac{\partial h_l(x)}{\partial c_t}\frac{h_T(x)}{(1-h_l(x))}\right) \\
&= \kappa\sigma_t f_T(x)\left(\frac{h'(M_T)(1-\xi)^{T-t-1}}{h_T(x)} - \sum_{l=t+1}^{T-1}\frac{h'(M_l)(1-\xi)^{l-t-1}}{(1-h_l(x))}\right)
\end{aligned}$$

For a spabilized path with $M_t = M$ for all t, let $h := h(M)$ and $h' := h'(M)$. Then

$$\begin{aligned}
\frac{\partial f_T(x)}{\partial c_t} &= h'\kappa\sigma_t(1-h)^{T-1}\left((1-\xi)^{T-t-1} - \frac{h}{(1-h)}\sum_{l=t+1}^{T-1}(1-\xi)^{l-t-1}\right) \\
&= \frac{h'\kappa\sigma_t(1-h)^{T-1}}{\xi}\left\{\left(\xi + \frac{h}{1-h}\right)(1-\xi)^{T-t-1} - \frac{h}{1-h}\right\}.
\end{aligned}$$

Hence

$$\lim_{h\to 0}\frac{\partial f_T(x)}{\partial c_t} = h'\kappa\sigma_t(1-\xi)^{T-t-1}. \tag{A.11}$$

Similarly,

$$\frac{\partial f_T(x)}{\partial c_{t+1}} = \frac{h'\kappa\sigma_{t+1}(1-h)^{T-1}}{\xi}\left\{\left(\xi + \frac{h}{1-h}\right)(1-\xi)^{T-t-2} - \frac{h}{1-h}\right\}.$$

and thus

$$\lim_{h\to 0}\frac{\partial f_T(x)}{\partial c_{t+1}} = h'\kappa\sigma_{t+1}(1-\xi)^{T-t-2}. \tag{A.12}$$

Notice that since $\Phi_t^{-1}(x;K) \geq 0$ for any h,

$$\begin{aligned}
\lim_{h\to 0}\rho_t^{ex}(x) &= \frac{v'(c_t)}{v'(c_{t+1})}\frac{1-Kv(c_{t+1})}{1-Kv(c_t)}\left(1 + \lim_{h\to 0}\Phi_t^{-1}(x;K)\right) - 1 \\
&\geq \frac{v'(c_t)}{v'(c_{t+1})}\frac{1-Kv(c_{t+1})}{1-Kv(c_t)} - 1 \\
&= \rho_t^{de}(x),
\end{aligned} \tag{A.13}$$

which is positive because $\rho_t^{de}(x) > \sigma_t/\sigma_{t+1} - 1$ and $\sigma_t \geq \sigma_{t+1}$. Also notice

$$\begin{aligned}
U_{t+1} &= \sum_{k=1}^{t+1} v(c_k)\prod_{\tau=1}^{k-1}(1-Kv(c_\tau)) \\
&\leq \sum_{k=1}^{T} v(c_k)\prod_{\tau=1}^{k-1}(1-Kv(c_\tau)) = U_T
\end{aligned} \tag{A.14}$$

for $T \geq t+2$. Therefore, if

$$1 - \frac{\sigma_{t+1}}{\sigma_t}(1 + \rho_t^{de}(x)) < 0,$$

then (A.11), (A.12), (A.13) and (A.14) imply

$$\begin{aligned}
&\lim_{h \to 0} \Theta_t(x) \frac{\partial W(x)}{\partial c_{t+1}} \\
&\quad = U_{t+1} \lim_{h \to 0} \frac{\partial f_{t+1}(x)}{\partial c_t} \\
&\qquad + \sum_{T=t+2}^{\infty} U_T \left(\lim_{h \to 0} \frac{\partial f_T(x)}{\partial c_t} - (1 + \lim_{h \to 0} \rho_t^{ex}(x)) \lim_{h \to 0} \frac{\partial f_T(x)}{\partial c_{t+1}} \right) \\
&\quad = h' \kappa \sigma_t U_{t+1} \\
&\qquad + h' \kappa \sigma_t \left[1 - \frac{1}{1-\xi} \frac{\sigma_{t+1}}{\sigma_t} (1 + \lim_{h \to 0} \rho_t^{ex}(x)) \right] \sum_{T=t+2}^{\infty} (1-\xi)^{T-t-1} U_T \\
&\quad \leq h' \kappa \sigma_t U_{t+1} \\
&\qquad + h' \kappa \sigma_t \underbrace{\left[1 - \frac{1}{1-\xi} \frac{\sigma_{t+1}}{\sigma_t} (1 + \rho_t^{de}(x)) \right]}_{<0} \sum_{T=t+2}^{\infty} (1-\xi)^{T-t-1} U_T \\
&\quad \leq h' \kappa \sigma_t U_{t+1} \\
&\qquad + h' \kappa \sigma_t \left[1 - \frac{1}{1-\xi} \frac{\sigma_{t+1}}{\sigma_t} (1 + \rho_t^{de}(x)) \right] \sum_{T=t+2}^{\infty} (1-\xi)^{T-t-1} U_{t+1} \\
&\quad = \frac{h' \kappa \sigma_t U_{t+1}}{\xi} \left(1 - \frac{\sigma_{t+1}}{\sigma_t} (1 + \rho_t^{de}(x)) \right) \\
&\quad < 0,
\end{aligned}$$

which means there exists $h^* > 0$ such that $\Theta_t(x) < 0$ for all $h < h^*$.

B Assumptions for Numerical Simulation

B.1 Exogenous Trends

The exogenous trends for population, total factor productivity, carbon intensity are specified as follows:

$$\begin{aligned} N_{t+1} &= (1+g_t^n)N_t \quad \text{with} \quad g_{t+1}^n = (1-\delta^n)g_t^n, \\ \psi_{t+1} &= (1+g_t^\psi)\psi_t, \quad \text{with} \quad g_{t+1}^\psi = (1-\delta^\psi)g_t^\psi, \\ \sigma_{t+1} &= (1-g_t^\sigma)\sigma_t \quad \text{with} \quad g_{t+1}^\sigma = (1-\delta^\sigma)g_t^\sigma. \end{aligned}$$

The population trend is based on the projection provided by the United Nations Population Division[1]. I directly input the projected population for the first three periods and then chose g_{2025}^n and δ^n such that the model matches the future prediction thereafter. The parameters governing productivity and carbon intensity are the same as Nordhaus (2008).

The marginal abatement cost is assumed to decline overtime. On this point, I follow Nordhaus (2008) and specify α_t as

$$\alpha_t = \frac{\sigma_t}{\zeta} m \left[\nu + (1-g^m)^t(1-\nu)\right],$$

where $\nu \in (0,1)$.

B.2 Data & Parameter Values

The emission data of carbon dioxide was taken from International Energy Annual of US Energy Information Administration[2]. Following Ikefuji *et al.* (2010), I used sulfur emission as a representative index of aerosols in the atmosphere. The sulfur emission data is based on the work of Stern (2005)[3]. The GDP data of World Economic Outlook Databases of International Monetary Fund[4] was used for the initial output. The data of carbon concentration comes from SCRIPPS CO_2 of Mauna Loa Observatory[5]. All of the parameter values used in this analysis is listed in Table 1 below.

[1] http://esa.un.org/unpd/wpp2008/index.htm
[2] http://www.eia.doe.gov/environment.html
[3] http://www.sterndavidi.com/datasite.html
[4] http://www.imf.org/external/index.htm
[5] http://scrippsco2.ucsd.edu/data/atmospheric_co2.html

Symbol	Value	Description	Unit
M_{1750}	596	Carbon concentration (1750)	GtC
M_{2005}	809	Carbon concentration (2005)	GtC
Z_{2005}	14.482	Mean temperature (2005)	ºC
K_{2005}	91	Capital stock (2005)	trill US$
N_{2025}	6468	Population (2025)	million
ψ_{2005}	3.189	Total factor productivity (2005)	—
σ_{2005}	0.187	Emission-output ratio (2005)	—
m	0.9	Parameter in cost function	—
g^n_{2025}	0.0798	Population growth rate	—
g^ψ_{2005}	0.092	Growth rate of TFP (2005)	—
g^σ_{2005}	-0.0724	De-carbonization rate (2005)	—
g^m	0.05	Convergence rate of carbon abatement cost	—
δ^n	0.656	Convergence rate of population growth	—
δ^ψ	0.001	Convergence rate of TFP	—
δ^σ	-0.0296	Convergence rate of de-carbonization (2005)	—
ν	0.5	Ratio of initial to final abatement cost	—
ϵ	0.3	Capital's share of income	—
υ	0.1	Capital depreciation rate per decade	—
$\bar{d}$	0.00284	Damage coefficient	—
τ^*	13.71	Desirable mean temperature	ºC
ξ	0.0524	Carbon depreciation rate per decade	—
κ	0.47	Carbon retention rate per decade	—
τ_0	-4.561	Parameter in temperature equation	—
τ_1	0.786	Parameter in temperature equation	—
τ_2	1.206	Parameter in temperature equation	—
τ_3	-0.001	Parameter in temperature equation	—
τ_4	-0.201	Parameter in temperature equation	—
τ_5	0.024	Parameter in temperature equation	—
α	0.651	Parameter in cost function	—
ζ	2.8	Parameter in cost function	—
ω_0	12.559	Parameter in utility function	—
ω_1	0.01	Parameter in utility function	—
ω_2	1.2	Parameter in utility function	—
$\bar{h}$	5×10^{-6}	Parameter in hazard rate function	—
β	2.75×10^{-11}	Parameter in hazard rate function	—

Table 1: Data & Parameter Values

C Results for Different K

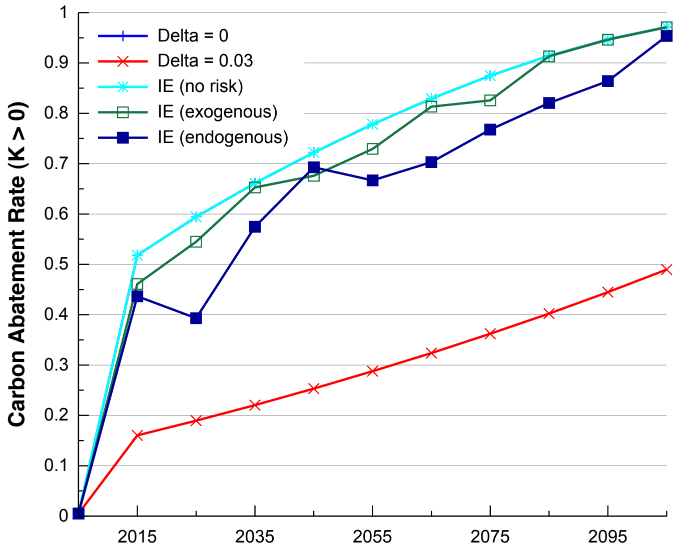

Figure 6: Optimal Carbon Abatement Paths ($K = 1 \times 10^{-6}$)

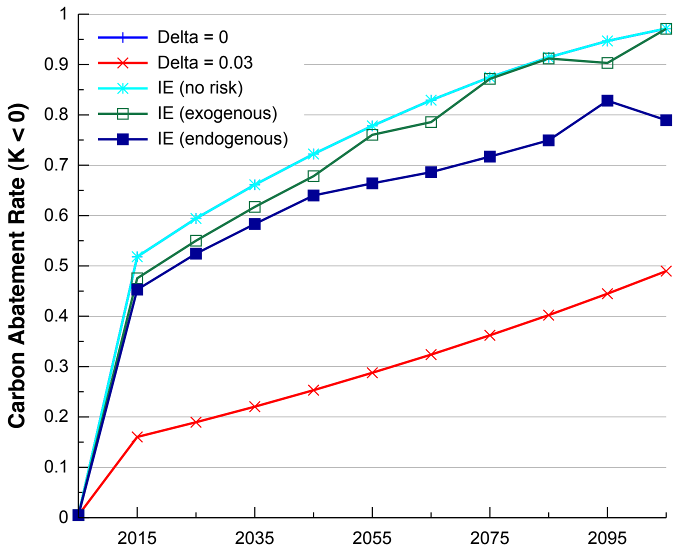

Figure 7: Optimal Carbon Abatement Paths ($K = -1 \times 10^{-6}$)

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